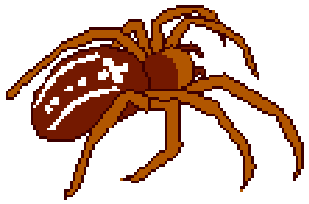


Pajek

Pajek XXL

Pajek 3XL

Programs for Analysis and Visualization of Very Large Networks

Reference Manual

List of commands with short explanation
version 5.09

Andrej Mrvar and Vladimir Batagelj

Ljubljana, June 15, 2020

PdfLaTex version October 1, 2003

Andrej Mrvar
Faculty of Social Sciences
University of Ljubljana, Slovenia

http://mrvar.fdv.uni-lj.si/

andrej.mrvar@fdv.uni-lj.si

Vladimir Batagelj
Department of Mathematics, FMF
University of Ljubljana, Slovenia

http://vlado.fmf.uni-lj.si/

vladimir.batagelj@fmf.uni-lj.si

Contents

1 Pajek

Pajek is a program, for Windows, for analysis and visualization of *large networks* having some thousands or even millions of vertices. In Slovenian language the word *pajek* means spider. The latest version of **Pajek** is freely available, for noncommercial use, at its home page:

http://mrvar.fdv.uni-lj.si/pajek/

We started the development of **Pajek** in November 1996. **Pajek** is implemented in Delphi (Pascal). Some procedures were contributed by Matjaž Zaveršnik.

The main motivation for development of **Pajek** was the observation that there exist several sources of large networks that are already in machine-readable form. **Pajek** should provide tools for analysis and visualization of such networks: collaboration networks, organic molecule in chemistry, protein-receptor interaction networks, genealogies, Internet networks, citation networks, diffusion (AIDS, news, innovations) networks, data-mining (2-mode networks), etc. See also collection of large networks at:

http://vlado.fmf.uni-lj.si/pub/networks/data/

The design of **Pajek** is based on our previous experiences gained in development of graph data structure and algorithms libraries Graph and X-graph, collection of network analysis and visualization programs STRAN, RelCalc, Draw, Energ, and SGML-based graph description markup language NetML.

http://vlado.fmf.uni-lj.si/pub/networks/default.htm

Pajek-XXL and **Pajek-3XL** are a special editions of program **Pajek**. Their memory consumption is much lower. For the same network they need at least 2-3 times less physical memory than **Pajek**. Operations that are memory intensive (e.g. generating random networks, extracting, shrinking,...) are also faster. In the rest of the manual commands that are specific to **Pajek-XXL** and **Pajek-3XL** are written in brown font color.
For details on **Pajek-XXL** and **Pajek-3XL** see:

http://mrvar.fdv.uni-lj.si/pajek/PajekXXL.htm

or its mirror:

http://mrvar2.fdv.uni-lj.si/pajek/PajekXXL.htm

Figure 1: ***Pajek/Spider***

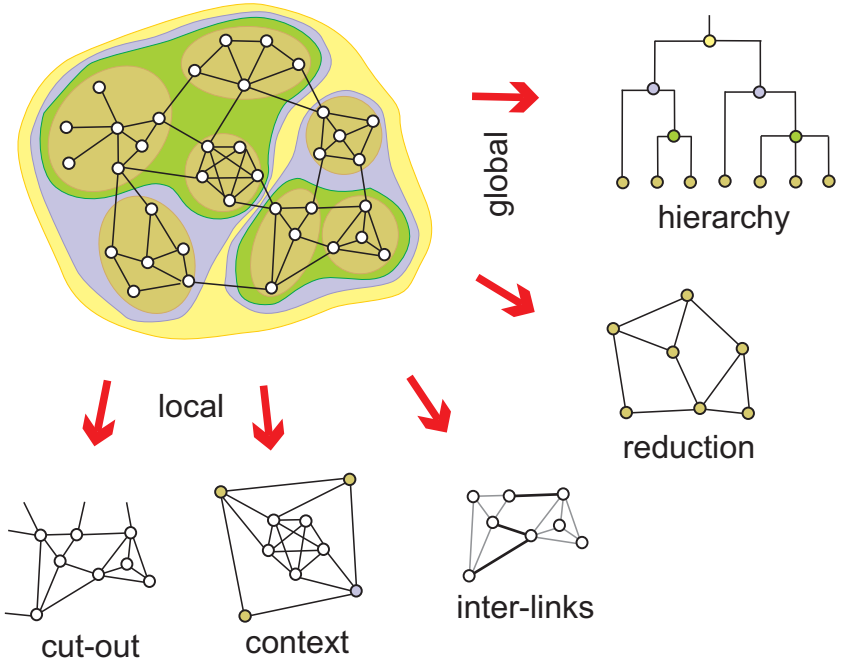


Figure 2: *Approaches to deal with large networks*

The main goals in the design of **Pajek** are:

- to support abstraction by (*recursive*) *decomposition* of a large network into several smaller networks that can be treated further using more sophisticated

methods;

- to provide the user with some powerful *visualization* tools;
- to implement a selection of *efficient* (*subquadratic*) *algorithms* for analysis of large networks.

With **Pajek** we can: *find* clusters (components, neighbourhoods of 'important' vertices, cores, etc.) in a network, *extract* vertices that belong to the same clusters and *show* them separately, possibly with the parts of the context (detailed local view), *shrink* vertices in clusters and show relations among clusters (global view).

Besides ordinary (directed, undirected, mixed) networks **Pajek** supports also *multi-relational networks*, *2-mode networks* (bipartite (valued) graphs – networks between two disjoint sets of vertices), and *temporal networks* (dynamic graphs – networks changing over time).

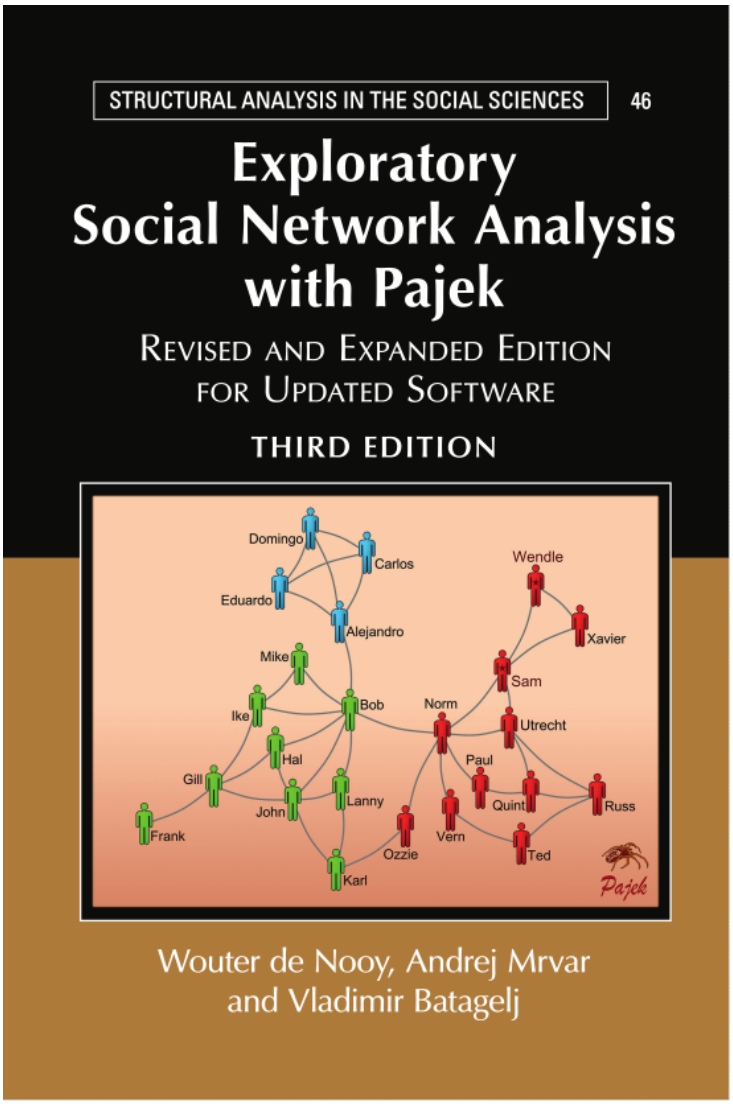


Figure 3: ***Pajek textbook***

This manual provides short explanations of all procedures implemented in the last version of **Pajek**. The novice users we advise to read the **Pajek** textbook [31]

> de Nooy W., Mrvar A., Batagelj V. (2018) *Exploratory Social Network Analysis With* **Pajek**. Revised and Expanded Edition for Updated Software. Third Edition. Structural Analysis in the Social Sciences 46, Cambridge University Press, 2018.

For an overview of *network analysis with* **Pajek** see the NICTA workshop slides

[5].

2 Data objects

In **Pajek** six types of objects are used:

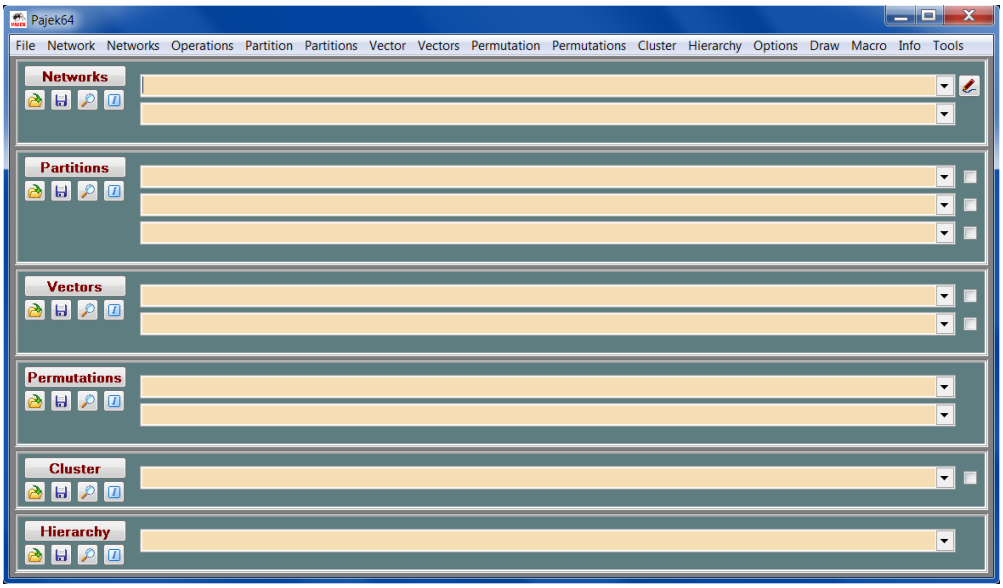


Figure 4: **Pajek**'s Main Window

1. **Networks** – main objects (vertices and lines). Default extension: `.net`. Network can be presented on input file in different ways:
   - using arcs/edges (e.g. 1 2 – line from 1 to 2)
   - using arcslists/edgeslists (e.g. 1 2 3 – line from 1 to 2 and from 1 to 3)
   - matrix format
   - UCINET, GEDCOM, chemical formats...

   Additional information for network drawing can be included in input file as well. This is explained in the section Exports to EPS/SVG/VRML.
2. **Partitions** – they tell for each vertex to which class vertex belong. Default extension: `.clu`.
3. **Permutations** – reordering of vertices. Default extension: `.per`.
4. **Clusters** – subset of vertices (e.g. one class from partition). Default extension: `.cls`.
5. **Hierarchies** – hierarchically ordered vertices. Example:

```
          Root
       g1       g2
    g11 g12   v5,v6,v7
v1,v2   v3,v4
```

Root has two subgroups – g1 and g2. g2 is a leaf – cluster with vertices v5,v6 and v7. g1 has two subgroups – g11 and g12. . . Default extension: **.hie**.

6. **Vectors** – they tell for each vertex some numerical property (real number). Default extension: **.vec**.

By double clicking on selected network, partition,... you can show the object on screen.

The procedures in **Pajek**'s main window (see Figure 4) are organized according to the types of data objects they use as input.

Permutations, partitions and vectors can be used to store properties of vertices measured in different scales: ordered, nominal (categorical) and numeric.

Figure 5: ***Spider web***; Photo: Vladimir Batagelj.

Table 1: List of time events.

Event	Explanation
TI t	initial events – following events happen when time point *t* starts
TE t	end events – following events happen when time point *t* is finished
AV vns	add vertex *v* with label *n* and properties *s*
HV v	hide vertex *v*
SV v	show vertex *v*
DV v	delete vertex *v*
AA uvs	add arc *(u,v)* with properties *s*
HA uv	hide arc *(u,v)*
SA uv	show arc *(u,v)*
DA uv	delete arc *(u,v)*
AE uvs	add edge *(u:v)* with properties *s*
HE uv	hide edge *(u:v)*
SE uv	show edge *(u:v)*
DE uv	delete edge *(u:v)*
CV vs	change vertex property – change property of vertex *v* to *s*
CA uvs	change arc property – change property of arc *(u,v)* to *s*
CE uvs	change edge property – change property of edge *(u:v)* to *s*
CT uv	change type – change (un)directedness of line *(u,v)*
CD uv	change direction of arc *(u,v)*
PE uvs	replace pair of arcs *(u,v)* and *(v,u)* by single edge *(u:v)* with properties *s*
AP uvs	add pair of arcs *(u,v)* and *(v,u)* with properties *s*
DP uv	delete pair of arcs *(u,v)* and *(v,u)*
EP uvs	replace edge *(u:v)* by pair of arcs *(u,v)* and *(v,u)* with properties *s*

3 Main Window Tools

3.1 File

Input/Output manipulation with the six data objects.

- **Network – N**
  - **Read** – Read network from Ascii/Unicode UTF8 file.
  - **Read Time Events** – Read temporal network described using time events. See Table 1.

    List of properties *s* can be empty as well. If several edges (arcs) can connect two vertices, additional tag like *:k* (*k*-th line) must be given to

determine to which line the command applies. E.g. command `HE:3 14 37` results in hiding the third edge connecting vertices `14` and `37`. Example of time network described using time events:

```
*Vertices 3
*Events
TI 1
AV 2 "b"
TE 3
HV 2
TI 4
AV 3 "e"
TI 5
AV 1 "a"
TI 6
AE 1 3 1
TI 7
SV 2
AE 1 2 1
TE 7
DE 1 2
DV 2
TE 8
DE 1 3
TE 10
HV 1
TI 12
SV 1
TE 14
DV 1
```

See also other possibility: description of time network using time intervals.

- **Save** – Save selected network to Ascii/Unicode UTF8 file.
  If network represents Ore graph with the following five relations (arcs):
  1. Wi→Hu, 2. Mo→Da, 3. Mo→So, 4. Fa→Da, 5. Fa→So
  it can be stored as GEDCOM file.
  The other possibility is Pajek Ore graph: 1.Fa→Ch, 2.Mo→Ch, 3.Hu-Wi (edge), or 1.Pa→Ch, 3.Hu-Wi (edge).
- **Save as Time Events** – Save temporal network in time events format.
- **View/Edit** – Edit network. Choose vertex, show its neighbors and then:
  - add new lines to/from selected vertex (by left mouse double clicking on Newline);
  - delete lines (by left mouse double clicking);
  - change value of line (by single right mouse clicking);

  - subdivide line to two orthogonal lines using new invisible vertex (by single middle mouse clicking).

- **Change Label** of selected network.
- **Dispose** selected network from memory.
- **Export as Matrix to [EPS, SVG + PDF]** – export matrix to EPS (SVG and PDF) format:
  - **Original** – use default numbering (for 1-mode and 2-mode networks).
  - **Using Partition** – use current partition. In the text window number and density of lines among classes (and vertices in selected two classes) are displayed. Additionally matrix is exported to EPS where density is expressed using shadowing:
    1. **Structural** – Densities are normalized according to maximum possible number of lines among classes (suitable for dense networks).
    2. **Delta** – Densities are normalized according to vertices having the highest number of *input* and *output* neighbors in classes (suitable for sparse networks).
  - **Using Permutation** – use current permutation to reorder rows / columns. Option can be used for 1-mode and 2-mode networks.
  - **Using Permutation + Partition** – use current permutation and partition. Lines are drawn to divide different classes defined by partition. Option can be used for 1-mode and 2-mode networks.
  - **Options** – Select additional parameters when exporting to EPS.
    - **Diamonds for Negative Values, Circles for 0** – Squares are used for posititive values, diamonds for negative and circles for value 0 (useful for black and white printing).
    - **Diamonds, Circles and Lines in GreyScale** – Diamonds, circles and dividing lines are drawn in greyscale (not in red, green and blue).
    - **Labels on Top/Right** – Labels are written on the top and on the right of the matrix - suitable for longer labels.
    - **Only Black Borders** – All squares in matrix have black borders, otherwise dark squares will have white and light squares will have black borders.
    - **Thick Boundary Line** – Use thicker line for dividing clusters.

  - **Large Squares/Diamonds/Circles** – Use larger or smaller squares, diamonds, and circles.
  - **Use Partition Colors for Vertex Labels** – Labels of vertices are displayed using partition colors.

- **VertexID - ID** - vertex identificators.
  - **Read** identificators from Ascii file.
  - **Save** selected identificators to Ascii file.
  - **Save to NET File as Vertex Labels** – Store identificators to a NET file that can be used later by `Network/Transform/Add/Vertex Labels/from File(s)`.
  - **View/Edit** VertexIDs (change identificators).
  - **Change Label** of selected identificators.
  - **Dispose** selected identificators from memory.

- **Partition – C**
  - **Read** partition from Ascii file.
  - **Save** selected partition to Ascii file.
  - **View/Edit** partition (put vertices to classes).
  - **Change Label** of selected partition.
  - **Dispose** selected partition from memory.

- **Vector – V**
  - **Read** vector from Ascii file.
  - **Save** selected vector(s) to Ascii file. If cluster representing vector id's is present, all vectors with corresponding id numbers will be saved to the same output file. Vector's id can be added to cluster by pressing *V* on the selected vector (empty cluster should be created first). All vectors must have the same dimensions.
  - **View/Edit** vector (change components of vector).
  - **Change Label** of selected vector.
  - **Dispose** selected vector from memory.

- **Permutation – P**
  - **Read** permutation from Ascii file.

  - **Save** selected permutation to Ascii file.
  - **View/Edit** permutation (interchange positions of two vertices).
  - **Change Label** of selected permutation.
  - **Dispose** selected permutation from memory.

- **Cluster – S** (list of selected vertices)
  - **Read** cluster from Ascii file.
  - **Save** selected cluster to Ascii file.
  - **View/Edit** cluster (add and delete vertices).
  - **Change Label** of selected cluster.
  - **Dispose** selected cluster from memory.

- **Hierarchy – H**
  - **Read** hierarchy from Ascii file.
  - **Save** selected hierarchy to Ascii file.
  - **View/Edit** hierarchy (change types and names of nodes, or show vertices (and subtree) belonging to selected node). Nodes can be pushed up and down within hierarcy.
  - **Change Label** of selected hierarchy.
  - **Dispose** selected hierarchy from memory.
  - **Export as Dendrogram to [EPS, SVG + PDF]** – Draw dendrogram of hierarchy in EPS (SVG and PDF). Works for binary hierarchies only. Dissimilarities must be stored in names of nodes of hierarchy between [and]. These are obtained automatically when obtaining hierarchies using hierarchical clustering or clustering with relational constraint.

- **Pajek Project File – *.paj**
  - **Read Pajek** project file (file containing all possible **Pajek** data objects – networks, partitions, permutations, clusters, hierarchies and vectors).
  - **Save** all currently loaded objects as a **Pajek** project file.

- **Ini File**
  - **Load** – Use selected configuration of **Pajek** which is stored in the file (`*.ini`).

  - **Save** – Save the current configuration of **Pajek** into a file (`*.ini`).

- **Exit** program.

3.2 Network

Operations, for which only a network (or nothing in case of random networks) is needed as input.

- **Create Random Network** – Generate random network of selected dimension
  - **Total No. of Arcs** – Generate random directed network of selected dimension and given number of arcs.
  - **Vertices Output Degree** – Generate random directed network of selected dimension and output degree of each vertex in given range.
  - **Bernoulli/Poisson** – Generate undirected, directed, acyclic, bipartite or 2-mode random network according to model defined by Bernoulli / Poisson: each line is selected with the given probability p. Instead of p, which is for large and sparse networks (very) small number, in **Pajek** a more intuitive *average degree* $\overline{d}$ is used. They are connected with relations $\overline{d} = \frac{1}{n}\sum_{v\in V}\deg(v) = \frac{2m}{n}$ and $m = pM$ where $n = |V|$, $m = |L|$ and M is the number of lines in maximal possible network – for example, for undirected graphs $M = n(n-1)$.
  - **Scale Free** – Generate scale free undirected, directed or acyclic network. The procedure is based on a refinement of the model for generating *scale free network*s, proposed in [56]. At each step of the growth a new vertex and k edges are added to the network N. The endpoints of the edges are randomly selected among all vertices according to the probability

$$\Pr(v) = \alpha\frac{\text{indeg}(v)}{|E|} + \beta\frac{\text{outdeg}(v)}{|E|} + \gamma\frac{1}{|V|}$$

    where $\alpha + \beta + \gamma = 1$. It is easy to check that $\sum_{v\in V}\Pr(v) = 1$.
  - **Small World** – Generate Small world random network. See [12].
  - **Extended Model** – Generate random network according to extended model defined by Albert and Barabasi [3].

- **Create New Network** – Create a new network from selected network (or create empty or complete network).

- **Empty Network** - Create new network with given number of vertices and no lines.
- **Complete Network** - Create undirected or directed network with given number of vertices and all possible lines.
- **with Bi-Connected Components stored as Relation Numbers** – Biconnected Components of selected network. Articulation points belong to several classes, so the result cannot be stored in partition – biconnected components are stored in hierarchy. In newer Pajek versions they are stored also as multiple relations network. Minimal number of vertices in components can be selected. Partition containing vertices belonging to exactly one bicomponent, vertices outside bicomponents and articulation points is also produced: vertices outside bicomponents get class zero, each bicomponent is numbered consecutively (from 1 to number of bicomponents) and articulation points get class number 999999998. Additionally vector containing articulation points is produced: number of biconnected components to which each vertex belongs is given.
- **with Ring Counts stored as Line Values** - how many times each line belongs to predefined rings
  - **3-Rings** – For each line count number of 3-rings to which the line belongs.
    - **Undirected** – for undirected networks – count undirected 3-rings.
    - **Directed** – for directed networks – count **cyclic**, **transitive**, or all 3-rings, or count how many times each line is a transitive shortcut (see Figure 10).
  - **4-Rings** – For each line count number of 4-rings to which the line belongs.
    - **Undirected** – for undirected networks – count undirected 4-rings.
    - **Directed** – for directed networks – count **cyc**lic, **dia**monds, **gen**ealogical, **tra**nsitive, or all 4-rings, or count how many times each line is a transitive shortcut (see Figure 11).
- **SubNetwork with Paths**
  - **One shortest Path between Two Vertices** – Find the shortest path between two vertices. Result is new network. Values on lines can be taken into account (if they present distances between vertices) or not (graph theoretical distance). The latter possibility is faster.

  - **All Shortest Paths between Two Vertices** – Find all shortest paths between two vertices. Result is new network. Values on lines can be taken into account (if they present distances between vertices) or not (graph theoretical distance). The latter possibility is faster.
  - **Walks with Limited Length between Two Vertices** – Find all walks between two vertices with limited maximum length.
  - **Geodesics Matrices*** – Compute the shortest path length matrix and the geodesics count matrix (*for small networks only!*).
  - **Info on Diameter** – Find diameter – the length of the longest shortest path in network and corresponding two vertices. Full search is performed, so the operation may be slow for very large networks (number of vertices larger than 2000).

- **SubNetwork with Flows**
  - **Maximum Flow between Two Vertices** – Find maximum flow between selected two vertices (algorithm looks for paths to be saturated and among them it always selects the shortest path). Algorithm can be used in the technical area (actual flow, values on lines mean capacities) or for analysing graphs (if all values are 1). Result is a new network, containing the two vertices and lines contributing to maximum flow between them.
  - **Maximum Flow between Pairs in Cluster*** – Find maximum flow among vertices determined by cluster. Result is a new network, where a value on line means maximum flow between corresponding two vertices. Algorithm is slow: Use it on smaller networks or clusters with limited number of vertices only!

- **Transform**
  - **Transpose 1-Mode** – Transposed network of selected 1-mode network – change direction of arcs.
  - **Remove**
    - **Selected Vertices** – Remove selected vertices from network.
    - **all Edges** – Remove all edges from selected network.
    - **all Arcs** – Remove all arcs from selected network.
    - **Multiple Lines** – Remove all multiple lines from selected network.
      1. **Sum Values** – Values of all deleted lines are added to not deleted line between corresponding two vertices.

2. **Number of Lines** – Value of line between two vertices in a new network correspond to the number of lines between the two vertices in original network.
3. **Min Value** – Minimum value of all lines between two vertices is selected.
4. **Max Value** – Maximum value of all lines between two vertices is selected.
5. **Single Line** – Value of line between two vertices in a new network is 1.

· **Loops** – Remove all loops from selected network.

· **Lines with Value**

1. **lower than** – Remove all lines with value lower than specified value.
2. **higher than** – Remove all lines with value higher than specified value.
3. **within interval** – Remove all lines with values within specified interval.

· **all Arcs from each Vertex except**

1. **k with Lowest Line Values** – Sort lines around vertices in ascending order according to output line values. Keep only selected number of lines with lowest values.
2. **k with Highest Line Values** – Sort lines around vertices in descending order according to output line values. Keep only selected number of lines with highest values.
3. **Keep Arcs with Value equal to the k-th Value** – Determine what to do with lines with value equal to the k-th value (remove them or not).

· **Triangle** – Remove arcs belonging to lower or upper triangle.

∗ **Add** additional vertices, lines or vertices/lines labels to network.

· **Vertices** – Copy network to new network. Dimension can be enlarged for selected number of vertices (additional vertices without lines are added).

· **Vertex Labels**

1. **Default** – Replace current labels of vertices by default vertices labels (*v1, v2...*).
2. **from File(s)** – Replace the default vertices labels (*v1, v2...*) by labels given in a file. In case of a 2-mode network you can select two different files with labels (one for each

mode). This is particularly important for 2-mode networks obtained by network multiplications, where labels for each mode are stored in different files.

3. **and Descriptions from File(s)** – Replace the default vertices labels (*v1, v2...*) by labels given in a file. Again, in case of a 2-mode network you can select two different files with labels. Additional descriptions like vertex shapes, colors etc. are replaced too.

· **Line Labels as Line Values** – replace labels of lines (or create new if there are no) with line values. Number of decimal places is the same as used in Draw window for marking lines with line values.

· **Sibling edges** – Add sibling edges to vertices with a common

1. **Input** – arc-ancestor
2. **Output** – arc-descendant

∗ **Edges** → **Arcs** – Convert all edges to arcs (in both directions) (make directed network).

∗ **Arcs** → **Edges**

· **All** – Convert all arcs to edges (make undirected network).

· **Bidirected only** – Convert only arcs in both directions to edges:

1. **Sum Values** – Value of the new edge is the sum of values of both arcs.
2. **Min Value** – Value of the new edge is the smaller of values of arcs.
3. **Max Value** – Value of the new edge is the larger of values of arcs.

∗ **Bidirected Arcs** → **Arc**

· **Select Min Value** – If there exist bidirected arcs between two vertices retain only the arc with lower value and remove the arc with higher value. If both values are equal replace both arcs with an edge.

· **Select Max Value** – If there exist bidirected arcs between two vertices retain only the arc with higher value and remove the arc with lower value. If both values are equal replace both arcs with an edge.

∗ **Line Values** – Transformations of line values:

· **Set All Line Values to 1** – Set all line values to 1.

- **Recode** – Display frequency distribution of line values according to selected intervals and recode line values in this way.
- **Multiply by** a constant.
- **Add Constant** to line values.
- **Constant** – min or max of line value and selected constant.
- **Absolute** line values.
- **Absolute + Sqrt** – square root of line values.
- **Truncate** – truncated line values.
- **Exp** – exponent of line values.
- **Ln** – natural logarithm of line values.
- **Power** – selected power of line values.
- **Normalize**
  1. **Sum** – normalize so that the sum of line values will be 1
  2. **Max** – normalize so that the maximum line value will be 1
- **Replace Arc Values with Ranks** – In directed network: sort lines around vertices according to line values in ascending or descending order and replace line values with ranks.

* **Reduction**
  - **Degree** – (Recursively) delete from network all vertices with degree lower than selected value (according to Input, Output or All degree). Operation can be limited to selected cluster.
  - **Pathfinder*** – removing lines of a weighted network where values of lines are dissimilarities. Only lines which do not violate the triangle inequality are kept. Additional parameter r (real number larger than 0) need to be provided. r parameter defines the metric used for computing the distance of paths (Minkowski distance):

$$a \boxed{\mathrm{r}} b = \sqrt[r]{a^r + b^r}$$

Usually we use $r = 1$, $r = 2$, or $r = \infty$:

$$r = 1 \Rightarrow a \boxed{\mathrm{r}} b = a + b$$

$$r = 2 \Rightarrow a \boxed{\mathrm{r}} b = \sqrt{a^2 + b^2}$$

$$r = \infty \Rightarrow a \boxed{\mathrm{r}} b = \max(a, b).$$

According to complexity of the algorithm it can be applied to medium sized networks (networks with up to approx. 10000

vertices). Details:
http://en.wikipedia.org/wiki/Pathfinder_network

- **Hierarchy** – Recursively delete from network all vertices that have only 0 or 1 neighbor. Results: simpler network and hierarchy with deleted vertices. Original network can be later restored (if we forget directions of lines).
- **Subdivisions** – Recursively delete from network all vertices that have exactly 2 neighbors (together with corresponding two lines) and (instead of that) add direct line between these two neighbors. Result is simpler network (for drawing). Original network cannot be restored!
- **Design (flow graph)** Reduction of all structural parts of network according to McCabe (for programs – flow graphs) [51].

* **Replace Loops with Duplicates of Vertices** – Transform any network with loops to a network without loops in the following way:
  - For each vertex with a loop create a duplicate of this vertex (without a loop).
  - Link the duplicate vertex to original vertex with line value equal to loop value.
  - *Optionally*: Link the duplicate vertex also to all vertices to which original vertex is linked (with the same value as they are linked to original vertex).
* **1-Mode to 2-Mode** – Generate 2-mode network from any network.
* **Incidence Network** – Transform Pajek network (adjacency matrix) to *incidence matrix*. Incidence matrix can be used to generate linegraph.
* **Sort Lines** –
  - **Neighbors around Vertices** – For each vertex sort lines connected to it in ascending order according to the other end-vertex.
  - **Line Values** – Sort lines in ascending or descending order according to line values.
  - **Line Values around Vertices** – For each vertex sort lines connected to it in ascending or descending order according to line values.

- **Create Partition** – Partitioning Network. Result is a Partition.

  – **Degree**

  - ∗ **Input** – Number of incoming lines for each vertex.
  - ∗ **Output** – Number of outgoing lines for each vertex.
  - ∗ **All** – Total number of lines incident to each vertex.
- **Components**
  - ∗ **Weak** – Weak Components of selected network.
  - ∗ **Strong** – Strong Components of selected network.
  - ∗ **Strong-Periodic** – Strong Periodic Components of selected network - strongly connected components are further divided according to periods.
- k**-Neighbors** – Select all vertices
  - ∗ **Input** ...from which we can reach selected vertex in at most k-steps.
  - ∗ **Output** ...that can be reached from selected vertex in at most k-steps.
  - ∗ **All** ...Input + Output (forget direction of lines)
    Result is partition where vertices are in class numbers equal to the distance from given vertex, vertices that cannot be reached from selected vertex are in class number 999999998. After you get a partition you can extract subnetwork.
- k**-Core** – k-core is a subnetwork of given network where each vertex has at least k neighbors in the same core according to:
  - ∗ **Input** ... lines coming into vertex.
  - ∗ **Output** ... lines going out of vertex.
  - ∗ **All** ... all neighbors.
- **Valued Core** – Generalized k-core: Instead of counting lines (neighbors) use values of lines. sum of lines or maximum value can be used when computing valued core:
  Sum valued core of threshold *val* is a subnetwork of given network where the sum of values of lines to (from) the members of the same core is at least *val*.
  Max valued core of threshold *val* is a subnetwork of given network where the maximum value of all lines to (from) the members of the same core is at least *val*.
  Threshold values must be given in advance. Two different ways to determine thresholds:
  - ∗ **First Threshold and Step** – Select first threshold value and step in which to increase threshold.

  - **Selected Thresholds** – Thresholds (increasing numbers) are given using vector.

  Additionally (for 1-mode networks), Input, Output or All valued cores can be used.

- **Communities** – Find communities in the network.
  - **Louvain Method** – Find communities using the Louvain method. Line values are taken into account. Algorithm runs in several levels. If network is signed (at least one line has a negative value) a special version of algorithm is called. Resolution as additional parameter can be entered (resolution 1 means standard Louvain method, higher resolutions produce larger number of clusters, lower resolutions produce lower number of clusters). There are two algorithms available:
    - **Multi-Level Coarsening + Single Refinement** - which performs only refinement of the partition obtained in the last level (the coarsest partition).
    - **Multi-Level Coarsening + Multi-Level Refinement** - which iterativelly performs coarsening and later refinement phases for each obtained level. Modularities obtained by this algorithm are usually higher. For details of this algorithm see: http://dl.acm.org/citation.cfm?id=1970376
  - **VOS Clustering** – Similar to Louvain method: both methods have resolution parameter, but in Louvain method *modularity* is optimized while in VOS Clustering *VOS quality function* is optimized. For details see VOSviewer page: http://www.vosviewer.com/. Line values are always taken as positive, therefore this algorithm is not suitable for signed networks. Two algorithms are available as well:
    - **Multi-Level Coarsening + Single Refinement** - which performs only refinement of the partition obtained in the last level (the coarsest partition).
    - **Multi-Level Coarsening + Multi-Level Refinement** - which iterativelly performs coarsening and later refinement phases for each obtained level. VOS qualities obtained by this algorithm are usually higher.
- **Islands** – Partition vertices of network with values on lines (weights) to cohesive clusters (weights inside clusters must be larger than weights to neighborhood): the height of vertex (vector) is defined as the maximum weight of the neighbor lines. Two options:

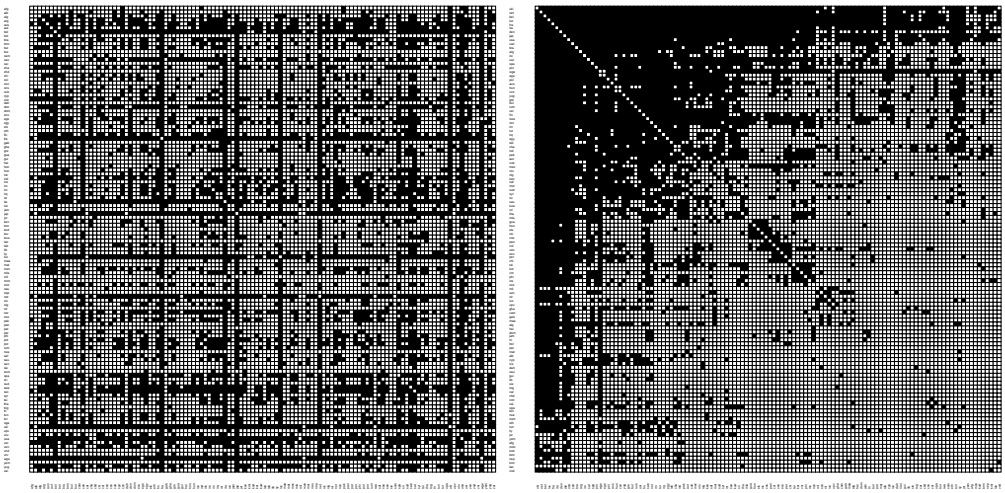


Figure 6: World trade. Orderings: alphabetical and determined by clustering

- **Line Weights**
- **Line Weights [Simple]**

New network with only lines constituting islands can be generated if **Generate Network with Islands** is checked.

– **Blockmodeling*** – Generalized blockmodeling of 1-mode and 2-mode networks [7, 35]. For details see Section 7 on page 100. Descriptions of models are stored on MDL files. See also block types on page 65.

- **Random Start** – Start the optimization with random partition(s).
- **Optimize Partition** – Show the criterion function for selected partition and optimize it.
- **Restricted Options** – Show only selected part of options (sufficient for most users) or all options.
- **Short Report** – Show only main results of optimization in Report window (sufficient for most users) or detailed, long report.

– **Vertex Labels** – Partition vertices so that vertices with same labels get the same cluster numbers (e.g. useful for molecule).

– **Vertex Labels matching Regular Expression** – Create binary partition according to regular expression on vertex labels. Some examples of regular expressions:
- ^D - for labels starting with 'D';
- a$ - for labels ending with 'a';
- edu - for labels containing 'edu'.

Vertices with labels that match regular expression are in cluster 1, other vertices in cluster 0.
`http://docwiki.embarcadero.com/RADStudio/Seattle/en/Regular_Expressions`

- **Vertex Shapes** – Partition vertices with same shapes (ellipse, box, diamond) to the same class numbers (used in genealogy to show gender).
- **Bow-Tie** – Partition vertices of directed network (graph structure of the web) to the following classes: 1 – LSCC, 2 – IN, 3 – OUT, 4 – TUBES, 5 – TENDRILS, 0 – OTHERS.
- **Default Labels Partition** – Input is network with default vertex labels: e.g., $v3$, $v9$,... Result is a partition of selected dimension, where vertices defined by numbers stored in vertex labels (e.g., 3, 9,...) go to cluster 1, other vertices go to cluster 0.
  Operation can be used to make other objects (e.g. partitions, vectors, ...) compatible with a network, if network is reduced by several operations (e.g. extractions).
- p**-Cliques** Partition network according to p-Cliques (partition to clusters where vertices have at least proportion p (number between 0 and 1) neighbors inside the cluster.
  - **Strong** ... for directed network.
  - **Weak** ... for undirected network.

- **Create Vector** – Get vector from network
  - **Centrality** – Result is a vector containing selected centrality measure of each vertex and centralisation index of the whole network [65, p. 169-219].
    - **Degree**
      - **Input** – Number of lines into vertices.
      - **Output** – Number of lines out of vertices.
      - **All** – Number of neighbors of vertices.
    - **Weighted Degree** – Weighted degree: sum of line values according to incoming, outgoing or all lines.
    - **Closeness** centrality (Sabidussi).
      1. **Input** – centrality of each vertex according to distances of other vertices to selected vertex.
      2. **Output** – centrality of each vertex according to distances of selected vertex to all other vertices.

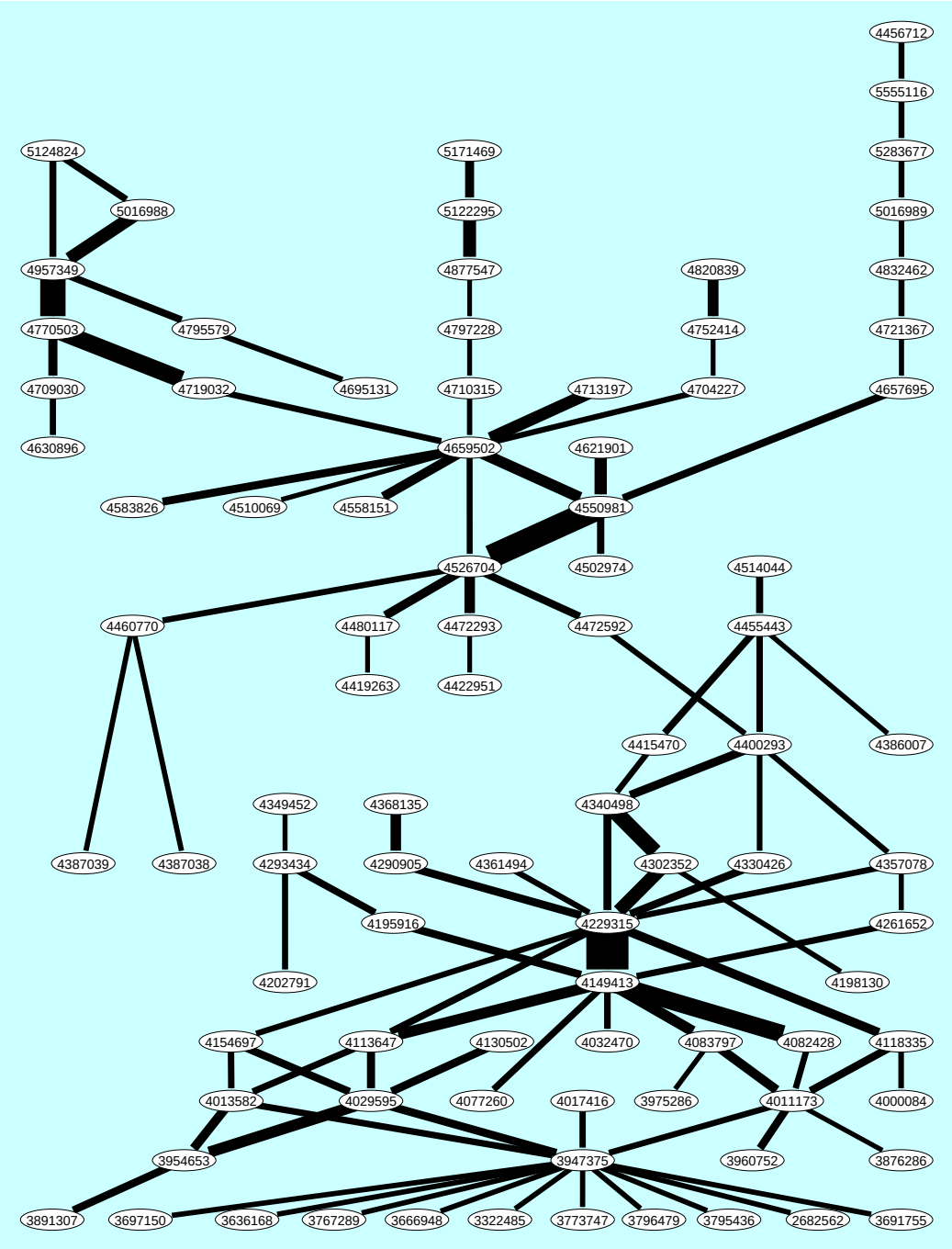


Figure 7: US Patents – Main island 'liquid-crystal display'

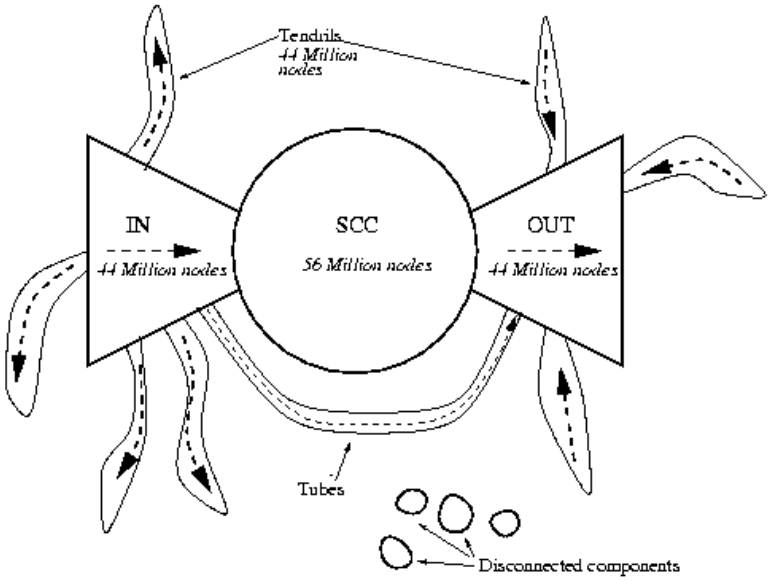


Figure 8: Bow-tie – Graph structure in the web [26]

3. **All** – forget direction of lines – consider network as undirected.

* **Betweenness** centrality (Freeman).
* **Hubs-Authorities** – In directed networks we can usually identify two types of important vertices: *hubs* and *authorities* [47]. A vertex is a good hub, if it points to many good authorities, and it is a good authority, if it is pointed to by many good hubs. In obtained partition value 1 means, that the vertex is a good authority, value 2 means, that the vertex is a good authority and a good hub, and value 3 means, that the vertex is a good hub.
* **Proximity Prestige** – For each vertex compute its domain according to *input*, *output* or *all* neighbors. Results are:
  - Partition containing size of domain - number of reachable vertices.
  - Vector containing the normalized size of domain - normalization is done by total number of vertices – 1.
  - Vector containing the average distance from/to domain.
  - Vector containing the Proximity Prestige.
* **Laplace** – Laplacian centrality. Details:

www.scirp.org/journal/PaperDownload.aspx?paperID=27402

  - **Line Values**
    - **Min** – Find minimum value of incoming, outgoing or all lines connected to selected vertex.
    - **Max** – Find maximum value of incoming, outgoing or all lines connected to selected vertex.
  - **Centers** – Find centers in a graph using 'robbery' algorithm: vertices that have higher degrees (are stronger) than their neighbors steal from them:
    - at the beginning give to vertices initial strength according to their degrees, or start with value 1
    - when 'weak' vertex is found, neighbors steal from it according to their strengths, or they steal the same amount

– **Clustering Coefficients** – Compute different inherent tendency coefficients in undirected network:

  Let $\deg(v)$ denotes degree of vertex v, $|E(G_1(v))|$ number of lines among vertices in 1-neighborhood of vertex v, MaxDeg maximum degree of vertex in a network, and $|E(G_2(v))|$, number of lines among vertices in 1 and 2-neighborhood of vertex v.

  - CC_1 – coefficients considering only 1-neighborhood:

$$CC_1(v) = \frac{2|E(G_1(v))|}{\deg(v)\cdot(\deg(v)-1)} \qquad CC_1'(v) = \frac{\deg(v)}{\mathrm{MaxDeg}} CC_1(v)$$

  - CC_2 – coefficients considering 2-neighborhood

$$CC_2(v) = \frac{|E(G_1(v))|}{|E(G_2(v))|} \qquad CC_2'(v) = \frac{\deg(v)}{\mathrm{MaxDeg}} CC_2(v)$$

  If $\deg(v) \leq 1$ all coefficients for vertex v get missing value (999999998). Watts-Strogatz Clustering Coefficient (Transitivity) and Network Clustering Coefficient are also reported.

– **Structural Holes** – Burt's measure of constraint (structural holes) [27, page 54-55]. Results are:

  - network p_{ij}: the proportion of the value of i's relation(s) with j compared to the total value of all relations of i. where a_{ij} is the value of the line from i to j

$$p_{ij} = \frac{a_{ij} + a_{ji}}{\sum_k (a_{ik} + a_{ki})}$$

  - network containing dyadic constraint c_{ij} – the constraint of absent primary holes around j on i: Explanation: Contact j constrains your i's entrepreneurial opportunities to the extent that:
    (a) you've made a large investment of time and energy to reach j, and
    (b) j is surrounded by few structural holes with which you could negotiate to get a favorable return on the investment.

$$c_{ij} = (p_{ij} + \sum_{k,k\neq i,k\neq j} p_{ik}p_{kj})^2$$

  - vector containing aggregate constraint C_i: $C_i = \sum_j c_{ij}$, $C_i = 1$ for isolated vertices.

- **Generalized Core** – k-cores generalized.
  - **Degree** – ordinary cores.
  - **Sum** – taking values of lines into account (sum of values of lines inside pcore).
  - **Max** – taking values of lines into account (max of values of lines inside pcore).
- **Distribution of Distances*** – Compute distribution of lengths of the shortest paths and average path length among all reachable pairs of vertices in network. Take all vertices as starting points.
- **Get Loops** – store values of loops to vector.
- **Get Coordinate** – x, y, or z coordinate of network. You can also get all coordinates at once - possibility to have more than 3 coordinates, coordinates must contain character . (dot).

- **Create Permutation** – compute permutations of given network.
  - **Depth First** – Depth first numbering of selected network...
    - **Strong** ... taking directions of arcs into account.
    - **Weak** ... forget directions (or undirected network).
  - **Breadth First** – Breadth first numbering of selected network...
    - **Strong** ... taking directions of arcs into account.
    - **Weak** ... forget directions (or undirected network).
  - **Reverse Cuthill-McKee** – RCM numbering. See paper.
  - **Core + Degree** – Numbering in decreasing order according to all core partition. Within the same core number vertices are ordered in decreasing order according to number of neighbors which have the same or higher core number.

- **Create Hierarchy** – hierarchical decomposition of given network.
  - **Clustering*** – Hierarchical clustering procedure. Input is dissimilarity network (matrix), which can be obtained using *Operations/Cluster/Dissimilarity/Network based* or read from input file.
    - **Run** – Hierarchical clustering procedure. Result is hierarchy with nested clusters and dendrogram in EPS.
    - **Options** – Select method for hierarchical clustering procedure (general, minimum, maximum, average, ward, squared ward).
  - **Symmetric-Acyclic** – Symmetric-Acyclic decomposition of network. Result is hierarchy with nested clusters [33].
  - **Clustering with Relational Constraint** – Hierarchical clustering with relational constraint procedure. See:
    Ferligoj A., Batagelj V. (1983): Some types of clustering with relational constraints. *Psychometrika*, **48(4)**, 541-552.
    Only dissimilarities among vertices that are linked are taken into account what enables to find clusterings very fast also for large networks.
    Input is network with dissimilarities, which can be obtained using *Operations/Dissimilarity/Network* or *Vector based* or read from input file.
    - **Run** – Results are: a partition representing tree: fathers of nodes; and two vectors: describing heights of nodes and number of vertices in subtree respectivelly. If network has n vertices then obtained partitions and vectors have dimension 2*n-1. Note that this objects are not compatible with original network, you must use *Make Partition* to get compatible results.
      Options – Select method for hierarchical clustering with relational constraint (*minimum*, *maximum*, or *average*) and strategy (*strict*, *leader*, or *tolerant*).
    - **Make Partition** – From obtained partition representing tree generate partition compatible with original network
      - **using Threshold determined by Vector** – From obtained partition representing tree and one of the two vectors (all have dimension 2*n-1) generate partition compatible with original network by giving threshold value.
      - **with selected Size of Clusters** – From obtained partition representing tree and given number of vertices in clusters generate partition compatible with original network.

    * **Extract Subtree as Hierarchy** – Extract subtree from obtained Partition by giving the root as Pajek Hierarchy.

- **2-Mode Network** – Operations that are specific to 2-mode networks.

  – **Partition into 2 Modes** – Partition of vertices of a 2-mode network into two subsets.

  – **Transpose 2-Mode** - Interchange Rows and Cols of given 2-mode network.

  – **2-Mode to 1-Mode** – Generate an ordinary (1-mode) network from 2-mode (affiliation) network. Result is a valued network. To store a 2-mode network in input file use **Pajek** or Ucinet format (look at **Davis.dat** from Ucinet dataset).

    * **Rows** – Result is a network with relations among row elements (actors). The value of line tells number of common events of the two actors.

    * **Columns** – Result is network with relations among column elements (events). The value of a line tells number of actors that took part in both events.

    * **Special** – Instead of multiplication minimun (or maximum) value is used.

    * **Include Loops** – If checked, loops with value telling the total number of events for each actor (total number of actors for each event), are added.

    * **Multiple Lines** – Generate nonvalued 1-mode network, where multiple lines among vertices can exist. The label of the generated line corresponds to the label of the event/actor that served to induce the line. If partition of the same dimension is present, multirelational network can be generated.

    * **Normalize 1-Mode** – Normalize the obtained 1-Mode network. 1-Mode network must be obtained with option **include loops** checked, and **multiple lines** not checked:

$$\begin{aligned}
\text{Geo}_{ij} &= \frac{a_{ij}}{\sqrt{a_{ii}a_{jj}}} \\
\text{Input}_{ij} &= \frac{a_{ij}}{a_{jj}} \\
\text{Output}_{ij} &= \frac{a_{ij}}{a_{ii}} \\
\text{Min}_{ij} &= \frac{a_{ij}}{\min(a_{ii}, a_{jj})} \\
\text{Max}_{ij} &= \frac{a_{ij}}{\max(a_{ii}, a_{jj})} \\
\text{MinDir}_{ij} &= \begin{cases} \frac{a_{ij}}{a_{ii}} & a_{ii} \leq a_{jj} \\ 0 & \text{otherwise} \end{cases} \\
\text{MaxDir}_{ij} &= \begin{cases} \frac{a_{ij}}{a_{jj}} & a_{ii} \leq a_{jj} \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

The obtained network is usually not sparse. To make it sparser use **`Network/Transform/Remove/lines with value/lower than`**.

- ∗ **Rows=Cols** – Transform 2-Mode network with the same subsets of vertices to 1-Mode network.
- ∗ **Cols=0** – Transform 2-Mode network to 1-Mode network by setting number of columns to 0. The result is the same as changing for example `*Vertices 32 18` to `*Vertices 32` in input network file.

– **Core** – Find core in a 2-mode network.

- ∗ **2-Mode** – Core partition of a 2-mode network. Given minimum degree in first (k_1) and minimum degree in second subset (k_2) a new partition is generated where 0 means that vertex does not belong to the core of prespecified k_1 and k_2, 1 means that vertex belongs to that core.
- ∗ **2-Mode Review** – Given starting values of k_1 and k_2 the following list is computed:
  k_1 k_2 Rows Cols Comp
  where k_1 is minimum degree in the first, k_2 minimum degree in the second subset, $Rows$ and $Cols$ are number of vertices in first and second subset respectivelly and $Comp$, number of connected components in network induced by k_1 and k_2. k_1 and k_2 are incremented until the resulting network is empty.

  - **2-Mode Border** – Compute only border values of $k1$ and $k2$ for a given 2-mode network.
  - **Valued 2-Mode Core** – Valued core partition (according to line values) of a 2-mode network. Given minimum valued degree in first (k_1) and minimum valued degree in second subset (k_2) a new partition is generated where 0 means that vertex does not belong to the valued core of prespecified k_1 and k_2, 1 means that vertex belongs to that core.

- **Important Vertices** – Generalization of hubs and authorities algorithm for 2-mode networks – find important vertices in first and second subset.
- **Info** – Basic information of 2-mode network, e.g. 2-mode density.

- **Multiple Relations Network** – Operations that are specific to multiple relations networks.

  - **Select Relation(s) into One Network** – Keep only selected relation(s) of a given network and remove all other relations. Result is only one network with selected relations.
  - **Extract Relation(s) into Separate Network(s)** – Extract one or more relations from selected multiple relations network. For each selected relation a new network is generated.
  - **Canonical Numbering** – Enumerate relations with sequential numbers 1, 2,...
  - **Generate 3-Mode Network** – generate a 3-mode network from 1-mode or 2-mode multirelational network. For each line in multirelational network `r: i j v` (line from `i` to `j` with value `v`, relation number is `r`) generate the following three lines (triangle):

    * 1-mode networks:
      ```
      i N+j v
      i 2N+r v
      N+j 2N+r v
      ```
    * 2-mode networks:
      ```
      i j v
      i N+M+r v
      j N+M+r v
      ```

    where `N` is cardinality of the first mode and `M` cardinality of the second mode.

  - **Line Values $->$ Relation Numbers** – Store line values as relation numbers (absolute truncated values).
  - **Relation Numbers $->$ Line Values** – Store relation numbers as line values.
  - **Change Relation Number - Label** – Change selected relation number to new relation number with corresponding label.
  - **Info** – General information about multiple relations network
    - number of relations
    - number of arcs, edges and total number of lines for each relation

- **Acyclic Network** – Operations that are specific to acyclic networks.
  - **Create Partition** – Create Partition from acyclic network.
    - **Depth Partition**
      - **Acyclic** – Partition acyclic network according to depths of vertices.
      - **Genealogical** – Partition network that represents genealogy according to layers of vertices.
      - **Generational** – Partition network that represents genealogy according to layers of vertices. The same as genealogical partition but with less layers.
    - **Select First Vertices** – Select first vertices of a network and put them to different clusters (starting with 1) in a partition. This partition can be later used as input for computing the **Genetic Structure** of a network.
    - **Select Last Vertices** – Select last vertices of a network and put them to different clusters (starting with 1) in a partition.
  - **Create Weighted Network + Vector** – Compute weights (importances) of vertices (papers) and arcs (citations) in acyclic network.
    - **Traversal Weights** – If network represents citation network, traversal weights of arcs (citations) and vertices (articles) can be computed. Results are:
      - Network with values on ars representing importance of citations.
      - Vector with number of different incoming paths.
      - Vector with number of different outgoing paths.
      - Vector with traversal weights (importance of vertices - articles).

Different methods of assigning traversal weights [43]:

· **Search Path Count (SPC)** – Compute from Source to Sink vertices.

· **Search Path Link Count (SPLC)** – Each vertex is considered as a Source vertex.

· **Search Path Node Pair (SPNP)**.

In case of larger networks traversal weights might become huge integers. In this case weights can be normalized (using flow or maximum value) or logged.

∗ **Probabilistic Flow** – Result is vector and new network with values on arcs. Vector values tell us what is the probabilty that a random walk starting in the source vertices goes through the selected vertex. Arc values tell us what is the probabilty that a random walk starting in the source vertices goes through the selected arc.

– **Create (Sub)Network** – Extract important paths from weighted acyclic network.

∗ **Main paths** – Search for main paths in a citation network with traversal weights (or any other acyclic network with weights on arcs). For details see [49].

· **Local Search** – in each step of the search select the arc(s) with the highest weight that are incident to the current arc. Tolerance (number between 0 and 1) can be used. Tolerance 0 means that in each step only arcs with the highest weights are taken into account, while larger tolerance means that also a bit smaller weights are considered.

1. **Forward** – search for local main path(s) from Source to Sink vertices.
2. **Backward** – search for local main path(s) in the opposite direction (from Sink to Source vertices).
3. **Key-Route** – select some (most important) arcs (called key-routes) and search for main paths from both ends of the key-routes.
4. **Through Vertices in Cluster** – search for local main path(s) passing through vertices selected in Cluster.

· **Global Search** – search for main path(s) with the highest overall sum of weights.

1. **Standard** – search for main path(s) from source to sink vertices with the highest overall sum of weights. This is method is also called Critical Path Method (CPM).

2. **Key-Route** – search for all main paths containing selected key-routes with the highest overall sum of weights.
3. **Through Vertices in Cluster** – search for main path(s) passing through vertices selected in Cluster with the highest overall sum of weights.

· **Mark Main Paths as Multirelational Network** – The original network (network with traversal weights) is transformed to a multirelational network: arcs belonging to main path(s) get relation number 2, all other arcs get relation number 1.

Additional result is a binary partition with vertices on the main path(s) in cluster 1 and all other vertices in cluster 0.

∗ **Critical Path Method (CPM)** – Find the critical path in acyclic network – result is new network containing the critical path. Algorithm can be used in the area of project planning but also for analysing acyclic graphs. Additional networks containing total and free delay times of activities are generated. The two networks are multirelational: arcs belonging to critical path get relation number 2. Two vectors (partitions) are generated as well: First containing the earliest possible times of coming into given states and the second containing the latest feasible times of coming into given states.

– **Create Permutation** – Create Permutation of (an almost) acyclic network.

∗ **Topological Ordering** – Topological ordering of acyclic network.

∗ **Becker's Heuristic** – Becker's heuristic for constructing a linear ordering of an almost acyclic network. Similar to topological ordering of acyclic networks but can be applied to non-acyclic networks as well. It is supposed to be used on directed networks with large number of small strong components.

– **Transform** – Transformations of (almost) acyclic networks.

∗ **Add Source and Sink** – Add unique first and last vertex to acyclic network (new network has two artificial vertices).

∗ **Preprint Transformation** – Transform an 'almost' acyclic network to 'true' acyclic network in the following way: Each paper u from a strong component is duplicated with its 'preprint' version u'. The papers inside strong component cite preprints, and the preprints cite the outside papers. Each strong component is replaced by the corresponding complete directed bipartite subnetwork papers $\times$ preprints.

- **+ Partition** – Acyclic Network and Partition needed as input for these operations.
  - ∗ **Create Vectors with Genetic Structure** – Compute genetic structure of given acyclic network according to given partition (of first vertices). Partition can be obtained using **Network / Acyclic Network / Create Partition / Select First Vertices**. As result we get one vector for each cluster number in a partition. Be careful: if number of clusters is too high computation can take a long time. Additional result is the dominant gene partition and the dominant gene vector.
- **Info** – Report number of first and last vertices, minimum and maximum degree, number of components, size of the largest component and (in case of p-graphs) also Relinking Index.

• **Temporal Network** – Operations that are specific to temporal networks.

- **Generate in Time** – Generate network in specified time(s) or interval. Input first time, last time and step (integers).
  Additional parameters when vertices and lines are active should be given in network to perform this operation. They must be given between signs `[` and `]`:
  − is used to divide lower and upper limit of interval,
  `,` is used to separate intervals,
  ⋆ means infinity. Example:

```
*Vertices 3
1 "a" [5-10,12-14]
2 "b" [1-3,7]
3 "e" [4-*]
*Edges
1 2 1 [7]
1 3 1 [6-8]
```

  Vertex 'a' is active from times 5 to 10, and 12 to 14, vertex 'b' in times 1 to 3 and in time 7, vertex 'e' from time 4 on. Line from 1 to 2 is active only in time 7, line from 1 to 3 in times 6 to 8.
  The lines and vertices in a temporal network should satisfy the *consistency* condition: if a line is active in time t then also its end-vertices are active in time t. When generating time slices of a given temporal network only 'consistent' lines are generated.
  Note that time records should always be written as last in the row where vertices / lines are defined.

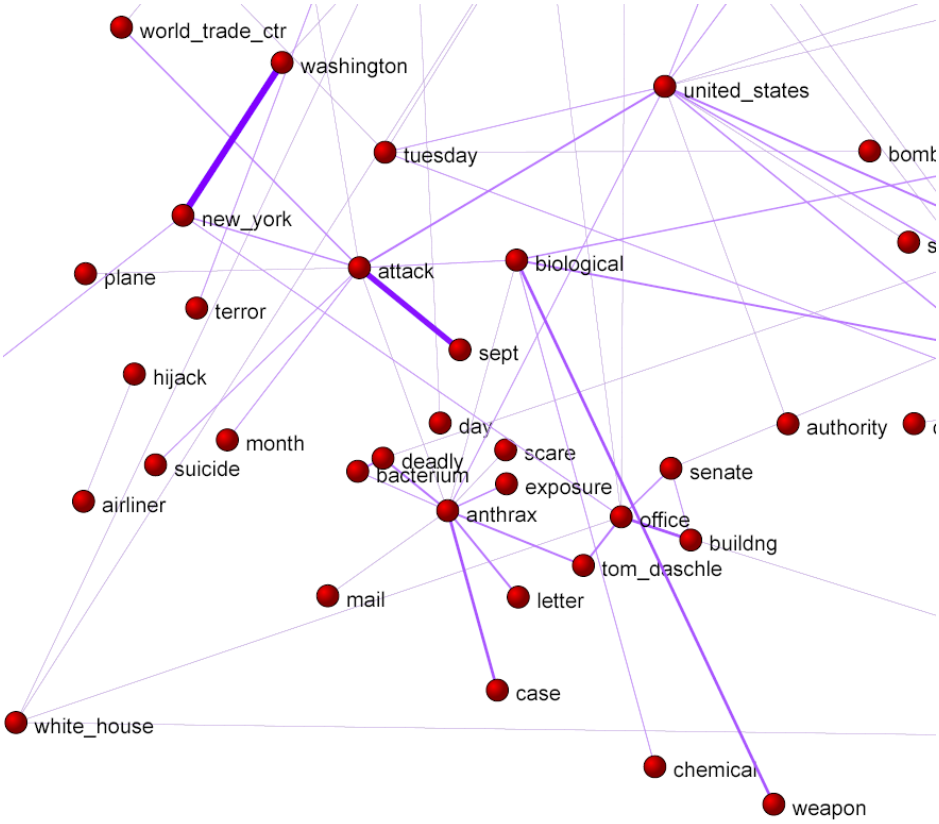


Figure 9: Part of Reuters Terror News network on the 36th day.

See also other possibility of describing time network: description of time network using time events.

* **All** – Generate all networks in specified times.
* **Only Different** – Generate network in specified time only if the new network will differ in at least one vertex or line from the last network which was generated.
* **Interval** – Generate network with vertices and lines present in selected interval.

– **Info** – Report number of vertices and lines in each time point of a temporal network.

- **Signed Network** – Operations that are specific to signed networks (networks with both positive and negative lines).

  – **Set All Positive Line Values to 1, Negative to -1** – Set all positive line values to 1, and all negative line values to -1.

  – **Create Partition** – Create Partition form network (and initial random partition).

    * **Doreian-Mrvar method*** – Relocation algorithm for partitioning signed graphs (graphs with positive and negative values on lines representing friends and enemies, for example). Given partition is optimized to get as much as possible positive lines inside classes and negative lines between classes. Another algorithm does not distinguish between diagonal and off-diagonal blocks: each block can be positive, negative, or null. If number of repetitions is higher than 1, initial partitions into given number of classes are chosen randomly for every repetition separately. If program finds several optimal solutions, all are reported. For more details about algorithm see Doreian and Mrvar [32].
      Option can be used for two mode signed graphs as well: input is two mode partition. In this case algorithm tries to find as 'clear' as possible positive, negative, and null blocks.
      If *Prespecification* is checked user can define a prespecified model by entering letters **P**, **N**, or **0** to cells (to require positive, negative or null blocks) or leave cells empty (in this case the block can be of any type). In this case for each vertex selected cluster can be prespecified/prohibited. For each cluster minimum and maximum number of vertices it contains can also be prespecified.

By setting penalty for small null blocks to some nonzero value, we try to get null blocks as large as possible.

  - **Zeleny Approach*** – Partitioning signed networks by optimizing Morale scores [69].
  - **Louvain method** – Standard Louvain method modified to be used on signed networks. For details see description of Louvain method.

- **Info** – Compute Structural Balance Inconsistency and Signed Modularity of given signed network according to partition. Also Signed E-I Indices (positive and negative) and Morale Indices [69] are computed.

- **Info** – Information about network
  - **General** – General information about network
    - number of vertices
    - number of arcs, edges and loops
    - density of lines
    - average degree
    - sort lines according to their values (ascending or descending) to find the most/least important lines.
  - **Line Values** – Frequency distribution of line values.
  - **Triadic Census** – Number of different triads in network. See book of Faust and Wasserman [65] and Figure 15 on page 68.
  - **Degree Assortativity** – indices of a given network.
  - **Vertex Label -> Vertex Number** – Find vertex number by giving (part of) its label, or find vertex label for given vertex number.
  - **Line -> Rank of its Value** – Find the rank of selected line according to its value.
  - **Indices** – Different indices on network (mostly chemical).

3.3 Networks

Operations on two networks.

- **Union of Lines** – Fuse selected networks. Result is a multiple relations network. If you want to get union of networks, multiple lines must still be deleted. Networks must match in dimension or: If one network has m vertices and other n vertices and $m < n$ then in network with n vertices first m vertices must match with vertices in network with m vertices.

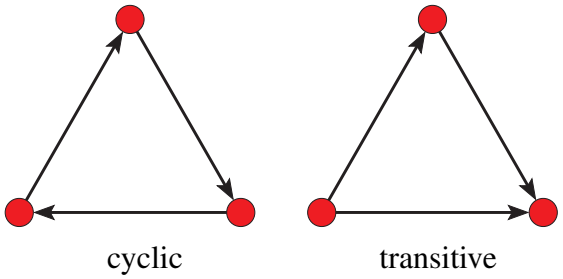


Figure 10: Lines belonging to cyclic and transitive (shortcut) 3-rings

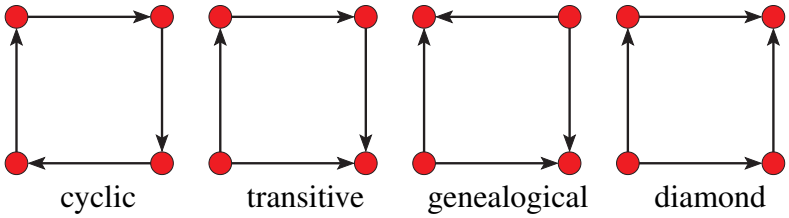


Figure 11: Types of directed 4-rings on arcs

- **Union of Vertices** – Add the second network at the end of first network.
- **Cross-Intersection** – Intersection of selected networks. Networks must match in dimension or: If one network has m vertices and other n vertices and $m < n$ then in network with n vertices first m vertices must match with vertices in network with m vertices. Values of lines in intercept can be *sum, difference, product, quotient, min, or max* of both values. We can also take value from *the first* or value from *the second* network.
- **Intersection of Multiple Relations Networks** – Intersection of selected networks where relation numbers are taken into account.
- **Cross-Difference** – Difference of selected networks.
- **Difference of Multiple Relations Networks** – Difference of selected networks where relation numbers are taken into account.
- **Multiply Networks** - multiply selected 1-mode or 2-mode networks (that match criteria for multiplication). If one network is a 2-mode network and one a 1-mode network, the latter must be transformed to 2-mode network

(**Network/Create New Network/1-Mode to 2-Mode**) before multiplication (othewise warning for non-compatible networks will be displayed).

- **Multiply Networks - Get Loops only** - returns only diagonal obtained by multiplication of selected 1-mode networks (or 2-mode networks where result is a 1-mode network).

- **Fragment (First in Second)** – Find all instances of fragment (determined by network 1) in network 2.

  - **Induced** – there should be no additional lines between vertices in instance of fragment to match (stronger condition) otherwise additional lines can be present (weaker).
  - **Negative lines inside fragment not allowed** – there should be no line with negative value between vertices in instance of fragment to match. A negative line means that the two vertices should not belong to the same fragment. This option is visible only if *Induced* is not checked.
  - **Labeled** – labels must match (e.g. atoms in molecule). Labels are determined by classes (colors) in partition - first partition and second partition must be selected before searching for labeled fragments. First partition determines 'labels' of first network (fragment), second partition determines 'labels' of second (original) network.
  - **Check values of lines** – values of lines must match (e.g. in genealogy values represent sex: 1 – man, 2 – woman).
  - **Check relation number** – relation numbers must match.
  - **Check only cluster** – only fragments are searched. where first vertex is one of the vertices in cluster.
  - **Extract subnetwork** – produce additional result: extract subnetwork containing vertices belonging to fragments and corresponding lines.
    - **Retain all vertices after extracting** – in extracted network the same vertices as in original network are present, only lines which do not belong to any fragment are removed.
  - **Same vertices determine one fragment at most** – how fragments on the same set of vertices are treated: *if not checked* – fragments with the same set of vertices are allowed.
    - **Create Hierarchy with fragments** – result of fragment searching is also a Hierarchy with vertices in fragments (available only if **Same vertices determine one fragment at most** is not checked.

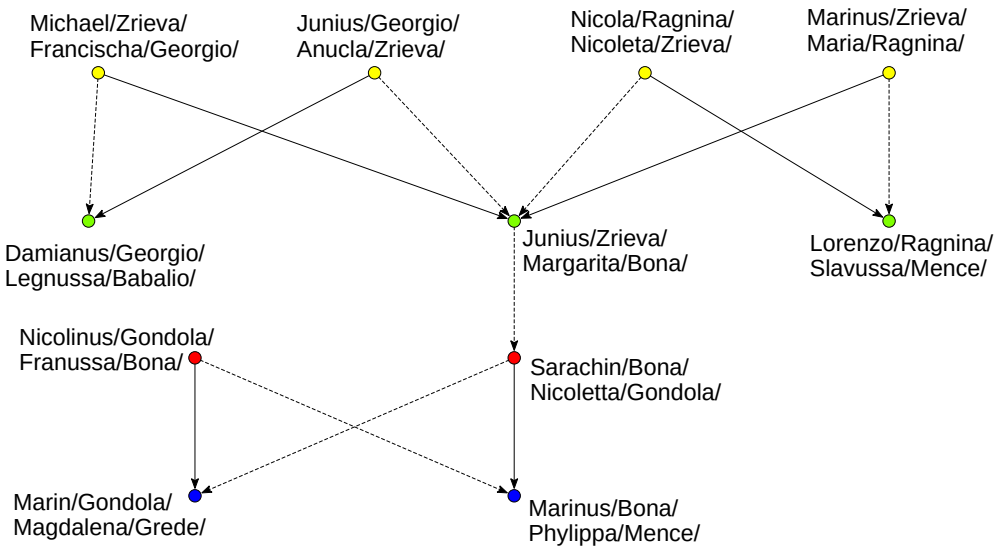


Figure 12: Fragments – Marriages among relatives in Ragusa

  - **Repeating vertices in fragment allowed** – same vertices can appear in fragment more than once (e.g. in cycles).
    if not checked: found fragments always have the same number of vertices as original fragment
    if checked: some of found fragments can have less vertices than original fragment

- **Match Vertex Labels** - For given two networks generate two partitions: Positions of vertex labels of the first network in the second network and positions of vertex labels of the second network in the first network (0 means that the corresponding label does not exist in another network).
  Operation is useful for combining two networks where some (but not all) vertex labels are the same.
  Partitions and vectors can be made compatible with extracted subnetworks using this operation as well.

- **Shrink coordinates (First to Second)** - Useful if you shrink network, draw shrunk network separately, and then apply all coordinates to vertices in original network (vertices in same class get the same coordinates). Replace coordinates in network 2 using coordinates of shrunk network 1. Shrinking can be determined using

  - **Partition** or

  - **Hierarchy**

3.4 VertexID

Operations, for which only list of vertex identificators (VertexID) is needed as input.

- **Default VertexIDs** – Generate default identificators of selected dimension and initial value.
- **Copy VertexIDs to Partition** – Copy identificators to partition.
- **Expand VertexIDs to Partition** – Result is a partition of selected dimension, where vertices defined by identificators from VertexID go to cluster 1, other vertices go to cluster 0.
- **Info** – General information about identificators: dimension, the lowest and the highest value.

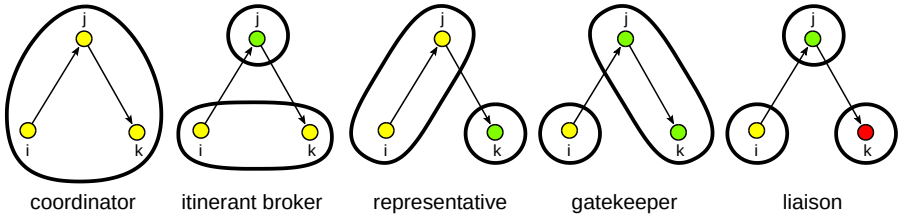
3.5 Operations

One Network and something else is needed as input.

- **Network + VertexID** – Operations on Network and VertexID.
  - **Shrink Network** – Shrink network according to selected VertexID.
- **Network + Partition** – Operations on Network and Partition (and Cluster in case of Count Neighbour Clusters; and Permutation in case of Coloring).
  - **Extract** – Extract subnetwork(s) according to selected partition:
    - **SubNetwork Induced by Union of Selected Clusters** – Extract subnetwork according to selected partition (extract range of clusters from partition). Result always consists of only one subnetwork. Extracted partition is produced as additional result.
    - **SubNetworks Induced by Each Selected Cluster Separately** – Extract subnetworks induced by selected clusters of partition. Several subnetworks are extracted using a single command. Each selected cluster produces one induced subnetwork.

  - **2-Mode Network** – Extract 2-mode network from 1-mode network: first and second mode are determined by given set of clusters in partition.
  - **to GEDCOM** – Extract sub-genealogy according to selected partition (weakly connected component) to new GEDCOM file (genealogy must be read as Ore graph).

- **Shrink Network** – Shrink network according to selected partition. Vertices in class 0 are (by default) left unchanged, others are shrunk. Results are shrunken network and shrunken partition. Before starting shrinking, select appropriate blockmodel in Options menu. Default is just number of lines between shrunk vertices that must be present in original network, to cause a line in a new network.
- **Brokerage Roles** - For each vertex j count five brokerage roles (*coordinator, itinerant broker, representative, gatekeeper* and *liaison*) according to given partition.



- **Internal/External Cluster** – For selected network and partition two vectors (called *Internal* and *External*) are generated. Dimension of vectors is equal to the largest value (cluster) in partiton. For each cluster i from the partition the two vectors give:
  - **Min Line Value** - $Internal[i]$ is minimum line value where both endvertices are in cluster i. $External[i]$ is minimum line value where only one of its endvertices is in cluster i.
  - **Max Line Value** - $Internal[i]$ is maximum line value where both endvertices are in cluster i. $Exernal[i]$ is maximum line value where only one of its endvertices is in cluster i.
- **Expand Partition**
  - **Greedy Partition** – Put vertices with unknown class number (0) in the same class as selected vertices in partition if
    - **Input** ...we can reach selected vertices in at most k-steps.
    - **Output** ...we can come to vertices from selected vertices in at most k-steps.

· **All** ...Input + Output (forget direction of lines)

Classes are joined if one vertex should belong to more classes.

∗ **Influence Partition** – Put every vertex with unknown class number (0) in given partition in the same class as is the class of the closest vertex. If several vertices with known class number have the same distance, the highest value is used.

∗ **Make Multiple Relations Network** – Transform network to a multiple relation network using a partition: if both endvertices of a line belong to the same class in partition the multiple relations tag will be equal to the class number of endvertices, otherwise it will be 0, class number of initial or class number of terminal vertex.

– **Identify** – Identify (reorder and/or join some units).

– **Refine Partition** Refine partition according to selected network (reachability).

∗ **Strong** ... for directed network.

∗ **Weak** ... for undirected network.

– **Leader Partition** – find clusters of vertices of network inside layers.

– **Petri** – Execute Petri net according to starting marking of places determined by partition. Number of places in network is equal to dimension of partition. Places must be defined first ($1..m$) then transitions ($m+1..n$). What to do if more than one transition can fire? Two possibilities:

∗ **Random** – Transition is chosen randomly.

∗ **Complete** – Complete tree of all possible transitions is built - result is hierarchy. You can choose the maximum depth of the tree, or execute Petri net as long as possible.

Try for example `petri2` from the book of Peterson [57, page 21] or `petri52` (see Figure 13) data.

– **Transform** – Transformations of network according to Partition.

∗ **Remove Lines** – Removing lines according to partition.

· **Inside Clusters** – Remove all lines with incident vertices in the same (selected) cluster(s).

· **Between Clusters** – Remove all lines with incident vertices in different clusters.

· **Between Two Clusters**

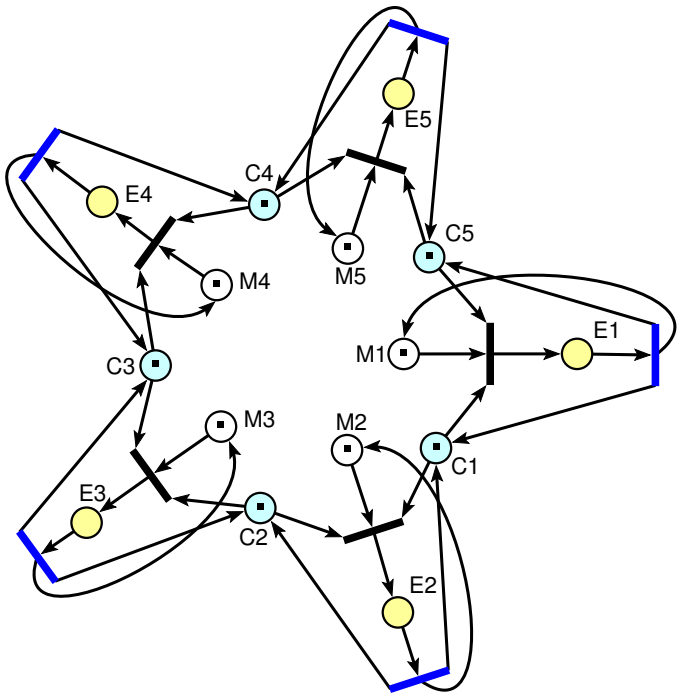


Figure 13: Petri net

1. **Arcs** – Remove all arcs pointing from first to second cluster.
2. **Edges** – Remove all edges between the selected two clusters.

· **Inside Clusters with value**

1. **lower than Vector value** – Remove all lines inside clusters (determined by a Partition) with value lower than the value specified in a Vector.
2. **higher than Vector value** – Remove all lines inside clusters (determined by a Partition) with value higher than the value specified in a Vector.

Dimension of a Vector must be equal to the highest cluster number in a Partition.

∗ **Add Time Intervals determined by Partitions** – change network to temporal network using two partitions: first partition determines initial time point, second determines terminal time point of each vertex.

∗ **Direction** – Convert to directed network where all arcs are pointing from

· **Lower->Higher** class number.

· **Higher->Lower** class number.

Lines inside classes may be deleted or not.

– **+ Cluster** – Operations that need Network, Partition and Cluster as input.

∗ **Count Neighbour Clusters** – For selected Network and Partition one Vector is generated for each cluster number in Cluster. In each obtained Vector the number of vertices of selected cluster in the neighbourhood of each vertex is given. Cluster numbers for which we want to obtain results as Vectors must be provided in Cluster. Note the exception: In this case Cluster represents set of cluster numbers and not set of vertices numbers.

– **+ Permutation** – Operations that need Network, Partition and Permutation as input.

∗ **Coloring**

· **Create New** – Sequential coloring of vertices in order determined by permutation. Result depends on selected permutation significantly.

· **Complete Old** – Complete partial coloring of vertices in order determined by permutation. For example some vertices can be colored by hand, but most of the vertices are still uncolored (in class 0). In this way you can help program to produce better coloring.

– **Info** – Info on Partition of given Network.

∗ **Modularity** – Network modularity according to Partition.

∗ **VOS Quality** – VOS Quality according to Partition.

∗ **E-I Index** – E-I Index according to Partition (weighted or unweighted).

- **Network + Vector** – Operations on Network and Vector (and Cluster in case of Diffusion Partition, and Partition and Permutation in case of Harmonic Function).

  – **Network * Vector** – Ordinary multiplication of matrix (network) by vector. Result is a new vector.

  – **Vector # Network** – Result is a new network:

  ∗ **Input** – Multiplying incoming arcs in network by corresponding vector values - multiplying *i*-th column of matrix by *i*-th component of vector.

  ∗ **Output** – Multiplying outgoing arcs in network by corresponding vector values - multiplying *i*-th row of matrix by *i*-th component of vector.

  – **Neighbours**

  ∗ **Sum** – For each vertex compute the sum of vector values of its neighbors according to

  · **Input** – neighbours

  · **Output** – neighbours

  · **All** – neighbours

  ∗ **Min** – For each vertex compute the minimum vector value of its neighbors according to

  · **Input** – neighbours

  · **Output** – neighbours

  · **All** – neighbours

  ∗ **Max** – For each vertex compute the maximum vector value of its neighbors according to

  · **Input** – neighbours

  - **Output** – neighbours
  - **All** – neighbours

– **Islands** – Partition vertices to cohesive clusters according to weights of vertices determined by a vector.

  * **Vertex Weights** – Vertex island is a cluster of vertices of given network with weighted vertices where the weights of the vertices on the island are larger than the weights of the vertices in the neighborhood. The weights are also called heights.
  * **Vertex Weights [Simple]** – Simple vertex island is vertex island with only one top.

– **Transform** – Transformations of network according to Vector.

  * **Put Loops** – put vector values as loops (arcs or edges) in current network.
  * **Put Coordinate** – put vector as x, y, or z coordinate, or put it as polar radius or polar angle of vertices in network layout.
  * **Vector(s) -> Line Values** – Replace line values with result of some operation (sum, difference, multiplication, division, minimum, maximum) on vector(s) values in corresponding terminal and initial vertices. Line value can also be replaced by a vector value in initial or terminal vertex of the line.

– **+ Cluster** – Operations that need Network, Vector and Cluster as input.

  * **Diffusion Partition** – Compute diffusion partition according to thresholds given in vector. Vertices in selected cluster are considered to adopt in time 1.

– **+ Partition + Permutation** – Operations that need Network, Vector, Partition and Permutation as input.

  * **Harmonic Function** – See Bollobas [25, page 328]. Let (G, a) be a connected weighted graph, with weight function $a(x, y)$, and let S is subset of vertices $V(G)$. A function $f : V(G) \to \mathbb{R}$ is said to be harmonic on (G, a), with boundary S, if

$$f(x) = \frac{1}{A(x)} \sum_y (a(x, y) f(y)), \qquad \forall x \in V(G) \setminus S$$

$$A(x) = \sum_y a(x, y)$$

    Implementation in **Pajek**:

· function f is determined by vector
· weight function $a(x, y)$ is given by (valued) network
· subset S is determined by partition – vertices in class 1 are in subset S (fixed vertices), other vertices are in $V(G) \setminus S$
· additionally, permutation determines the order of vertices in computations.

In **Pajek** you can compute the harmonic function once or iterativelly - as long as difference between successive functions become small enough. Components of vector that represents function f can be modified immediately when they are computed or only at the end of each iteration (after all components are computed). Procedure can be run according to:

· **Input** – neighbors
· **Output** – neighbors
· **All** – neighbors

– **Info** – indices of network obtained using one or two vectors (one for initial, another for the terminal vertex of the line).

1. **Assortativity** – index of a network according to one or two vectors. If two vectors are used, first defines numerical value for the initial vertex of the link, the second defines numerical value for the terminal vertex of the link. In case only one vector is used, it defines both (the values for the initial and terminal vertices).

- **Network + Scalar** – Operations on Network and Scalar (Vector with dimension 1).

  – **Line Values** – Modifying line values according to given scalar using operations Add, Subtract, Multiply, Divide, Min, Max.

- **Network + Permutation** – Operations on Network and Permutation.

  – **Reorder Network** – Reorder vertices in network according to selected permutation.
  – **Numbering*** – Improve given permutation according to network.
    * **Travelling Salesman** – Can be applied to dissimilarity matrix, or modified matrix representing network (fill diagonal and change 0 in the matrix with some large numbers):
      · **Run** – Run 3-OPT algorithm for solving Travelling Salesman Problem.

· **Options** – Put selected value on diagonal, add some artificial vertices, and incident lines with large values, change value 0 with selected (large) value.

∗ **Seriaton** – Starting with network and (random) permutation improve the permutation using seriation algorithm from Murtagh [54, page 11-16].

· **1-Mode** – for ordinary (1-Mode) networks

· **2-Mode** – for 2-Mode networks

∗ **Clumping** – Starting with network and (random) permutation improve the permutation using clumping algorithm from Murtagh [54, page 11-16].

· **1-Mode** – for ordinary (1-Mode) networks

· **2-Mode** – for 2-Mode networks

∗ **R-Enumeration** – Starting with network and (random) permutation find such permutation that enumeration of neighbor vertices are as close to each other as possible.

- **Network + Cluster** – Operations on Network and Cluster.

  – **Extract SubNetwork** – Extract sub-network according to selected cluster.

  – **Dissimilarity***

  ∗ **Network based** – Compute selected dissimilarity matrix (d_1, d_2, d_3 or d_4) among vertices in cluster according to number of common neighbors. *Corrected Euclidean-like* d_5 and *Manhattan-like* d_6 dissimilarities can be computed as well [13]. The obtained matrix can be used further for hierarchical clustering procedure. You can include vertex v to its own neighborhood or not and display in report window only upper triangle / undirected or complete matrix /directed (if number of vertices is low).

  N_v is a set of input, output or all neighbors of vertex v; $+$ stands for symmetric sum, $\cup$ stands for set union and $\setminus$ stands for set difference; $|$ stands for set cardinality; 1st maxdegree and 2nd maxdegree are the largest degree and the second largest degree in network, respectively.

$$d_1(u,v) = \frac{|N_u + N_v|}{1\text{st maxdegree} + 2\text{nd maxdegree}}$$

$$
\begin{aligned}
d_2(u,v) &= \frac{|N_u + N_v|}{|N_u \cup N_v|} \\
d_3(u,v) &= \frac{|N_u + N_v|}{|N_u| + |N_v|} \\
d_4(u,v) &= \frac{\max(|N_u \setminus N_v|, |N_v \setminus N_u|)}{\max(|N_u|, |N_v|)} \\
d_5(u,v) &= \sqrt{\sum_{\substack{s=1 \\ s\neq u,v}}^{n} ((q_{us} - q_{vs})^2 + (q_{su} - q_{sv})^2) + p \cdot ((q_{uu} - q_{vv})^2 + (q_{uv} - q_{vu})^2)} \\
d_6(u,v) &= \sum_{\substack{s=1 \\ s\neq u,v}}^{n} (|q_{us} - q_{vs}| + |q_{su} - q_{sv}|) + p \cdot (|q_{uu} - q_{vv}| + |q_{uv} - q_{vu}|)
\end{aligned}
$$

Dissimilarities d_5 and d_6 are based on some matrix $\mathbf{Q} = [q_{uv}]$ on vertices – for example on adjacency matrix or on distance matrix. The parameter p is usually set to value 1 or 2. In the case $N_u = N_v = 0$ we set all dissimilarities d_1 - d_4 to 1.
If **Among all linked Vertices only** is checked dissimilarities are computed as line values of given network.

* **Vector based – Euclidean, Manhattan, Canberra, or (1-Cosine)/2** dissimilarities among Vectors determined by Cluster are computed as line values of given network.

– k**-Neighbors from Vertices in Cluster** – For each vertex in given Cluster compute distances
  * **Input** ...from all other vertices.
  * **Output** ...to all other vertices.
  * **All** ...from/to all other vertices (forget direction of lines).
    Vertices that cannot be reached from selected vertex are on distance 999999998. As result we get one Vector for each vertex in Cluster.

– **Distribution of Distances from Vertices in Cluster*** – Compute distribution of lengths of the shortest paths and average path length among all reachable pairs of vertices in network. Only distances from vertices selected by Cluster are computed.

– **Transform** – Transformations of network according to Cluster.
  * **Add Arcs from Vertex to Cluster** – add arcs from selected vertex to all vertices in Cluster.
  * **Add Arcs from Cluster to Vertex** – add arcs from all vertices in Cluster to selected vertex.

- **Network + Hierarchy** – Operations on Network and Hierarchy.
  - **Shrink Network** – Shrink network according to selected hierarchy. Nodes in hierarchy that are *Closed* are shrunk to new vertex. *Cut* nodes are shrunk to virtual vertex. *Border* nodes are not shrunk, but they are not visible. Vertices belonging to other nodes are left unchanged. Type of shrinking (blockmodel) can be selected in Options menu.
  - **Expand Reduction** – Restore original network from reduced network (hierarchical reduction!) and appropriate hierarchy (result is always undirected network).
- **VertexID + Partition** – Operations on Vertex ID and Partition.
  - **Extract SubVertexIDs** – Extract only identificators that satisfy given criterion (are in specified interval) determined by a Partition.
- **Partition + Permutation** – Operations on Partition and Permutation.
  - **Reorder Partition** – according to Permutation. The same result can be obtained using functional compositition **Permutation*Partition**: Let f be a permutation and g a partition. The result is new partition r defined in the following way: $r[v] = (f * g)[v] = g[f[v]]$.
  - **Functional Composition Partition*Permutation** – Let f be a partition and g a permutation. The result is new partition r defined in the following way: $r[v] = (f * g)[v] = g[f[v]]$.
- **Partition + Cluster** – Operations on Partition and Cluster.
  - **Binarize Partition** – Binarize Partition according to Cluster - make binary partition of the same dimension as the given partition, vertices that are in cluster numbers determined by the cluster will go to class 1 other to class 0. Note the exception: In this case cluster represents set of cluster numbers and not set of vertices numbers.
- **Vector + Partition** – Operations on Vector and Partition.
  - **Extract Subvector** – Extract SubVector from given Vector - criterion is class in the selected Partition.
  - **Shrink Vector** – Shrink vector values according to clusters of Partition to new Vector – adjusting Vector to shrunken network. When shrinking several values to one value, sum of values, mean, min, max or median value can be used.

  - **Functional Composition Partition*Vector** – Let f be a partition and g a vector. The result is new vector r defined in the following way: $r[v] = (f * g)[v] = g[f[v]]$.

- **Vector + Permutation** – Operations on Vector and Permutation.

  - **Reorder Vector** – according to Permutation. The same result can be obtained using functional composition **Permutation*Vector**: Let f be a permutation and g a vector. The result is new vector r defined in the following way: $r[v] = (f * g)[v] = g[f[v]]$.

3.6 Partition

Only Partition is needed as input.

- **Create Constant Partition** – Create constant partition of selected dimension. Default dimension is the size of selected network (if there is one in memory).
- **Create Identity Partition** – Create identity partition ($C[i] := i$).
- **Create Random Partition** – Create random one or two mode partition.
- **Binarize Partition** – Make binary (0-1) partition from selected partition.
- **Fuse Clusters** – Fuse selected cluster numbers to a new cluster.
- **Canonical Partition**

  - **with Vertex 1 in Cluster 1** – Transform partition to its canonical (unique) form (vertex 1 is always in cluster 1, the next vertex with smallest number that is not in the same cluster as vertex 1 is in cluster 2...).
  - **with Decreasing Frequencies** – Transform partition to its canonical (unique) form (in cluster 1 the old cluster with the highest frequency will be set, in cluster 2 the old cluster with the second highest frequency...).

- **Make Network** – Generate network from partition.

  - **Random Network** – Generate random network where degrees of vertices are determined using partition.

    * **Undirected** – partition gives degrees of vertices in undirected network.

    - ∗ **Input** – partition gives input degrees of vertices.
    - ∗ **Output** – partition gives output degrees of vertices.
  - **2-Mode Network** – Generate 2-mode network: first set consists of vertices ($v_1 \ldots v_n$), second set consists of clusters ($c_0 \ldots c_m$). If vertex i is in cluster j the line from v_i to c_j is generated. If option *Existing Clusters only* is selected only clusters containing at least one vertex are generated as vertices in the second set.

- **Copy to VertexID** – Convert partition clusters to vertex identificators.
- **Make Permutation** – Make permutation from selected partition. (first all vertices with the lowest class number, ...)
- **Make Cluster** – Transform partition to cluster.
  - **Vertices from Selected Clusters** – Put vertices from selected clusters in Partition to Cluster.
  - **Random Representatives of each Cluster** – Randomly select one vertex (representative) from each cluster in Partition. Store representatives to Cluster. Representatives can be later used as pivots in Pivot MDS.
- **Make Hierarchy** – Transform partition to hierarchy (nested or not).
- **Copy to Vector** – Copy partition to vector ($V[i] := C[i]$).
- **Count, Min-Max Vector** – info about cluster frequencies and minimum and maximum vector value according to given partition.
- **Info** – General information about partition. Sort vertices according to their class numbers (ascending or descending) to see the most important vertices. Frequency distribution of class numbers. Average, median and standard deviation of class numbers are also given.

3.7 Partitions

Operations on two partitions. Two partitions must be selected before performing operations.

- **Extract SubPartition (Second from First)** – Extract from first partition only vertices that satisfy criterion (are in specified interval) determined by second partition. This operation is useful when we have partition that actually saves some information about vertices (for example gender). When

you get (extract) some smaller part of the network (for example vertices that are on distances less than 3 from selected vertex), information about gender would be lost without performing the same operation (extraction) on partition.

- **Add (First+Second)** – Add two partitions (useful for example when combining Input and Output neighbors in acyclic networks).
- **Min (First, Second)** – Minimum of two partitions.
- **Max (First, Second)** – Maximum of two partitions.
- **Fuse Partitions** – Fuse two partitions – add second to the end of the first (useful for 2-mode networks).
- **Expand Partition** – Expand partition to higher (original) dimension.
  - **First according to Second (Shrink)** – Expand first partition according to shrinking determined by second partition.
  - **Insert First into Second according to Third (Extract)** – The current partition was obtained by extracting selected classes defined by the second partition from the first partition. This sub-partition was modified. Using this operation we can insert this modified sub-partition back to the first partition.
- **Intersection of Partitions** – intersection of selected partitions.
- **Cover with** – Let p be a partition, b a binary partition, and c selected cluster number. Result is new partition q determined in the following way:
  if $b(v) = 0$ **then** $q(v) = p(v)$ **else** $q(v) = c$.
- **Merge Partitions** – Let p and q be partitions and b a binary partition. Result is new partition s determined in the following way:
  if $b(v) = 0$ **then** $s(v) = p(v)$ **else** $s(v) = q(v)$.
- **Make Random Network** – generate random network with input degrees determined by the first and output degrees by the second partition.
- **Functional Composition First*Second** – Let f and g be two partitions. The result is new partition r defined in the following way: $r[v] = (f * g)[v] = g[f[v]]$.
- **Info** – Bivariate statistical measures between selected partitions:

  - **Cramer's V, Rajski, Adjusted Rand Index** – Report contingency table, compute Cramer's V, Rajski coefficients, and Adjusted Rand Index.
  - **Spearman Rank** correlation coefficient.

3.8 Vector

Operations using vector.

- **Create Constant Vector** – Create constant vector (vector with all values equal to selected value) of selected dimension. Default dimension is the size of selected network (if there is one in memory).
- **Create Scalar** – Create Scalar (Vector with dimension 1) from given Vector.
- **Make Partition** – Convert vector to partition:
  - **by Intervals** – according to selected dividing numbers in vector vertices get appropriate class numbers. Intervals can be given by:
    * **First Threshold and Step** – Select first threshold and step in which to increase threshold.
    * **Selected Thresholds** – Select all thresholds or number of classes (#) in advance.
  - **Copy to Partition by Truncating (Abs)** – partition is absolute and truncated vector.
- **Make Permutation** – Convert vector to permutation - sorting permutation.
- **Make Cluster** – Convert vector to cluster – select vertices with vector values lower/higher than selected value.
- **Make 2-Mode Network** – Convert vector to 2-mode network (row or col).
- **Transform** – Transformations of given vector:
  - **Multiply by** a constant.
  - **Add Constant** to vector values.
  - **Absolute** values of its elements.
  - **Absolute + Sqrt** – square root of its absolute components.
  - **Truncate** – truncated vector.
  - **Exp** – exponential of vector.

  - **Ln** – natural logarithm of vector.
  - **Power** – selected power of vector.
  - **Normalize**
    - **Sum** – normalize so that the sum of elements is 1.
    - **Max** – normalize so that the maximum element will have value 1.
    - **Standardize** – normalize so that arithmetic mean will be 0 and standard deviation 1.
  - **Invert** – inverse values of vector.
  - **Cumulatives** – new vector u with cumulatives of vector v is computed in the following way: $u_1 = v_1, u_i = u_{i-1} + v_i, i > 1$.

- **Missing Values** – Select how values larger than 999.999.997 should be treated in operations on Vector: as valid values or as missing values?

- **Info** – General information about vector: Vertices sorted according to their values, average, median, standard deviation and frequency distribution of vector values into given number of classes (# – number of classes or selected dividing values can be given).

3.9 Vectors

Operations on two vectors. Two vectors must be selected before performing operations. Vectors must be of the same dimension, or one vector can be scalar.

- **Add (First+Second)** – sum of selected vectors.

- **Subtract (First-Second)** – difference of selected vectors.

- **Multiply (First*Second)** – product of selected vectors.

- **Divide (First/Second)** – division of selected vectors.

- **Min (First, Second)** – smaller elements in selected vectors.

- **Max (First, Second)** – bigger elements in selected vectors.

- **Fuse Vectors** – fusion of vectors.

- **Linear Regression** – fit the two vectors using linear regression. Results are: regression line, linear estimates of second vector and corresponding errors.

- **Transform** – two vectors to another two vectors:

  - **Cartesian → Polar** – First vector must contain x-coordinates second y-coordinates. Results are: vector containing polar radius and vector containing polar angles in degrees.
  - **Polar → Cartesian** – First vector must contain polar radius second polar angles in degrees. Results are: vector containing x-coordinates and vector containing y-coordinates.

  Results can be (de)normalized to enable direct use in Draw window.

- **Missing Values** – Select how values larger than 999.999.997 should be treated in operations on Vectors: as valid values or as missing values?

- **Info** – Pearson correlation coefficient between selected vectors.

3.10 Permutation

Only permutation is needed as input.

- **Create Identity Permutation** – Create identity permutation of selected dimension. Default dimension is the size of selected network (if there is one in memory).

- **Create Random Permutation** – Create random 1 or 2-mode permutation of selected dimension. Default dimension is the size of selected network (if there is one in memory).

- **Inverse Permutation** – Create inverse permutation of selected permutation.

- **Mirror Permutation** – Create mirroring permutation of selected permutation (sort in opposite direction).

- **Make Partition** – Create partition from permutation.

  - **Into Given Number of Clusters** – Create partition into given number of clusters.
  - **Orbits** – Create partition according to orbits in permutation.

- **Copy to Vector** – Copy permutation to vector.

- **Info** – Check if permutation is valid and return number of orbits.

3.11 Permutations

Operation on two permutations.

- **Fuse Permuations** – Fuse two permutations – add second to the end of the first (useful for 2-mode networks).
- **Functional Composition First*Second** – Let f and g be two permutations. The result is new permutation r defined in the following way: $r[v] = (f * g)[v] = g[f[v]]$. If dimensions of permutations are different, result is a partition.

3.12 Cluster

Only cluster is needed as input.

- **Create Empty Cluster** – Create cluster without vertices.
- **Create Complete Cluster** – Create cluster with values 1..n.
- **Create Random Cluster** – Create random cluster with selected maximum value and number of values.
- **Make Partition** – Transform cluster to partition.
- **Info** – General information about cluster: dimension, the lowest and the highest value.

3.13 Hierarchy

Only hierarchy is needed as input.

- **Extract Cluster** – Extract cluster from hierarchy - the cluster is whole subtree of selected node in hierarchy.
- **Make Network** – Converts hierarchy to network (use it for example to draw hierarchy – drawing by layers). Closed nodes are also taken into account.
- **Make Partition** – Converts hierarchy to partition (according to closed nodes).
- **Make Permutation** – Converts hierarchy to permutation.
- **Info** – General information about hierarchy. Operation is possible only if node numbers are integers. It returns number of vertices in nodes of hierarchy (on first level).

3.14 Options

- **Read - Write**
  - **Threshold** – Value of line must be higher (absolutely) than the given threshold to generate line between two vertices.
  - **Large Network (Vertices)** – Select the threshold number of vertices that defines very large network. For such networks Pajek (for some operations), asks if the old network can be destroyed and replaced by the network that is obtained as a result. In this way we can spare a lot of memory. This is also threshold value for which quantiles in **Vector/Info** are computed (for large vectors where several values are equal this operations might be slow).
  - **x / 0 =** Specify the result when dividing nonzero value with 0.
  - **0 / 0 =** Specify the result when dividing 0 with 0.
  - **Ignore Missing Values in menu Vector and Vectors** – When doing operations in menu Vector or Vectors or computing descriptive statistics in Vector/Info treat missing values (values larger that 999999997) as valid numbers.
  - **Use Scientific format for small/large real numbers** – if numbers are too big or too small they are written in scientific format (e.g. 2.4E+015 or 2.4E-015).
  - **Save Files as Unicode UTF8** – Instead of saving Pajek objects and delimited files as ASCII files, save files as Unicode UTF8 files.
    - **With BOM** – Saving as UTF8 files with or without BOM.

    Some facts about saving specific files to UTF8:
    - Pajek needs BOM to recognize UTF8 input files. In case of networks containing labels with nonenglish letters saving as UTF8 + BOM is recommended.
    - Exports to SVG and X3D are always generated in format UTF8 without BOM (they are **xml** based - format is determined explicitelly in the first line of the file).
    - Reports are always saved in format UTF8 (with our without BOM).
    - For *Delimited Files* BOM is selected/omitted in **Tools / Delimited File Properties**.
  - **Auto Report** – Automatically report all text results to file rep1.rep.
  - **Read - Save vertices labels** – Read / Save also labels, coordinates, and other descriptions of vertices or not. If vertices labels are not read

(recommended if network is very large and vertices labels are long) they can be imported later from input file using
Network / Transform / Add / Vertex Labels / from File.

- **Save coordinates of vertices** – Save coordinates of vertices to network file (or not).
- **Save complete vertex description** – When saving network to output file for each vertex complete description will be written, even if consequent vertices have the same descriptions (e.g. shapes, time intervals...).
- **Check equality of vertex descriptions by reading** – Enables users to speed up reading large network files according to descriptions of vertices: Check this option to save space when exactly the same descriptions of vertices are repeated often (e.g. shapes of vertices). Uncheck this option to save time when there are several different desctriptions of vertices in input file (e.g. time stamps in temporal networks).
- **Check equality of line descriptions by reading** – Enables users to speed up reading large network files according to descriptions of lines: Check this option to save space when exactly the same descriptions of lines are repeated often (e.g. line pattern Dots/Solid). Uncheck this option to save time when there are several different desctriptions of lines in input file (e.g. time stamps in temporal networks).
- **Max. vertices to draw** – Maximum number of vertices in network to allow drawing (to prevent long waiting).
- **Ore: Different relations for male and female links** – When reading genealogy as Ore graph generate 2 different types of arcs: arc with relation number 1 (also value 1) represents (god)father to child relation, arc with with relation number 2 (also value 2) represents (god)mother to child relation.
- **Ore: Generate Godparent relation** – When reading genealogy as Ore graph generate also godparent relation (relation number 4) or godfather (relation number 4) and godmother (relation number 5) relations.
- **GEDCOM – Pgraph** – Use pgraph format (nodes are couples or individuals) when reading genealogies (D. R. White), otherwise nodes are only individuals.
- **Bipartite Pgraph** – Generate bipartite pgraph that has squares for marriages, triangles and circles for individuals.

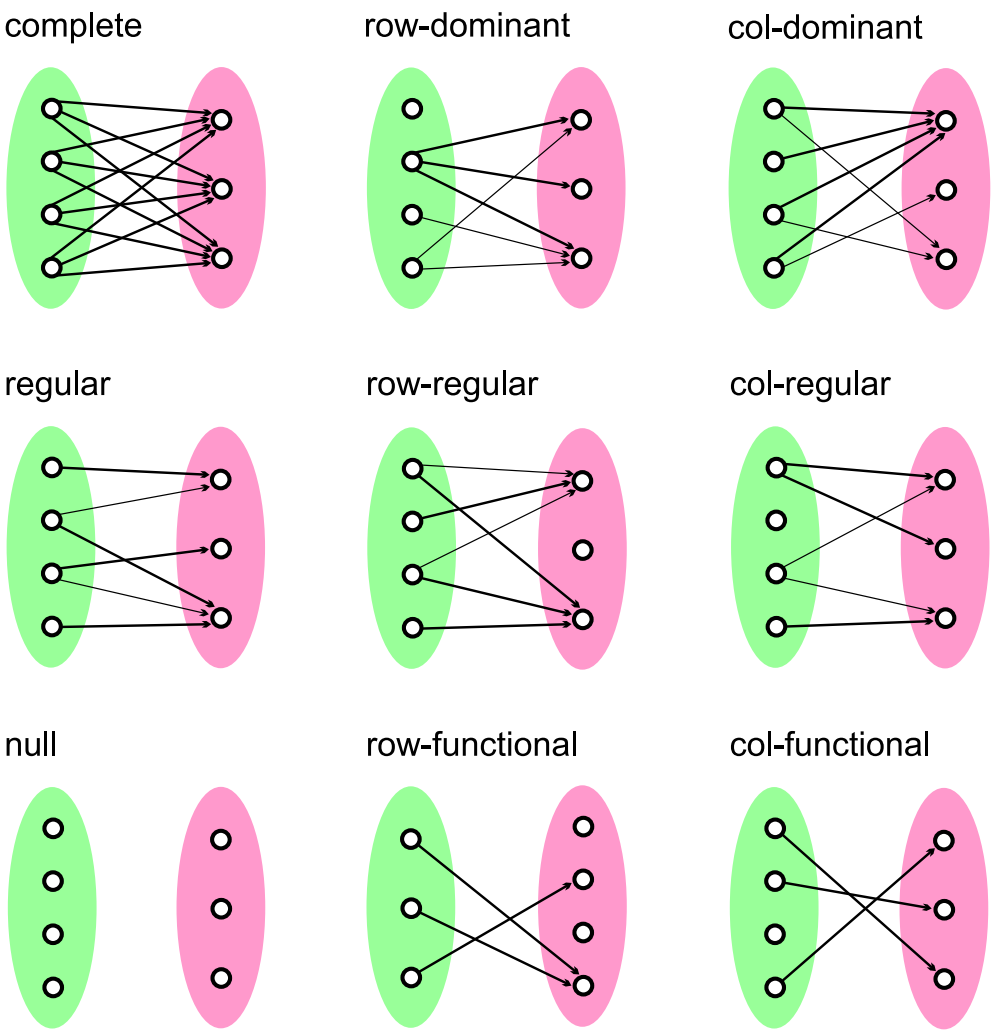


Figure 14: Generalized block types

  - **Pgraph+labels** – Attach also labels of lines to pgraph, when reading GEDCOM file.

- **Background Colors** - Select colors for different backgrounds and color for panels.

- **Font (Typeface, Style, Size, Color)**

  - **Select Proportional Font** – Select proportional font for displaying Unicode characters (e.g. in Draw window).

  - **Select Monospaced Font** – Select non-proportional font for displaying Unicode characters (e.g. in Report window).
  - **Default Fonts** – Use *Arial Unicode MS (bold, Maroon)* for proportional, and *Courier New (Bold, Maroon)* for monospaced font.

- **Blockmodel** – Select type of blockmodel for shrinking. Possibilities are:
  - 0..Min Number of Links
  - 1..Null
  - 2..Complete
  - 3..Row-Dominant
  - 4..Col-Dominant
  - 5..Row-Regular
  - 6..Col-Regular
  - 7..Regular
  - 8..Row-Functional
  - 9..Col-Functional
  - 10..Degree Density

  Look in Batagelj [7] and Doreian, Batagelj, Ferligoj [35].

- **Hide Drawing Icons** – Hide Drawing icons in main Pajek window.
- **Use Old Style Dialogs** – If Windows 7 have problems with opening/saving files check this option.
- **Refresh Objects** – Refresh all objects that are loaded in memory at the moment in Pajek.

3.15 Info

- **Child Windows** - Manually restoring up / closing child Pajek Info windows (Network, Partition, Vector, Permutation, Cluster, Hierarchy, Draw) and showing / hiding Report window.
- **Memory** – Information about available and used physical memory.
- **About** – Information about Pajek version, authors, copyrights...

3.16 Tools

- **Pajek** – direct calling ordinary Pajek after extracting some smaller subnetwork using PajekXXL. Also the right vertex labels and other vertex descriptions (e.g. shapes, coordinates) can be attached to network that is sent to Pajek. In addition to extracted subnetwork, corresponding Partition and/or Vector can be sent from PajekXXL directly to Pajek too.

- **R**

  - **Send to R** – Call statistical package R [58] with one vector/network, vectors/networks selected by cluster or all currently available vectors and/or networks.
  - **Locate R** – Locate position of statistical program R (Rgui.exe or Rterm.exe) on the disk.

- **SPSS**

  - **Send to SPSS** – Call statistical package SPSS with one partition, vector or network, partitions/vectors selected by cluster or all currently available partitions and vectors.
  - **Locate SPSS** – Locate position of statistical program SPSS (runsyntx.exe) on the disk.

- **Excel**

  - **Send to Excel** – Call spreadsheet program Excel with Network, Partitions, and/or Vectors.
  - **Locate Excel** – Locate position of program Excel (Excel.exe) on the disk.

- **Export to Delimited File** – Export Networks, Partitions, and Vectors to delimited file. This file can then be imported to other programs, like statistical packages, Excel...

- **Delimited File Properties** – Determine properties of generated delimited file (properties are used also when exporting to Excel).

  - **Delimiter** – Select delimiter to be used when exporting to delimited files or Excel. Delimiter can be *Tab*, *Semicolon*, *Comma* or *Space*. Different delimiters can be used for *ASCII* and *UTF8* files:

    - ∗ **ASCII** - Select delimiter to be used when **Save Files as Unicode UTF8 with BOM** is not checked in **Options / Read - Write**. Default delimiter for these kind of files is *Tab*.
    - ∗ **UTF8** - Select delimiter to be used when **Save Files as Unicode UTF8 with BOM** is checked in **Options / Read - Write**. Default delimiter for these kind of files is *Semicolon*.
  - **Add BOM to UTF8 files** – Add BOM (Byte Order Mark) at the beginning of delimited file when exporting as UTF8 files.

- **Web Browser** – Select which web browser to open when (in Draw window) clicking on vertex with *Shift* and *Right mouse button*.
- **Inkscape** – Locate where program **Inkscape** is installed (exports of matrices and dendrograms to SVG and PDF are partially created by Inkscape). Information which version of Inkscape (lower than 1.0 or 1.0 and higher) is installed must also be provided.
- **Add Program** – add new executable program with specified parameters to the tools menu.
- **Edit Parameters** – edit parameters of selected external program.
- **Remove Program** – remove selected external program from the tools menu.

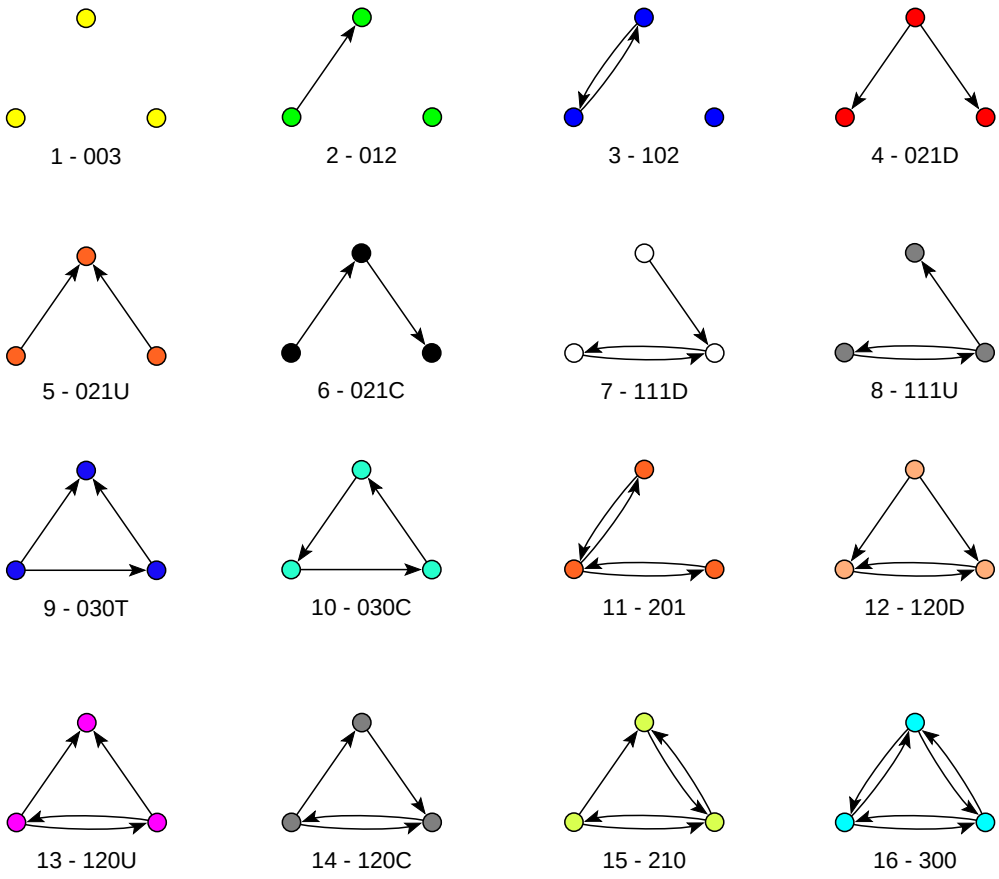


Figure 15: All different triads.

4 Draw Window Tools

4.1 Main Window Draw Tool

- **Network** - Draw Network. A new window is open, where a new menu appears. You can edit network by hand (move vertices using left mouse button), select a part of the picture (using right mouse button and select the area), edit lines that belong to selected vertex by clicking on vertex using right mouse button, spin picture using keys X, Y, Z, S, x, y, z, s. Description of Draw window menu:

- **Network + First Partition** – Similar to **Draw / Network**. Colors of vertices represent the classes in selected partition. Additionally you can put selected vertex or selected vertices into given class in partition (classes are shown using different colors) by clicking on middle mouse button (or Shift+left button) (increment class), or together with Alt (or Alt+left button) - decrement class number. In Figure 20 on page 106 you can see which color represents selected class.

  Some additional menu items that were already described appeared (you can draw network according to layers from partition and optimise energy using fixed vertices determined using partition). It is also possible to move the selected class (by clicking close to vertex from that partition).

- **Network + First Vector** – Sizes of vertices are determined using selected vector.

- **Network + First Vector + Second Vector** – Sizes of vertices are determined using selected two vectors (first for width second for height).

- **Network + First Partition + First Vector** – Colors of vertices are determined using selected partition, sizes of vertices are determined using selected vector.

- **Network + First Partition + First Vector + Second Vector** – Colors of vertices are determined using selected partition, sizes of vertices are determined using selected two vectors (first for width second for height).

- **Network + Create Null Partition** – Create null partition and draw network using it.

4.2 Layout

Generate layout of the network.

- **Circular** – Position vertices on circle
  1. **Original** – in order determined by the network.
  2. **using Permutation** – in order determined by current permutation.
  3. **using Partition** – Create separate circles for clusters in selected partition.
     (a) **Different Centers** – Center of the circle is determined by the arithmetic mean of positions of vertices in the cluster.
     (b) **Concentric** – All circles are concentric – center is in the middle of the Draw window.
  4. **Random** – in random order.
- **Energy** – Automatic layout generation using spring embedders.
  1. **Kamada-Kawai** – algorithm for automatic layout generation in the plane.
     (a) **Free** – Every position in the plane is possible.
     (b) **Separate Components** – Optimize each component separatelly and tile components at the end.
     (c) **Optimize inside Clusters only** – Optimize only subnetworks induced by clusters (e.g. communities).
     (d) **Fix first and last** – First and last vertex are fixed in opposite corners.
     (e) **Fix One vertex in the middle** - Select vertex which will be fixed in the middle of the picture.
     (f) **Selected group only** – Only selected part of the picture is taking into account during optimisation.
     (g) **Fix selected vertices** - Selected vertices (from partition) are fixed on given positions). This item is visible only if Draw partition is active.
  2. **Fruchterman-Reingold** – another algorithm for automatic layout generation (faster than Kamada-Kawai).
     (a) **2D** – optimisation in plane.
     (b) **3D** – optimisation in space.
     (c) **Factor** – Input factor for optimal distance among vertices when using Fruchterman Reingold optimisation.
  3. **Starting positions** – for energy drawing (random, circular, given positions on plane xy, given z coordinates).

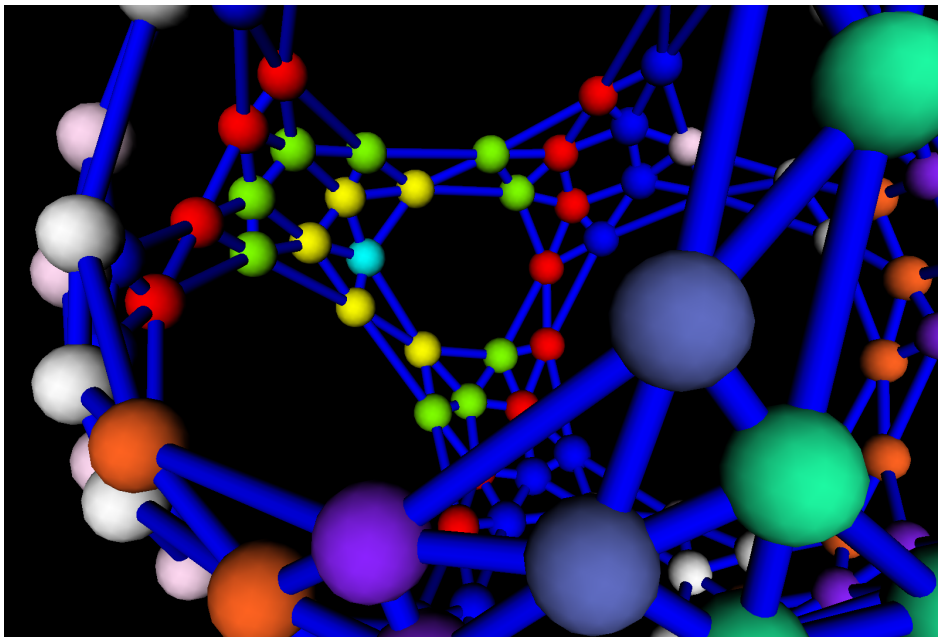


Figure 16: VRML display of layout determined by eigenvectors

- **Pivot MDS** – Automatic layout generation with Pivot MDS (Brandes and Pich). Algorithm can produce 2D and 3D layouts. Pivots can be selected by random or provided in advance using Cluster. Algorithm is faster than spring embedders and can be used for larger networks (some 100's thousands of vertices).

- **VOS Mapping** – Obtaining 2D ot 3D layouts using Visualization of Similarities Mapping algorithm (Van Eck and Waltman). Algorithm produces nice layouts for dense weighted networks where line values represent similarities. For networks with more than 20.000 vertices a lot of memory is needed and it can be very slow. Two parameters are used to control the optimization: *Number of Restarts* of optimization (usually only one restart is enough) and *Maximum Number of Iterations in each Restart* (usually 100 iterations are enough).

- **EigenValues** – Drawing using eigenvalues/eigenvectors (Lanczos algorithm). Values of lines can be taken into account or not.

    1. **1 1 1** – Select 2 or 3 eigenvalues and algorithm will compute corresponding eigenvectors. Eigenvalues may be multiple so there are many possibilities. Some examples

(a) **1 1 1** – compute 3 eigenvectors that correspond to the first eigenvalue

(b) **1 1 2** – compute 2 eigenvectors that correspond to the first eigenvalue and 1 that correspond to the second

(c) **1 2 2** – compute 1 eigenvector that correspond to the first eigenvalue and 2 that correspond to the second

(d) **1 2 3** – compute 1 eigenvector that correspond to the first, 1 that correspond to the second and 1 that correspond to the third eigenvalue

(e) **1 1** – compute 2 eigenvectors that correspond to the first eigenvalue (2D picture)

- **Tile Components** – Tile weakly connected components in a plane.

4.3 Layers

Visible only if Draw partition is active. Draw in layers according to partition.

- **Type of Layout** – Select type of picture (2D – layers in y direction, or 3D – layers in z direction). According to that, appropriate menu appears.
- **In y direction** – Draw vertices in layers (y coordinate) inside layers draw vertices centered-equidistantly (x coordinate), z coordinate is 0.5 for all vertices.
- **In y direction+random in x** – Same as first option, only vertices are put on layers in random order not according to vertex numbers.
- **In z direction** – Draw layers in z direction, live x and y coordinates as they are.
- **In z direction + random in xy** – Draw layers in z direction, x and y coordinates get random values
- **Averaging x coordinate** – Use it after vertices are put on layers in 2D. Iterativelly compute average x-coordinate of all neighbors and normalize. Good approximation of global picture, but vertices are put to close to each other. Use it on all vertices or only on selected one.
- **Averaging x and y coordinates** – Use it after vertices are put on layers in 3D. Iterativelly compute average x and y coordinates of all neighbors and normalize. Good approximation of global picture, but vertices are put to close to each other. Use it on all vertices or only on selected one.

- **Tile in x direction** – After averaging x coordinate vertices are put to close to each other, so using this option vertices are repositioned to minimal distance described in resolution.

- **Tile in xy plane** – Same as previous, only this option is used when drawing 3D pictures.

- **Optimize layers in x direction** – Optimize layout in layers using minimization of the total length of lines.

    1. Forward – go from first to last layer. In the current layer optimize only layers having numbers equal or one smaller as the current layer number.
    2. Backward – go from last to first layer. In the current layer optimize only layers having numbers equal or one larger as the current layer number.
    3. Complete – go from first to last layer. In the current layer optimize only layers having numbers equal or one smaller or one larger as the current layer number.

- **Optimize layers in xy plane** – Same as previous, only this option is used when drawing 3D pictures.

- **Resolution** – How many additional positions are available on layers. Works only if Pgraph is selected. The higher the resolution, the better is the result of optimization, but also slower.

4.4 GraphOnly

Show complete picture without labels and arrows.

4.5 Previous

Draw previous network, and/or partition and/or vector which are loaded in **Pajek** (depending on selection in Options/PreviousNext/Apply to).

4.6 Redraw

Redraw network.

4.7 Next

Draw next network, and/or partition and/or vector which are loaded in **Pajek** (depending on selection in Options/ PreviousNext/ Apply to).

4.8 ZoomOut

Zoom out (visible only when zooming in layout).

4.9 Options

Additional options for picture layout.

- **Transform** – Transformations of picture.
  1. **Fit area**
     (a) **max(x), max(y), max(z)** – Draw picture as big as possible to fit area (resize each coordinate to fit in picture independently).
     (b) **max(x,y,z)** – Resize to fit in area but keep real proportions (resize according to largest distance in all three coordinates, e.g. molecule).
  2. **Resize** – Resize picture (or selected part of it) in all three directions for selected factor.
  3. **Translate** – Translate picture (or selected part of it) in space.
  4. **Reflect y axis** – Reflect picture (or selected part of it) around y axis.
  5. **Set z-coordinate to 0.5** – make layout in plane.
  6. **Rotate 2D** – Rotate picture (or selected part of it) in xy plane.
  7. **FishEye** – FishEye transformation (cartesian or polar). If no vertex is selected the middle point of the window (or selected part of it) is used as a focus, otherwise selected vertex is used as a focus.
  8. **Resize Clusters Area** – Resize (expand or shrink) clusters around its center for selected factor.
- **Values of lines** – Meaning of values of lines during energy drawing or eigenvectors computing (no meaning, similarities, dissimilarities (distances)).
- **Mark Vertices Using** – Labels can be marked using
  1. **Labels**
  2. **Numbers**

3. **No Labels**
4. **No Labels no Arrows**
5. **Mark Cluster Only** – only vertices belonging to the current Cluster are labeled in the layout.
6. **Vector Values** – vertices are additionally marked with one or two vector values if one or two vectors of the same size are selected.
7. **Clusters of Second Partition** – vertices are additionally marked with cluster numbers if second partition of the same size is selected.
8. **Cluster Symbols of Second Partition** – vertices are additionally marked with cluster symbols if second partition of the same size is selected. If layout is exported to SVG or EPS symbols are written in the middle of the vertex (in EPS only ASCII symbols can be used).
9. **Labels as Tooltips** – vertex labels (and additional properties) are displayed as a tooltip when mouse cursor touches the vertex (and not as a text next to vertex).
10. **Labels Centered** – Display labels of vertices centered in the middle of vertices.

- **Lines** – Select the way the lines are drawn:

  1. **Draw Lines**
     (a) **Edges** – draw edges or not
     (b) **Arcs** – draw arcs or not
     (c) **Relations** – draw all lines (leave empty string) or just lines belonging to selected relations, e.g. 1-3,6,10-15.
  2. **Mark Lines**
     (a) **No** – do not mark lines
     (b) **with Labels**
     (c) **with Values**
  3. **Different Widths** – if checked, the width of the lines will be determined by their line values.
  4. **GreyScale** – if checked, the color in grayscale of lines will be determined by their line values.

- **Size** – Determine size of vertices, width of vertices border, width of lines, size of arrows, size of symbols and font or turn them to default values.
  Sizes of vertices can be set to autosize (0-average). They can be read from

input file (x_fact and y_fact) or determined by one vector (currently selected) or the two currently selected vectors. FontSize can be proportional to values stored in third partition.

- **Colors** – Determine color of background, vertices, border of vertices, edges, arcs and font (of vertices labels and lines labels) or turn them to default values. Colors of symbols can be determined using the third partition. Colors of edges/arcs can represent relation number of lines. You can select which color should represent given class using **Relation Colors**. You can also use colors of vertices (**ic Red**), border of vertices (**bc Blue**), arcs and edges (**c Green**) as defined on input file (see Figure 19, page 105). FontColor can be determined by values stored in second partition. Additionally to predefined colors it is possible to specify colors using RGB (e.g. RGBFF0000, or RGB(1,0,0)) and CMYK (e.g. CMYK00FF0000, or CMYK(0,1,0,0)) format. Note that there should be no spaces in string defining a color.

  Example NET file:

```
*Vertices      9
     1 "a" ic Pink             bc Black
     2 "b" ic CMYK(0,0,1,0.0)  bc CMYKFF000000
     3 "c" ic Cyan             bc Yellow
     4 "d" ic Purple           bc Orange
     5 "e" ic Orange           bc Brown
     6 "f" ic Magenta          bc Green
     7 "g" ic Brown            bc Magenta
     8 "h" ic RGB(1,0,0)       bc Blue
     9 "i" ic Green            bc Magenta
*Arcs
      1       2       1 c RGB0000FF
      2       3       1 c Red
      3       4       1 c Black
      4       5       1 c Yellow
      5       6       1 c Gray
      6       7       1 c Cyan
      7       8       1 c Magenta
      8       9       1 c Purple
      9       1       1 c Brown
```

- **Symbols for Partition Clusters** – Assign symbols to partition clusters. Symbols can be any Unicode characters.

- **Layout** – Layout options

  1. **Redraw** – Redraw current network:
     - **during Vertex Moving** - while vertices are moving;
     - **after Vertex Moving** - after vertices were moved;
     - **on Layout Paint** - if Draw window was paint;

- **on Layout Resized** - if Draw window was resized.
- **Rendering Refresh Period** - period in which network rendering is refreshed while drawing (large) network. In this case **Wait** notification is also visible in top roght corner while drawing network. If we select 0, there will be no refreshing and no notification. Note that some options work only if no refreshing is selected (e.g. rotating layout by pressing keys 'x', 'y', 'z', or 's', using ScrollBars).

2. **Real xy proportions** – The draw window has always square shape or not.
3. **Arrows in the Middle** – Draw arrows in the middle of lines – not at terminal vertices.
4. **Size of vertex 0** – How to handle vertices of size 0 (size of vertex is determined in input file or using vector)?
   (a) **Hide vertex** – vertices of size 0 are shown or not.
   (b) **Hide attached lines** – lines with one end in vertex of size 0 are shown or not.
5. **Decimal Places** – How many decimal places to use when marking vertices using vectors.
6. **Show SubLabel** – Select the position of the vertices sublabel that is shown in network layouts.
7. **Labels with Transparent Background** – Select displaying labels with transparent background.

- **ScrollBar On/Off** – Show/Hide the scrollbars in the top left corner of Draw window. When part of picture is selected, scrollbar is used for moving. When whole picture is selected, scrollbar is used for spinning (like pressing keys X, Y, Z, x, y, z – spinning around axis defined by the key, and S – spin around selected normal)

- **Interrupt** – Interrupt period during optimisation (stop every ? second, or not)

- **Previous/Next** – Select parameters when using Previous or Next commands for drawing sequence of networks in draw window:

  1. **Max. number** – How many networks to show in sequence. If the number is higher than number of existing networks the sequence will stop earlier.
  2. **Seconds to wait** – Seconds to wait between the two layouts.

3. **Optimize Layouts** – Optimize the current layout or not. If the sequence of networks is obtained from the same network, it is useful to choose Energy/Starting Positions/ Given xy to start optimization with existing coordinates.

   (a) **Kamada-Kawai** – Optimize the current layout using Kamada-Kawai algorithm.

   (b) **2D Frucht. Rein.** – Optimize the current layout using 2D Frucht. Rein. algorithm.

   (c) **3D Frucht. Rein.** – Optimize the current layout using 3D Frucht. Rein. algorithm.

   (d) **No** – Do not optimize the layout, just show the picture.

4. **Apply to** – Which object (Network, Partition, Vector) will change when Previous or Next is selected:

   (a) **Network** – The previous / next network in memory is drawn. If Draw/Network+Partition is selected and the new network matches in dimension with selected partition, the same partition will determine colors of vertices of the new network. If Draw/Network+Vector is selected and the new network matches in dimension with selected vector, the same vector will determine sizes of vertices of the new network.
   Option can be used to show several networks of equal size using the same partition/vector.

   (b) **Partition** – The previous / next partition in memory is selected. If Draw/Network+-Partition is selected: The same network is drawn using previous / next partition (network and partition must match in size).
   Option can be used to show several partitions of selected network.

   (c) **Vector** – The previous / next vector in memory is selected. If Draw/Network+Vector is selected: The same network is drawn using previous / next vector (network and vector must match in size).
   Option can be used to show several vectors of selected network.

   By checking several objects (Network, Partition, Vector) at the same time previous / next networks will be drawn using previous / next partitions and (or) vectors at the same time. All consequent selected objects must match in size.

4.10 Export

Export layout of the network to one of the following two or three dimensional formats:

- **2D** – two dimensional exports:
  - **EPS/PS** – Export to EPS format (with or without Clip, or WYSIWYG [What You See Is What You Get – exported EPS picture is similar to picture in Draw window – except that colors are Black/White or color determined by partition]). **PS** – Export to PS format (similar to EPS but without header).
  - **SVG** – Export to SVG (Scalable Vector Graphics) format. Additional controls over layout can be included in SVG or HTML. Linear or radial gradients (continuously smooth color transitions from one color to another) can be selected as well – up to three background colors can be selected in Export/Options window.
    1. **General** – Export to SVG without possibility to choose parts of the picture.
    2. **Labels/Arcs/Edges** – Export with possibility to turn labels, arcs and/or edges on/off.
    3. **Partition** – Export to SVG using one or two partitions. If network is drawn using a partition and second partition is not selected then first partition determines clusters and colors. But, if two partitions are selected and second partition is not used for symbols or label colors then first partition will determine colors, the second clusters.
       (a) **Classes** – User can turn selected classes and lines among classes on/off.
       (b) **Classes with semi-lines** – User can turn selected classes on/off. Lines among classes are drawn as semi-lines.
       (c) **Nested Classes** – Upper classes are nested in lower – whenever selected class is turned on all higher classes are turned on too, and all lower classes are turned off (suitable for showing cores, for example).
    4. **Line Values** – Export to SVG using values of lines. Threshold values or number of classes must be given. If you input #n, n classes of equal size will be generated. According to obtained thresholds, subsets of lines (and incident vertices) are defined and can be turned on/off inside web browser.

(a) **Classes** – User can turn lines of selected value and incident vertices on/off.

(b) **Nested classes** – User can turn lines of selected value or higher (lower) and incident vertices on (off).

(c) **Options** – Additional options to emphasize the values of lines using some visual properties.
Different Colors – Subsets of lines are drawn using colors which are used for partitioning vertices too.
Using GreyScale – The darkness of a line corresponds to its value.
Different Widths – The width of a line corresponds to its value.

5. **Multiple Relations Network** – Export multiple relations network to SVG. User can show/hide one or more relations in the layout.
6. **Current and all Subsequent** – If checked the current network and all subsequent networks will be exported to SVG. For each network separate html file is generated. Files are given the following names: file0001.htm, file0002.htm, ..., file9999.htm. Generated html files get additional links (Previous/Next) to transition among them. If also next partitions / vectors fit in dimension to dimension of networks, partitions will determine color of vertices, vectors will determine sizes of vertices. *Subsequent* is applied to any combination of [Network, Partition, Vector] (one of the three objects only, any pair of them or all three of them) according to selection in *Options/Previous/Next/ Apply to* in Draw window.

– **JPEG** – Export to JPEG/JPG format. Compression quality (0-100) and greyscale can be selected.

– **Bitmap** – Export to Windows bitmap (bmp) format.

– **VOSviewer** – Call VOSviewer (1.5.2 or higher) with Pajek Network, Partition and/or Vector.

- **3D** – three dimensional exports:

  – **X3D** – Export to X3D (XML based 3D computer graphics, the successor of VRML) format. X3D format can be imported to `https://www.shapeways.com/` for 3D printing.

  – **Kinemages** – Export to Kinemages format with balls or labels. You need Mage or King viewer to watch it. A free copy of the Mage software can be downloaded from its site [59].

1. **Current Network Only** – Export only current network to Kinemages. Two partitions defined by Partitions menu can be used - one for generations one for colors.
2. **Current and all Subsequent** – Export current network and all subsequent networks (use commands `KINEMAGE/Next` or `Ctrl N` in Mage). If also next partitions/vectors fit in dimension to dimension of networks, partitions will determine color of vertices, vectors will determine sizes of vertices. *Subsequent* is applied to any combination of [Network, Partition, Vector] (one of the three objects only, any pair of them or all three of them) according to selection in *Options/Previous/Next/ Apply to* in Draw window.
3. **Multiple Relations Network** – Export with possibility to hide / show selected relations.

– **VRML** – Export to VRML (Virtual Reality) format. For examining it you need a VRML viewer such as Cortona [29] or (older) Cosmo player [30].

– **MDL MOL file** – Export to MDL Molfile format. You need Chime plugin (Chemscape Chime) for Netscape to explore it [52].

- **Options** – EPS, SVG, X3D and VRML default options (see section on Exports to EPS/SVG/X3D/VRML).

- **Append to Pajek Project File** – Add current network to the end of selected project file (project file is formatted in the right way to be imported to program **PajekToSvgAnim**).

  – **Select file** – Select the project file.

  – **Append** – Append to the project file.

  Useful comment: **Macro / Repeat Last Command** can be used to send a sequence of networks to the selected project file (instead of adding networks manually one by one).

4.11 Spin

- **Spin around** – Spin network around selected normal.

- **Perspective** – Distant vertices are drawn smaller (or not).

- **Normal** – Normal vector to spin around.

- **Step in degrees** – Step in degrees when showing rotation.

4.12 Move

Give additional constraints on hand vertex moving:

- **Fix** – Fix (do not allow) moving in x or y direction, or do not allow changing distance from center (circulating).
- **Grid** – Define (x, y) positions on grid, these become feasible positions for vertices during moving by hand.
- **Circle** – Define (x, y) positions on concentric circles, these become feasible positions for vertices during moving by hand.
- **Grasp** – Determine which additional vertices are moving when clicking with left mouse button close to vertex in given class. Vertices that will be moved are in the:
  1. **Closest Class Only**
  2. **Closest Class and Higher**
  3. **Closest Class and Lower**

4.13 Info

Select aesthetic properties of the current layout to compute:

- Closest vertices
- Smallest angle
- Shortest/Longest line
- Number of crossings of lines
- Vertex closest to line
- All properties
- Correlation between distances among vertices in Draw window and their geodesics distances

4.14 FishEye

Turns Draw window into a FishEye mode (on of off): Hovering the mouse pointer over Draw window magnifies area aroud the mouse pointer. To keep the obtained layout click somewhere in Draw window.

- **Cartesian** – use cartesian FishEye transformation.
- **Polar** – use polar FishEye transformation.
- **Factor** – select the amount of movement.
- **Exit** – exit FishEye mode and restore old coordinates.

5 Exports to EPS/SVG/X3D/VRML

5.1 Defaults

If you have no labels in Draw window when you call Export there will also be no labels in EPS/SVG picture, otherwise numbers/labels like in Draw window will be shown. If you are looking at the picture using Draw/Partition, the same colors will be automatically used in EPS/SVG picture too.

5.2 Parameters in EPS, SVG, X3D, and VRML Defaults Window

Window is divided into 5 frames, two on the left and three on the right.

Note that settings you made in this window are overwritten if the parameters are specified in Pajek input (NET) file.

Top frame on the left – EPS/SVG Vertex Default

This frame defines default drawing of vertices when we export layouts to EPS and SVG:

- **Interior Color** – interior color of vertices (see Figure 19, page 105). If drawing using Partition is used, partition colors will overwrite the specified interior color.
- **x/y Ratio** – ratio between size of vertex in x and y direction (e.g., value 1 for circle, value larger/smaller than 1 for ellipse).
- **Symbol Size** – relative or absolute size of the symbol displayed in the middle of the vertex. If symbol size is set to some positive value (e.g. 0.75), this value is relative (according to vertex size): Final size of the symbol is computed by taking into account vertex size and default symbol size. In this way vertices with different sizes have different sizes of symbols as well. If symbol size is set to negative value (e.g. -25) this value (absolute) is also the final size of symbol. In this way vertices with different sizes have all the same sizes of symbols.
- **Border Color** – color of the borderline of vertices.
- **Border Width** – width of the borderline of vertices.
- **Shape Angle** – rotation of shapes in degrees.

- **Label Color** – color of label of vertices.
- **Font Size** – size of font of vertices labels.
- **Label Angle** – angle in which vertex labels will be displayed: if angle is smaller than 360, it means relative according to horizontal line (0 – horizontally); otherwise it is relative to center of the layout (360 – concentrically). The latter is useful when all vertices are drawn in concentric circle(s) using **Layout/Circular**.
- **Label Position: Radius /Angle** – position where the label will be displayed:
  - **Radius** – distance of beginning of vertex label from vertex center – first polar parameter.
  - **Angle** – position of vertex label in degrees - second polar parameter (0..360).

  If radius is equal to 0, the label will be centered, otherwise it will be left aligned on the specified position.
- **Shape** – default shape of vertex (*ellipse*, *box*, *diamond*, *triangle*, *house*, *man*, or *woman*).
- **Shapes file** – default shapes file, double click to change it.
- **Export options overwrite shapes file** – if checked: options selected in this window overwrite options defined in shapes file, otherwise default values for each shape are defined in selected shapes file.

Bottom frame on the left – *EPS/SVG Line Default*

This frame defines default drawing of lines when we export layouts to EPS and SVG:

- **Edge Color** – color of edges.
- **Edge Width** – width of edges.
- **Arc Color** – color of arcs.
- **Arc Width** – width of arcs.
- **Pattern** – pattern for drawing lines (*Solid* or *Dots*).

- **Arrow Size** – size of arrows.
- **Arrow Position** – distance of arrow from terminal vertex: If distance is between 0 and 1, it means relative distance according to arc length, e.g.: 0 – arrow touching the terminal vertex; 0.5 – arrow in the middle of the arc. If distance is larger than 1 it means absolute distance from the terminal vertex (this is useful if you want to have all arcs on the same distance from terminal vertex, regardless of arcs length)
- **Label Color** – color of line labels.
- **Label Angle** – angle in which label of line will be displayed: if angle is smaller than 360, it means relative to direction of line (0 – parallel to line); otherwise it is relative to horizontal line (360 – horizontally).
- **Label Position** – position of the center of line label – position is point on the line - distance of the center of line label from terminal vertex (see also **Arrow Position**)
- **Fontsize** – size of font of line labels.
- **Label Position: Radius /Angle** – position where the center of line label will be displayed according (relative) to **Label Position**:
  - **Radius** – distance of center of line label from point defined by **Label Position** - first polar parameter.
  - **Angle** – position of label in degrees - second polar parameter (0..360).
- **Bezier Curves** – provide *angle* and *velocity* at initial and terminal vertices (Bezier parameters). If both angles are 0, all lines will be drawn as straight lines, otherwises selected Bezier curves will be used for drawing lines.
- **Straight Lines** – drawing all lines (incliding bidirectional and multiple lines) without curves.

Top frame on the right

This frame defines some additional defaults when we export layouts to EPS, SVG, X3D, or VRML:

- **EPS, SVG, X3D, VRML Size of Vertices** – default size of vertices when exporting to X3D/VRML (valid for EPS and SVG exports as well).
- **X3D/VRML Size of Lines** – width of lines in X3D/VRML.

- **SVG: Opacity of Vertices** – a number between 0.00 (vertices drawn fully transparent) and 1.00 (vertices drawn fully opaque - default).
- **SVG: Opacity of Lines** – a number between 0.00 (lines drawn fully transparent) and 1.00 (lines drawn fully opaque - default).
- **SVG: Vertices 3D Effect** – if selected, gradient (linear or radial) will be applied to get 3D look of vertices.
- **SVG Tooltips for Labels of Vertices/Lines/Clusters** – if selected no labels for vertices/lines are displayed. Instead a tooltip will appear when mouse hovers the pointer over vertex/line/cluster. Title of selected cluster is list of all vertices in cluster.
- **EPS, SVG: Lines finished at Vertex Border** – when drawing vertices transparently, it is better to finish drawing lines at the border of vertex instead of drawing lines till the middle of the vertex.
- **EPS: Use RGB colors instead of CMYK** – in description of colors in EPS file RGB is used (default is CMYK).

Middle frame on the right – *Background Colors*

This frame defines background color when exporting to EPS/SVG/X3D/VRML and gradients (continuously smooth color transitions from one color to another) when exporting layouts to SVG/X3D:

- **Bckg. Color 1** – Background color for layout in EPS/SVG/X3D/VRML.
- **Bckg. Color 2** – the second color for SVG/X3D export. *No* – means without second color, otherwise selected gradient will be used.
- **Bckg. Color 3** – the third color for SVG/X3D export – gradient.
- **Gradients** – type of gradient to use in SVG (*No*, *Linear*, or *Radial*). In X3D only *Radial* gradient is supported.

Bottom frame on the right

This frame defines some additional defaults when we export layouts to EPS:

- **Left, Right, Top, Bottom** – additional border around layout when exporting to EPS (only when *EPS Clip* format is selected)

- **Border Color** – color of borderline of layout. It is used for export to SVG as well. *No* – means without borderline.
- **Border Radius** – radius of borderline of layout (if greater than 0 it means oval instead of rectangle).
- **Border Width** – width of borderline of layout.

Figure 17: ***Spider / Križevec***; Photo: Stana.

5.3 Exporting pictures to EPS/SVG – defining parameters in input file

Definition of Network (and its picture) on Input ASCII File

For every vertex and line we can specify in details how it should be drawn (colors, shapes, sizes, patterns, rotations, widths...).

A kind of standardized language is used for describing networks. The following reserved words are used:

1. ***Vertices** n – definition of vertices follows. n is number of vertices. Each vertex is described using following description line:

   vertex_num label [x y z] [shape] [changes of default parameters]

   Explanation:

   - vertex_num – vertex number $(1, 2, 3 \ldots n)$
   - label – if label starts with character A..Z or 0..9 first blank determines end of the label (e.g., vertex1), labels consisting of more words must be enclosed in pair of special characters (e.g., "vertex 1")
   - x, y, z – coordinates of vertex (between 0 and 1)
   - shape – shape of the vertex. Shapes are defined in file SHAPES.CFG (ellipse, box, diamond, triangle, cross, empty, house, man, or woman). In Draw window house is replaced by a triangle, man by a box, and woman by an ellipse. Shape is described with several parameters:
     - sh – replace sh with ellipse, box, diamond, triangle, cross, empty, house, man, or woman. This is the name of PostScript procedure that actually draws object (procedure is defined in drawnet.pro).
     - **s_size** – default size
     - **x_fact** – magnification in x direction
     - **y_fact** – magnification in y direction
     - **phi** – rotation in degrees of object in + direction (0..360)
     - **r** – parameter used for rectangle and diamond for describing radius of corners ($r = 0$ – rectangle, $r > 0$ – roundangle)
     - **q** – parameter used for diamonds – ratio between top and middle side of diamond (try q 0.01, q 0.5, q 2, ...)
     - **ic** – interior color of vertex. See Figure 19, page 105 for the list of possible colors.
     - **bc** – boundary color of vertex
     - **bw** – boundary width of vertex

- **lc** – label color
- **la** – label angle in degrees (0..360)
- **lr** – distance of beginning of vertex label from vertex center (radius – first polar parameter)
- **lphi** – position of label in degrees (0..360) (angle phi – second polar parameter)
- **fos** – font size
- **font** – PostScript font used for writing labels (Helvetica, Courier, ...)
- **HOOKS** – positions where edges can join the selected shape - according to s_size. Three different ways to specify these positions:
  (a) **CART** – x y – positions in Cartesian coordinates (x,y)
  (b) **POLAR** – r phi – positions in polar coordinates, phi is positive angle (0..360)
  (c) **CIRC** – r phi1 – iteration of positions in polar coordinates r – radius, $phi = k * phi1$, $k = 1, 2, ..; k * phi1 \leq 360$

Default values can be changed for each vertex in definition line, example:
1 "vertex one" 0.3456 0.1234 0.5 box ic White fos 20

Explanation: White box will represent vertex 1, label (vertex one) will be displayed using font size 20.

2. ***Arcs** (or ***Edges**) – definition of arcs (edges). Format:

   v1 v2 value [additional parameters]

   Explanation:

   - v1 – initial vertex number
   - v2 – terminal vertex number
   - value – value of arc from v1 to v2

   These three parameters must always be present. If no other parameter is specified, the default arc will be black, straight, solid arc with following exceptions:

   - if value is negative, dotted line will be used instead of solid,
   - if arc is a loop (arc to itself) bezier loop will be drawn,
   - if bidirected arc exists two curved bezier arcs will be drawn.

Arrow will be drawn at the end of the edge (at terminal vertex).

As we mentioned, hooks are used to specify exact position where line joins vertices.

Additional parameters:

- **w** – width of line
- **c** – color of line
- **p** – pattern of line (Solid, Dots)
- **s** – size of arrow
- **a** – type (shape) of arrow (A or B)
- **ap** – position of arrow
  - $ap = 0$ – arrow at terminal vertex
  - $0 < ap \leq 1$ – proportional distance from terminal vertex (according to line length)
  - $ap > 1$ – absolute distance
- **l** – line label (e.g. ”line 1 2”)
- **lp** – label position (look at *ap*)
- **lr** – label radius (position of center text from point on edge)
- **lphi** – label radius (angle of center text according to point on edge) (lr and lphi are polar coordinates)
- **lc** – label color
- **la** – label angle
  ($0 < la < 360$ – relative to edge, $la \geq 360$ – absolute angle according to x axis)
- **fos** – font size of label
- **font** – PostScript font used for writing labels (Helvetica, Courier, ...)
- **h1** – hook at initial vertex (0 – center, -1 the closest, 1, 2.. user defined)
- **h2** – hook at terminal vertex
- **a1** – angle at initial vertex (Bezier)
- **k1** – velocity at initial vertex (Bezier)
- **k2** – velocity at terminal vertex (Bezier)
- **a2** – angle at terminal vertex (Bezier)

Special shapes of lines can be defined using combinations of alpha1, k1, alpha2, k2:

- $alpha1 = alpha2 = 0, k1 \geq 0, k2 \geq 0$ – straight line (default)
- $alpha1 = alpha2 = 0, k1 = -1, k2 > 0$ – oval edge with radius $k2$ (measure of radius is absolute as explained above)
- $alpha1 = alpha2 = 0, k1 = -1, k2 < 0$ – second possible oval edge with radius $-k2$
- $alpha1 = alpha2 = 0, k1 = -2, k2 > 0$ – circular arc with radius $k2$ in positive direction
- $alpha1 = alpha2 = 0, k1 = -2, k2 < 0$ – second possible circular arc with radius $-k2$ in positive direction
- $alpha1 = alpha2 = 0, k1 = -3, k2 > 0$ – circular arc with radius $k2$ in negative direction
- $alpha1 = alpha2 = 0, k1 = -3, k2 < 0$ – second possible circular arc with radius $-k2$ in negative direction
- $alpha1 = alpha2 = 0, k1 = -4$ – double edge
- $alpha1$ or $alpha2 \neq 0$, $k1 > 0$, $k2 > 0$ – Bezier curve (if $alpha1$ and $alpha2$ have different signs line goes from one to another side of straight line connecting both vertices, if $alpha1$ and $alpha2$ have the same sign – line stays on the same side of straight line connecting both vertices)

3. **`*Edges`** – definition of edges.

   The same parameters as for arcs can be used, except that type (*a*), size (*s*) and position (*ap*) have no meaning for edges.

PS and EPS

- PS – Export to Postscript (.PS) without header (drawnet.pro). Use it to spare space, if you have many pictures and your word processor enables you to define the header separately (like in LaTeX).

- EPS – Export into file with Encapsulated PostScript description (.EPS). Drawnet.pro is already inserted in the beginning. The picture is complete, you can include it into text, make it bigger or smaller (without losing quality), rotate it, print it on Postscript printer, view it with GhostScript viewer, convert it to PDF, JPG...

An example

```
*Vertices 4
1 "ellipse"  0.120 0.285 0.5 ellipse x_fact 5 y_fact 3 fos 20 ic LightYellow lc Red
2 "box"      0.818 0.246 0.5 box x_fact 5 y_fact 3 fos 20 ic LightCyan lc Blue
3 "diamond"  0.368 0.779 0.5 diamond x_fact 7 y_fact 3 fos 20 ic LightGreen lc Black
4 "triangle" 0.835 0.833 0.5 triangle x_fact 9 y_fact 3 fos 20 ic LightOrange lc Brown lr 30 lphi 200
*Arcs
 1 1  1 h2 0 w 3 c Blue s 2 a1 -130 k1 0.6 a2 -130 k2 0.6 ap 0.25 l "Bezier loop" lc OliveGreen fos 20 lr 13 lp 0.5 la 360
 2 1  1 h2 1 a1 120 k1 1 a2 10 k2 0.8 ap 0 l "Bezier arc" lphi 270 la 180 lr 13 lp 0.5
 1 2  1 h2 -1 a1 40 k1 2 a2 -30 k2 0.8 ap 0 l "Bezier arc" lphi 90 la 0 lp 0.75
 4 2 -1 l "Straight dotted arc" p Dots c Red
*Edges
 1 3  1 l "Straight edge" lp 0.4
 3 4  1 l "Straight edge"
```

You have to set some options in Pajek's draw window: for example
`Options / Lines / Mark Lines / with labels`
to activate the display of line labels.

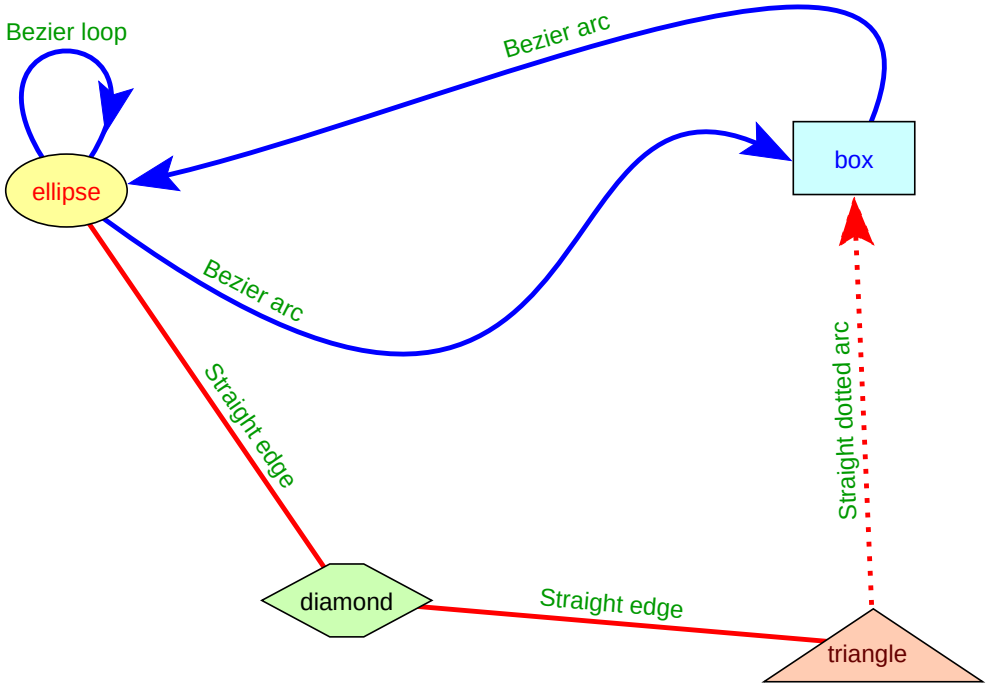


Figure 18: Example picture

5.4 Using Unicode in Pajek's pictures

Sometimes we would like to use in Pajek's picture some characters from unusual alphabets (special symbols, Cyrillic, Greek, Arabic, Chinese, ...). They are available in Unicode. Pajek supports Unicode UTF8 files with BOM from version 2.00 on.

Some hints on using Unicode in Pajek:

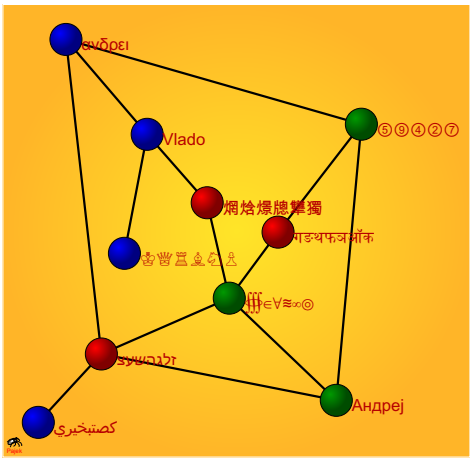
- Using the free Unicode editor BabelPad we prepare the Unicode version of the `*.NET` file. Use the U tool (Unicode table) to enter the Unicode characters.

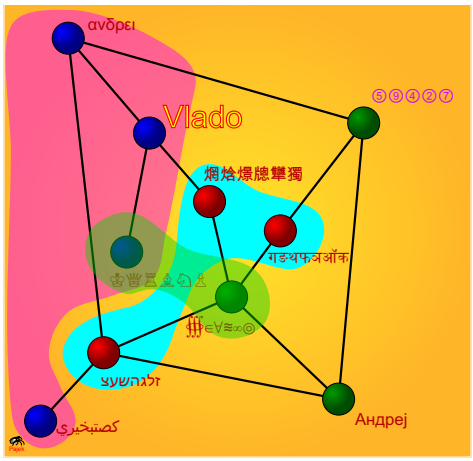
- Using the BabelPad option `File/Save as/UTF8` and checking `Byte Order Mark` you save network in a Unicode file.
  You can use also `File/Save as/ASCII plus decimal NCR`. In this way we transform Unicode file into an ASCII file in which the Unicode characters are represented using numeric codes `&#dddd;`.

  Any of the two files generated can be read by Pajek and proper Unicode characters will appear in Draw, Report and other windows provided that you select proper proportional and monospaced font in `Options/Select Font` in Pajek Main menu. We suggest to use `Arial Unicode MS` or `Lucida Sans Unicode` (one of them usually comes with Windows) for proportional font and GNU Unifont for monospaced font. `Courier New` can be used too.

  When saving networks they are stored as ASCII files by default (nonASCII characters are stored as `&#dddd;`) unless you check **Options/Read-Write/Save Files as Unicode UTF8 with BOM**.

  The file is used in Pajek 2.00 (or higher) to produce picture. The picture is exported in the SVG format.





- The SVG picture can be opened in the free drawing program InkScape. Here we can make some additional enhancements of the picture.

- In InkScape we can, using the option `File/Save as/pdf` or `eps`, save the picture in the desired format PDF or EPS or some other) with labels containing Unicode characters. Note that the PDF (1.4 or higher) also supports the transparency. The EPS file can be inserted in Word document.

Similar approach can be used also for 3D models in X3D. For their inspection we recommend the use of Instant Player.

6 Using Macros in **Pajek**

6.1 What is a Macro?

A macro enables you to record a sequence of primitive **Pajek** commands into a file. You can use this file later to execute the saved sequence of commands without selecting one by one.

6.2 How to record a Macro?

1. First load all objects (networks, partitions,...) which will be used by a macro.
2. Select objects which will be used in macro (for example, network which will be first used must be shown in Network ComboBox).
3. Choose Macro/Record and select the name of macro file (default extension is **.mcr**).
4. Use **Pajek** as usual to define a sequence of commands. We advise to add additional comments to macro file using Macro/Add Message. When running macro this comments are displayed in Report window and help you to follow the macro execution.
5. At the end select Macro/Record again to stop recording.

6.3 How to execute the Macro?

1. First load object(s) that will be used as input in macro execution.
2. Choose Macro/Play and select the macro file.

6.4 Example

The following macro performs topological sort of an acyclic network:

```
NETBEGIN 2
CLUBEGIN 1
PERBEGIN 1
CLSBEGIN 1
HIEBEGIN 1
VECBEGIN 1
NETPARAM 1
```

```
Msg Depth Partition
C 1 DEP 1 (10)
Msg Make Permutation
P 1 MPER 1 (10)
Msg Reordering network
N 2 REOR 1 1 (10)
```

The first seven commands store the current state of ComboBoxes. The acyclic network on which we want to execute topological sort must be at the top of Network ComboBox before starting the macro.

6.5 List of macros available in installation file

6.5.1 Macros prepared for genealogies and other acyclic networks

- **Path** – find shortest chain (undirected path) between the two vertices.
- **Descendants** – find all vertices 'later' than selected vertex (in a p-graph).
- **Ancestors** – find all vertices 'before' the selected vertex (in a p-graph).
- **Cognatic3** – find vertices three generations 'before' and three generations 'after' the selected vertex (in a p-graph).
- **Cognatic** – find all reachable vertices 'before' and all 'after' the selected vertex (in a p-graph).
- **Layers, Layers1, Layers2** – draw acyclic network in layers (different algorithms).
- **zLayers** – draw acyclic network in layers in z direction.
- **LongestPatrilineage** – find the longest patrilineage in Ore-genealogy. The genealogy must be read with the option **`Ore graph: 1-Male, 2-Female links checked`**.
- **LongestMatrilineage** – find the longest matrilineage in Ore-genealogy. The genealogy must be read with the option **`Ore graph: 1-Male, 2-Female links checked`**.
- **NumDescendants** – for each vertex compute the size of output domain (number of descendants in Ore-graph).
- **NumAncestors** – for each vertex compute the size of input domain (number of ancestors in Ore-graph).

6.5.2 Macros prepared for computing derived kinship relations

Pajek generates three relations when reading genealogy as Ore graph with the option **Ore graph: 1-Male, 2-Female links checked:**

1 : *is a father of*
2 : *is a mother of*
3 : *is a spouse of*

Other kinship relations can be obtained from these relations by running macros. Gender partition is also result of reading genealogy and is input to the following macros.

4 : *is a parent of*
5 : *is a child of*
6 : *is a son of*
7 : *is a daughter of*
8 : *is a husband of*
9 : *is a wife of*
10 : *is a sibling of*
11 : *is a brother of*
12 : *is a sister of*
13 : *is an uncle of*
14 : *is an aunt of*
15 : *is a semisibling of*

Using macro **add_all_relations** you can add all above relations at once.
As a test/example file you can use ***/pajek/data/family.ged**

6.6 Repeating session

During program execution all commands are written to file `*.log`. In this way you can repeat any execution by running selected log file (*Macro/Repeat Last Command*). If you change in the log file a name of a file to ?, program will ask for name when running logfile next time (so you can repeat the same sequence of steps – logfile with different input data). If startup logfile (Pajek.log) exists (in the same directory as Pajek.exe), it is automatically executed every time when **Pajek** is run.

6.7 Run Command

'Run Command' enables us to run again one of the already executed commands with new parameters. First select one of the executed commands from the list, change its parameters, and finally execute the command with updated parameters.

6.8 Repeating last command

Macro submenu enables to run the last command executed by Pajek several times applied to different successive objects, as well.

Example: after loading 100 networks in Pajek, execute degree partition on the first network and run Repeat Last Command by typing 99 to compute degree partition on all other networks.

Commands that include several objects can be run as well (e.g. extracting subnetworks according to selected partition(s)). There are 3 different possibilities for extracting from the set of networks (possibilities are selected by fixing appropriate objects):

- for all networks extracting is determined by the same partition (incrementing applied to network, not applied to partition – partition is fixed)
- for all networks extracting is determined by another partition (incrementing applied to network and to partition – nothing is fixed)
- for one network several extractions are determined by different partitions (incrementing not applied to network, applied to partition – network is fixed)

Important: Always first execute the command on the first loaded object(s). Repeating will start with object(s) that have object number(s) one higher than the object(s) on which the command was executed.

If the result of the command is also a constant, all constants are stored in a vector. In Pajek the following constants exist: number of vertices, arcs, edges, network densities, centralization indices, diameter, relinking index, correlation and contingency coefficients, number of fragments, main core number, number of components, length of critical path, maximum flow, distance between vertices, number of islands, minimum and maximum value in partition / hierarchy, minimum, maximum, arithmetic mean, median and standard deviation in vector.

6.9 Storing constant(s) reported by last command to Scalar(s)

During execution of some commands one or more constants are written to Report window (e.g. all Info's, Components (Number, Size of the largest), Network centralizations, Network clustering coefficient, Diameter,...). We can store the constants obtained by the *last* command to Scalars (Vectors of dimension 1) and use them later (for example to normalize networks or vectors).

7 Blockmodeling in Pajek

The blockmodeling option is an embedding of the programs for optimization approach to generalized blockmodeling MODEL2 and TwoMODEL from package STRAN – STRucture ANalysis [9] into **Pajek**.

The blockmodeling command seeks for the best partition of a given network satisfying given types of blocks – generalized blockmodeling [7, 35]. When seeking for the best partition it starts from the random partition into given number of clusters and tries to optimize it using two local transformations: moving of vertices from one to another cluster and interchaging (transposing) two vertices that belong to different clusters. The blockmodel can be built inside Pajek (if *User Defined* is selected) and/or its description can be stored in MDL file. The impact of errors in each block can be controlled using penalty weights.

The option supports also generalized blockmodeling of two-mode networks [34].

The maximum size of a network on which the command can be applied is 1000 vertices. But the real limit is time complexity – already on 100 vertices optimization can last some hours.

The results of the command are stored as partitions. They can be displayed as a picture

`Draw / Network+Partition`

and

`Layout / Energy / Kamada-Kawai / Free`

or

`Layout / Circular / using Partition`

The result can be displayed also in the matrix form. This requires two steps:

`Partition / Make Permutation`

`File / Network / Export as Matrix to EPS /`
`Using Permutation + Partition` enter file name; yes

7.1 MDL files

The structure of a MDL file is evident from the following example

```
*MODEL Tina
 9
 0 3 100  0 1 2 3 4
*CONSTRAINTS
 1 100  2 1
 4 100  1 3
*EOM
```

The first character in each line should be a star * or a blank.
The last character in a line should not be a blank.

A number in the second line is number of clusters. In the case of **two-mode network** two numbers should be given in this line – number of row-clusters and number of col-clusters. The other data are the same for both type of networks. The following lines have the structure

$$i \quad j \quad penalty \qquad t_1 \quad t_2 \ldots t_k$$

When $i, j > 0$ the line prescribes that the block (i, j) can be of types $t_1, t_2 \ldots t_k$. The types are coded as follows

```
0  -  - null                7 rfn - row-function
1 com - complete            8 cfn - col-function
2 rdo - row-dominant        9 den - density
3 cdo - col-dominant       10 dnc - do not care
4 reg - regular            11 one - non-null
5 rre - row-regular        12 sym - symmetric
6 cre - col-regular
```

Lines with $i = 0$ defines the types of of parts of model matrix:

- $j = 0$: diagonal (for one-mode networks only);
- $j = 1$: upper triangle (for one-mode networks only);
- $j = 2$: lower triangle (for one-mode networks only);
- $j = 3$: complete matrix.

In the case of several lines describing the same block the last prescription prevails.

Lines following `*CONSTRAINTS` define additional constraints in blockmodel. Constraints have the form

$$k \quad penalty \qquad i \quad j$$

with the following meaning

- $k = 1$: $i \in C_j$ – vertex i belongs to cluster C_j;
- $k = 2$: $i \notin C_j$ – vertex i does not belong to cluster C_j;
- $k = 3$: $C(i) = C(j)$ – vertices i and j belong to the same cluster;
- $k = 4$: $C(i) \neq C(j)$ – vertices i and j belong to different clusters;

- $k = 5$: $i \leq |C(j)|$ – cluster C_j contains at least i vertices;
- $k = 6$: $i \geq |C(j)|$ – cluster C_j contains at most i vertices.

Be careful in the case of two mode network for constraints of type $k = 5$ and $k = 6$ (constraints on min/max number of vertices in selected cluster): If there are r row-clusters and c col-clusters in blockmodel, then use numbers in the range $1 \ldots r$ to define constraint on row-cluster and numbers in the range $r + 1 \ldots r + c$ to define constraint on col-cluster.

The violations of constraints contribute to criterion function with a term

$$+ \quad \text{\# of violations} \quad \times \quad \text{penalty}$$

The values of penalties have to be in the range 0 to 1000.

7.2 Examples of MDL files

7.2.1 Regular blocks

```
*MODEL Regular
 10
 0 3   1  0 1 4
*EOM
```

7.2.2 Diagonal blocks (clustering)

```
*MODEL Diagonal
 10
 0 3 100  0
 0 0   1  0 1 4
*EOM
```

7.2.3 Acyclic model (up)

```
*MODEL Hierarchy
 9
 0 1   1  0 5 6
 0 0  10  0 1 4 12
 0 2 100  0
*EOM
```

7.2.4 Acyclic model with symmetric clusters (down)

```
*MODEL SymHiera
 9
 0 0  10  0 1 12
 0 1 100  0
 0 2   1  0 11
*EOM
```

7.2.5 Center-Periphery

```
*MODEL Center-Periphery
 2
 0 3   1   0 11
 2 2  10   0
 1 1 100   0 1 4
*EOM
```

7.2.6 Regular path

```
*MODEL Regular Path
 9
 0 0 10  0 1 4
 1 2 10  0 1 4
 2 3 10  0 1 4
 3 4 10  0 1 4
 4 5 10  0 1 4
 5 6 10  0 1 4
 6 7 10  0 1 4
 7 8 10  0 1 4
 8 9 10  0 1 4
*EOM
```

7.2.7 Regular chain

```
*MODEL Regular Chain
 9
 0 0 10  0 1 4
 1 2 10  0 1 4
 2 3 10  0 1 4
 3 4 10  0 1 4
 4 5 10  0 1 4
```

```
 5  6 10   0 1 4
 6  7 10   0 1 4
 7  8 10   0 1 4
 8  9 10   0 1 4
 2  1 10   0 1 4
 3  2 10   0 1 4
 4  3 10   0 1 4
 5  4 10   0 1 4
 6  5 10   0 1 4
 7  6 10   0 1 4
 8  7 10   0 1 4
 9  8 10   0 1 4
*EOM
```

7.2.8 2-mode 'standard model' for Davis.net

```
*MODEL UserDefined
 2 3
  1  1          1 1
  1  2          1 1
  1  3        100 0
  2  1        100 0
  2  2          1 1
  2  3          1 1
*EOM
```

8 Colors in **Pajek**

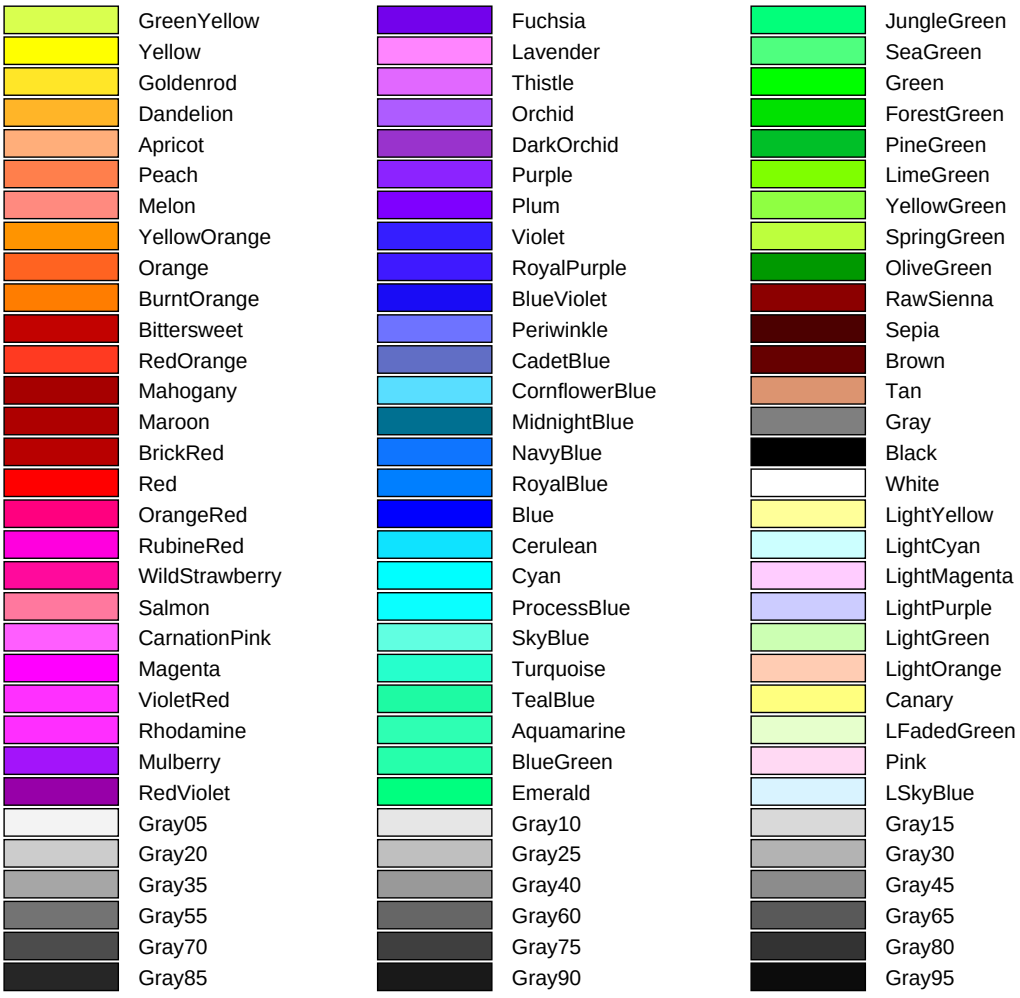


Figure 19: Colors in **Pajek**.

Figure 20: Partition colors.

9 Citing Pajek

For citing **Pajek** you may consider:

- W. de Nooy, A. Mrvar, V. Batagelj: *Exploratory Social Network Analysis with Pajek: Revised and Expanded Edition for Updated Software. Third Edition.* Structural Analysis in the Social Sciences 46, Cambridge University Press, 2018. ISBN:9781108462273. CUP, Amazon.

- A. Mrvar, V. Batagelj: *Pajek – Program for Large Network Analysis*. Home page: **http://mrvar.fdv.uni-lj.si/pajek/**

- Mrvar, A. and Batagelj, V.: *Analysis and visualization of large networks with program package Pajek*. Complex Adaptive Systems Modeling, 4:6. 2016. SpringerOpen.

- V. Batagelj, A. Mrvar: *Pajek – Analysis and Visualization of Large Networks.* In Jünger, M., Mutzel, P. (Eds.): Graph Drawing Software. Springer (series Mathematics and Visualization), Berlin 2003. 77-103. ISBN 3-540-00881-0. Springer, Amazon, preprint

- V. Batagelj, A. Mrvar: *Pajek – Program for Large Network Analysis*. Connections, **21**(1998)2, 47-57. preprint

Figure 21: ***Gartenkreuzspinne / Araneus diadematus*; Photo: Stefan Ernst**

References

[1] Adobe SVG viewer. http://www.adobe.com/svg/
http://www.adobe.com/svg/viewer/install

[2] Ahmed, A., Batagelj, V., Fu, X., Hong, S.-H., Merrick, D., Mrvar, A. (2007): Visualisation and analysis of the Internet movie database. Asia-Pacific Symposium on Visualisation 2007 (IEEE Cat. No. 07EX1615), 17-24.

[3] Albert R., Barabasi A.L.: Topology of evolving networks: local events and universality.
http://xxx.lanl.gov/abs/cond-mat/0005085

[4] Batagelj V.: Papers on network analysis.
http://vlado.fmf.uni-lj.si/pub/networks/doc/

[5] Batagelj V.: Workshop on Network Analysis, Sydney, Australia: 14th to 17th June 2005; at Nicta (National ICT Australia).
http://vlado.fmf.uni-lj.si/pub/networks/doc/#NICTA

[6] Batagelj V.: Some new procedures in Pajek. Dagstuhl seminar 05361, Dagstuhl, Germany, Sept 5-9, 2005.
http://vlado.fmf.uni-lj.si/pub/networks/doc/dagstuhl/NewProcs.pdf

[7] Batagelj, V. (1997) Notes on blockmodeling. *Social Networks* **19**, 143-155.

[8] Batagelj V.: Efficient Algorithms for Citation Network Analysis.
http://arxiv.org/abs/cs.DL/0309023

[9] Batagelj V.: **MODEL 2**. http://vlado.fmf.uni-lj.si/pub/networks/

[10] Batagelj V. (2009): Social Network Analysis, Large-Scale. R.A. Meyers, ed., Encyclopedia of Complexity and Systems Science, Springer 2009: 8245-8265. http://www.springerlink.com/content/tp3w7237m4624462/

[11] Batagelj V. (2009): Complex Networks, Visualization of. R.A. Meyers, ed., Encyclopedia of Complexity and Systems Science, Springer 2009: 1253-1268. http://www.springerlink.com/content/m472707688618h17/

[12] Batagelj V., Brandes U. (2005): Efficient Generation of Large Random Networks. *Physical Review* E **71**, 036113, 1-5.

[13] Batagelj, V., Ferligoj, A., and Doreian, P. (1992), Direct and Indirect Methods for Structural Equivalence, *Social Networks*, **14**, 63–90.

[14] Batagelj, V., Doreian, P., and Ferligoj, A. (1992) An Optimizational Approach to Regular Equivalence. *Social Networks* **14**, 121-135.

[15] Batagelj, V, Ferligoj, A, Doreian, P (2007): Indirect blockmodeling of 3-way networks. Selected contributions in data analysis and classification, Springer, Berlin, 151-159.

[16] Batagelj, V., Kejžar, N., Korenjak-Černe, S. (2008): Analysis of the Customers? Choice Networks: An Application on Amazon Books and CDs Data. Metodološki zvezki/Advances in Methodology and Statistics **4** (2): 191-204.

[17] Batagelj V., Mrvar A.: Pajek.
http://vlado.fmf.uni-lj.si/pub/networks/pajek/

[18] Batagelj V., Mrvar A. (2000) Some Analyses of Erdős Collaboration Graph. Social Networks, **22**, 173-186

[19] Batagelj V., Mrvar A. (2001) A Subquadratic Triad Census Algorithm for Large Sparse Networks with Small Maximum Degree. Social Networks, **23**, 237-243

[20] Batagelj V., Mrvar A. (2008) Analysis of Kinship Relations With Pajek. Social Science Computer Review **26**(2), 224-246, 2008.

[21] Batagelj V., Mrvar A., Zaveršnik M. (1999) Partitioning Approach to Visualization of Large Graphs. In: Kratochvil J. (Ed.) GD'99, Štiřin Castle, Czech Republic. LNCS **1731**. Springer-Verlag, 90-97.

[22] Batagelj V., Mrvar A., Zaveršnik M. (2002) Network analysis of texts. Language Technologies, Ljubljana, p. 143-148.
http://nl.ijs.si/isjt02/zbornik/sdjt02-24bbatagelj.pdf

[23] Batagelj V., Zaveršnik M. (2011): Fast algorithms for determining (generalized) core groups in social networks. Advances in Data Analysis and Classification. Springer

[24] Batagelj, V. and Zaveršnik, M. (2007): Short Cycles Connectivity. Discrete Math 307 (3-5): 310-318.
http://arxiv.org/abs/cs.DS/0308011

[25] Bollobas B.: Random Walks on Graphs,

[26] Broder A. etal. (2000): Graph structure in the web.
http://www.almaden.ibm.com/cs/k53/www9.final/

[27] Burt R.S. (1992): *Structural Holes*. The Social Structure of Competition. Cambridge MA: Harvard University Press.

[28] Butts, C.T. (2002) sna: Tools for Social Network Analysis. `http://cran.at.r-project.org/src/contrib/PACKAGES.html#sna`

[29] Cortona Player (2006) `http://www.parallelgraphics.com/products/cortona/`

[30] Cosmo Player (2002) `http://ca.com/cosmo/`

[31] de Nooy W., Mrvar A., Batagelj V. (2018) *Exploratory Social Network Analysis With* **Pajek**. Revised and Expanded Edition for Updated Software. Third Edition. Structural Analysis in the Social Sciences 46, Cambridge University Press, 2018. ISBN:9781108462273. CUP, 2018, Chinese - Amazon, 2014, Japanese - Amazon, 2009.

[32] Doreian P., Mrvar A. (1996) A Partitioning Approach to Structural Balance. Social Networks, **18**. 149-168

[33] Doreian, P., Batagelj, V., Ferligoj, A. (2000) Symmetric-acyclic decompositions of networks. *J. classif.*, **17**(1), 3-28.

[34] Doreian P., Batagelj V., Ferligoj A. (2004) Generalized blockmodeling of two-mode network data. *Social Networks* **26**, 29-53.

[35] Doreian P., Batagelj V., Ferligoj A.: *Generalized Blockmodeling*, Structural Analysis in the Social Sciences 25, Cambridge University Press, 2005. ISBN:0521840856. CUP, Amazon.

[36] Dremelj P., Mrvar A., Batagelj V. (2002) Analiza rodoslova dubrovačkog vlasteoskog kruga pomoću programa **Pajek**. Anali Dubrovnik XL, HAZU, Zagreb, Dubrovnik, 105-126 (in Croat).

[37] GEDCOM 5.5. `http://homepages.rootsweb.com/~pmcbride/gedcom/55gctoc.htm`

[38] Ghostscript, Ghostview and GSview. `http://www.cs.wisc.edu/~ghost/`

[39] Gibbons A. (1985) Algorithmic Graph Theory. Cambridge University Press.

[40] Grossman J. (2002) The Erdős Number Project. `http://www.oakland.edu/~grossman/erdoshp.html`

[41] Hall, B.H., Jaffe, A.B. and Tratjenberg M.: The NBER U.S. Patent Citations Data File. NBER Working Paper 8498 (2001).
http://www.nber.org/patents/

[42] Hamberger, K., Houseman, M., Daillant, I., White, D.R., Barry, L. (2004) *Matrimonial Ring Structures*. Math. & Sci. hum. / Mathematics and Social Sciences, **42(4)**, 83-119.
http://www.ehess.fr/revue-msh/pdf/N168R965.pdf

[43] Hummon, N.P., Doreian, P. (1989) Connectivity in a citation network: The development of DNA theory. *Social Networks*, **11**, 39–63.

[44] Jones B. (2002). Computational geometry database.
http://compgeom.cs.uiuc.edu/~jeffe/compgeom/biblios.html

[45] Kejžar, N., Korenjak Černe, S., Batagelj, V. (2010) Network Analysis of Works on Clustering and Classification from Web of Science. Classification as a Tool for Research. Hermann Locarek-Junge, Claus Weihs eds. Proceedings of IFCS 2009. Studies in Classification, Data Analysis, and Knowledge Organization, Part 3, 525-536, Springer, Berlin, 2010. Springer

[46] Kejžar, N., Nikoloski, Z., Batagelj, V. (2008): Probabilistic Inductive Classes of Graphs. Journal of Mathematical Sociology **32**: 85-109.

[47] Kleinberg J. (1998) Authoritative sources in a hyperlinked environment. In Proc 9th ACMSIAM Symposium on Discrete Algorithms, p. 668-677.
http://www.cs.cornell.edu/home/kleinber/auth.ps

[48] Knuth, D. E. (1993) *The Stanford GraphBase*. Stanford University, ACM Press, New York. ftp://labrea.stanford.edu/pub/sgb/

[49] Liu, J.S. and L.Y.Y. Lu (2012) An Integrated Approach for Main Path Analysis: Development of the Hirsch Index as an Example. Journal of the American Society for Information Science and Technology, 63(3), p. 528-542
http://onlinelibrary.wiley.com/doi/10.1002/asi.21692/abstract

[50] LOCKS: CRA Analyses of News Stories on the Terrorist Attack. Arizona State University. http://locks.asu.edu/terror/

[51] McCabe, T. Computer Science Approaches: Visualization Tools and Software Metrics. in Survey Automation. NAP, 2003, p. 116-136.
http://books.nap.edu/books/0309089301/html/116.html

[52] MDL Information Systems, Inc. (2002) http://www.mdli.com/

[53] James Moody home page (2002)
http://www.soc.sbs.ohio-state.edu/jwm/

[54] Murtagh, F. (1985) Multidimensional Clustering Algorithms, *Compstat lectures*, **4**, Vienna: Physica-Verlag.

[55] Pajek's datasets:
http://vlado.fmf.uni-lj.si/pub/networks/data/

[56] D.M. Pennock etal. (2002) Winners dont't take all, PNAS, **99**/8, 5207-5211.

[57] Peterson J. L.: *Petri Net Theory and the Modeling of Systems*.

[58] The R Project for Statistical Computing. http://www.r-project.org/

[59] Richardson D.C., Richardson J.S. (2002) The Mage Page.
http://kinemage.biochem.duke.edu/index.html

[60] Scott, J. (2000) Social Network Analysis: A Handbook, 2nd edition. London: Sage Publications.

[61] Seidman S. B. (1983) Network structure and minimum degree, *Social Networks*, **5**, 269–287.

[62] Tarjan, R. E. (1983) *Data Structures and Network Algorithms*. Society for Industrial and Applied Mathematics Philadelphia, Pennsylvania.

[63] UCINET (2002) http://www.analytictech.com/

[64] The United States Patent and Trademark Office.
http://patft.uspto.gov/netahtml/srchnum.htm

[65] Wasserman S., Faust K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge University Press, Cambridge.

[66] White D.R., Batagelj V., Mrvar A. (1999) Analyzing Large Kinship and Marriage Networks with Pgraph and **Pajek**. Social Science Computer Review, **17 (3)**, 245-274

[67] Wilson, R.J., Watkins, J.J. (1990) *Graphs: An Introductory Approach*. New York: John Wiley and Sons.

[68] W3C SVG page. http://www.w3.org/Graphics/SVG

[69] Zeleny, L.D. (1947): Selection of compatible flying partners. *American Journal of Sociology*, **52**, 424-431.