Handling Seasonality

About Our Autoregression Attempt

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- It worked reasonably well
- ...But the error signal was very noisy
- And some anomalies went completely undetected

I bet you know a possible cause

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About our autoregression attempt:

- It worked reasonably well
- ...But the error signal was very noisy
- And some anomalies went completely undetected

I bet you know a possible cause

...And some problems are in fact caused by the presence of a period

- Periodic-like behavior in time series is referred to as seasonality
- In principle, linear regression can deal with periodic data
- ...As long as the period is shorter than the observed window
- However, other periods are still a problem

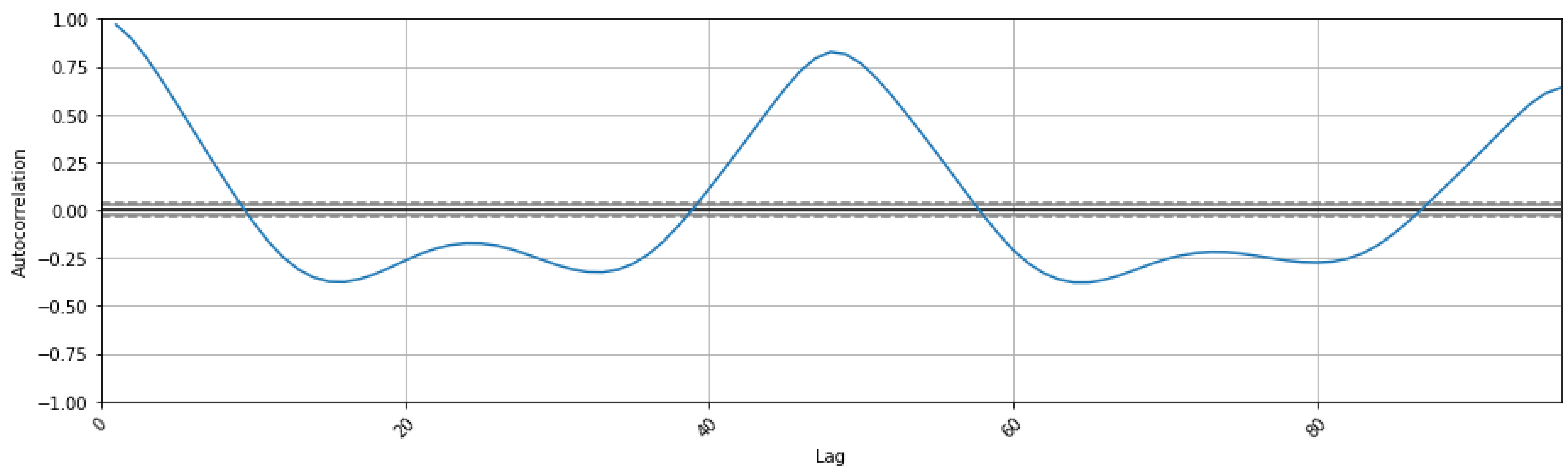
We will now explore the idea of getting rid of the periodicity

Identification of the Seasonality

The first step to removing seasonality is its identification

- One way to do it is using an autocorrelation plot (for the training data)

In [4]:
```
nab.plot_autocorrelation(data_tr, max_lag=96, figsize=figsize)
```



By looking up to > 48 lags, the daily period is evident

Identification of the Seasonality

However, using an autocorrelation plot has its problems:

Peaks repeat:

- The peak at 48 unit lag will repeat at 96 and so on
- How can we say whether it's a single period or multiple periods?

Multiple periods may disturb each other:

- A shorter period may make a longer one less evident
- Non co-prime periods hide each other

Trends cause even more noise:

- If the data has a trend (e.g. generally increasing or decreasing)...
- ...All correlations due to seasonal behavior will be weaker

Identification of the Seasonality

A better method consist in using frequence analysis

In particular, we can compute the Fast Fourier Transform of our signal

- Without going too deep, a FFT is decomposition of a discrete signal...
- ...Into a discrete set of sine/cosine components

Given a signal x with N samples, its FFT has also N components

The amplitudes $\boldsymbol{y}$ of the components are defined so that:

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} y_k e^{2\pi i \frac{kn}{N}}$$

I.e. the whole signal can be reconstructed from its decomposition

- x_n is the n-th element in the discrete signal
- Each amplitude y_k is assigned to a frequency

FFT in numpy

In numpy, we can compute an FFT via:

```
In [5]: from numpy.fft import fft
        y = fft(data_tr['value'])
```

We can get the corresponding frequencies with:

```
In [6]: from numpy.fft import fftfreq
        f = fftfreq(len(data_tr))
        print(f)
```

```
[ 0.          0.00018116  0.00036232 ... -0.00054348 -0.00036232
 -0.00018116]
```

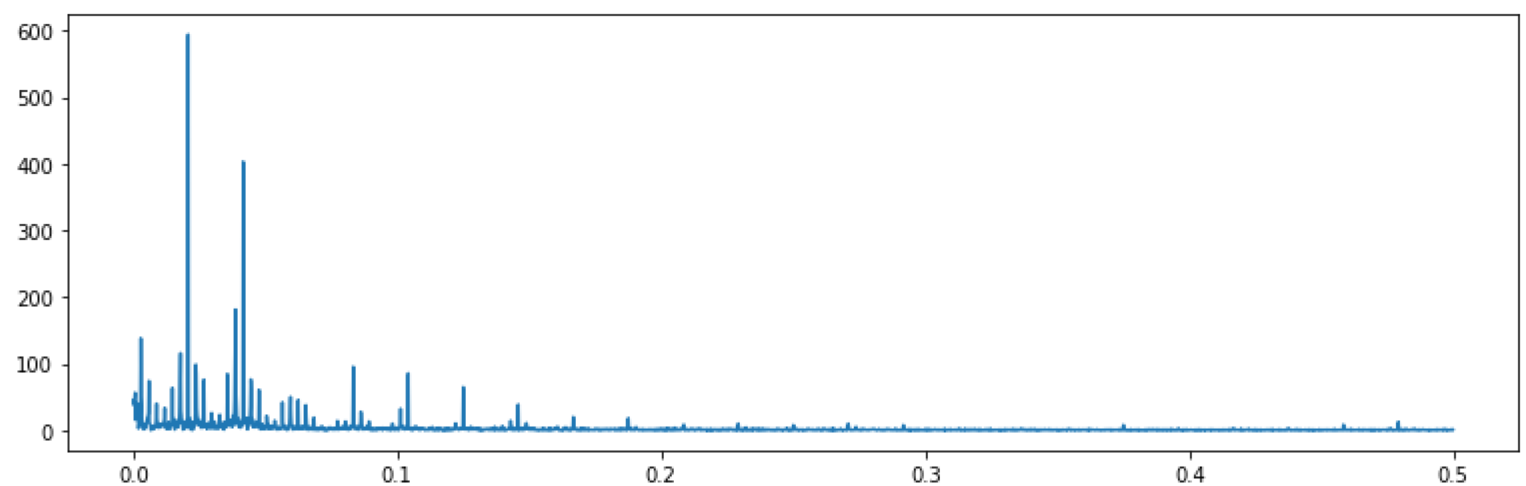
- The frequencies are in cycles/time unit (30 minutes, in our case)
- A different sampling frequency may be specified, if you wish

FFT in numpy

For a real-valued signal, the FFT is always symmetric

- So, we care only about the positive frequencies (by convention, the first half)
- The 0 frequency (constant component) is also not relevant for us

In [8]:
```
n = len(data_tr)
plt.figure(figsize=figsize)
plt.plot(f[1:n//2], np.abs(y[1:n//2]))
plt.xticks()
plt.show()
```



Getting the Peak Frequencies

Let us select all freqecies with amplitude greater than 100

First, we focus on the positive frequencies

```
In [9]: yp = y[1:n//2]
        fp = f[1:n//2]
```

Then, we select the frequencies with (absolute) amplitude above 100:

```
In [10]: fp_peak = fp[np.abs(yp) >= 100]
```

...And we invert them to obtain the periods (in time steps):

```
In [11]: 1/fp_peak

Out[11]: array([345.        ,  55.75757576,  48.        ,  25.91549296,
                 25.79439252,  24.        ])
```

Getting the Peak Frequencies

Let's discard the duplicates and consider them one by one

```
In [12]: np.unique(np.round(1/fp_peak))

Out[12]: array([ 24.,  26.,  48.,  56., 345.])
```

There are a few clusters:

- 345: this is a weekly period (345 / 48 = 7.18). Rounded to 48*7 = 336
- 56, 48: this is the main, daily period. Rounded to 48
- 24: this is a half-day period. Rounded to 24

Getting the Peak Frequencies

The periods are not co-prime

- ..I.e. they are multiples one of each other
- In this situation, the longest period tends to be the most important one

In fact, the longest period conveys information about its sub-multiples

- Taxis requested at 7am last week...
- ...Are likely similar to those requested at 7am yesterday
- The same goes for all sub-multiples

In our situation:

- All periods are (roughly) submultiples of one week
- ...So we can focus on the weekly period

Differencing

A simple approach to handle seasonality consist in differencing

Our predictor becomes the sum of two predictors (i.e. a type of ensemble):

- The linear regression model, $f(x, \theta)$
- A predictor $g(x, d)$ that outputs the target exactly d steps before:

$$g(x_i, d) = \hat{y}_{i-d}$$

Where:

- x_i is the value of the series for the i-th time step
- d is the period, $\hat{y}_{i-d}$ is the target value d time steps earlier

The idea is to let the a dedicated predictor handle the period

Differencing

With the differencing approach we wish to have

$$g(x_i, d) + f(x_i, \theta) \simeq \hat{y}_i$$

- Where $\hat{y}_i$ is the target for the i-th time step (e.g. the value of the next step)

About learning the (ensemble) predictor:

- Training $g(x, d)$ first requires to identify the period
- Once we know d, the we can rewrite the equation as:

$$f(x_i, d) \simeq \hat{y}_i - g(x_i, d)$$

- The target for the linear regressor is obtained by taking the original target
- ...And subtracting the target d step earlier

Differencing

The use of a difference operator is the reason for the name "differencing"

We obtain the linear regressor targets by subtraction:

In [13]:
```
periods = [1, 336]
deltas = []
data_d = data.copy()
for d in periods:
    delta = data_d.iloc[:-d]
    data_d = data_d.iloc[d:] - delta.values
    deltas.append(delta)
```

- The `values` field returns the data as a numpy array (without an index)
- We use it to keep the index of `data_d.iloc[d:]`
- We store all subtracted vectors so we can reverse the transformation
    - I.e. so that we can later compute the value of the $g(x, d)$ predictor

Differencing

The use of a difference operator is the reason for the name "differencing"

We obtain the linear regressor targets by subtraction:

In [14]:
```
periods = [1, 336]
deltas = []
data_d = data.copy()
for d in periods:
    delta = data_d.iloc[:-d]
    data_d = data_d.iloc[d:] - delta.values
    deltas.append(delta)
```

We have also added a "1" period

- This is not captured by frequency analysis via FFT
- ...But most signals have some "inertia"
- ...Hence, just repeating the last value has a strong predicting value

The act of just predicting the last value has even name: persistence predictor

Differencing

We can wrap the algorithm in a function

This is available in the `nab` module

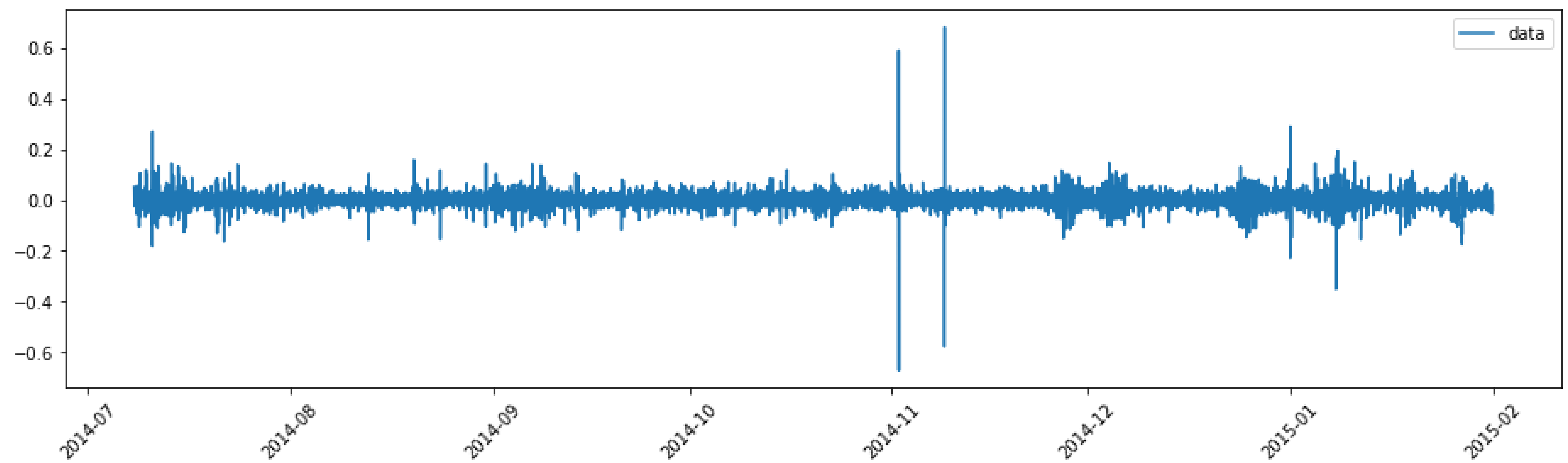
```
def apply_differencing(data, lags):
    deltas = {}
    data_d = data.copy()
    for d in lags:
        delta = data_d.iloc[:-d]
        data_d = data_d.iloc[d:] - delta.values
        deltas[d] = delta
    return data_d, deltas
```

```
In [15]: data_d, deltas = nab.apply_differencing(data, periods)
```

Differencing

We can now plot the resulting series

```
In [17]: nab.plot_series(data_d, figsize=figsize)
```



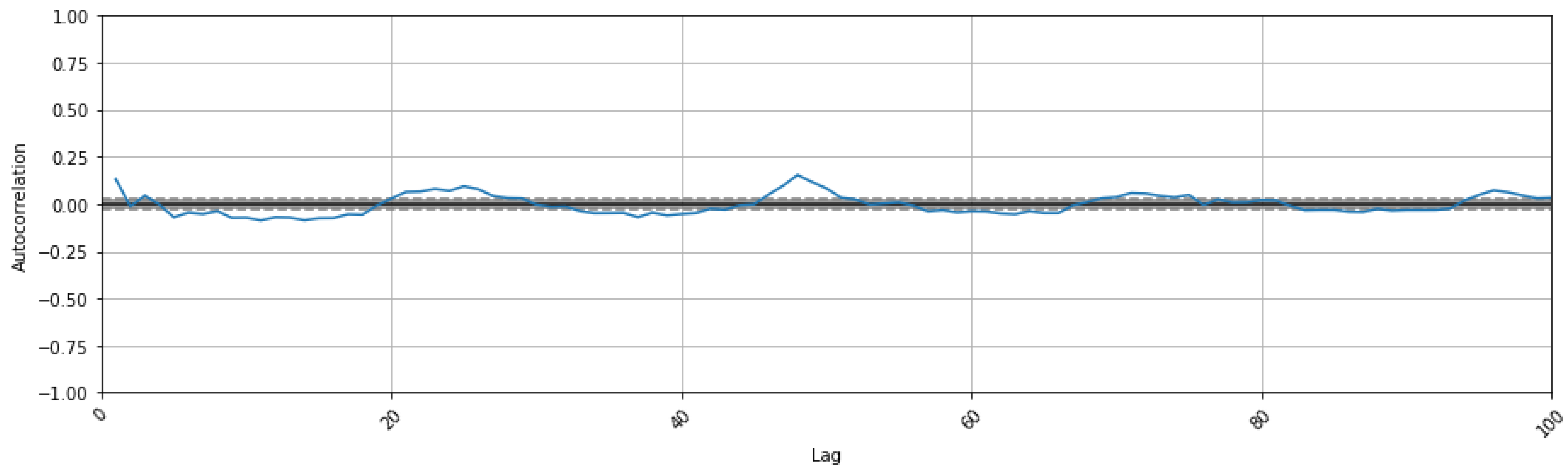
- Periods are no longer visible!

Preparing the Data

Now we need to choose a sliding window length

For this task, the best tool is still an autocorrelation plot

In [19]:
```
nab.plot_autocorrelation(data_d, figsize=figsize)
```



- Correlations are now very weak: we can use a small window

Preparing the Data

Then, we can actually apply the sliding window

In [20]:
```
wlen_d = 3
wdata_d = nab.sliding_window_1D(data_d, wlen=wlen_d)
wdata_d.head()
```

Out[20]:

	0	1	2
timestamp			
2014-07-08 01:30:00	0.050538	0.038159	-0.024660
2014-07-08 02:00:00	0.038159	-0.024660	-0.025088
2014-07-08 02:30:00	-0.024660	-0.025088	-0.001284
2014-07-08 03:00:00	-0.025088	-0.001284	0.005400
2014-07-08 03:30:00	-0.001284	0.005400	0.001877

...And then separate input and output

In [21]:
```
wdata_d_out = data_d.iloc[wlen_d:]['value']
wdata_d_in = wdata_d.iloc[:-1]
```

Fitting the Autoregressor

Now, we can train again our regressor

In [22]:
```
wdata_d_in_tr = wdata_d_in[wdata_d_in.index < train_end]
wdata_d_out_tr = wdata_d_out[wdata_d_out.index <= train_end]

reg_d = LinearRegression()
reg_d.fit(wdata_d_in_tr, wdata_d_out_tr);
```

Then we obtain the predictions:

In [23]:
```
pred_d = reg_d.predict(wdata_d_in)
pred_d = pd.Series(index=wdata_d_out.index, data=pred_d)
```

- These are predictions for the signal after differencing
- They do not reflect the scale of the original signal
- ...And therefore they are not yet enough

Fitting the Autoregressor

Differencing leads to a more complex predictor

- We have the differencing operation + the Linear Regression model
- Once again, we have an ensemble

Getting the actual predictions requires to reverse the differencing operation

In [24]:
```
dsum = 0
pred = pred_d.copy()
for i, d in reversed(list(enumerate(periods))):
    delta = deltas[i].values.reshape((-1,))
    pred = pred + delta[wlen_d+dsum:]
    dsum +=  d
targets = data.iloc[wlen_d+dsum:]['value']
```

- We do it by summing the right-most part of the residual vectors we stored
  - For this we visit the periods in reverse order
  - ...And keep a sum (`dsum`) of the processed periods
- We also need to take into account the window for the Linear Regression model

Fitting the Autoregressor

We can wrap even this operation in a function

This is available in the `nab` module

```
def deapply_differencing(pred, deltas, lags, extra_wlen=0):
    dsum = 0
    pred_dd = pred.copy()
    for i, d in reversed(list(enumerate(lags))):
        delta = deltas[i].values.reshape((-1,))
        pred_dd = pred_dd + delta[extra_wlen+dsum:]
        dsum +=  d
    return pred_dd
```

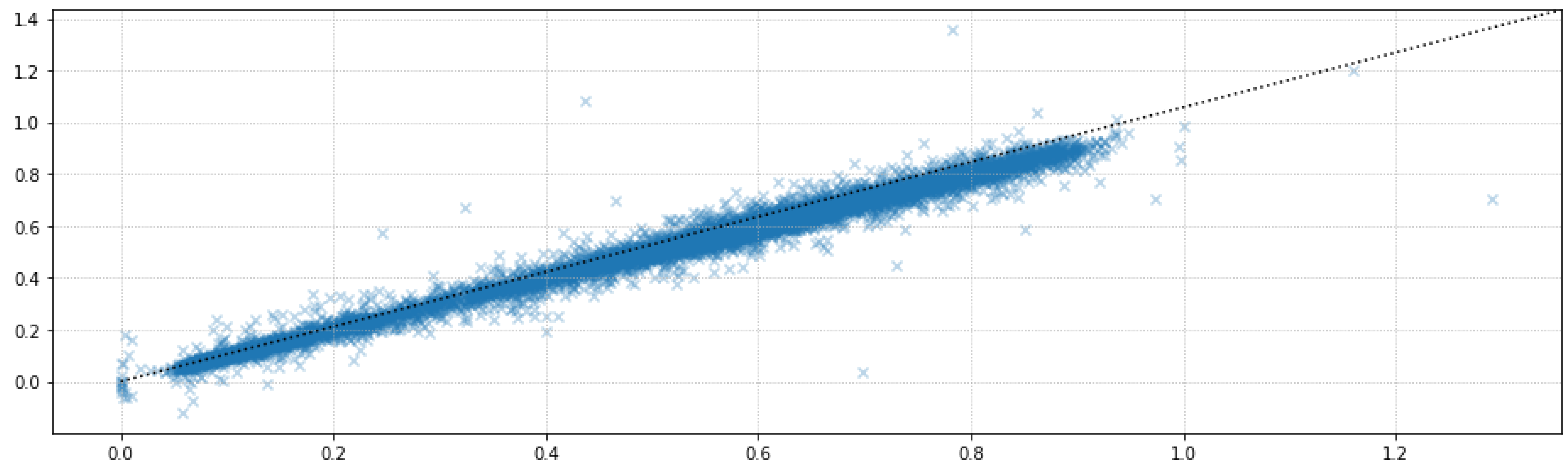
In [25]:

```
pred = nab.deapply_differencing(pred_d, deltas, periods, wlen_d)
```

Prediction Quality

We can check the prediction quality via a scatter plot:

In [27]:
```
nab.plot_prediction_scatter(targets, pred, figsize=figsize)
```



...And via the R2 score:

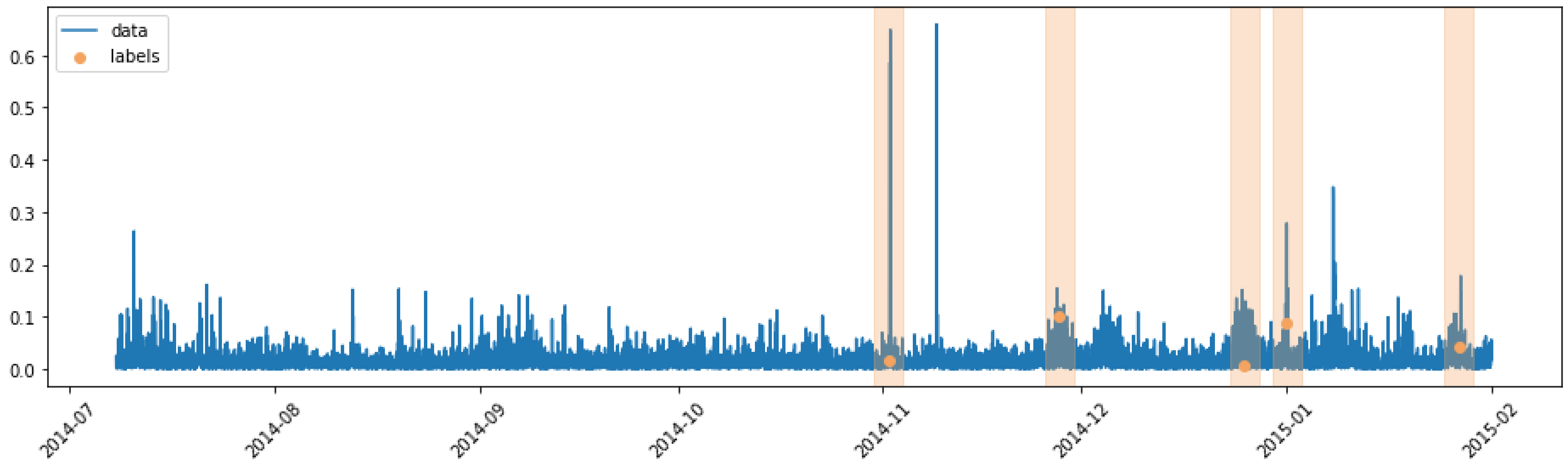
In [28]:
```
r2_score(targets, pred)
```

Out[28]: 0.9818505405355401

Alarm Signal

Now we compute the errors to obtain our signal

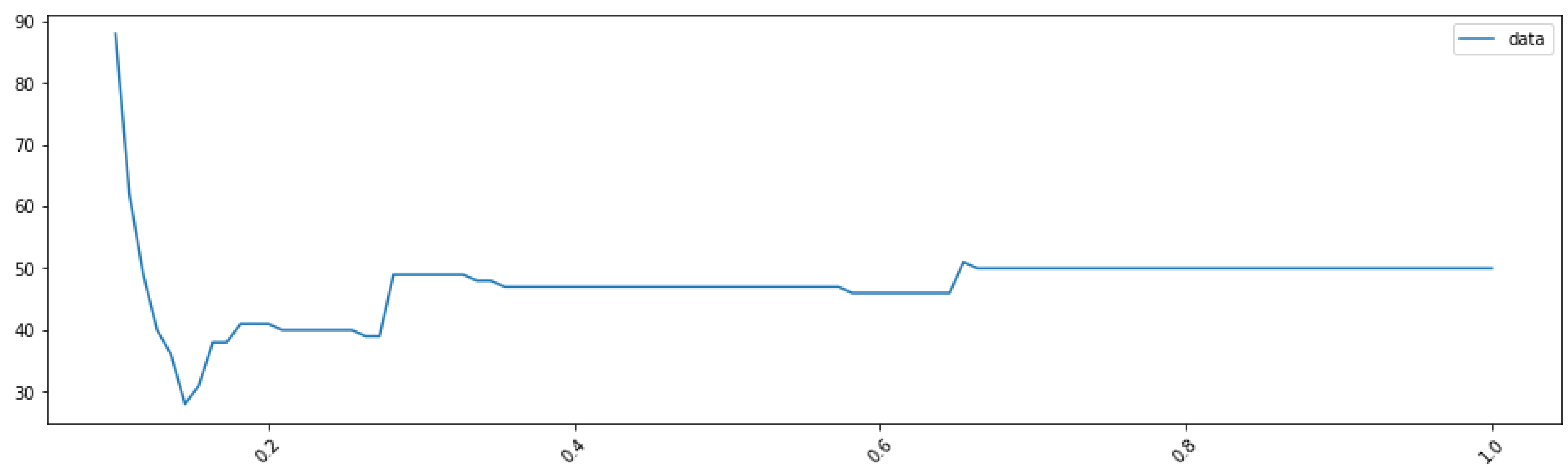
In [30]:
```
err_d = targets - pred
signal_d = np.abs(err_d)
nab.plot_series(signal_d, labels, windows, figsize=figsize)
```



Effect of the Threshold

Here's the cost surface w.r.t. the threshold

In [32]:
```
thr_range = np.linspace(0.1, 1, 100)
cost_range = [cmodel.cost(signal_d, labels, windows, thr)
              for thr in thr_range]
cost_range = pd.Series(index=thr_range, data=cost_range)
nab.plot_series(cost_range, figsize=figsize)
```



Threshold Optimization

We can choose the value of θ in the usual way

In [33]:
```
signal_d_opt = signal_d[signal_d.index < val_end]
labels_opt = labels[labels < val_end]
windows_opt = windows[windows['end'] < val_end]

best_thr, best_cost = nab.opt_thr(signal_d_opt, labels_opt,
                                  windows_opt,  cmodel, thr_range)
print(f'Best threshold: {best_thr:.3f}, corresponding cost: {best_cost}')
```

```
Best threshold: 0.155, corresponding cost: 10
```

Over all the dataset

In [34]:
```
ctst = cmodel.cost(signal_d, labels, windows, best_thr)
print(f'Cost on the whole dataset {ctst}')
```

```
Cost on the whole dataset 31
```

Considerations

We have seen just one way to handle seasonality:

- Another method consists in augmenting the input data...
- ...With a frequency component in the form $w_1 \cos \pi\omega t + w_2 \sin \pi\omega t$
  - ω corresponds to the frequency we want to capture
  - w_1 and w_2 are weights to be tuned by Linear Regression
- In practice, we add one column for $\cos \pi\omega t$ and one for $\sin \pi\omega t$

Differencing is an idea with a broader scope:

- It is at the heart of so-called "integrated" linear approaches for time series
- ...It's the "I" in ARIMA models
  - ...Which BTW stands for Auto Regressive Integrated Moving Average

Considerations

Seasonality and time-dependent models rely on periods

- ...But they do different things!
- In one case we model the period effect, and then we remove it
  - E.g. differencing, using a frequency component
- In time-indexed models we learn models that focus on specific times
  - There is no explicit time model
  - ...Or, better, the time model is a lookup table with distinct predictors
- Hard to say a priori which approach can work best

Both approaches not tied to Linear Regression and KDE

Removing seasonality is related to gradient boosting

- I.e. to the idea of building an ensemble as a sequence of (summed) predictors
- ..Each one trying to correct the mistake of the sub-sequence that precedes it