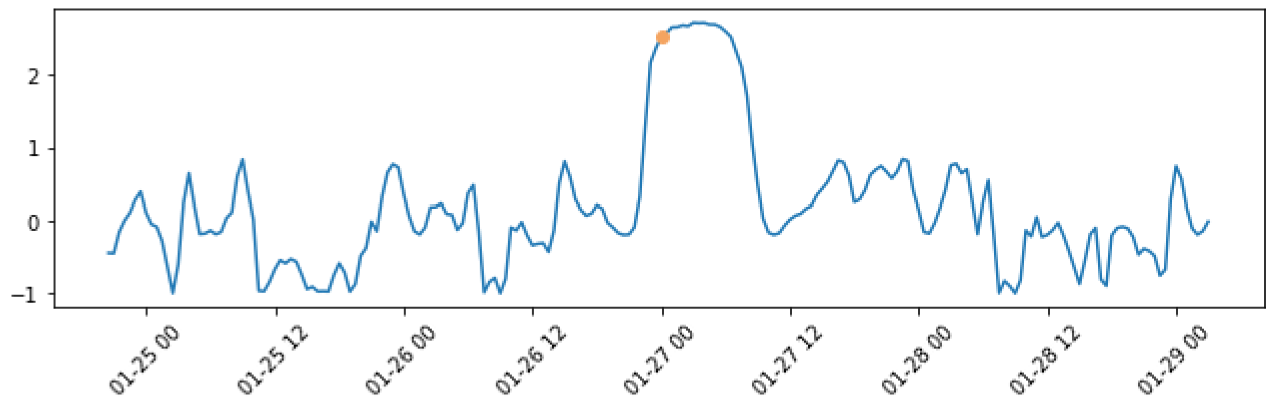
Combining Observations

Combining Observations

An anomaly may be linked to a sequence of observations

In [2]:
```
zstart = windows.loc[4]['begin']
zend = windows.loc[4]['end']
zsignal = signal[(signal.index >= zstart) & (signal.index < zend)]
nab.plot_series(zsignal, labels=labels.loc[4:5])
```



The `loc` field in pandas addresses the index of a `DataFrame` or `Series`

Combining Observations

An anomaly may be linked to a sequence of observations

It's a frequent case in real life:

- Isolated outliers may be due to measurement noise
  - e.g. faulty sensors, human mistakes
- Real anomalies often persist for a while

In this case it may be worth to combine multiple probabilities

- We will start by seeing a simple approach
- ...Which makes the assumption that the observations are i.i.d.
  - I.i.d.: Independent and Identically Distributed

Combining Observations

- Let $\mathbf{x}$ be a random variable corresponding to $\boldsymbol{n}$ subsequent observations
- We can formulate our new detection condition as follows:

$$P(\mathbf{x}) \leq \theta^n$$

Since we are assuming i.i.d. observations, we get:

$$\prod_{i=1}^{n} P(x_i) < \theta^n$$

With a log transformation:

$$\sum_{i=1}^{n} \log P(x_i) < n \log \theta$$

Combining Observations

Finally, we get:

$$\frac{1}{n}\sum_{i=1}^{n}\log P(x_i) < \theta$$

Intuitively:

- Considering multiple (independet, identical) observations
- ...It's the same as smoothing our signal using a moving average

Convolutions

We can implement the smoothing via a convolution:

Given a sequence $\{x_i\}_{i=1}^{n}$ and a sequence $\{f_j\}_{j=1}^{m}$ (a filter)

- A convolution is an operation that "slides" the filter over the main series
- ...And computes dot products to yield a third sequence $\{y_k\}_{k=m}^{n}$ s.t.:

$$y_k = f \cdot \begin{pmatrix} x_{k-m} & x_{k-m+1} & x_{k-m+2} & \ldots & x_k \end{pmatrix}$$

- I.e. the filter is applied to the first m terms...
- ...Then we move one time step forward and we repeat

Normally we need at least m values before the first filter application

- Hence, the output series will be shorter than the input one
- This is depicted by having the y sequence start from index m
- There are other ways (not discussed) to handle the series boundaries

Convolutions

We want to compute a moving average

...Which is just the average of the last few values:

- Let m be the length of the time window for the moving average
- Let us choose as filter: $\left(\frac{1}{m}, \frac{1}{m}, \dots\right)$

The convolution will compute an output sequence $\{y_k\}_{k=m}^{m}$, s.t.:

$$y_k = \frac{1}{m} \sum_{i=k-m}^{k} x_i$$

This is exactly what we need!

Convolutions

First we build the filter:

In [3]:
```
avg_win_len = 24
flt = np.ones(avg_win_len) / avg_win_len
flt
```

Out[3]:
```
array([0.04166667, 0.04166667, 0.04166667, 0.04166667, 0.04166667,
       0.04166667, 0.04166667, 0.04166667, 0.04166667, 0.04166667,
       0.04166667, 0.04166667, 0.04166667, 0.04166667, 0.04166667,
       0.04166667, 0.04166667, 0.04166667, 0.04166667, 0.04166667,
       0.04166667, 0.04166667, 0.04166667, 0.04166667])
```

The we apply the convolution:

In [4]:
```
smooth_signal = -np.convolve(ldens, flt, mode='valid')
smooth_signal_idx = data.index[avg_win_len-1:]
smooth_signal = pd.Series(index=smooth_signal_idx, data=smooth_signal)
```

- The convolution needs n observations before it can be first applied
- Hence, we need to update the index for our smoothed signal

Convolutions

In pandas, we can streamline this process via the `rolling` iterator

```
In [5]: signal.rolling(avg_win_len)

Out[5]: Rolling [window=24,center=False,axis=0,method=single]
```

- This will iterate over the series by groups of `avg_win_len` observations
- Then we can compute the average with:

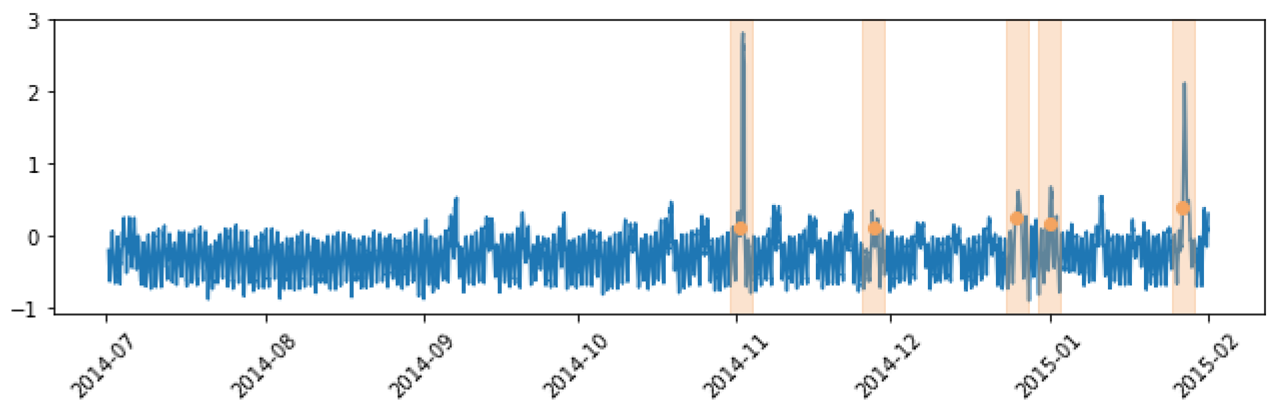
```
In [6]: signal.rolling(avg_win_len).mean()

Out[6]: timestamp
        2014-07-01 00:00:00         NaN
        2014-07-01 00:30:00         NaN
        2014-07-01 01:00:00         NaN
        2014-07-01 01:30:00         NaN
        2014-07-01 02:00:00         NaN
                                  ...
        2015-01-31 21:30:00    0.104761
        2015-01-31 22:00:00    0.159036
        2015-01-31 22:30:00    0.232360
        2015-01-31 23:00:00    0.280827
        2015-01-31 23:30:00    0.307642
        Length: 10320, dtype: float64
```

Combining Observations

Let's plot the smoothed signal

In [7]:
```
nab.plot_series(smooth_signal, labels=labels, windows=windows)
```



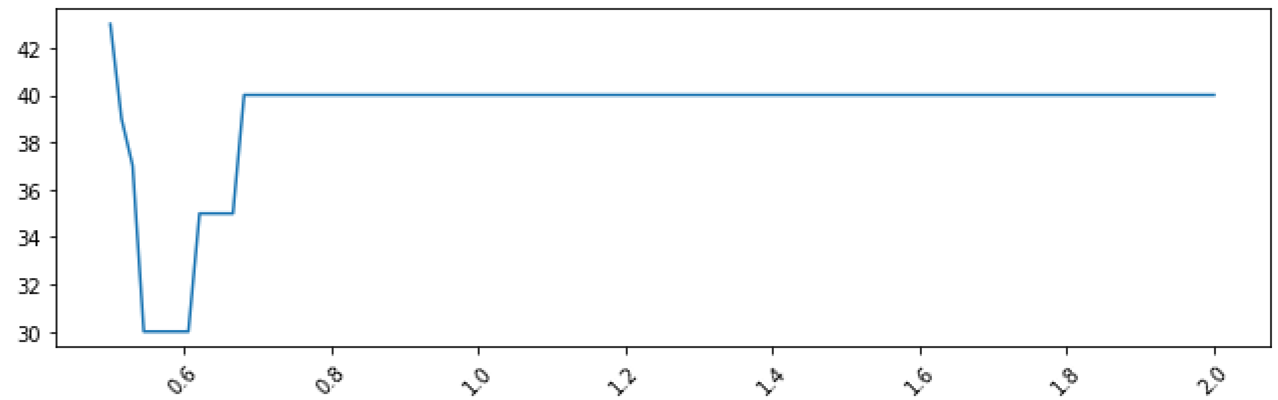
Some anomalies are now more evident!

Threshold Effect

We can now measure the effect of changing the threshold

First, we consider the "idealized" cost surface

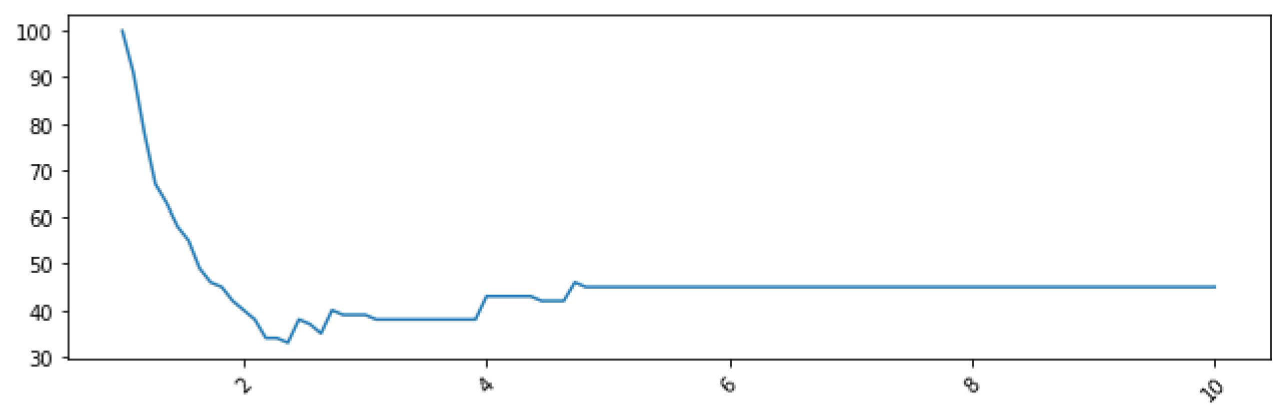
In [8]:
```
smooth_thr_range = np.linspace(0.5, 2, 100)
smooth_cost_range = [cmodel.cost(smooth_signal, labels, windows, thr)
              for thr in smooth_thr_range]
smooth_cost_range = pd.Series(index=smooth_thr_range, data=smooth_cost_range)
nab.plot_series(smooth_cost_range)
```



Threshold Effect

It is worth to compare it with our orignial cost surface:

In [9]:
```
thr_range = np.linspace(1, 10, 100)
cost_range = [cmodel.cost(signal, labels, windows, thr) for thr in thr_range]
cost_range = pd.Series(index=thr_range, data=cost_range)
nab.plot_series(cost_range)
```



The minimum is a bit lower with the smoothed signal

Threshold Optimization

We can now optimize the threshold

In [10]:
```
smooth_signal_opt = smooth_signal[smooth_signal.index < val_end]
smooth_best_thr, smooth_best_cost = nab.opt_thr(smooth_signal_opt, labels_opt,
                                    windows_opt,  cmodel, smooth_thr_range)
print(f'Best threshold: {smooth_best_thr}, corresponding cost: {smooth_best_cost}')
```

```
Best threshold: 0.5303030303030303, corresponding cost: 15
```

- Same cost as before on the trainining set
- On the whole dataset, however:

In [11]:
```
smooth_ctst = cmodel.cost(smooth_signal, labels, windows, smooth_best_thr)
print(f'Cost on the whole dataset {smooth_ctst}')
```

```
Cost on the whole dataset 37
```

- The cost with our original approach used to be 45

Some Considerations

It may worth combining multiple observations:

- If we notice that the data is noise
- ...Or if we are interested in persistent anomalies

Combining multiple observations with an i.i.d. assumption...

- ...Is equivalent to smoothing with a moving average
- ...And the other way round!
- Warning: the assumption may not be valid

The approach introduces an extra parameter (window length):

- In principle, we should optimize over that as well
- We skipped that part for sake of simplicity