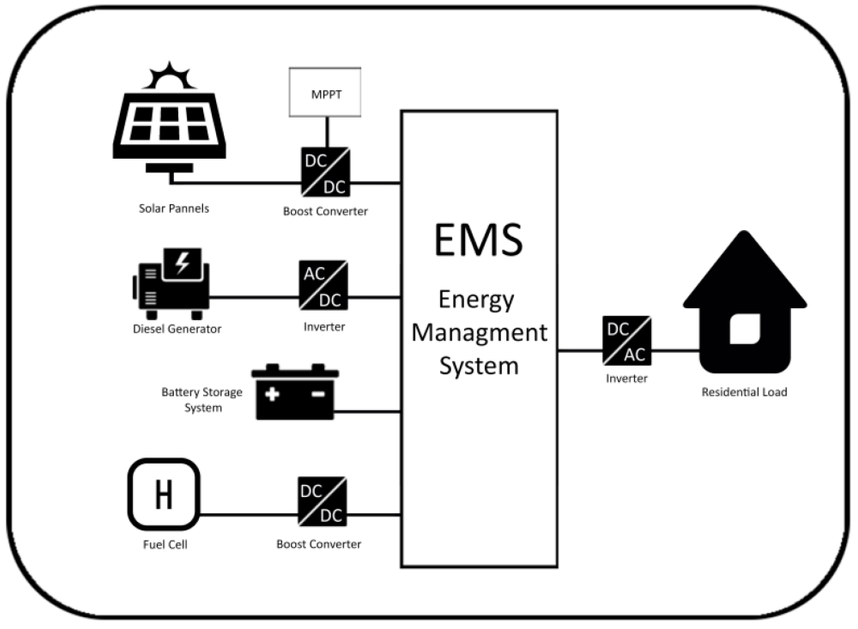
05. Last Leg of the Journey

Multi-Stage Stochastic Optimization

What if we have a sequence of decision stages?

Consider for example and Energy Management System:



- We need to make some decisions (using a generator, buyng from the grid...)
- ...Then observe how uncertainty unfolds
- ...Based on that, we make another round of decisions and so on

Multi-Stage Stochastic Optimization

We will also assume that there are non-trivial constraints

- This setup is called multi-stage stochastic optimization
- ...Or also online stochastic optimization, or sequential decision making

There are a few possible solution approaches

One approach consist in using scenarios, again

- ...But since there are many stages, the decisions variables branch out
- A solution is called a policy tree, which is very expensive to compute

A second approach consists in using anticipatory algorithms

- We iteratively solve an optimization problem with a bit of look-ahead
- Several examples can be found in [1]

[1] Hentenryck, Pascal Van, and Russell Bent. Online stochastic combinatorial optimization. The MIT Press, 2006.

Formalization

Formally, this setup is well captured by a constrained Markov Decision Process (MDP)

In particular, we will consider a constrained $\langle X, Z, P^0, P, f, F \rangle$ be an MDP with:

- A set of possible (observable) states X
- A set of possible decisions Z
- A distribution $P^0(X)$ for the initial state
- A distribution $P(X \mid X, Z)$ for the possible state transitions
- A cost function $f(z, x, x^+)$
- A feasible space $F(x)$ which depends on the state

Some comments:

- The next state depends on the current state and decisions
- The cost depends on the current state and decisions, and on the next state

Formalization

Within this framework, we can formalize a multi-stage problem

Our goal is to define a solution policy π^* from a set of candidates Π s.t.:

$$\pi^* = \text{argmin}_{\pi \in \Pi} \; \mathbb{E}_{x^0 \sim P^0, x^{t+1} \sim P(X|x^t, z^t)} \left[\sum_{t=1}^{eoh} f(z^t, x^t, x^{t+1}) \right]$$

$$\text{subject to: } z^t = \pi(x^t)$$

$$z^t \in F(x^t)$$

This is very complex problem:

- We are not searching for a fixed solution, but for a policy
- The decisions can be anything (including discrete and combinatorial)
- ...They affect the state at the next stage (endogenous uncertainty)
- ...And they should be feasible according to hard constraints

Solution Approach Wanted

Normally, with an MDP we may turn to Reinforcement Learning

...But in this case there are a couple of difficulties:

- Handling constraints (hard ones in particular) in RL is challenging
- Handling combinatorial decisions in RL is very challenging

Let's recap our situation

- Classical approaches from stochastic optimization have poor scalability
- RL approaches have poor support for constraints and combinatorial spaces

Can we use DFL in this scenario?

[1] García, Javier, and Fernando Fernández. "A comprehensive survey on safe reinforcement learning." Journal of Machine Learning Research 16.1 (2015): 1437-1480.

DFL and RL (UNIFY)

Indeed we can, and at this point it's not even that difficult

The trick is simply to decompose the policy π, leading to:

$$\theta^* = \text{argmin}_\theta \; \mathbb{E}_{x^0 \sim P^0, x^{t+1} \sim P(X|x^t, z^t)} \left[\sum_{t=1}^{eoh} f(z^t, x^t, x^{t+1}) \right]$$

$$\text{subject to: } z^t = z^*(y^t, x^t)$$

$$y^t = h(x^t, \theta)$$

Intuitively:

- We use a ML model to output a set of virtual parameters y
- ...Then we compute z^k by solving a constrained optimization problem
- The ML model take care of uncertianty
- The optimization problem take care of the constraints

DFL and RL (UNIFY)

We use the generalized, surrogate-based approach to compute z^*

In particular, we have:

$$z^*(y, x) = \text{argmin}_z \{ \tilde{f}(z, y, x) \mid z \in \tilde{F}(y, x) \}$$

- Depending on our choice for the virtual parameters
- We will need to craft the surrogate cost $\tilde{f}$ and feasible space $\tilde{F}$
- The original constraints are satisfied as long as $z \in \tilde{F}(y, x) \Rightarrow z \in F(x)$

The surrogate terms can usually be designed by tweaking a bit $\tilde{f}$ and $\tilde{F}$

The overall idea is that the ML model guides the optimizer,
exactly as in normal DFL

DFL and RL (UNIFY)

For training, we can rely on a simple reformulation

In particular, we define a new unconstrained MDP $\langle \boldsymbol{X}, \boldsymbol{\Theta}, \boldsymbol{P}^0, \boldsymbol{P}, \boldsymbol{g} \rangle$ such that:

- The set of states is the same as before
- The set of states is the set $\boldsymbol{\Theta}$ of possible training parameters
- The state transition distributions are the same as before
- The cost function is defined as:

$$g(y, x, x^+) = f(z^*(y), x, x^+)$$

Intuitively, we treat the solver as part of the environment

This new MDP can be addressed by any RL learning approach
so we can benfit from recent advances in such field

DFL and RL (UNIFY)

This setup is the most general we have seen so far

It can be used to address a wide number of problem types

- Optimization with parameters that need to be estimated
- One-stage stochastic programming
- Two-state stochastic programming
- Sequential decision making with constraints
- In principle, also black-box optimization and parameter tuning
- ...Though it probably would not a good fit for such cases

You can find it described in [1], under the name **UNIFY**

[2] Silvestri, Mattia, et al. "UNIFY: a Unified Policy Designing Framework for Solving Constrained Optimization Problems with Machine Learning." arXiv preprint arXiv:2210.14030 (2022).

An Example

Let's consider the Energy Management System example in detail

Every 15 minutes, we need to adjust power flow to/from a set of nodes

- Nodes can be generators, demand points, or the grid
- One special node represents a storage system

The decisions z^t at time t include:

- A vector of power flows z^t_{nodes} to/from the main nodes
- A power flow $z^t_{storage}$ to/from the storage system

The state x^t at time t is given by:

- The power x^t_{power} generated by some nodes (e.g. PV plants)
- The demand x^t_{demand} for some nodes (e.g. production sites or housing)
- The storage charge level $x^t_{storage}$

An Example

The transition distribution P is defined by:

- A distribution $\boldsymbol{P_{power}}$ of the yield of renewable energy generators
- A distribution $\boldsymbol{P_{demand}}$ of the demand
- The deterministic transition $\boldsymbol{x_{storage}^{t+1} = x_{storage}^{t} + \eta z_{storage}}$

The feasible space $F(x^t)$ is defined via:

- Flow capacity constraints: $l \leq z^t \leq t$
- Flow balance constraints: $1^T z + x_{power} - x_{demand} = 0$
- Storage capacity constraints $0 \leq x_{storage}^{t} + \eta z_{storage} \leq C$

The cost $f(z^t, x^t, x^{t+1})$ is given by:

$$f(z^t, x^t, x^{t+1}) = c^T z_{nodes}$$

- There is no cost associate to demands, renewable generators, and the storage

The Optimization Problem

We can compute $z^*(y, x)$ by solvig the following LP

$$\begin{aligned} \text{argmin}_z \; & c^T z_{nodes} + y z_{storage} \\ \text{subject to: } & l \leq z^t \leq t \\ & 1^T z + x_{power} - x_{demand} = 0 \\ & 0 \leq x^t_{storage} + \eta z_{storage} \leq C \end{aligned}$$

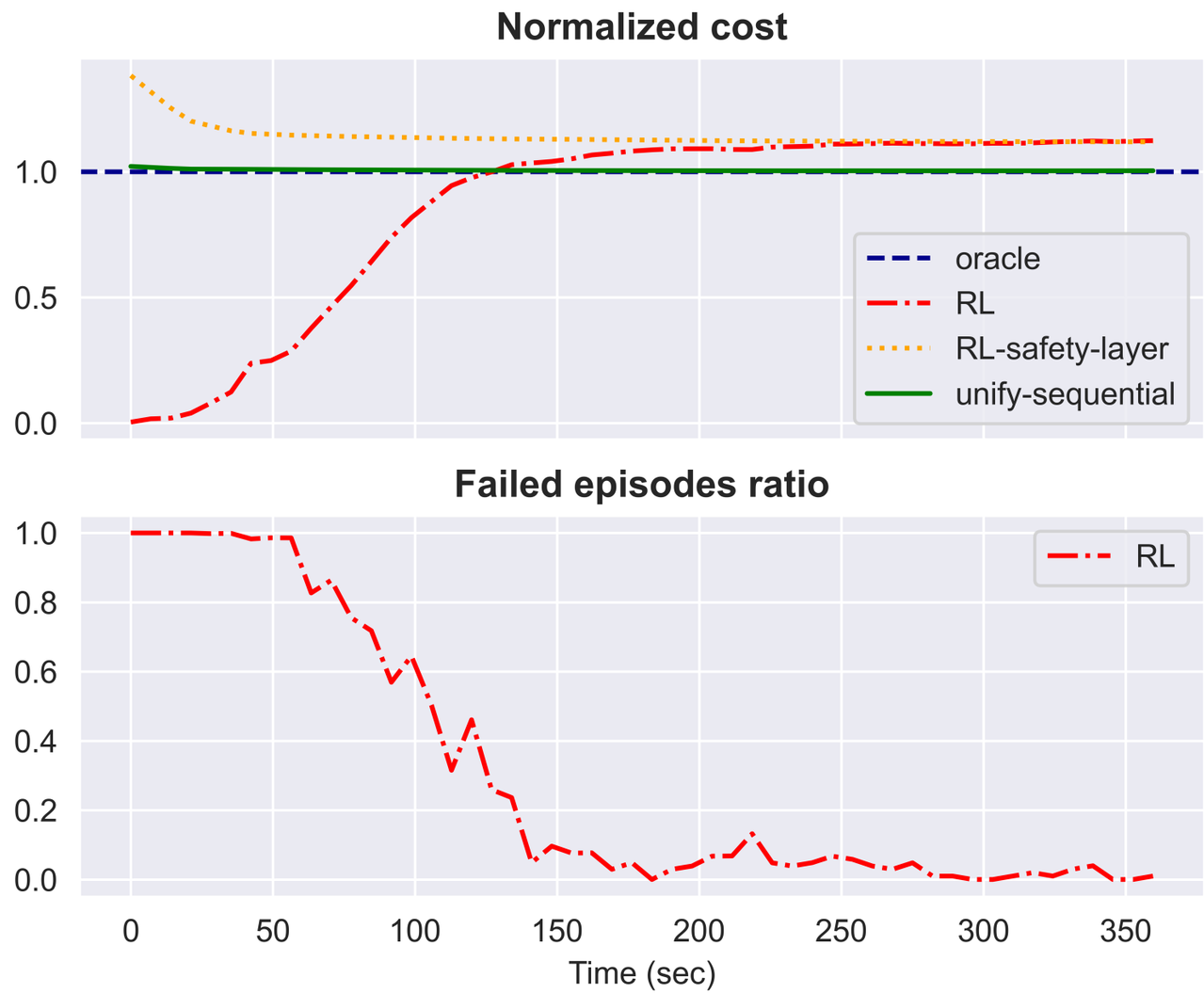
The main alteration is that a virtual cost is associated to the storage system

- If $y > 0$, the solve will tend to charge the storage
- If $y < 0$, the solve will tend to draw power from the storage
- ...So that the ML model can alter the decisions

Without the virtual cost, the storage system would never be charged

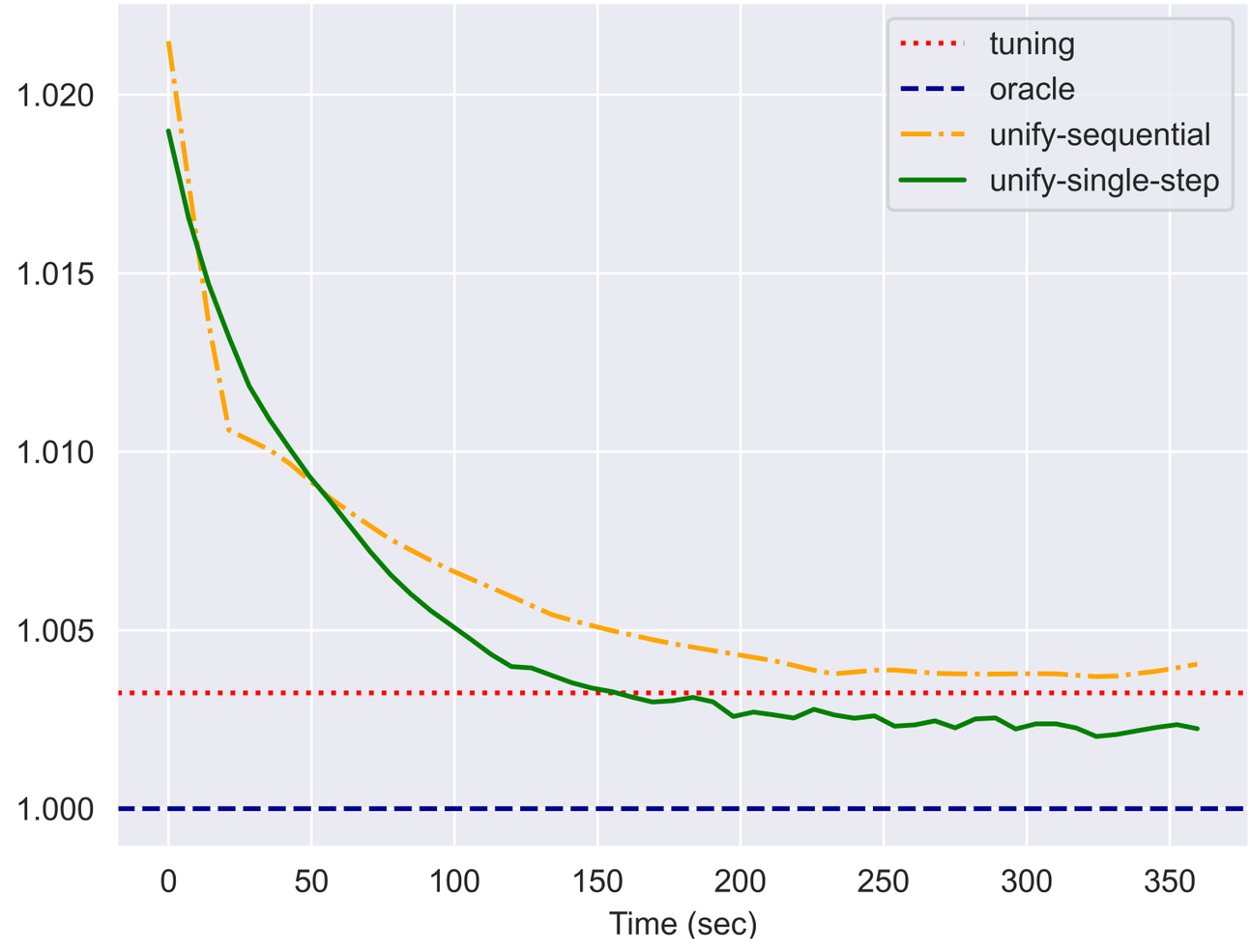
Some Results

Here's a comparison with some constrained RL methods



Some Results

And here's a comparison with a specialized stochastic optimization approach



Some Final Thoughts

If you retain one idea from our ramble, makes sure it is this:

DFL can be used for way more than one purpose!

You just need to stretch it a little bit ;-)

Where next?

- We can reap what we haven't sowed! Let's test more RL algos (spoiler: started)
- Scalability is still a big issue
- We need more (and more realistic) applications
- ...

Thanks for your patience! Any question?