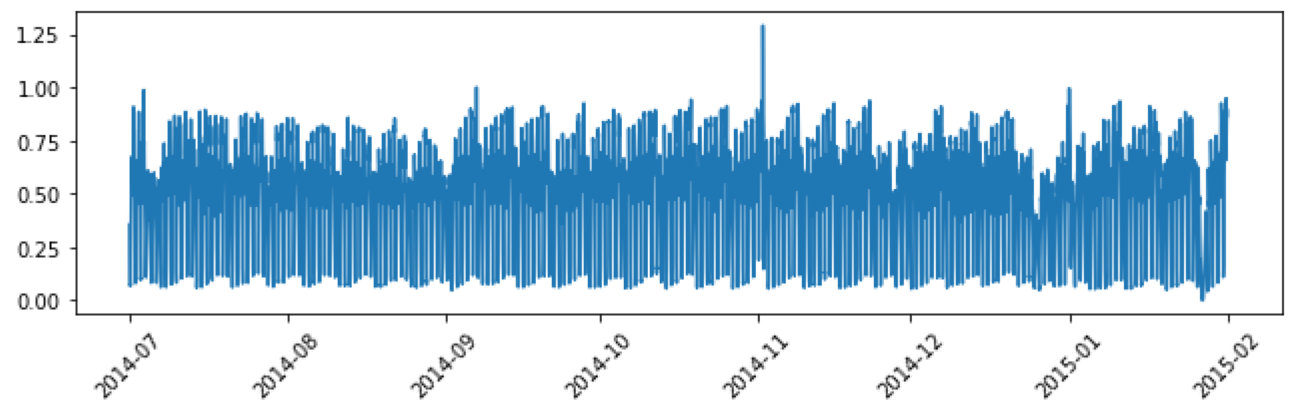
A Time-Dependent Estimator

Looking More Closely

Let us look again at our (normalized) data:

In [2]:
```
nab.plot_series(data)
```



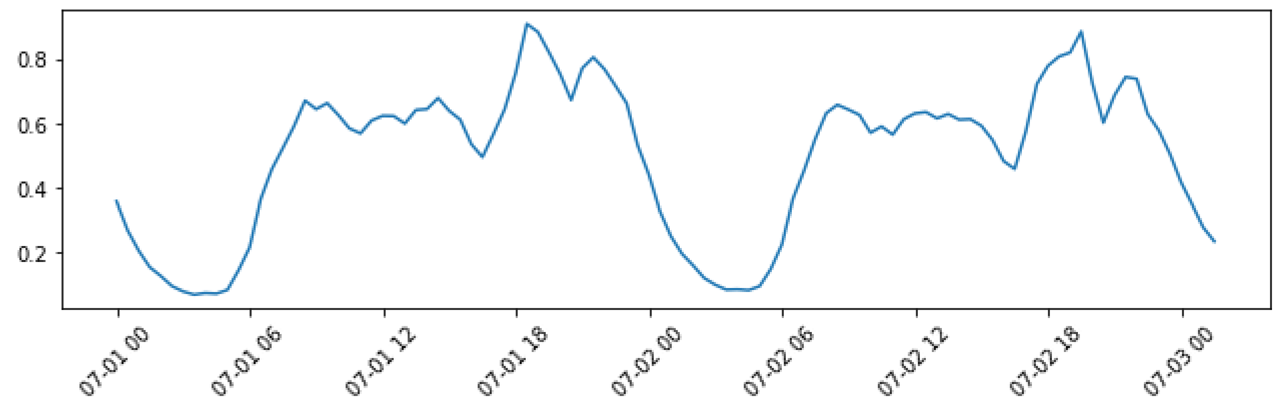
- There is a recurring pattern!
- Can we take advantage of that?

Analyzing the Pattern

Our signal is almost periodic

...I.e. the pattern is time-based

In [3]:
```
nab.plot_series(data.iloc[:100])
```



Determine the Period

How to determine the period of a series?

A useful tool: autocorrelation plots

- Consider a range of possible lags
- For each lag value l:
  - Make a copy of the series and shift it by l time steps
  - Compute the Pearson Correlation Coefficient with the original series
- Plot the correlation coefficients over the lag values

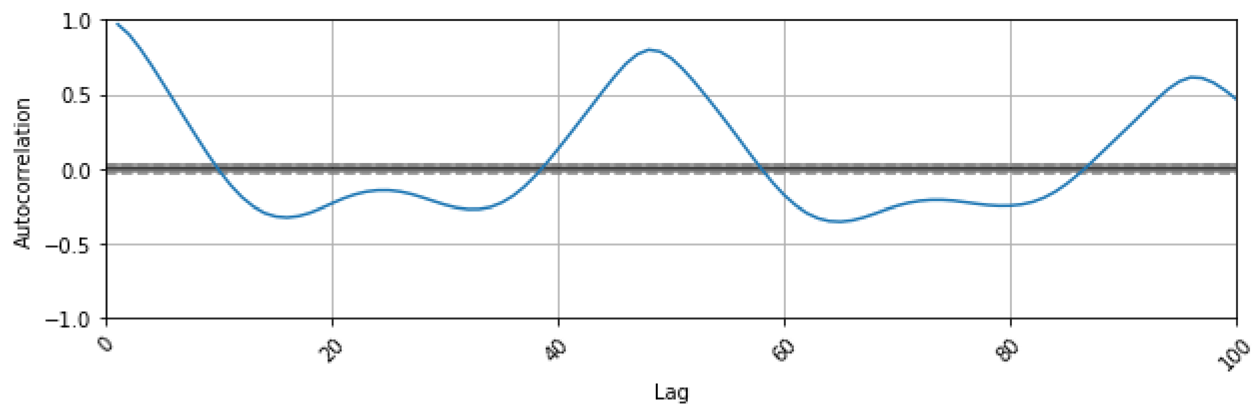
Then we look at the resulting plot:

- If there are peaks corresponding to existing periods
- Positive peaks denote strong correlations
- Negative peaks denote strong negative correlations

Determine the Period

Let's see an autocorrelation plot for our data:

```
In [4]: nab.plot_autocorrelation(data, max_lag=100)
```



- There is strong peak at 48
- A time step is 30 minutes ⇒ there is a period of 24 hours
- We will disregard the horizontal bars

Multivariate-Distribution

One way to look at that:

- The distribution depends on the time of the day
- Equivalently: our x variable has two components
  - The first component x_1 is the time of the day
  - The second component x_2 is the value

Let us extract (from the index) this new information:

In [5]:
```
dayhour = (data.index.hour + data.index.minute / 60)
dayhour = dayhour / 23.5 # normalize
```

We can then add it as a separate column to the data:

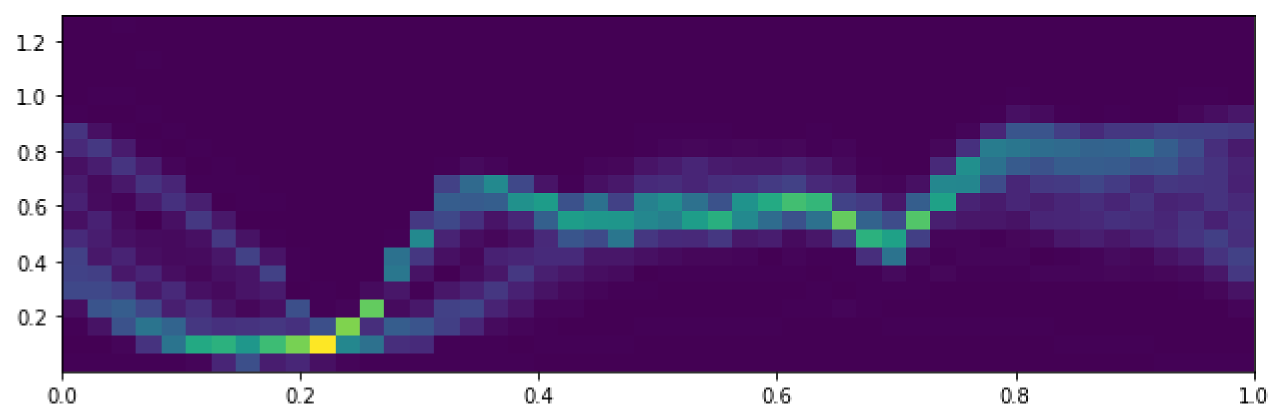
In [6]:
```
data2 = data.copy()
data2['dayhour'] = dayhour
```

Multivariate Distribution

Let us examine the resulting multivariate distribution

We can use a 2D histogram:

```
In [7]: nab.plot_histogram2d(data2['dayhour'], data2['value'], bins=(48, 20))
```

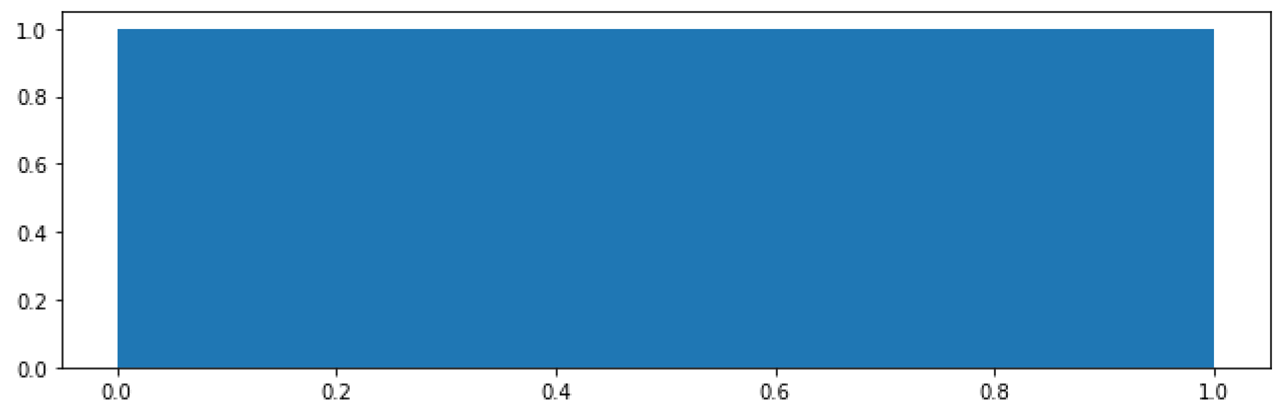


- x = time, y = value, color = frequency of occurrence

Multivariate Distribution

The distribution of the time component is uniform:

In [8]:
```
nab.plot_histogram(data2['dayhour'], bins=48)
```



Time-Dependent Estimator

We can use this information to build a time-dependent estimator

When we run the the estimator:

- The time component (i.e. x_1) is completely predictable
- The value component (i.e. x_2) may be anomalous

Hence, we can define our anomaly condition as:

$$P(x_2 \mid x_1) \leq \theta$$

Using the definition of conditional probability:

$$\frac{P(x_2, x_1)}{P(x_1)} \leq \theta$$

Time-Dependent Estimator

However, since $P(x_1)$ is uniform

...It can be incorporated in the threshold:

$$P(x_2, x_1) \leq \theta'$$

Where $\theta' = \theta P(x_1)$

KDE cannot learn natively conditional probabilities

If we need to use them anyway, there are two strategies:

- Using the Bayes theorem (like we did)
  - In the general case, we will need a second estimator for $P(x_1)$
- If x_1 is discrete, we can learn a KDE estimator for each x_1 value

Bandwidth Choice in Multivariate KDE

We now need to learn our multivariate KDE estimator

First, we need to choose a bandwidth

- We cannot use the (univariate) rule of thumb
- ...But we can use another technique

Let $\tilde{x}$ be a validation set of m examples:

Assuming independent observations, its likelihood (estimated probability) is:

$$L(\tilde{\mathbf{x}}, \hat{\mathbf{x}}, h) = \prod_{i=1}^{m} f(\tilde{x}_i, \hat{\mathbf{x}}, h)$$

- f is the estimator, $\hat{\mathbf{x}}$ the training set, h is the bandwidth
- We can choose h so as to maximize the likelihood!

Bandwidth Choice in Multivariate KDE

A simple approach consist in using grid search

- It's the same approach that we used for optimizing the threshold
- scikit learn provides a convenient implementation
- ...Which resorts to cross-fold validation to define $\tilde{x}$

First, we build a grid search optimizer:

```
In [9]: from sklearn.model_selection import GridSearchCV

        # Build the grid search optimizer
        params = {'bandwidth': np.linspace(0.001, 0.01, 10)}
        opt = GridSearchCV(KernelDensity(kernel='gaussian'), params, cv=5)
```

- The first argument of `GridSearchCV` is the (type of) estimator
- The `params` dictionary specifies the range to be explored
- `cv = 5` specifies the number folds for cross-validation

Bandwidth Choice in Multivariate KDE

Next, we:

- Separate the training set
- "Fit" the `GridSearchCV`, which will run the optimization loop

```
In [10]: # Split training data
         data2_tr = data2[data2.index < train_end]
         opt.fit(data2_tr);
```

Not surprisingly, the process takes some time

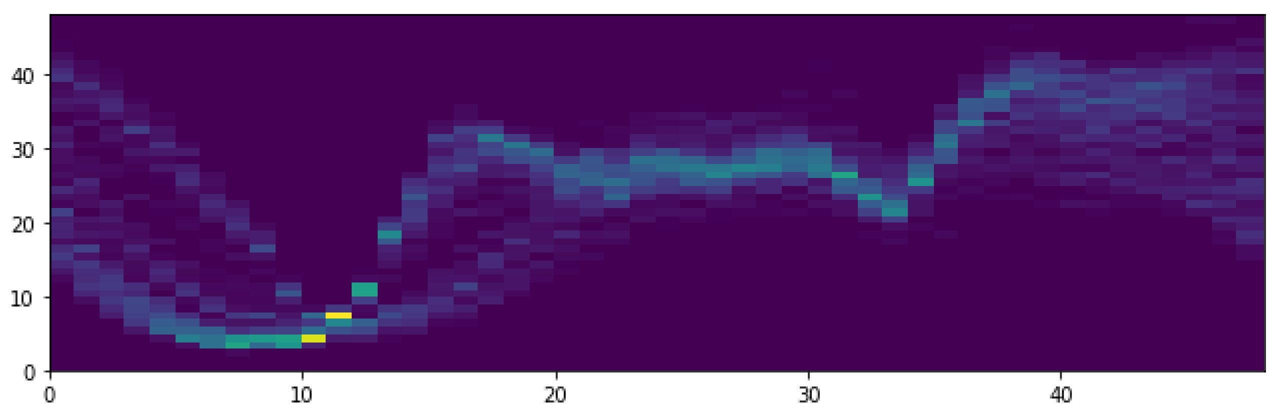
Then we can access the best parameters with:

```
In [11]: opt.best_params_

Out[11]: {'bandwidth': 0.006}
```

Fitting the Estimator

Finally, we can fit the estimator

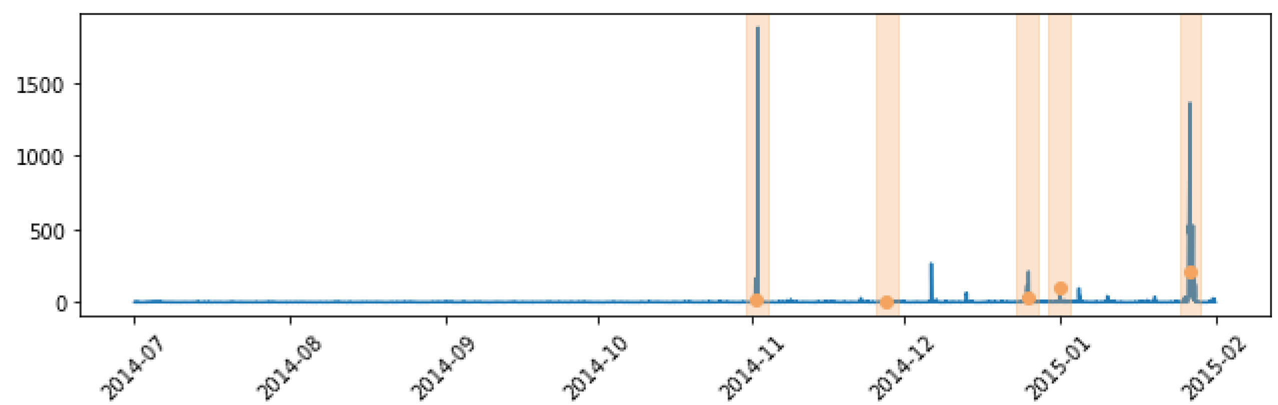
In [12]:
```
h = opt.best_params_['bandwidth']
kde2 = KernelDensity(kernel='gaussian', bandwidth=h)
kde2.fit(data2_tr)

xr = np.linspace(0, 1, 48)
yr = np.linspace(0, 1, 48)
nab.plot_density_estimator_2D(kde2, xr, yr)
```



Alarm Signal

Let us obtain the alarm signal

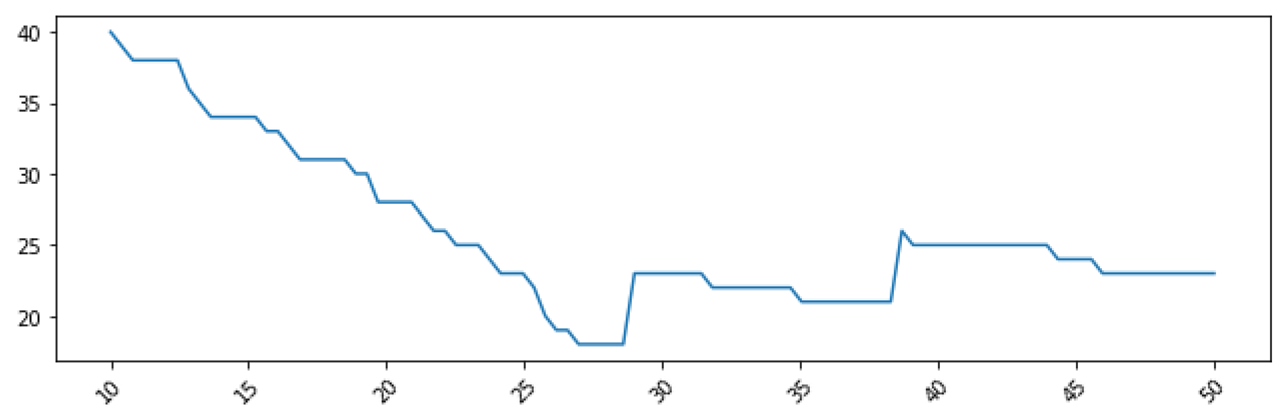
In [13]:
```
ldens2 = kde2.score_samples(data2)
signal2 = pd.Series(index=data2.index, data=-ldens2)
nab.plot_series(signal2, labels=labels, windows=windows)
```



Effect of the Threshold

Let us see the response surface w.r.t. the threshold

In [14]:
```
thr2_range = np.linspace(10, 50, 100)
cost2_range = [cmodel.cost(signal2, labels, windows, thr)
               for thr in thr2_range]
cost2_range = pd.Series(index=thr2_range, data=cost2_range)
nab.plot_series(cost2_range)
```



This is considerably better than before!

Threshold Optimization

Now, let us optimize our threshold:

```
In [15]: signal2_opt = signal2[signal2.index < val_end]
         best_thr2, best_cost2 = nab.opt_thr(signal2_opt, labels_opt,
                                             windows_opt,  cmodel, thr2_range)
         print(f'Best threshold: {best_thr2}, corresponding cost: {best_cost2}')

         Best threshold: 25.757575757575758, corresponding cost: 9
```

On the whole dataset:

```
In [16]: c2tst = cmodel.cost(signal2, labels, windows, best_thr2)
         print(f'Cost on the whole dataset {c2tst}')

         Cost on the whole dataset 20
```

- It was 45 for the first approach and 37 for the second

Considerations

Time-dependencies in the data should be exploited

- We always know what time it is: it's free information!
- Additional information can be used to improve our predictions
- In fact, our time dependent estimator is based on a conditional probability

Open Problem: There is a second period in the data

- Can you figure out which one?
- How to exploit it?

If you wish, you can investigate this at home

- There will be no evaluation
- ...But it's a good occasion to practice and learn