

# RUL Prediction as Regression

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# RUL Prediction as Regression

Let's start from **the most straightforward formulation** of a RUL problem

- We will predict the RUL using a regression approach
- ...And trigger maintenance when the estimated RUL becomes too low, i.e.:

$$f(x, \lambda) < \theta$$

- $f$  is the regressor, with parameter vector  $\lambda$
- The threshold  $\theta$  must account for possible estimation errors

**We will focus on the hardest of the four datasets (to reduce training times):**

```
In [4]: data_by_src = cmapss.split_by_field(data, field='src')
        dt = data_by_src['train_FD004']
```

# Training and Test Data

## We now need to define our training and test data

In a practical setting:

- Some run-to-failure experiments will form the training set
- Others run-to-failure experiments will be used for testing

I.e. we split **whole experiments** rather than individual examples!

## Each run-to-failure experiment in our data is associated to a machine

Let's check how many we have:

```
In [5]: print(f'Number of machines: {len(dt.machine.unique())}')
```

```
Number of machines: 249
```

- This is actually a very large number
- In most practical setting, **much fewer** experiments will be available

# Training and Test Data

**Let's use 75% of the machine for training, the rest for testing**

First, we partition the machine indexes:

```
In [6]: tr_ratio = 0.75
        np.random.seed(42)
        machines = dt.machine.unique()
        np.random.shuffle(machines)

        sep = int(tr_ratio * len(machines))
        tr_mcn = machines[:sep]
        ts_mcn = machines[sep:]
```

Then, we partition the dataset itself (via a helper function):

```
In [7]: tr, ts = cmapss.partition_by_machine(dt, tr_mcn)
```

# Training and Test Data

## Let's have a look at the training data

In [8]:

```
tr
```

Out[8]:

	src	machine	cycle	p1	p2	p3	s1	s2	s3	s4	...	s13	s14	s15	s16
0	train_FD004	461	1	42.0049	0.8400	100.0	445.00	549.68	1343.43	1112.93	...	2387.99	8074.83	9.3335	0.02
1	train_FD004	461	2	20.0020	0.7002	100.0	491.19	606.07	1477.61	1237.50	...	2387.73	8046.13	9.1913	0.02
2	train_FD004	461	3	42.0038	0.8409	100.0	445.00	548.95	1343.12	1117.05	...	2387.97	8066.62	9.4007	0.02
3	train_FD004	461	4	42.0000	0.8400	100.0	445.00	548.70	1341.24	1118.03	...	2388.02	8076.05	9.3369	0.02
4	train_FD004	461	5	25.0063	0.6207	60.0	462.54	536.10	1255.23	1033.59	...	2028.08	7865.80	10.8366	0.02
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
60989	train_FD004	708	180	35.0019	0.8409	100.0	449.44	556.28	1377.65	1148.96	...	2387.77	8048.91	9.4169	0.02
60990	train_FD004	708	181	0.0023	0.0000	100.0	518.67	643.95	1602.98	1429.57	...	2388.27	8122.44	8.5242	0.03
60991	train_FD004	708	182	25.0030	0.6200	60.0	462.54	536.88	1268.01	1067.09	...	2027.98	7865.18	10.9790	0.02
60992	train_FD004	708	183	41.9984	0.8414	100.0	445.00	550.64	1363.76	1145.72	...	2387.48	8069.84	9.4607	0.02
60993	train_FD004	708	184	0.0013	0.0001	100.0	518.67	643.50	1602.12	1430.34	...	2388.33	8120.43	8.4998	0.03

45385 rows × 28 columns

# Training and Test Data

...And at the test data

In [9]: ts

Out[9]:

	src	machine	cycle	p1	p2	p3	s1	s2	s3	s4	...	s13	s14	s15	s16
<b>321</b>	train_FD004	462	1	41.9998	0.8400	100.0	445.00	548.99	1341.82	1113.16	...	2387.98	8082.37	9.3300	0.02
<b>322</b>	train_FD004	462	2	9.9999	0.2500	100.0	489.05	604.23	1498.00	1299.54	...	2388.07	8125.46	8.6088	0.03
<b>323</b>	train_FD004	462	3	42.0079	0.8403	100.0	445.00	549.11	1351.47	1126.43	...	2387.93	8082.11	9.2965	0.02
<b>324</b>	train_FD004	462	4	42.0077	0.8400	100.0	445.00	548.77	1345.81	1116.64	...	2387.88	8079.41	9.3200	0.02
<b>325</b>	train_FD004	462	5	24.9999	0.6200	60.0	462.54	537.00	1259.55	1043.95	...	2028.13	7867.08	10.8841	0.02
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
<b>61244</b>	train_FD004	709	251	9.9998	0.2500	100.0	489.05	605.33	1516.36	1315.28	...	2388.73	8185.69	8.4541	0.03
<b>61245</b>	train_FD004	709	252	0.0028	0.0015	100.0	518.67	643.42	1598.92	1426.77	...	2388.46	8185.47	8.2221	0.03
<b>61246</b>	train_FD004	709	253	0.0029	0.0000	100.0	518.67	643.68	1607.72	1430.56	...	2388.48	8193.94	8.2525	0.03
<b>61247</b>	train_FD004	709	254	35.0046	0.8400	100.0	449.44	555.77	1381.29	1148.18	...	2388.83	8125.64	9.0515	0.02
<b>61248</b>	train_FD004	709	255	42.0030	0.8400	100.0	445.00	549.85	1369.75	1147.45	...	2388.66	8144.33	9.1207	0.02

15864 rows × 28 columns

# Standardization/Normalization

Now, we need to make the range of each columns more uniform

We will **standardize** all parameters and sensor inputs:

```
In [10]: trmean = tr[dt_in].mean()
trstd = tr[dt_in].std().replace(to_replace=0, value=1) # handle static fields

ts_s = ts.copy()
ts_s[dt_in] = (ts_s[dt_in] - trmean) / trstd
tr_s = tr.copy()
tr_s[dt_in] = (tr_s[dt_in] - trmean) / trstd
```

We will **normalize** the RUL values (i.e. our regression target)

```
In [12]: trmaxrul = tr['rul'].max()

ts_s['rul'] = ts['rul'] / trmaxrul
tr_s['rul'] = tr['rul'] / trmaxrul
```

# Standardization/Normalization

Let's check the results

In [13]: `tr_s.describe()`

Out [13]:

	machine	cycle	p1	p2	p3	s1	s2	s3	s4
count	45385.000000	45385.000000	4.538500e+04	4.538500e+04	4.538500e+04	4.538500e+04	4.538500e+04	4.538500e+04	4.538500e+04
mean	582.490955	133.323896	2.894775e-16	1.302570e-16	1.178889e-16	4.664830e-15	2.522791e-15	1.727041e-15	1.727041e-15
std	71.283034	89.568561	1.000000e+00	1.000000e+00	1.000000e+00	1.000000e+00	1.000000e+00	1.000000e+00	1.000000e+00
min	461.000000	1.000000	-1.623164e+00	-1.838222e+00	-2.381839e+00	-1.055641e+00	-1.176507e+00	-1.646830e+00	-1.646830e+00
25%	521.000000	62.000000	-9.461510e-01	-1.031405e+00	4.198344e-01	-1.055641e+00	-8.055879e-01	-6.341243e-01	-6.341243e-01
50%	585.000000	123.000000	6.868497e-02	4.154560e-01	4.198344e-01	-3.917563e-01	-6.336530e-01	-4.718540e-01	-4.718540e-01
75%	639.000000	189.000000	1.218855e+00	8.661917e-01	4.198344e-01	6.926385e-01	7.407549e-01	7.495521e-01	7.495521e-01
max	708.000000	543.000000	1.219524e+00	8.726308e-01	4.198344e-01	1.732749e+00	1.741030e+00	1.837978e+00	1.837978e+00

8 rows × 27 columns



# Regression Model

## We can now define a regression model

We will use a feed-forward neural network (MLP):

```
In [14]: def build_regressor(hidden):  
    input_shape = (len(dt_in), )  
    model_in = keras.Input(shape=input_shape, dtype='float32')  
    x = model_in  
    for h in hidden:  
        x = layers.Dense(h, activation='relu')(x)  
    model_out = layers.Dense(1, activation='linear')(x)  
    model = keras.Model(model_in, model_out)  
    return model
```

- The `hidden` argument is a list of sizes for the hidden layers
- ...E.g. `hidden = [64, 32]`
- By setting `hidden=[]` we obtain a simple Linear Regression approach

# Training

## We can now train our model

```
In [15]: nn = build_regressor(hidden=[])
nn.compile(optimizer='Adam', loss='mse')
cb = [callbacks.EarlyStopping(patience=10, restore_best_weights=True)]
history = nn.fit(tr_s[dt_in], tr_s['rul'], validation_split=0.2,
                callbacks=cb, batch_size=32, epochs=20, verbose=1)
```

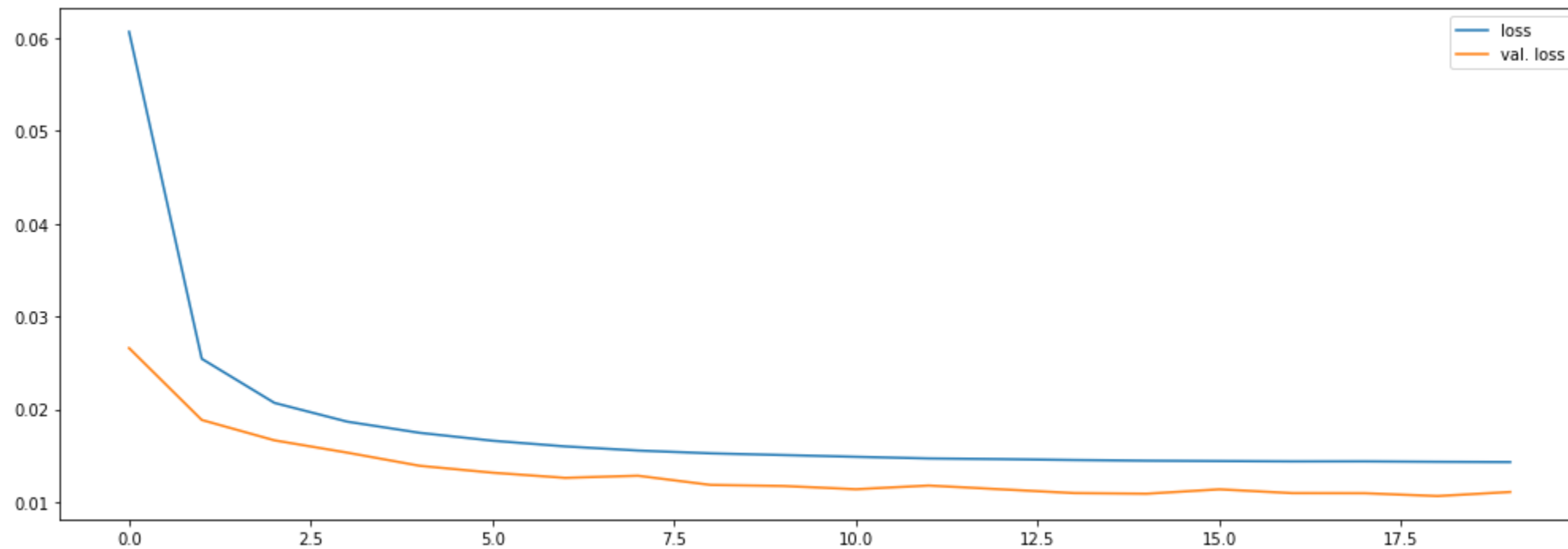
```
Epoch 1/20
1135/1135 [=====] - 1s 606us/step - loss: 0.0607 - val_loss: 0.0266
Epoch 2/20
1135/1135 [=====] - 1s 510us/step - loss: 0.0254 - val_loss: 0.0188
Epoch 3/20
1135/1135 [=====] - 1s 520us/step - loss: 0.0207 - val_loss: 0.0166
Epoch 4/20
1135/1135 [=====] - 1s 516us/step - loss: 0.0187 - val_loss: 0.0153
Epoch 5/20
1135/1135 [=====] - 1s 521us/step - loss: 0.0175 - val_loss: 0.0139
Epoch 6/20
1135/1135 [=====] - 1s 510us/step - loss: 0.0166 - val_loss: 0.0132
Epoch 7/20
1135/1135 [=====] - 1s 515us/step - loss: 0.0160 - val_loss: 0.0126
Epoch 8/20
1135/1135 [=====] - 1s 508us/step - loss: 0.0155 - val_loss: 0.0128
Epoch 9/20
1135/1135 [=====] - 1s 442us/step - loss: 0.0152 - val_loss: 0.0118
Epoch 10/20
1135/1135 [=====] - 0s 434us/step - loss: 0.0151 - val_loss: 0.0117
Epoch 11/20
```

# Training

Here's the loss evolution over time and its final value

```
In [16]: cmapss.plot_training_history(history, figsize=figsize)
trl, vll = history.history["loss"][-1], np.min(history.history["val_loss"])
print(f'Final loss: {trl:.4f} (training), {vll:.4f} (validation)')
```

Final loss: 0.0143 (training), 0.0106 (validation)



# Training

## Let's try with a more complex model

```
In [17]: nn = build_regressor(hidden=[32, 32])
nn.compile(optimizer='Adam', loss='mse')
history = nn.fit(tr_s[dt_in], tr_s['rul'], validation_split=0.2,
                callbacks=cb, batch_size=32, epochs=20, verbose=1)
```

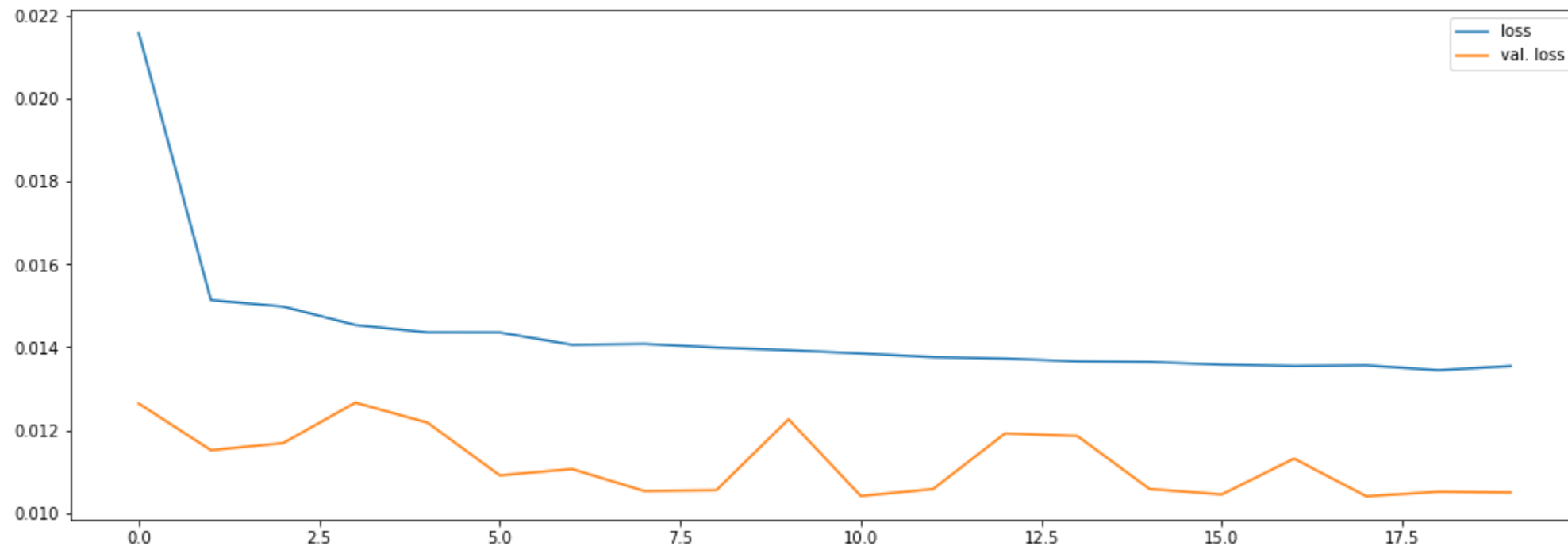
```
Epoch 1/20
1135/1135 [=====] - 1s 687us/step - loss: 0.0216 - val_loss: 0.0126
Epoch 2/20
1135/1135 [=====] - 1s 622us/step - loss: 0.0151 - val_loss: 0.0115
Epoch 3/20
1135/1135 [=====] - 1s 624us/step - loss: 0.0150 - val_loss: 0.0117
Epoch 4/20
1135/1135 [=====] - 1s 624us/step - loss: 0.0145 - val_loss: 0.0127
Epoch 5/20
1135/1135 [=====] - 1s 627us/step - loss: 0.0144 - val_loss: 0.0122
Epoch 6/20
1135/1135 [=====] - 1s 623us/step - loss: 0.0144 - val_loss: 0.0109
Epoch 7/20
1135/1135 [=====] - 1s 634us/step - loss: 0.0141 - val_loss: 0.0111
Epoch 8/20
1135/1135 [=====] - 1s 628us/step - loss: 0.0141 - val_loss: 0.0105
Epoch 9/20
1135/1135 [=====] - 1s 630us/step - loss: 0.0140 - val_loss: 0.0106
Epoch 10/20
1135/1135 [=====] - 1s 556us/step - loss: 0.0139 - val_loss: 0.0123
Epoch 11/20
1135/1135 [=====] - 1s 630us/step - loss: 0.0139 - val_loss: 0.0104
```

# Training

## Let's check the loss behavior and compare it to Linear Regression

```
In [18]: cmapss.plot_training_history(history, figsize=figsize)
trl, vll = history.history["loss"][-1], np.min(history.history["val_loss"])
print(f'Final loss: {trl:.4f} (training), {vll:.4f} (validation)')
```

Final loss: 0.0135 (training), 0.0104 (validation)



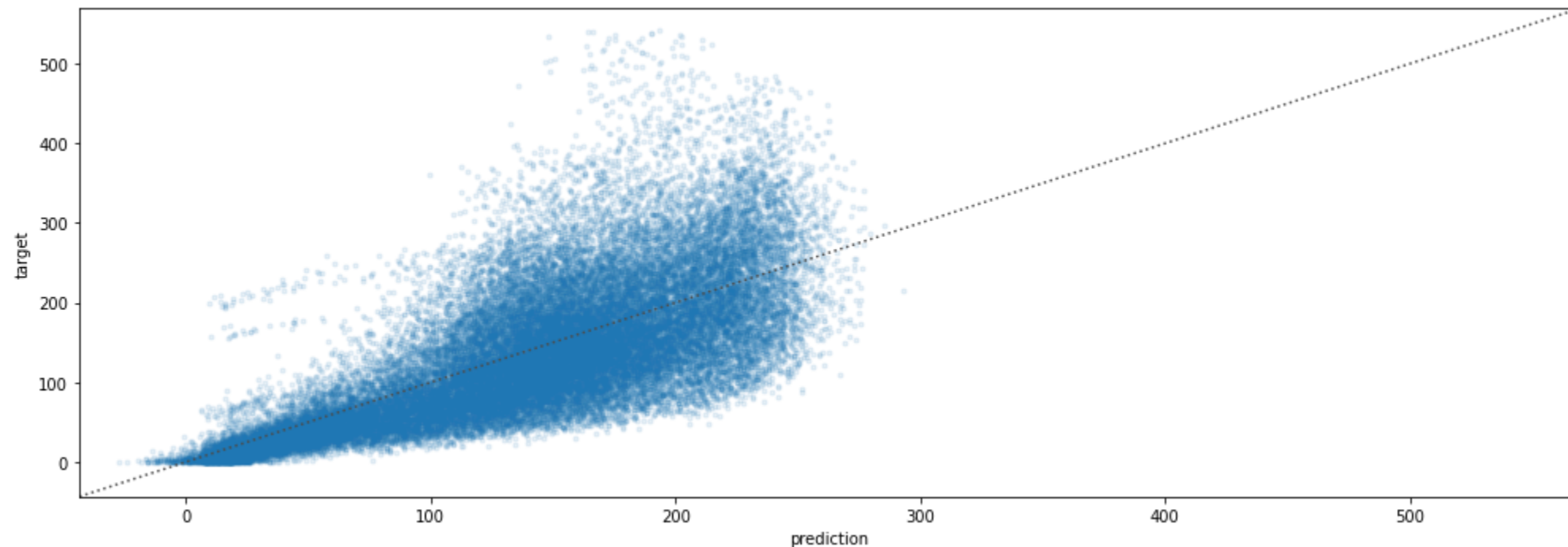
- There is some improvement w.r.t. pure Linear Regression

# Predictions

We can now obtain the predictions and evaluate their quality

```
In [19]: tr_pred = nn.predict(tr_s[dt_in]).ravel() * trmaxrul  
cmapss.plot_pred_scatter(tr_pred, tr['rul'], figsize=figsize)  
print(f'R2 score: {r2_score(tr["rul"], tr_pred)}')
```

R2 score: 0.5425721983671121

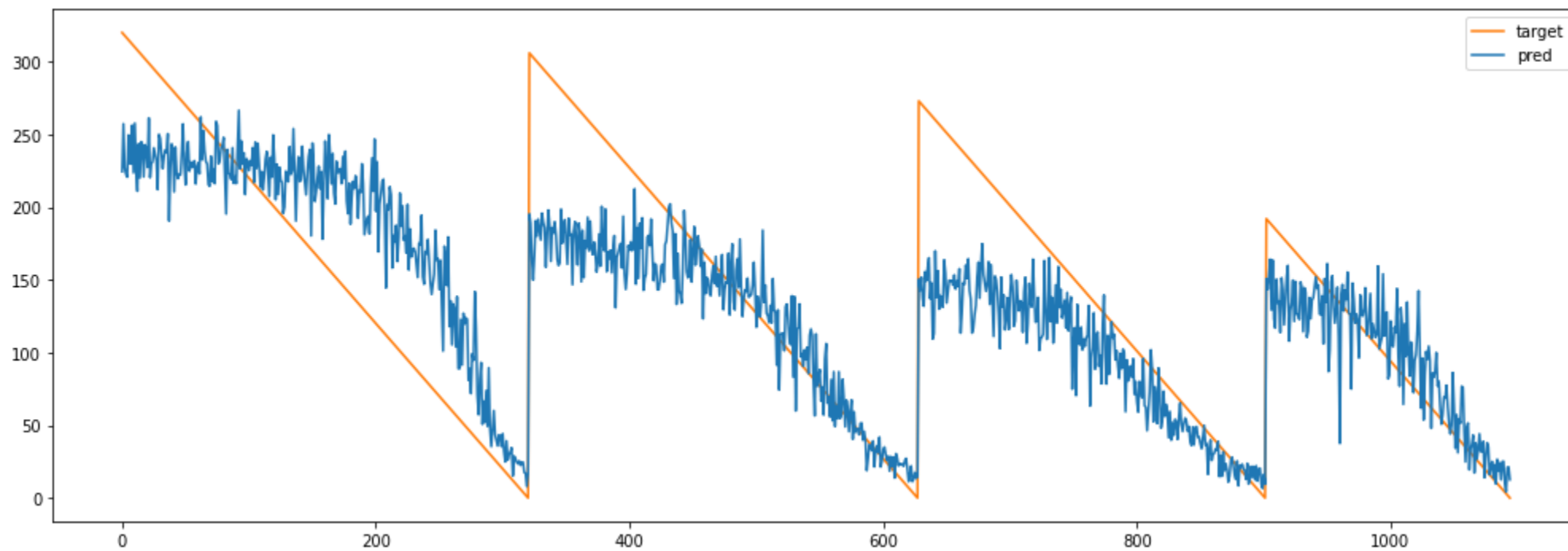


# Predictions

**The results so far are not comforting**

...But it's worth seeing what is going on over time:

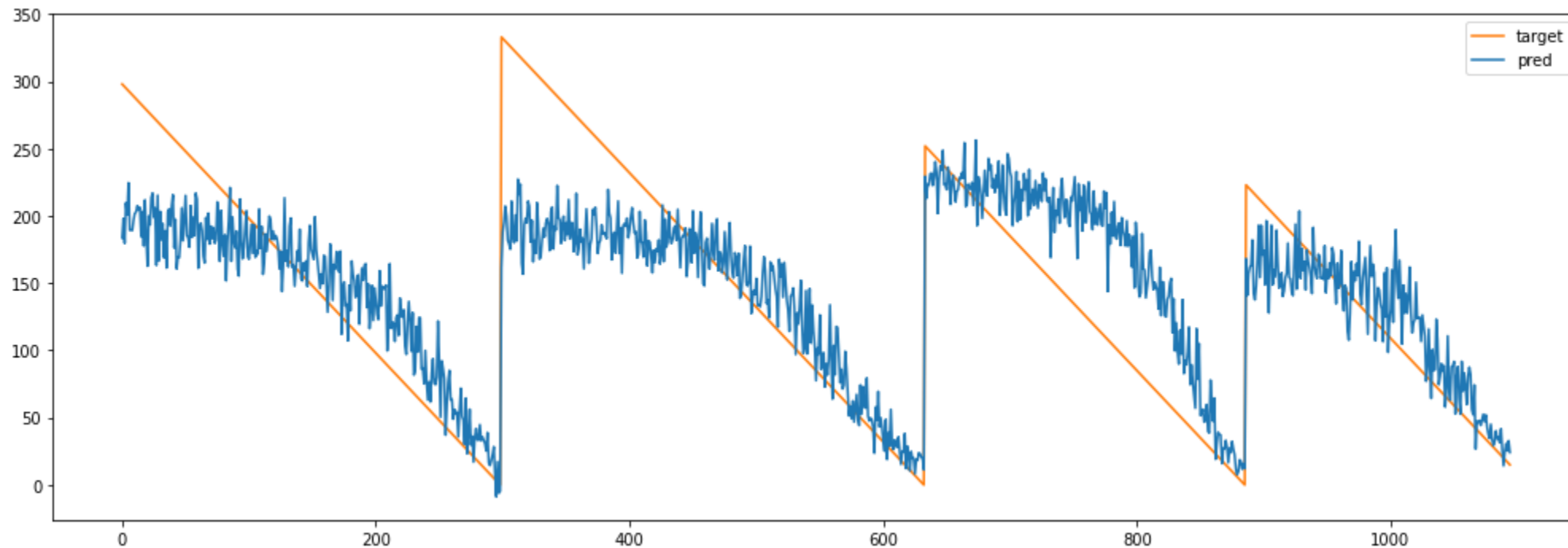
```
In [20]: stop = 1095  
cmapss.plot_rul(tr_pred[:stop], tr['rul'][:stop], figsize=figsize)
```



# Predictions

The situation is similar on the test set:

```
In [21]: ts_pred = nn.predict(ts_s[dt_in]).ravel() * tmaxrul  
cmapss.plot_rul(ts_pred[:stop], ts['rul'][:stop], figsize=figsize)
```





# Quality Evaluation

## Let's try to recap the situation

- Our accuracy is quite poor
- ...Especially for large RUL values

## Possible reasons:

- Large RUL value are somewhat scarce on the dataset
- Fault effects become noticeable only after a while

## But perhaps we don't care! Our goal is **not a high accuracy**

- We just need to **stop at the right time**
- ...And our model may still be good enough for that

**For a proper evaluation, we need a **cost model****

# Cost Model

**We will assume that:**

- Whenever a turbine operates for a time step, we gain a profit of 1 unit
- A failure costs  $C$  units (i.e. the equivalent of  $C$  operation days)
- We never trigger maintenance before  $s$  time steps

**Let  $x_k$  be the times series for machine  $k$ , and  $I_k$  its set of time steps**

With our RUL based policy:

- The time step when we trigger maintenance is given by:

$$\min\{i \in I_k \mid f(x_{ki}) < \theta\}$$

- A failure occurs if:

$$f(x_{ki}) \geq \theta \quad \forall i \in I_k$$

## Cost Model

The whole cost formula **for a single machine** will be:

$$cost(f, x_k, \theta) = op\_profit(f(x_k), \theta) + fail\_cost(f(x_k), \theta)$$

Where:

$$op\_profit(f(x_k), \theta) = -\max(0, \min\{i \in I_k \mid f(x_{ki}) < \theta\} - s)$$

$$fail\_cost(f(x_k), \theta) = \begin{cases} C & \text{if } f(x_{ki}) \geq \theta \quad \forall i \in I_k \\ 0 & \text{otherwise} \end{cases}$$

- $s$  units of machine operation are guaranteed
- ...So we gain over the default policy only if we stop after that
- Profit is modeled as a negative cost

**For the total cost, we need to sum over all machines**

# Cost Model

Normally, we would proceed as follows

- $s$  is determined by the fixed maintenance schedule
- $C$  must be determined by discussing with the customer

In our example, we will derive both from data

First, we collect all failure times

```
In [22]: failtimes = dt.groupby('machine')['cycle'].max()  
failtimes.head()
```

```
Out[22]: machine  
461      321  
462      299  
463      307  
464      274  
465      193  
Name: cycle, dtype: int64
```

# Cost Model

Then, we define  $s$  and  $C$  based on statistics

```
In [23]: print(failtimes.describe())  
safe_interval = failtimes.min()  
maintenance_cost = failtimes.max()
```

```
count    249.00000  
mean     245.97992  
std       73.11080  
min      128.00000  
25%      190.00000  
50%      234.00000  
75%      290.00000  
max      543.00000  
Name: cycle, dtype: float64
```

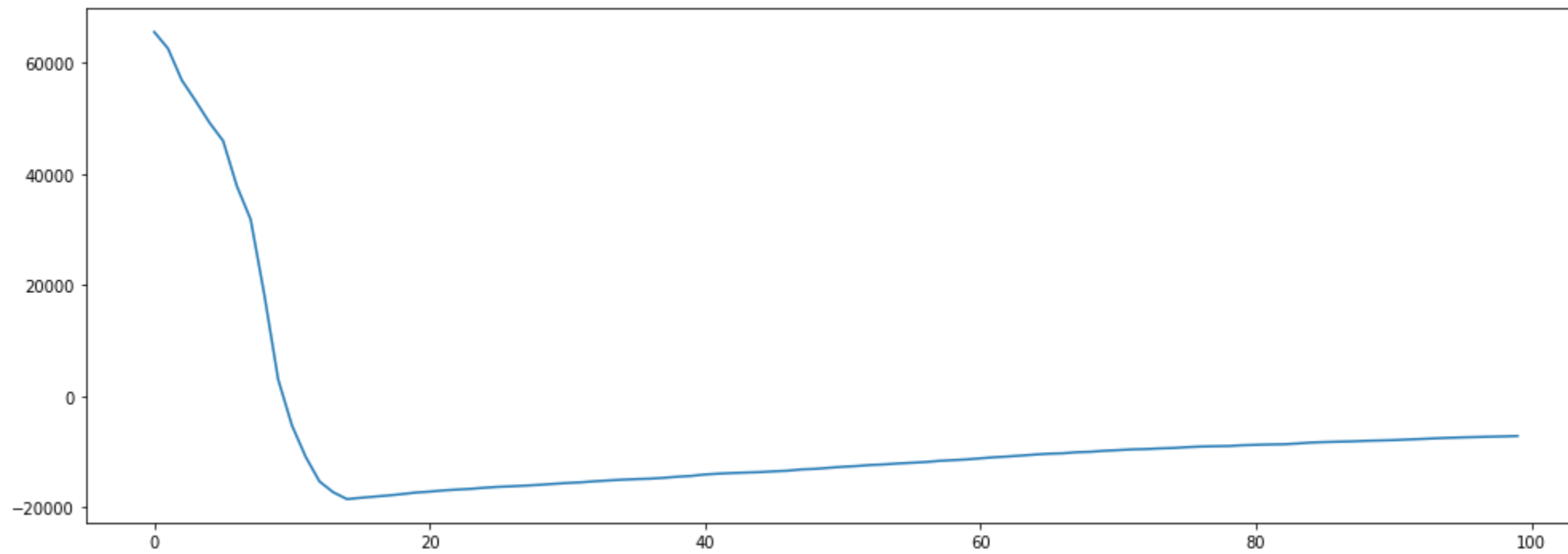
- For the safe interval  $s$ , we choose the minimum failure time
- For the maintenance cost  $C$  we choose the largest failure time

# Threshold Choice

We can then choose the threshold  $\theta$  as usual

```
In [26]: cmodel = cmapss.RULCostModel(maintenance_cost=maintenance_cost, safe_interval=safe_interval)
th_range = np.arange(0, 100)
tr_thr = cmapss.opt_threshold_and_plot(tr['machine'].values, tr_pred, th_range, cmodel, figsize=
print(f'Optimal threshold for the training set: {tr_thr}')
```

Optimal threshold for the training set: 14



# Evaluation

## Let's see how we fare in terms of cost

```
In [40]: tr_c, tr_f, tr_sl = cmodel.cost(tr['machine'].values, tr_pred, tr_thr, return_margin=True)
         ts_c, ts_f, ts_sl = cmodel.cost(ts['machine'].values, ts_pred, tr_thr, return_margin=True)
         print(f'Cost: {tr_c} (training), {ts_c} (test)')
```

```
Cost: -18474 (training), -6298 (test)
```

We can also evaluate the margin for improvement:

```
In [41]: print(f'Avg. fails: {tr_f/len(tr_mcn)} (training), {ts_f/len(ts_mcn)} (test)')
         print(f'Avg. slack: {tr_sl/len(tr_mcn):.2f} (training), {ts_sl/len(ts_mcn):.2f} (test)')
```

```
Avg. fails: 0.0 (training), 0.015873015873015872 (test)
```

```
Avg. slack: 16.87 (training), 14.03 (test)
```

- Slack = distance between when we stop and the failure
- The results are actually quite good!
- ...And we also generalize fairly well

# Sequence Input in Neural Models

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# Sequence Input in Neural Models

## Feeding more time steps to our NN may improve the results

- Intuitively, sequences provide information about the **trend**
- This may allow a better RUL estimate w.r.t. using only the current state
- E.g. we may gauge how quickly the component is deteriorating

## We will try to build a model capable to processing such input

But first, we need to apply a sliding window:

```
In [30]: wlen = 3
          tr_sw, tr_sw_m, tr_sw_r = cmapss.sliding_window_by_machine(tr_s, wlen, dt_in)
          ts_sw, ts_sw_m, ts_sw_r = cmapss.sliding_window_by_machine(ts_s, wlen, dt_in)
```

- This must be done on a **per machine** basis
- Windows **should not mix data** belonging to different machines!

# Sliding Window for Multivariate Data

The `sliding_window_by_machine` relies internally on:

```
def sliding_window_2D(data, wlen, stride=1):  
    # Get shifted_tables_  
    m = len(data)  
    lt = [data.iloc[i:m-wlen+i+1:stride, :].values for i in range(wlen)]  
    # Reshape to _add a new axis_  
    s = lt[0].shape  
    for i in range(wlen):  
        lt[i] = lt[i].reshape(s[0], 1, s[1])  
    # Concatenate  
    wdata = np.concatenate(lt, axis=1)  
    return wdata
```

- It's similar to our code for the univariate case
- The output is a tensor with shape `(n_windows, w_len, n_dims)`

# Sliding Window for Multivariate Data

Let's look in deeper detail at the returned data structures

`tr_sw` contain the actual sliding window data:

```
In [29]: tr_sw[0]
```

```
Out[29]: array([[ 1.21931469,  0.86619169,  0.41983436, -1.05564063, -0.79621447,
                 -0.70080293, -0.74549387, -1.1386061 , -1.08249848, -0.99389823,
                 -0.11421637, -0.6315044 , -0.67586863, -0.36411574, -0.98910425,
                  0.41889575,  0.08700467,  0.05991388, -0.69502688, -0.63793104,
                 -0.11268403,  0.41983436, -1.03117521, -1.03187757],
                [-0.26962527,  0.41609996,  0.41983436,  0.6926385 ,  0.71397375,
                  0.56288953,  0.29808726,  0.36365649,  0.3710279 ,  0.33249075,
                  0.65388538,  0.56210134, -0.20641916,  0.32893584,  0.33156802,
                  0.41687122, -0.24758681, -0.12925879, -0.69502688,  0.47652818,
                  0.65613725,  0.41983436,  0.35321893,  0.35869109],
                [ 1.21924025,  0.86908928,  0.41983436, -1.05564063, -0.8157647 ,
                 -0.70372248, -0.7109787 , -1.1386061 , -1.08433606, -0.98831315,
                 -0.11380415, -0.64524209, -0.67586863, -0.37335643, -0.99026013,
                  0.41874002, -0.00870947,  0.14931194, -0.69502688, -0.67388133,
                 -0.11268403,  0.41983436, -1.04527086, -1.02276728]])
```

- 3 times steps per example
- 24 dimensions per time step

# Sliding Window for Multivariate Data

Let's look in deeper detail at the returned data structures

`tr_sw_m` contains the corresponding machine values

```
In [32]: tr_sw_m
```

```
Out[32]: array([461, 461, 461, ..., 708, 708, 708], dtype=int64)
```

- The structure is a plain numpy array

`tr_sw_r` contains the RUL values

```
In [33]: tr_sw_r
```

```
Out[33]: array([0.58671587, 0.58487085, 0.58302583, ..., 0.00369004, 0.00184502,  
               0.          ])
```

- Again, the structure is a plain numpy array

# 1D Convolutions in Keras

The chosen format is ideal for **1D convolutions** in keras

We can define a simple 1D convolutional NN as follows:

```
In [34]: def build_cnn_regressor(wlen):  
    input_shape = (wlen, len(dt_in))  
    model_in = keras.Input(shape=input_shape, dtype='float32')  
    model_out = layers.Conv1D(32, kernel_size=3, activation='relu')(model_in)  
    model_out = layers.Flatten()(model_out)  
    model_out = layers.Dense(32, activation='relu')(model_out)  
    model_out = layers.Dense(1, activation='linear')(model_out)  
    model = keras.Model(model_in, model_out)  
    return model
```

- For simplicity, we consider a fixed architecture
- Each convolution in this case will consider 3 time steps
- We need to "flatten" the input before the fully connected layers
  - Flattening = getting rid of the temporal structure

# CNN Training

## Let's train our CNN

```
In [35]: nn2 = build_cnn_regressor(wlen)
nn2.compile(optimizer='Adam', loss='mse')
history2 = nn2.fit(tr_sw, tr_sw_r, validation_split=0.2,
                  callbacks=cb,
                  batch_size=32, epochs=20, verbose=1)
```

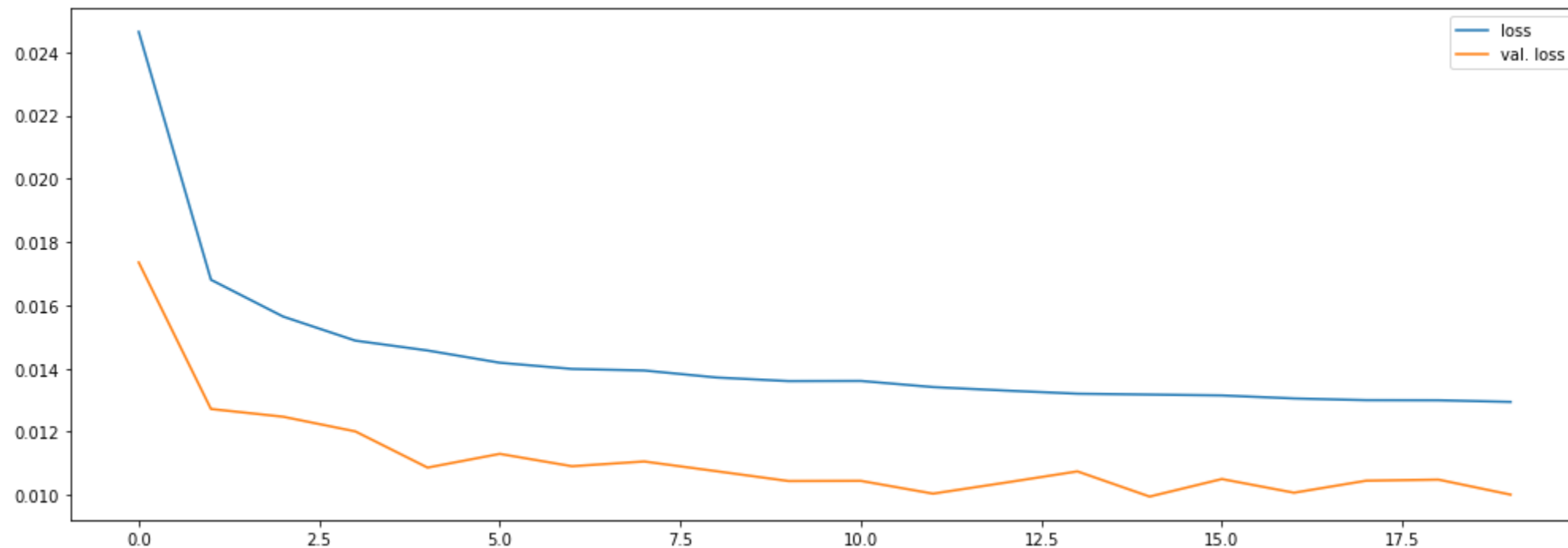
```
Epoch 1/20
1126/1126 [=====] - 1s 711us/step - loss: 0.0247 - val_loss: 0.0174
Epoch 2/20
1126/1126 [=====] - 1s 643us/step - loss: 0.0168 - val_loss: 0.0127
Epoch 3/20
1126/1126 [=====] - 1s 645us/step - loss: 0.0156 - val_loss: 0.0125
Epoch 4/20
1126/1126 [=====] - 1s 643us/step - loss: 0.0149 - val_loss: 0.0120
Epoch 5/20
1126/1126 [=====] - 1s 649us/step - loss: 0.0146 - val_loss: 0.0109
Epoch 6/20
1126/1126 [=====] - 1s 648us/step - loss: 0.0142 - val_loss: 0.0113
Epoch 7/20
1126/1126 [=====] - 1s 645us/step - loss: 0.0140 - val_loss: 0.0109
Epoch 8/20
1126/1126 [=====] - 1s 646us/step - loss: 0.0139 - val_loss: 0.0111
Epoch 9/20
1126/1126 [=====] - 1s 653us/step - loss: 0.0137 - val_loss: 0.0108
Epoch 10/20
1126/1126 [=====] - 1s 656us/step - loss: 0.0136 - val_loss: 0.0104
Epoch 11/20
```

# CNN Training

Let's check the loss behavior and compare it to the MLP model

```
In [36]: cmapss.plot_training_history(history2, figsize=figsize)
trl2, vl12 = history2.history["loss"][-1], np.min(history2.history["val_loss"])
print(f'Final loss: {trl2:.4f} (training), {vl12:.4f} (validation)')
```

Final loss: 0.0129 (training), 0.0100 (validation)



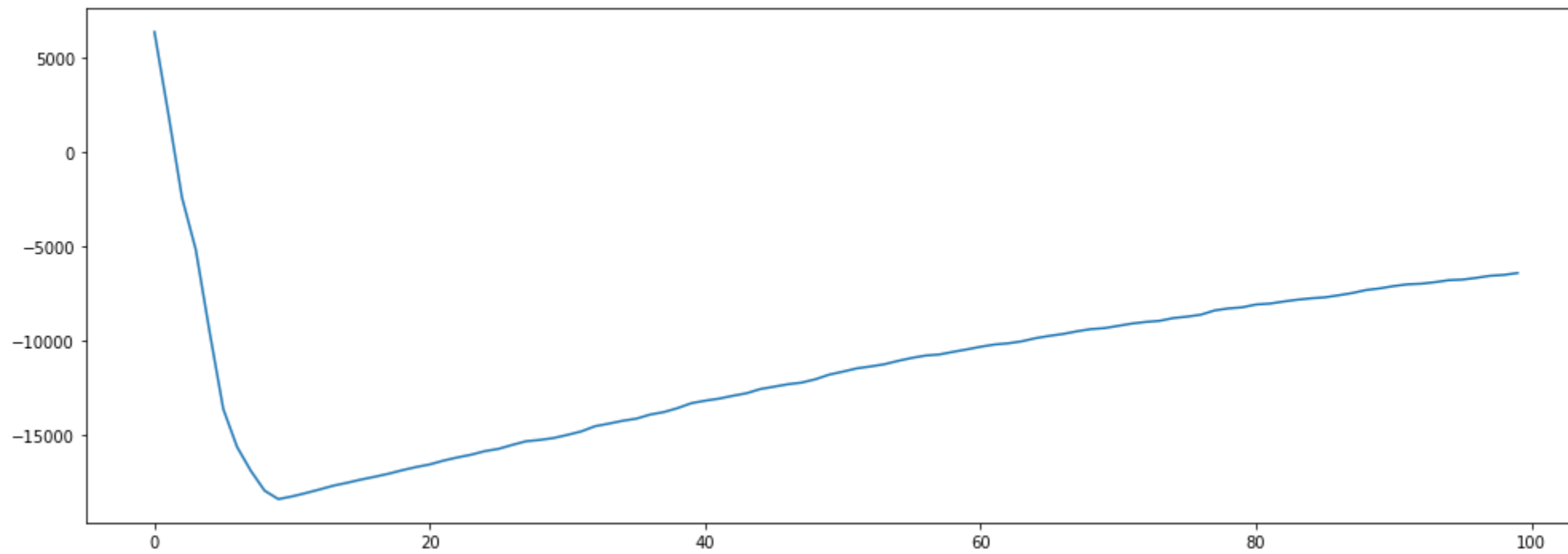
- Not a lot of improvement, apparently, but some!

# Threshold Optimization

Now we can proceed by choosing a threshold

```
In [37]: tr_pred2 = nn2.predict(tr_sw).ravel() * trmaxrul  
ts_pred2 = nn2.predict(ts_sw).ravel() * trmaxrul  
tr_thr2 = cmapss.opt_threshold_and_plot(tr_sw_m, tr_pred2, th_range, cmodel, figsize=figsize)  
print(f'Optimal threshold for the training set: {tr_thr2}')
```

Optimal threshold for the training set: 9





# Evaluation

## Let's see how the CNN fares in terms of cost

```
In [38]: tr_c2, tr_f2, tr_sl2 = cmodel.cost(tr_sw_m, tr_pred2, tr_thr2, return_margin=True)
ts_c2, ts_f2, ts_sl2 = cmodel.cost(ts_sw_m, ts_pred2, tr_thr2, return_margin=True)
print(f'Cost: {tr_c2} (training), {ts_c2} (test)')
print(f'Avg. fails: {tr_f2/len(tr_mcn)} (training), {ts_f2/len(ts_mcn)} (test)')
print(f'Avg. slack: {tr_sl2/len(tr_mcn):.2f} (training), {ts_sl2/len(ts_mcn):.2f} (test)')
```

Cost: -18439 (training), -6866 (test)  
Avg. fails: 0.0 (training), 0.0 (test)  
Avg. slack: 15.09 (training), 12.83 (test)

Which is (more or less) on par with our MLP approach, for which we had:

```
In [39]: print(f'Cost: {tr_c} (training), {ts_c} (test)')
print(f'Avg. fails: {tr_f/len(tr_mcn)} (training), {ts_f/len(ts_mcn)} (test)')
print(f'Avg. slack: {tr_sl/len(tr_mcn):.2f} (training), {ts_sl/len(ts_mcn):.2f} (test)')
```

Cost: -18474 (training), -6298 (test)  
Avg. fails: 0.0 (training), 0.015873015873015872 (test)  
Avg. slack: 16.87 (training), 14.03 (test)

# Considerations and Take-Home Messages

## A first approach for RUL-based maintenance policies

- Build a RUL predictor
- Trigger maintenance when the prediction is below a threshold
- In practice: a simple two-stage sequential optimization process

## Don't evaluate the performance in term of accuracy: it's a trap!

- We don't care about accuracy when the RUL is high
- We only care about when we stop
- We need a **cost model**

## Sequence input may help

- But don't automatically jump for that!
- First **think**: would considering multiple step provide more useful information?