MATTEO LOMBARDI

1. $f(x,y) = e^{x^2+y} - x^2 + e^{-(y+1)}$

- $\nabla f(x,y)$
  - $\partial_x f = \partial_x(e^{x^2+y}) - 2x = 2x(e^{x^2+y} - 1)$
    - $\partial_x(e^{x^2+y}) = 2xe^{x^2+y}$

$f(x) = e^x \quad f'(x) = e^x$

$g(x) = x^2+y \quad g'(x) = 2x$

- $\partial_y f = e^{x^2+y} + \partial_y(e^{-(y+1)}) = e^{x^2+y} - e^{-(y+1)}$

$f(y) = e^y \quad f'(y) = e^y$

$g(y) = -y-1 \quad g'(y) = -1$

$\nabla f(x,y) = \left(2x(e^{x^2+y}-1),\ e^{x^2+y} - e^{-(y+1)}\right)$

- $\nabla f(x,y) = 0$

$$\begin{cases} 2x(e^{x^2+y}-1) = 0 \\ e^{x^2+y} - e^{-(y+1)} = 0 \end{cases}$$

$x = 0$ (A) $\vee$ $x^2+y = 0 \Rightarrow y = -x^2$ (B)

(A) $x = 0 \Rightarrow e^y - e^{-y-1} = 0 \quad e^y = e^{-y-1} \quad y = -y-1$

$2y = -1 \quad y = -\frac{1}{2} \quad \left(0, -\frac{1}{2}\right)$

(B) $y = -x^2 \Rightarrow e^{x^2-x^2} - e^{x^2-1} = 0 \quad e^{x^2-1} = 1 \quad x^2-1 = 0$

$x^2 = 1 \quad x = \pm 1 \quad (-1,-1) \quad (+1,-1)$

$y = -1 \quad y = -1$

I PUNTI CRITICI SONO: $(0, -\frac{1}{2}), (-1,-1), (+1,-1)$

- $Hf(x,y) = \begin{pmatrix} \partial_x(2x(e^{x^2+y}-1)), & \partial_y(2x(e^{x^2+y}-1)) \\ \partial_x(e^{x^2+y} - e^{-(y+1)}), & \partial_y(e^{x^2+y} - e^{-(y+1)}) \end{pmatrix}$

- $\partial_x(2x(e^{x^2+y}-1)) = 2e^{x^2+y} - 2 + 4x^2 e^{x^2+y}$

  $f(x) = 2x \qquad f'(x) = 2$

  $g(x) = e^{x^2+y} - 1 \qquad g'(x) = 2x e^{x^2+y}$

- $\partial_y(2x(e^{x^2+y}-1)) = \partial_y(2xe^{x^2+y} - 2x) = 2xe^{x^2+y}$

  $= \partial_x(e^{x^2+y} - e^{-(y+1)})$

  PER IL TEOREMA DI SCHWARZ

- $\partial_y(e^{x^2+y} - e^{-(y+1)}) = e^{x^2+y} + e^{-(y+1)}$

$$Hf(x,y) = \begin{pmatrix} 2e^{x^2+y} - 2 + 4x^2 e^{x^2+y} & 2xe^{x^2+y} \\ 2xe^{x^2+y} & e^{x^2+y} + e^{-(y+1)} \end{pmatrix}$$

- $(0, -\frac{1}{2})$ $\quad Hf(0, -\frac{1}{2}) = \begin{pmatrix} 2e^{-\frac{1}{2}} - 2 & 0 \\ 0 & e^{-\frac{1}{2}} + e^{-\frac{1}{2}} \end{pmatrix}$

$$= \begin{pmatrix} \frac{2}{\sqrt{e}} - 2 & 0 \\ 0 & \frac{2}{\sqrt{e}} \end{pmatrix}$$

$\det(Hf(0, -\frac{1}{2}))$

$= \underbrace{\left(\frac{2}{\sqrt{e}} - 2\right)}_{<0} \underbrace{\left(\frac{2}{\sqrt{e}}\right)}_{>0} < 0 \Rightarrow$

$Hf(0, -\frac{1}{2})$ è INDEFINITA

$\Rightarrow$

$\boxed{(0, -\frac{1}{2}) \text{ è UN PUNTO DI SELLA}}$

• $(-1,-1)$ $\quad Hf(-1,-1) = \begin{pmatrix} 2-2+4 & -2 \\ -2 & 1+1 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ -2 & 2 \end{pmatrix}$

$$\left.\begin{array}{l} \det(Hf(-1,-1)) = 8-4 = 4 > 0 \\ a = 3 > 0 \end{array}\right\} \Rightarrow Hf(-1,-1) > 0$$

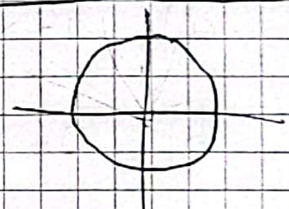
$$\Rightarrow \boxed{(-1,-1) \text{ PUNTO DI MINIMO RELATIVO}}$$

• $(+1,-1)$ $\quad Hf(+1,-1) = \begin{pmatrix} 2-2+4 & 2 \\ 2 & 1+1 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 2 & 2 \end{pmatrix}$

$$\left.\begin{array}{l} \det(Hf(+1,-1)) = 8-4 = 4 > 0 \\ a = 4 > 0 \end{array}\right\} \Rightarrow Hf(+1,-1) > 0$$

$$\Rightarrow \boxed{(+1,-1) \text{ PUNTO DI MINIMO RELATIVO}}$$

2. $f(x,y) = \sin(\pi + xy^2) \qquad (\bar{x},\bar{y}) = \left(-\frac{\pi}{6}, 1\right)$



• $v_{MAX} = \dfrac{\nabla f(\bar{x},\bar{y})}{|\nabla f(\bar{x},\bar{y})|}$

• $\nabla f(x,y) = \left(y^2 \cos(\pi + xy^2),\ 2xy\cos(\pi + xy^2) \right)$

• $\nabla f\left(-\frac{\pi}{6}, 1\right) = \left(\cos\left(\pi - \frac{\pi}{6}\right),\ -\frac{\cancel{2}\pi}{\cancel{6}_3} \cos\left(\pi - \frac{\pi}{6}\right) \right)$

$= \left(\cos\left(\frac{5}{6}\pi\right),\ -\frac{\pi}{3}\cos\left(\frac{5}{6}\pi\right) \right) = \left(-\frac{\sqrt{3}}{2},\ \frac{\pi}{3}\frac{\sqrt{3}}{2} \right)$

$= \left(-\frac{\sqrt{3}}{2},\ \frac{\sqrt{3}\pi}{6} \right)$

$\cancel{v_{MAX} = -\frac{\sqrt{3}}{2}}$ $\quad$ • $\left|\nabla f\left(-\frac{\pi}{6}, 1\right)\right| = \sqrt{\frac{3}{4} + \frac{3\pi^2}{36_{12}}} = \sqrt{\frac{9+\pi^2}{12}}$

$v_{MAX} = \left(-\frac{\sqrt{3}}{2} \cdot \sqrt{\frac{12}{9+\pi^2}},\ \frac{\sqrt{3}}{6}\pi \cdot \sqrt{\frac{12}{9+\pi^2}} \right)$

$= \left(-\frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}\sqrt{3}}{\sqrt{9+\pi^2}},\ \frac{\cancel{\sqrt{3}}}{\cancel{3}\cancel{6}}\pi \cdot \frac{\cancel{2}\cancel{\sqrt{3}}}{\sqrt{9+\pi^2}} \right) = \boxed{\left(-\frac{3}{\sqrt{9+\pi^2}},\ \frac{\pi}{\sqrt{9+\pi^2}} \right)}$

- $\dfrac{\partial f_1}{\partial v_{MAX}}\left(-\dfrac{\pi}{6}, 1\right) = \|\nabla f_1(-\frac{\pi}{6}, 1)\|_2 = \boxed{\sqrt{\dfrac{9+\pi^2}{12}}}$

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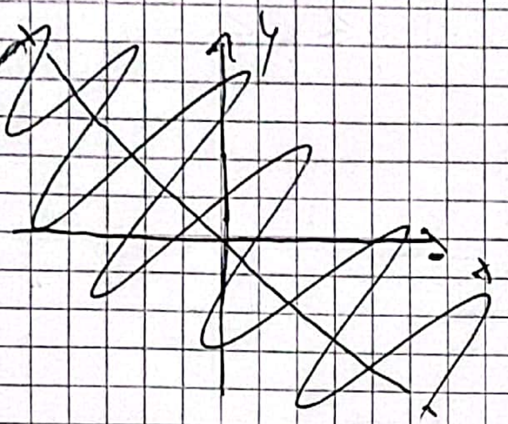
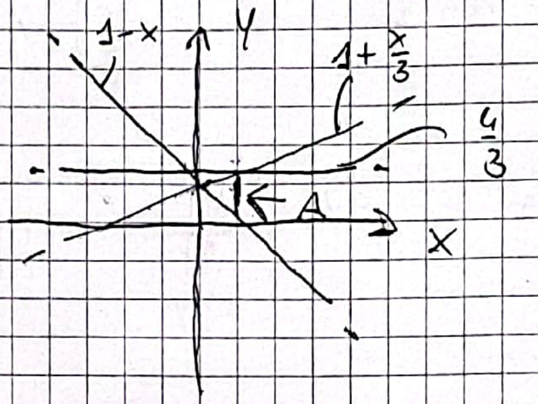
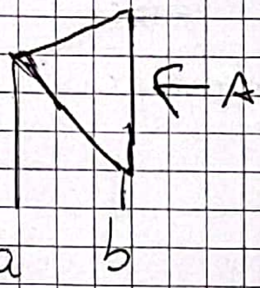
3. $A = \{(x,y) \in \mathbb{R}^2 \mid 1-x \le y \le 1+\frac{x}{3} \le \frac{4}{3}\}$







TROVIAMO a E b:

a) $1-x = 1+\frac{x}{3} \Rightarrow x = 0$

b) $1+\frac{x}{3} = \frac{4}{3} \Rightarrow 3+x = 4 \Rightarrow x = 1$

$A = \{(x,y) \in \mathbb{R}^2 \mid x \in [0,1],\ 1-x \le y \le 1+\frac{x}{3}\}$

↑ INSIEME y-SEMPLICE

$$\int_A \frac{x}{\sqrt{y+x}}\,dx\,dy = \int_0^1 \left(\int_{1-x}^{1+\frac{x}{3}} \frac{x}{\sqrt{y+x}}\,dy\right)dx$$

$$= \int_0^1 x \left(\int_{1-x}^{1+\frac{x}{3}} \frac{1}{\sqrt{y+x}}\,dy\right)dx = \int_0^1 x \left[2\sqrt{y+x}\right]_{1-x}^{1+\frac{x}{3}} dx$$

$f(y) = 2\sqrt{y}$ $\quad f'(y) = \frac{1}{\sqrt{y}} = y^{-\frac{1}{2}}$

$g(y) = y+x$ $\quad g'(y) = 1$

$$= \int_0^1 x\left(2\sqrt{1+\frac{4}{3}x} - 2\sqrt{1}\right) dx$$

$$= \int_0^1 x\left(2\sqrt{1+\frac{4}{3}x} - 2\right) dx = 2\int_0^1 x\sqrt{1+\frac{4}{3}x}\,dx - 2\int_0^1 x\,dx$$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ A $\qquad\qquad\qquad$ B

$$A = \frac{2}{\sqrt{3}}\int_0^1 x\sqrt{3+4x}\,dx = \frac{2}{\sqrt{3}}\int_0^1 \frac{t-3}{16}\sqrt{t}\,dt$$

$$t = 3+4x \qquad x = \frac{t-3}{4} \qquad dx = \frac{1}{4}dt$$

$$= \frac{2}{\sqrt{3}}\int_0^1 \frac{t\sqrt{t} - 3\sqrt{t}}{16}\,dt \quad \ldots$$

$\leftarrow$ POSSIAMO LASCIARE I CALCOLI