Arrival Prediction

Arrival Prediction

We can now frame our arrival prediction problem

We want to predict the number of arrivals in the next interval

- We will focus on predicting the total number of arrivals
- The same models can be applied to any of the individual counts

Technically, this is a regression problem

- ...Which does not mean that an MSE is the best choice
- It makes more sense to check the target distribution first

Main issue: our regressor will learn a conditional probability distribution

- ...So, that's the kind of distribution that we should check, in principle
- It is a difficult task, since we do not know yet which input we are goig to use

However, we already know that some features are likely to be useful

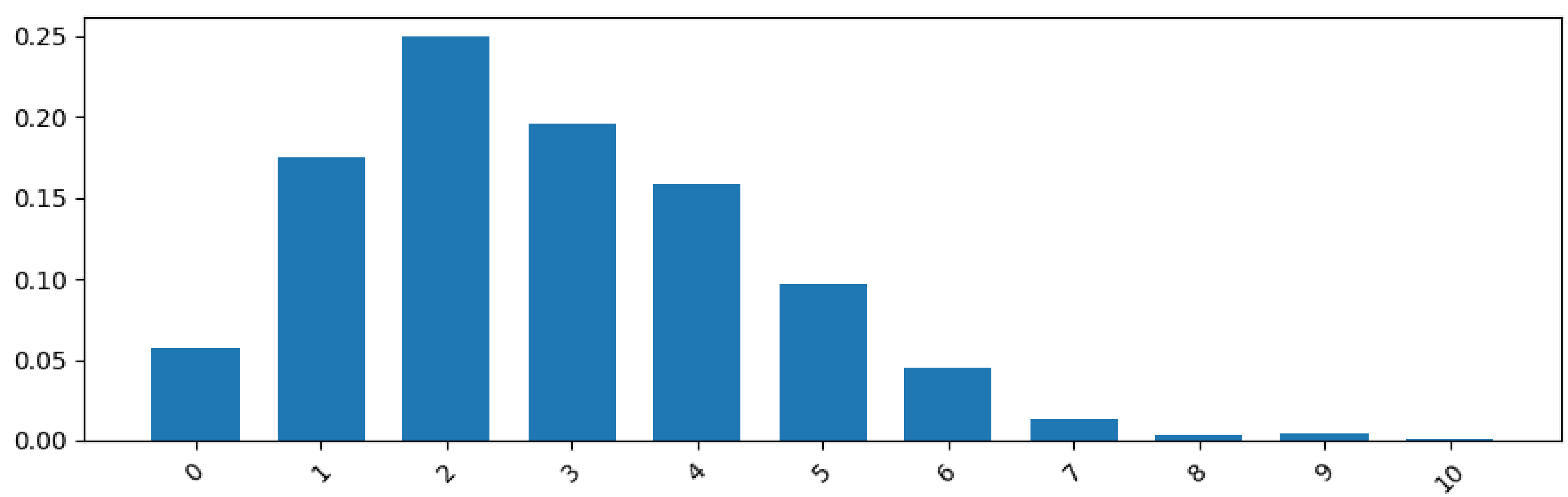
Analyzing the Conditional Arrival Distribution

...In particuar, we know that the hour of the day is a good predictor

Let's check the (conditional) distribution for 6am:

In [37]:
```
tmp = codes_b[codes_b.index.hour == 6]['total']
tmpv = tmp.value_counts(sort=False, normalize=True).sort_index()
er.plot_bars(tmpv, figsize=figsize)
```

Figure



- This is not a normal distribution

Poisson Distribution

When we need to count occurrencies over time...

It's almost always worth checking the Poisson distribution, which models:

- The number of occurrences of a certain event in a given interval
- ...Assuming that these events are independent
- ...And they occur at a constant rate

In our case:

- The independence assumption is reasonable (arrivals do not affect each other)
- The constant rate is true for the conditional probability
- ...Assuming that our predictor is good enough

Intuitively: the predictor will estimate the constant rate

Poisson Distribution

The Poisson distribution is defined by a single parameter λ

λ is the rate of occurrence of the events

- The distribution has a discrete support
- The Probability Mass Function is:

$$p(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

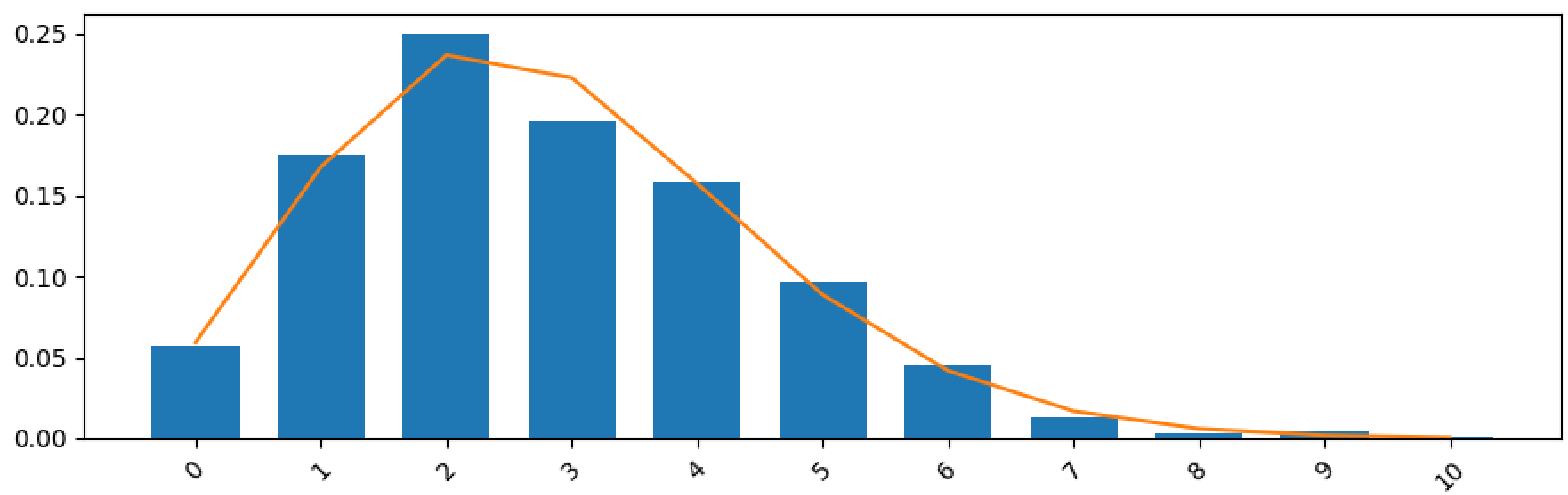
- Both the mean and the standard deviation have the same value (i.e. λ)
- The distribution skewness is $\lambda^{-\frac{1}{2}}$
  - For low λ values, there is a significant positive skew (to the left)
  - The distribution becomes less skewed for large λ

Fitted Poisson Distribution

Let's try to fit a Poisson distribution over our target

In [39]:
```
mu = tmp.mean()
dist = stats.poisson(mu)
x = np.arange(tmp.min(), tmp.max()+1)
er.plot_bars(tmpv, figsize=figsize, series=pd.Series(index=x, data=dist.pmf(x)))
```

Figure



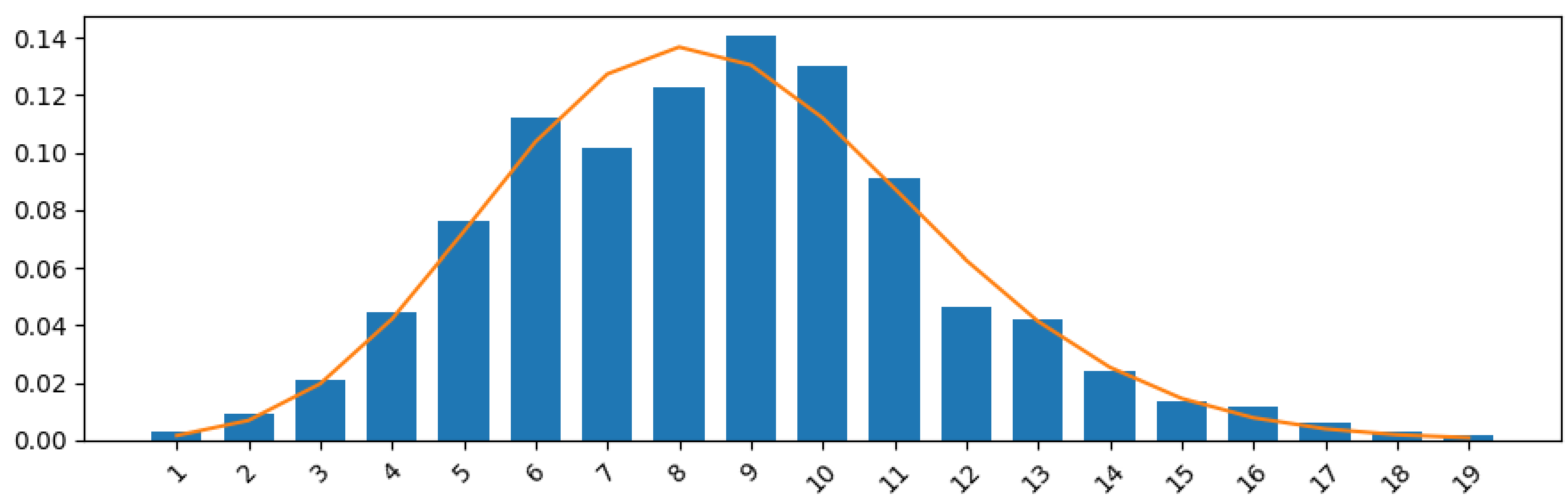
It's a very good match!

Fitted Poisson Distribution

Let's try for 8AM (closer to the peak)

In [41]:
```
tmp = codes_b[codes_b.index.hour == 8]['total']
tmpv = tmp.value_counts(sort=False, normalize=True).sort_index()
mu = tmp.mean()
dist = stats.poisson(mu)
x = np.arange(tmp.min(), tmp.max()+1)
er.plot_bars(tmpv, figsize=figsize, series=pd.Series(index=x, data=dist.pmf(x)))
```

Figure



Fitted Poisson Distribution

...And finally for the peak itself

In [47]:
```
tmp = codes_b[codes_b.index.hour == 11]['total']
tmpv = tmp.value_counts(sort=False, normalize=True).sort_index()
mu = tmp.mean()
dist = stats.poisson(mu)
x = np.arange(tmp.min(), tmp.max()+1)
er.plot_bars(tmpv, figsize=figsize, series=pd.Series(index=x, data=dist.pmf(x)))
```

Figure

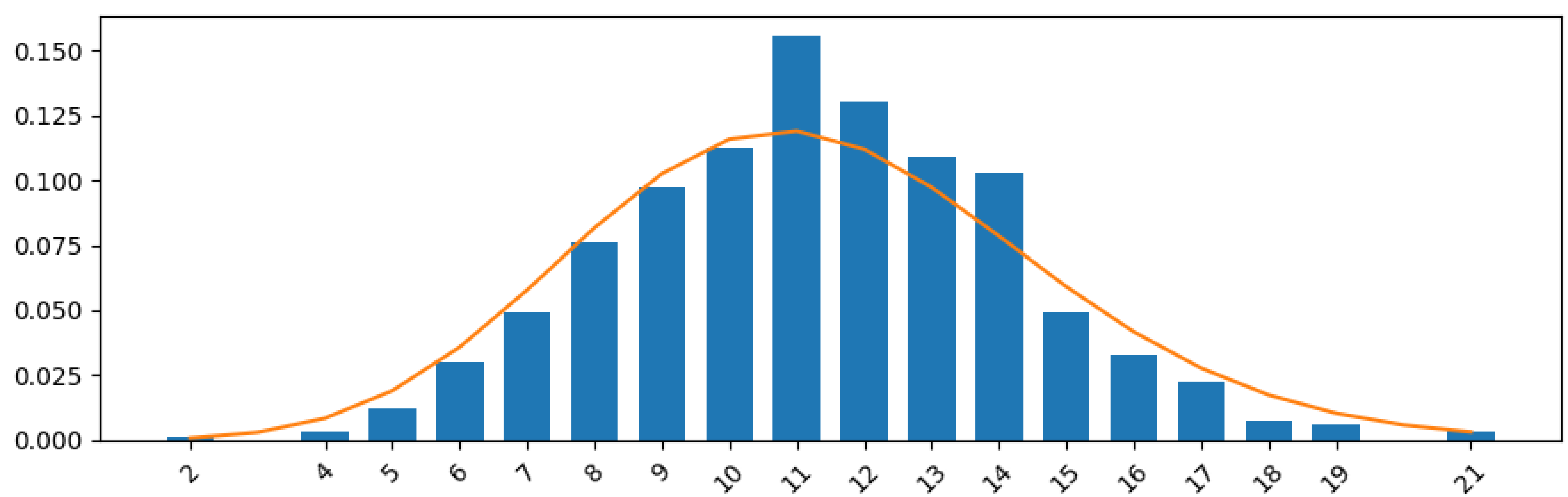


Table Model

Rate Table

We know that:

- A Poisson distribution is a good fit for our target...
- ...But the rate depends (at least) on the hour of the day

The simplest way to take this into account is using a lookup table

- The table will contain average arrival values for each hour of the day
- ...But first, we need to separate the training and test data

In [49]:
```
sep = '2019-01-01'
tr_data = codes_b[codes_b.index < sep]
ts_data = codes_b[codes_b.index >= sep]
```

Rate Table

We can then build our "rate table"

In [51]:
```
rate_table = tr_data.groupby(tr_data.index.hour).mean()
rate_table.head()
```

Out[51]:

	green	red	white	yellow	total
Triage					
0	2.265753	0.219178	0.367123	1.054795	3.906849
1	1.915068	0.213699	0.331507	0.860274	3.320548
2	1.734247	0.202740	0.287671	0.780822	3.005479
3	1.515068	0.153425	0.232877	0.687671	2.589041
4	1.334247	0.134247	0.197260	0.717808	2.383562

- We are computing (average) rates for all the codes
- This will enable changing focus to a different target, if we so wish

Predictions

We can not obtain the predictions

- We need to associate each example withe the correct rates
- The rates themselves are then the predictions

In [52]:
```
def preds_from_rate_table(data, rate_table):
    tmp = data.copy()
    tmp['hour'] = data.index.hour
    tmp = tmp.join(rate_table, on='hour', lsuffix='_orig')
    return tmp[rate_table.columns]

tr_preds = preds_from_rate_table(tr_data, rate_table)
ts_preds = preds_from_rate_table(ts_data, rate_table)
```

- We use a `join` operation to associate examples and rates

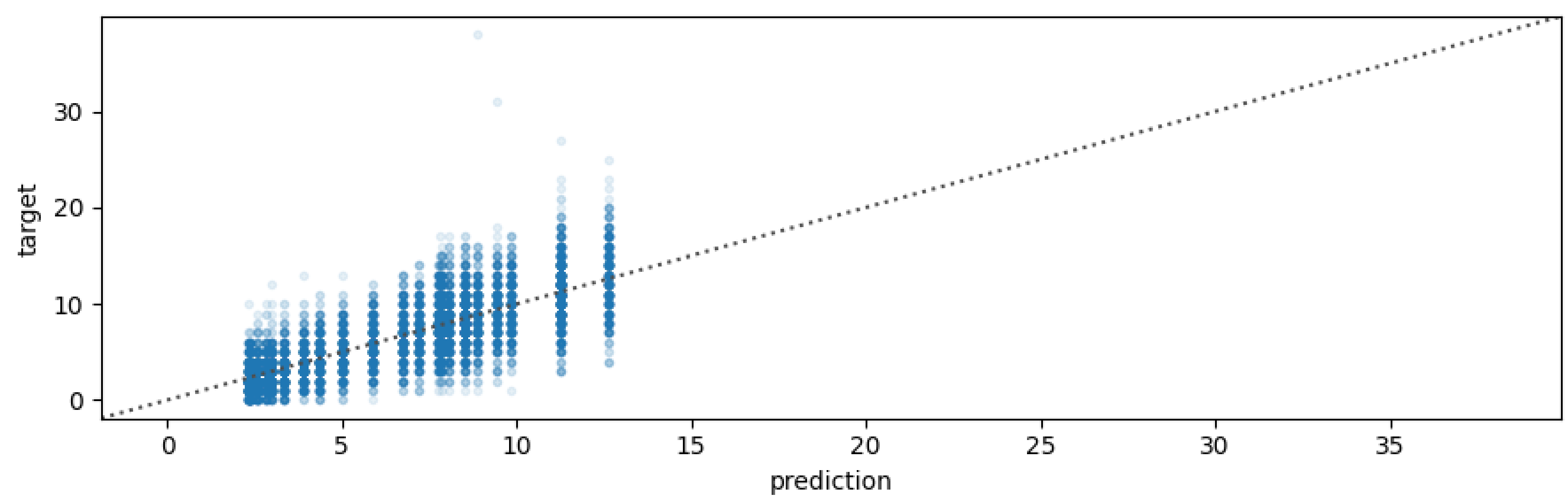
Evaluation

We can now evaluate the predictions. First for the training set:

```
In [53]: slabel = 'total'
         er.plot_pred_scatter(tr_preds[slabel], tr_data[slabel], figsize=figsize)
         print(f'R2: {metrics.r2_score(tr_data[slabel], tr_preds[slabel])}')
         print(f'MAE: {metrics.mean_absolute_error(tr_data[slabel], tr_preds[slabel])}')
```

```
R2: 0.5940116186135868
MAE: 1.945866016138112
```

Figure



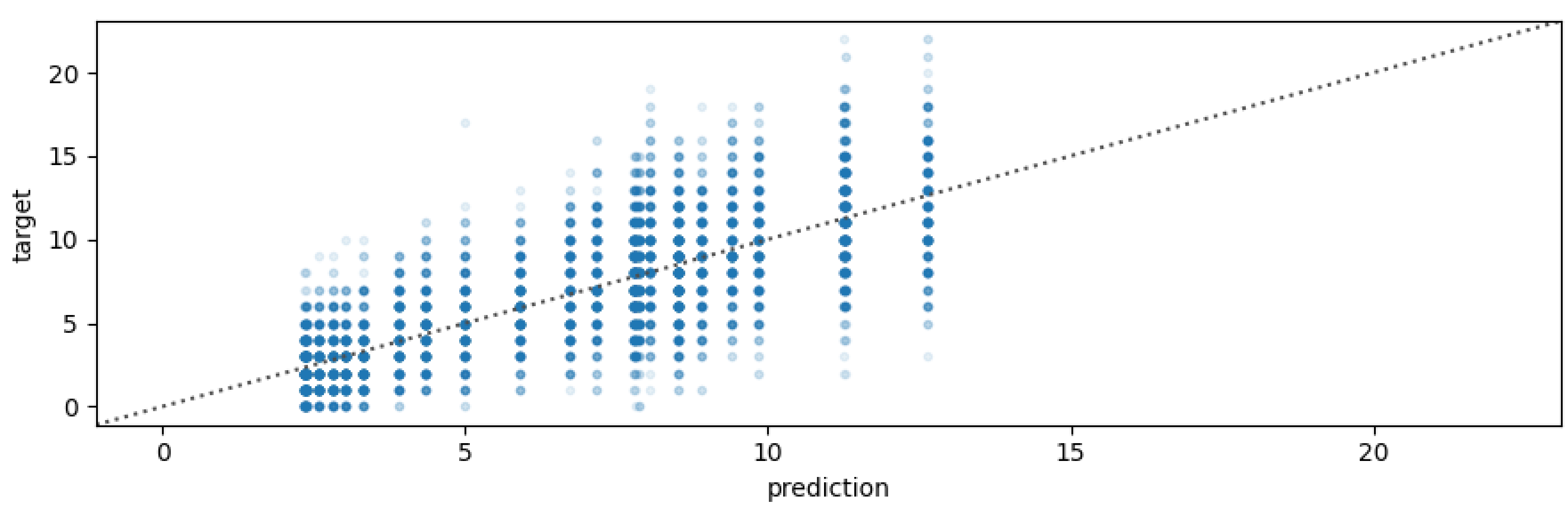
Evaluation

...And then for the test set:

In [54]:
```
er.plot_pred_scatter(ts_preds['total'], ts_data['total'], figsize=figsize)
print(f'R2: {metrics.r2_score(ts_data[slabel], ts_preds[slabel])}')
print(f'MAE: {metrics.mean_absolute_error(ts_data[slabel], ts_preds[slabel])}')
```

```
R2: 0.5878063024665628
MAE: 1.9621410117760154
```

Figure



These results will be our baseline

Probabilistic Neural Model

Probabilistic Neural Model

We will now try to learn a hybrid probabilistic-neural model

Using a neural network makes adding inputs easier

- We know that there is a slight decreasing trend along week days
- ...So let's try adding the week day as a numerical input

```
In [65]: tr_data_in = keras.utils.to_categorical(tr_data.index.hour)
         tr_data_in = np.hstack((tr_data_in, (tr_data.index.weekday/7).values.reshape(-1,1) ))

         ts_data_in = keras.utils.to_categorical(ts_data.index.hour.values)
         ts_data_in = np.hstack((ts_data_in, (ts_data.index.weekday/7).values.reshape(-1,1)))
```

- We use a categorical encoding for the day hour, due to its non-linear effect
- ...But it is (mostly) fine to use the week day as number, due to its linear effect

The Architecture

First we define our architecture

```
In [66]: def build_probabilistic_regressor(input_shape, hidden):
             model_in = keras.Input(shape=input_shape, dtype='float32')
             x = model_in
             for h in hidden:
                 x = layers.Dense(h, activation='relu')(x)
             log_rate = layers.Dense(1, activation='linear')(x)
             lf = lambda t: tfp.distributions.Poisson(log_rate=t, force_probs_to_zero_outside_support=Fal
             model_out = tfp.layers.DistributionLambda(lf)(log_rate)
             model = keras.Model(model_in, model_out)
             return model

         nn2 = build_probabilistic_regressor(tr_data_in.shape[1], hidden=[32])
```

- We use an MLP, plus a `DistributionLambda` layer for the Poisson distribution
- We will try with a shallow network

The Architecture

We can also plot the architecture

In [67]:
```
nn2.summary()
```

```
Model: "model_3"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_4 (InputLayer)         [(None, 25)]              0
_________________________________________________________________
dense_6 (Dense)              (None, 32)                832
_________________________________________________________________
dense_7 (Dense)              (None, 1)                 33
_________________________________________________________________
distribution_lambda_3 (Distr multiple                  0
=================================================================
Total params: 865
Trainable params: 865
Non-trainable params: 0
_________________________________________________________________
```

- More parameters than the table (and initially it will work worse)

Training

We train the model for maximum likelihood, as usual

In [68]:

```python
negloglikelihood = lambda y_true, dist: -dist.log_prob(y_true)
nn2.compile(optimizer='Adam', loss=negloglikelihood)
cb = [callbacks.EarlyStopping(patience=10, restore_best_weights=True)]
history2 = nn2.fit(tr_data_in, tr_data['total'].values.astype(np.float32),
                   validation_split=0.2, callbacks=cb, batch_size=32, epochs=50, verbose=1)
```

```
Epoch 1/50
219/219 [==============================] - 1s 1ms/step - loss: 4.4935 - val_loss: 2.4800
Epoch 2/50
219/219 [==============================] - 0s 710us/step - loss: 2.3570 - val_loss: 2.2709
Epoch 3/50
219/219 [==============================] - 0s 720us/step - loss: 2.2711 - val_loss: 2.2778
Epoch 4/50
219/219 [==============================] - 0s 733us/step - loss: 2.2643 - val_loss: 2.2699
Epoch 5/50
219/219 [==============================] - 0s 719us/step - loss: 2.2639 - val_loss: 2.2661
Epoch 6/50
219/219 [==============================] - 0s 726us/step - loss: 2.2618 - val_loss: 2.2677
Epoch 7/50
219/219 [==============================] - 0s 710us/step - loss: 2.2620 - val_loss: 2.2683
Epoch 8/50
219/219 [==============================] - 0s 718us/step - loss: 2.2610 - val_loss: 2.2710
Epoch 9/50
219/219 [==============================] - 0s 721us/step - loss: 2.2594 - val_loss: 2.2669
Epoch 10/50
219/219 [==============================] - 0s 718us/step - loss: 2.2597 - val_loss: 2.2688
Epoch 11/50
```

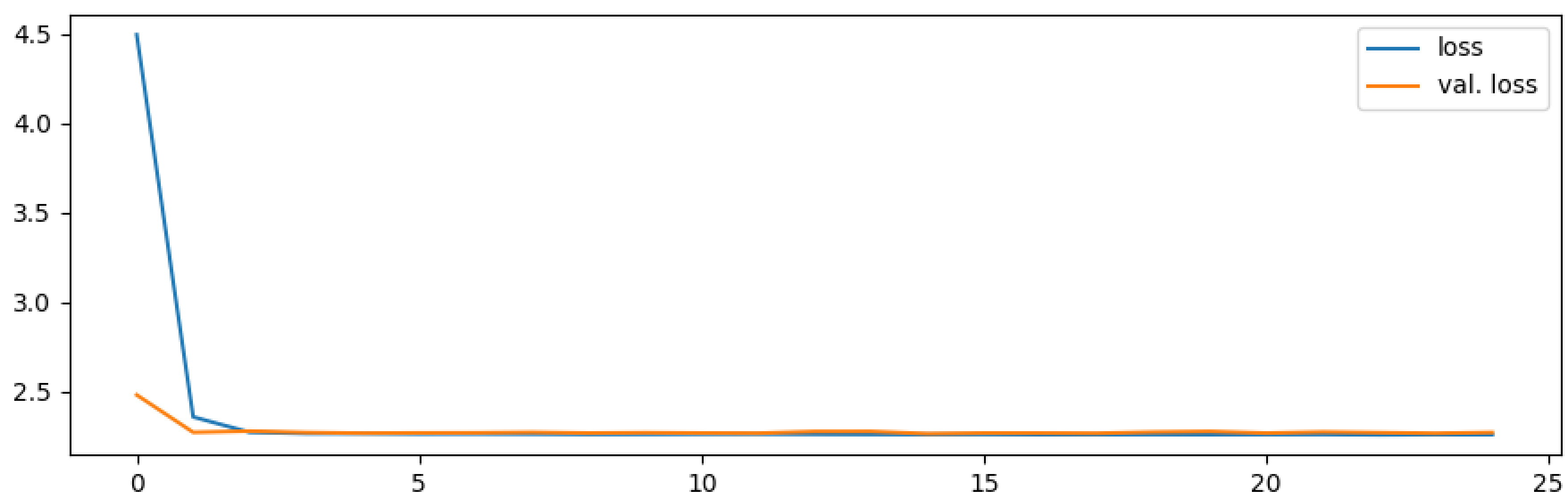
Training

We check the loss behavior over time

In [69]:
```
er.plot_training_history(history2, figsize=figsize)
tr2, vl2 = history2.history["loss"][-1], np.min(history2.history["val_loss"])
print(f'Loss: {tr2:.4f} (training, final), {vl2:.4f} (validation, best)')
```

```
Loss: 2.2576 (training, final), 2.2632 (validation, best)
```

Figure



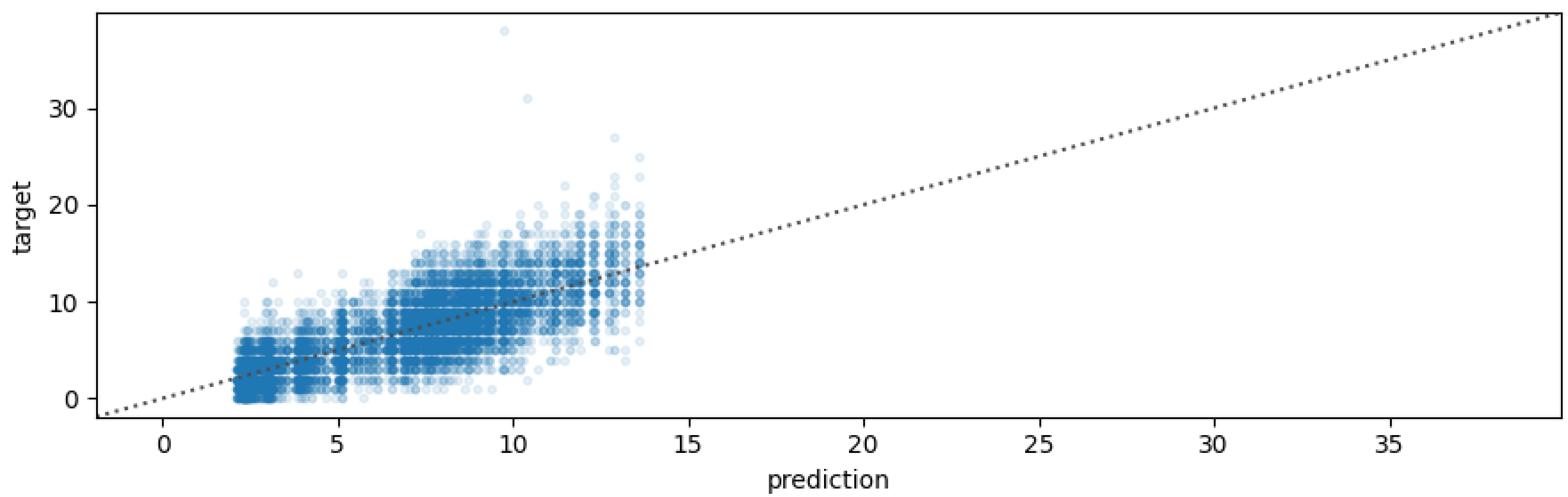
Evaluation

We can now obtain and evaluate the predictions. On the training set:

```
In [70]: tr_preds2 = nn2(tr_data_in).mean()
         slabel = 'total'
         er.plot_pred_scatter(tr_preds2, tr_data[slabel], figsize=figsize)
         print(f'R2: {metrics.r2_score(tr_data[slabel], tr_preds2)}')
         print(f'MAE: {metrics.mean_absolute_error(tr_data[slabel], tr_preds2)}')
```

```
R2: 0.6059787926961983
MAE: 1.9191851117839551
```

Figure



Evaluation

And we get similar results for the test set

In [71]:
```
ts_preds2 = nn2(ts_data_in).mean()
er.plot_pred_scatter(ts_preds2, ts_data['total'], figsize=figsize)
print(f'R2: {metrics.r2_score(ts_data[slabel], ts_preds2)}')
print(f'MAE: {metrics.mean_absolute_error(ts_data[slabel], ts_preds2)}')
```

```
R2: 0.6006004538247001
MAE: 1.9296061691681021
```

Figure 1