

# DATA MINING AND ANALYSIS

## Fundamental Concepts and Algorithms

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## Preface

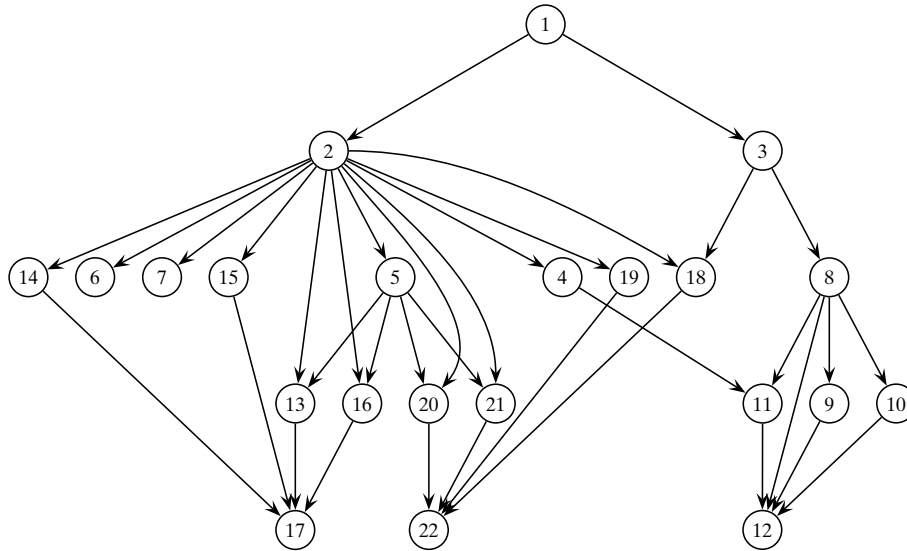
This book is an outgrowth of data mining courses at Rensselaer Polytechnic Institute (RPI) and Universidade Federal de Minas Gerais (UFMG); the RPI course has been offered every Fall since 1998, whereas the UFMG course has been offered since 2002. Although there are several good books on data mining and related topics, we felt that many of them are either too high-level or too advanced. Our goal was to write an introductory text that focuses on the fundamental algorithms in data mining and analysis. It lays the mathematical foundations for the core data mining methods, with key concepts explained when first encountered; the book also tries to build the intuition behind the formulas to aid understanding.

The main parts of the book include exploratory data analysis, frequent pattern mining, clustering, and classification. The book lays the basic foundations of these tasks, and it also covers cutting-edge topics such as kernel methods, high-dimensional data analysis, and complex graphs and networks. It integrates concepts from related disciplines such as machine learning and statistics and is also ideal for a course on data analysis. Most of the prerequisite material is covered in the text, especially on linear algebra, and probability and statistics.

The book includes many examples to illustrate the main technical concepts. It also has end-of-chapter exercises, which have been used in class. All of the algorithms in the book have been implemented by the authors. We suggest that readers use their favorite data analysis and mining software to work through our examples and to implement the algorithms we describe in text; we recommend the R software or the Python language with its NumPy package. The datasets used and other supplementary material such as project ideas and slides are available online at the book's companion site and its mirrors at RPI and UFMG:

- <http://dataminingbook.info>
- <http://www.cs.rpi.edu/~zaki/dataminingbook>
- <http://www.dcc.ufmg.br/dataminingbook>

Having understood the basic principles and algorithms in data mining and data analysis, readers will be well equipped to develop their own methods or use more advanced techniques.



**Figure 0.1.** Chapter dependencies

### Suggested Roadmaps

The chapter dependency graph is shown in Figure 0.1. We suggest some typical roadmaps for courses and readings based on this book. For an undergraduate-level course, we suggest the following chapters: 1–3, 8, 10, 12–15, 17–19, and 21–22. For an undergraduate course without exploratory data analysis, we recommend Chapters 1, 8–15, 17–19, and 21–22. For a graduate course, one possibility is to quickly go over the material in Part I or to assume it as background reading and to directly cover Chapters 9–22; the other parts of the book, namely frequent pattern mining (Part II), clustering (Part III), and classification (Part IV), can be covered in any order. For a course on data analysis the chapters covered must include 1–7, 13–14, 15 (Section 2), and 20. Finally, for a course with an emphasis on graphs and kernels we suggest Chapters 4, 5, 7 (Sections 1–3), 11–12, 13 (Sections 1–2), 16–17, and 20–22.

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