Практика 2.

(1) $\dot{x} = -\varepsilon \sin(x)$; $x^* - ?$

$\dot{x} = 0$

$0 = -\varepsilon \sin(x)$

1) $\varepsilon = 0 \Rightarrow x(t) = Const$

2) $x = \pi k, k \in \mathbb{Z}$

I: $k = 0$

Лин. систем около $x^* = 0$

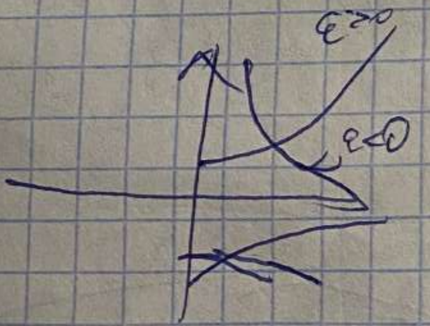
$$\dot{x} = \left.\frac{\partial F(\varepsilon, x)}{\partial x}\right|_{x = x^*} \cdot (x - x^*) = -\varepsilon \cdot \cos(0)\,(x - 0) = -\varepsilon x$$

$\dot{x} = -\varepsilon x$

$x = Ce^{-\varepsilon t}$

При $\varepsilon > 0$ x - убыв. эксп.

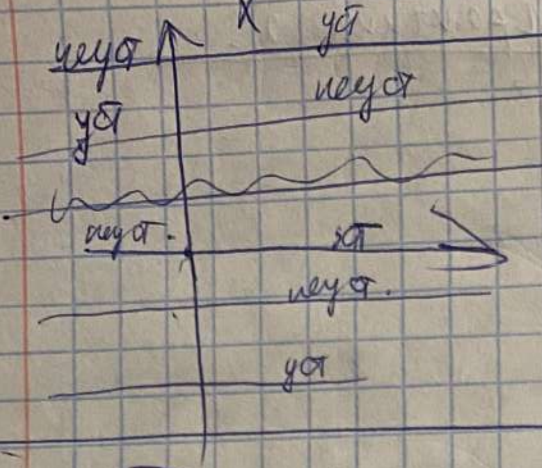
$\varepsilon < 0$ - x - возр эксп.



II. $k = 1 \Rightarrow \dot{x} = -\varepsilon \cos\pi (x - \pi) = \varepsilon(x - \pi)$

$x = Ce^{\varepsilon t} + \pi$ $\quad \varepsilon > 0$ x - возр. эксп

$\varepsilon < 0$, $\dot{x}$ - убыв. экспоненты



— бифур. диаграмма.

2.1. $\frac{dx}{dt} = \varepsilon - x^2$ $\Big|\ x_1^* = 0,\ x_2^* = \varepsilon$

1) $x_1^* = 0$

$$\dot{x} = (\varepsilon - 2x)\,x\,\Big|_{x=0} = \varepsilon x$$

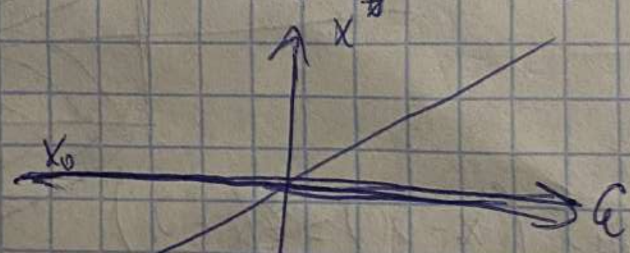
$$x = Ce^{\varepsilon t}$$

2) $x_2^* = \varepsilon$

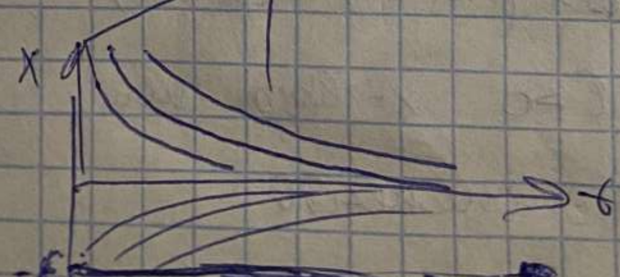
$$\dot{x} = (\varepsilon - 2x^*)(x - x^*) = -\varepsilon(x - \varepsilon)$$

$$\dot{x} = -\varepsilon x - \varepsilon^2$$

$$x = Ce^{-\varepsilon t} + \varepsilon$$



— бифур. диаграмма



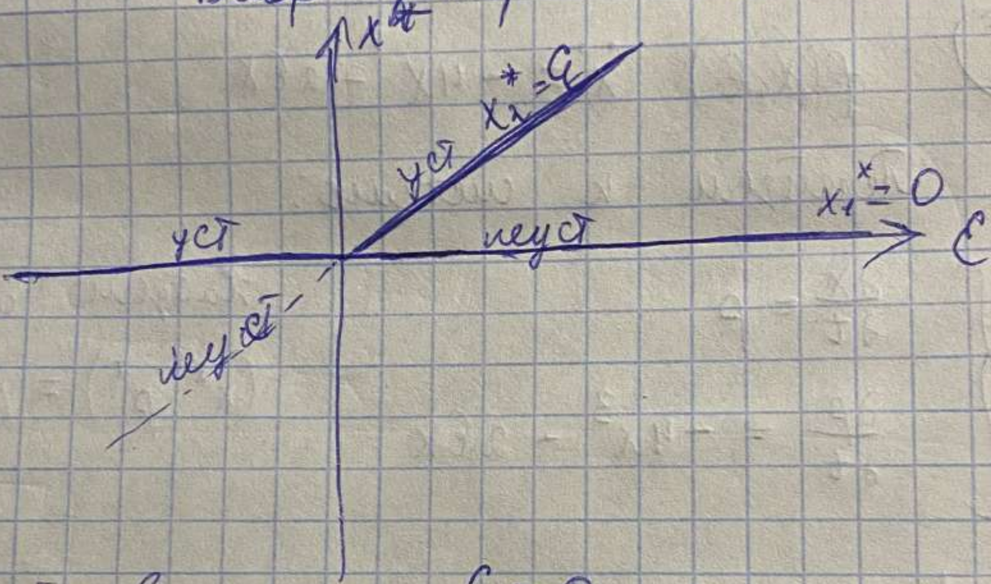
— бифур.

3) $\frac{\partial x}{\partial t} = \varepsilon x - x^2$

Найдём особые точки:

$x_1^* = \varepsilon$, $x_2^* = 0$. Строим БД:

Бифур. диаграмма.



x_1^* устойчива при $\varepsilon > 0$ и неуст. при $\varepsilon < 0$

x_2^* устойчива при $\varepsilon < 0$ и неуст. при $\varepsilon > 0$

(4) $\frac{\partial x}{\partial t} = \varepsilon x - x^3$

Найдём точки особые:

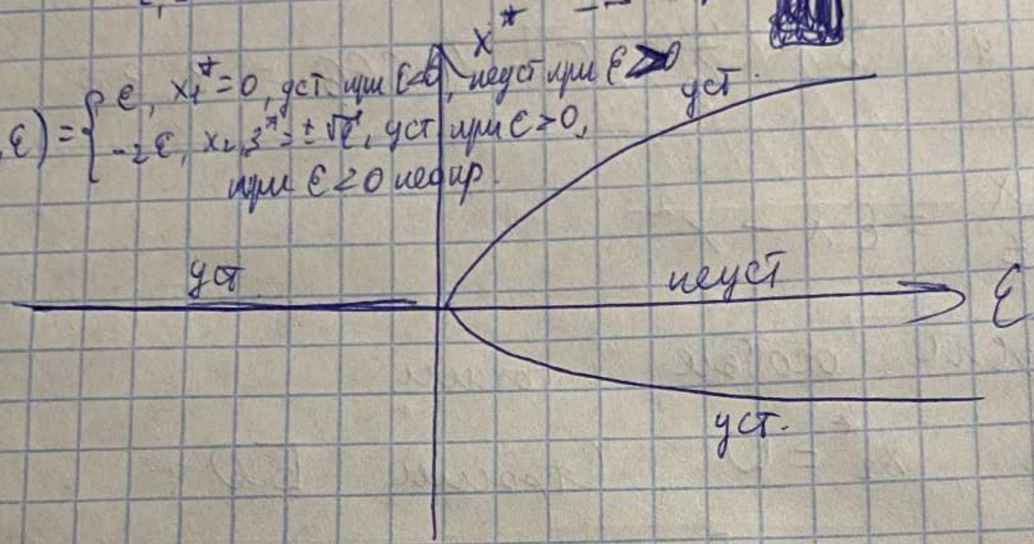
$x_1^* = 0$

$x_{2,3}^* = \pm\sqrt{\varepsilon}$

$f'(x,\varepsilon) = \varepsilon - 3x^2 = \begin{cases} \varepsilon, & x_1^* = 0 \\ -2\varepsilon, & x_{2,3}^* = \pm\sqrt{\varepsilon} \end{cases}$

$\Rightarrow$

$f'(x,\varepsilon) = \begin{cases} \varepsilon, & x_1^* = 0, \text{ уст при } \varepsilon < 0, \text{ неуст при } \varepsilon \geq 0 \\ -2\varepsilon, & x_{2,3}^* = \pm\sqrt{\varepsilon}, \text{ уст при } \varepsilon > 0, \\ & \text{при } \varepsilon < 0 \text{ неопр} \end{cases}$



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(5) ~~$f(x,\varepsilon)$~~ $\ddot{x} = -4x^3 - 2\varepsilon x$

Перейдём к системе:

$$\begin{cases} \frac{\partial x}{\partial t} = p \\ \frac{\partial p}{\partial t} = -4x^3 - 2\varepsilon x \end{cases}$$

Найдём особые точки

$(x^*, p^*) = (0, 0)$

Линеаризуем:

$$\begin{cases} \frac{\partial x}{\partial t} = p \\ \frac{\partial p}{\partial t} = (-12x^{*2} - 2\varepsilon)(x - 0)\Big|_{x^* = 0} = -2\varepsilon x \end{cases}$$

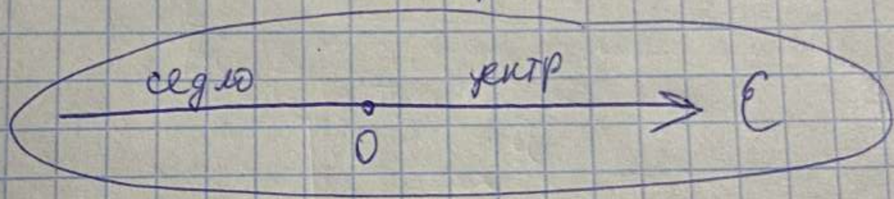
$$\begin{cases} \frac{\partial x}{\partial t} = p \\ \frac{\partial p}{\partial t} = -2\varepsilon x \end{cases}$$

Матрица системы:

$$A = \begin{pmatrix} 0 & 1 \\ -2\varepsilon & 0 \end{pmatrix} \Rightarrow \det(A - \lambda E) = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ -2\varepsilon & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 + 2\varepsilon = 0$$

$$\lambda_{1,2} = \pm i\sqrt{2\varepsilon}$$ ~~[illegible]~~



⑥
$$\begin{cases} \frac{\partial x_1}{\partial t} = \varepsilon x_1 \\ \frac{\partial x_2}{\partial t} = -x_2 \end{cases}$$

Запишем матрицу системы

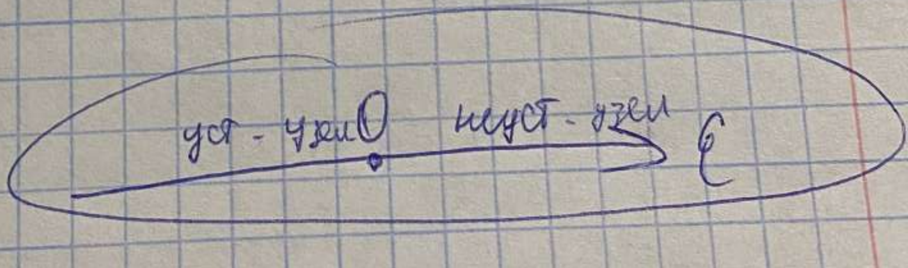
$$A = \begin{pmatrix} \varepsilon & 0 \\ 0 & -1 \end{pmatrix}$$

$$\det(A - \lambda E) = \begin{vmatrix} \varepsilon - \lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} = 0$$

$$(\varepsilon - \lambda)(-1-\lambda) = 0$$

$$\lambda_1 = \varepsilon$$

$$\lambda_2 = -1$$



$$⑦\ \begin{cases} \dot{x} = \varepsilon x - y - x(x^2 + y^2) \\ \dot{y} = x + \varepsilon y - y(x^2+y^2) \end{cases}$$

$$\{x(t), y(t)\} \longrightarrow \{r(t), \varphi(t)\}$$

$$x(t) = r(t)\cdot\cos(\varphi)$$
$$y(t) = r(t)\cdot\sin(\varphi)$$

$$\dot{x}(t) = \dot{r}(t)\cdot\cos\varphi - r(t)\cdot\sin\varphi\cdot\dot{\varphi}(t)$$
$$\dot{y}(t) = \dot{r}(t)\sin\varphi + r(t)\cos\varphi\cdot\dot{\varphi}(t)$$

$$\left.\begin{aligned} \cdot\sin\varphi\ \Big|\ & \dot{r}\cos\varphi - r\sin\varphi\cdot\dot{\varphi} = \varepsilon r\cos\varphi - r\sin\varphi - r\cos\varphi\cdot r^2 \\ \cos\varphi\ \Big|\ & \dot{r}\sin\varphi + r\cos\varphi\cdot\dot{\varphi} = r\cos\varphi + \varepsilon r\sin\varphi - r\sin\varphi\cdot r^2 \end{aligned}\right\}$$

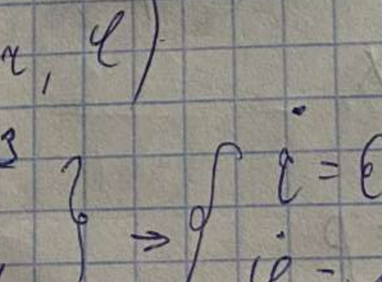
$$\begin{cases} \dot{r} = Q(r, \varphi) \\ \dot{\varphi} = P(r, \varphi) \end{cases}$$

1) $\cdot\sin$ $\cdot\cos$ и слож

2) $\cdot\cos$ $\cdot\sin$ и вычесть

$$\begin{cases} \dot{r} = \varepsilon r - r^3 \\ -r\dot{\varphi} = -r \end{cases} \rightarrow \begin{cases} \dot{r} = \varepsilon r - r^3 \\ \dot{\varphi} = 1 \end{cases} \quad r_1^* = 0,\ r_{23}^* = \sqrt{\varepsilon}$$

$r \geq 0$



$0 < \varphi < 2\pi$

$\dot{r} = (\varepsilon - 3r^{*2})(r - r^*)$

$r^* = 0$

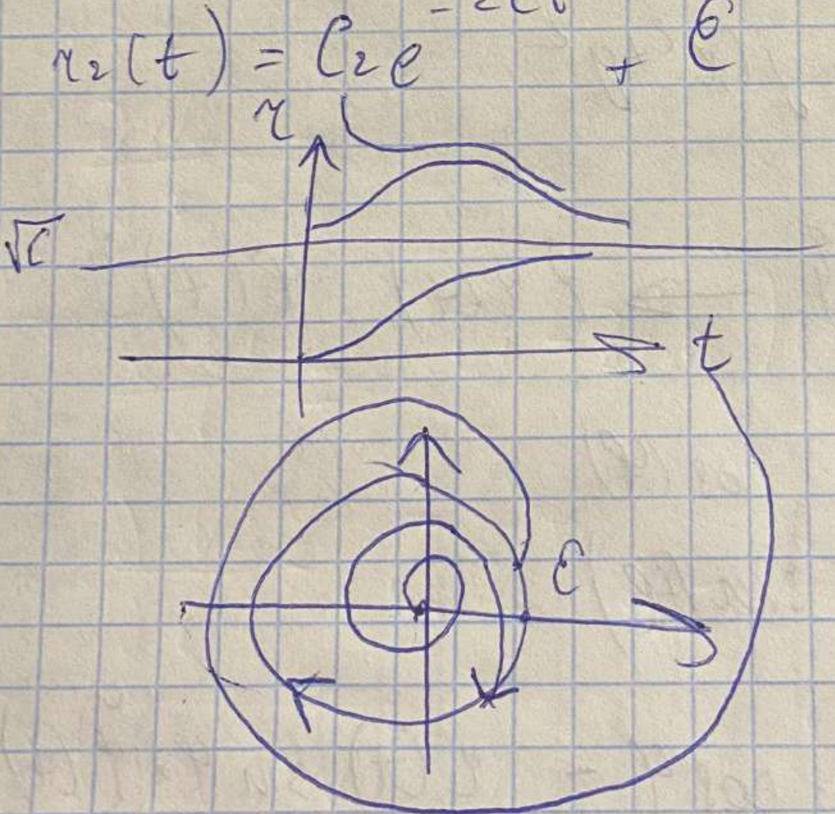
$\dot{r} = \varepsilon r$

$r^* = \sqrt{\varepsilon}$

$\dot{r} = -2\varepsilon(r - \sqrt{\varepsilon})$

$r_1(t) = C_1 e^{\varepsilon t}$

$r_2(t) = C_2 e^{-2\varepsilon t} + c$



Предельный цикл