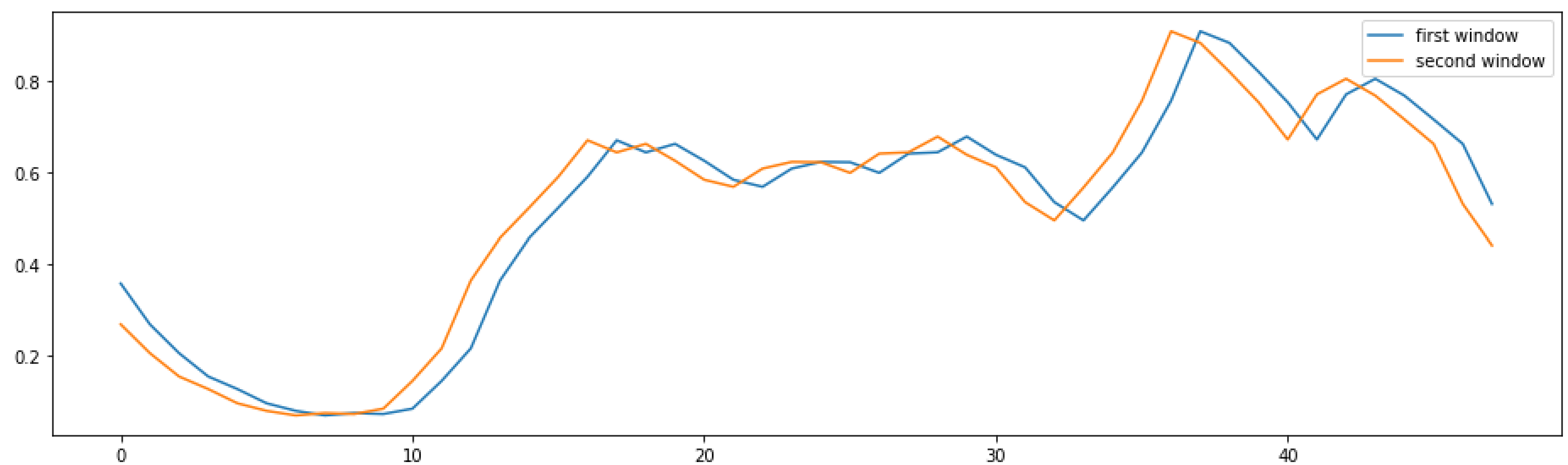
Time Indexed Models

Spotting the Problem

Our KDE estimator is learning from all the training data

This means it will learn from both these series:

In [5]:
```
plt.figure(figsize=figsize)
plt.plot(wdata.iloc[0], label='first window')
plt.plot(wdata.iloc[1], label='second window')
plt.legend()
plt.tight_layout()
```



Spotting the Problem

Let us consider the first two window applications

- In the first window, the observations are x_0, x_1 and so on
- In the second window, the observations are x_1, x_2 and so on

x_0 is number of taxis as 00:00, x_1 at 00:30, and so on

- Hence, the first observation in the first window corresponds to 00:00
- ...But in the second window corresponds to 00:30

Our estimator learns a distribution for the observations:

- Moving the window forward changes "who is who"
- We learn the distribution of x_0 (and its correlations) multiple times!

The learning problem is still well defined, but also very complex

We will see how to improve things by adding time information

Exploiting Time

We have mentioned two ways to exploit time information in KDE:

The first approach consists in making time (of the day) an additional variable:

- Say $\mathbf{x}$ is the vector of our observations and z is the time
- Then we learn an estimator for $P(\mathbf{x}, z)$ and one for $P(z)$
- So that we can compute $P(\mathbf{x} \mid z) = P(\mathbf{x}, z)/P(z)$

In our case $P(z)$ is trivial (uniform)

This is what we did in the previous lecture:

- We managed to use the time information
- ...But we had to add one dimension

This makes the estimation problem more complex

- ...And we are having enough trouble as it is

Exploiting Time

We have discussed two ways to exploit time information in KDE:

The second approach consists in learning many KDE estimators:

- Each KDE estimator is specialized for a given time (e.g. 00:00, 00:30, 01:00...)
- In other words: our estimator becomes an ensemble
- Whole estimation = time-based switch + time-specialized KDE

Let us see the properties of this approach

- The whole model is more complex and the approach requires discrete time
- Each KDE estimator works with smaller amounts of data
- ...But the individual density estimation problems are easier!

Let us investigate this approach

Learning a 23:30 Estimator

Let us make a test by learning an estimator for 23:30

First, we separate the training data:

In [6]:
```
wdata_tr = wdata[wdata.index < train_end]
wdata_tr.head()
```

Out[6]:

	0	1	2	3	4	5	6	7	8	9	...	38	
timestamp													
2014-07-01 23:30:00	0.357028	0.267573	0.204458	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	...	0.883252	0.8
2014-07-02 00:00:00	0.267573	0.204458	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	0.082804	...	0.819939	0.7
2014-07-02 00:30:00	0.204458	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	0.082804	0.143680	...	0.753136	0.6
2014-07-02 01:00:00	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	0.082804	0.143680	0.214862	...	0.671452	0.7
2014-07-02 01:30:00	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	0.082804	0.143680	0.214862	0.363448	...	0.770454	0.8

5 rows × 48 columns

Learning a 23:30 Estimator

Let us make a test by learning an estimator for 23:30

Then, we focus on sequences corresponding to 23:30

In [7]:
```
wdata_tr_2330 = wdata_tr.iloc[0::48]
wdata_tr_2330.head()
```

Out[7]:

	0	1	2	3	4	5	6	7	8	9	...	38	
timestamp													
2014-07-01 23:30:00	0.357028	0.267573	0.204458	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	...	0.883252	0.8
2014-07-02 23:30:00	0.440194	0.327429	0.249267	0.194811	0.158694	0.119646	0.098541	0.083462	0.084615	0.081816	...	0.819379	0.8
2014-07-03 23:30:00	0.416357	0.347743	0.277088	0.233694	0.191815	0.144306	0.107661	0.097060	0.103579	0.101307	...	0.987225	0.7
2014-07-04 23:30:00	0.513318	0.473941	0.412702	0.373391	0.328581	0.276693	0.237053	0.216574	0.186251	0.147302	...	0.493827	0.4
2014-07-05 23:30:00	0.578672	0.533006	0.475455	0.412702	0.362361	0.301287	0.263721	0.233629	0.210944	0.145557	...	0.598624	0.5

5 rows × 48 columns

Learning a 23:30 Estimator

Then we proceed as usual

We choose a bandwidth:

```
In [8]: grid = GridSearchCV(KernelDensity(kernel='gaussian'),
                            {'bandwidth': np.linspace(0.01, 0.1, 20)}, cv = 5)
        grid.fit(wdata_tr_2330)
        grid.best_params_

Out[8]: {'bandwidth': 0.03368421052631579}
```

...And we train the estimator:

```
In [9]: h = grid.best_params_['bandwidth']
        kde_2330 = KernelDensity(kernel='gaussian', bandwidth=h)
        kde_2330.fit(wdata_tr_2330);
```

- We just need to repeat this for each (meaningful) time value

Learning the Ensemble

In our case, all unique hour values are meaningful

```
In [10]: day_hours = wdata_tr.index.hour + wdata_tr.index.minute / 60
         day_hours = day_hours.unique()
         print(day_hours)

Float64Index([23.5,  0.0,  0.5,  1.0,  1.5,  2.0,  2.5,  3.0,  3.5,  4.0,  4.5,
               5.0,  5.5,  6.0,  6.5,  7.0,  7.5,  8.0,  8.5,  9.0,  9.5, 10.0,
              10.5, 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5,
              16.0, 16.5, 17.0, 17.5, 18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0,
              21.5, 22.0, 22.5, 23.0],
             dtype='float64', name='timestamp')
```

- `unique` in pandas returns a `Series` with all unique values
- It does not matter how we measure time
- ...We only care about having 48 discrete steps

Learning the Ensemble

Then we learn 48 specialized estimators

In [11]:
```
kde = {}
for hidx, hour in enumerate(day_hours):
    tmp_data = wdata_tr.iloc[hidx::48]
    kde[hour] = KernelDensity(kernel='gaussian', bandwidth=h)
    kde[hour].fit(tmp_data)
```

- For each unique time value, we separate a subset of the training data
- Then we build and learn a KDE estimator

We chose to store everything in a dictionary:

In [12]:
```
print(str(kde)[:256], '...}')
```

```
{23.5: KernelDensity(bandwidth=0.03368421052631579), 0.0: KernelDensity(bandwidth=0.0336842105
2631579), 0.5: KernelDensity(bandwidth=0.03368421052631579), 1.0: KernelDensity(bandwidth=0.03
368421052631579), 1.5: KernelDensity(bandwidth=0.03368421052631579), ...}
```

Generating the Signal

The we can generate the alarm signal

- In a practical implementation we should do this step by step
- ...But for an evaluation purpose it is easier to do it all at once

```
In [13]: ldens_list = []
         for hidx, hour in enumerate(day_hours):
             tmp_data = wdata.iloc[hidx::48]
             tmp_ldens = kde[hour].score_samples(tmp_data)
             tmp_ldens = pd.Series(index=tmp_data.index, data=tmp_ldens)
             ldens_list.append(tmp_ldens)
```

- For each unique time value, we separate a subset of the whole data
- Then we obtain the estimated (log) probabilities

The process is faster than before!

- ...Because each KDE estimator is trained a smaller dataset

Generating the Signal

All signals are stored in a list

- We need to concatenate them all in single `DataFrame`
- Then we can sort all rows by timestamp (it's the index)

```
In [14]: ldens = pd.concat(ldens_list, axis=0)
         ldens = ldens.sort_index()
         signal = -ldens
         signal.head()

Out[14]: timestamp
         2014-07-01 23:30:00   -113.900870
         2014-07-02 00:00:00   -113.909604
         2014-07-02 00:30:00   -113.909604
         2014-07-02 01:00:00   -113.909604
         2014-07-02 01:30:00   -113.909604
         dtype: float64
```

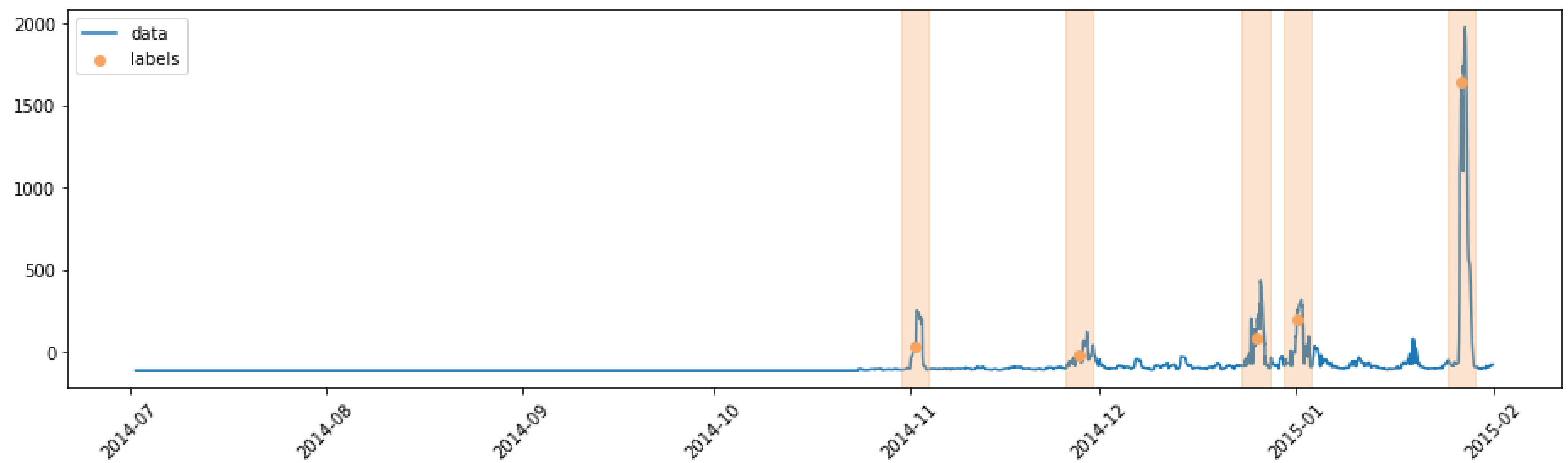
A suggestion: always do concatenations in a single step in `pandas`

It's way faster than appending `DataFrame` objects one by one

Generating the Signal

Now we can plot out signal:

```
In [15]: nab.plot_series(signal, labels=labels, windows=windows, figsize=figsize)
```

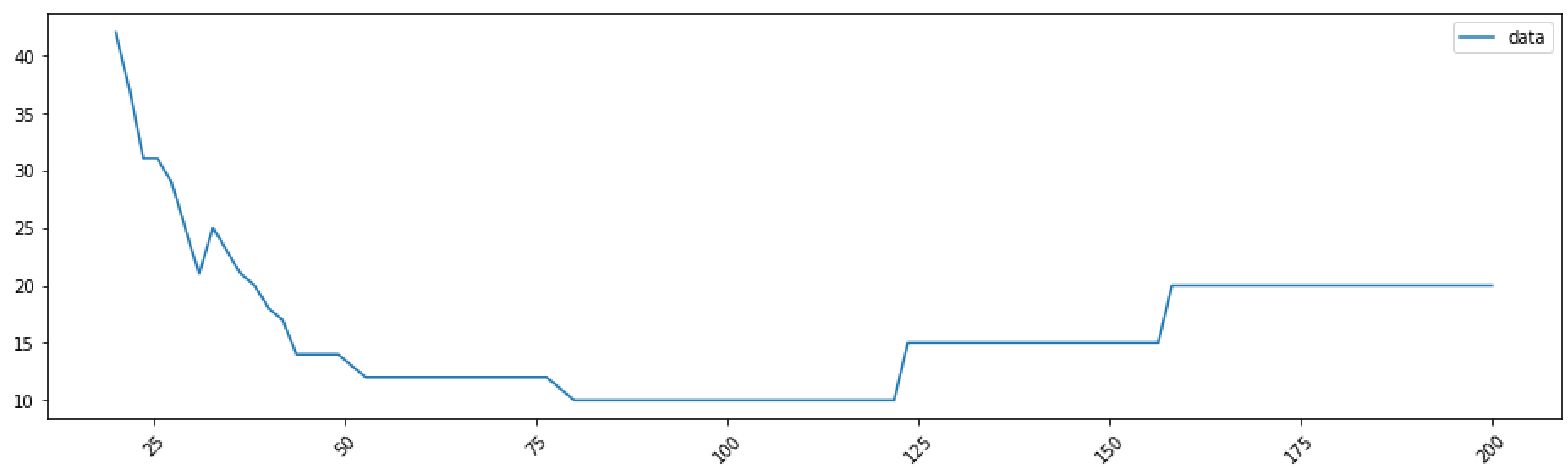


It's way more stable than before!

Effect of the Threshold

Now, as usual, we examine the response surface

In [16]:
```
thr_range = np.linspace(20, 200, 100)
cost_range = [cmodel.cost(signal, labels, windows, thr)
              for thr in thr_range]
cost_range = pd.Series(index=thr_range, data=cost_range)
nab.plot_series(cost_range, figsize=figsize)
```



Threshold Optimization

Then we optimize the threshold:

In [17]:
```
signal_opt = signal[signal.index < val_end]
labels_opt = labels[labels < val_end]
windows_opt = windows[windows['end'] < val_end]

best_thr, best_cost = nab.opt_thr(signal_opt, labels_opt,
                                  windows_opt,  cmodel, thr_range)
print(f'Best threshold: {best_thr}, corresponding cost: {best_cost}')
```

```
Best threshold: 43.63636363636364, corresponding cost: 10
```

Let us see the cost on the whole dataset:

In [18]:
```
ctst = cmodel.cost(signal, labels, windows, best_thr)
print(f'Cost on the whole dataset {ctst}')
```

```
Cost on the whole dataset 14
```

Considerations

Some considerations and take-home messages

Be mindful of trade-offs

- We wanted to exploit some new information, and we had to choose between:
  - Using all data, but complicating the learning problem
  - Loosing some data, but simplifying the learning problem
- We chose the latter, but perhaps even the other approach had a chance!

Careful with time-dependent models

- Using raw, absolute, time as a feature may lead to overfitting
- ...Since every sample has its own time
- So, exploiting time relies on the existend ot meaningful periods