Silesian University
of Technology

FACULTY OF AUTOMATIC CONTROL, ELECTRONICS
AND COMPUTER SCIENCE

Advanced Optimization Methods

Constrained dynamic optimization (penalty methods)

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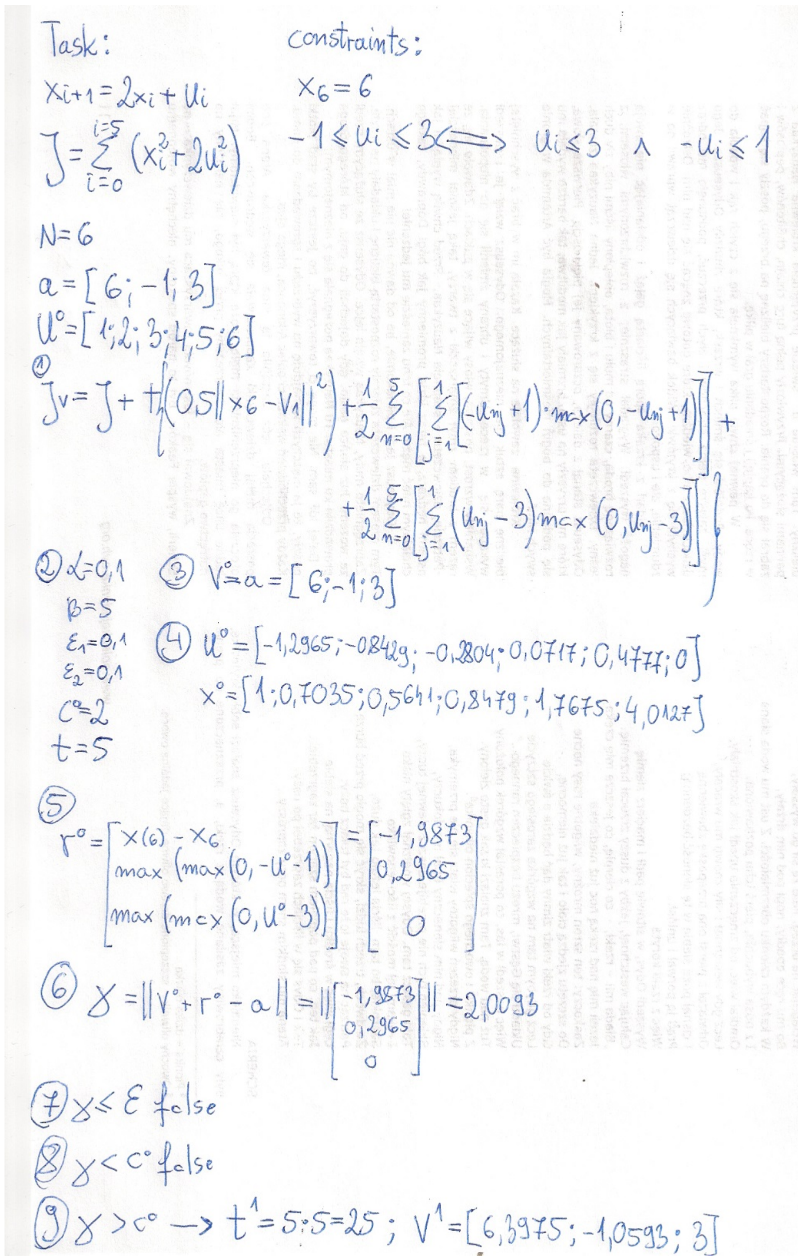
Year 1, semester 1, Macro Data Science

June 1, 2020

1 Task

The task was to determine optimal control for a system described by state equation, with given x_0 and constraints. The equations are specified in the instruction in problem number 3. Selected x_6 parameter was $x_6 = 6$. In accordance with Professor's e-mail, the constraint $x_3 = 5$ was removed.

2 Hand-written part of solution


Task:

$x_{i+1} = 2x_i + u_i$

$J = \sum_{i=0}^{i=5} (x_i^2 + 2u_i^2)$

constraints:

$x_6 = 6$

$-1 \leq u_i \leq 3 \iff u_i \leq 3 \wedge -u_i \leq 1$

$N = 6$

$a = [6; -1; 3]$

$U^\circ = [1; 2; 3; 4; 5; 6]$

(1) $J_v = J + t_h \Big(0{,}5 \|x_6 - V_1\|^2 \Big) + \frac{1}{2} \sum_{n=0}^{5} \Big[\sum_{j=1}^{1} \big[(-u_{nj} + 1) \cdot \max(0, -u_{nj} + 1) \big] \Big] + \frac{1}{2} \sum_{n=0}^{5} \Big[\sum_{j=1}^{1} (u_{nj} - 3) \max(0, u_{nj} - 3) \Big]$

(2) $\alpha = 0{,}1$, $\beta = 5$, $\varepsilon_1 = 0{,}1$, $\varepsilon_2 = 0{,}1$, $c^\circ = 2$, $t = 5$

(3) $V^\circ = a = [6; -1; 3]$

(4) $U^\circ = [-1{,}2965; -0{,}8429; -0{,}2804; 0{,}0717; 0{,}4777; 0]$

$x^\circ = [1; 0{,}7035; 0{,}5641; 0{,}8479; 1{,}7675; 4{,}0127]$

(5) $r^\circ = \begin{bmatrix} x(6) - x_6 \\ \max(\max(0, -U^\circ - 1)) \\ \max(\max(0, U^\circ - 3)) \end{bmatrix} = \begin{bmatrix} -1{,}9873 \\ 0{,}2965 \\ 0 \end{bmatrix}$

(6) $\gamma = \|V^\circ + r^\circ - a\| = \left\| \begin{bmatrix} -1{,}9873 \\ 0{,}2965 \\ 0 \end{bmatrix} \right\| = 2{,}0093$

(7) $\gamma < \varepsilon$ false

(8) $\gamma < c^\circ$ false

(9) $\gamma > c^\circ \rightarrow t^1 = 5 \cdot 5 = 25$; $V^1 = [6{,}3975; -1{,}0593; 3]$

Figure 1: Hand written part of solutions with selected parameters

Selected initial values of control vector: $u_0 = 1, u_1 = 2, u_2 = 3, u_3 = 4, u_4 = 5, u_5 = 6$

3 Results

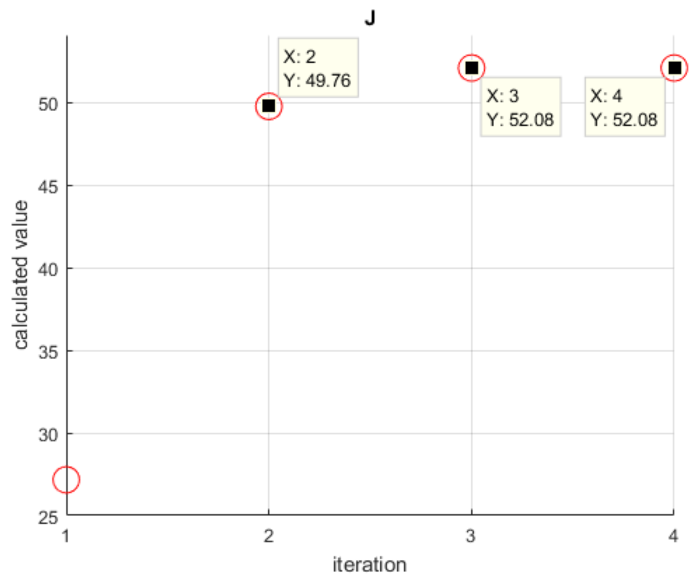


Figure 2: Results obtained for performance index J

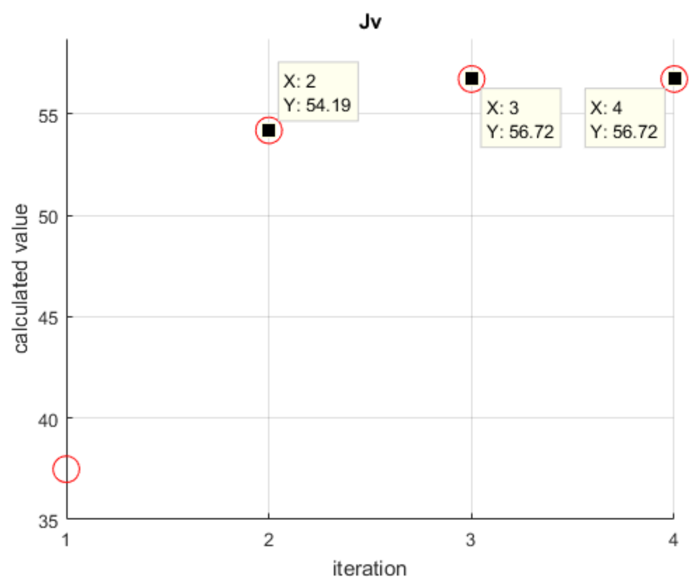


Figure 3: Results obtained for penalty functional

```
X values, x(1): 1, x(2): 0.82318, x(3): 0.74425, x(4): 1.2235, x(5): 2.6204, x(6): 5.9826
U values, u(1): -1.1768, u(2): -0.90211, u(3): -0.26497, u(4): 0.17336, u(5): 0.74178, ...
    u(6): -1.9861e-05
```

4 Conclusions

The algorithm as usual needed small amount of iterations to achieve the solution. As can be seen the constraints are not fully satisfied, the u_1 still violates them therefore the value of penalty functional is still greater than the function being minimized. The algorithm has stopped after meeting both stop criteria - achieving absolute value of consecutive penalty functionals being subtracted smaller than given accuracy and calculated gamma also being smaller than given accuracy. As expected, the value of found x_6 is within given precision, which was set to 0.1.

5 Source code

main.m

```
clc
clear all
%%%%%%
x_1 = 1;
x_6 = 6;
n=6;
%%%%%% defining parameters and initial values%%%%%%
alfa = 0.1;
beta = 5;
epsilon = 0.1;
epsilon_2 = 0.1;
c = 2;
t = 5;
a = [x_6, -1, 3];                                           %values of constraints, x_6=7, ...
    -1<u_i<3
v = a;                                                      %step 3
u0 = [1,2,3,4,5,6]                                          %random values of initial control
u=u0
i = 1;

while true
    Jv_function = @(u)calculate_penalty(x_1, u, n, v, t);   %step 1
    u = fminsearch(Jv_function, u);                         %finding un, step 4
    [Jv(i), x, J(i)] = calculate_penalty(x_1, u, n, v, t);  %finding xn, step 4
    r = calculate_r(x(6), v, u);                            %step 5
    gamma = norm(v+r-a);                                    %step 6

    if gamma > epsilon                                      %step 7
        if gamma < c                                        %step 8
            v= a-r;
            c = alfa*beta;
        else                                                %step 9
            t = beta*t;
            v = a - r / beta;
        end
        gamma = norm(v+r-a);
    end

    if (i >1)                                               %step 10
        j_jv = abs(prev_Jv - Jv(i))
        if gamma ≤ epsilon && j_jv < epsilon_2
            break
        end
    end

    prev_Jv=Jv(i);
    i=i+1;
```

```
end

visualize_results(x,u,J,Jv)
```

calculate_penalty.m

```
function [Jv, x, J] = calculate_penalty(x0 , u, n, v, t)
x = zeros (1, n);
x(1) = x0;
for i = 2:n
    x(i) = 2*x(i-1) + u(i-1);
end
J = sum(x.^2 + 2*u.^2);
R_1 = 0.5 * norm(x(6) - v(1))^2; %v(1) corresponds to x_6, etc
R_2 = sum(sum((-u + v(2)).* max(0, -u + v(2)))) ;
R_3 = sum(sum((u - v(3)).* max(0, u - v(3)))) ;
Jv = J + t*(R_1 + R_2 + R_3);
end
```

calculate_r.m

```
function r = calculate_r(x6,v,u)
    r = [0,0,0];
    r(1) = x6 - v(1);                      %for constraint x6 = 1
    r(2) = max(max(0, -u + v(2)));    %for u_i ≤ 14.5
    r(3) = max(max(0, u - v(3)));     %for u_i ≥ 14.5
end
```

visualize_results.m

```
function r = visualize_results(x, u, J, Jv)
display_x = ['X values, x(1): ', num2str(x(1)), ...
                     ', x(2): ', num2str(x(2)), ...
                     ', x(3): ', num2str(x(3)), ...
                     ', x(4): ', num2str(x(4)), ...
                     ', x(5): ', num2str(x(5)), ...
                     ', x(6): ', num2str(x(6))];

display_u = ['U values, u(1): ', num2str(u(1)), ...
                     ', u(2): ', num2str(u(2)), ...
                     ', u(3): ', num2str(u(3)), ...
                     ', u(4): ', num2str(u(4)), ...
                     ', u(5): ', num2str(u(5)), ...
                     ', u(6): ', num2str(u(6))];
disp(display_x)
disp(display_u)
x_vector = [1:length(Jv)];
figure
scatter(x_vector, Jv, 200, 'red')
grid on
title("Jv")
xlabel("iteration")
ylabel("calculated value")
ax = gca;
ax.XTick = x_vector;
ax.YLim(2) = max(Jv)+2;

figure
scatter (x_vector, J, 200, 'red')
grid on
title("J")
```

```
xlabel("iteration")
ylabel("calculated value")
ax = gca;
ax.XTick = x_vector;
ax.YLim(2) = max(J)+2;
end
```