

Q1

1.) $m(a+bX) = a + bm(X) :$

$$m(a+bX) = \frac{1}{N} \sum_{i=1}^N (a+bx_i) = \frac{Na}{N} + \frac{b}{N} \sum_{i=1}^N x_i = a + bm(X) \quad \checkmark$$

2.) $\text{cov}(X, a+bY) = b \times \text{cov}(X, Y)$

$$\begin{aligned} \text{cov}(X, a+bY) &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X)) (a + by_i - \underbrace{m(a+bY)}_{a + bm(Y)}) \\ &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X)) (b(y_i - m(Y))) \\ &= b \frac{1}{N} \sum_{i=1}^N (x_i - m(X)) (y_i - m(Y)) = b \text{cov}(X, Y) \quad \checkmark \end{aligned}$$

3.) $\text{cov}(a+bX, a+bX) = b^2 \text{cov}(X, X), \quad \text{cov}(X, X) = s^2$

$$\begin{aligned} \text{cov}(X, X) &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X)) (x_i - m(X)) \\ &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))^2 = s^2 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{cov}(a+bX, a+bX) &= \frac{1}{N} \sum_{i=1}^N (a+bx_i - \underbrace{m(a+bX)}_{a + bm(X)}) (a+bx_i - \underbrace{m(a+bX)}_{a + bm(X)}) \\ &= \frac{1}{N} \sum_{i=1}^N (b(x_i - m(Y))) (b(x_i - m(Y))) \\ &= \frac{1}{N} \sum_{i=1}^N (b(x_i - m(Y)))^2 = b^2 \frac{1}{N} \sum_{i=1}^N (x_i - m(X))^2 \\ &= b^2 s^2 \\ &= b^2 \text{cov}(X, X) \quad \checkmark \end{aligned}$$

4.) $\text{median}(g(X)) = g(\text{median}(X))$

Same, because median is middle value, it doesn't matter if a transformation is applied since it will still be in the middle. It'll hold for quantiles, but not IQR, or range.

$$5.) m(g(x)) = g(m(x))$$

No, only works for linear functions. Not for x^2 or anything else.

$$5(x^2) \neq (5x)^2 \quad , \quad \text{if } x = 2, \text{ then } 20 \neq 100$$