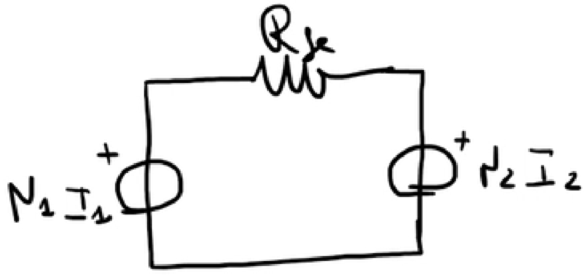
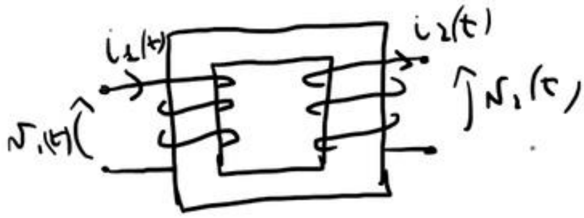
Circuito equivalente a parametri concentrati



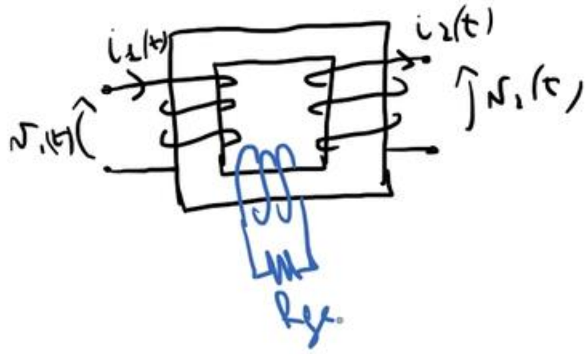
$$N_1 I_1 = \mathcal{R}_{fe}\,\Phi + N_2 I_2$$

$$\Rightarrow \quad N_1 I_1 - N_2 I_2 = \mathcal{R}_{fe}\,\Phi$$

Consideriamo le perdite nel ferro. Rifacciamo il trasformatore.



Come possiamo rappresentare una perdita per effetto Joule? Tramite una resistenza. Inseriamo un avvolgimento a cui collegare una resistenza.



- Trasf. ideale

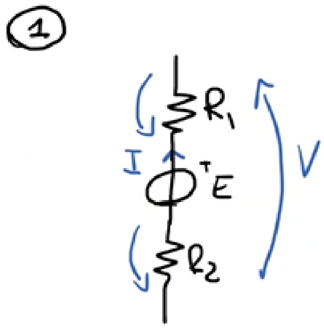
$$R_1 = R_2 = L_{d1} = L_{d2} = \mathcal{R}_{fe} = 0$$

$$\Rightarrow \begin{cases} v_1(t) = N_1 \dfrac{\partial \Phi}{\partial t} \\ v_2(t) = N_2 \dfrac{\partial \Phi}{\partial t} \end{cases} \Rightarrow \frac{v_1(t)}{v_2(t)} = \frac{N_1}{N_2} = K$$

k è il rapporto di trasformazione del trasformatore.

$$N_1 \bar{I}_1 = N_2 \bar{I}_2 \Rightarrow \frac{\bar{I}_1}{\bar{I}_2} = \frac{N_2}{N_1} = \frac{1}{K}$$

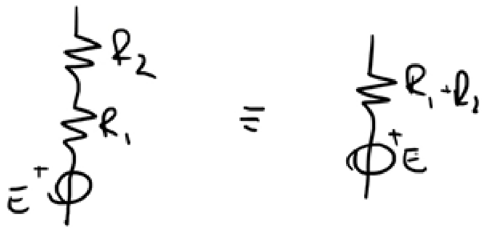
Chiarimenti



Le resistenze sono in serie.

$$-R_1 I + E - R_2 I = -R_1 I - R_2 I + E =$$
$$= -(R_1 + R_2) I + E$$

Il circuito equivale a:



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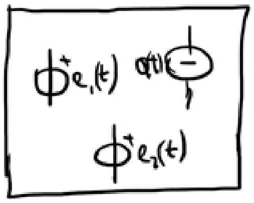

$$e_1(t) = 3\cos(\omega t + 30°)\ V$$
$$e_2(t) = 2\sin(\omega t + 45°)\ V$$
$$a(t) = \sin(\omega t - 30°)\ A$$

Abbiamo coseni e seni, tuttavia, nel dominio fasoriale deve essere tutto trasformato in seno o coseno.

Attenzione, nei fasori si usa il valore efficace.

I Fasori:

$$\underline{E}_1 = \frac{3}{\sqrt{2}} \angle 30°$$

Trasformo il seno in coseno, il seno è in anticipo rispetto al coseno di 90°.

$$e_2(t) = 2\cos(\omega t + 45° - 90°) = 2\cos(\omega t - 45°) \rightarrow \underline{E}_2 = \frac{2}{\sqrt{2}} \angle -45°$$

$$a(t) = \sin(\omega t - 30°) = \cos(\omega t - 30° - 90°) = \cos(\omega t - 120°)$$
$$\Rightarrow \underline{A} = \frac{1}{\sqrt{2}} \angle -120°$$

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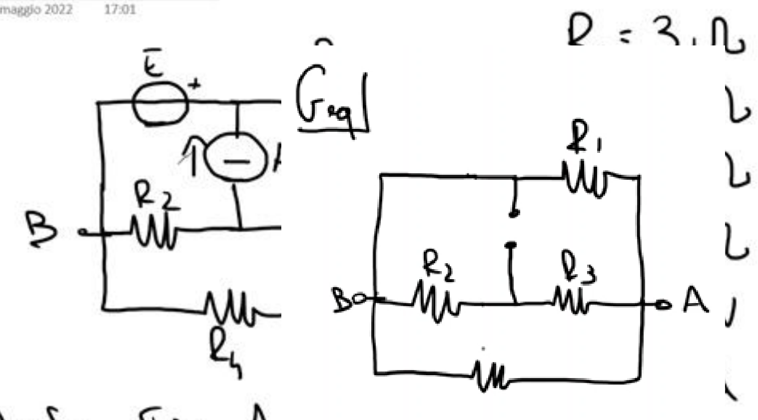


$G_{eq} = ?$

$I_{eq} = ?$

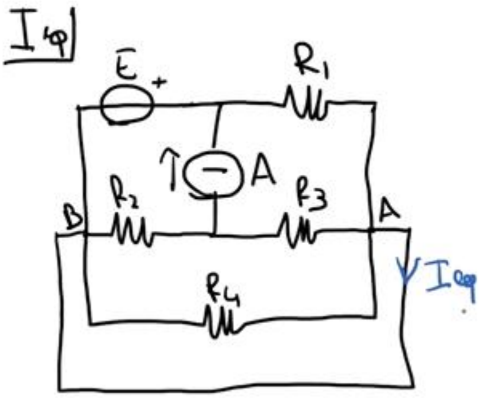
Norton tra A e B

$$G_{eq} = G_{R_1} + G_{R_4} + G_{R_2 R_3} =$$

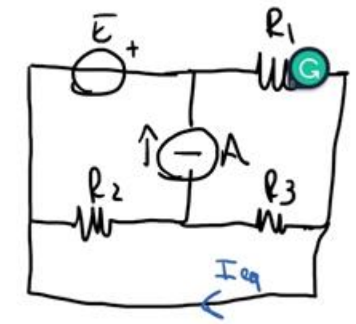
$$= \frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_2 + R_3} =$$

$$= \frac{1}{3} + \frac{1}{2} + \frac{1}{4} = \frac{4+6+3}{12} = \frac{13}{12} \, [S]$$

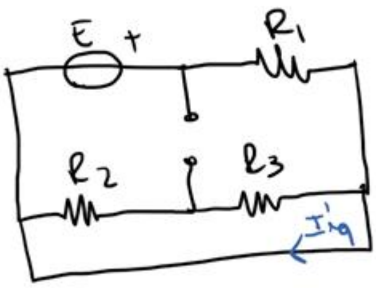
Calcoliamo la corrente equivalente, siamo in Norton, si calcola la corrente di cortocircuito.



$\equiv$

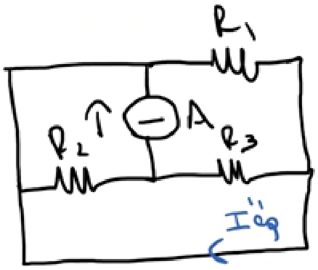


E]



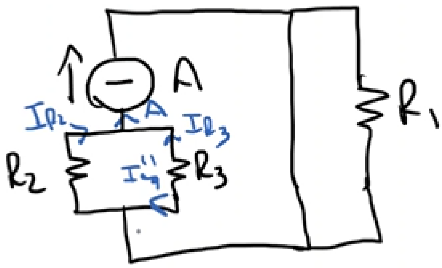
$$I'_{eq} = \frac{E}{R_1} = \frac{1}{3}\ A$$

A]



$$I''_{eq} = -\frac{R_2}{R_2+R_3}\cdot A$$

$$= -2\cdot\frac{1}{4} = -\frac{1}{2}\ A$$

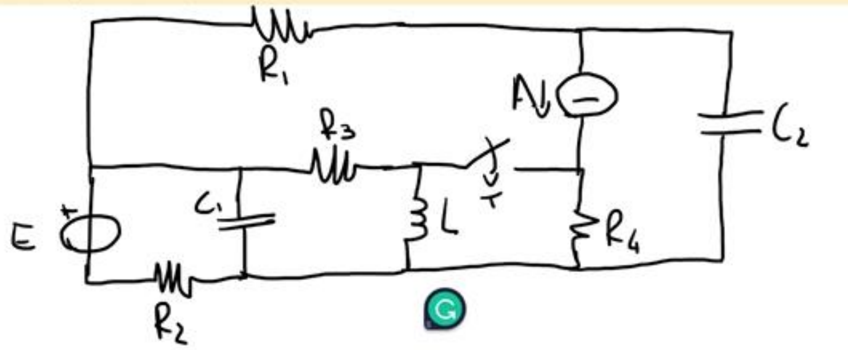


$$A = I_{R_2} + I_{R_3}$$

$$I_{eq} = I'_{eq} + I''_{eq} = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = -\frac{1}{6}\ A$$

Dobbiamo sempre mantenere il verso della I equivalente, da A verso B.

Esercizio



$E = 10\ V$

$A = 2{,}5\ A$

$R_1 = R_2 = R_3 = R_4 = 4\ \Omega$

$L = 0{,}1\ H$

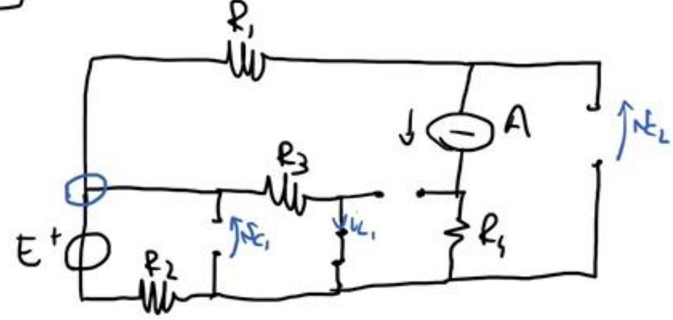
$C_1 = C_2 = 200\ \mu F$

$t = 0^-$: ε in ogni elemento con memoria

$t = 0^+$: di_L/dt

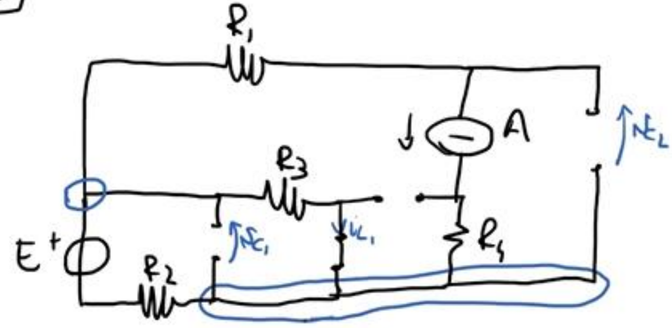
$t = \infty$: ε in ogni elem. con memoria

$t = 0^-$

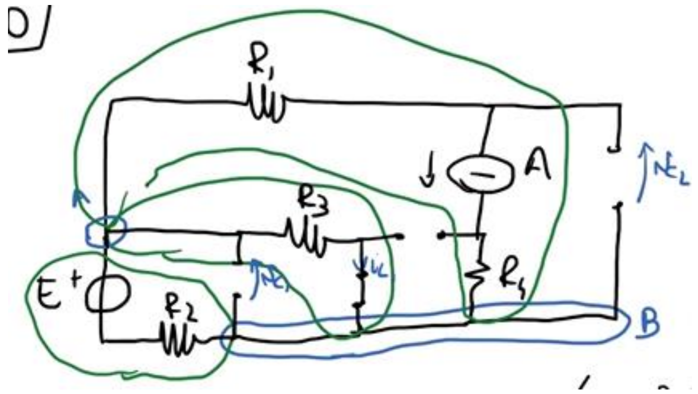


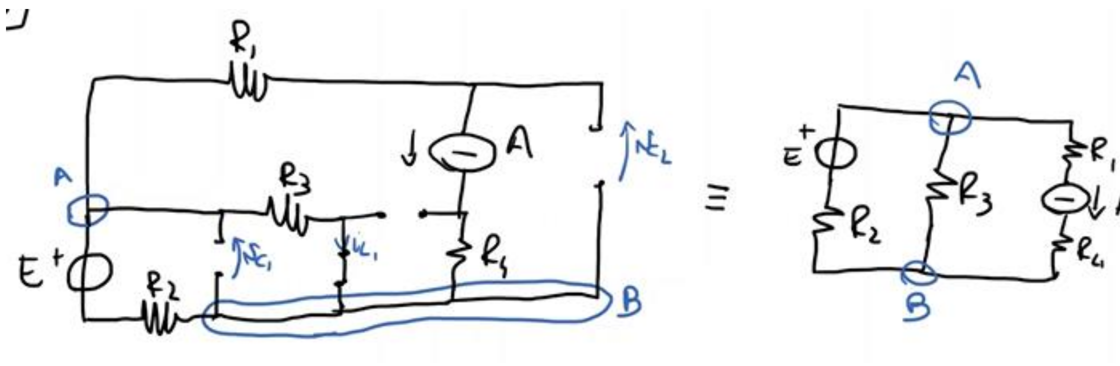
Abbiamo 3 nodi funzionali.

$t = 0^-$



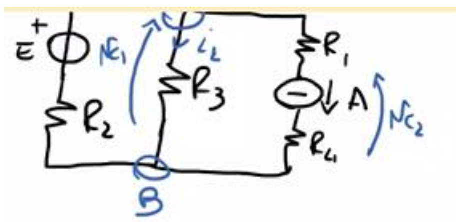
In verde, ci sono i 3 rami.





Potremmo risolverlo con Millman. Le correnti di cortocircuito di ogni ramo diviso le impedenze danno la tensione.

$$v_{AB} = \frac{E/R_2 - A}{1/R_2 + 1/R_3} = \frac{10/4 - 2{,}5}{1/4 + 1/4} = \frac{10-10}{2} = 0 \text{ V}$$



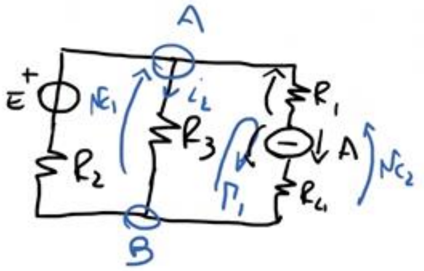
$$v_{C1}(t=0^-) = v_{AB} = 0 \text{ V}$$

$$i_L(t=0^-) = \frac{v_{AB}}{R_3} = 0 \text{ A}$$

$$v_{C2} = v_{AB} - v_{R1} = v_{AB} - R_1 A = 0 - 4 \cdot 2{,}5 = -10 \text{ V}$$

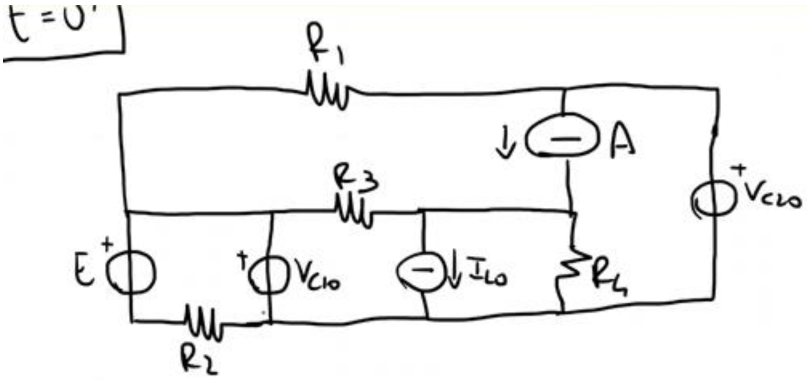
$$\mathcal{E}_{C_1} = \frac{1}{2} C_1 v_{C_1}^2 = 0 \text{ J}$$
$$\mathcal{E}_{C_2} = \frac{1}{2} C_2 v_{C_2}^2 = \frac{1}{2} \cdot 200 \cdot 10^{-6} \cdot (10)^2 = 10 \text{ mJ}$$
$$\mathcal{E}_{L} = \frac{1}{2} L i^2 = 0 \text{ J}$$

Scriviamo la LKT alla maglia M1.

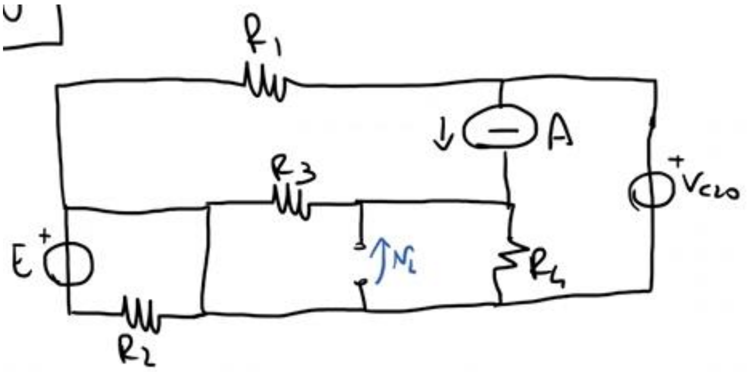


$$\text{LKT } M1: \quad v_{AB} - A \cdot R_1 + v_A - A \cdot R_4 = 0$$
$$\Rightarrow v_A = v_{AB} + A R_1 - A \cdot R_4 = 0 + 2{,}5 \cdot 4 + 2{,}5 \cdot 4 =$$
$$= +20 \text{ V}$$

$$v_{C_2} = A \cdot R_4 - v_A = 2{,}5 \cdot 4 - 20 = -10 \text{ V}$$

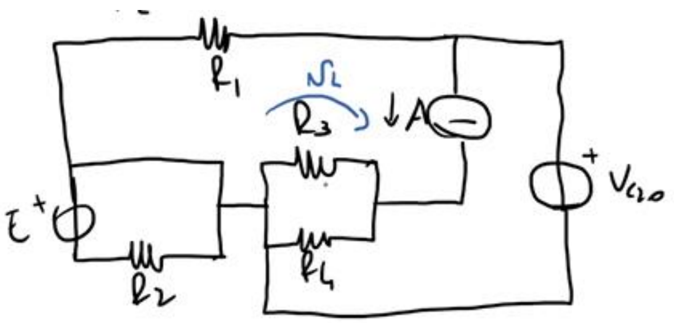


Tuttavia, sappiamo che a 0 abbiamo un corto e un aperto.



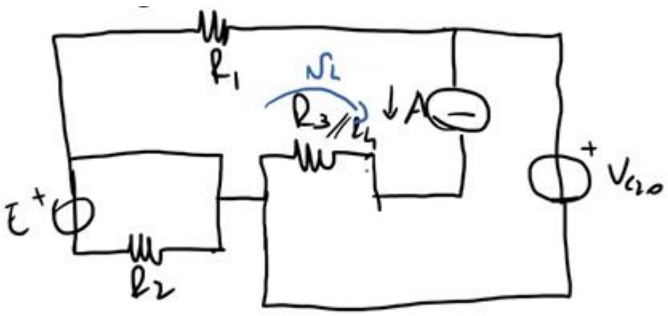
$$\frac{di_L}{dt} = \frac{v_L}{L}$$

Dobbiamo calcolare la VL.



$$v_L = A \cdot R_3 /\!/ R_4 = 2{,}5 \cdot 2 = 5 \ V$$

$$\frac{di_L}{dt} = \frac{v_L}{L} = \frac{5}{0{,}1} [A/s] = 50 [A/s]$$



VL è semplicemente A per R3 parallelo a R4, perché A circola solo in R3 parallelo a R4.

Per t uguale a infinito, si applica lo stesso procedimento.

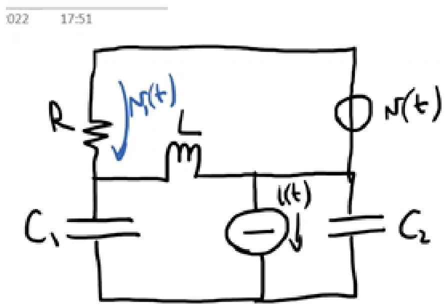
$t = \infty)$

$$\mathcal{E}_{C_1} = 0\ J$$

$$\mathcal{E}_{L} = 0{,}3125\ J$$

$$\mathcal{E}_{C_2} = 10\ mJ$$

Esercizio



$i(t) = \cos(t - 120°)\ A$

$v(t) = 3 \sin(t - 90°)\ V$

$R_1 = 2\ \Omega \qquad L = 1\ H$

$C_1 = 3\ F \qquad C_2 = 1\ F$

1) Fasori di $i(t)$ e $v(t)$
2) Calcolare tutte le impedenze
3) Calcolare l'impedenza equivalente vista da $i(t)$
4) Calcolare il fasore $\underline{V}_1$
5) Calcolare $v_1(t)$

① $\underline{I} = \frac{1}{\sqrt{2}} \angle -120°$

$\underline{V} = \frac{3}{\sqrt{2}} \angle -180°$

② $\underline{Z}_{R_1} = 2\ \Omega$

$\underline{Z}_{C_1} = -\frac{j}{\omega C} = -\frac{1}{3} j$

$\underline{Z}_{C_2} = -j$

$\underline{Z}_L = j$

③ $\underline{Z}_{eq} = 0{,}9 + 0{,}2j$

④ $\underline{V}_1 = 1{,}85 \angle -77{,}05°$

⑤ $v_1(t) = 2{,}62 \cos(t - 77{,}05°)$