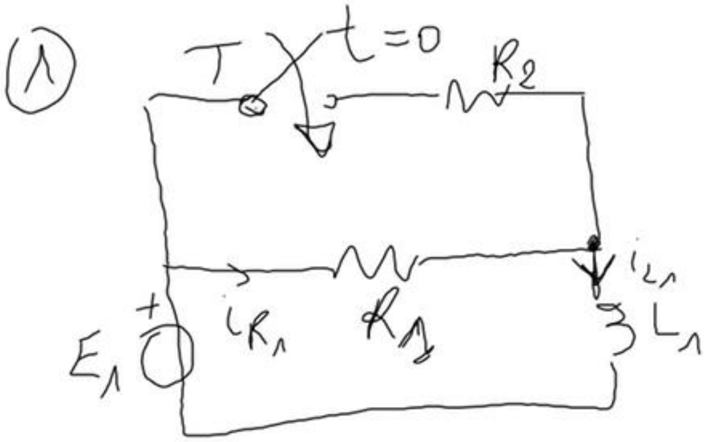
Esercizio



$$R_1 = R_2 = 2\,\Omega$$
$$L_1 = 1\,mH$$
$$E_1 = 10\,V$$

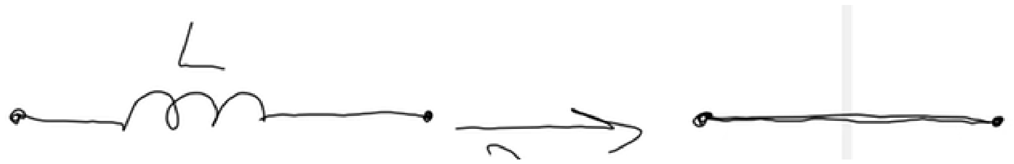
$t = 0^-$ → RETE A REGIME

$$\frac{\partial}{\partial t} = 0$$

Conoscendo la relazione costitutiva:

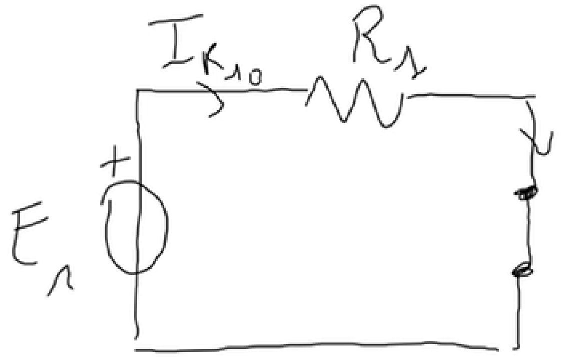
$$v_L = L\frac{di_L}{dt}$$

Essendo la tensione nulla su 2 nodi, abbiamo un corto.



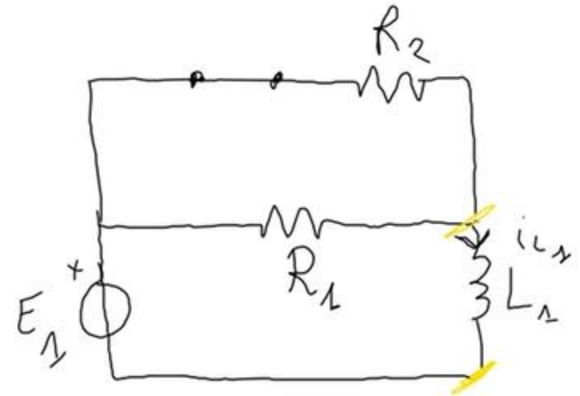
Invece, per i condensatori:

$$i_c = C\frac{dv_c}{dt}$$



$$i_{L_1}\Big|_{t=0^-} = I_{L_{10}} = \frac{E_1}{R_1} = I_{R_{10}} = 5\ A$$

E nel caso t uguale a 0+:



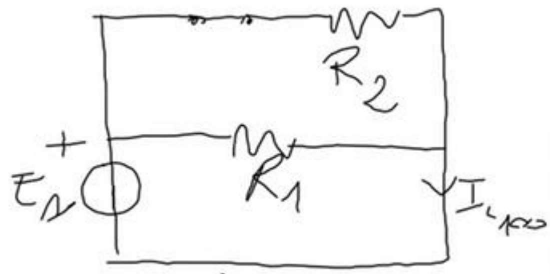
$$i_{L_1}(t) = I_{L_{1\infty}} + (I_{L_{10}} - I_{L_{1\infty}})e^{-t/\tau}$$

$$\tau = L_1/R_{eq}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1}{2} = 1\,\Omega$$

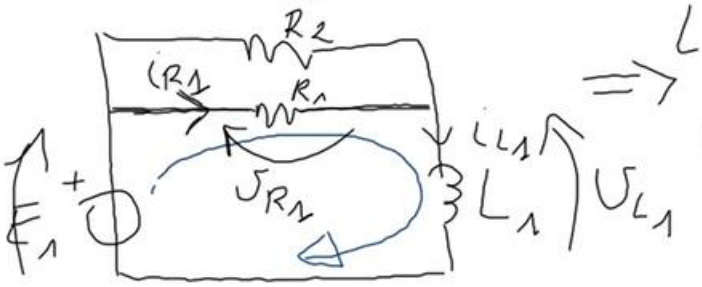
Infine:

$t \to \infty \Rightarrow$



$$I_{L_1\infty} = \frac{E_1}{R_1 // R_2} = 10\,A$$

Applichiamo la LKT (potevamo usare il partitore di corrente).



$\Rightarrow$ LKT:

$$E_1 = v_{R_1} + v_{L_1}$$

$$v_{L_1} = L_1 \frac{di_{L_1}}{dt} = L_1 \frac{d}{dt}\left[10 - 5e^{-t/\tau}\right] =$$

$$i_{R_1} = \frac{E_1 - V_{L_1}}{R_1} = \frac{10 - 5e^{-t/\tau}}{R_1} =$$

$$= 5 - 2{,}5e^{-\frac{t}{1m}}$$

$$W_{L_1 0^-} = W_{L_1 0^+}$$

$$\frac{1}{2} L_1 I_{L_1 0^-}^2 = \frac{1}{2} L_1 I_{L_1 0^+}^2$$

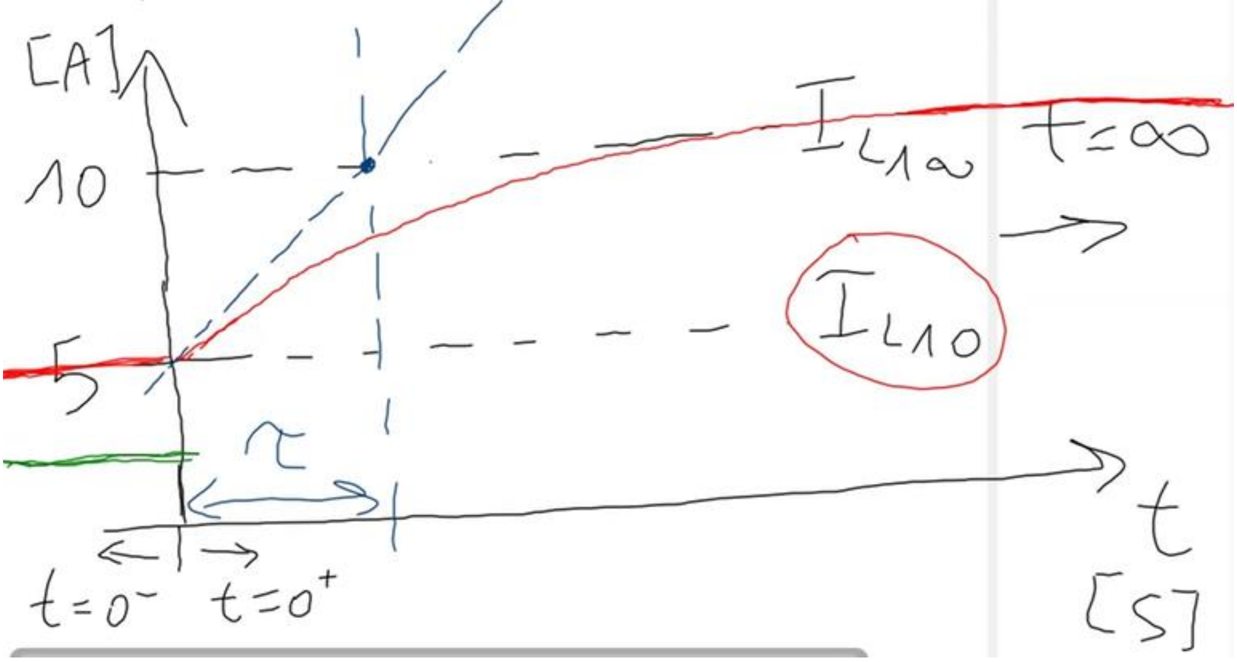
$$V_{C_{0^-}} = V_{C_{0^+}}$$

$$v_{L_1} = L_1 \frac{d i_{L_1}}{dt} = L_1 \frac{d}{dt}\left[\cancel{10} - 5e^{-t/\tau}\right] =$$

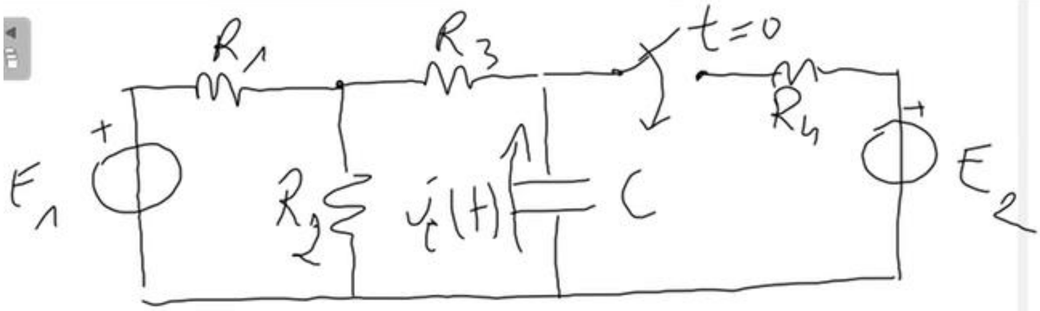
$$= -L_1\, 5\, e^{-t/\tau}\left(-\frac{1}{\tau}\right) =$$

$$= -R_{eq}\, 5\, e^{-t/\tau} \underset{\substack{\uparrow \\ R_{eq} = 1\Omega}}{=} -5\, e^{-\frac{t}{1m}}$$

Evoluzione



Esercizio

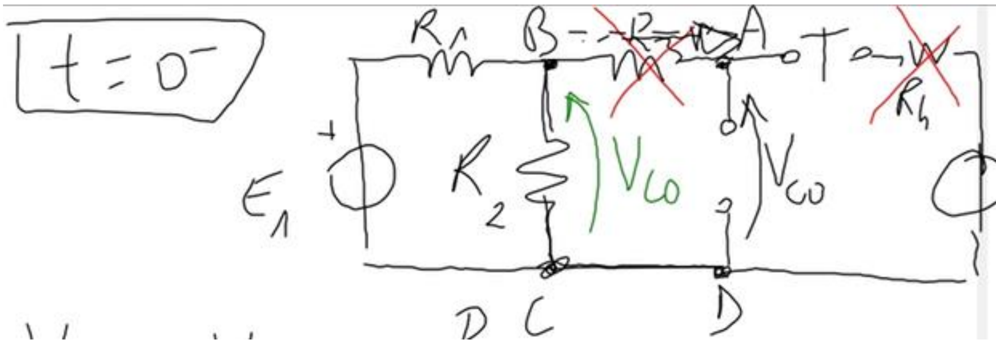


$E_1 = 8\,V \quad E_2 = 2\,V \quad R_1 = R_2 = 60\,k\Omega$

$R_3 = 30\,k\Omega \quad R_4 = 20\,k\Omega$

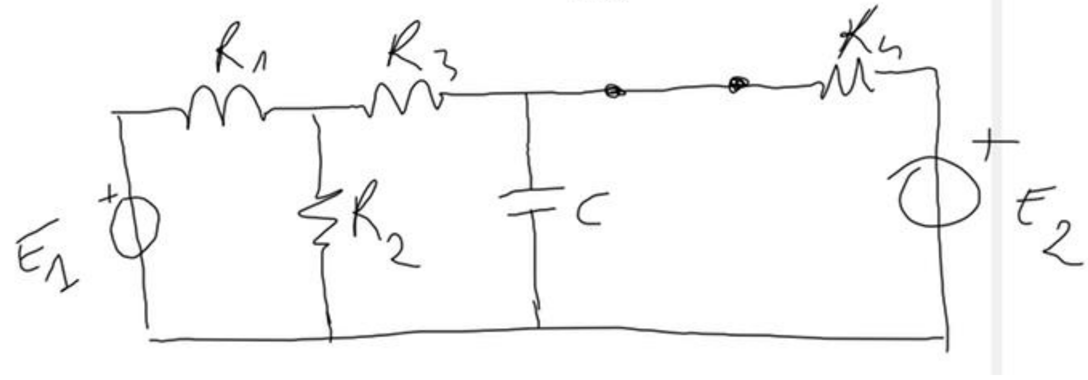
$t = 0^-$ REGIME

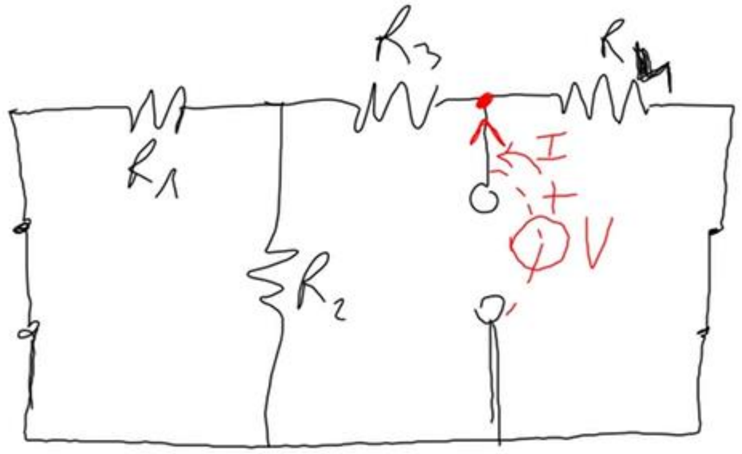
TROVA $v_c(t)$



$$V_{c0} = V_{R_2} = \frac{R_2}{R_1 + R_2} E_1 = 4\,V$$

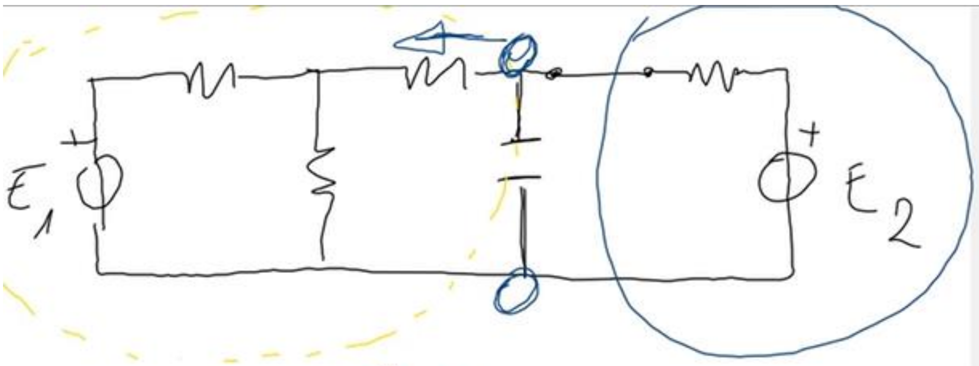
$\boxed{t=0^+}$ $V_{C0^+} = V_{C0^-} = V_{C0} = 4V$



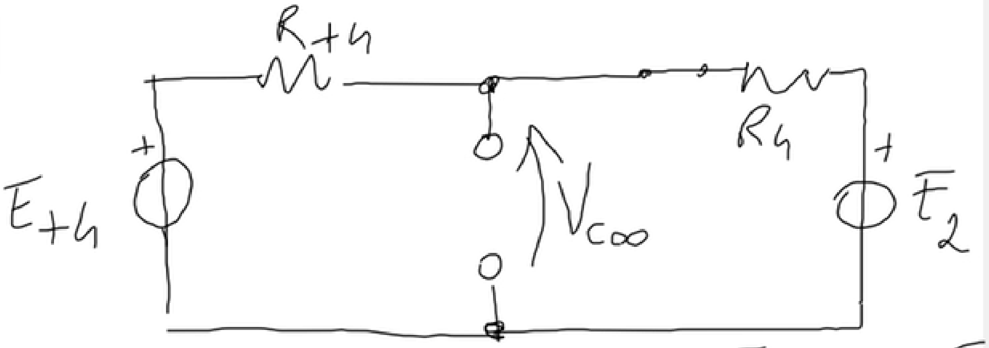


$$R_{eq} = (R_1 // R_2 + R_3) // R_4$$

$$R_{eq} = \left(\frac{R_1 R_2}{R_1 + R_2} + R_3 \right) // R_4$$



$$E_{th} = \frac{R_2}{R_1 + R_2} E_1$$

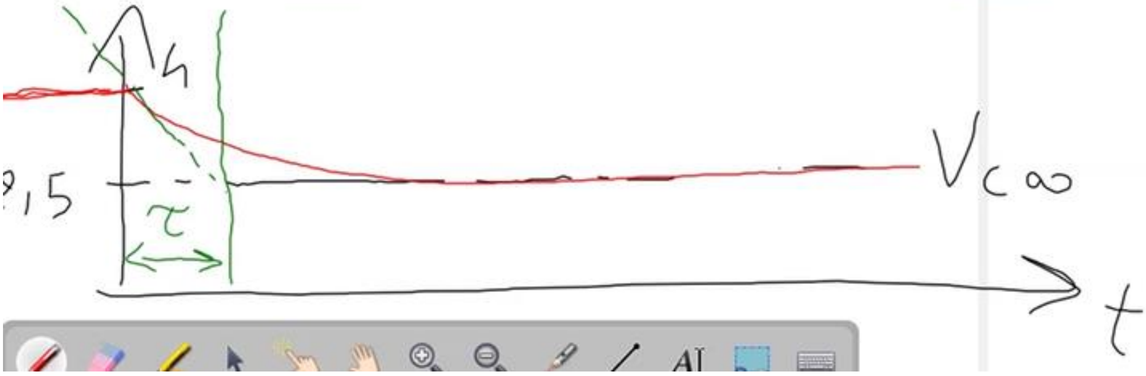


Millman $\rightarrow V_{c\infty} = \dfrac{\dfrac{E_{th}}{R_{th}} + \dfrac{E_2}{R_h}}{\dfrac{1}{R_{th}} + \dfrac{1}{R_h}}$

$= 2{,}5\ V$

$$v_C(t) = V_{C\infty} + (V_{C0} - V_{C\infty})\, e^{-t/\tau}$$

$$= \boxed{2,5 + 1,5\, e^{-t/30\,\mathrm{ns}}}$$



$$W_C(t = t^*) = \frac{1}{2} W_{C\infty} = \frac{1}{2}\left(\frac{1}{2} C V_{\infty}^2\right) =$$

$$= \overset{1/2}{\cancel{\tfrac{1}{4}}}\, \cancel{C}\, V_{\infty}^2 = \frac{1}{\cancel{2}} \cancel{C}\, v_c^2(t^*)$$

$$V_{\infty} = 4\,V > 0 \implies v_c(t^*) = \sqrt{\frac{1}{2} V_{C\infty}^2}$$

$$= \frac{V_{\infty}}{\sqrt{2}}$$

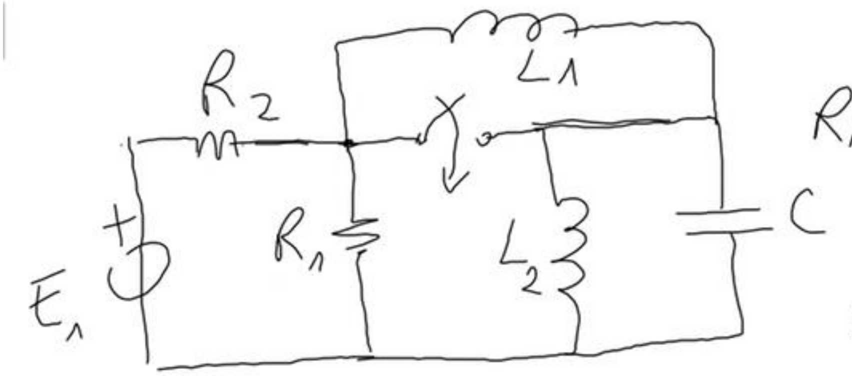
$$\frac{V_{co}}{\sqrt{2}} = 2{,}5 + 1{,}5\, e^{-t^*/30\,ms}$$

$$\Rightarrow \cancel{1{,}5}\, e^{-t^*/\tau} = \left(\frac{V_{co}}{\sqrt{2}} - 2{,}5\right)\frac{1}{1{,}5}$$

$$\Rightarrow t^* = -\ln\left(\frac{V_{co}}{\sqrt{2}} - 2{,}5\right)\frac{\tau}{1{,}5}$$

$$= 15{,}57\,ms$$

Esercizio



Dati:

$R_1 = R_2 = 10\,\Omega$

$L_1 = L_2 = 0{,}1\,H$

$C = 200\,\mu F$

$E_1 = 20\,V$

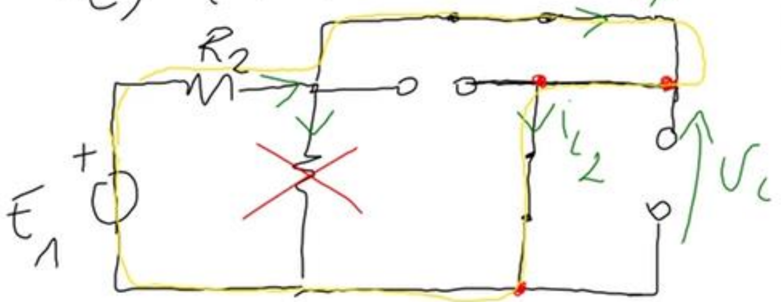
Trova

a) W_{L1}, W_{L2}, W_C per $t = 0^-$ (REGIME)

b) $\frac{di_{L1}}{dt}$ & $\frac{di_{L2}}{dt}$ per $t = 0^+$

c) W_{L1}, W_{L2}, W_C per $t \to \infty$ (REGIME)

a) $t = 0^-$



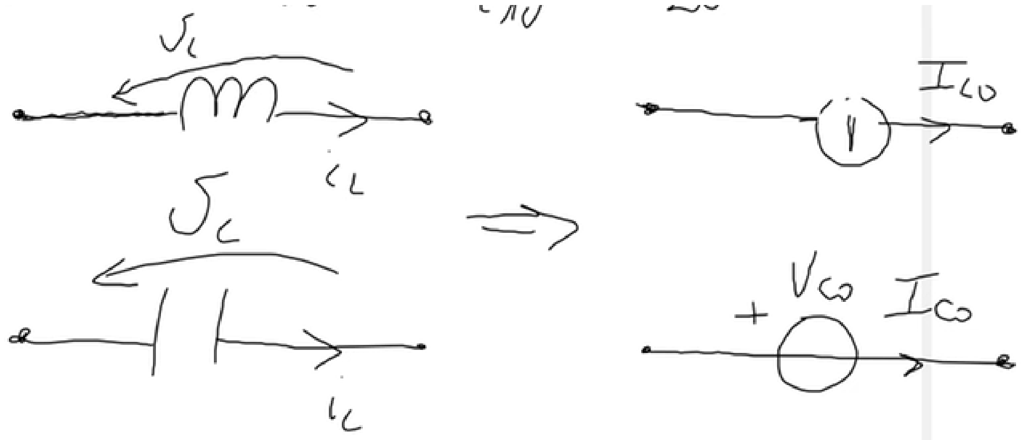
$V_{C0} = 0\,V$

$$I_{L10} = I_{L20} = \frac{E_1}{R_2} = 2\,A$$

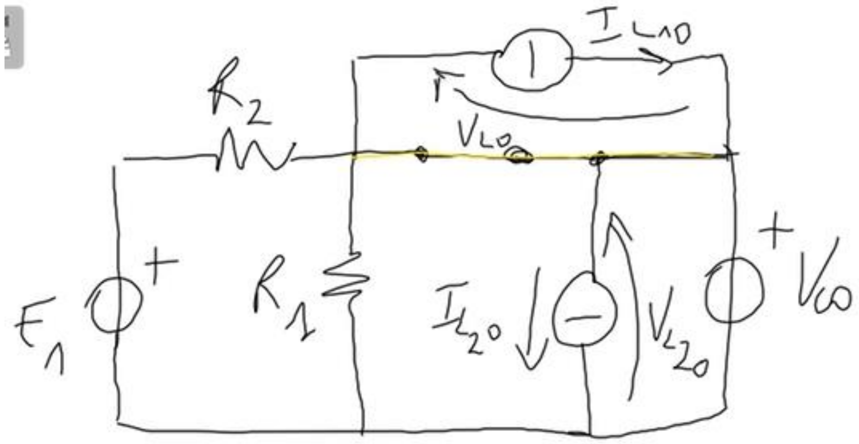
$$W_{C0} = \frac{1}{2} C V_{C0}^2 = 0 \qquad W_{L10} = W_{L2} = \frac{1}{2} \cdot 0{,}1 \cdot 2^2 = 0{,}2\,J$$

$t = 0^+$ V_{C0} I_{L10} I_{L20}

Dobbiamo tener conto degli effetti delle condizioni iniziali sul circuito.



Sostituisco i componenti nel circuito.



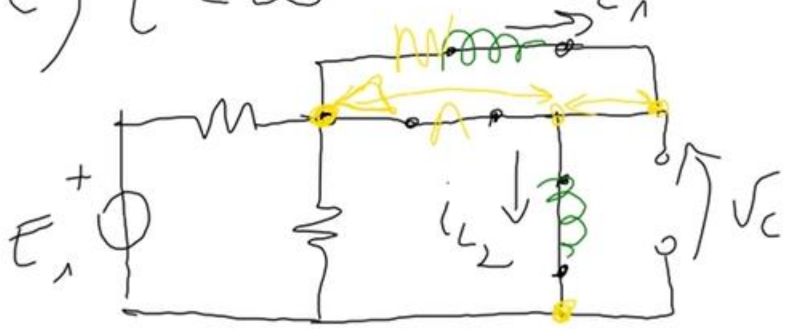
LKT

$$V_{L20} - V_{C0} = 0$$

$$\left.\frac{di_{L1}}{dt}\right|_{t=0^+} = \frac{V_{L10}}{L_1} = 0 \quad \left[\frac{A}{s}\right]$$

$$\left.\frac{di_{L2}}{dt}\right|_{t=0^+} = \frac{V_{L20}}{L_2} = 0$$

c) $t = \infty$



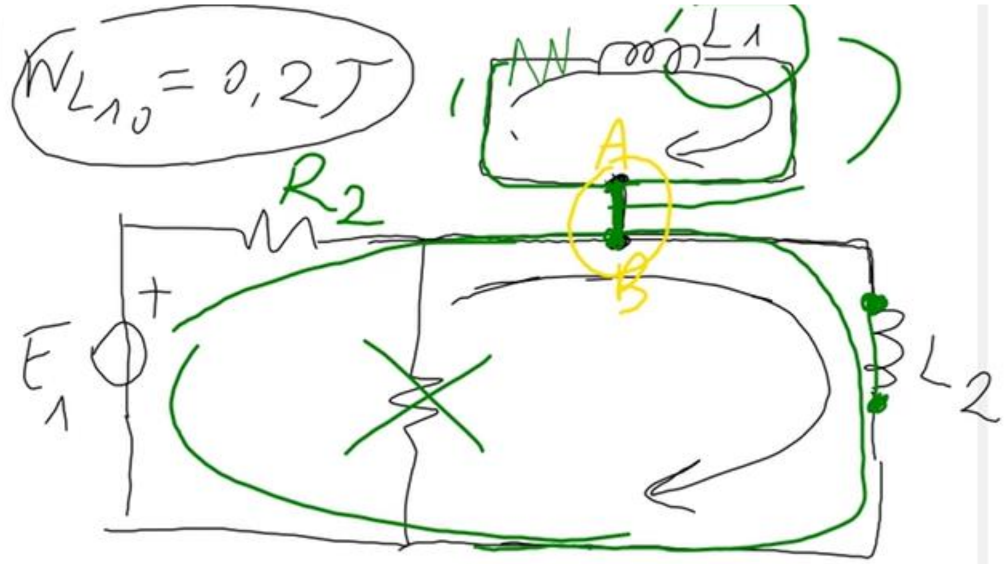
$$\frac{R}{R+R} = \frac{0}{0}$$

$$W_{C\infty} = \frac{1}{2} C V_{C\infty}^2 = 0\ J$$

$$W_{L1\infty} = \frac{1}{2} L_1 I_{L_1\infty}^2$$

$$W_{L2\infty} = \frac{1}{2} L_2 I_{L_2\infty}^2$$

$$W_{L_{10}} = 0{,}2\ J$$



$$\frac{1}{2} L i^2 \Rightarrow I_{L_1\infty} = I_{L_{10}}$$

Nei circuiti incernierati la corrente non circola.