Anomaly Detection via Autoregression

Drawbacks

We managed to use KDE for anomaly detection to a good effect

But there are some drawbacks:

- Training was pretty fast (except for the bandwidth optimization)
- ...but obtaining predictions was a bit slow

Latency may be an issue:

- In our case, time steps are 30 minutes long
- ...Which gives ample time to make predictions
- In other problems, there are strict latency constraints
  - E.g. graceful thermal throttling of a multi-core CPU
  - E.g. speed control in an industrial pump

What if KDE turns up to be too slow?

Anomaly Detection via Autoregression

Let us see an alternative approach for anomaly detection:

The main idea is to build an autoregressor, say $f(x)$:

- Autoregressor = a predictor for the next step in the time series
- We can then use the prediction error as an alarm signal

$$|f(x) - y| \geq \theta$$

Why doing that?

- This trick allows us to use any regression approach for anomaly detection
- E.g. Linear Regression, (Ensembles of) Decision Trees, Neural Networks...

As usual, we will make an attempt with the simplest possible approach:

I.e. we are going to use Linear Regression

Linear Regression

A few known things about Linear Regression

1) It's a supervised learning approach (training set = $\hat{x}$, target = $\hat{y}$)

2) The goal is typically to fit a linear combination of the input features, i.e.:

$$f(x, w) = w_0 + \sum_{j=1}^{n} w_i x_i$$

- w is a vector of $n + 1$ weights, w_0 is used a constant and it's called intercept
- If the signal to be predicted has 0-mean, w_0 can be omitted

3) Training is done via the Least Squares method:

$$\text{argmin}_w \| f(\hat{x}, w) - \hat{y} \|_2^2$$

Linear Regression

A few less-known things about Linear Regression

1) The least squares method can be optimally solved in polynomial time

- It's a convex, unconstrained, numerical optimization problem
- Solving a linear systems of equations yields an optimal solution
- Gradient descent gets arbitrarily close

2) The least squares method works for "non-linear" functions

- In fact, you can fit any function in the form:

$$f(x, w) = w_0 + \sum_{j=1}^{n} w_i K(x)$$

- K is called in this case a basis function and can be non-linear

Linear Regression

A few less-known things about Linear Regression

In the least squares method we minimize:

$$\sum_{i=1}^{m}(f(\hat{x}_i, w) - \hat{y}_i)^2$$

Multiplying everything for a scalar does not change the optimal point:

$$\sum_{i=1}^{m}\frac{1}{2}(f(\hat{x}_i, w) - \hat{y}_i)^2$$

Linear Regression

A few less-known things about Linear Regression

Then, with a few manipulations we get:

$$\sum_{i=1}^{m} \log e^{\frac{1}{2}(f(\hat{x}_i,w)-\hat{y}_i)^2}$$

...And then:

$$\log\left(\prod_{i=1}^{m} e^{\frac{1}{2}(f(\hat{x}_i,w)-\hat{y}_i)^2}\right)$$

Linear Regression

A few less-known things about Linear Regression

Once, again we make a few modifications that do not change the optimal solution:

$$\log\left(\prod_{i=1}^{m} \frac{1}{\sigma\sqrt{2\pi}} e^{\left(\frac{f(\hat{x}_i, w) - \hat{y}_i}{\sigma}\right)^2}\right)$$

- Where $\boldsymbol{\sigma}$ is constant

This is the log likelihood of $\hat{y}$, assuming a normal distribution!

3) So the predictions can be interpreted as solutions of $\mathbf{argmax}_y P(\hat{y} \mid \mathbf{x})$

- I.e. the most likely values, assuming that the target
- ...Follows a a Normal distribution with a fixed variance

This is an important (often) hidden assumption of Linear Regression

Linear Regression

It does not hold only for Linear Regression

Given an observation x:

Any regressor f(x) trained for a MSE loss outputs a conditional Maximum A Posteriori (MAP), with the assumption that the target has fixed variance

- MAP the most likely value of an estimated probability distribution
- The distribution is conditional, since x is observed
- The fixed variance assumption is called homoscedasticity
- If the data is not really homoscedastic, the regressor will work less well

Preparing The Dataset

Before using Linear Regression, we need to build the target vector

In [3]:
```
wdata_out = data.iloc[wlen:]['value']
wdata_out.head()
```

Out[3]:
```
timestamp
2014-07-02 00:00:00    0.440194
2014-07-02 00:30:00    0.327429
2014-07-02 01:00:00    0.249267
2014-07-02 01:30:00    0.194811
2014-07-02 02:00:00    0.158694
Name: value, dtype: float64
```

- The last known output will be at the end of the series
- So we need to remove the last row from our sliding window dataset

In [4]:
```
wdata_in = wdata.iloc[:-1]
```

Fitting the Linear Model

Next, we need to separate the training set:

```
In [5]: wdata_in_tr = wdata_in[wdata_in.index < train_end]
        wdata_out_tr = wdata_out[wdata_out.index <= train_end] # Notice the "<=" sign
```

- `wdata_in` and `wdata_out` have different indices (hence the "<=" sign)

Then, we can train our predictor:

```
In [6]: from sklearn.linear_model import LinearRegression
        reg = LinearRegression()
        reg.fit(wdata_in_tr, wdata_out_tr);
```

- Predictions are associated to the `wdata_out` index

Obtaining the Predictions

We can now obtain the predictions:

In [7]:
```
%time pred_out = reg.predict(wdata_in)
pred_out = pd.Series(index=wdata_out.index, data=pred_out)
```

```
CPU times: user 5.12 ms, sys: 3.09 ms, total: 8.21 ms
Wall time: 2.83 ms
```

- The process is now very fast!
- Time can be measures via the `%time` ipython magic

For comparison, let us see the time for KDE (the univariate estimator)

In [8]:
```
%time ldens = kde.score_samples(data)
signal = pd.Series(index=data.index, data=-ldens)
```

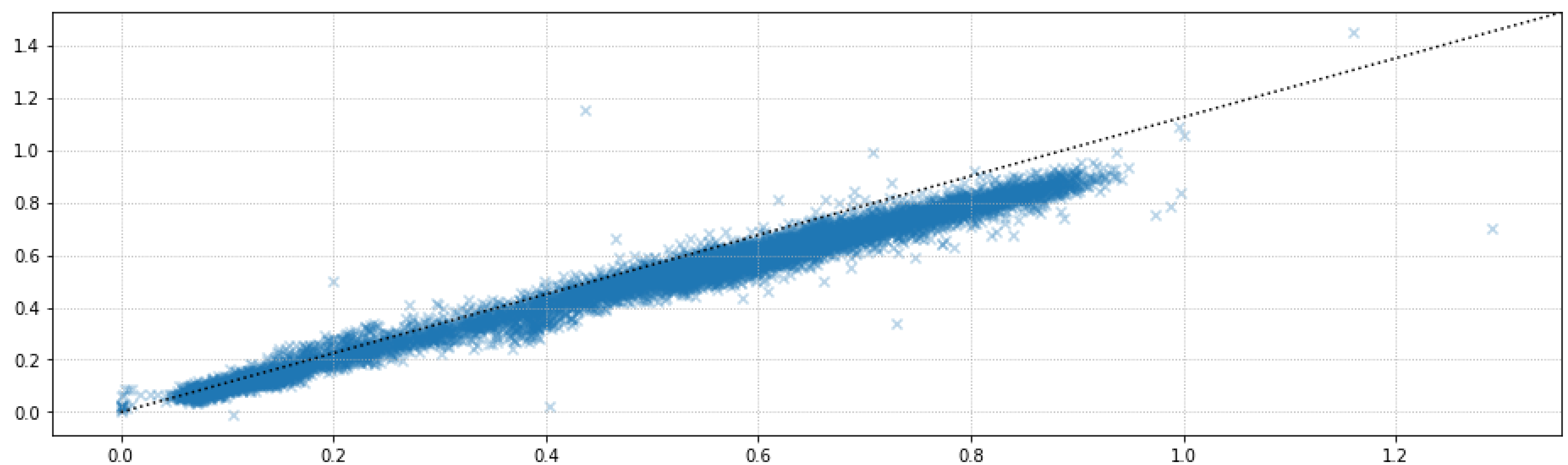
```
CPU times: user 2.92 s, sys: 192 µs, total: 2.92 s
Wall time: 2.93 s
```

Prediction Quality

Let us have a look at the prediction quality:

In [10]:
```
nab.plot_prediction_scatter(wdata_out, pred_out, figsize=figsize)
print(f'R2: {r2_score(wdata_out, pred_out):.3f}')
```

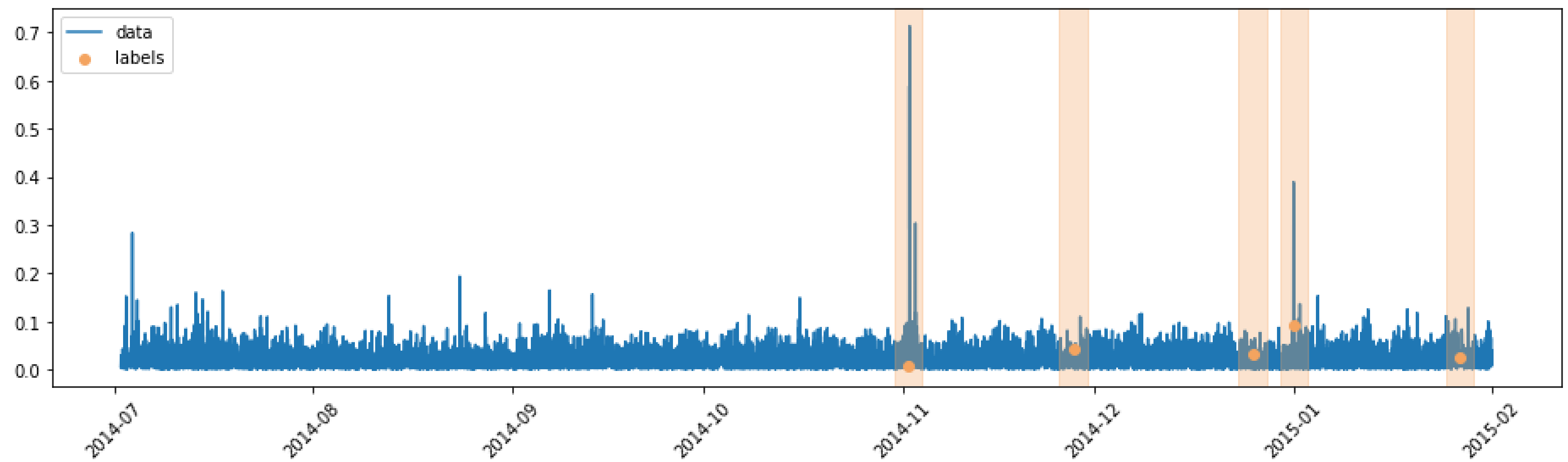
```
R2: 0.979
```



Alarm Signal

We now just need to compute the errors to obtain our signal

In [12]:
```
err = wdata_out - pred_out
signal = np.abs(err)
nab.plot_series(signal, labels, windows, figsize=figsize)
```



- It does not seem to be particularly good

Threshold Optimization

We can proceed as usual (threshold optimization and evaluation)

In [13]:
```
thr_range = np.linspace(0.1, 1, 100)
signal_opt = signal[signal.index < val_end]
labels_opt = labels[labels < val_end]
windows_opt = windows[windows['end'] < val_end]
best_thr, best_cost = nab.opt_thr(signal_opt, labels_opt,
                                  windows_opt,  cmodel, thr_range)
print(f'Best threshold: {best_thr:.3f}, corresponding cost: {best_cost}')
```

```
Best threshold: 0.291, corresponding cost: 15
```

Over all the dataset

In [14]:
```
ctst = cmodel.cost(signal, labels, windows, best_thr)
print(f'Cost on the whole dataset {ctst}')
```

```
Cost on the whole dataset 35
```

Considerations

Some considerations and take-home messages

An alternative way to perform Anomaly Detection:

- Train an autoregressor and use the (absolute) error as an alarm signal
- You can use any regression approach
- Typicallly faster signal generations

We are not restricted to absolute errors:

- We can use other functions, with different results
- A better interpretation will come in a few lectures

The least squares method:

- Assumes the predictions is Normally distributed with fixed variance
- The fixed variance is an important limitation