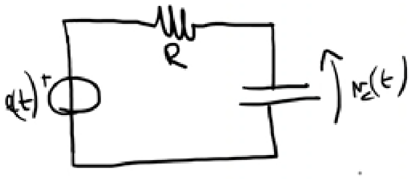
Filtro passa-basso



Vogliamo vedere come la tensione sul condensatore dipende dalla tensione di ingresso.

$$\underline{H}(\omega) = \frac{\underline{V_c}(\omega)}{\underline{E}(\omega)}$$

$$\underline{V_c} = \underline{E} \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1}\underline{E}$$

$$\underline{H} = \frac{1}{1 + j\omega CR}$$

Possiamo calcolare il modulo e la fase di H.

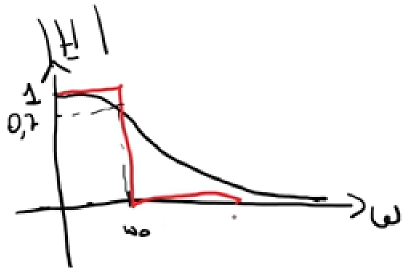
$$\underline{H} = \frac{1}{1+j\omega CR} \rightarrow |\underline{H}| = \frac{1}{\sqrt{1+\omega^2C^2R^2}}$$

$$\rightarrow \underline{H} = \frac{1}{1+j\omega CR} \cdot \frac{1-j\omega CR}{1-j\omega CR} =$$

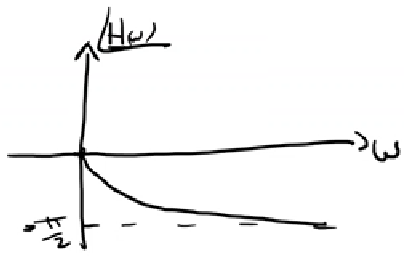
$$= \frac{1-j\omega CR}{1+\omega^2C^2R^2} \rightarrow \angle\underline{H} = \tan^{-1}\left(\frac{-\omega CR}{1}\right)$$

Andiamo a graficare il modulo di H.

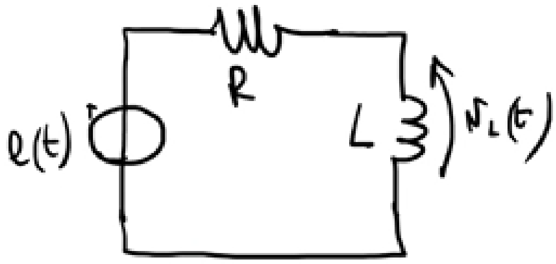
Perché 0.7? Perché a omega0 abbiamo un guadagno di 0.7, perdiamo il 30% del segnale. La cosa ideale che vorremmo è questa, ma non si può ottenere a meno che non si ragioni con ordini superiori, ma non è questo il caso.



Vediamo la fase. Se omega è minore di omega0, possiamo considerare che queste frequenze escono, non vengono tagliate.



Filtro passa-alto



Definiamo la H.

$$\underline{H}(\omega) = \frac{\underline{V}_L}{\underline{E}}$$

V di L è un partitore tra impedenze.

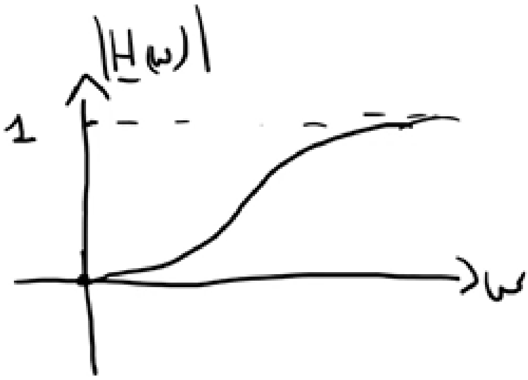
$$\underline{V}_L = \frac{j\omega L}{R + j\omega L}\,\underline{E} \rightarrow \underline{H}(\omega) = \frac{j\omega L}{R + j\omega L}$$

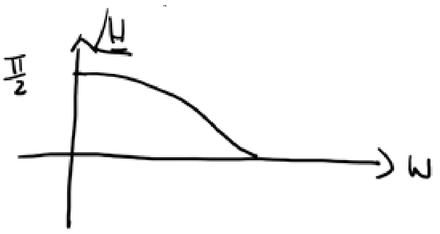
$$|\underline{H}| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$

$$\underline{H} = \frac{j\omega L}{R + j\omega L} \cdot \frac{R - j\omega L}{R - j\omega L} = \frac{j\omega L R + \omega^2 L^2}{R^2 + \omega^2 L^2}$$

$$\angle \underline{H} = \mathrm{Tan}^{-1}\left(\frac{\omega L R}{\omega^2 L^2}\right)$$

Vediamo il modulo e la fase di H.

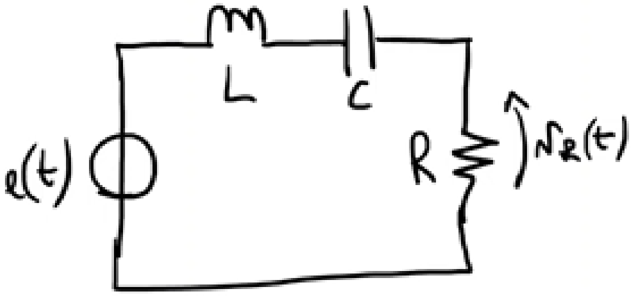




$\angle H$
$\frac{\pi}{2}$
ω

Filtro passa-banda

Consideriamo una serie di un induttore, un condensatore e un resistore.



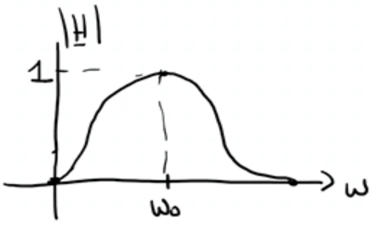
$$\underline{H}(\omega) = \frac{\underline{V}_R}{\underline{E}}$$

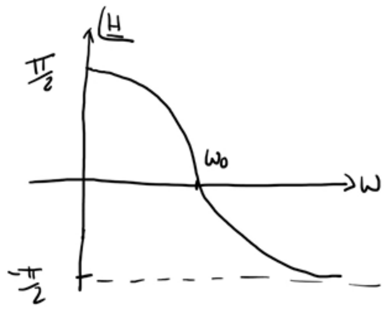
$$\underline{V}_R = \frac{R}{R + j\omega L - \frac{j}{\omega C}} \underline{E} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$|\underline{H}| = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

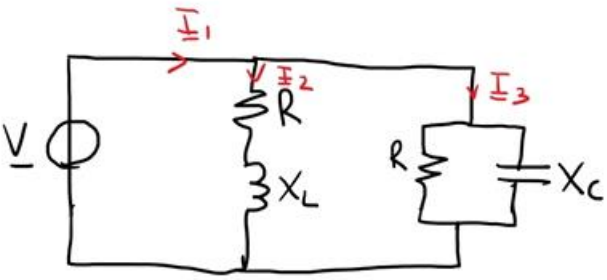
$$\underline{H} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \cdot \frac{R - j\left(\omega L - \frac{1}{\omega C}\right)}{R - j\left(\omega L - \frac{1}{\omega C}\right)} =$$

$$= \frac{R^2 - jR\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \rightarrow \angle\underline{H} = \tan^{-1}\left(\frac{-R\left(\omega L - \frac{1}{\omega C}\right)}{R^2}\right)$$





Esercizio



$V = 100\,V$

$R = 10\,\Omega$

$X_L = 10\,\Omega$

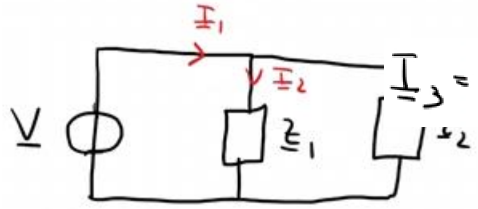
$X_C = -10\,\Omega$

$\underline{I}_1 = ?\quad \underline{I}_2 = ?\quad \underline{I}_3 = ?$

$$\underline{V} = 100\, e^{j0}$$

$$\underline{Z}_1 = j10 + 10\,\Omega$$

$$\underline{Z}_2 = \frac{-10j \cdot 10}{-10j + 10} = \frac{-100j}{10 - 10j} = 5 - 5j$$



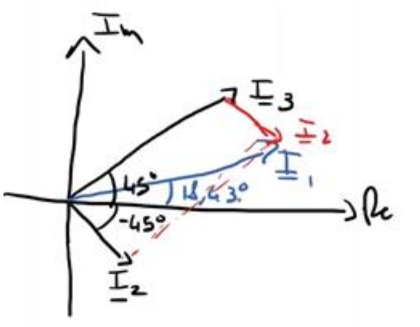
$$\underline{I}_3 = 10 + 10j \qquad \frac{\underline{V}}{\underline{Z}_1} = 7{,}07\, e^{-j45^\circ} = 5 - 5j$$

$$\underline{I}_1 = \underline{I}_2 + \underline{I}_3 = (10 + 5) + j(10 - 5) = 15 + 5j$$

$$= 15{,}81 \angle 18{,}43^\circ$$

$$\underline{I}_3 = 10 + 10j = 14{,}14 \angle 45^\circ$$

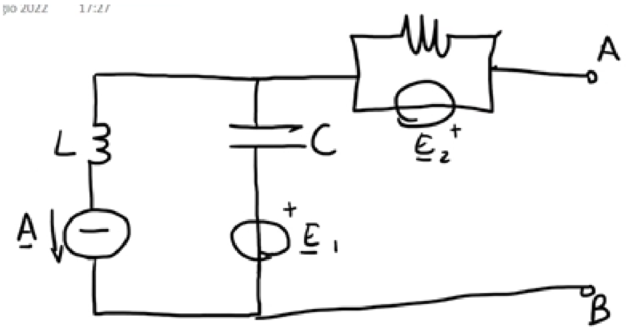
Rappresentiamo i fasori nel piano complesso.



La somma dei fasori è fattibile anche in questo modo, tutto ci deve tornare ovviamente. Il risultato ha una fase minore e un modulo maggiore dei due.

Esercizio

gio 2022 17:27

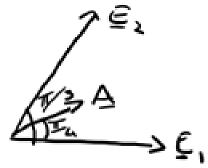


$E_1 = 5\ V \qquad E_2 = 10\ V$

$A = 3A \qquad f = 50\ Hz$

$L = 10\ mH \qquad C = 20\ mF$

$R = 10\ \Omega$



$= 5\ V$

$\underline{E}_1 = 5\ V$

$\underline{E}_2 = 10\, e^{\frac{\pi}{3}j}\ V$

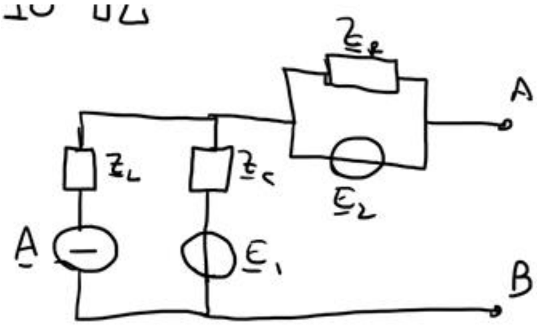
$\underline{A} = 3\, e^{\frac{\pi}{4}j}\ A$

$\underline{Z}_L = \pi j\ \Omega$

$\underline{Z}_C = -\frac{j}{2\pi}\ \Omega$

$\underline{Z}_R = 10\ \Omega$

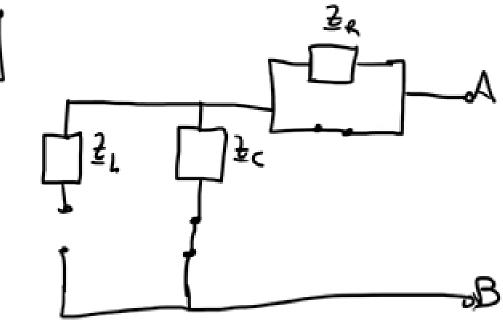
10 Ω

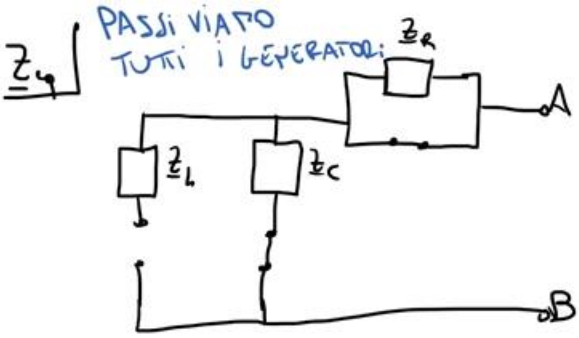


$\underline{A} = 3\, e^{\frac{\pi}{4}j}\ A$

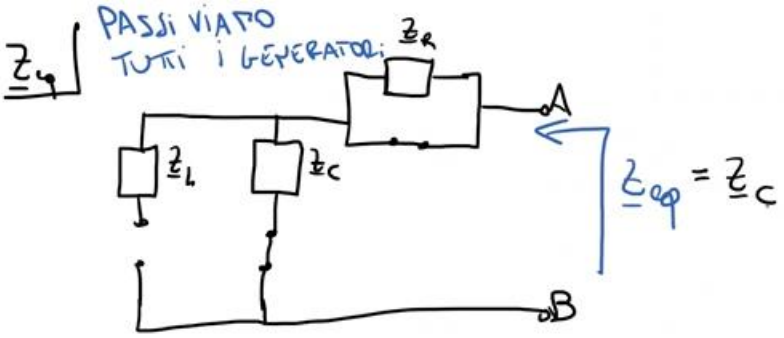
$\underline{Z}_L = \pi j\ \Omega \rightarrow \underline{Z}_L = jX_L = j\omega L = j\, 2\pi \cdot 50 \cdot 10 \cdot 10^{-3} = j\,\pi \cdot \cancel{(100 \cdot 10^{-3})}$

$\underline{Z}_{eq}|$

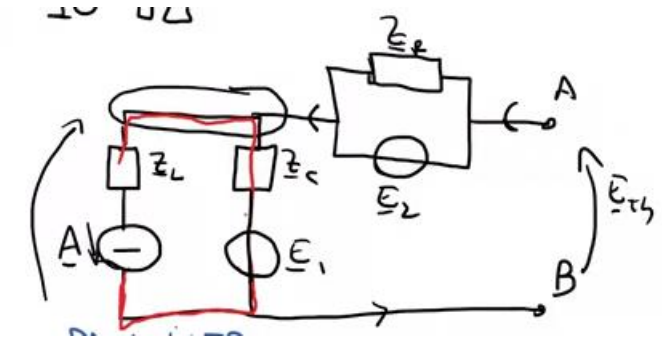




Quando calcoliamo l’impedenza equivalente, passiviamo tutti i generatori.



Facciamo la LKT alla maglia rossa per calcolare la E equivalente.



Attenzione: prima di fare LKT o sovrapposizione degli effetti, abbiamo diversi modi di risolvere un circuito, scegliamo quale conviene di più.

$\underline{E}_{eq}$]

$$\underline{E}_1 - \underline{V}_C + \underline{E}_2 - \underline{E}_{TH} = 0$$

Facciamo anche la LKC sul nodo evidenziato.

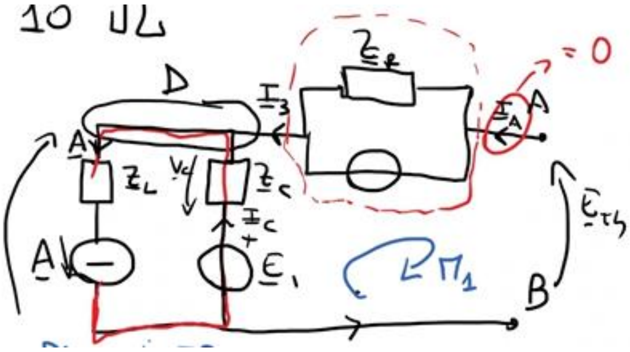
$\underline{E}_{TH}$

$$\underline{E}_1 - \underline{V}_C + \underline{E}_2 - \underline{E}_{TH} = 0$$

$$\begin{cases} \underline{E}_1 - \underline{Z}_C \underline{I}_C + \underline{E}_2 - \underline{E}_{TH} = 0 & \text{LKT } \Pi 1 \\ \underline{I}_3 + \underline{I}_C - \underline{A} = 0 & \text{LKC } D \end{cases}$$

$$\underline{I}_3 = 0 \Rightarrow \underline{I}_C = \underline{A}$$

10 UL



$\underline{I}_A = \underline{I}_3$

$\Rightarrow \underline{I}_3 = 0$

$$\underline{E}_{TH} = \underline{E}_1 + \underline{E}_2 - \underline{Z}_C \underline{A} = 13{,}20 \angle 43^\circ \text{ V}$$

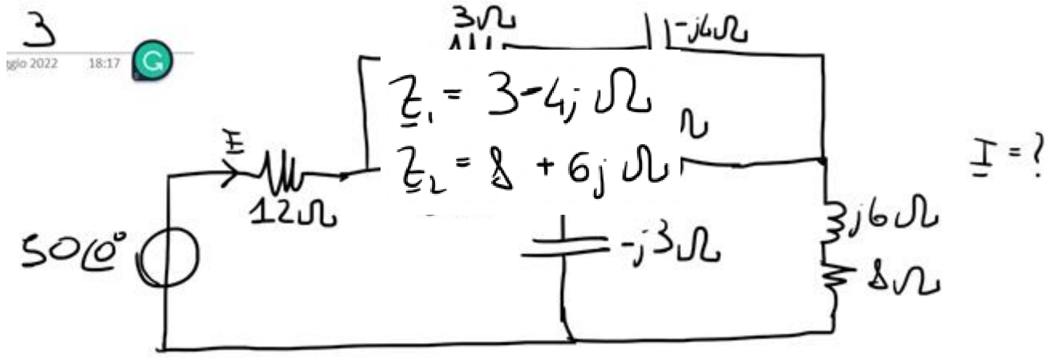
$$e_{th}(t) = 13{,}20\sqrt{2} \cos(\omega t + 43^\circ) \text{ V}$$

$$t^* = \frac{T}{4} \rightarrow e_{th}(t^*) = 13{,}2\sqrt{2} \cos\left(\frac{2\pi}{\cancel{T}} \cdot \frac{\cancel{T}}{4} + 43^\circ\right) \text{ V}$$

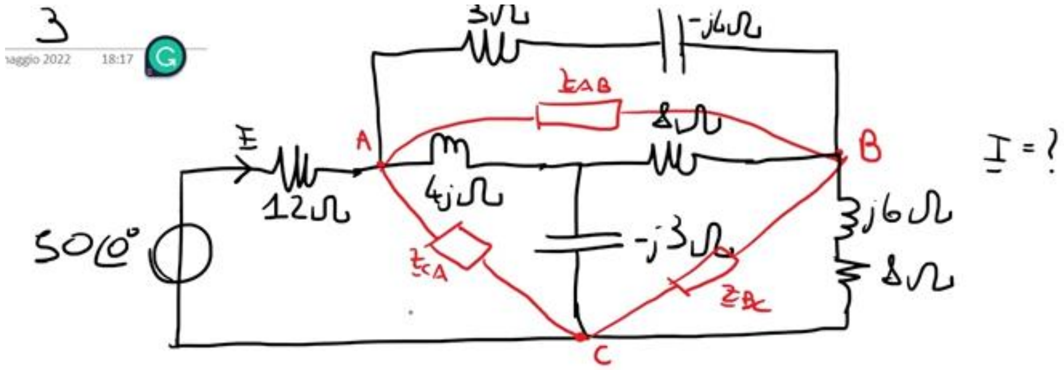
$$= 13{,}2\sqrt{2} \cos(90^\circ + 43^\circ) \text{ V} = 13{,}2\sqrt{2} \cos(133^\circ)$$

$$= -12{,}73 \text{ V}$$

Esercizio



Abbiamo un collegamento a stella, lo trasformiamo in un collegamento a triangolo.



$$\underline{Z}_{AB} = \frac{\underline{Z}_A \cdot \underline{Z}_B + \underline{Z}_B \underline{Z}_C + \underline{Z}_C \cdot \underline{Z}_A}{\underline{Z}_C}$$

$$\underline{Z}_{BC} = \frac{\underline{Z}_A \cdot \underline{Z}_B + \underline{Z}_B \cdot \underline{Z}_C + \underline{Z}_C \cdot \underline{Z}_A}{\underline{Z}_A}$$

$$\underline{Z}_{CA} = \frac{\underline{Z}_A \cdot \underline{Z}_B + \underline{Z}_B \cdot \underline{Z}_C + \underline{Z}_C \cdot \underline{Z}_A}{\underline{Z}_B}$$

$$\underline{Z}_A \cdot \underline{Z}_B + \underline{Z}_B \cdot \underline{Z}_C + \underline{Z}_C \cdot \underline{Z}_A = 4j \cdot 8 + 8(-3j) + (4j)(-3j) =$$

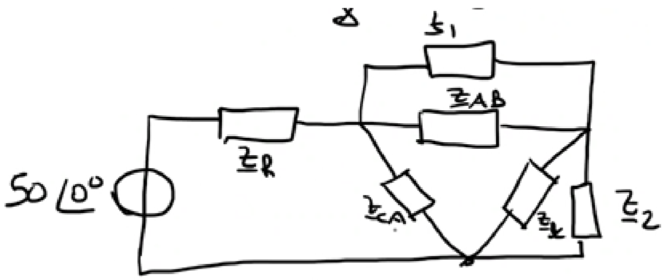
$$= 32j - 24j + 12 = 12 + 8j$$

$$\underline{Z}_{AB} = \frac{12+8j}{-3j} = \frac{12j-8}{3} = -\frac{8}{3} + 4j$$

C'era un modo più veloce della stella-triangolo? Solo se le impedenze fossero state uguali. In questo caso, no.

$$\underline{Z}_{BC} = \frac{12+8j}{4j} = \frac{12j-8}{-4} = 2-3j$$

$$\underline{Z}_{CA} = \frac{12+8j}{8} = \frac{3}{2}+j$$



$$\underline{Z}_{BC} \,//\, \underline{Z}_2$$

$$\underline{Z}_{AB} \,//\, \underline{Z}_1$$

$$\underline{Z}_{BC} \,//\, \underline{Z}_2 = \frac{(8+j6)(2-3j)}{10+3j} = 2{,}79-2{,}04j \ \Omega$$

$$\underline{Z}_{AB} \,//\, \underline{Z}_1 = \frac{(3-4j)(-\frac{8}{3}+4j)}{3-\cancel{4j}-\frac{8}{3}+\cancel{4j}} = 24+68j \ \Omega$$

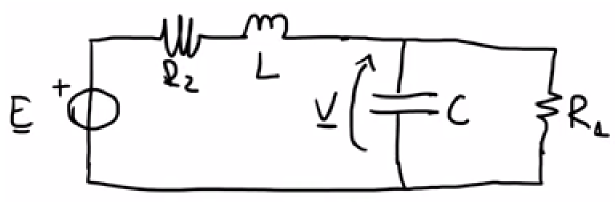
$$\underline{Z}_{eq_2} = \underline{Z}_{BC}//\underline{Z}_2 + \underline{Z}_{AB}//\underline{Z}_1 = 26{,}79+65{,}96j \ \Omega$$

$$\underline{Z}_{eq_3} = \underline{Z}_{CA} // \underline{Z}_{eq_2} = \frac{(26{,}79+65{,}96j)(\frac{3}{2}+j)}{\ldots} = 1{,}46+j$$

$$\underline{Z}_{eq} = \underline{Z}_{eq_3} + \underline{Z}_R = 13{,}46+j \ \Omega$$

$$\underline{I} = \frac{\underline{E}}{\underline{Z}_{eq}} = \frac{50\angle 0°}{13{,}46+j} = 3{,}69-0{,}27j \quad A$$

Esercizio



$$V = 400\ V \qquad L = 0{,}3\ H$$

$$R_1 = 50\ \Omega \qquad R_2 = 20\ \Omega$$

$$C = 30\ \mu F \qquad f = 50\ Hz$$

Calcolare la potenza generata da E.

Ci sono 2 metodi per risolverlo. Vediamo il primo. Dobbiamo calcolare una potenza complessa.

$$\underline{N}_E = \underline{E} \cdot \underline{I}^*$$

Dobbiamo calcolare la E e la I.

1) Calcolo $\underline{I}$

$$\underline{I} = \frac{V}{\underline{Z}_{eq}} \qquad \underline{Z}_{eq} = \frac{R_1 (jX_c)}{R_1 + jX_c}$$

$$\underline{Z}_{eq} = \frac{R_1 (jX_c)}{R_1 + jX_c} = \frac{-R_1 \left(j \frac{1}{\omega C}\right)}{R_1 + j\left(-\frac{1}{\omega C}\right)} =$$

$$= - \frac{50 \left(j \frac{1}{314{,}16 \cdot 30 \cdot 10^{-6}}\right)}{50 - j \frac{1}{314{,}16 \cdot 30 \cdot 10^{-5}}} =$$

$$= 41 - 19j\ \Omega$$

$$\underline{I} = \frac{400}{41 - 19j} = 8{,}03 + j\,3{,}72\ A$$

2) Calcolo $\underline{E}$

$$\underline{E} = \underline{V} + \underline{Z}_2 \cdot \underline{I}$$

$$\hookrightarrow \underline{Z}_2 = R_2 + jX_L = 20 + j\omega L = 20 + j94,2 \ \Omega$$

$$\underline{E} = 400 + (20 + 94,2j)(8,03 + 3,72j) = 210,18 + 830,83j \ V$$

Ora che abbiamo E ed I, andiamo a calcolare N.

3) Calcolare $\underline{N}$

$$\underline{N} = \underline{E} \cdot \underline{I}^* = 4,78 + 5,89j \ kVA$$

4,78 è in kilowatt, 5,89 è in kilovoltampere reattivi, si misura in kilovoltampere.