Python code for Artificial Intelligence: Foundations of Computational Agents

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http://aipython.org http://artint.info

Contents

Chapter 1

Python for Artificial Intelligence

1.1 Why Python?

We use Python because Python programs can be close to pseudo-code. It is designed for humans to read.

Python is reasonably efficient. Efficiency is usually not a problem for small examples. If your Python code is not efficient enough, a general procedure to improve it is to find out what is taking most the time, and implement just that part more efficiently in some lower-level language. Most of these lower-level languages interoperate with Python nicely. This will result in much less programming and more efficient code (because you will have more time to optimize) than writing everything in a low-level language. You will not have to do that for the code here if you are using it for course projects.

1.2 Getting Python

You need Python 3 (`http://python.org/`) and matplotlib (`http://matplotlib.org/`) that runs with Python 3. This code is *not* compatible with Python 2 (e.g., with Python 2.7).

Download and istall the latest Python 3 release from `http://python.org/`. This should also install *pip*3. You can install matplotlib using

```
pip3 install matplotlib
```

in a terminal shell (not in Python). That should "just work". If not, try using `pip` instead of `pip3`.

The command `python` or `python3` should then start the interactive python shell. You can quit Python with a control-D or with `quit()`.

To upgrade matplotlib to the latest version (which you should do if you install a new version of Python) do:

```
pip3 install --upgrade matplotlib
```

We recommend using the enhanced interactive python **ipython** (`http://ipython.org/`). To install ipython after you have installed python do:

```
pip3 install ipython
```

1.3 Running Python

We assume that everything is done with an interactive Python shell. You can either do this with an IDE, such as IDLE that comes with standard Python distributions, or just running ipython3 (or perhaps just ipython) from a shell.

Here we describe the most simple version that uses no IDE. If you download the zip file, and cd to the "`aipython`" folder where the .py files are, you should be able to do the following, with user input following `:` . The first ipython3 command is in the operating system shell (note that the `-i` is important to enter interactive mode), with user input in bold:

```
ipython -i searchGeneric.py
Python 3.6.5 (v3.6.5:f59c0932b4, Mar 28 2018, 05:52:31)
Type 'copyright', 'credits' or 'license' for more information
IPython 6.2.1 -- An enhanced Interactive Python. Type '?' for help.
Testing problem 1:
7 paths have been expanded and 4 paths remain in the frontier
Path found: a --> b --> c --> d --> g
Passed unit test

In [1]: searcher2 = AStarSearcher(searchProblem.acyclic_delivery_problem) #A*

In [2]: searcher2.search()  # find first path
16 paths have been expanded and 5 paths remain in the frontier
Out[2]: o103 --> o109 --> o119 --> o123 --> r123

In [3]: searcher2.search()  # find next path
21 paths have been expanded and 6 paths remain in the frontier
Out[3]: o103 --> b3 --> b4 --> o109 --> o119 --> o123 --> r123

In [4]: searcher2.search()  # find next path
28 paths have been expanded and 5 paths remain in the frontier
Out[4]: o103 --> b3 --> b1 --> b2 --> b4 --> o109 --> o119 --> o123 --> r123

In [5]: searcher2.search()  # find next path
No (more) solutions. Total of 33 paths expanded.
```

```
In [6]:
```

You can then interact at the last prompt.

There are many textbooks for Python. The best source of information about python is `https://www.python.org/`. We will be using Python 3; please download the latest release. The documentation is at `https://docs.python.org/3/`.

The rest of this chapter is about what is special about the code for AI tools. We will only use the Standard Python Library and matplotlib. All of the exercises can be done (and should be done) without using other libraries; the aim is for you to spend your time thinking about how to solve the problem rather than searching for pre-existing solutions.

1.4 Pitfalls

It is important to know when side effects occur. Often AI programs consider what would happen or what may have happened. In many such cases, we don't want side effects. When an agent acts in the world, side effects are appropriate.

In Python, you need to be careful to understand side effects. For example, the inexpensive function to add an element to a list, namely *append*, changes the list. In a functional language like Haskell or Lisp, adding a new element to a list, without changing the original list, is a cheap operation. For example if x is a list containing n elements, adding an extra element to the list in Python (using *append*) is fast, but it has the side effect of changing the list x. To construct a new list that contains the elements of x plus a new element, without changing the value of x, entails copying the list, or using a different representation for lists. In the searching code, we will use a different representation for lists for this reason.

1.5 Features of Python

1.5.1 Lists, Tuples, Sets, Dictionaries and Comprehensions

We make extensive uses of lists, tuples, sets and dictionaries (dicts). See `https://docs.python.org/3/library/stdtypes.html`

One of the nice features of Python is the use of list comprehensions (and also tuple, set and dictionary comprehensions).

(*fe* for *e* in *iter* if *cond*)

enumerates the values *fe* for each *e* in *iter* for which *cond* is true. The "if cond" part is optional, but the "for" and "in" are not optional. Here *e* has to be a variable, *iter* is an iterator, which can generate a stream of data, such as a list, a set, a range object (to enumerate integers between ranges) or a file. *cond*

is an expression that evaluates to either True or False for each *e*, and *fe* is an expression that will be evaluated for each value of *e* for which *cond* returns *True*.

The result can go in a list or used in another iteration, or can be called directly using *next*. The procedure *next* takes an iterator returns the next element (advancing the iterator) and raises a StopIteration exception if there is no next element. The following shows a simple example, where user input is prepended with >>>

```
>>> [e*e for e in range(20) if e%2==0]
[0, 4, 16, 36, 64, 100, 144, 196, 256, 324]
>>> a = (e*e for e in range(20) if e%2==0)
>>> next(a)
0
>>> next(a)
4
>>> next(a)
16
>>> list(a)
[36, 64, 100, 144, 196, 256, 324]
>>> next(a)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
StopIteration
```

Notice how *list*(*a*) continued on the enumeration, and got to the end of it.

Comprehensions can also be used for dictionaries. The following code creates an index for list *a*:

```
>>> a = ["a","f","bar","b","a","aaaaa"]
>>> ind = {a[i]:i for i in range(len(a))}
>>> ind
{'a': 4, 'f': 1, 'bar': 2, 'b': 3, 'aaaaa': 5}
>>> ind['b']
3
```

which means that `'b'` is the 3rd element of the list.

The assignment of *ind* could have also be written as:

```
>>> ind = {val:i for (i,val) in enumerate(a)}
```

where *enumerate* returns an iterator of $(index, value)$ pairs.

1.5.2 Functions as first-class objects

Python can create lists and other data structures that contain functions. There is an issue that tricks many newcomers to Python. For a local variable in a function, the function uses the last value of the variable when the function is

called, not the value of the variable when the function was defined (this is called "late binding"). This means if you want to use the value a variable has when the function is created, you need to save the current value of that variable. Whereas Python uses "late binding" by default, the alternative that newcomers often expect is "early binding", where a function uses the value a variable had when the function was defined, can be easily implemented.

Consider the following programs designed to create a list of 5 functions, where the *i*th function in the list is meant to add *i* to its argument:[1]

pythonDemo.py — Some tricky examples

```
11 fun_list1 = []
12 for i in range(5):
13     def fun1(e):
14         return e+i
15     fun_list1.append(fun1)
16
17 fun_list2 = []
18 for i in range(5):
19     def fun2(e,iv=i):
20         return e+iv
21     fun_list2.append(fun2)
22
23 fun_list3 = [lambda e: e+i for i in range(5)]
24
25 fun_list4 = [lambda e,iv=i: e+iv for i in range(5)]
26
27 i=56
```

Try to predict, and then test to see the output, of the output of the following calls, remembering that the function uses the latest value of any variable that is not bound in the function call:

pythonDemo.py — (continued)

```
29 # in Shell do
30 ## ipython -i pythonDemo.py
31 # Try these (copy text after the comment symbol and paste in the Python
       prompt):
32 # print([f(10) for f in fun_list1])
33 # print([f(10) for f in fun_list2])
34 # print([f(10) for f in fun_list3])
35 # print([f(10) for f in fun_list4])
```

In the first for-loop, the function *fun* uses *i*, whose value is the last value it was assigned. In the second loop, the function *fun*2 uses *iv*. There is a separate *iv* variable for each function, and its value is the value of *i* when the function was defined. Thus *fun*1 uses late binding, and *fun*2 uses early binding. *fun_list*3

[1] Numbered lines are Python code available in the code-directory, `aipython`. The name of the file is given in the gray text above the listing. The numbers correspond to the line numbers in that file.

and *fun_list*4 are equivalent to the first two (except *fun_list*4 uses a different *i* variable).

One of the advantages of using the embedded definitions (as in *fun*1 and *fun*2 above) over the lambda is that is it possible to add a __doc__ string, which is the standard for documenting functions in Python, to the embedded definitions.

1.5.3 Generators and Coroutines

Python has generators which can be used for a form of coroutines.

The `yield` command returns a value that is obtained with `next`. It is typically used to enumerate the values for a `for` loop or in generators. (The `yield` command can also be used for coroutines, but we only us it for genertors in AIPython.)

A version of the built-in *range*, with 2 or 3 arguments (and positive steps) can be implemented as:

pythonDemo.py — (continued)

```
37 def myrange(start, stop, step=1):
38     """enumerates the values from start in steps of size step that are
39     less than stop.
40     """
41     assert step>0, "only positive steps implemented in myrange"
42     i = start
43     while i<stop:
44         yield i
45         i += step
46
47 print("list(myrange(2,30,3)):",list(myrange(2,30,3)))
```

Note that the built-in *range* is unconventional in how it handles a single argument, as the single argument acts as the second argument of the function. Note also that the built-in range also allows for indexing (e.g., $range(2,30,3)[2]$ returns 8), which the above implementation does not. However *myrange* also works for floats, which the built-in range does not.

Exercise 1.1 Implement a version of *myrange* that acts like the built-in version when there is a single argument. (Hint: make the second argument have a default value that can be recognized in the function.)

Yield can be used to generate the same sequence of values as in the example of Section 1.5.1:

pythonDemo.py — (continued)

```
49 def ga(n):
50     """generates square of even nonnegative integers less than n"""
51     for e in range(n):
52         if e%2==0:
53             yield e*e
54 a = ga(20)
```

The sequence of next(a), and list(a) gives exactly the same results as the comprehension in Section 1.5.1.

It is straightforward to write a version of the built-in *enumerate*. Let's call it *myenumerate*:

pythonDemo.py — (continued)

```
56 def myenumerate(enum):
57     for i in range(len(enum)):
58         yield i,enum[i]
```

Exercise 1.2 Write a version of *enumerate* where the only iteration is "for val in enum". Hint: keep track of the index.

1.6 Useful Libraries

1.6.1 Timing Code

In order to compare algorithms, we often want to compute how long a program takes; this is called the **runtime** of the program. The most straightforward way to compute runtime is to use *time.perf_counter*(), as in:

```
import time
start_time = time.perf_counter()
compute_for_a_while()
end_time = time.perf_counter()
print("Time:", end_time - start_time, "seconds")
```

Note that `time.perf_counter()` measures clock time; so this should be done without user interaction between the calls. On the console, you should do:

```
start_time = time.perf_counter(); compute_for_a_while(); end_time = time.perf_counter()
```

If this time is very small (say less than 0.2 second), it is probably very inaccurate, and it may be better to run your code many times to get a more accurate count. For this you can use *timeit* (`https://docs.python.org/3/library/timeit.html`). To use timeit to time the call to *foo.bar*(*aaa*) use:

```
import timeit
time = timeit.timeit("foo.bar(aaa)",
                     setup="from __main__ import foo,aaa", number=100)
```

The setup is needed so that Python can find the meaning of the names in the string that is called. This returns the number of seconds to execute *foo.bar*(*aaa*) 100 times. The variable *number* should be set so that the runtime is at least 0.2 seconds.

You should not trust a single measurement as that can be confounded by interference from other processes. *timeit.repeat* can be used for running *timit* a few (say 3) times. Usually the minimum time is the one to report, but you should be explicit and explain what you are reporting.

1.6.2 Plotting: Matplotlib

The standard plotting for Python is matplotlib (`http://matplotlib.org/`). We will use the most basic plotting using the pyplot interface.

Here is a simple example that uses everything we will use.

pythonDemo.py — (continued)

```
60 import matplotlib.pyplot as plt
61
62 def myplot(min,max,step,fun1,fun2):
63     plt.ion() # make it interactive
64     plt.xlabel("The x axis")
65     plt.ylabel("The y axis")
66     plt.xscale('linear') # Makes a 'log' or 'linear' scale
67     xvalues = range(min,max,step)
68     plt.plot(xvalues,[fun1(x) for x in xvalues],
69              label="The first fun")
70     plt.plot(xvalues,[fun2(x) for x in xvalues], linestyle='--',color='k',
71              label=fun2.__doc__) # use the doc string of the function
72     plt.legend(loc="upper right") # display the legend
73
74 def slin(x):
75     """y=2x+7"""
76     return 2*x+7
77 def sqfun(x):
78     """y=(x-40)^2/10-20"""
79     return (x-40)**2/10-20
80
81 # Try the following:
82 # from pythonDemo import myplot, slin, sqfun
83 # import matplotlib.pyplot as plt
84 # myplot(0,100,1,slin,sqfun)
85 # plt.legend(loc="best")
86 # import math
87 # plt.plot([41+40*math.cos(th/10) for th in range(50)],
88 #          [100+100*math.sin(th/10) for th in range(50)])
89 # plt.text(40,100,"ellipse?")
90 # plt.xscale('log')
```

At the end of the code are some commented-out commands you should try in interactive mode. Cut from the file and paste into Python (and remember to remove the comments symbol and leading space).

1.7 Utilities

1.7.1 Display

In this distribution, to keep things simple and to only use standard Python, we use a text-oriented tracing of the code. A graphical depiction of the code could

override the definition of *display* (but we leave it as a project).

The method *self.display* is used to trace the program. Any call

$$self.display(level, to\_print \ldots)$$

where the level is less than or equal to the value for *max_display_level* will be printed. The *to_print* ... can be anything that is accepted by the built-in *print* (including any keyword arguments).

The definition of *display* is:

display.py — A simple way to trace the intermediate steps of algorithms.

```
11 class Displayable(object):
12     """Class that uses 'display'.
13     The amount of detail is controlled by max_display_level
14     """
15     max_display_level = 1 # can be overridden in subclasses
16 
17     def display(self,level,*args,**nargs):
18         """print the arguments if level is less than or equal to the
19         current max_display_level.
20         level is an integer.
21         the other arguments are whatever arguments print can take.
22         """
23         if level <= self.max_display_level:
24             print(*args, **nargs) ##if error you are using Python2 not
                   Python3
```

Note that *args* gets a tuple of the positional arguments, and *nargs* gets a dictionary of the keyword arguments). This will not work in Python 2, and will give an error.

Any class that wants to use *display* can be made a subclass of *Displayable*.

To change the maximum display level to say 3, for a class do:

$$Classname.max\_display\_level = 3$$

which will make calls to *display* in that class print when the value of *level* is less than-or-equal to 3. The default display level is 1. It can also be changed for individual objects (the object value overrides the class value).

The value of *max_display_level* by convention is:

0 display nothing

1 display solutions (nothing that happens repeatedly)

2 also display the values as they change (little detail through a loop)

3 also display more details

4 and above even more detail

In order to implement more sophisticated visualizations of the algorithm, we add a **visualize** "decorator" to the methods to be visualized. The following code ignores the decorator:

display.py — (continued)

```
26 def visualize(func):
27     """A decorator for algorithms that do interactive visualization.
28     Ignored here.
29     """
30     return func
```

1.7.2 Argmax

Python has a built-in *max* function that takes a generator (or a list or set) and returns the maximum value. The *argmax* method returns the index of an element that has the maximum value. If there are multiple elements with the maximum value, one if the indexes to that value is returned at random. *argmaxe* assumes an enumeration; a generator of $(element, value)$ pairs, as for example is generated by the built-in $enumerate(list)$ for lists or $dict.items()$ for dicts.

utilities.py — AIPython useful utilities

```
11 import random
12 import math
13
14 def argmaxall(gen):
15     """gen is a generator of (element,value) pairs, where value is a real.
16     argmaxall returns a list of all of the elements with maximal value.
17     """
18     maxv = -math.inf     # negative infinity
19     maxvals = []    # list of maximal elements
20     for (e,v) in gen:
21         if v>maxv:
22             maxvals,maxv = [e], v
23         elif v==maxv:
24             maxvals.append(e)
25     return maxvals
26
27 def argmaxe(gen):
28     """gen is a generator of (element,value) pairs, where value is a real.
29     argmaxe returns an element with maximal value.
30     If there are multiple elements with the max value, one is returned at
           random.
31     """
32     return random.choice(argmaxall(gen))
33
34 def argmax(lst):
35     """returns maximum index in a list"""
36     return argmaxe(enumerate(lst))
37 # Try:
```

```
# argmax([1,6,3,77,3,55,23])

def argmaxd(dct):
   """returns the arx max of a dictionary dct"""
   return argmaxe(dct.items())
# Try:
# arxmaxd({2:5,5:9,7:7})
```

Exercise 1.3 Change argmax to have an optional argument that specifies whether you want the "first", "last" or a "random" index of the maximum value returned. If you want the first or the last, you don't need to keep a list of the maximum elements.

1.7.3 Probability

For many of the simulations, we want to make a variable True with some probability. $flip(p)$ returns True with probability p, and otherwise returns False.

utilities.py — (continued)

```
def flip(prob):
    """return true with probability prob"""
    return random.random() < prob
```

1.7.4 Dictionary Union

This is now | in Python 3.9, so will be replaced.

The function $dict\_union(d1, d2)$ returns the union of dictionaries $d1$ and $d2$. If the values for the keys conflict, the values in $d2$ are used. This is similar to $dict(d1, **d2)$, but that only works when the keys of $d2$ are strings.

utilities.py — (continued)

```
def dict_union(d1,d2):
    """returns a dictionary that contains the keys of d1 and d2.
    The value for each key that is in d2 is the value from d2,
    otherwise it is the value from d1.
    This does not have side effects.
    """
    d = dict(d1)  # copy d1
    d.update(d2)
    return d
```

1.8 Testing Code

It is important to test code early and test it often. We include a simple form of **unit test**. The value of the current module is in `__name__` and if the module is run at the top-level, it's value is `"__main__"`. See `https://docs.python.org/3/library/__main__.html`.

The following code tests `argmax` and `dict_union`, but only when if `utilities` is loaded in the top-level. If it is loaded in a module the test code is not run.

In your code you should do more substantial testing than we do here, in particular testing the boundary cases.

utilities.py — (continued)

```
59 def test():
60     """Test part of utilities"""
61     assert argmax(enumerate([1,6,55,3,55,23])) in [2,4]
62     assert dict_union({1:4, 2:5, 3:4},{5:7, 2:9}) == {1:4, 2:9, 3:4, 5:7}
63     print("Passed unit test in utilities")
64 
65 if __name__ == "__main__":
66     test()
```

Chapter 2

Agents and Control

This implements the controllers described in Chapter 2.

In this version the higher-levels call the lower-levels. A more sophisticated version may have them run concurrently (either as coroutines or in parallel). The higher-levels calling the lower-level works in simulated environments when there is a single agent, and where the lower-level are written to make sure they return (and don't go on forever), and the higher level doesn't take too long (as the lower-levels will wait until called again).

2.1 Representing Agents and Environments

An agent observes the world, and carries out actions in the environment, it also has an internal state that it updates. The environment takes in actions of the agents, updates it internal state and returns the percepts.

In this implementation, the state of the agent and the state of the environment are represented using standard Python variables, which are updated as the state changes. The percepts and the actions are represented as variable-value dictionaries.

An agent implements the $go(n)$ method, where n is an integer. This means that the agent should run for n time steps.

In the following code `raise NotImplementedError()` is a way to specify an abstract method that needs to be overidden in any implemented agent or environment.

agents.py — Agent and Controllers

```
11 import random
12
13 class Agent(object):
14     def __init__(self,env):
```

```
15          """set up the agent"""
16          self.env=env
17
18      def go(self,n):
19          """acts for n time steps"""
20          raise NotImplementedError("go") # abstract method
```

The environment implements a $do(action)$ method where *action* is a variable-value dictionary. This returns a percept, which is also a variable-value dictionary. The use of dictionaries allows for structured actions and percepts.

Note that *Environment* is a subclass of *Displayable* so that it can use the *display* method described in Section 1.7.1.

agents.py — (continued)

```
22 from display import Displayable
23 class Environment(Displayable):
24     def initial_percepts(self):
25         """returns the initial percepts for the agent"""
26         raise NotImplementedError("initial_percepts") # abstract method
27
28     def do(self,action):
29         """does the action in the environment
30         returns the next percept """
31         raise NotImplementedError("do") # abstract method
```

2.2 Paper buying agent and environment

To run the demo, in folder "aipython", load "agents.py", using e.g., `ipython -i agents.py`, and copy and paste the commented-out commands at the bottom of that file. This requires Python 3 with `matplotlib`.

This is an implementation of the paper buying example.

2.2.1 The Environment

The environment state is given in terms of the *time* and the amount of paper in *stock*. It also remembers the in-stock history and the price history. The percepts are the price and the amount of paper in stock. The action of the agent is the number to buy.

Here we assume that the prices are obtained from the *prices* list plus a random integer in range $[0, max\_price\_addon)$ plus a linear "inflation". The agent cannot access the price model; it just observes the prices and the amount in stock.

agents.py — (continued)

```
33 class TP_env(Environment):
```

```
    prices = [234, 234, 234, 234, 255, 255, 275, 275, 211, 211, 211,
    234, 234, 234, 234, 199, 199, 275, 275, 234, 234, 234, 234, 255,
    255, 260, 260, 265, 265, 265, 265, 270, 270, 255, 255, 260, 260,
    265, 265, 150, 150, 265, 265, 270, 270, 255, 255, 260, 260, 265,
    265, 265, 265, 270, 270, 211, 211, 255, 255, 260, 260, 265, 265,
    260, 265, 270, 270, 205, 255, 255, 260, 260, 265, 265, 265, 265,
    270, 270]
    max_price_addon = 20 # maximum of random value added to get price

    def __init__(self):
        """paper buying agent"""
        self.time=0
        self.stock=20
        self.stock_history = [] # memory of the stock history
        self.price_history = [] # memory of the price history

    def initial_percepts(self):
        """return initial percepts"""
        self.stock_history.append(self.stock)
        price = self.prices[0]+random.randrange(self.max_price_addon)
        self.price_history.append(price)
        return {'price': price,
                'instock': self.stock}

    def do(self, action):
        """does action (buy) and returns percepts (price and instock)"""
        used = pick_from_dist({6:0.1, 5:0.1, 4:0.2, 3:0.3, 2:0.2, 1:0.1})
        bought = action['buy']
        self.stock = self.stock+bought-used
        self.stock_history.append(self.stock)
        self.time += 1
        price = (self.prices[self.time%len(self.prices)] # repeating pattern
                 +random.randrange(self.max_price_addon) # plus randomness
                 +self.time//2)                          # plus inflation
        self.price_history.append(price)
        return {'price': price,
                'instock': self.stock}
```

The *pick_from_dist* method takes in a *item* : *probability* dictionary, and returns one of the items in proportion to its probability.

agents.py — (continued)

```
def pick_from_dist(item_prob_dist):
    """ returns a value from a distribution.
    item_prob_dist is an item:probability dictionary, where the
        probabilities sum to 1.
    returns an item chosen in proportion to its probability
    """
    ranreal = random.random()
    for (it,prob) in item_prob_dist.items():
        if ranreal < prob:
```

```
81              return it
82          else:
83              ranreal -= prob
84      raise RuntimeError(str(item_prob_dist)+" is not a probability
            distribution")
```

2.2.2 The Agent

The agent does not have access to the price model but can only observe the current price and the amount in stock. It has to decide how much to buy.

The belief state of the agent is an estimate of the average price of the paper, and the total amount of money the agent has spent.

agents.py — (continued)

```
86  class TP_agent(Agent):
87      def __init__(self, env):
88          self.env = env
89          self.spent = 0
90          percepts = env.initial_percepts()
91          self.ave = self.last_price = percepts['price']
92          self.instock = percepts['instock']
93
94      def go(self, n):
95          """go for n time steps
96          """
97          for i in range(n):
98              if self.last_price < 0.9*self.ave and self.instock < 60:
99                  tobuy = 48
100             elif self.instock < 12:
101                 tobuy = 12
102             else:
103                 tobuy = 0
104             self.spent += tobuy*self.last_price
105             percepts = env.do({'buy': tobuy})
106             self.last_price = percepts['price']
107             self.ave = self.ave+(self.last_price-self.ave)*0.05
108             self.instock = percepts['instock']
```

Set up an environment and an agent. Uncomment the last lines to run the agent for 90 steps, and determine the average amount spent.

agents.py — (continued)

```
110 env = TP_env()
111 ag = TP_agent(env)
112 #ag.go(90)
113 #ag.spent/env.time ## average spent per time period
```

2.2.3 Plotting

The following plots the price and number in stock history:

agents.py — (continued)

```
115 import matplotlib.pyplot as plt
116
117 class Plot_prices(object):
118     """Set up the plot for history of price and number in stock"""
119     def __init__(self, ag,env):
120         self.ag = ag
121         self.env = env
122         plt.ion()
123         plt.xlabel("Time")
124         plt.ylabel("Number in stock.
                Price.")
125
126     def plot_run(self):
127         """plot history of price and instock"""
128         num = len(env.stock_history)
129         plt.plot(range(num),env.stock_history,label="In stock")
130         plt.plot(range(num),env.price_history,label="Price")
131         #plt.legend(loc="upper left")
132         plt.draw()
133
134 # pl = Plot_prices(ag,env)
135 # ag.go(90); pl.plot_run()
```

2.3 Hierarchical Controller

To run the hierarchical controller, in folder "aipython", load "agentTop.py", using e.g., `ipython -i agentTop.py`, and copy and paste the commands near the bottom of that file. This requires Python 3 with `matplotlib`.

In this implementation, each layer, including the top layer, implements the environment class, because each layer is seen as an environment from the layer above.

We arbitrarily divide the environment and the body, so that the environment just defines the walls, and the body includes everything to do with the agent. Note that the named locations are part of the (top-level of the) agent, not part of the environment, although they could have been.

2.3.1 Environment

The environment defines the walls.

agentEnv.py — Agent environment

```
import math
from agents import Environment

class Rob_env(Environment):
    def __init__(self,walls = {}):
        """walls is a set of line segments
              where each line segment is of the form ((x0,y0),(x1,y1))
        """
        self.walls = walls
```

2.3.2 Body

The body defines everything about the agent body.

agentEnv.py — (continued)

```
import math
from agents import Environment
import matplotlib.pyplot as plt
import time

class Rob_body(Environment):
    def __init__(self, env, init_pos=(0,0,90)):
        """ env is the current environment
        init_pos is a triple of (x-position, y-position, direction)
            direction is in degrees; 0 is to right, 90 is straight-up, etc
        """
        self.env = env
        self.rob_x, self.rob_y, self.rob_dir = init_pos
        self.turning_angle = 18 # degrees that a left makes
        self.whisker_length = 6 # length of the whisker
        self.whisker_angle = 30 # angle of whisker relative to robot
        self.crashed = False
        # The following control how it is plotted
        self.plotting = True    # whether the trace is being plotted
        self.sleep_time = 0.05  # time between actions (for real-time
            plotting)
        # The following are data structures maintained:
        self.history = [(self.rob_x, self.rob_y)] # history of (x,y)
            positions
        self.wall_history = []  # history of hitting the wall

    def percepts(self):
        return {'rob_x_pos':self.rob_x, 'rob_y_pos':self.rob_y,
               'rob_dir':self.rob_dir, 'whisker':self.whisker() ,
                   'crashed':self.crashed}
    initial_percepts = percepts # use percept function for initial percepts
        too

    def do(self,action):
        """ action is {'steer':direction}
```

```
52          direction is 'left', 'right' or 'straight'
53          """
54          if self.crashed:
55              return self.percepts()
56          direction = action['steer']
57          compass_deriv =
                {'left':1,'straight':0,'right':-1}[direction]*self.turning_angle
58          self.rob_dir = (self.rob_dir + compass_deriv +360)%360 # make in
                range [0,360)
59          rob_x_new = self.rob_x + math.cos(self.rob_dir*math.pi/180)
60          rob_y_new = self.rob_y + math.sin(self.rob_dir*math.pi/180)
61          path = ((self.rob_x,self.rob_y),(rob_x_new,rob_y_new))
62          if any(line_segments_intersect(path,wall) for wall in
                self.env.walls):
63              self.crashed = True
64              if self.plotting:
65                  plt.plot([self.rob_x],[self.rob_y],"r*",markersize=20.0)
66                  plt.draw()
67          self.rob_x, self.rob_y = rob_x_new, rob_y_new
68          self.history.append((self.rob_x, self.rob_y))
69          if self.plotting and not self.crashed:
70              plt.plot([self.rob_x],[self.rob_y],"go")
71              plt.draw()
72              plt.pause(self.sleep_time)
73          return self.percepts()
```

This detects if the whisker and the wall intersect. It's value is returned as a percept.

agentEnv.py — (continued)

```
75      def whisker(self):
76          """returns true whenever the whisker sensor intersects with a wall
77          """
78          whisk_ang_world = (self.rob_dir-self.whisker_angle)*math.pi/180
79              # angle in radians in world coordinates
80          wx = self.rob_x + self.whisker_length * math.cos(whisk_ang_world)
81          wy = self.rob_y + self.whisker_length * math.sin(whisk_ang_world)
82          whisker_line = ((self.rob_x,self.rob_y),(wx,wy))
83          hit = any(line_segments_intersect(whisker_line,wall)
84                    for wall in self.env.walls)
85          if hit:
86              self.wall_history.append((self.rob_x, self.rob_y))
87              if self.plotting:
88                  plt.plot([self.rob_x],[self.rob_y],"ro")
89                  plt.draw()
90          return hit
91
92  def line_segments_intersect(linea,lineb):
93      """returns true if the line segments, linea and lineb intersect.
94      A line segment is represented as a pair of points.
95      A point is represented as a (x,y) pair.
```

```
96      """
97      ((x0a,y0a),(x1a,y1a)) = linea
98      ((x0b,y0b),(x1b,y1b)) = lineb
99      da, db = x1a-x0a, x1b-x0b
100     ea, eb = y1a-y0a, y1b-y0b
101     denom = db*ea-eb*da
102     if denom==0:  # line segments are parallel
103         return False
104     cb = (da*(y0b-y0a)-ea*(x0b-x0a))/denom # position along line b
105     if cb<0 or cb>1:
106         return False
107     ca = (db*(y0b-y0a)-eb*(x0b-x0a))/denom # position along line a
108     return 0<=ca<=1
109
110 # Test cases:
111 # assert line_segments_intersect(((0,0),(1,1)),((1,0),(0,1)))
112 # assert not line_segments_intersect(((0,0),(1,1)),((1,0),(0.6,0.4)))
113 # assert line_segments_intersect(((0,0),(1,1)),((1,0),(0.4,0.6)))
```

2.3.3 Middle Layer

The middle layer acts like both a controller (for the environment layer) and an environment for the upper layer. It has to tell the environment how to steer. Thus it calls $env.do(\cdot)$. It also is told the position to go to and the timeout. Thus it also has to implement $do(\cdot)$.

agentMiddle.py — Middle Layer

```
11 from agents import Environment
12 import math
13
14 class Rob_middle_layer(Environment):
15     def __init__(self,env):
16         self.env=env
17         self.percepts = env.initial_percepts()
18         self.straight_angle = 11 # angle that is close enough to straight
               ahead
19         self.close_threshold = 2 # distance that is close enough to arrived
20         self.close_threshold_squared = self.close_threshold**2 # just
               compute it once
21
22     def initial_percepts(self):
23         return {}
24
25     def do(self, action):
26         """action is {'go_to':target_pos,'timeout':timeout}
27         target_pos is (x,y) pair
28         timeout is the number of steps to try
29         returns {'arrived':True} when arrived is true
30             or {'arrived':False} if it reached the timeout
```

```
        """
        if 'timeout' in action:
            remaining = action['timeout']
        else:
            remaining = -1  # will never reach 0
        target_pos = action['go_to']
        arrived = self.close_enough(target_pos)
        while not arrived and remaining != 0:
            self.percepts = self.env.do({"steer":self.steer(target_pos)})
            remaining -= 1
            arrived = self.close_enough(target_pos)
        return {'arrived':arrived}
```

This determines how to steer depending on whether the goal is to the right or the left of where the robot is facing.

agentMiddle.py — (continued)

```
    def steer(self,target_pos):
        if self.percepts['whisker']:
            self.display(3,'whisker on', self.percepts)
            return "left"
        else:
            gx,gy = target_pos
            rx,ry = self.percepts['rob_x_pos'],self.percepts['rob_y_pos']
            goal_dir = math.acos((gx-rx)/math.sqrt((gx-rx)*(gx-rx)
                                                   +(gy-ry)*(gy-ry)))*180/math.pi
            if ry>gy:
                goal_dir = -goal_dir
            goal_from_rob = (goal_dir -
                self.percepts['rob_dir']+540)%360-180
            assert -180 < goal_from_rob <= 180
            if goal_from_rob > self.straight_angle:
                return "left"
            elif goal_from_rob < -self.straight_angle:
                return "right"
            else:
                return "straight"

    def close_enough(self,target_pos):
        gx,gy = target_pos
        rx,ry = self.percepts['rob_x_pos'],self.percepts['rob_y_pos']
        return (gx-rx)**2 + (gy-ry)**2 <= self.close_threshold_squared
```

2.3.4 Top Layer

The top layer treats the middle layer as its environment. Note that the top layer is an environment for us to tell it what to visit.

agentTop.py — Top Layer

```
from agentMiddle import Rob_middle_layer
```

```
12 from agents import Environment
13
14 class Rob_top_layer(Environment):
15     def __init__(self, middle, timeout=200, locations = {'mail':(-5,10),
16                          'o103':(50,10), 'o109':(100,10),'storage':(101,51)}
                                ):
17         """middle is the middle layer
18         timeout is the number of steps the middle layer goes before giving
               up
19         locations is a loc:pos dictionary
20             where loc is a named location, and pos is an (x,y) position.
21         """
22         self.middle = middle
23         self.timeout = timeout # number of steps before the middle layer
               should give up
24         self.locations = locations
25
26     def do(self,plan):
27         """carry out actions.
28         actions is of the form {'visit':list_of_locations}
29         It visits the locations in turn.
30         """
31         to_do = plan['visit']
32         for loc in to_do:
33             position = self.locations[loc]
34             arrived = self.middle.do({'go_to':position,
                   'timeout':self.timeout})
35             self.display(1,"Arrived at",loc,arrived)
```

2.3.5 Plotting

The following is used to plot the locations, the walls and (eventually) the movement of the robot. It can either plot the movement if the robot as it is going (with the default *env.plotting* = *True*), or not plot it as it is going (setting *env.plotting* = *False*; in this case the trace can be plotted using *pl.plot_run*()).

agentTop.py — (continued)

```
37 import matplotlib.pyplot as plt
38
39 class Plot_env(object):
40     def __init__(self, body,top):
41         """sets up the plot
42         """
43         self.body = body
44         plt.ion()
45         plt.clf()
46         plt.axes().set_aspect('equal')
47         for wall in body.env.walls:
48             ((x0,y0),(x1,y1)) = wall
```

```
49            plt.plot([x0,x1],[y0,y1],"-k",linewidth=3)
50        for loc in top.locations:
51            (x,y) = top.locations[loc]
52            plt.plot([x],[y],"k<")
53            plt.text(x+1.0,y+0.5,loc) # print the label above and to the
                  right
54        plt.plot([body.rob_x],[body.rob_y],"go")
55        plt.draw()
56
57    def plot_run(self):
58        """plots the history after the agent has finished.
59        This is typically only used if body.plotting==False
60        """
61        xs,ys = zip(*self.body.history)
62        plt.plot(xs,ys,"go")
63        wxs,wys = zip(*self.body.wall_history)
64        plt.plot(wxs,wys,"ro")
65        #plt.draw()
```

The following code plots the agent as it acts in the world:

agentTop.py — (continued)

```
67 from agentEnv import Rob_body, Rob_env
68
69 env = Rob_env({((20,0),(30,20)), ((70,-5),(70,25))})
70 body = Rob_body(env)
71 middle = Rob_middle_layer(body)
72 top = Rob_top_layer(middle)
73
74 # try:
75 # pl=Plot_env(body,top)
76 # top.do({'visit':['o109','storage','o109','o103']})
77 # You can directly control the middle layer:
78 # middle.do({'go_to':(30,-10), 'timeout':200})
79 # Can you make it crash?
```

Exercise 2.1 The following code implements a robot trap. Write a controller that can escape the "trap" and get to the goal. See textbook for hints.

agentTop.py — (continued)

```
81 # Robot Trap for which the current controller cannot escape:
82 trap_env = Rob_env({((10,-21),(10,0)), ((10,10),(10,31)),
       ((30,-10),(30,0)),
83                     ((30,10),(30,20)), ((50,-21),(50,31)),
                           ((10,-21),(50,-21)),
84                     ((10,0),(30,0)), ((10,10),(30,10)), ((10,31),(50,31))})
85 trap_body = Rob_body(trap_env,init_pos=(-1,0,90))
86 trap_middle = Rob_middle_layer(trap_body)
87 trap_top = Rob_top_layer(trap_middle,locations={'goal':(71,0)})
88
89 # Robot trap exercise:
```

```
# pl=Plot_env(trap_body,trap_top)
# trap_top.do({'visit':['goal']})
```

Chapter 3

Searching for Solutions

3.1 Representing Search Problems

A search problem consists of:

- a start node
- a neighbors function that given a node, returns an enumeration of the arcs from the node
- a specification of a goal in terms of a Boolean function that takes a node and returns true if the node is a goal
- a (optional) heuristic function that, given a node, returns a non-negative real number. The heuristic function defaults to zero.

As far as the searcher is concerned a node can be anything. If multiple-path pruning is used, a node must be hashable. In the simple examples, it is a string, but in more complicated examples (in later chapters) it can be a tuple, a frozen set, or a Python object.

In the following code `raise NotImplementedError()` is a way to specify that this is an abstract method that needs to be overridden to define an actual search problem.

searchProblem.py — representations of search problems

```
11 class Search_problem(object):
12     """A search problem consists of:
13     * a start node
14     * a neighbors function that gives the neighbors of a node
15     * a specification of a goal
16     * a (optional) heuristic function.
```

```
17      The methods must be overridden to define a search problem."""
18
19      def start_node(self):
20          """returns start node"""
21          raise NotImplementedError("start_node") # abstract method
22
23      def is_goal(self,node):
24          """is True if node is a goal"""
25          raise NotImplementedError("is_goal") # abstract method
26
27      def neighbors(self,node):
28          """returns a list of the arcs for the neighbors of node"""
29          raise NotImplementedError("neighbors") # abstract method
30
31      def heuristic(self,n):
32          """Gives the heuristic value of node n.
33          Returns 0 if not overridden."""
34          return 0
```

The neighbors is a list of arcs. A (directed) arc consists of a *from_node* node and a *to_node* node. The arc is the pair $\langle$*from_node, to_node*$\rangle$, but can also contain a non-negative *cost* (which defaults to 1) and can be labeled with an *action*.

searchProblem.py — (continued)

```
36  class Arc(object):
37      """An arc has a from_node and a to_node node and a (non-negative)
            cost"""
38      def __init__(self, from_node, to_node, cost=1, action=None):
39          assert cost >= 0, ("Cost cannot be negative for"+
40                             str(from_node)+"->"+str(to_node)+", cost:
                                   "+str(cost))
41          self.from_node = from_node
42          self.to_node = to_node
43          self.action = action
44          self.cost=cost
45
46      def __repr__(self):
47          """string representation of an arc"""
48          if self.action:
49              return str(self.from_node)+" --"+str(self.action)+"-->
                    "+str(self.to_node)
50          else:
51              return str(self.from_node)+" --> "+str(self.to_node)
```

3.1.1 Explicit Representation of Search Graph

The first representation of a search problem is from an explicit graph (as opposed to one that is generated as needed).

An **explicit graph** consists of

- a list or set of nodes
- a list or set of arcs
- a start node
- a list or set of goal nodes
- (optionally) a dictionary that maps a node to a heuristic value for that node

To define a search problem, we need to define the start node, the goal predicate, the neighbors function and the heuristic function.

searchProblem.py — (continued)

```
53 class Search_problem_from_explicit_graph(Search_problem):
54     """A search problem consists of:
55     * a list or set of nodes
56     * a list or set of arcs
57     * a start node
58     * a list or set of goal nodes
59     * a dictionary that maps each node into its heuristic value.
60     * a dictionary that maps each node into its (x,y) position
61     """
62 
63     def __init__(self, nodes, arcs, start=None, goals=set(), hmap={},
           positions={}):
64         self.neighs = {}
65         self.nodes = nodes
66         for node in nodes:
67             self.neighs[node]=[]
68         self.arcs = arcs
69         for arc in arcs:
70             self.neighs[arc.from_node].append(arc)
71         self.start = start
72         self.goals = goals
73         self.hmap = hmap
74         self.positions = positions
75 
76     def start_node(self):
77         """returns start node"""
78         return self.start
79 
80     def is_goal(self,node):
81         """is True if node is a goal"""
82         return node in self.goals
83 
84     def neighbors(self,node):
85         """returns the neighbors of node"""
86         return self.neighs[node]
87 
```

```
88      def heuristic(self,node):
89          """Gives the heuristic value of node n.
90          Returns 0 if not overridden in the hmap."""
91          if node in self.hmap:
92              return self.hmap[node]
93          else:
94              return 0
95
96      def __repr__(self):
97          """returns a string representation of the search problem"""
98          res=""
99          for arc in self.arcs:
100             res += str(arc)+". "
101         return res
```

The following is used for the depth-first search implementation below.

searchProblem.py — (continued)

```
103     def neighbor_nodes(self,node):
104         """returns an iterator over the neighbors of node"""
105         return (path.to_node for path in self.neighs[node])
```

3.1.2 Paths

A searcher will return a path from the start node to a goal node. A Python list is not a suitable representation for a path, as many search algorithms consider multiple paths at once, and these paths should share initial parts of the path. If we wanted to do this with Python lists, we would need to keep copying the list, which can be expensive if the list is long. An alternative representation is used here in terms of a recursive data structure that can share subparts.

A path is either:

- a node (representing a path of length 0) or
- a path, *initial* and an arc, where the *from_node* of the arc is the node at the end of *initial*.

These cases are distinguished in the following code by having $arc = None$ if the path has length 0, in which case *initial* is the node of the path. Python `yield` is used for enumerations only

searchProblem.py — (continued)

```
107 class Path(object):
108     """A path is either a node or a path followed by an arc"""
109
110     def __init__(self,initial,arc=None):
111         """initial is either a node (in which case arc is None) or
112         a path (in which case arc is an object of type Arc)"""
113         self.initial = initial
```

```
114         self.arc=arc
115         if arc is None:
116             self.cost=0
117         else:
118             self.cost = initial.cost+arc.cost
119 
120     def end(self):
121         """returns the node at the end of the path"""
122         if self.arc is None:
123             return self.initial
124         else:
125             return self.arc.to_node
126 
127     def nodes(self):
128         """enumerates the nodes for the path.
129         This starts at the end and enumerates nodes in the path
                backwards."""
130         current = self
131         while current.arc is not None:
132             yield current.arc.to_node
133             current = current.initial
134         yield current.initial
135 
136     def initial_nodes(self):
137         """enumerates the nodes for the path before the end node.
138         This starts at the end and enumerates nodes in the path
                backwards."""
139         if self.arc is not None:
140             yield from self.initial.nodes()
141 
142     def __repr__(self):
143         """returns a string representation of a path"""
144         if self.arc is None:
145             return str(self.initial)
146         elif self.arc.action:
147             return (str(self.initial)+"\n --"+str(self.arc.action)
148                     +"--> "+str(self.arc.to_node))
149         else:
150             return str(self.initial)+" --> "+str(self.arc.to_node)
```

3.1.3 Example Search Problems

The first search problem is one with 5 nodes where the least-cost path is one with many arcs. See Figure 3.1. Note that this example is used for the unit tests, so the test (in `searchGeneric`) will need to be changed if this is changed.

searchProblem.py — (continued)

```
152 problem1 = Search_problem_from_explicit_graph(
153     {'a','b','c','d','g'},
154     [Arc('a','c',1), Arc('a','b',3), Arc('c','d',3), Arc('c','b',1),
```

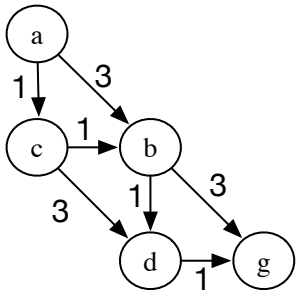


Figure 3.1: problem1

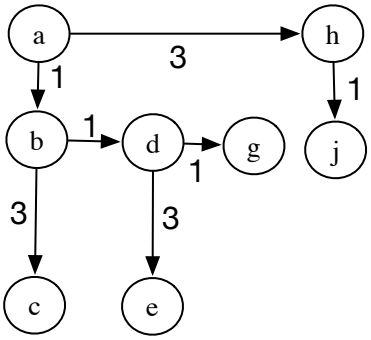


Figure 3.2: problem2

```
155         Arc('b','d',1), Arc('b','g',3), Arc('d','g',1)],
156     start = 'a',
157     goals = {'g'},
158     positions={'a': (0, 0), 'b': (1, 1), 'c': (0,1), 'd': (1,2), 'g':
            (2,2)})
```

The second search problem is one with 8 nodes where many paths do not lead to the goal. See Figure 3.2.

searchProblem.py — (continued)

```
159 problem2 = Search_problem_from_explicit_graph(
160     {'a','b','c','d','e','g','h','j'},
161     [Arc('a','b',1), Arc('b','c',3), Arc('b','d',1), Arc('d','e',3),
162         Arc('d','g',1), Arc('a','h',3), Arc('h','j',1)],
163     start = 'a',
164     goals = {'g'},
165     positions={'a': (0, 0), 'b': (0, 1), 'c': (0,4), 'd': (1,1), 'e': (1,4),
166                   'g': (2,1), 'h': (3,0), 'j': (3,1)})
```

The third search problem is a disconnected graph (contains no arcs), where the start node is a goal node. This is a boundary case to make sure that weird cases work.

searchProblem.py — (continued)

```
problem3 = Search_problem_from_explicit_graph(
    {'a','b','c','d','e','g','h','j'},
    [],
    start = 'g',
    goals = {'k','g'})
```

The acyclic_delivery_problem is the delivery problem described in Example 3.4 and shown in Figure 3.2 of the textbook.

searchProblem.py — (continued)

```
acyclic_delivery_problem = Search_problem_from_explicit_graph(
    {'mail','ts','o103','o109','o111','b1','b2','b3','b4','c1','c2','c3',
     'o125','o123','o119','r123','storage'},
     [Arc('ts','mail',6),
        Arc('o103','ts',8),
        Arc('o103','b3',4),
        Arc('o103','o109',12),
        Arc('o109','o119',16),
        Arc('o109','o111',4),
        Arc('b1','c2',3),
        Arc('b1','b2',6),
        Arc('b2','b4',3),
        Arc('b3','b1',4),
        Arc('b3','b4',7),
        Arc('b4','o109',7),
        Arc('c1','c3',8),
        Arc('c2','c3',6),
        Arc('c2','c1',4),
        Arc('o123','o125',4),
        Arc('o123','r123',4),
        Arc('o119','o123',9),
        Arc('o119','storage',7)],
    start = 'o103',
    goals = {'r123'},
    hmap = {
        'mail' : 26,
        'ts' : 23,
        'o103' : 21,
        'o109' : 24,
        'o111' : 27,
        'o119' : 11,
        'o123' : 4,
        'o125' : 6,
        'r123' : 0,
        'b1' : 13,
        'b2' : 15,
        'b3' : 17,
        'b4' : 18,
        'c1' : 6,
        'c2' : 10,
```

```
        'c3' : 12,
        'storage' : 12
        }
    )
```

The `cyclic_delivery_problem` is the delivery problem described in Example 3.8 and shown in Figure 3.6 of the textbook. This is the same as `acyclic_delivery_problem`, but almost every arc also has its inverse.

searchProblem.py — (continued)

```
cyclic_delivery_problem = Search_problem_from_explicit_graph(
    {'mail','ts','o103','o109','o111','b1','b2','b3','b4','c1','c2','c3',
     'o125','o123','o119','r123','storage'},
     [  Arc('ts','mail',6), Arc('mail','ts',6),
        Arc('o103','ts',8), Arc('ts','o103',8),
        Arc('o103','b3',4),
        Arc('o103','o109',12), Arc('o109','o103',12),
        Arc('o109','o119',16), Arc('o119','o109',16),
        Arc('o109','o111',4), Arc('o111','o109',4),
        Arc('b1','c2',3),
        Arc('b1','b2',6), Arc('b2','b1',6),
        Arc('b2','b4',3), Arc('b4','b2',3),
        Arc('b3','b1',4), Arc('b1','b3',4),
        Arc('b3','b4',7), Arc('b4','b3',7),
        Arc('b4','o109',7),
        Arc('c1','c3',8), Arc('c3','c1',8),
        Arc('c2','c3',6), Arc('c3','c2',6),
        Arc('c2','c1',4), Arc('c1','c2',4),
        Arc('o123','o125',4), Arc('o125','o123',4),
        Arc('o123','r123',4), Arc('r123','o123',4),
        Arc('o119','o123',9), Arc('o123','o119',9),
        Arc('o119','storage',7), Arc('storage','o119',7)],
    start = 'o103',
    goals = {'r123'},
    hmap = {
        'mail' : 26,
        'ts' : 23,
        'o103' : 21,
        'o109' : 24,
        'o111' : 27,
        'o119' : 11,
        'o123' : 4,
        'o125' : 6,
        'r123' : 0,
        'b1' : 13,
        'b2' : 15,
        'b3' : 17,
        'b4' : 18,
        'c1' : 6,
        'c2' : 10,
        'c3' : 12,
```

```
260         'storage' : 12
261         }
262     )
```

3.2 Generic Searcher and Variants

To run the search demos, in folder "aipython", load "searchGeneric.py" , using e.g., ipython -i searchGeneric.py, and copy and paste the example queries at the bottom of that file. This requires Python 3.

3.2.1 Searcher

A *Searcher* for a problem can be asked repeatedly for the next path. To solve a problem, we can construct a *Searcher* object for the problem and then repeatedly ask for the next path using *search*. If there are no more paths, *None* is returned.

searchGeneric.py — Generic Searcher, including depth-first and A*

```
11 from display import Displayable, visualize
12
13 class Searcher(Displayable):
14     """returns a searcher for a problem.
15     Paths can be found by repeatedly calling search().
16     This does depth-first search unless overridden
17     """
18     def __init__(self, problem):
19         """creates a searcher from a problem
20         """
21         self.problem = problem
22         self.initialize_frontier()
23         self.num_expanded = 0
24         self.add_to_frontier(Path(problem.start_node()))
25         super().__init__()
26
27     def initialize_frontier(self):
28         self.frontier = []
29
30     def empty_frontier(self):
31         return self.frontier == []
32
33     def add_to_frontier(self,path):
34         self.frontier.append(path)
35
36     @visualize
37     def search(self):
38         """returns (next) path from the problem's start node
39         to a goal node.
```

```
40          Returns None if no path exists.
41          """
42          while not self.empty_frontier():
43              path = self.frontier.pop()
44              self.display(2, "Expanding:",path,"(cost:",path.cost,")")
45              self.num_expanded += 1
46              if self.problem.is_goal(path.end()): # solution found
47                  self.display(1, self.num_expanded, "paths have been expanded
                        and",
48                               len(self.frontier), "paths remain in the
                                   frontier")
49                  self.solution = path # store the solution found
50                  return path
51              else:
52                  neighs = self.problem.neighbors(path.end())
53                  self.display(3,"Neighbors are", neighs)
54                  for arc in reversed(list(neighs)):
55                      self.add_to_frontier(Path(path,arc))
56                  self.display(3,"Frontier:",self.frontier)
57          self.display(1,"No (more) solutions. Total of",
58                       self.num_expanded,"paths expanded.")
```

Note that this reverses the neigbours so that it implements depth-first search in an intutive manner (expanding the first neighbor first), and *list* is needed if the neighboure are generated. Reversing the neighbours might not be required for other methods. The calls to *reversed* and *list* can be removed, and the algothihm still implements depth-fist search.

Exercise 3.1 When it returns a path, the algorithm can be used to find another path by calling *search*() again. However, it does not find other paths that go through one goal node to another. Explain why, and change the code so that it can find such paths when *search*() is called again.

3.2.2 Frontier as a Priority Queue

In many of the search algorithms, such as A^* and other best-first searchers, the frontier is implemented as a priority queue. Here we use the Python's built-in priority queue implementations, `heapq`.

Following the lead of the Python documentation, `http://docs.python.org/3.3/library/heapq.html`, a frontier is a list of triples. The first element of each triple is the value to be minimized. The second element is a unique index which specifies the order when the first elements are the same, and the third element is the path that is on the queue. The use of the unique index ensures that the priority queue implementation does not compare paths; whether one path is less than another is not defined. It also lets us control what sort of search (e.g., depth-first or breadth-first) occurs when the value to be minimized does not give a unique next path.

The variable *frontier_index* is the total number of elements of the frontier that have been created. As well as being used as a unique index, it is useful for statistics, particularly in conjunction with the current size of the frontier.

searchGeneric.py — (continued)

```
import heapq      # part of the Python standard library
from searchProblem import Path

class FrontierPQ(object):
    """A frontier consists of a priority queue (heap), frontierpq, of
       (value, index, path) triples, where
    * value is the value we want to minimize (e.g., path cost + h).
    * index is a unique index for each element
    * path is the path on the queue
    Note that the priority queue always returns the smallest element.
    """

    def __init__(self):
        """constructs the frontier, initially an empty priority queue
        """
        self.frontier_index = 0 # the number of items ever added to the
            frontier
        self.frontierpq = [] # the frontier priority queue

    def empty(self):
        """is True if the priority queue is empty"""
        return self.frontierpq == []

    def add(self, path, value):
        """add a path to the priority queue
        value is the value to be minimized"""
        self.frontier_index += 1 # get a new unique index
        heapq.heappush(self.frontierpq,(value, -self.frontier_index, path))

    def pop(self):
        """returns and removes the path of the frontier with minimum value.
        """
        (_,_,path) = heapq.heappop(self.frontierpq)
        return path
```

The following methods are used for finding and printing information about the frontier.

searchGeneric.py — (continued)

```
    def count(self,val):
        """returns the number of elements of the frontier with value=val"""
        return sum(1 for e in self.frontierpq if e[0]==val)

    def __repr__(self):
        """string representation of the frontier"""
        return str([(n,c,str(p)) for (n,c,p) in self.frontierpq])
```

```
101
102     def __len__(self):
103         """length of the frontier"""
104         return len(self.frontierpq)
105
106     def __iter__(self):
107         """iterate through the paths in the frontier"""
108         for (_,_,path) in self.frontierpq:
109             yield path
```

3.2.3 A^* Search

For an A^* **Search** the frontier is implemented using the FrontierPQ class.

searchGeneric.py — (continued)

```
111 class AStarSearcher(Searcher):
112     """returns a searcher for a problem.
113     Paths can be found by repeatedly calling search().
114     """
115
116     def __init__(self, problem):
117         super().__init__(problem)
118
119     def initialize_frontier(self):
120         self.frontier = FrontierPQ()
121
122     def empty_frontier(self):
123         return self.frontier.empty()
124
125     def add_to_frontier(self,path):
126         """add path to the frontier with the appropriate cost"""
127         value = path.cost+self.problem.heuristic(path.end())
128         self.frontier.add(path, value)
```

Code should always be tested. The following provides a simple **unit test**, using problem1 as the the default problem.

searchGeneric.py — (continued)

```
130 import searchProblem as searchProblem
131
132 def test(SearchClass, problem=searchProblem.problem1,
        solutions=[['g','d','b','c','a']] ):
133     """Unit test for aipython searching algorithms.
134     SearchClass is a class that takes a problemm and implements search()
135     problem is a search problem
136     solutions is a list of optimal solutions
137     """
138     print("Testing problem 1:")
139     schr1 = SearchClass(problem)
140     path1 = schr1.search()
```

```
141     print("Path found:",path1)
142     assert path1 is not None, "No path is found in problem1"
143     assert list(path1.nodes()) in solutions, "Shortest path not found in
            problem1"
144     print("Passed unit test")
145 
146 if __name__ == "__main__":
147     #test(Searcher)
148     test(AStarSearcher)
149 
150 # example queries:
151 # searcher1 = Searcher(searchProblem.acyclic_delivery_problem) # DFS
152 # searcher1.search() # find first path
153 # searcher1.search() # find next path
154 # searcher2 = AStarSearcher(searchProblem.acyclic_delivery_problem) # A*
155 # searcher2.search() # find first path
156 # searcher2.search() # find next path
157 # searcher3 = Searcher(searchProblem.cyclic_delivery_problem) # DFS
158 # searcher3.search() # find first path with DFS. What do you expect to
        happen?
159 # searcher4 = AStarSearcher(searchProblem.cyclic_delivery_problem) # A*
160 # searcher4.search() # find first path
```

Exercise 3.2 Change the code so that it implements (i) best-first search and (ii) lowest-cost-first search. For each of these methods compare it to A^* in terms of the number of paths expanded, and the path found.

Exercise 3.3 In the *add* method in *FrontierPQ* what does the "-" in front of *frontier_index* do? When there are multiple paths with the same f-value, which search method does this act like? What happens if the "-" is removed? When there are multiple paths with the same value, which search method does this act like? Does it work better with or without the "-"? What evidence did you base your conclusion on?

Exercise 3.4 The searcher acts like a Python iterator, in that it returns one value (here a path) and then returns other values (paths) on demand, but does not implement the iterator interface. Change the code so it implements the iterator interface. What does this enable us to do?

3.2.4 Multiple Path Pruning

To run the multiple-path pruning demo, in folder "aipython", load "searchMPP.py" , using e.g., `ipython -i searchMPP.py`, and copy and paste the example queries at the bottom of that file.

The following implements A^* with multiple-path pruning. It overrides *search*() in *Searcher*.

searchMPP.py — Searcher with multiple-path pruning

```
11 from searchGeneric import AStarSearcher, visualize
12 from searchProblem import Path
```

```
13
14 class SearcherMPP(AStarSearcher):
15     """returns a searcher for a problem.
16     Paths can be found by repeatedly calling search().
17     """
18     def __init__(self, problem):
19         super().__init__(problem)
20         self.explored = set()
21
22     @visualize
23     def search(self):
24         """returns next path from an element of problem's start nodes
25         to a goal node.
26         Returns None if no path exists.
27         """
28         while not self.empty_frontier():
29             path = self.frontier.pop()
30             if path.end() not in self.explored:
31                 self.display(2, "Expanding:",path,"(cost:",path.cost,")")
32                 self.explored.add(path.end())
33                 self.num_expanded += 1
34                 if self.problem.is_goal(path.end()):
35                     self.display(1, self.num_expanded, "paths have been
                           expanded and",
36                                  len(self.frontier), "paths remain in the
                                       frontier")
37                     self.solution = path # store the solution found
38                     return path
39                 else:
40                     neighs = self.problem.neighbors(path.end())
41                     self.display(3,"Neighbors are", neighs)
42                     for arc in neighs:
43                         self.add_to_frontier(Path(path,arc))
44                     self.display(3,"Frontier:",self.frontier)
45         self.display(1,"No (more) solutions. Total of",
46                      self.num_expanded,"paths expanded.")
47
48 from searchGeneric import test
49 if __name__ == "__main__":
50     test(SearcherMPP)
51
52 import searchProblem
53 # searcherMPPcdp = SearcherMPP(searchProblem.cyclic_delivery_problem)
54 # print(searcherMPPcdp.search()) # find first path
```

Exercise 3.5 Implement a searcher that implements cycle pruning instead of multiple-path pruning. You need to decide whether to check for cycles when paths are added to the frontier or when they are removed. (Hint: either method can be implemented by only changing one or two lines in SearcherMPP. Hint: there is a cyle if `path.end() in path.initial_nodes()`) Compare no pruning, multiple path

pruning and cycle pruning for the cyclic delivery problem. Which works better in terms of number of paths expanded, computational time or space?

3.3 Branch-and-bound Search

To run the demo, in folder "aipython", load "searchBranchAndBound.py", and copy and paste the example queries at the bottom of that file.

Depth-first search methods do not need an a priority queue, but can use a list as a stack. In this implementation of branch-and-bound search, we call *search* to find an optimal solution with cost less than bound. This uses depth-first search to find a path to a goal that extends *path* with cost less than the bound. Once a path to a goal has been found, that path is remembered as the *best_path*, the bound is reduced, and the search continues.

searchBranchAndBound.py — Branch and Bound Search

```
11 from searchProblem import Path
12 from searchGeneric import Searcher
13 from display import Displayable, visualize
14
15 class DF_branch_and_bound(Searcher):
16     """returns a branch and bound searcher for a problem.
17     An optimal path with cost less than bound can be found by calling
           search()
18     """
19     def __init__(self, problem, bound=float("inf")):
20         """creates a searcher than can be used with search() to find an
               optimal path.
21         bound gives the initial bound. By default this is infinite -
               meaning there
22         is no initial pruning due to depth bound
23         """
24         super().__init__(problem)
25         self.best_path = None
26         self.bound = bound
27
28     @visualize
29     def search(self):
30         """returns an optimal solution to a problem with cost less than
               bound.
31         returns None if there is no solution with cost less than bound."""
32         self.frontier = [Path(self.problem.start_node())]
33         self.num_expanded = 0
34         while self.frontier:
35             path = self.frontier.pop()
36             if path.cost+self.problem.heuristic(path.end()) < self.bound:
37                 # if path.end() not in path.initial_nodes(): # for cycle
                       pruning
```

```
38              self.display(3,"Expanding:",path,"cost:",path.cost)
39              self.num_expanded += 1
40              if self.problem.is_goal(path.end()):
41                  self.best_path = path
42                  self.bound = path.cost
43                  self.display(2,"New best path:",path," cost:",path.cost)
44              else:
45                  neighs = self.problem.neighbors(path.end())
46                  self.display(3,"Neighbors are", neighs)
47                  for arc in reversed(list(neighs)):
48                      self.add_to_frontier(Path(path, arc))
49      self.display(1,"Number of paths expanded:",self.num_expanded,
50                     "(optimal" if self.best_path else "(no", "solution
                           found)")
51      self.solution = self.best_path
52      return self.best_path
```

Note that this code used *reversed* in order to expand the neighbors of a node in the left-to-right order one might expect. It does this because $pop()$ removes the rightmost element of the list. The call to *list* is there because reversed only works on lists and tuples, but the neighbours can be generated.

Here is a unit test and some queries:

searchBranchAndBound.py — (continued)

```
54 from searchGeneric import test
55 if __name__ == "__main__":
56     test(DF_branch_and_bound)
57
58 # Example queries:
59 import searchProblem
60 # searcherb1 = DF_branch_and_bound(searchProblem.acyclic_delivery_problem)
61 # print(searcherb1.search())    # find optimal path
62 # searcherb2 = DF_branch_and_bound(searchProblem.cyclic_delivery_problem,
       bound=100)
63 # print(searcherb2.search())    # find optimal path
```

Exercise 3.6 Implement a branch-and-bound search uses recursion. Hint: you don't need an explicit frontier, but can do a recursive call for the children.

Exercise 3.7 After the branch-and-bound search found a solution, Sam ran search again, and noticed a different count. Sam hypothesized that this count was related to the number of nodes that an $A*$ search would use (either expand or be added to the frontier). Or maybe, Sam thought, the count for a number of nodes when the bound is slightly above the optimal path case is related to how A^* would work. Is there relationship between these counts? Are there different things that it could count so they are related? Try to find the most specific statement that is true, and explain why it is true.

To test the hypothesis, Sam wrote the following code, but isn't sure it is helpful:

searchTest.py — code that may be useful to compare A* and branch-and-bound

```
from searchGeneric import Searcher, AStarSearcher
from searchBranchAndBound import DF_branch_and_bound
from searchMPP import SearcherMPP

DF_branch_and_bound.max_display_level = 1
Searcher.max_display_level = 1

def run(problem,name):
    print("\n\n*******",name)

    print("\nA*:")
    asearcher = AStarSearcher(problem)
    print("Path found:",asearcher.search()," cost=",asearcher.solution.cost)
    print("there are",asearcher.frontier.count(asearcher.solution.cost),
          "elements remaining on the queue with
              f-value=",asearcher.solution.cost)

    print("\nA* with MPP:"),
    msearcher = SearcherMPP(problem)
    print("Path found:",msearcher.search()," cost=",msearcher.solution.cost)
    print("there are",msearcher.frontier.count(msearcher.solution.cost),
          "elements remaining on the queue with
              f-value=",msearcher.solution.cost)

    bound = asearcher.solution.cost+0.01
    print("\nBranch and bound (with too-good initial bound of", bound,")")
    tbb = DF_branch_and_bound(problem,bound) # cheating!!!!
    print("Path found:",tbb.search()," cost=",tbb.solution.cost)
    print("Rerunning B&B")
    print("Path found:",tbb.search())

    bbound = asearcher.solution.cost*2+10
    print("\nBranch and bound (with not-very-good initial bound of",
        bbound, ")")
    tbb2 = DF_branch_and_bound(problem,bbound) # cheating!!!!
    print("Path found:",tbb2.search()," cost=",tbb2.solution.cost)
    print("Rerunning B&B")
    print("Path found:",tbb2.search())

    print("\nDepth-first search: (Use ^C if it goes on forever)")
    tsearcher = Searcher(problem)
    print("Path found:",tsearcher.search()," cost=",tsearcher.solution.cost)


import searchProblem
from searchTest import run
if __name__ == "__main__":
    run(searchProblem.problem1,"Problem 1")
#   run(searchProblem.acyclic_delivery_problem,"Acyclic Delivery")
#   run(searchProblem.cyclic_delivery_problem,"Cyclic Delivery")
```

```
58 # also test some graphs with cycles, and some with multiple least-cost
       paths
```

Chapter 4

Reasoning with Constraints

4.1 Constraint Satisfaction Problems

4.1.1 Variables

A **variable** consists of a name, a domain and an optional (x,y) position (for displaying). The domain of a variable is a list or a tuple, as the ordering will matter in the representation of constraints.

cspProblem.py — Representations of a Constraint Satisfaction Problem

```
11 import random
12 import matplotlib.pyplot as plt
13
14 class Variable(object):
15     """A random variable.
16     name (string) - name of the variable
17     domain (list) - a list of the values for the variable.
18     Variables are ordered according to their name.
19     """
20
21     def __init__(self, name, domain, position=None):
22         """Variable
23         name a string
24         domain a list of printable values
25         position of form (x,y)
26         """
27         self.name = name # string
28         self.domain = domain # list of values
29         self.position = position if position else (random.random(),
               random.random())
30         self.size = len(domain)
31
```

```
32      def __str__(self):
33          return self.name
34
35      def __repr__(self):
36          return self.name # f"Variable({self.name})"
```

4.1.2 Constraints

A **constraint** consists of:

- A tuple (or list) of variables is called the **scope**.
- A **condition** is a Boolean function that takes the same number of arguments as there are variables in the scope. The condition must have a `__name__` property that gives a printable name of the function; built-in functions and functions that are defined using *def* have such a property; for other functions you may need to define this property.
- An optional name
- An optional (x, y) position

cspProblem.py — (continued)

```
38  class Constraint(object):
39      """A Constraint consists of
40      * scope: a tuple of variables
41      * condition: a Boolean function that can applied to a tuple of values
            for variables in scope
42      * string: a string for printing the constraints. All of the strings
            must be unique.
43      for the variables
44      """
45      def __init__(self, scope, condition, string=None, position=None):
46          self.scope = scope
47          self.condition = condition
48          if string is None:
49              self.string = self.condition.__name__ + str(self.scope)
50          else:
51              self.string = string
52          self.position = position
53
54      def __repr__(self):
55          return self.string
```

An **assignment** is a *variable*:*value* dictionary.

If *con* is a constraint, *con.holds*(*assignment*) returns True or False depending on whether the condition is true or false for that assignment. The assignment *assignment* must assigns a value to every variable in the scope of the constraint *con* (and could also assign values other variables); *con.holds* gives an error if

not all variables in the scope of *con* are assigned in the assignment. It ignores variables in *assignment* that are not in the scope of the constraint.

In Python, the $*$ notation is used for unpacking a tuple. For example, $F(*(1,2,3))$ is the same as $F(1,2,3)$. So if t has value $(1,2,3)$, then $F(*t)$ is the same as $F(1,2,3)$.

cspProblem.py — (continued)

```
57     def can_evaluate(self, assignment):
58         """
59         assignment is a variable:value dictionary
60         returns True if the constraint can be evaluated given assignment
61         """
62         return all(v in assignment for v in self.scope)
63 
64     def holds(self,assignment):
65         """returns the value of Constraint con evaluated in assignment.
66 
67         precondition: all variables are assigned in assignment, ie
               self.can_evaluate(assignment) is true
68         """
69         return self.condition(*tuple(assignment[v] for v in self.scope))
```

4.1.3 CSPs

A constraint satisfaction problem (CSP) requires:

- *variables*: a list or set of variables
- *constraints*: a set or list of constraints.

Other properties are inferred from these:

- *variables* is the set of variables.
- *var_to_const* is a mapping from variables to set of constraints, such that *var_to_const*[*var*] is the set of constraints with *var* in the scope.

cspProblem.py — (continued)

```
71 class CSP(object):
72     """A CSP consists of
73     * a title (a string)
74     * variables, a set of variables
75     * constraints, a list of constraints
76     * var_to_const, a variable to set of constraints dictionary
77     """
78     def __init__(self, title, variables, constraints):
79         """title is a string
80         variables is set of variables
81         constraints is a list of constraints
```

```
82          """
83          self.title = title
84          self.variables = variables
85          self.constraints = constraints
86          self.var_to_const = {var:set() for var in self.variables}
87          for con in constraints:
88              for var in con.scope:
89                  self.var_to_const[var].add(con)
90
91      def __str__(self):
92          """string representation of CSP"""
93          return str(self.title)
94
95      def __repr__(self):
96          """more detailed string representation of CSP"""
97          return f"CSP({self.title}, {self.variables}, {([str(c) for c in
                self.constraints])})"
```

csp.consistent(*assignment*) returns true if the assignment is consistent with each of the constraints in *csp* (i.e., all of the constraints that can be evaluated evaluate to true). Note that this is a local consistency with each constraint; it does *not* imply the CSP is consistent or has a solution.

cspProblem.py — (continued)

```
99      def consistent(self,assignment):
100         """assignment is a variable:value dictionary
101         returns True if all of the constraints that can be evaluated
102                       evaluate to True given assignment.
103         """
104         return all(con.holds(assignment)
105                    for con in self.constraints
106                    if con.can_evaluate(assignment))
```

The **show** method uses matplotlib to show the graphical structure of a constraint network.

cspProblem.py — (continued)

```
108     def show(self):
109         plt.ion()  # interactive
110         ax = plt.figure().gca()
111         ax.set_axis_off()
112         plt.title(self.title)
113         var_bbox = dict(boxstyle="round4,pad=1.0,rounding_size=0.5")
114         con_bbox = dict(boxstyle="square,pad=1.0",color="green")
115         for var in self.variables:
116             if var.position is None:
117                 var.position = (random.random(), random.random())
118         for con in self.constraints:
119             if con.position is None:
120                 con.position = tuple(sum(var.position[i] for var in
                        con.scope)/len(con.scope)
```

```
                                        for i in range(2))
            bbox = dict(boxstyle="square,pad=1.0",color="green")
            for var in con.scope:
                ax.annotate(con.string, var.position, xytext=con.position,
                                arrowprops={'arrowstyle':'-'},bbox=con_bbox,
                                ha='center')
        for var in self.variables:
            x,y = var.position
            plt.text(x,y,var.name,bbox=var_bbox,ha='center')
```

4.1.4 Examples

In the following code $ne\_$, when given a number, returns a function that is true when its argument is not that number. For example, if $f = ne\_(3)$, then $f(2)$ is True and $f(3)$ is False. That is, $ne\_(x)(y)$ is true when $x \neq y$. Allowing a function of multiple arguments to use its arguments one at a time is called **currying**, after the logician Haskell Curry. Functions used as conditions in constraints require names (so they can be printed).

cspExamples.py — Example CSPs

```
from cspProblem import Variable, CSP, Constraint
from operator import lt,ne,eq,gt

def ne_(val):
    """not equal value"""
    # nev = lambda x: x != val # alternative definition
    # nev = partial(neq,val)  # another alternative definition
    def nev(x):
        return val != x
    nev.__name__ = str(val)+"!="  # name of the function
    return nev
```

Similarly $is\_(x)(y)$ is true when $x = y$.

cspExamples.py — (continued)

```
def is_(val):
    """is a value"""
    # isv = lambda x: x == val # alternative definition
    # isv = partial(eq,val)   # another alternative definition
    def isv(x):
        return val == x
    isv.__name__ = str(val)+"=="
    return isv
```

The CSP, $csp0$ has variables X, Y and Z, each with domain $\{1,2,3\}$. The constraints are $X < Y$ and $Y < Z$.

cspExamples.py — (continued)

```
X = Variable('X', {1,2,3})
Y = Variable('Y', {1,2,3})
```

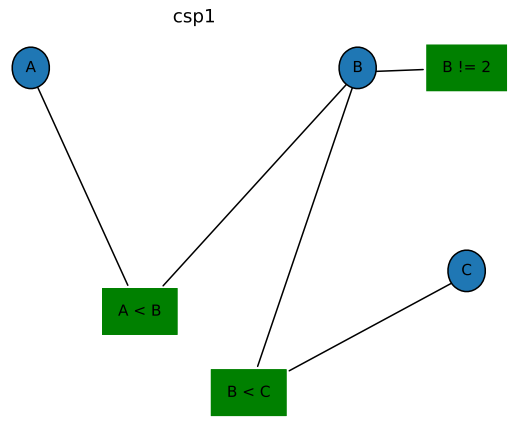


Figure 4.1: csp1.show()

```
Z = Variable('Z', {1,2,3})
csp0 = CSP("csp0", {X,Y,Z},
          [ Constraint([X,Y],lt),
            Constraint([Y,Z],lt)])
```

The CSP, *csp*1 has variables A, B and C, each with domain $\{1,2,3,4\}$. The constraints are $A < B$, $B \neq 2$ and $B < C$. This is slightly more interesting than *csp*0 as it has more solutions. This example is used in the unit tests, and so if it is changed, the unit tests need to be changed.

cspExamples.py — (continued)

```
A = Variable('A', {1,2,3,4}, position=(0.2,0.9))
B = Variable('B', {1,2,3,4}, position=(0.8,0.9))
C = Variable('C', {1,2,3,4}, position=(1,0.4))
C0 = Constraint([A,B], lt, "A < B", position=(0.4,0.3))
C1 = Constraint([B], ne_(2), "B != 2", position=(1,0.9))
C2 = Constraint([B,C], lt, "B < C", position=(0.6,0.1))
csp1 = CSP("csp1", {A, B, C},
          [C0, C1, C2])
```

The next CSP, *csp*2 is Example 4.9 of the textbook; the domain consistent network (after applying the unary constraints) is shown in Figure **??**. Note that we use the same variables as the previous example and add two more.

cspExamples.py — (continued)

```
D = Variable('D', {1,2,3,4}, position=(0,0.4))
E = Variable('E', {1,2,3,4}, position=(0.5,0))
csp2 = CSP("csp2", {A,B,C,D,E},
          [ Constraint([B], ne_(3), "B != 3", position=(1,0.9)),
            Constraint([C], ne_(2), "C != 2", position=(1,0.2)),
```

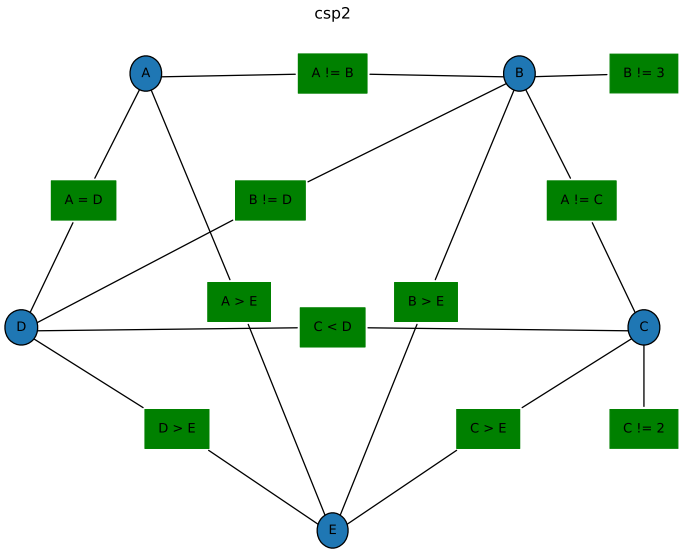


Figure 4.2: csp2.show()

```
53          Constraint([A,B], ne, "A != B"),
54          Constraint([B,C], ne, "A != C"),
55          Constraint([C,D], lt, "C < D"),
56          Constraint([A,D], eq, "A = D"),
57          Constraint([A,E], gt, "A > E"),
58          Constraint([B,E], gt, "B > E"),
59          Constraint([C,E], gt, "C > E"),
60          Constraint([D,E], gt, "D > E"),
61          Constraint([B,D], ne, "B != D")])
```

The following example is another scheduling problem (but with multiple answers). This is the same a scheduling 2 in the original AIspace.org consistency app.

cspExamples.py — (continued)

```
63 csp3 = CSP("csp3", {A,B,C,D,E},
64           [Constraint([A,B], ne, "A != B"),
65            Constraint([A,D], lt, "A < D"),
66            Constraint([A,E], lambda a,e: (a-e)%2 == 1, "A-E is odd"), #
                  A-E is odd
67            Constraint([B,E], lt, "B < E"),
68            Constraint([D,C], lt, "D < C"),
69            Constraint([C,E], ne, "C != E"),
70            Constraint([D,E], ne, "D != E")])
```

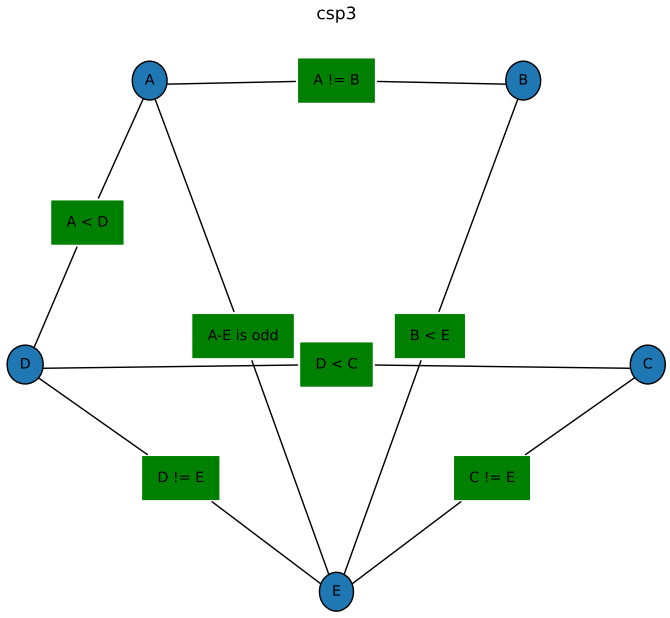


Figure 4.3: csp3.show()

The following example is another abstract scheduling problem. What are the solutions?

cspExamples.py — (continued)

```
72 def adjacent(x,y):
73     """True when x and y are adjacent numbers"""
74     return abs(x-y) == 1
75
76 csp4 = CSP("csp4", {A,B,C,D,E},
77            [Constraint([A,B], adjacent, "adjacent(A,B)"),
78             Constraint([B,C], adjacent, "adjacent(B,C)"),
79             Constraint([C,D], adjacent, "adjacent(C,D)"),
80             Constraint([D,E], adjacent, "adjacent(D,E)"),
81             Constraint([A,C], ne, "A != C"),
82             Constraint([B,D], ne, "B != D"),
83             Constraint([C,E], ne, "C != E")])
```

The following examples represent the crossword shown in Figure 4.5.

In the first representation, the variables represent words. The constraint imposed by the crossword is that where two words intersect, the letter at the intersection must be the same. The method `meet_at` is used to test whether two words intersect with the same letter. For example, the constraint `meet_at(2,0)`

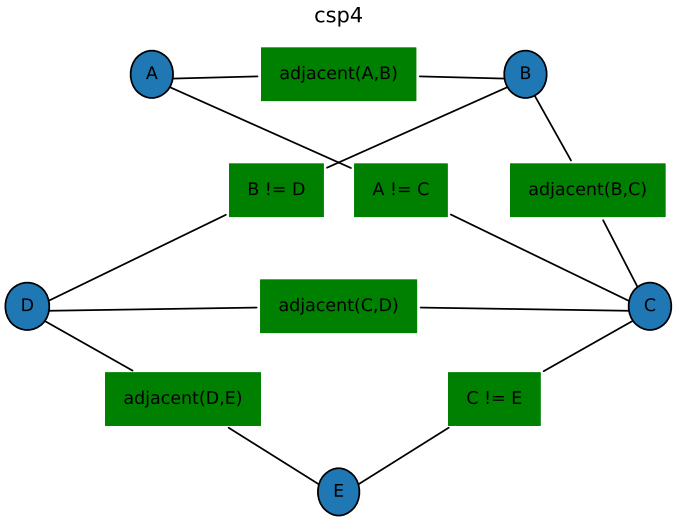
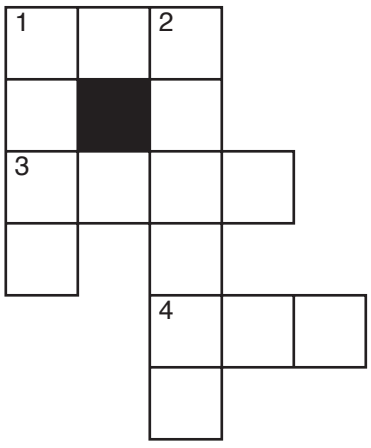


Figure 4.4: csp4.show()



Words:
ant, big, bus, car, has, book, buys, hold, lane, year, ginger, search, symbol, syntax.

Figure 4.5: crossword1: a crossword puzzle to be solved

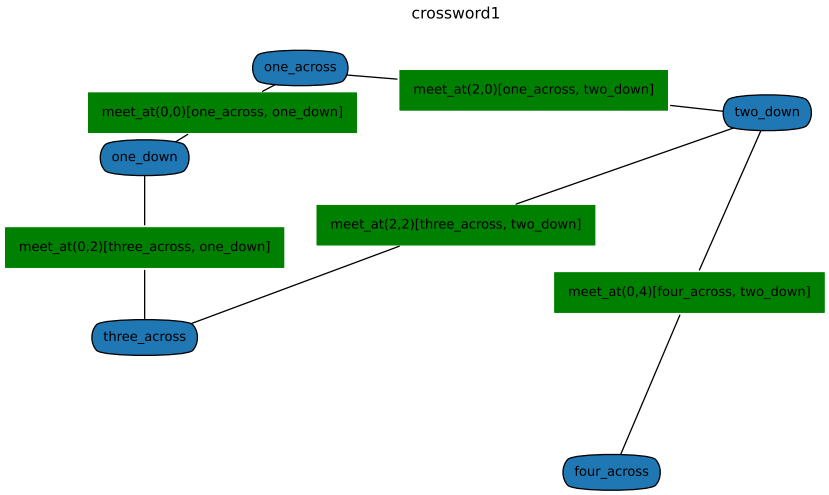


Figure 4.6: crossword1.show()

means that the third letter (at position 2) of the first argument is the same as the first letter of the second argument. This is shown in Figure 4.6.

cspExamples.py — (continued)

```
85  def meet_at(p1,p2):
86      """returns a function of two words that is true
87                 when the words intersect at postions p1, p2.
88      The positions are relative to the words; starting at position 0.
89      meet_at(p1,p2)(w1,w2) is true if the same letter is at position p1 of
            word w1
90           and at position p2 of word w2.
91      """
92      def meets(w1,w2):
93          return w1[p1] == w2[p2]
94      meets.__name__ = "meet_at("+str(p1)+','+str(p2)+')'
95      return meets
96
97  one_across = Variable('one_across', {'ant', 'big', 'bus', 'car', 'has'},
        position=(0.3,0.9))
98  one_down = Variable('one_down', {'book', 'buys', 'hold', 'lane', 'year'},
        position=(0.1,0.7))
99  two_down = Variable('two_down', {'ginger', 'search', 'symbol', 'syntax'},
        position=(0.9,0.8))
100 three_across = Variable('three_across', {'book', 'buys', 'hold', 'land',
        'year'}, position=(0.1,0.3))
101 four_across = Variable('four_across',{'ant', 'big', 'bus', 'car', 'has'},
        position=(0.7,0.0))
```

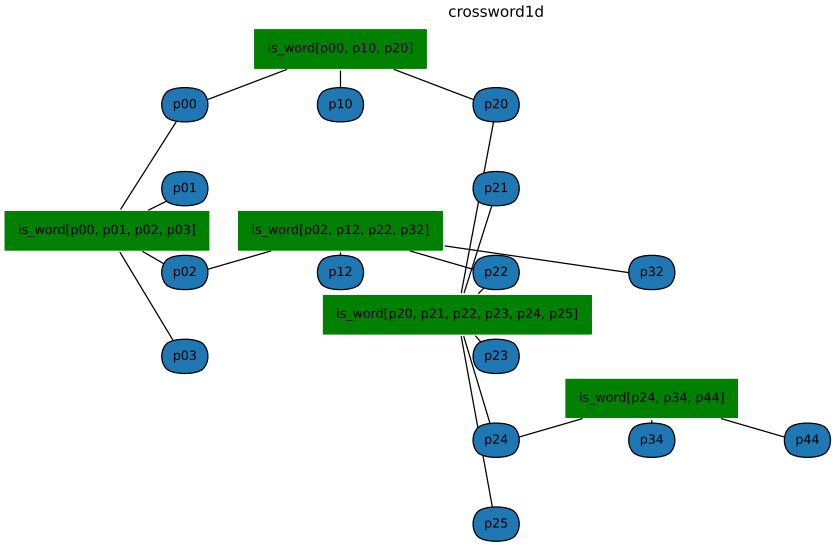


Figure 4.7: crossword1d.show()

```
crossword1 = CSP("crossword1",
                 {one_across, one_down, two_down, three_across,
                     four_across},
                 [Constraint([one_across,one_down], meet_at(0,0)),
                  Constraint([one_across,two_down], meet_at(2,0)),
                  Constraint([three_across,two_down], meet_at(2,2)),
                  Constraint([three_across,one_down], meet_at(0,2)),
                  Constraint([four_across,two_down], meet_at(0,4))])
```

In an alternative representation of a crossword (the "dual" representation), the variables represent letters, and the constraints are that adjacent sequences of letters form words. This is shown in Figure 4.7.

cspExamples.py — (continued)

```
words = {'ant', 'big', 'bus', 'car', 'has','book', 'buys', 'hold',
        'lane', 'year', 'ginger', 'search', 'symbol', 'syntax'}

def is_word(*letters, words=words):
    """is true if the letters concatenated form a word in words"""
    return "".join(letters) in words

letters = {"a", "b", "c", "d", "e", "f", "g", "h", "i", "j", "k", "l",
  "m", "n", "o", "p", "q", "r", "s", "t", "u", "v", "w", "x", "y",
```

```
  "z"}

# pij is the variable representing the letter i from the left and j down
    (starting from 0)
p00 = Variable('p00', letters, position=(0.1,0.85))
p10 = Variable('p10', letters, position=(0.3,0.85))
p20 = Variable('p20', letters, position=(0.5,0.85))
p01 = Variable('p01', letters, position=(0.1,0.7))
p21 = Variable('p21', letters, position=(0.5,0.7))
p02 = Variable('p02', letters, position=(0.1,0.55))
p12 = Variable('p12', letters, position=(0.3,0.55))
p22 = Variable('p22', letters, position=(0.5,0.55))
p32 = Variable('p32', letters, position=(0.7,0.55))
p03 = Variable('p03', letters, position=(0.1,0.4))
p23 = Variable('p23', letters, position=(0.5,0.4))
p24 = Variable('p24', letters, position=(0.5,0.25))
p34 = Variable('p34', letters, position=(0.7,0.25))
p44 = Variable('p44', letters, position=(0.9,0.25))
p25 = Variable('p25', letters, position=(0.5,0.1))

crossword1d = CSP("crossword1d",
                {p00, p10, p20, # first row
                 p01, p21, # second row
                 p02, p12, p22, p32, # third row
                 p03, p23, #fourth row
                 p24, p34, p44, # fifth row
                 p25 # sixth row
                 },
                [Constraint([p00, p10, p20], is_word,
                    position=(0.3,0.95)), #1-across
                 Constraint([p00, p01, p02, p03], is_word,
                     position=(0,0.625)), # 1-down
                 Constraint([p02, p12, p22, p32], is_word,
                     position=(0.3,0.625)), # 3-across
                 Constraint([p20, p21, p22, p23, p24, p25], is_word,
                     position=(0.45,0.475)), # 2-down
                 Constraint([p24, p34, p44], is_word,
                     position=(0.7,0.325)) # 4-across
                 ])
```

Exercise 4.1 How many assignments of a value to each variable are there for each of the representations of the above crossword? Do you think an exhaustive enumeration will work for either one?

The queens problem is a puzzle on a chess board, where the idea is to place a queen on each column so the queens cannot take each other: there are no two queens on the same row, column or diagonal. The **n-queens problem** is a generalization where the size of the board is an $n \times n$, and n queens have to be placed.

Here is a representation of the n-queens problem, where the variables are the columns and the values are the rows in which the queen is placed. The original queens problem on a standard (8×8) chess board is `n_queens(8)`

cspExamples.py — (continued)

```
def queens(ri,rj):
    """ri and rj are different rows, return the condition that the queens
        cannot take each other"""
    def no_take(ci,cj):
        """is true if queen at (ri,ci) cannot take a queen at (rj,cj)"""
        return ci != cj and abs(ri-ci) != abs(rj-cj)
    return no_take

def n_queens(n):
    """returns a CSP for n-queens"""
    columns = list(range(n))
    variables = [Variable(f"R{i}",columns) for i in range(n)]
    return CSP("n-queens",
               variables,
                [Constraint([variables[i], variables[j]], queens(i,j))
                     for i in range(n) for j in range(n) if i != j])

# try the CSP n_queens(8) in one of the solvers.
# What is the smallest n for which there is a solution?
```

Exercise 4.2 How many constraints does this representation of the n-queens problem produce? Can it be done with fewer constraints? Either explain why it can't be done with fewer constraints, or give a solution using fewer constraints.

Unit tests

The following defines a **unit test** for csp solvers, by default using example csp1.

cspExamples.py — (continued)

```
def test_csp(CSP_solver, csp=csp1,
             solutions=[{A: 1, B: 3, C: 4}, {A: 2, B: 3, C: 4}]):
    """CSP_solver is a solver that takes a csp and returns a solution
    csp is a constraint satisfaction problem
    solutions is the list of all solutions to csp
    This tests whether the solution returned by CSP_solver is a solution.
    """
    print("Testing csp with",CSP_solver.__doc__)
    sol0 = CSP_solver(csp)
    print("Solution found:",sol0)
    assert sol0 in solutions, "Solution not correct for "+str(csp)
    print("Passed unit test")
```

Exercise 4.3 Modify *test* so that instead of taking in a list of solutions, it checks whether the returned solution actually is a solution.

Exercise 4.4 Propose a test that is appropriate for CSPs with no solutions. Assume that the test designer knows there are no solutions. Consider what a CSP solver should return if there are no solutions to the CSP.

Exercise 4.5 Write a unit test that checks whether all solutions (e.g., for the search algorithms that can return multiple solutions) are correct, and whether all solutions can be found.

4.2 A Simple Depth-first Solver

The first solver searches through the space of partial assignments. This takes in a CSP problem and an optional variable ordering, which is a list of the variables in the CSP. It returns a generator of the solutions (see Python documentation on `yield` for enumerations).

cspDFS.py — Solving a CSP using depth-first search.

```
11 from cspExamples import csp1,csp2,test_csp, crossword1, crossword1d
12
13 def dfs_solver(constraints, context, var_order):
14     """generator for all solutions to csp.
15     context is an assignment of values to some of the variables.
16     var_order is  a list of the variables in csp that are not in context.
17     """
18     to_eval = {c for c in constraints if c.can_evaluate(context)}
19     if all(c.holds(context) for c in to_eval):
20         if var_order == []:
21             yield context
22         else:
23             rem_cons = [c for c in constraints if c not in to_eval]
24             var = var_order[0]
25             for val in var.domain:
26                 yield from dfs_solver(rem_cons, context|{var:val},
                       var_order[1:])
27
28 def dfs_solve_all(csp, var_order=None):
29     """depth-first CSP solver to return a list of all solutions to csp.
30     """
31     if var_order == None:  # use an arbitrary variable order
32         var_order = list(csp.variables)
33     return list( dfs_solver(csp.constraints, {}, var_order))
34
35 def dfs_solve1(csp, var_order=None):
36     """depth-first CSP solver to find single solution or None if there are
           no solutions.
37     """
38     if var_order == None:  # use an arbitrary variable order
39         var_order = list(csp.variables)
40     gen = dfs_solver(csp.constraints, {}, var_order)
41     try:    # Python generators raise an exception if there are no more
           elements.
```

```
42          return next(gen)
43      except StopIteration:
44          return None
45
46  if __name__ == "__main__":
47      test_csp(dfs_solve1)
48
49  #Try:
50  # dfs_solve_all(csp1)
51  # dfs_solve_all(csp2)
52  # dfs_solve_all(crossword1)
53  # dfs_solve_all(crossword1d) # warning: may take a *very* long time!
```

Exercise 4.6 Instead of testing all constraints at every node, change it so each constraint is only tested when all of it variables are assigned. Given an elimination ordering, it is possible to determine when each constraint needs to be tested. Implement this. Hint: create a parallel list of sets of constraints, where at each position i in the list, the constraints at position i can be evaluated when the variable at position i has been assigned.

Exercise 4.7 Estimate how long `dfs_solve_all(crossword1d)` will take on your computer. To do this, reduce the number of variables that need to be assigned, so that the simplifies problem can be solved in a reasonable time (between 0.1 second and 10 seconds). This can be done by reducing the number of variables in `var_order`, as the program only splits on these. How much more time will it take if the number of variables is increased by 1? (Try it!) Then extrapolate to all of the variables. See Section 1.6.1 for how to time your code. Would making the code 100 times faster or using a computer 100 times faster help?

4.3 Converting CSPs to Search Problems

To run the demo, in folder "`aipython`", load "`cspSearch.py`", and copy and paste the example queries at the bottom of that file.

The next solver constructs a search space that can be solved using the search methods of the previous chapter. This takes in a CSP problem and an optional variable ordering, which is a list of the variables in the CSP. In this search space:

- A node is a *variable* : *value* dictionary which does not violate any constraints (so that dictionaries that violate any conmtratints are not added).
- An arc corresponds to an assignment of a value to the next variable. This assumes a static ordering; the next variable chosen to split does not depend on the context. If no variable ordering is given, this makes no attempt to choose a good ordering.

cspSearch.py — Representations of a Search Problem from a CSP.

```
from cspProblem import CSP, Constraint
from searchProblem import Arc, Search_problem
from utilities import dict_union

class Search_from_CSP(Search_problem):
    """A search problem directly from the CSP.

    A node is a variable:value dictionary"""
    def __init__(self, csp, variable_order=None):
        self.csp=csp
        if variable_order:
            assert set(variable_order) == set(csp.variables)
            assert len(variable_order) == len(csp.variables)
            self.variables = variable_order
        else:
            self.variables = list(csp.variables)

    def is_goal(self, node):
        """returns whether the current node is a goal for the search
        """
        return len(node)==len(self.csp.variables)

    def start_node(self):
        """returns the start node for the search
        """
        return {}
```

The *neighbors*(*node*) method uses the fact that the length of the node, which is the number of variables already assigned, is the index of the next variable to split on. Note that we do no need to check whether there are no more variables to split on, as the nodes are all consistent, by construction, and so when there are no more variables we have a solution, and so don't need the neighbours.

cspSearch.py — (continued)

```
    def neighbors(self, node):
        """returns a list of the neighboring nodes of node.
        """
        var = self.variables[len(node)] # the next variable
        res = []
        for val in var.domain:
            new_env = dict_union(node,{var:val}) #dictionary union
            if self.csp.consistent(new_env):
                res.append(Arc(node,new_env))
        return res
```

The unit tests relies on a solver. The following procedure creates a solver using search that can be tested.

cspSearch.py — (continued)

```
from cspExamples import csp1,csp2,test_csp, crossword1, crossword1d
```

```
50 from searchGeneric import Searcher
51
52 def solver_from_searcher(csp):
53     """depth-first search solver"""
54     path = Searcher(Search_from_CSP(csp)).search()
55     if path is not None:
56         return path.end()
57     else:
58         return None
59
60 if __name__ == "__main__":
61     test_csp(solver_from_searcher)
62
63 ## Test Solving CSPs with Search:
64 searcher1 = Searcher(Search_from_CSP(csp1))
65 #print(searcher1.search()) # get next solution
66 searcher2 = Searcher(Search_from_CSP(csp2))
67 #print(searcher2.search()) # get next solution
68 searcher3 = Searcher(Search_from_CSP(crossword1))
69 #print(searcher3.search()) # get next solution
70 searcher4 = Searcher(Search_from_CSP(crossword1d))
71 #print(searcher4.search()) # get next solution (warning: slow)
```

Exercise 4.8 What would happen if we constructed the new assignment by assigning $node[var] = val$ (with side effects) instead of using dictionary union? Give an example of where this could give a wrong answer. How could the algorithm be changed to work with side effects? (Hint: think about what information needs to be in a node).

Exercise 4.9 Change neighbors so that it returns an iterator of values rather than a list. (Hint: use *yield*.)

4.4 Consistency Algorithms

To run the demo, in folder "aipython", load "cspConsistency.py", and copy and paste the commented-out example queries at the bottom of that file.

A *Con_solver* is used to simplify a CSP using arc consistency.

cspConsistency.py — Arc Consistency and Domain splitting for solving a CSP

```
11 from display import Displayable
12
13 class Con_solver(Displayable):
14     """Solves a CSP with arc consistency and domain splitting
15     """
16     def __init__(self, csp, **kwargs):
17         """a CSP solver that uses arc consistency
18         * csp is the CSP to be solved
```

```
        * kwargs is the keyword arguments for Displayable superclass
        """
        self.csp = csp
        super().__init__(**kwargs) # Or Displayable.__init__(self,**kwargs)
```

The following implementation of arc consistency maintains the set *to_do* of (variable, constraint) pairs that are to be checked. It takes in a domain dictionary and returns a new domain dictionary. It needs to be careful to avoid side effects (by copying the *domains* dictionary and the *to_do* set).

cspConsistency.py — (continued)

```
    def make_arc_consistent(self, orig_domains=None, to_do=None):
        """Makes this CSP arc-consistent using generalized arc consistency
        orig_domains is the original domains
        to_do is a set of (variable,constraint) pairs
        returns the reduced domains (an arc-consistent variable:domain
            dictionary)
        """
        if orig_domains is None:
            orig_domains = {var:var.domain for var in self.csp.variables}
        if to_do is None:
            to_do = {(var, const) for const in self.csp.constraints
                     for var in const.scope}
        else:
            to_do = to_do.copy() # use a copy of to_do
        domains = orig_domains.copy()
        self.display(2,"Performing AC with domains", domains)
        while to_do:
            var, const = self.select_arc(to_do)
            self.display(3, "Processing arc (", var, ",", const, ")")
            other_vars = [ov for ov in const.scope if ov != var]
            new_domain = {val for val in domains[var]
                          if self.any_holds(domains, const, {var: val},
                              other_vars)}
            if new_domain != domains[var]:
                self.display(4, "Arc: (", var, ",", const, ") is
                    inconsistent")
                self.display(3, "Domain pruned", "dom(", var, ") =",
                    new_domain,
                              " due to ", const)
                domains[var] = new_domain
                add_to_do = self.new_to_do(var, const) - to_do
                to_do |= add_to_do    # set union
                self.display(3, " adding", add_to_do if add_to_do else
                    "nothing", "to to_do.")
            self.display(4, "Arc: (", var, ",", const, ") now consistent")
        self.display(2, "AC done. Reduced domains", domains)
        return domains

    def new_to_do(self, var, const):
        """returns new elements to be added to to_do after assigning
```

```
59          variable var in constraint const.
60          """
61          return {(nvar, nconst) for nconst in self.csp.var_to_const[var]
62                  if nconst != const
63                  for nvar in nconst.scope
64                  if nvar != var}
```

The following selects an arc. Any element of *to_do* can be selected. The selected element needs to be removed from *to_do*. The default implementation just selects which ever element *pop* method for sets returns. A user interface could allow the user to select an arc. Alternatively a more sophisticated selection could be employed (or just a stack or a queue).

cspConsistency.py — (continued)

```
66      def select_arc(self, to_do):
67          """Selects the arc to be taken from to_do .
68          * to_do is a set of arcs, where an arc is a (variable,constraint)
                pair
69          the element selected must be removed from to_do.
70          """
71          return to_do.pop()
```

The value of `new_domain` is the subset of the domain of `var` that is consistent with the assignment to the other variables. It might be easier to understand the following code, which treats unary (with no other variables in the constraint) and binary (with one other variables in the constraint) constraints as special cases (this can replace the assignment to `new_domain` in the above code):

```
if len(other_vars)==0:          # unary constraint
    new_domain = {val for val in domains[var]
                  if const.holds({var:val})}
elif len(other_vars)==1:        # binary constraint
    other = other_vars[0]
    new_domain = {val for val in domains[var]
           if any(const.holds({var: val,other:other_val})
                  for other_val in domains[other])}
else:                           # general case
    new_domain = {val for val in domains[var]
        if self.any_holds(domains, const, {var: val}, other_vars)}
```

any_holds is a recursive function that tries to finds an assignment of values to the other variables (*other_vars*) that satisfies constraint *const* given the assignment in *env*. The integer variable *ind* specifies which index to *other_vars* needs to be checked next. As soon as one assignment returns *True,* the algorithm returns *True*. Note that it has side effects with respect to *env*; it changes the values of the variables in *other_vars*. It should only be called when the side effects have no ill effects.

cspConsistency.py — (continued)

```
    def any_holds(self, domains, const, env, other_vars, ind=0):
        """returns True if Constraint const holds for an assignment
        that extends env with the variables in other_vars[ind:]
        env is a dictionary
        Warning: this has side effects and changes the elements of env
        """
        if ind == len(other_vars):
            return const.holds(env)
        else:
            var = other_vars[ind]
            for val in domains[var]:
                # env = dict_union(env,{var:val}) # no side effects!
                env[var] = val
                if self.any_holds(domains, const, env, other_vars, ind + 1):
                    return True
            return False
```

4.4.1 Direct Implementation of Domain Splitting

The following is a direct implementation of domain splitting with arc consistency that uses recursion. It finds one solution if one exists or returns False if there are no solutions.

cspConsistency.py — (continued)

```
    def solve_one(self, domains=None, to_do=None):
        """return a solution to the current CSP or False if there are no
            solutions
        to_do is the list of arcs to check
        """
        new_domains = self.make_arc_consistent(domains, to_do)
        if any(len(new_domains[var]) == 0 for var in new_domains):
            return False
        elif all(len(new_domains[var]) == 1 for var in new_domains):
            self.display(2, "solution:", {var: select(
                new_domains[var]) for var in new_domains})
            return {var: select(new_domains[var]) for var in new_domains}
        else:
            var = self.select_var(x for x in self.csp.variables if
                len(new_domains[x]) > 1)
            if var:
                dom1, dom2 = partition_domain(new_domains[var])
                self.display(3, "...splitting", var, "into", dom1, "and",
                    dom2)
                new_doms1 = copy_with_assign(new_domains, var, dom1)
                new_doms2 = copy_with_assign(new_domains, var, dom2)
                to_do = self.new_to_do(var, None)
                self.display(3, " adding", to_do if to_do else "nothing",
                    "to to_do.")
                return self.solve_one(new_doms1, to_do) or
                    self.solve_one(new_doms2, to_do)
```

```
111
112     def select_var(self, iter_vars):
113         """return the next variable to split"""
114         return select(iter_vars)
115
116 def partition_domain(dom):
117     """partitions domain dom into two.
118     """
119     split = len(dom) // 2
120     dom1 = set(list(dom)[:split])
121     dom2 = dom - dom1
122     return dom1, dom2
```

The domains are implemented as a dictionary that maps each variables to its domain. Assigning a value in Python has side effects which we want to avoid. *copy_with_assign* takes a copy of the domains dictionary, perhaps allowing for a new domain for a variable. It creates a copy of the CSP with an (optional) assignment of a new domain to a variable. Only the domains are copied.

cspConsistency.py — (continued)

```
124 def copy_with_assign(domains, var=None, new_domain={True, False}):
125     """create a copy of the domains with an assignment var=new_domain
126     if var==None then it is just a copy.
127     """
128     newdoms = domains.copy()
129     if var is not None:
130         newdoms[var] = new_domain
131     return newdoms
```

cspConsistency.py — (continued)

```
133 def select(iterable):
134     """select an element of iterable. Returns None if there is no such
            element.
135
136     This implementation just picks the first element.
137     For many of the uses, which element is selected does not affect
            correctness,
138     but may affect efficiency.
139     """
140     for e in iterable:
141         return e # returns first element found
```

Exercise 4.10 Implement of *solve_all* that is like *solve_one* but returns the set of all solutions.

Exercise 4.11 Implement *solve_enum* that enumerates the solutions. It should use Python's *yield* (and perhaps *yield from*).

Unit test:

cspConsistency.py — (continued)

```
from cspExamples import test_csp
def ac_solver(csp):
    "arc consistency (solve_one)"
    return Con_solver(csp).solve_one()

if __name__ == "__main__":
    test_csp(ac_solver)
```

4.4.2 Domain Splitting as an interface to graph searching

An alternative implementation is to implement domain splitting in terms of the search abstraction of Chapter 3.

A node is domains dictionary.

cspConsistency.py — (continued)

```
from searchProblem import Arc, Search_problem

class Search_with_AC_from_CSP(Search_problem,Displayable):
    """A search problem with arc consistency and domain splitting

    A node is a CSP """
    def __init__(self, csp):
        self.cons = Con_solver(csp) #copy of the CSP
        self.domains = self.cons.make_arc_consistent()

    def is_goal(self, node):
        """node is a goal if all domains have 1 element"""
        return all(len(node[var])==1 for var in node)

    def start_node(self):
        return self.domains

    def neighbors(self,node):
        """returns the neighboring nodes of node.
        """
        neighs = []
        var = select(x for x in node if len(node[x])>1)
        if var:
            dom1, dom2 = partition_domain(node[var])
            self.display(2,"Splitting", var, "into", dom1, "and", dom2)
            to_do = self.cons.new_to_do(var,None)
            for dom in [dom1,dom2]:
                newdoms = copy_with_assign(node,var,dom)
                cons_doms = self.cons.make_arc_consistent(newdoms,to_do)
                if all(len(cons_doms[v])>0 for v in cons_doms):
                    # all domains are non-empty
                    neighs.append(Arc(node,cons_doms))
                else:
```

```
                    self.display(2,"...",var,"in",dom,"has no solution")
        return neighs
```

Exercise 4.12 When splitting a domain, this code splits the domain into half, approximately in half (without any effort to make a sensible choice). Does it work better to split one element from a domain?

Unit test:

cspConsistency.py — (continued)

```
from cspExamples import test_csp
from searchGeneric import Searcher

def ac_search_solver(csp):
    """arc consistency (search interface)"""
    sol = Searcher(Search_with_AC_from_CSP(csp)).search()
    if sol:
        return {v:select(d) for (v,d) in sol.end().items()}

if __name__ == "__main__":
    test_csp(ac_search_solver)
```

Testing:

cspConsistency.py — (continued)

```
from cspExamples import csp1, csp2, csp3, csp4, crossword1, crossword1d

## Test Solving CSPs with Arc consistency and domain splitting:
#Con_solver.max_display_level = 4 # display details of AC (0 turns off)
#Con_solver(csp1).solve_one()
#searcher1d = Searcher(Search_with_AC_from_CSP(csp1))
#print(searcher1d.search())
#Searcher.max_display_level = 2 # display search trace (0 turns off)
#searcher2c = Searcher(Search_with_AC_from_CSP(csp2))
#print(searcher2c.search())
#searcher3c = Searcher(Search_with_AC_from_CSP(crossword1))
#print(searcher3c.search())
#searcher4c = Searcher(Search_with_AC_from_CSP(crossword1d))
#print(searcher4c.search())
```

4.5 Solving CSPs using Stochastic Local Search

To run the demo, in folder "aipython", load "cspSLS.py", and copy and paste the commented-out example queries at the bottom of that file. This assumes Python 3. Some of the queries require matplotlib.

This implements both the two-stage choice, the any-conflict algorithm and a random choice of variable (and a probabilistic mix of the three).

Given a CSP, the stochastic local searcher (*SLSearcher*) creates the data structures:

- *variables_to_select* is the set of all of the variables with domain-size greater than one. For a variable not in this set, we cannot pick another value from that variable.
- *var_to_constraints* maps from a variable into the set of constraints it is involved in. Note that the inverse mapping from constraints into variables is part of the definition of a constraint.

cspSLS.py — Stochastic Local Search for Solving CSPs

```
from cspProblem import CSP, Constraint
from searchProblem import Arc, Search_problem
from display import Displayable
import random
import heapq

class SLSearcher(Displayable):
    """A search problem directly from the CSP..

    A node is a variable:value dictionary"""
    def __init__(self, csp):
        self.csp = csp
        self.variables_to_select = {var for var in self.csp.variables
                                    if len(var.domain) > 1}
        # Create assignment and conflicts set
        self.current_assignment = None # this will trigger a random restart
        self.number_of_steps = 0 #number of steps after the initialization
```

restart creates a new total assignment, and constructs the set of conflicts (the constraints that are false in this assignment).

cspSLS.py — (continued)

```
    def restart(self):
        """creates a new total assignment and the conflict set
        """
        self.current_assignment = {var:random_choice(var.domain) for
                                   var in self.csp.variables}
        self.display(2,"Initial assignment",self.current_assignment)
        self.conflicts = set()
        for con in self.csp.constraints:
            if not con.holds(self.current_assignment):
                self.conflicts.add(con)
        self.display(2,"Number of conflicts",len(self.conflicts))
        self.variable_pq = None
```

The *search* method is the top-level searching algorithm. It can either be used to start the search or to continue searching. If there is no current assignment,

it must create one. Note that, when counting steps, a restart is counted as one step.

This method selects one of two implementations. The argument *pob_best* is the probability of selecting a best variable (one involving the most conflicts). When the value of *prob_best* is positive, the algorithm needs to maintain a priority queue of variables and the number of conflicts (using *search_with_var_pq*). If the probability of selecting a best variable is zero, it does not need to maintain this priority queue (as implemented in *search_with_any_conflict*).

The argument *prob_anycon* is the probability that the any-conflict strategy is used (which selects a variable at random that is in a conflict), assuming that it is not picking a best variable. Note that for the probability parameters, any value less that zero acts like probability zero and any value greater than 1 acts like probability 1. This means that when $prob\_anycon = 1.0$, a best variable is chosen with probability *prob_best*, otherwise a variable in any conflict is chosen. A variable is chosen at random with probability $1 - prob\_anycon - prob\_best$ as long as that is positive.

This returns the number of steps needed to find a solution, or *None* if no solution is found. If there is a solution, it is in *self.current_assignment*.

cspSLS.py — (continued)

```
42    def search(self,max_steps, prob_best=0, prob_anycon=1.0):
43        """
44        returns the number of steps or None if these is no solution.
45        If there is a solution, it can be found in self.current_assignment
46
47        max_steps is the maximum number of steps it will try before giving
              up
48        prob_best is the probability that a best variable (one in most
              conflict) is selected
49        prob_anycon is the probability that a variable in any conflict is
              selected
50        (otherwise a variable is chosen at random)
51        """
52        if self.current_assignment is None:
53            self.restart()
54            self.number_of_steps += 1
55            if not self.conflicts:
56                self.display(1,"Solution found:", self.current_assignment,
                      "after restart")
57                return self.number_of_steps
58        if prob_best > 0: # we need to maintain a variable priority queue
59            return self.search_with_var_pq(max_steps, prob_best,
                  prob_anycon)
60        else:
61            return self.search_with_any_conflict(max_steps, prob_anycon)
```

Exercise 4.13 This does an initial random assignment but does not do any random restarts. Implement a searcher that takes in the maximum number of walk

steps (corresponding to existing *max_steps*) and the maximum number of restarts, and returns the total number of steps for the first solution found. (As in *search*, the solution found can be extracted from the variable *self.current_assignment*).

4.5.1 Any-conflict

If the probability of picking a best variable is zero, the implementation need to keeps track of which variables are in conflicts.

cspSLS.py — (continued)

```
63    def search_with_any_conflict(self, max_steps, prob_anycon=1.0):
64        """Searches with the any_conflict heuristic.
65        This relies on just maintaining the set of conflicts;
66        it does not maintain a priority queue
67        """
68        self.variable_pq = None # we are not maintaining the priority queue.
69                                # This ensures it is regenerated if
70                                #   we call search_with_var_pq.
71        for i in range(max_steps):
72            self.number_of_steps +=1
73            if  random.random() < prob_anycon:
74                con = random_choice(self.conflicts) # pick random conflict
75                var = random_choice(con.scope) # pick variable in conflict
76            else:
77                var = random_choice(self.variables_to_select)
78            if len(var.domain) > 1:
79                val = random_choice([val for val in var.domain
80                                     if val is not
                                         self.current_assignment[var]])
81                self.display(2,self.number_of_steps,":
                      Assigning",var,"=",val)
82                self.current_assignment[var]=val
83                for varcon in self.csp.var_to_const[var]:
84                    if varcon.holds(self.current_assignment):
85                        if varcon in self.conflicts:
86                            self.conflicts.remove(varcon)
87                    else:
88                        if varcon not in self.conflicts:
89                            self.conflicts.add(varcon)
90                self.display(2,"   Number of conflicts",len(self.conflicts))
91            if not self.conflicts:
92                self.display(1,"Solution found:", self.current_assignment,
93                               "in", self.number_of_steps,"steps")
94                return self.number_of_steps
95        self.display(1,"No solution in",self.number_of_steps,"steps",
96                    len(self.conflicts),"conflicts remain")
97        return None
```

Exercise 4.14 This makes no attempt to find the best alternative value for a variable. Modify the code so that after selecting a variable it selects a value the reduces

the number of conflicts by the most. Have a parameter that specifies the probability that the best value is chosen.

4.5.2 Two-Stage Choice

This is the top-level searching algorithm that maintains a priority queue of variables ordered by (the negative of) the number of conflicts, so that the variable with the most conflicts is selected first. If there is no current priority queue of variables, one is created.

The main complexity here is to maintain the priority queue. This uses the dictionary *var_differential* which specifies how much the values of variables should change. This is used with the updatable queue (page 79) to find a variable with the most conflicts.

cspSLS.py — (continued)

```
99      def search_with_var_pq(self,max_steps, prob_best=1.0, prob_anycon=1.0):
100         """search with a priority queue of variables.
101         This is used to select a variable with the most conflicts.
102         """
103         if not self.variable_pq:
104             self.create_pq()
105         pick_best_or_con = prob_best + prob_anycon
106         for i in range(max_steps):
107             self.number_of_steps +=1
108             randnum = random.random()
109             ## Pick a variable
110             if randnum < prob_best: # pick best variable
111                 var,oldval = self.variable_pq.top()
112             elif randnum < pick_best_or_con: # pick a variable in a conflict
113                 con = random_choice(self.conflicts)
114                 var = random_choice(con.scope)
115             else: #pick any variable that can be selected
116                 var = random_choice(self.variables_to_select)
117             if len(var.domain) > 1: # var has other values
118                 ## Pick a value
119                 val = random_choice([val for val in var.domain if val is not
120                                      self.current_assignment[var]])
121                 self.display(2,"Assigning",var,val)
122                 ## Update the priority queue
123                 var_differential = {}
124                 self.current_assignment[var]=val
125                 for varcon in self.csp.var_to_const[var]:
126                     self.display(3,"Checking",varcon)
127                     if varcon.holds(self.current_assignment):
128                         if varcon in self.conflicts: #was incons, now consis
129                             self.display(3,"Became consistent",varcon)
130                             self.conflicts.remove(varcon)
131                             for v in varcon.scope: # v is in one fewer
                                    conflicts
```

```
                            var_differential[v] =
                                var_differential.get(v,0)-1
                    else:
                        if varcon not in self.conflicts: # was consis, not now
                            self.display(3,"Became inconsistent",varcon)
                            self.conflicts.add(varcon)
                            for v in varcon.scope: # v is in one more
                                conflicts
                                var_differential[v] =
                                    var_differential.get(v,0)+1
                self.variable_pq.update_each_priority(var_differential)
                self.display(2,"Number of conflicts",len(self.conflicts))
            if not self.conflicts: # no conflicts, so solution found
                self.display(1,"Solution found:",
                    self.current_assignment,"in",
                            self.number_of_steps,"steps")
                return self.number_of_steps
        self.display(1,"No solution in",self.number_of_steps,"steps",
                    len(self.conflicts),"conflicts remain")
        return None
```

create_pq creates an updatable priority queue of the variables, ordered by the number of conflicts they participate in. The priority queue only includes variables in conflicts and the value of a variable is the *negative* of the number of conflicts the variable is in. This ensures that the priority queue, which picks the minimum value, picks a variable with the most conflicts.

cspSLS.py — (continued)

```
    def create_pq(self):
        """Create the variable to number-of-conflicts priority queue.
        This is needed to select the variable in the most conflicts.

        The value of a variable in the priority queue is the negative of the
        number of conflicts the variable appears in.
        """
        self.variable_pq = Updatable_priority_queue()
        var_to_number_conflicts = {}
        for con in self.conflicts:
            for var in con.scope:
                var_to_number_conflicts[var] =
                    var_to_number_conflicts.get(var,0)+1
        for var,num in var_to_number_conflicts.items():
            if num>0:
                self.variable_pq.add(var,-num)
```

cspSLS.py — (continued)

```
def random_choice(st):
    """selects a random element from set st.
    It will be more efficient to convert to a tuple or list only once."""
    return random.choice(tuple(st))
```

Exercise 4.15 This makes no attempt to find the best alternative value for a variable. Modify the code so that after selecting a variable it selects a value the reduces the number of conflicts by the most. Have a parameter that specifies the probability that the best value is chosen.

Exercise 4.16 These implementations always select a value for the variable selected that is different from its current value (if that is possible). Change the code so that it does not have this restriction (so it can leave the value the same). Would you expect this code to be faster? Does it work worse (or better)?

4.5.3 Updatable Priority Queues

An **updatable priority queue** is a priority queue, where key-value pairs can be stored, and the pair with the smallest key can be found and removed quickly, and where the values can be updated. This implementation follows the idea of `http://docs.python.org/3.5/library/heapq.html`, where the updated elements are marked as removed. This means that the priority queue can be used unmodified. However, this might be expensive if changes are more common than popping (as might happen if the probability of choosing the best is close to zero).

In this implementation, the equal values are sorted randomly. This is achieved by having the elements of the heap being $[val, rand, elt]$ triples, where the second element is a random number. Note that Python requires this to be a list, not a tuple, as the tuple cannot be modified.

cspSLS.py — (continued)

```
170 class Updatable_priority_queue(object):
171     """A priority queue where the values can be updated.
172     Elements with the same value are ordered randomly.
173 
174     This code is based on the ideas described in
175     http://docs.python.org/3.3/library/heapq.html
176     It could probably be done more efficiently by
177     shuffling the modified element in the heap.
178     """
179     def __init__(self):
180         self.pq = []  # priority queue of [val,rand,elt] triples
181         self.elt_map = {} # map from elt to [val,rand,elt] triple in pq
182         self.REMOVED = "*removed*" # a string that won't be a legal element
183         self.max_size=0
184 
185     def add(self,elt,val):
186         """adds elt to the priority queue with priority=val.
187         """
188         assert val <= 0,val
189         assert elt not in self.elt_map, elt
190         new_triple = [val, random.random(),elt]
191         heapq.heappush(self.pq, new_triple)
192         self.elt_map[elt] = new_triple
```

```
193
194     def remove(self,elt):
195         """remove the element from the priority queue"""
196         if elt in self.elt_map:
197             self.elt_map[elt][2] = self.REMOVED
198             del self.elt_map[elt]
199
200     def update_each_priority(self,update_dict):
201         """update values in the priority queue by subtracting the values in
202         update_dict from the priority of those elements in priority queue.
203         """
204         for elt,incr in update_dict.items():
205             if incr != 0:
206                 newval = self.elt_map.get(elt,[0])[0] - incr
207                 assert newval <= 0,
                        str(elt)+":"+str(newval+incr)+"-"+str(incr)
208                 self.remove(elt)
209                 if newval != 0:
210                     self.add(elt,newval)
211
212     def pop(self):
213         """Removes and returns the (elt,value) pair with minimal value.
214         If the priority queue is empty, IndexError is raised.
215         """
216         self.max_size = max(self.max_size, len(self.pq)) # keep statistics
217         triple = heapq.heappop(self.pq)
218         while triple[2] == self.REMOVED:
219             triple = heapq.heappop(self.pq)
220         del self.elt_map[triple[2]]
221         return triple[2], triple[0] # elt, value
222
223     def top(self):
224         """Returns the (elt,value) pair with minimal value, without
                removing it.
225         If the priority queue is empty, IndexError is raised.
226         """
227         self.max_size = max(self.max_size, len(self.pq)) # keep statistics
228         triple = self.pq[0]
229         while triple[2] == self.REMOVED:
230             heapq.heappop(self.pq)
231             triple = self.pq[0]
232         return triple[2], triple[0] # elt, value
233
234     def empty(self):
235         """returns True iff the priority queue is empty"""
236         return all(triple[2] == self.REMOVED for triple in self.pq)
```

4.5.4 Plotting Runtime Distributions

Runtime_distribution uses matplotlib to plot runtime distributions. Here the runtime is a misnomer as we are only plotting the number of steps, not the time. Computing the runtime is non-trivial as many of the runs have a very short runtime. To compute the time accurately would require running the same code, with the same random seed, multiple times to get a good estimate of the runtime. This is left as an exercise.

cspSLS.py — (continued)

```
238 import matplotlib.pyplot as plt
239
240 class Runtime_distribution(object):
241     def __init__(self, csp, xscale='log'):
242         """Sets up plotting for csp
243         xscale is either 'linear' or 'log'
244         """
245         self.csp = csp
246         plt.ion()
247         plt.xlabel("Number of Steps")
248         plt.ylabel("Cumulative Number of Runs")
249         plt.xscale(xscale) # Makes a 'log' or 'linear' scale
250
251     def plot_runs(self,num_runs=100,max_steps=1000, prob_best=1.0,
            prob_anycon=1.0):
252         """Plots num_runs of SLS for the given settings.
253         """
254         stats = []
255         SLSearcher.max_display_level, temp_mdl = 0,
                SLSearcher.max_display_level # no display
256         for i in range(num_runs):
257             searcher = SLSearcher(self.csp)
258             num_steps = searcher.search(max_steps, prob_best, prob_anycon)
259             if num_steps:
260                 stats.append(num_steps)
261         stats.sort()
262         if prob_best >= 1.0:
263             label = "P(best)=1.0"
264         else:
265             p_ac = min(prob_anycon, 1-prob_best)
266             label = "P(best)=%.2f, P(ac)=%.2f" % (prob_best, p_ac)
267         plt.plot(stats,range(len(stats)),label=label)
268         plt.legend(loc="upper left")
269         #plt.draw()
270         SLSearcher.max_display_level= temp_mdl #restore display
```

4.5.5 Testing

cspSLS.py — (continued)

```
from cspExamples import test_csp
def sls_solver(csp,prob_best=0.7):
    """stochastic local searcher (prob_best=0.7)"""
    se0 = SLSearcher(csp)
    se0.search(1000,prob_best)
    return se0.current_assignment
def any_conflict_solver(csp):
    """stochastic local searcher (any-conflict)"""
    return sls_solver(csp,0)

if __name__ == "__main__":
    test_csp(sls_solver)
    test_csp(any_conflict_solver)

from cspExamples import csp1, csp2, crossword1, crossword1d

## Test Solving CSPs with Search:
#se1 = SLSearcher(csp1); print(se1.search(100))
#se2 = SLSearcher(csp2); print(se2.search(1000,1.0)) # greedy
#se2 = SLSearcher(csp2); print(se2.search(1000,0)) # any_conflict
#se2 = SLSearcher(csp2); print(se2.search(1000,0.7)) # 70% greedy; 30%
    any_conflict
#SLSearcher.max_display_level=2 #more detailed display
#se3 = SLSearcher(crossword1); print(se3.search(100),0.7)
#p = Runtime_distribution(csp2)
#p.plot_runs(1000,1000,0) # any_conflict
#p.plot_runs(1000,1000,1.0) # greedy
#p.plot_runs(1000,1000,0.7) # 70% greedy; 30% any_conflict
```

Exercise 4.17 Modify this to plot the runtime, instead of the number of steps. To measure runtime use *timeit* (`https://docs.python.org/3.5/library/timeit.html`). Small runtimes are inaccurate, so timeit can run the same code multiple times. Stochastic local algorithms give different runtimes each time called. To make the timing meaningful, you need to make sure the random seed is the same for each repeated call (see `random.getstate` and `random.setstate` in `https://docs.python.org/3.5/library/random.html`). Because the runtime for different seeds can vary a great deal, for each seed, you should start with 1 iteration and multiplying it by, say 10, until the time is greater than 0.2 seconds. Make sure you plot the average time for each run. Before you start, try to estimate the total runtime, so you will be able to tell if there is a problem with the algorithm stopping.

4.6 Discrete Optimization

A `SoftConstraint` is a constraint, but where the condition is a real-valued function. Because we did not force the condition to be Boolean, we can makejust reuse the `Constraint` class.

cspSoft.py — Representations of Soft Constraints

```
from cspProblem import Variable, Constraint, CSP
```

```
class SoftConstraint(Constraint):
    """A Constraint consists of
    * scope: a tuple of variables
    * function: a real-valued function that can applied to a tuple of values
    * string: a string for printing the constraints. All of the strings
        must be unique.
    for the variables
    """
    def __init__(self, scope, function, string=None, position=None):
        Constraint.__init__(self, scope, function, string, position)

    def value(self,assignment):
        return self.holds(assignment)
```

cspSoft.py — (continued)

```
A = Variable('A', {1,2}, position=(0.2,0.9))
B = Variable('B', {1,2,3}, position=(0.8,0.9))
C = Variable('C', {1,2}, position=(0.5,0.5))
D = Variable('D', {1,2}, position=(0.8,0.1))

def c1fun(a,b):
    if a==1: return (5 if b==1 else 2)
    else: return (0 if b==1 else 4 if b==2 else 3)
c1 = SoftConstraint([A,B],c1fun,"c1")
def c2fun(b,c):
    if b==1: return (5 if c==1 else 2)
    elif b==2: return (0 if c==1 else 4)
    else: return (2 if c==1 else 0)
c2 = SoftConstraint([B,C],c2fun,"c2")
def c3fun(b,d):
    if b==1: return (3 if d==1 else 0)
    elif b==2: return 2
    else: return (2 if d==1 else 4)
c3 = SoftConstraint([B,D],c3fun,"c3")

def penalty_if_same(pen):
    "returns a function that gives a penalty of pen if the arguments are
        the same"
    return lambda x,y: (pen if (x==y) else 0)

c4 = SoftConstraint([C,A],penalty_if_same(3),"c4")

scsp1 = CSP("scsp1", {A,B,C,D}, [c1,c2,c3,c4])

### The second soft CSP has an extra variable, and 2 constraints
E = Variable('E', {1,2}, position=(0.1,0.1))

c5 = SoftConstraint([C,E],penalty_if_same(3),"c5")
c6 = SoftConstraint([D,E],penalty_if_same(2),"c6")
scsp2 = CSP("scsp1", {A,B,C,D,E}, [c1,c2,c3,c4,c5,c6])
```

4.6.1 Branch-and-bound Search

Here we specialize the branch-and-bound algorithm (Section 3.3 on page 47).

cspSoft.py — (continued)

```
60 from display import Displayable, visualize
61 import math
62
63 class DF_branch_and_bound_opt(Displayable):
64     """returns a branch and bound searcher for a problem.
65     An optimal assignment with cost less than bound can be found by calling
           search()
66     """
67     def __init__(self, csp, bound=math.inf):
68         """creates a searcher than can be used with search() to find an
               optimal path.
69         bound gives the initial bound. By default this is infinite -
               meaning there
70         is no initial pruning due to depth bound
71         """
72         super().__init__()
73         self.csp = csp
74         self.best_asst = None
75         self.bound = bound
76
77     def optimize(self):
78         """returns an optimal solution to a problem with cost less than
               bound.
79         returns None if there is no solution with cost less than bound."""
80         self.num_expanded=0
81         self.cbsearch({}, 0, self.csp.constraints)
82         self.display(1,"Number of paths expanded:",self.num_expanded)
83         return self.best_asst, self.bound
84
85     def cbsearch(self, asst, cost, constraints):
86         """finds the optimal solution that extends path and is less the
               bound"""
87         self.display(2,"cbsearch:",asst,cost,constraints)
88         can_eval = [c for c in constraints if c.can_evaluate(asst)]
89         rem_cons = [c for c in constraints if c not in can_eval]
90         newcost = cost + sum(c.value(asst) for c in can_eval)
91         self.display(2,"Evaluaing:",can_eval,"cost:",newcost)
92         if newcost < self.bound:
93             self.num_expanded += 1
94             if rem_cons==[]:
95                 self.best_asst = asst
96                 self.bound = newcost
97                 self.display(1,"New best assignment:",asst," cost:",newcost)
98             else:
99                 var = next(var for var in self.csp.variables if var not in
                       asst)
```

```
100                 for val in var.domain:
101                     self.cbsearch({var:val}|asst, newcost, rem_cons)
102
103 # bnb = DF_branch_and_bound_opt(scsp1)
104 # bnb.max_display_level=3 # show more detail
105 # bnb.optimize()
```

Chapter 5

Propositions and Inference

5.1 Representing Knowledge Bases

A clause consists of a head (an atom) and a body. A body is represented as a list of atoms. Atoms are represented as strings.

logicProblem.py — Representations Logics

```
11 class Clause(object):
12     """A definite clause"""
13
14     def __init__(self,head,body=[]):
15         """clause with atom head and lost of atoms body"""
16         self.head=head
17         self.body = body
18
19     def __str__(self):
20         """returns the string representation of a clause.
21         """
22         if self.body:
23             return self.head + " <- " + " & ".join(self.body) + "."
24         else:
25             return self.head + "."
```

An askable atom can be asked of the user. The user can respond in English or French or just with a "y".

logicProblem.py — (continued)

```
27 class Askable(object):
28     """An askable atom"""
29
30     def __init__(self,atom):
31         """clause with atom head and lost of atoms body"""
```

```
32          self.atom=atom
33
34      def __str__(self):
35          """returns the string representation of a clause."""
36          return "askable " + self.atom + "."
37
38  def yes(ans):
39      """returns true if the answer is yes in some form"""
40      return ans.lower() in ['yes', 'yes.', 'oui', 'oui.', 'y', 'y.'] #
            bilingual
```

A knowledge base is a list of clauses and askables. In order to make top-down inference faster, this creates a dictionary that maps each atoms into the set of clauses with that atom in the head.

logicProblem.py — (continued)

```
42  from display import Displayable
43
44  class KB(Displayable):
45      """A knowledge base consists of a set of clauses.
46      This also creates a dictionary to give fast access to the clauses with
            an atom in head.
47      """
48      def __init__(self, statements=[]):
49          self.statements = statements
50          self.clauses = [c for c in statements if isinstance(c, Clause)]
51          self.askables = [c.atom for c in statements if isinstance(c,
                Askable)]
52          self.atom_to_clauses = {} # dictionary giving clauses with atom as
                head
53          for c in self.clauses:
54              if c.head in self.atom_to_clauses:
55                  self.atom_to_clauses[c.head].add(c)
56              else:
57                  self.atom_to_clauses[c.head] = {c}
58
59      def clauses_for_atom(self,a):
60          """returns set of clauses with atom a as the head"""
61          if a in self.atom_to_clauses:
62              return self.atom_to_clauses[a]
63          else:
64              return set()
65
66      def __str__(self):
67          """returns a string representation of this knowledge base.
68          """
69          return '\n'.join([str(c) for c in self.statements])
```

Here is a trivial example (I think therefore I am) using in the unit tests:

logicProblem.py — (continued)

```
71  triv_KB = KB([
```

```
72      Clause('i_am', ['i_think']),
73      Clause('i_think'),
74      Clause('i_smell', ['i_exist'])
75      ])
```

Here is a representation of the electrical domain of the textbook:

logicProblem.py — (continued)

```
77  elect = KB([
78      Clause('light_l1'),
79      Clause('light_l2'),
80      Clause('ok_l1'),
81      Clause('ok_l2'),
82      Clause('ok_cb1'),
83      Clause('ok_cb2'),
84      Clause('live_outside'),
85      Clause('live_l1', ['live_w0']),
86      Clause('live_w0', ['up_s2','live_w1']),
87      Clause('live_w0', ['down_s2','live_w2']),
88      Clause('live_w1', ['up_s1', 'live_w3']),
89      Clause('live_w2', ['down_s1','live_w3' ]),
90      Clause('live_l2', ['live_w4']),
91      Clause('live_w4', ['up_s3','live_w3' ]),
92      Clause('live_p_1', ['live_w3']),
93      Clause('live_w3', ['live_w5', 'ok_cb1']),
94      Clause('live_p_2', ['live_w6']),
95      Clause('live_w6', ['live_w5', 'ok_cb2']),
96      Clause('live_w5', ['live_outside']),
97      Clause('lit_l1', ['light_l1', 'live_l1', 'ok_l1']),
98      Clause('lit_l2', ['light_l2', 'live_l2', 'ok_l2']),
99      Askable('up_s1'),
100     Askable('down_s1'),
101     Askable('up_s2'),
102     Askable('down_s2'),
103     Askable('up_s3'),
104     Askable('down_s2')
105     ])
106
107 # print(kb)
```

The following knowledge base is false of the intended interpretation. One of the clauses is wrong; can you see which one? We will show how to debug it.

logicProblem.py — (continued)

```
108 elect_bug = KB([
109     Clause('light_l2'),
110     Clause('ok_l1'),
111     Clause('ok_l2'),
112     Clause('ok_cb1'),
113     Clause('ok_cb2'),
114     Clause('live_outside'),
115     Clause('live_p_2', ['live_w6']),
```

```
116     Clause('live_w6', ['live_w5', 'ok_cb2']),
117     Clause('light_l1'),
118     Clause('live_w5', ['live_outside']),
119     Clause('lit_l1', ['light_l1', 'live_l1', 'ok_l1']),
120     Clause('lit_l2', ['light_l2', 'live_l2', 'ok_l2']),
121     Clause('live_l1', ['live_w0']),
122     Clause('live_w0', ['up_s2','live_w1']),
123     Clause('live_w0', ['down_s2','live_w2']),
124     Clause('live_w1', ['up_s3', 'live_w3']),
125     Clause('live_w2', ['down_s1','live_w3' ]),
126     Clause('live_l2', ['live_w4']),
127     Clause('live_w4', ['up_s3','live_w3' ]),
128     Clause('live_p_1', ['live_w3']),
129     Clause('live_w3', ['live_w5', 'ok_cb1']),
130     Askable('up_s1'),
131     Askable('down_s1'),
132     Askable('up_s2'),
133     Clause('light_l2'),
134     Clause('ok_l1'),
135     Clause('light_l2'),
136     Clause('ok_l1'),
137     Clause('ok_l2'),
138     Clause('ok_cb1'),
139     Clause('ok_cb2'),
140     Clause('live_outside'),
141     Clause('live_p_2', ['live_w6']),
142     Clause('live_w6', ['live_w5', 'ok_cb2']),
143     Clause('ok_l2'),
144     Clause('ok_cb1'),
145     Clause('ok_cb2'),
146     Clause('live_outside'),
147     Clause('live_p_2', ['live_w6']),
148     Clause('live_w6', ['live_w5', 'ok_cb2']),
149     Askable('down_s2'),
150     Askable('up_s3'),
151     Askable('down_s2')
152     ])
153
154 # print(kb)
```

5.2 Bottom-up Proofs (with askables)

fixed_point computes the fixed point of the knowledge base *kb*.

logicBottomUp.py — Bottom-up Proof Procedure for Definite Clauses

```
11 from logicProblem import yes
12
13 def fixed_point(kb):
14     """Returns the fixed point of knowledge base kb.
```

```
    """
    fp = ask_askables(kb)
    added = True
    while added:
        added = False  # added is true when an atom was added to fp this
            iteration
        for c in kb.clauses:
            if c.head not in fp and all(b in fp for b in c.body):
                fp.add(c.head)
                added = True
                kb.display(2,c.head,"added to fp due to clause",c)
    return fp

def ask_askables(kb):
    return {at for at in kb.askables if yes(input("Is "+at+" true? "))}
```

The following provides a trivial **unit test**, by default using the knowledge base `triv_KB`:

logicBottomUp.py — (continued)

```
from logicProblem import triv_KB
def test(kb=triv_KB, fixedpt = {'i_am','i_think'}):
    fp = fixed_point(kb)
    assert fp == fixedpt, "kb gave result "+str(fp)
    print("Passed unit test")
if __name__ == "__main__":
    test()

from logicProblem import elect
# elect.max_display_level=3 # give detailed trace
# fixed_point(elect)
```

Exercise 5.1 It is not very user-friendly to ask all of the askables up-front. Implement ask-the-user so that questions are only asked if useful, and are not re-asked. For example, if there is a clause $h \leftarrow a \wedge b \wedge c \wedge d \wedge e$, where c and e are askable, c and e only need to be asked if a, b, d are all in fp and they have not been asked before. Askable e only needs to be asked if the user says "yes" to c. Askable c doesn't need to be asked if the user previously replied "no" to e.

This form of ask-the-user can ask a different set of questions than the top-down interpreter that asks questions when encountered. Give an example where they ask different questions (neither set of questions asked is a subset of the other).

Exercise 5.2 This algorithm runs in time $O(n^2)$, where n is the number of clauses, for a bounded number of elements in the body; each iteration goes through each of the clauses, and in the worst case, it will do an iteration for each clause. It is possible to implement this in time $O(n)$ time by creating an index that maps an atom to the set of clauses with that atom in the body. Implement this. What is its complexity as a function of n and b, the maximum number of atoms in the body of a clause?

Exercise 5.3 It is possible to be asymptotically more efficient (in terms of the number of elements in a body) than the method in the previous question by noticing that each element of the body of clause only needs to be checked once. For example, the clause $a \leftarrow b \wedge c \wedge d$, needs only be considered when b is added to fp. Once b is added to fp, if c is already in pf, we know that a can be added as soon as d is added. Implement this. What is its complexity as a function of n and b, the maximum number of atoms in the body of a clause?

5.3 Top-down Proofs (with askables)

prove$(kb, goal)$ is used to prove *goal* from a knowledge base, *kb*, where a *goal* is a list of atoms. It returns *True* if $kb \vdash goal$. The *indent* is used when displaying the code (and doesn't need to have a non-default value).

logicTopDown.py — Top-down Proof Procedure for Definite Clauses

```
from logicProblem import yes

def prove(kb, ans_body, indent=""):
    """returns True if kb |- ans_body
    ans_body is a list of atoms to be proved
    """
    kb.display(2,indent,'yes <-',' & '.join(ans_body))
    if ans_body:
        selected = ans_body[0] # select first atom from ans_body
        if selected in kb.askables:
            return (yes(input("Is "+selected+" true? "))
                    and  prove(kb,ans_body[1:],indent+" "))
        else:
            return any(prove(kb,cl.body+ans_body[1:],indent+" ")
                       for cl in kb.clauses_for_atom(selected))
    else:
        return True   # empty body is true
```

The following provides a simple **unit test** that is hard wired for `triv_KB`:

logicTopDown.py — (continued)

```
from logicProblem import triv_KB
def test():
    a1 = prove(triv_KB,['i_am'])
    assert a1, "triv_KB proving i_am gave "+str(a1)
    a2 = prove(triv_KB,['i_smell'])
    assert not a2, "triv_KB proving i_smell gave "+str(a2)
    print("Passed unit tests")
if __name__ == "__main__":
    test()
# try
from logicProblem import elect
# elect.max_display_level=3 # give detailed trace
# prove(elect,['live_w6'])
# prove(elect,['lit_l1'])
```

Exercise 5.4 This code can re-ask a question multiple times. Implement this code so that it only asks a question once and remembers the answer. Also implement a function to forget the answers.

Exercise 5.5 What search method is this using? Implement the search interface so that it can use A^* or other searching methods. Define an admissible heuristic that is not always 0.

5.4 Debugging and Explanation

Here we modify the top-down procedure to build a proof tree than can be traversed for explanation and debugging.

`prove_atom(kb,atom)` returns a proof for *atom* from a knowledge base *kb*, where a proof is a pair of the atom and the proofs for the elements of the body of the clause used to prove the atom. `prove_body(kb,body)` returns a list of proofs for list *body* from a knowledge base, *kb*. The *indent* is used when displaying the code (and doesn't need to have a non-default value).

logicExplain.py — Explaining Proof Procedure for Definite Clauses

```
from logicProblem import yes # for asking the user

def prove_atom(kb, atom, indent=""):
    """returns a pair (atom,proofs) where proofs is the list of proofs
       of the elements of a body of a clause used to prove atom.
    """
    kb.display(2,indent,'proving',atom)
    if atom in kb.askables:
        if yes(input("Is "+atom+" true? ")):
            return (atom,"answered")
        else:
            return "fail"
    else:
        for cl in kb.clauses_for_atom(atom):
            kb.display(2,indent,"trying",atom,'<-',' & '.join(cl.body))
            pr_body = prove_body(kb, cl.body, indent)
            if pr_body != "fail":
                return (atom, pr_body)
        return "fail"

def prove_body(kb, ans_body, indent=""):
    """returns proof tree if kb |- ans_body or "fail" if there is no proof
    ans_body is a list of atoms in a body to be proved
    """
    proofs = []
    for atom in ans_body:
        proof_at = prove_atom(kb, atom, indent+" ")
        if proof_at == "fail":
            return "fail" # fail if any proof fails
        else:
```

```
41              proofs.append(proof_at)
42      return proofs
```

The following provides a simple **unit test** that is hard wired for `triv_KB`:

logicExplain.py — (continued)

```
44  from logicProblem import triv_KB
45  def test():
46      a1 = prove_atom(triv_KB,'i_am')
47      assert a1, "triv_KB proving i_am gave "+str(a1)
48      a2 = prove_atom(triv_KB,'i_smell')
49      assert a2=="fail", "triv_KB proving i_smell gave "+str(a2)
50      print("Passed unit tests")
51  if __name__ == "__main__":
52      test()
53  # try
54  from logicProblem import elect, elect_bug
55  # elect.max_display_level=3 # give detailed trace
56  # prove_atom(elect, 'live_w6')
57  # prove_atom(elect, 'lit_l1')
```

The `interact(kb)` provides an interactive interface to explore proofs for knowledge base `kb`. The user can ask to prove atoms and can ask `how` an atom was proved.

To ask `how`, there must be a current atom for which there is a proof. This starts as the atom asked. When the user asks "how n" the current atom becomes the n-th element of the body of the clause used to prove the (previous) current atom. The command "up" makes the current atom the atom in the head of the rule containing the (previous) current atom. Thus "how n" moves down the proof tree and "up" moves up the proof tree, allowing the user to explore the full proof.

logicExplain.py — (continued)

```
59  helptext = """Commands are:
60  ask atom    ask is there is a proof for atom (atom should not be in quotes)
61  how         show the clause that was used to prove atom
62  how n       show the clause used to prove the nth element of the body
63  up          go back up proof tree to explore other parts of the proof tree
64  kb          print the knowledge base
65  quit        quit this interaction (and go back to Python)
66  help        print this text
67  """
68
69  def interact(kb):
70      going = True
71      ups = []   # stack for going up
72      proof="fail" # there is no proof to start
73      while going:
74          inp = input("logicExplain: ")
75          inps = inp.split(" ")
```

```
        try:
            command = inps[0]
            if command == "quit":
                going = False
            elif command == "ask":
                proof = prove_atom(kb, inps[1])
                if proof == "fail":
                    print("fail")
                else:
                    print("yes")
            elif command == "how":
                if proof=="fail":
                    print("there is no proof")
                elif len(inps)==1:
                   print_rule(proof)
                else:
                    try:
                        ups.append(proof)
                        proof = proof[1][int(inps[1])] #nth argument of rule
                        print_rule(proof)
                    except:
                        print('In "how n", n must be a number between 0
                            and',len(proof[1])-1,"inclusive.")
            elif command == "up":
                if ups:
                    proof = ups.pop()
                else:
                    print("No rule to go up to.")
                print_rule(proof)
            elif command == "kb":
                 print(kb)
            elif command == "help":
                print(helptext)
            else:
                print("unknown command:", inp)
                print("use help for help")
        except:
            print("unknown command:", inp)
            print("use help for help")

def print_rule(proof):
    (head,body) = proof
    if body == "answered":
        print(head,"was answered yes")
    elif body == []:
            print(head,"is a fact")
    else:
            print(head,"<-")
            for i,a in enumerate(body):
                print(i,":",a[0])
```

```
125
126 # try
127 # interact(elect)
128 # Which clause is wrong in elect_bug? Try:
129 # interact(elect_bug)
130 # logicExplain: ask lit_l1
```

The following shows an interaction for the knowledge base `elect`:

```
>>> interact(elect)
logicExplain: ask lit_l1
Is up_s2 true? no
Is down_s2 true? yes
Is down_s1 true? yes
yes
logicExplain: how
lit_l1 <-
0 : light_l1
1 : live_l1
2 : ok_l1
logicExplain: how 1
live_l1 <-
0 : live_w0
logicExplain: how 0
live_w0 <-
0 : down_s2
1 : live_w2
logicExplain: how 0
down_s2 was answered yes
logicExplain: up
live_w0 <-
0 : down_s2
1 : live_w2
logicExplain: how 1
live_w2 <-
0 : down_s1
1 : live_w3
logicExplain: quit
>>>
```

Exercise 5.6 The above code only ever explores one proof – the first proof found. Change the code to enumerate the proof trees (by returning a list all proof trees, or preferably using yield). Add the command "retry" to the user interface to try another proof.

5.5 Assumables

Atom a can be made assumable by including $Assumable(a)$ in the knowledge base. A knowledge base that can include assumables is declared with *KBA*.

logicAssumables.py — Definite clauses with assumables

```
from logicProblem import Clause, Askable, KB, yes

class Assumable(object):
    """An askable atom"""

    def __init__(self,atom):
        """clause with atom head and lost of atoms body"""
        self.atom = atom

    def __str__(self):
        """returns the string representation of a clause.
        """
        return "assumable " + self.atom + "."

class KBA(KB):
    """A knowledge base that can include assumables"""
    def __init__(self,statements):
        self.assumables = [c.atom for c in statements if isinstance(c,
            Assumable)]
        KB.__init__(self,statements)
```

The top-down Horn clause interpreter, *prove_all_ass* returns a list of the sets of assumables that imply *ans_body*. This list will contain all of the minimal sets of assumables, but can also find non-minimal sets, and repeated sets, if they can be generated with separate proofs. The set *assumed* is the set of assumables already assumed.

logicAssumables.py — (continued)

```
    def prove_all_ass(self, ans_body, assumed=set()):
        """returns a list of sets of assumables that extends assumed
        to imply ans_body from self.
        ans_body is a list of atoms (it is the body of the answer clause).
        assumed is a set of assumables already assumed
        """
        if ans_body:
            selected = ans_body[0] # select first atom from ans_body
            if selected in self.askables:
                if yes(input("Is "+selected+" true? ")):
                    return self.prove_all_ass(ans_body[1:],assumed)
                else:
                    return []  # no answers
            elif selected in self.assumables:
                return self.prove_all_ass(ans_body[1:],assumed|{selected})
            else:
                return [ass
```

```
                    for cl in self.clauses_for_atom(selected)
                    for ass in
                        self.prove_all_ass(cl.body+ans_body[1:],assumed)
                      ]  # union of answers for each clause with
                         head=selected
        else:             # empty body
            return [assumed]  # one answer

    def conflicts(self):
        """returns a list of minimal conflicts"""
        return minsets(self.prove_all_ass(['false']))
```

Given a list of sets, *minsets* returns a list of the minimal sets in the list. For example, *minsets*($[\{2,3,4\},\{2,3\},\{6,2,3\},\{2,3\},\{2,4,5\}]$) returns $[\{2,3\},\{2,4,5\}]$.

logicAssumables.py — (continued)

```
def minsets(ls):
    """ls is a list of sets
    returns a list of minimal sets in ls
    """
    ans = []    # elements known to be minimal
    for c in ls:
        if not any(c1<c for c1 in ls) and not any(c1 <= c for c1 in ans):
            ans.append(c)
    return ans

# minsets([{2, 3, 4}, {2, 3}, {6, 2, 3}, {2, 3}, {2, 4, 5}])
```

Warning: *minsets* works for a list of sets or for a set of (frozen) sets, but it does not work for a generator of sets. For example, try to predict and then test:

```
minsets(e for e in [{2, 3, 4}, {2, 3}, {6, 2, 3}, {2, 3}, {2, 4, 5}])
```

The diagnoses can be constructed from the (minimal) conflicts as follows. This also works if there are non-minimal conflicts, but is not as efficient.

logicAssumables.py — (continued)

```
def diagnoses(cons):
    """cons is a list of (minimal) conflicts.
    returns a list of diagnoses."""
    if cons == []:
        return [set()]
    else:
        return minsets([({e}|d)            # | is set union
                        for e in cons[0]
                        for d in diagnoses(cons[1:])])
```

Test cases:

logicAssumables.py — (continued)

```
electa = KBA([
    Clause('light_l1'),
```

```
    Clause('light_l2'),
    Assumable('ok_l1'),
    Assumable('ok_l2'),
    Assumable('ok_s1'),
    Assumable('ok_s2'),
    Assumable('ok_s3'),
    Assumable('ok_cb1'),
    Assumable('ok_cb2'),
    Assumable('live_outside'),
    Clause('live_l1', ['live_w0']),
    Clause('live_w0', ['up_s2','ok_s2','live_w1']),
    Clause('live_w0', ['down_s2','ok_s2','live_w2']),
    Clause('live_w1', ['up_s1', 'ok_s1', 'live_w3']),
    Clause('live_w2', ['down_s1', 'ok_s1','live_w3' ]),
    Clause('live_l2', ['live_w4']),
    Clause('live_w4', ['up_s3','ok_s3','live_w3' ]),
    Clause('live_p_1', ['live_w3']),
    Clause('live_w3', ['live_w5', 'ok_cb1']),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Clause('live_w5', ['live_outside']),
    Clause('lit_l1', ['light_l1', 'live_l1', 'ok_l1']),
    Clause('lit_l2', ['light_l2', 'live_l2', 'ok_l2']),
    Askable('up_s1'),
    Askable('down_s1'),
    Askable('up_s2'),
    Askable('down_s2'),
    Askable('up_s3'),
    Askable('down_s2'),
    Askable('dark_l1'),
    Askable('dark_l2'),
    Clause('false', ['dark_l1', 'lit_l1']),
    Clause('false', ['dark_l2', 'lit_l2'])
    ])
# electa.prove_all_ass(['false'])
# cs=electa.conflicts()
# print(cs)
# diagnoses(cs)      # diagnoses from conflicts
```

Exercise 5.7 To implement a version of *conflicts* that never generates non-minimal conflicts, modify *prove_all_ass* to implement iterative deepening on the number of assumables used in a proof, and prune any set of assumables that is a superset of a conflict.

Exercise 5.8 Implement *explanations*(*self*, *body*), where *body* is a list of atoms, that returns the a list of the minimal explanations of the body. This does not require modification of *prove_all_ass*.

Exercise 5.9 Implement *explanations*, as in the previous question, so that it never generates non-minimal explanations. Hint: modify *prove_all_ass* to implement iter-

ative deepening on the number of assumptions, generating conflicts and explanations together, and pruning as early as possible.

Chapter 6

Planning with Certainty

6.1 Representing Actions and Planning Problems

The STRIPS representation of an action consists of:

- the name of the action
- preconditions: a dictionary of *feature:value* pairs that specifies that the feature must have this value for the action to be possible
- effects: a dictionary of *feature:value* pairs that are made true by this action. In particular, a feature in the dictionary has the corresponding value (and not its previous value) after the action, and a feature not in the dictionary keeps its old value.

stripsProblem.py — STRIPS Representations of Actions

```
11 class Strips(object):
12     def __init__(self, name, preconds, effects, cost=1):
13         """
14         defines the STRIPS representation for an action:
15         * name is the name of the action
16         * preconds, the preconditions, is feature:value dictionary that
               must hold
17         for the action to be carried out
18         * effects is a feature:value map that this action makes
19         true. The action changes the value of any feature specified
20         here, and leaves other features unchanged.
21         * cost is the cost of the action
22         """
```

```
23          self.name = name
24          self.preconds = preconds
25          self.effects = effects
26          self.cost = cost
27
28      def __repr__(self):
29          return self.name
```

A STRIPS domain consists of:

- A set of actions.
- A dictionary that maps each feature into a set of possible values for the feature.
- A list of the actions

stripsProblem.py — (continued)

```
31  class STRIPS_domain(object):
32      def __init__(self, feature_domain_dict, actions):
33          """Problem domain
34          feature_domain_dict is a feature:domain dictionary,
35                 mapping each feature to its domain
36          actions
37          """
38          self.feature_domain_dict = feature_domain_dict
39          self.actions = actions
```

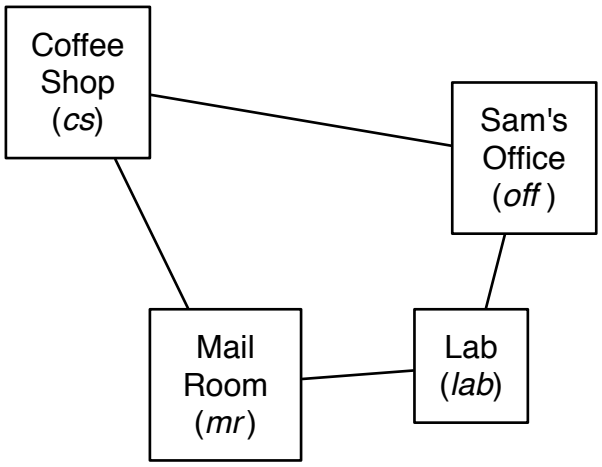
A planning problem consists of a planning domain, an initial state, and a goal. The goal does not need to fully specify the final state.

stripsProblem.py — (continued)

```
41  class Planning_problem(object):
42      def __init__(self, prob_domain, initial_state, goal):
43          """
44          a planning problem consists of
45          * a planning domain
46          * the initial state
47          * a goal
48          """
49          self.prob_domain = prob_domain
50          self.initial_state = initial_state
51          self.goal = goal
```

6.1.1 Robot Delivery Domain

The following specifies the robot delivery domain of Section 6.1, shown in Figure 6.1.



Features to describe states

RLoc – Rob's location
RHC – Rob has coffee
SWC – Sam wants coffee
MW – Mail is waiting
RHM – Rob has mail

Actions

mc – move clockwise
mcc – move counterclockwise
puc – pickup coffee
dc – deliver coffee
pum – pickup mail
dm – deliver mail

Figure 6.1: Robot Delivery Domain

stripsProblem.py — (continued)

```
boolean = {True, False}
delivery_domain = STRIPS_domain(
   {'RLoc':{'cs', 'off', 'lab', 'mr'}, 'RHC':boolean, 'SWC':boolean,
    'MW':boolean, 'RHM':boolean},       #feature:values dictionary
   { Strips('mc_cs', {'RLoc':'cs'}, {'RLoc':'off'}),
    Strips('mc_off', {'RLoc':'off'}, {'RLoc':'lab'}),
    Strips('mc_lab', {'RLoc':'lab'}, {'RLoc':'mr'}),
    Strips('mc_mr', {'RLoc':'mr'}, {'RLoc':'cs'}),
    Strips('mcc_cs', {'RLoc':'cs'}, {'RLoc':'mr'}),
    Strips('mcc_off', {'RLoc':'off'}, {'RLoc':'cs'}),
    Strips('mcc_lab', {'RLoc':'lab'}, {'RLoc':'off'}),
    Strips('mcc_mr', {'RLoc':'mr'}, {'RLoc':'lab'}),
    Strips('puc', {'RLoc':'cs', 'RHC':False}, {'RHC':True}),
    Strips('dc', {'RLoc':'off', 'RHC':True}, {'RHC':False, 'SWC':False}),
    Strips('pum', {'RLoc':'mr','MW':True}, {'RHM':True,'MW':False}),
    Strips('dm', {'RLoc':'off', 'RHM':True}, {'RHM':False})
   } )
```

stripsProblem.py — (continued)

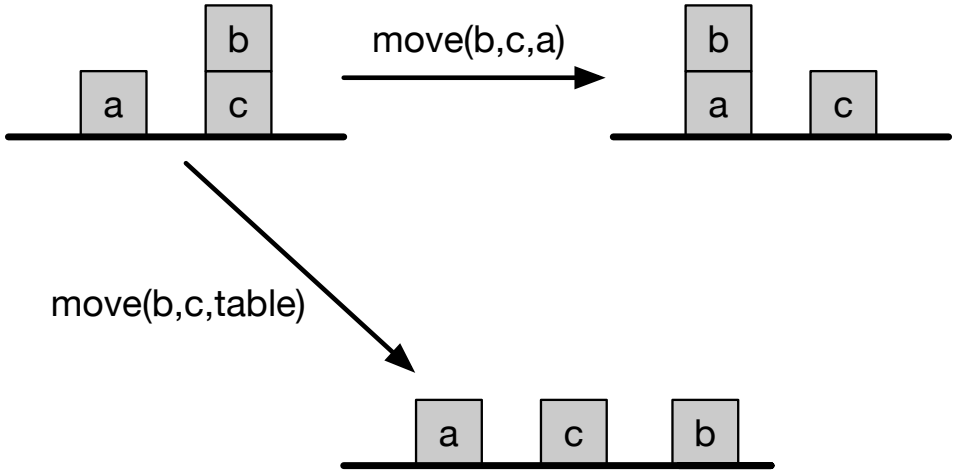


Figure 6.2: Blocks world with two actions

```
problem0 = Planning_problem(delivery_domain,
                            {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
                             'RHM':False},
                            {'RLoc':'off'})
problem1 = Planning_problem(delivery_domain,
                            {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
                             'RHM':False},
                            {'SWC':False})
problem2 = Planning_problem(delivery_domain,
                            {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
                             'RHM':False},
                            {'SWC':False, 'MW':False, 'RHM':False})
```

6.1.2 Blocks World

The blocks world consist of blocks and a table. Each block can be on the table or on another block. A block can only have one other block on top of it. Figure 6.2 shows 3 states with some of the actions between them.

A state is defined by the two features:

- *on* where $on(x) = y$ when block x is on block or table y
- *clear* where $clear(x) = True$ when block x has nothing on it.

There is one parameterized action

- $move(x, y, z)$ move block x from y to z, where y and z could be a block or the table.

To handle parameterized actions (which depend on the blocks involved), the actions and the features are all strings, created for the all combinations of the blocks. Note that we treat moving to a block separately from moving to the table, because the blocks needs to be clear, but the table always has room for another block.

stripsProblem.py — (continued)

```
### blocks world
def move(x,y,z):
    """string for the 'move' action"""
    return 'move_'+x+'_from_'+y+'_to_'+z
def on(x):
    """string for the 'on' feature"""
    return x+'_is_on'
def clear(x):
    """string for the 'clear' feature"""
    return 'clear_'+x
def create_blocks_world(blocks = {'a','b','c','d'}):
    blocks_and_table = blocks | {'table'}
    stmap = {Strips(move(x,y,z),{on(x):y, clear(x):True, clear(z):True},
                                {on(x):z, clear(y):True, clear(z):False})
                    for x in blocks
                    for y in blocks_and_table
                    for z in blocks
                    if x!=y and y!=z and z!=x}
    stmap.update({Strips(move(x,y,'table'), {on(x):y, clear(x):True},
                                {on(x):'table', clear(y):True})
                    for x in blocks
                    for y in blocks
                    if x!=y})
    feature_domain_dict = {on(x):blocks_and_table-{x} for x in blocks}
    feature_domain_dict.update({clear(x):boolean for x in blocks_and_table})
    return STRIPS_domain(feature_domain_dict, stmap)
```

The problem *blocks*1 is a classic example, with 3 blocks, and the goal consists of two conditions. See Figure 6.3. Note that this example is challenging because we can't achieve one of the goals and then the other; whichever one we achieve first has to be undone to achieve the second.

stripsProblem.py — (continued)

```
blocks1dom = create_blocks_world({'a','b','c'})
blocks1 = Planning_problem(blocks1dom,
    {on('a'):'table', clear('a'):True,
     on('b'):'c', clear('b'):True,
     on('c'):'table', clear('c'):False}, # initial state
    {on('a'):'b', on('c'):'a'}) #goal
```

The problem *blocks*2 is one to invert a tower of size 4.

stripsProblem.py — (continued)

```
blocks2dom = create_blocks_world({'a','b','c','d'})
```

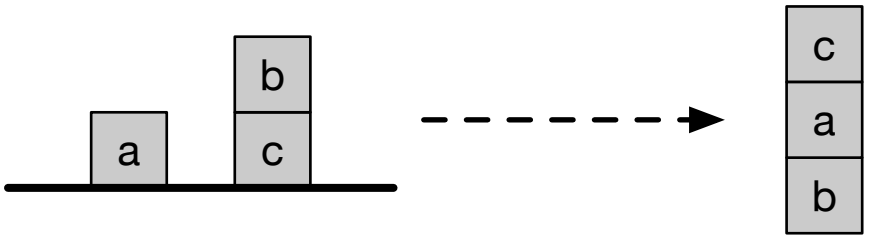


Figure 6.3: Blocks problem blocks1

```
tower4 = {clear('a'):True, on('a'):'b',
         clear('b'):False, on('b'):'c',
         clear('c'):False, on('c'):'d',
         clear('d'):False, on('d'):'table'}
blocks2 = Planning_problem(blocks2dom,
    tower4, # initial state
    {on('d'):'c',on('c'):'b',on('b'):'a'}) #goal
```

The problem *blocks*3 is to move the bottom block to the top of a tower of size 4.

stripsProblem.py — (continued)

```
blocks3 = Planning_problem(blocks2dom,
    tower4, # initial state
    {on('d'):'a', on('a'):'b', on('b'):'c'}) #goal
```

Exercise 6.1 Represent the problem of given a tower of 4 blocks (a on b on c on d on table), the goal is to have a tower with the previous top block on the bottom (b on c on d on a). Do not include the table in your goal (the goal does not care whether a is on the table). [Before you run the program, estimate how many steps it will take to solve this.] How many steps does an optimal planner take?

Exercise 6.2 Represent the domain so that $on(x, y)$ is a Boolean feature that is True when x is on y, Does the representation of the state need to not include negative *on* facts? Why or why not? (Note that this may depend on the planner; write your answer with respect to particular planners.)

Exercise 6.3 It is possible to write the representation of the problem without using *clear*, where $clear(x)$ means nothing is on x. Change the definition of the blocks world so that it does not use *clear* but uses *on* being false instead. Does this work better for any of the planners?

6.2 Forward Planning

To run the demo, in folder "aipython", load "stripsForwardPlanner.py", and copy and paste the commented-out example queries at the bottom of that file.

In a forward planner, a node is a state. A state consists of an assignment, which is a variable:value dictionary. In order to be able to do multiple-path pruning, we need to define a hash function, and equality between states.

stripsForwardPlanner.py — Forward Planner with STRIPS actions

```
from searchProblem import Arc, Search_problem
from stripsProblem import Strips, STRIPS_domain

class State(object):
    def __init__(self,assignment):
        self.assignment = assignment
        self.hash_value = None
    def __hash__(self):
        if self.hash_value is None:
            self.hash_value = hash(frozenset(self.assignment.items()))
        return self.hash_value
    def __eq__(self,st):
        return self.assignment == st.assignment
    def __str__(self):
        return str(self.assignment)
```

In order to define a search problem (page 33), we need to define the goal condition, the start nodes, the neighbours, and (optionally) a heuristic function. Here *zero* is the default heuristic function.

stripsForwardPlanner.py — (continued)

```
def zero(*args,**nargs):
    """always returns 0"""
    return 0

class Forward_STRIPS(Search_problem):
    """A search problem from a planning problem where:
    * a node is a state object.
    * the dynamics are specified by the STRIPS representation of actions
    """
    def __init__(self, planning_problem, heur=zero):
        """creates a forward search space from a planning problem.
        heur(state,goal) is a heuristic function,
           an underestimate of the cost from state to goal, where
           both state and goals are feature:value dictionaries.
        """
        self.prob_domain = planning_problem.prob_domain
        self.initial_state = State(planning_problem.initial_state)
        self.goal = planning_problem.goal
        self.heur = heur

    def is_goal(self, state):
        """is True if node is a goal.

        Every goal feature has the same value in the state and the goal."""
        return all(state.assignment[prop]==self.goal[prop]
```

```
                for prop in self.goal)

    def start_node(self):
        """returns start node"""
        return self.initial_state

    def neighbors(self,state):
        """returns neighbors of state in this problem"""
        return [ Arc(state, self.effect(act,state.assignment), act.cost,
            act)
                for act in self.prob_domain.actions
                if self.possible(act,state.assignment)]

    def possible(self,act,state_asst):
        """True if act is possible in state.
        act is possible if all of its preconditions have the same value in
            the state"""
        return all(state_asst[pre] == act.preconds[pre]
                   for pre in act.preconds)

    def effect(self,act,state_asst):
        """returns the state that is the effect of doing act given
            state_asst
        Python 3.9: return state_asst | act.effects"""
        new_state_asst = state_asst.copy()
        new_state_asst.update(act.effects)
        return State(new_state_asst)

    def heuristic(self,state):
        """in the forward planner a node is a state.
        the heuristic is an (under)estimate of the cost
        of going from the state to the top-level goal.
        """
        return self.heur(state.assignment, self.goal)
```

Here are some test cases to try.

stripsForwardPlanner.py — (continued)

```
from searchBranchAndBound import DF_branch_and_bound
from searchMPP import SearcherMPP
from stripsProblem import problem0, problem1, problem2, blocks1, blocks2,
    blocks3

# SearcherMPP(Forward_STRIPS(problem1)).search() #A* with MPP
# DF_branch_and_bound(Forward_STRIPS(problem1),10).search() #B&B
# To find more than one plan:
# s1 = SearcherMPP(Forward_STRIPS(problem1)) #A*
# s1.search() #find another plan
```

6.2.1 Defining Heuristics for a Planner

Each planning domain requires its own heuristics. If you change the actions, you will need to reconsider the heuristic function, as there might then be a lower-cost path, which might make the heuristic non-admissible.

Here is an example of defining a (not very good) heuristic for the coffee delivery planning domain.

First we define the distance between two locations, which is used for the heuristics.

stripsHeuristic.py — Planner with Heuristic Function

```
11 def dist(loc1, loc2):
12     """returns the distance from location loc1 to loc2
13     """
14     if loc1==loc2:
15         return 0
16     if {loc1,loc2} in [{'cs','lab'},{'mr','off'}]:
17         return 2
18     else:
19         return 1
```

Note that the current state is a complete description; there is a value for every feature. However the goal need not be complete; it does not need to define a value for every feature. Before checking the value for a feature in the goal, a heuristic needs to define whether the feature is defined in the goal.

stripsHeuristic.py — (continued)

```
21 def h1(state,goal):
22     """ the distance to the goal location, if there is one"""
23     if 'RLoc' in goal:
24         return dist(state['RLoc'], goal['RLoc'])
25     else:
26         return 0
27
28 def h2(state,goal):
29     """ the distance to the coffee shop plus getting coffee and delivering
           it
30     if the robot needs to get coffee
31     """
32     if ('SWC' in goal and goal['SWC']==False
33             and state['SWC']==True
34             and state['RHC']==False):
35         return dist(state['RLoc'],'cs')+3
36     else:
37         return 0
```

The maximum of the values of a set of admissible heuristics is also an admissible heuristic. The function `maxh` takes a number of heuristic functions as arguments, and returns a new heuristic function that takes the maximum of the values of the heuristics. For example, `h1` and `h2` are heuristic functions and so `maxh(h1,h2)` is also. `maxh` can take an arbitrary number of arguments.

stripsHeuristic.py — (continued)

```
def maxh(*heuristics):
    """Returns a new heuristic function that is the maximum of the
        functions in heuristics.
    heuristics is the list of arguments which must be heuristic functions.
    """
    # return lambda state,goal: max(h(state,goal) for h in heuristics)
    def newh(state,goal):
        return max(h(state,goal) for h in heuristics)
    return newh
```

The following runs the example with and without the heuristic.

stripsHeuristic.py — (continued)

```
##### Forward Planner #####
from searchMPP import SearcherMPP
from stripsForwardPlanner import Forward_STRIPS
from stripsProblem import problem0, problem1, problem2, blocks1, blocks2,
    blocks3

def test_forward_heuristic(thisproblem=problem1):
    print("\n***** FORWARD NO HEURISTIC")
    print(SearcherMPP(Forward_STRIPS(thisproblem)).search())

    print("\n***** FORWARD WITH HEURISTIC h1")
    print(SearcherMPP(Forward_STRIPS(thisproblem,h1)).search())

    print("\n***** FORWARD WITH HEURISTIC h2")
    print(SearcherMPP(Forward_STRIPS(thisproblem,h2)).search())

    print("\n***** FORWARD WITH HEURISTICs h1 and h2")
    print(SearcherMPP(Forward_STRIPS(thisproblem,maxh(h1,h2))).search())

if __name__ == "__main__":
    test_forward_heuristic()
```

Exercise 6.4 Try the forward planner with a heuristic function of just $h1$, with just $h2$ and with both. Explain how each one prunes or doesn't prune the search space.

Exercise 6.5 Create a better heuristic than $maxh(h1, h2)$. Try it for a number of different problems. In particular, try and include the following costs:

i) $h3$ is like $h2$ but also takes into account the case when *Rloc* is in goal.

ii) $h4$ uses the distance to the mail room plus getting mail and delivering it if the robot needs to get need to deliver mail.

iii) $h5$ is for getting mail when goal is for the robot to have mail, and then getting to the goal destination (if there is one).

Exercise 6.6 Create an admissible heuristic for the blocks world.

6.3 Regression Planning

To run the demo, in folder "aipython", load "stripsRegressionPlanner.py", and copy and paste the commented-out example queries at the bottom of that file.

In a regression planner a node is a subgoal that need to be achieved.

A *Subgoal* object consists of an assignment, which is *variable:value* dictionary. We make it hashable so that multiple path pruning can work. The hash is only computed when necessary (and only once).

stripsRegressionPlanner.py — Regression Planner with STRIPS actions

```
from searchProblem import Arc, Search_problem

class Subgoal(object):
    def __init__(self,assignment):
        self.assignment = assignment
        self.hash_value = None
    def __hash__(self):
        if self.hash_value is None:
            self.hash_value = hash(frozenset(self.assignment.items()))
        return self.hash_value
    def __eq__(self,st):
        return self.assignment == st.assignment
    def __str__(self):
        return str(self.assignment)
```

A regression search has subgoals as nodes. The initial node is the top-level goal of the planner. The goal for the search (when the search can stop) is a subgoal that holds in the initial state.

stripsRegressionPlanner.py — (continued)

```
from stripsForwardPlanner import zero

class Regression_STRIPS(Search_problem):
    """A search problem where:
    * a node is a goal to be achieved, represented by a set of propositions.
    * the dynamics are specified by the STRIPS representation of actions
    """

    def __init__(self, planning_problem, heur=zero):
        """creates a regression search space from a planning problem.
        heur(state,goal) is a heuristic function;
           an underestimate of the cost from state to goal, where
           both state and goals are feature:value dictionaries
        """
        self.prob_domain = planning_problem.prob_domain
        self.top_goal = Subgoal(planning_problem.goal)
        self.initial_state = planning_problem.initial_state
        self.heur = heur
```

```
44
45      def is_goal(self, subgoal):
46          """if subgoal is true in the initial state, a path has been found"""
47          goal_asst = subgoal.assignment
48          return all(self.initial_state[g]==goal_asst[g]
49                     for g in goal_asst)
50
51      def start_node(self):
52          """the start node is the top-level goal"""
53          return self.top_goal
54
55      def neighbors(self,subgoal):
56          """returns a list of the arcs for the neighbors of subgoal in this
                problem"""
57          goal_asst = subgoal.assignment
58          return [ Arc(subgoal, self.weakest_precond(act,goal_asst),
                    act.cost, act)
59                   for act in self.prob_domain.actions
60                   if self.possible(act,goal_asst)]
61
62      def possible(self,act,goal_asst):
63          """True if act is possible to achieve goal_asst.
64
65          the action achieves an element of the effects and
66          the action doesn't delete something that needs to be achieved and
67          the preconditions are consistent with other subgoals that need to
                be achieved
68          """
69          return ( any(goal_asst[prop] == act.effects[prop]
70                       for prop in act.effects if prop in goal_asst)
71                   and all(goal_asst[prop] == act.effects[prop]
72                           for prop in act.effects if prop in goal_asst)
73                   and all(goal_asst[prop]== act.preconds[prop]
74                           for prop in act.preconds if prop not in act.effects
                                and prop in goal_asst)
75                   )
76
77      def weakest_precond(self,act,goal_asst):
78          """returns the subgoal that must be true so goal_asst holds after
                act
79          should be:  act.preconds | (goal_asst - act.effects)
80          """
81          new_asst = act.preconds.copy()
82          for g in goal_asst:
83              if g not in act.effects:
84                  new_asst[g] = goal_asst[g]
85          return Subgoal(new_asst)
86
87      def heuristic(self,subgoal):
88          """in the regression planner a node is a subgoal.
```

```
89          the heuristic is an (under)estimate of the cost of going from the
                initial state to subgoal.
90          """
91          return self.heur(self.initial_state, subgoal.assignment)
```

stripsRegressionPlanner.py — (continued)

```
93  from searchBranchAndBound import DF_branch_and_bound
94  from searchMPP import SearcherMPP
95  from stripsProblem import problem0, problem1, problem2, blocks1, blocks2,
        blocks3
96
97  # SearcherMPP(Regression_STRIPS(problem1)).search() #A* with MPP
98  # DF_branch_and_bound(Regression_STRIPS(problem1),10).search() #B&B
```

Exercise 6.7 Multiple path pruning could be used to prune more than the current code. In particular, if the current node contains more conditions than a previously visited node, it can be pruned. For example, if $\{a : True, b : False\}$ has been visited, then any node that is a superset, e.g., $\{a : True, b : False, d : True\}$, need not be expanded. If the simpler subgoal does not lead to a solution, the more complicated one wont either. Implement this more severe pruning. (Hint: This may require modifications to the searcher.)

Exercise 6.8 It is possible that, as knowledge of the domain, that some assignment of values to variables can never be achieved. For example, the robot cannot be holding mail when there is mail waiting (assuming it isn't holding mail initially). An assignment of values to (some of the) variables is incompatible if no possible (reachable) state can include that assignment. For example, $\{'MW' : True, 'RHM' : True\}$ is an incompatible assignment. This information may be useful information for a planner; there is no point in trying to achieve these together. Define a subclass of *STRIPS_domain* that can accept a list of incompatible assignments. Modify the regression planner code to use such a list of incompatible assignments. Give an example where the search space is smaller.

Exercise 6.9 After completing the previous exercise, design incompatible assignments for the blocks world. (This should result in dramatic search improvements.)

6.3.1 Defining Heuristics for a Regression Planner

The regression planner can use the same heuristic function as the forward planner. However, just because a heuristic is useful for a forward planner does not mean it is useful for a regression planner, and vice versa. you should experiment with whether the same heuristic works well for both a a regression planner and a forward planner.

The following runs the same example as the forward planner with and without the heuristic defined for the forward planner:

stripsHeuristic.py — (continued)

```
69  ##### Regression Planner
70  from stripsRegressionPlanner import Regression_STRIPS
```

```
71
72 def test_regression_heuristic(thisproblem=problem1):
73     print("\n***** REGRESSION NO HEURISTIC")
74     print(SearcherMPP(Regression_STRIPS(thisproblem)).search())
75
76     print("\n***** REGRESSION WITH HEURISTICs h1 and h2")
77     print(SearcherMPP(Regression_STRIPS(thisproblem,maxh(h1,h2))).search())
78
79 if __name__ == "__main__":
80     test_regression_heuristic()
```

Exercise 6.10 Try the regression planner with a heuristic function of just $h1$ and with just $h2$ (defined in Section 6.2.1). Explain how each one prunes or doesn't prune the search space.

Exercise 6.11 Create a better heuristic than *heuristic_fun* defined in Section 6.2.1.

6.4 Planning as a CSP

> To run the demo, in folder "aipython", load "stripsCSPPlanner.py", and copy and paste the commented-out example queries at the bottom of that file. This assumes Python 3.

Here we implement the CSP planner assuming there is a single action at each step. This creates a CSP that can use any of the CSP algorithms to solve (e.g., stochastic local search or arc consistency with domain splitting).

This assumes the same action representation as before; we do not consider factored actions (action features), nor do we implement state constraints.

stripsCSPPlanner.py — CSP planner where actions are represented using STRIPS

```
11 from cspProblem import Variable, CSP, Constraint
12
13 class CSP_from_STRIPS(CSP):
14     """A CSP where:
15     * CSP variables are constructed for each feature and time, and each
           action and time
16     * the dynamics are specified by the STRIPS representation of actions
17     """
18
19     def __init__(self, planning_problem, number_stages=2):
20         prob_domain = planning_problem.prob_domain
21         initial_state = planning_problem.initial_state
22         goal = planning_problem.goal
23         # self.action_vars[t] is the action variable for time t
24         self.action_vars = [Variable(f"Action{t}", prob_domain.actions)
25                             for t in range(number_stages)]
26         # feat_time_var[f][t] is the variable for feature f at time t
27         feat_time_var = {feat: [Variable(f"{feat}_{t}",dom)
28                                 for t in range(number_stages+1)]
```

```
                        for (feat,dom) in
                            prob_domain.feature_domain_dict.items()}

        # initial state constraints:
        constraints = [Constraint((feat_time_var[feat][0],), is_(val))
                       for (feat,val) in initial_state.items()]

        # goal constraints on the final state:
        constraints += [Constraint((feat_time_var[feat][number_stages],),
                                   is_(val))
                        for (feat,val) in goal.items()]

        # precondition constraints:
        constraints += [Constraint((feat_time_var[feat][t],
            self.action_vars[t]),
                                   if_(val,act)) # feat@t==val if action@t==act
                        for act in prob_domain.actions
                        for (feat,val) in act.preconds.items()
                        for t in range(number_stages)]

        # effect constraints:
        constraints += [Constraint((feat_time_var[feat][t+1],
            self.action_vars[t]),
                                   if_(val,act)) # feat@t+1==val if
                                       action@t==act
                        for act in prob_domain.actions
                        for feat,val in act.effects.items()
                        for t in range(number_stages)]
        # frame constraints:

        constraints += [Constraint((feat_time_var[feat][t],
            self.action_vars[t], feat_time_var[feat][t+1]),
                                   eq_if_not_in_({act for act in
                                       prob_domain.actions
                                                  if feat in act.effects}))
                        for feat in prob_domain.feature_domain_dict
                        for t in range(number_stages) ]
        variables = set(self.action_vars) | {feat_time_var[feat][t]
                                             for feat in
                                                 prob_domain.feature_domain_dict
                                             for t in range(number_stages+1)}
        CSP.__init__(self, variables, constraints)

    def extract_plan(self,soln):
        return [soln[a] for a in self.action_vars]
```

The following methods return methods which can be applied to the particular environment.

For example, $is\_(3)$ returns a function that when applied to 3, returns True and when applied to any other value returns False. So $is\_(3)(3)$ returns *True*

and *is_*(3)(7) returns *False*.

Note that the underscore ('_') is part of the name; here we use it as the convention that it is a function that returns a function. This uses two different styles to define *is_* and *if_*; returning a function defined by *lambda* is equivalent to returning the embedded function, except that the embedded function has a name. The embedded function can also be given a docstring.

stripsCSPPlanner.py — (continued)

```
68 def is_(val):
69     """returns a function that is true when it is it applied to val.
70     """
71     #return lambda x: x == val
72     def is_fun(x):
73         return x == val
74     is_fun.__name__ = "value_is_"+str(val)
75     return is_fun
76 
77 def if_(v1,v2):
78     """if the second argument is v2, the first argument must be v1"""
79     #return lambda x1,x2: x1==v1 if x2==v2 else True
80     def if_fun(x1,x2):
81         return x1==v1 if x2==v2 else True
82     if_fun.__name__ = "if x2 is "+str(v2)+" then x1 is "+str(v1)
83     return if_fun
84 
85 def eq_if_not_in_(actset):
86     """first and third arguments are equal if action is not in actset"""
87     # return lambda x1, a, x2: x1==x2 if a not in actset else True
88     def eq_if_not_fun(x1, a, x2):
89         return x1==x2 if a not in actset else True
90     eq_if_not_fun.__name__ = "first and third arguments are equal if action
           is not in "+str(actset)
91     return eq_if_not_fun
```

Putting it together, this returns a list of actions that solves the problem *prob* for a given horizon. If you want to do more than just return the list of actions, you might want to get it to return the solution. Or even enumerate the solutions (by using *Search_with_AC_from_CSP*).

stripsCSPPlanner.py — (continued)

```
93 def con_plan(prob,horizon):
94     """finds a plan for problem prob given horizon.
95     """
96     csp = CSP_from_STRIPS(prob, horizon)
97     sol = Con_solver(csp).solve_one()
98     return csp.extract_plan(sol) if sol else sol
```

The following are some example queries.

stripsCSPPlanner.py — (continued)

```
100 from searchGeneric import Searcher
```

```
from stripsProblem import delivery_domain
from cspConsistency import Search_with_AC_from_CSP, Con_solver
from stripsProblem import Planning_problem, problem0, problem1, problem2,
    blocks1, blocks2, blocks3

# Problem 0
# con_plan(problem0,1) # should it succeed?
# con_plan(problem0,2) # should it succeed?
# con_plan(problem0,3) # should it succeed?
# To use search to enumerate solutions
#searcher0a = Searcher(Search_with_AC_from_CSP(CSP_from_STRIPS(problem0,
    1)))
#print(searcher0a.search()) # returns path to solution

## Problem 1
# con_plan(problem1,5) # should it succeed?
# con_plan(problem1,4) # should it succeed?
## To use search to enumerate solutions:
#searcher15a = Searcher(Search_with_AC_from_CSP(CSP_from_STRIPS(problem1,
    5)))
#print(searcher15a.search()) # returns path to solution

## Problem 2
#con_plan(problem2, 6) # should fail??
#con_plan(problem2, 7) # should succeed???

## Example 6.13
problem3 = Planning_problem(delivery_domain,
                            {'SWC':True, 'RHC':False}, {'SWC':False})
#con_plan(problem3,2) # Horizon of 2
#con_plan(problem3,3) # Horizon of 3

problem4 = Planning_problem(delivery_domain,{'SWC':True},
                              {'SWC':False, 'MW':False, 'RHM':False})

# For the stochastic local search:
#from cspSLS import SLSearcher, Runtime_distribution
# cspplanning15 = CSP_from_STRIPS(problem1, 5) # should succeed
#se0 = SLSearcher(cspplanning15); print(se0.search(100000,0.5))
#p = Runtime_distribution(cspplanning15)
#p.plot_runs(1000,1000,0.7) # warning will take a few minutes
```

6.5 Partial-Order Planning

To run the demo, in folder "aipython", load "stripsPOP.py", and copy and paste the commented-out example queries at the bottom of that file.

A partial order planner maintains a partial order of action instances. An action instance consists of a name and an index. We need action instances because the same action could be carried out at different times.

stripsPOP.py — Partial-order Planner using STRIPS representation

```
from searchProblem import Arc, Search_problem
import random

class Action_instance(object):
    next_index = 0
    def __init__(self,action,index=None):
        if index is None:
            index = Action_instance.next_index
            Action_instance.next_index += 1
        self.action = action
        self.index = index

    def __str__(self):
        return str(self.action)+"#"+str(self.index)

    __repr__ = __str__ # __repr__ function is the same as the __str__
        function
```

A node (as in the abstraction of search space) in a partial-order planner consists of:

- *actions*: a set of action instances.
- *constraints*: a set of (a_1, a_2) pairs, where a_1 and a_2 are action instances, which represents that a_1 must come before a_2 in the partial order. There are a number of ways that this could be represented. Here we represent the set of pairs that are in transitive closure of the *before* relation. This lets us quickly determine whether some before relation is consistent with the current constraints.
- *agenda*: a list of (s, a) pairs, where s is a (var, val) pair and a is an action instance. This means that variable *var* must have value *val* before a can occur.
- *causal_links*: a set of $(a0, g, a1)$ triples, where a_1 and a_2 are action instances and g is a (var, val) pair. This holds when action a_0 makes g true for action a_1.

stripsPOP.py — (continued)

```
class POP_node(object):
    """a (partial) partial-order plan. This is a node in the search
        space."""
    def __init__(self, actions, constraints, agenda, causal_links):
        """
```

```
        * actions is a set of action instances
        * constraints a set of (a0,a1) pairs, representing a0<a1,
          closed under transitivity
        * agenda list of (subgoal,action) pairs to be achieved, where
          subgoal is a (variable,value) pair
        * causal_links is a set of (a0,g,a1) triples,
          where ai are action instances, and g is a (variable,value) pair
        """
        self.actions = actions # a set of action instances
        self.constraints = constraints # a set of (a0,a1) pairs
        self.agenda = agenda  # list of (subgoal,action) pairs to be
            achieved
        self.causal_links = causal_links # set of (a0,g,a1) triples

    def __str__(self):
        return ("actions: "+str({str(a) for a in self.actions})+
                "\nconstraints: "+
                str({(str(a1),str(a2)) for (a1,a2) in self.constraints})+
                "\nagenda: "+
                str([(str(s),str(a)) for (s,a) in self.agenda])+
                "\ncausal_links:"+
                str({(str(a0),str(g),str(a2)) for (a0,g,a2) in
                    self.causal_links}) )
```

extract_plan constructs a total order of action instances that is consistent with the partial order.

stripsPOP.py — (continued)

```
    def extract_plan(self):
        """returns a total ordering of the action instances consistent
        with the constraints.
        raises IndexError if there is no choice.
        """
        sorted_acts = []
        other_acts = set(self.actions)
        while other_acts:
            a = random.choice([a for a in other_acts if
                    all(((a1,a) not in self.constraints) for a1 in
                        other_acts)])
            sorted_acts.append(a)
            other_acts.remove(a)
        return sorted_acts
```

POP_search_from_STRIPS is an instance of a search problem. As such, we need to define the start nodes, the goal, and the neighbors of a node.

stripsPOP.py — (continued)

```
from display import Displayable

class POP_search_from_STRIPS(Search_problem, Displayable):
    def __init__(self,planning_problem):
```

```
72          Search_problem.__init__(self)
73          self.planning_problem = planning_problem
74          self.start = Action_instance("start")
75          self.finish = Action_instance("finish")
76
77      def is_goal(self, node):
78          return node.agenda == []
79
80      def start_node(self):
81          constraints = {(self.start, self.finish)}
82          agenda = [(g, self.finish) for g in
                self.planning_problem.goal.items()]
83          return POP_node([self.start,self.finish], constraints, agenda, [] )
```

The *neighbors* method is a coroutine that enumerates the neighbors of a given node.

stripsPOP.py — (continued)

```
85      def neighbors(self, node):
86          """enumerates the neighbors of node"""
87          self.display(3,"finding neighbors of\n",node)
88          if node.agenda:
89              subgoal,act1 = node.agenda[0]
90              self.display(2,"selecting",subgoal,"for",act1)
91              new_agenda = node.agenda[1:]
92              for act0 in node.actions:
93                  if (self.achieves(act0, subgoal) and
94                     self.possible((act0,act1),node.constraints)):
95                      self.display(2," reusing",act0)
96                      consts1 =
                            self.add_constraint((act0,act1),node.constraints)
97                      new_clink = (act0,subgoal,act1)
98                      new_cls = node.causal_links + [new_clink]
99                      for consts2 in
                            self.protect_cl_for_actions(node.actions,consts1,new_clink):
100                         yield Arc(node,
101                                 POP_node(node.actions,consts2,new_agenda,new_cls),
102                                 cost=0)
103             for a0 in self.planning_problem.prob_domain.actions: #a0 is an
                    action
104                 if self.achieves(a0, subgoal):
105                     #a0 acheieves subgoal
106                     new_a = Action_instance(a0)
107                     self.display(2," using new action",new_a)
108                     new_actions = node.actions + [new_a]
109                     consts1 =
                            self.add_constraint((self.start,new_a),node.constraints)
110                     consts2 = self.add_constraint((new_a,act1),consts1)
111                     new_agenda1 = new_agenda + [(pre,new_a) for pre in
                            a0.preconds.items()]
112                     new_clink = (new_a,subgoal,act1)
```

```
113                 new_cls = node.causal_links + [new_clink]
114                 for consts3 in
                        self.protect_all_cls(node.causal_links,new_a,consts2):
115                     for consts4 in
                            self.protect_cl_for_actions(node.actions,consts3,new_clink):
116                         yield Arc(node,
117                                   POP_node(new_actions,consts4,new_agenda1,new_cls),
118                                   cost=1)
```

Given a casual link $(a0, subgoal, a1)$, the following method protects the causal link from each action in *actions*. Whenever an action deletes *subgoal*, the action needs to be before $a0$ or after $a1$. This method enumerates all constraints that result from protecting the causal link from all actions.

stripsPOP.py — (continued)

```
120     def protect_cl_for_actions(self, actions, constrs, clink):
121         """yields constraints that extend constrs and
122         protect causal link (a0, subgoal, a1)
123         for each action in actions
124         """
125         if actions:
126             a = actions[0]
127             rem_actions = actions[1:]
128             a0, subgoal, a1 = clink
129             if a != a0 and a != a1 and self.deletes(a,subgoal):
130                 if self.possible((a,a0),constrs):
131                     new_const = self.add_constraint((a,a0),constrs)
132                     for e in
                            self.protect_cl_for_actions(rem_actions,new_const,clink):
                            yield e # could be "yield from"
133                 if self.possible((a1,a),constrs):
134                     new_const = self.add_constraint((a1,a),constrs)
135                     for e in
                            self.protect_cl_for_actions(rem_actions,new_const,clink):
                            yield e
136             else:
137                 for e in
                        self.protect_cl_for_actions(rem_actions,constrs,clink):
                        yield e
138         else:
139             yield constrs
```

Given an action *act*, the following method protects all the causal links in *clinks* from *act*. Whenever *act* deletes *subgoal* from some causal link $(a0, subgoal, a1)$, the action *act* needs to be before $a0$ or after $a1$. This method enumerates all constraints that result from protecting the causal links from *act*.

stripsPOP.py — (continued)

```
141     def protect_all_cls(self, clinks, act, constrs):
142         """yields constraints that protect all causal links from act"""
143         if clinks:
```

```
144             (a0,cond,a1) = clinks[0] # select a causal link
145             rem_clinks = clinks[1:] # remaining causal links
146             if act != a0 and act != a1 and self.deletes(act,cond):
147                 if self.possible((act,a0),constrs):
148                     new_const = self.add_constraint((act,a0),constrs)
149                     for e in self.protect_all_cls(rem_clinks,act,new_const):
                            yield e
150                 if self.possible((a1,act),constrs):
151                     new_const = self.add_constraint((a1,act),constrs)
152                     for e in self.protect_all_cls(rem_clinks,act,new_const):
                            yield e
153             else:
154                 for e in self.protect_all_cls(rem_clinks,act,constrs): yield
                        e
155         else:
156             yield constrs
```

The following methods check whether an action (or action instance) achieves or deletes some subgoal.

stripsPOP.py — (continued)

```
158     def achieves(self,action,subgoal):
159         var,val = subgoal
160         return var in self.effects(action) and self.effects(action)[var] ==
                val
161
162     def deletes(self,action,subgoal):
163         var,val = subgoal
164         return var in self.effects(action) and self.effects(action)[var] !=
                val
165
166     def effects(self,action):
167         """returns the variable:value dictionary of the effects of action.
168         works for both actions and action instances"""
169         if isinstance(action, Action_instance):
170             action = action.action
171         if action == "start":
172             return self.planning_problem.initial_state
173         elif action == "finish":
174             return {}
175         else:
176             return action.effects
```

The constraints are represented as a set of pairs closed under transitivity. Thus if (a, b) and (b, c) are the list, then (a, c) must also be in the list. This means that adding a new constraint means adding the implied pairs, but querying whether some order is consistent is quick.

stripsPOP.py — (continued)

```
178     def add_constraint(self, pair, const):
179         if pair in const:
```

```
            return const
        todo = [pair]
        newconst = const.copy()
        while todo:
            x0,x1 = todo.pop()
            newconst.add((x0,x1))
            for x,y in newconst:
                if x==x1 and (x0,y) not in newconst:
                    todo.append((x0,y))
                if y==x0 and (x,x1) not in newconst:
                    todo.append((x,x1))
        return newconst

    def possible(self,pair,constraint):
        (x,y) = pair
        return (y,x) not in constraint
```

Some code for testing:

stripsPOP.py — (continued)

```
from searchBranchAndBound import DF_branch_and_bound
from searchMPP import SearcherMPP
from stripsProblem import problem0, problem1, problem2, blocks1, blocks2,
    blocks3

rplanning0 = POP_search_from_STRIPS(problem0)
rplanning1 = POP_search_from_STRIPS(problem1)
rplanning2 = POP_search_from_STRIPS(problem2)
searcher0 = DF_branch_and_bound(rplanning0,5)
searcher0a = SearcherMPP(rplanning0)
searcher1 = DF_branch_and_bound(rplanning1,10)
searcher1a = SearcherMPP(rplanning1)
searcher2 = DF_branch_and_bound(rplanning2,10)
searcher2a = SearcherMPP(rplanning2)
# Try one of the following searchers
# a = searcher0.search()
# a = searcher0a.search()
# a.end().extract_plan() # print a plan found
# a.end().constraints  # print the constraints
# SearcherMPP.max_display_level = 0 # less detailed display
# DF_branch_and_bound.max_display_level = 0 # less detailed display
# a = searcher1.search()
# a = searcher1a.search()
# a = searcher2.search()
# a = searcher2a.search()
```

Chapter 7

Supervised Machine Learning

This chapter is the first on machine learning. It covers the following topics:

- Data: how to load it, training and test sets
- Features: many of the features come directly from the data. Sometimes it is useful to construct fratures, e.g. $height > 1.9m$ might be a Boolean feature constructed from the real-values feature *height*. The next chapter is about how to learn these features; in this chapter we construct them by hand, in what is often known a **feature engineering**.
- Learning with no input features: this is the base case of many methods. What should we predict if we have no input features? This is basic knowledge that everyone doing machine learning should know.
- Decision tree learning: ane of the classic and simplest learning algorithms, which is the basis of many other algorithms.
- Cross validations and parameter tuning: methods to prevent overfitting.
- Linear regression and classification: other classic and simple techniques that often work well (particularly combined with feature learning or engineering).
- Boosting: combining simpler learning methods to make even better learners.

A good source of classic datasets is the UCI machine Learning Repository [Lichman, 2013]. The SPECT and car datasets are from this repository.

7.1 Representations of Data and Predictions

The code uses the following definitions and conventions:

- A **data set** is an enumeration of examples.
- An **example** is a list (or tuple) of feature values. The feature values can be numbers or strings.
- A **feature** is a function from examples into the range of the feature. We assume each feature has a variable `frange` that gives the range of the feature.

  A **Boolean feature** is a function from the examples into $\{False, True\}$. So, if f is a Boolean feature, $f.frange == [False, True]$, and if e is an example, $f(e)$ is either *True* or *False*.

  The `__doc__` variable of the function contains the docstring, a string description of the function.

learnProblem.py — A Learning Problem

```
11 import math, random
12 import csv
13 from display import Displayable
14
15 boolean = [False, True]
```

When creating a data set, we partition the data into a training set (*train*) and a test set (*test*). The target feature is the feature that we are making a prediction of.

learnProblem.py — (continued)

```
17 class Data_set(Displayable):
18     """ A data set consists of a list of training data and a list of test
           data.
19     """
20     seed = None #123456 # make it None for a different test set each time
21
22     def __init__(self, train, test=None, prob_test=0.30, target_index=0,
           header=None):
23         """A dataset for learning.
24         train is a list of tuples representing the training examples
25         test is the list of tuples representing the test examples
26         if test is None, a test set is created by selecting each
27             example with probability prob_test
28         target_index is the index of the target. If negative, it counts
               from right.
29             If target_index is larger than the number of properties,
30             there is no target (for unsupervised learning)
31         header is a list of names for the features
32         """
```

```
33        if test is None:
34            train,test = partition_data(train, prob_test, seed=self.seed)
35        self.train = train
36        self.test = test
37        self.display(1,f"Training set has {len(train)} examples. Number of
              columns in ",{len(e) for e in train})
38        self.display(1,f"Test set has {len(test)} examples. Number of
              columns in ",{len(e) for e in test})
39        self.prob_test = prob_test
40        self.num_properties = len(self.train[0])
41        if target_index < 0: #allows for -1, -2, etc.
42            target_index = self.num_properties + target_index
43        self.target_index = target_index
44        self.header = header
45        self.create_features()
46        self.display(1,"There are",len(self.input_features),"input
              features")
```

Initially we assume that the properties can be mapped directly into features. If all values are 0 or 1 they can be used as Boolean features. This can be overridden to allow for more general features.

learnProblem.py — (continued)

```
48    def create_features(self):
49        """create the input features and target feature.
50        This assumes that the features all have range {0,1}.
51        This should be overridden if the features have a different range.
52        """
53        self.input_features = []
54        for i in range(self.num_properties):
55            def feat(e,index=i):
56                return e[index]
57            if self.header:
58                feat.__doc__ = self.header[i]
59            else:
60                feat.__doc__ = "e["+str(i)+"]"
61            feat.frange = [0,1]
62            if i == self.target_index:
63                self.target = feat
64            else:
65                self.input_features.append(feat)
```

7.1.1 Evaluating Predictions

A **predictor** is a function that takes an example and makes a prediction on the value of the target feature.

An error measure takes a prediction and the actual value and returns a non-negative real number, such that the error for a dataset is the mean of the errors for each example. We assume that a lower error is better.

The function *evaluate_dataset* returns the average error for each example, where the error for each example depends on the evaluation criteria. Here we consider three evaluation criteria, the squared error (average of the square of the difference between the actual and predicted values), absolute errors(average of the absolute difference between the actual and predicted values) and the logloss (the a average negative log-likelihood, which can be interpreted as the number of bits to describe an example using a code based on the prediction treated as a probability).

learnProblem.py — (continued)

```
    def evaluate_dataset(self, data, predictor, error_measure):
        """Evaluates predictor on data according to the error_measure
        predictor is a function that takes an example and returns a
                prediction for the target feature.
        error_measure(prediction,actual) -> non-negative reals
        """
        if data:
            try:
                error = mean(error_measure(predictor(e), self.target(e))
                             for e in data)
            except ValueError:
                return float("inf") # infinity
            return error
```

The following evaluation criteria are defined. (Please keep the __doc__ strings a consistent length as they are used in tables.)

learnProblem.py — (continued)

```
def squared_error(prediction,actual):
    "squared error "
    return (prediction-actual)**2
def absolute_error(prediction, actual):
    "absolute error"
    return abs(prediction-actual)
def log_loss(prediction,actual):
    "logloss     "
    try:
      if actual==0:
        return -math.log2(1-prediction)
      else:
        return -math.log2(prediction)
    except ValueError:
        return float("inf") # infinity

evaluation_criteria = [squared_error, absolute_error, log_loss]
```

The following computes the mean of an enumeration, with an optional initial sum and initial count. This works for enumerations, even where *len*() is not defined, and only goes through the enumeration once. The obvious way to

compute a mean: $sum(enum)/len(enum)$ works for lists, but does not work for arbitrary enumerations.

learnProblem.py — (continued)

```
99  def mean(enum, isum=0, icount=0):
100     """returns the mean of enumeration enum,
101       isum is the initial sum, and icount is the initial count."""
102     for e in enum:
103         icount += 1
104         isum += e
105     return isum/icount
```

7.1.2 Creating Test and Training Sets

The following method partitions the data into a training set and a test set. Note that this does not guarantee that the test set will contain exactly a proportion of the data equal to *prob_test*.

[An alternative is to use *random.sample*() which can guarantee that the test set will contain exactly a particular proportion of the data. However this would require knowing how many elements are in the data set, which we may not know, as *data* may just be a generator of the data (e.g., when reading the data from a file).]

learnProblem.py — (continued)

```
107 def partition_data(data, prob_test=0.30, seed=None):
108     """partitions the data into a training set and a test set, where
109     prob_test is the probability of each example being in the test set.
110     """
111     train = []
112     test = []
113     if seed:    # given seed makes the partition consistent from run-to-run
114         random.seed(seed)
115     for example in data:
116         if random.random() < prob_test:
117             test.append(example)
118         else:
119             train.append(example)
120     return train, test
```

7.1.3 Importing Data From File

A data set is typically loaded from a file. The default here is that it loaded from a CSV (comma separated values) file, although the default separator can be changed. This assumes that all lines that contain the separator are valid data (so we only include those data items that contain more than one element). This allows for blank lines and comment lines that do not contain the separator.

However, it means that this method is not suitable for cases where there is only one feature.

Note that *data_all* and *data_tuples* are generators. *data_all* is a generator of a list of list of strings. This version assumes that CSV files are simple. The standard *csv* package, that allows quoted arguments, can be used by uncommenting the line for *data_all* and commenting out the following line. *data_tuples* contains only those lines that contain the delimiter (others lines are assumed to be empty or comments), and tries to convert the elements to numbers whenever possible.

This allows for some of the columns to be included; specified by *include_only*. Note that if *include_only* is specified, the target index is the column in the remaining columns.

learnProblem.py — (continued)

```
122 class Data_from_file(Data_set):
123     def __init__(self, file_name, separator=',', num_train=None,
            prob_test=0.3,
124                 has_header=False, target_index=0, boolean_features=True,
125                 categorical=[], include_only=None):
126         """create a dataset from a file
127         separator is the character that separates the attributes
128         num_train is a number n specifying the first n tuples are training,
                or None
129         prob_test is the probability an example should in the test set (if
                num_train is None)
130         has_header is True if the first line of file is a header
131         target_index specifies which feature is the target
132         boolean_features specifies whether we want to create Boolean
                features
133            (if False, it uses the original features).
134         categorical is a set (or list) of features that should be treated
                as categorical
135         include_only is a list or set of indexes of columns to include
136         """
137         self.boolean_features = boolean_features
138         with open(file_name,'r',newline='') as csvfile:
139             # data_all = csv.reader(csvfile,delimiter=separator) # for more
                    complicated CSV files
140             data_all = (line.strip().split(separator) for line in csvfile)
141             if include_only is not None:
142                 data_all = ([v for (i,v) in enumerate(line) if i in
                        include_only]
143                             for line in data_all)
144             if has_header:
145                 header = next(data_all)
146             else:
147                 header = None
148             data_tuples = (interpret_elements(d) for d in data_all if
                    len(d)>1)
```

```
149             if num_train is not None:
150                 # training set is divided into training then text examples
151                 # the file is only read once, and the data is placed in
                        appropriate list
152                 train = []
153                 for i in range(num_train):  # will give an error if
                        insufficient examples
154                     train.append(next(data_tuples))
155                 test = list(data_tuples)
156                 Data_set.__init__(self,train, test=test,
                        target_index=target_index,header=header)
157             else:    # randomly assign training and test examples
158                 Data_set.__init__(self,data_tuples, test=None,
                        prob_test=prob_test,
159                                   target_index=target_index, header=header)
160
161     def __str__(self):
162         if self.train and len(self.train)>0:
163             return ("Data: "+str(len(self.train))+" training examples, "
164                    +str(len(self.test))+" test examples, "
165                    +str(len(self.train[0]))+" features.")
166         else:
167             return ("Data: "+str(len(self.train))+" training examples, "
168                    +str(len(self.test))+" test examples.")
```

7.1.4 Creating Binary Features

Some of the algorithms require Boolean features or features with range $\{0, 1\}$. In order to be able to use these algorithms on datasets that allow for arbitrary ranges of input variables, we construct binary features from the attributes. This method overrides the method in *Data_set*.

There are 3 cases:

- When the range only has two values, we designate one to be the "true" value.
- When the values are all numeric, we assume they are ordered (as opposed to just being some classes that happen to be labelled with numbers) and construct Boolean features for splits of the data. That is, the feature is $e[ind] < cut$ for some value *cut*. We choose a number of *cut* values, up to a maximum number of cuts, given by *max_num_cuts*.
- When the values are not all numeric, we assume they are unordered, and create an indicator function for each value. An indicator function for a value returns true when that value is given and false otherwise. Note that we can't create an indicator function for values that appear in the test set but not in the training set because we haven't seen the test set.

For the examples in the test set with a value that doesn't appear in the training set for that feature, the indicator functions all return false.

learnProblem.py — (continued)

```
170     def create_features(self, max_num_cuts=8):
171         """creates boolean features from input features.
172         max_num_cuts is the maximum number of binary variables
173            to split a numerical feature into.
174         """
175         ranges = [set() for i in range(self.num_properties)]
176         for example in self.train:
177             for ind,val in enumerate(example):
178                 ranges[ind].add(val)
179         if self.target_index <= self.num_properties:
180             # If target_index is larger than the number of properties,
181             # there is no target (for unsupervised learning)
182             def target(e,index=self.target_index):
183                 return e[index]
184             if self.header:
185                 target.__doc__ = self.header[ind]
186             else:
187                 target.__doc__ = "e["+str(ind)+"]"
188             target.frange = ranges[self.target_index]
189             self.target = target
190         if self.boolean_features:
191             self.input_features = []
192             for ind,frange in enumerate(ranges):
193                 if ind != self.target_index and len(frange)>1:
194                     if len(frange) == 2:
195                         # two values, the feature is equality to one of them.
196                         true_val = list(frange)[1] # choose one as true
197                         def feat(e, i=ind, tv=true_val):
198                             return e[i]==tv
199                         if self.header:
200                             feat.__doc__ = self.header[ind]+"=="+str(true_val)
201                         else:
202                             feat.__doc__ = "e["+str(ind)+"]=="+str(true_val)
203                         feat.frange = boolean
204                         self.input_features.append(feat)
205                     elif all(isinstance(val,(int,float)) for val in frange):
206                         # all numeric, create cuts of the data
207                         sorted_frange = sorted(frange)
208                         num_cuts = min(max_num_cuts,len(frange))
209                         cut_positions = [len(frange)*i//num_cuts for i in
                                 range(1,num_cuts)]
210                         for cut in cut_positions:
211                             cutat = sorted_frange[cut]
212                             def feat(e, ind_=ind, cutat=cutat):
213                                 return e[ind_] < cutat
214
```

```
                        if self.header:
                            feat.__doc__ = self.header[ind]+"<"+str(cutat)
                        else:
                            feat.__doc__ = "e["+str(ind)+"]<"+str(cutat)
                        feat.frange = boolean
                        self.input_features.append(feat)
                else:
                    # create an indicator function for every value
                    for val in frange:
                        def feat(e, ind_=ind, val_=val):
                            return e[ind_] == val_
                        if self.header:
                            feat.__doc__ = self.header[ind]+"=="+str(val)
                        else:
                            feat.__doc__= "e["+str(ind)+"]=="+str(val)
                        feat.frange = boolean
                        self.input_features.append(feat)
        else: # boolean_features is off
            self.input_features = []
            for i in range(self.num_properties):
                def feat(e,index=i):
                    return e[index]
                if self.header:
                    feat.__doc__ = self.header[i]
                else:
                     feat.__doc__ = "e["+str(i)+"]"
                feat.frange = ranges[i]
                if i == self.target_index:
                    self.target = feat
                else:
                    self.input_features.append(feat)
```

Exercise 7.1 Change the code so that it splits using $e[ind] \leq cut$ instead of $e[ind] < cut$. Check boundary cases, such as 3 elements with 2 cuts. As a test case, make sure that when the range is the 30 integers from 100 to 129, and you want 2 cuts, the resulting Boolean features should be $e[ind] \leq 109$ and $e[ind] \leq 119$ to make sure that each of the resulting ranges is equal size.

Exercise 7.2 This splits on whether the feature is less than one of the values in the training set. Sam suggested it might be better to split between the values in the training set, and suggested using

$$cutat = (sorted\_frange[cut] + sorted\_frange[cut-1])/2$$

Why might Sam have suggested this? Does this work better? (Try it on a few data sets).

When reading from a file all of the values are strings. This next method tries to convert each values into a number (an int or a float) or Boolean, if it is possible.

learnProblem.py — (continued)

```
246 def interpret_elements(str_list):
247     """make the elements of string list str_list numerical if possible.
248     Otherwise remove initial and trailing spaces.
249     """
250     res = []
251     for e in str_list:
252         try:
253             res.append(int(e))
254         except ValueError:
255             try:
256                 res.append(float(e))
257             except ValueError:
258                 se = e.strip()
259                 if se in ["True","true","TRUE"]:
260                     res.append[True]
261                 if se in ["False","false","FALSE"]:
262                     res.append[False]
263                 else:
264                     res.append(e.strip())
265     return res
```

7.1.5 Augmented Features

Sometimes we want to augment the features with new features computed from the old features (eg. the product of features). Here we allow the creation of a new dataset from an old dataset but with new features. Note that these are sometimes called **kernel**s; mapping the original feature space into a new space, from which we can use standard learning tools. For those interested in the mathematics, read about support vector machines, which have neat way to do learning in the augmented space (the "kernel trick") that is beyond the scope of AIPython (currently).

A feature is a function of examples. A unary feature constructor takes a feature and returns a new feature. A binary feature combiner takes two features and returns a new feature.

learnProblem.py — (continued)

```
267 class Data_set_augmented(Data_set):
268     def __init__(self, dataset, unary_functions=[], binary_functions=[],
          include_orig=True):
269         """creates a dataset like dataset but with new features
270         unary_function is a list of unary feature constructors
271         binary_functions is a list of binary feature combiners.
272         include_orig specifies whether the original features should be
              included
273         """
274         self.orig_dataset = dataset
275         self.unary_functions = unary_functions
```

```
276         self.binary_functions = binary_functions
277         self.include_orig = include_orig
278         self.target = dataset.target
279         Data_set.__init__(self,dataset.train, test=dataset.test,
280                           target_index = dataset.target_index)
281
282     def create_features(self):
283         if self.include_orig:
284             self.input_features = self.orig_dataset.input_features.copy()
285         else:
286             self.input_features = []
287         for u in self.unary_functions:
288             for f in self.orig_dataset.input_features:
289                 self.input_features.append(u(f))
290         for b in self.binary_functions:
291             for f1 in self.orig_dataset.input_features:
292                 for f2 in self.orig_dataset.input_features:
293                     if f1 != f2:
294                         self.input_features.append(b(f1,f2))
```

The following are useful unary feature constructors and binary feature combiner.

learnProblem.py — (continued)

```
296 def square(f):
297     """a unary feature constructor to construct the square of a feature
298     """
299     def sq(e):
300         return f(e)**2
301     sq.__doc__ = f.__doc__+"**2"
302     return sq
303
304 def power_feat(n):
305     """given n returns a unary feature constructor to construct the nth
            power of a feature.
306     e.g., power_feat(2) is the same as square, defined above
307     """
308     def fn(f,n=n):
309         def pow(e,n=n):
310             return f(e)**n
311         pow.__doc__ = f.__doc__+"**"+str(n)
312         return pow
313     return fn
314
315 def prod_feat(f1,f2):
316     """a new feature that is the product of features f1 and f2
317     """
318     def feat(e):
319         return f1(e)*f2(e)
320     feat.__doc__ = f1.__doc__+"*"+f2.__doc__
321     return feat
```

```
322
323 def eq_feat(f1,f2):
324     """a new feature that is 1 if f1 and f2 give same value
325     """
326     def feat(e):
327         return 1 if f1(e)==f2(e) else 0
328     feat.__doc__ = f1.__doc__+"=="+f2.__doc__
329     return feat
330
331 def neq_feat(f1,f2):
332     """a new feature that is 1 if f1 and f2 give different values
333     """
334     def feat(e):
335         return 1 if f1(e)!=f2(e) else 0
336     feat.__doc__ = f1.__doc__+"!="+f2.__doc__
337     return feat
```

Example:

learnProblem.py — (continued)

```
339 # from learnProblem import Data_set_augmented,prod_feat
340 # data = Data_from_file('data/holiday.csv', num_train=19, target_index=-1)
341 ## data = Data_from_file('data/SPECT.csv', prob_test=0.5, target_index=0)
342 # dataplus = Data_set_augmented(data,[],[prod_feat])
343 # dataplus = Data_set_augmented(data,[],[prod_feat,neq_feat])
```

Exercise 7.3 For symmetric properties, such as product, we don't need both $f1 * f2$ as well as $f2 * f1$ as extra properties. Allow the user to be able to declare feature constructors as symmetric (by associating a Boolean feature with them). Change *construct_features* so that it does not create both versions for symmetric combiners.

7.2 Generic Learner Interface

A **learner** takes a dataset (and possibly other arguments specific to the method). To get it to learn, we call the *learn*() method. This implements *Displayable* so that we can display traces at multiple levels of detail (and perhaps with a GUI).

learnProblem.py — (continued)

```
344 from display import Displayable
345
346 class Learner(Displayable):
347     def __init__(self, dataset):
348         raise NotImplementedError("Learner.__init__") # abstract method
349
350     def learn(self):
351         """returns a predictor, a function from a tuple to a value for the
            target feature
```

```
352         """
353         raise NotImplementedError("learn") # abstract method
```

7.3 Learning With No Input Features

If we make the same prediction for each example, what prediction should we make? This can be used as a naive baseline; if a method does not do better than this, the the input features do not provide any useful information for the prediction. It is also the base case for some methods, such as decision-tree learning.

To run demo to compare different prediction methods on various evaluation criteria, in folder "aipython", load "learnNoInputs.py", using e.g., `ipython -i learnNoInputs.py`, and it prints some test results.

There are a few alternatives as to what could be allowed in a prediction:

- a point prediction, where we are only allowed to predict one of the values of the feature. For example, if the values of the feature are $\{0, 1\}$ we are only allowed to predict 0 or 1 or of the values are ratings in $\{1, 2, 3, 4, 5\}$, we can only predict one of these integers.
- a point prediction, where we are allowed to predict any value. For example, if the values of the feature are $\{0, 1\}$ we may be allowed to predict 0.3, 1, or even 1.7. For all of the criteria we can imagine, there is no point in predicting a value greater than 1 or less that zero (but that doesn't mean we can't), but it is often useful to predict a value between 0 and 1. If the values are ratings in $\{1, 2, 3, 4, 5\}$, we may want to predict 3.4.
- a probability distribution over the values of the feature. For each value v, we predict a non-negative number p_v, such that the sum over all predictions is 1.

The following code assumes the second of these, where we can make a point prediction of any value (although median will only predict one of the actual values for the feature). The third can be implemented by having multiple indicator functions for the target.

Here are some prediction functions that take in a dataset of number and returns a prediction for the next case. Note that median will average the two middle values when there are an even number of examples. The mode will pick one of the values arbitrarily (here the larger) when more than one value has the maximum number of elements. So the median of $[0, 1]$ is 0.5, but the mode is 1.

learnNoInputs.py — Learning ignoring all input features

```
11 from learnProblem import squared_error, absolute_error, log_loss, mean
```

```
12 import math, random, statistics
13 import utilities # argmax for (element,value) pairs
14
15 class Predict(object):
16     """The class of prediction methods for a list of numbers
17     Please make the doc strings the same length, because they are used in
           tables.
18     Note that we don't need self argument, as we are creating Predict
           objects,
19     To use call Predict.laplace(data) etc."""
20
21     def mean(data):
22         "mean       "
23         return mean(data)
24
25     def bounded_mean(data, bound=0.01):
26         "bounded mean"
27         return min(max(mean(data),bound),1-bound)
28
29     def laplace(data):
30         "Laplace    "  # for Boolean (or 0/1 data only)
31         return mean(data, isum=1, icount=2)
32
33     def mode(data):
34         "mode       "
35         counts = {}
36         for e in data:
37             if e in counts:
38                 counts[e] += 1
39             else:
40                 counts[e] = 1
41         return utilities.argmaxe(counts.items())
42
43     def median(data):
44         "median     "
45         return statistics.median(data)
46
47     all = [mean, bounded_mean, laplace, mode, median]
```

7.3.1 Evaluation

To evaluate a point prediction, we first generate some data from a simple (Bernoulli) distribution, where there are two possible values, 0 and 1 for the target feature. Given *prob*, a number in the range $[0, 1]$, this generate some training and test data where *prob* is the probability of each example being 1. To generate a 1 with probability *prob*, we generarate a random number in range [0,1] and return 1 if that number is less than *prob*.

learnNoInputs.py — (continued)

```
49 def evaluate(train_size, predictor, error_measure, num_samples=10000,
       test_size=10 ):
50     """return the average error when
51    train_size is the number of training examples
52    predictor(training) -> [0,1]
53    error_measure(prediction,actual) -> non-negative reals
54    """
55     error = 0
56     for sample in range(num_samples):
57         prob = random.random()
58         training = [1 if random.random()<prob else 0 for i in
               range(train_size)]
59         prediction = predictor(training)
60         test = (1 if random.random()<prob else 0 for i in range(test_size))
61         error += sum( error_measure(prediction,actual) for actual in
               test)/test_size
62     return error/num_samples
```

Let's evaluate the predictions of the possible selections according to the different evaluation criteria, for various training sizes.

learnNoInputs.py — (continued)

```
64 def test_no_inputs(error_measures = [squared_error, absolute_error,
       log_loss]):
65     for train_size in [1,2,3,4,5,10,20,100,1000]:
66         print("For training size",train_size,":")
67         print("  Predictor","\t".join(error_measure.__doc__ for
68                                       error_measure in
                                           error_measures),sep="\t")
69         for predictor in Predict.all:
70             print(f"  {predictor.__doc__}",
71                       "\t".join("{:.7f}".format(evaluate(train_size,
                              predictor, error_measure))
72                                 for error_measure in
                                       error_measures),sep="\t")
73
74 if __name__ == "__main__":
75     test_no_inputs()
```

Exercise 7.4 Wich predictor works best for low counts when the error is

(a) Squared error

(b) Absolute error

(c) Log loss

You may need to try this a few times to make sure your answer is supported by the evidence. Does the difference from the other methods get more ot less as the number of examples grow?

Exercise 7.5 Suggest some other predictions that only take the tarining data. Does your method do better than the given methods? A simple way to get other

predictors is to vary the threshold of bounded average, or to change the pseodo-counts of the Laplace method (use other numbers instead of 1 and 2).

7.4 Decision Tree Learning

To run the decision tree learning demo, in folder "aipython", load "learnDT.py", using e.g., `ipython -i learnDT.py`, and it prints some test results. To try more examples, copy and paste the commented-out commands at the bottom of that file. This requires Python 3 with `matplotlib`.

The decision tree algorithm does binary splits, and assumes that all input features are binary functions of the examples. It stops splitting if there are no input features, the number of examples is less than a specified number of examples or all of the examples agree on the target feature.

learnDT.py — Learning a binary decision tree

```
from learnProblem import Learner, squared_error, absolute_error, log_loss, mean
from learnNoInputs import Predict
import math

class DT_learner(Learner):
    def __init__(self,
                 dataset,
                 split_to_optimize=log_loss,       # to minimize for at each split
                 leaf_prediction=Predict.mean, # what to use for point prediction at leaves
                 train=None,                  # used for cross validation
                 min_number_examples=10):
        self.dataset = dataset
        self.target = dataset.target
        self.split_to_optimize = split_to_optimize
        self.leaf_prediction = leaf_prediction
        self.min_number_examples = min_number_examples
        if train is None:
            self.train = self.dataset.train
        else:
            self.train = train

    def learn(self):
        return self.learn_tree(self.dataset.input_features, self.train)
```

The main recursive algorithm, takes in a set of input features and a set of training data. It first decides whether to split. If it doesn't split, it makes a point prediction, ignoring the input features.

It splits unless:

- there are no more input features
- there are fewer examples than *min_number_examples*,
- all the examples agree on the value of the target, or
- the best split makes all examples in the same partition.

If it splits, it selects the best split according to the evaluation critereon (assuming that is the only split it gets to do), and returns the condition to split on (in the variable *split*) and the corresponding partition of the examples.

learnDT.py — (continued)

```
35      def learn_tree(self, input_features, data_subset):
36          """returns a decision tree
37          for input_features is a set of possible conditions
38          data_subset is a subset of the data used to build this (sub)tree
39
40          where a decision tree is a function that takes an example and
41          makes a prediction on the target feature
42          """
43          if (input_features and len(data_subset) >=
                self.min_number_examples):
44              first_target_val = self.target(data_subset[0])
45              allagree = all(self.target(inst)==first_target_val for inst in
                    data_subset)
46              if not allagree:
47                  split, partn = self.select_split(input_features, data_subset)
48                  if split: # the split succeeded in splitting the data
49                      false_examples, true_examples = partn
50                      rem_features = [fe for fe in input_features if fe !=
                            split]
51                      self.display(2,"Splitting on",split.__doc__,"with
                            examples split",
52                                   len(true_examples),":",len(false_examples))
53                      true_tree = self.learn_tree(rem_features,true_examples)
54                      false_tree = self.learn_tree(rem_features,false_examples)
55                      def fun(e):
56                          if split(e):
57                              return true_tree(e)
58                          else:
59                              return false_tree(e)
60                      #fun = lambda e: true_tree(e) if split(e) else
                            false_tree(e)
61                      fun.__doc__ = ("if "+split.__doc__+" then
                            ("+true_tree.__doc__+
62                                     ") else ("+false_tree.__doc__+")")
63                      return fun
64          # don't expand the trees but return a point prediction
65          prediction = self.leaf_prediction(self.target(e) for e in
                data_subset)
```

```
66          def leaf_fun(e):
67              return prediction
68          leaf_fun.__doc__ = "{:.7f}".format(prediction)
69          return leaf_fun
```

learnDT.py — (continued)

```
71      def select_split(self, input_features, data_subset):
72          """finds best feature to split on.
73
74          input_features is a non-empty list of features.
75          returns feature, partition
76          where feature is an input feature with the smallest error as
77                judged by split_to_optimize or
78                feature==None if there are no splits that improve the error
79          partition is a pair (false_examples, true_examples) if feature is
                not None
80          """
81          best_feat = None # best feature
82          # best_error = float("inf") # infinity - more than any error
83          best_error = training_error(self.dataset.target, data_subset,
84                                      self.split_to_optimize,
                                            self.leaf_prediction)
85          best_partition = None
86          for feat in input_features:
87              false_examples, true_examples = partition(data_subset,feat)
88              if false_examples and true_examples: #both partitons are
                    non-empty
89                  err = (training_error(self.dataset.target, false_examples,
90                                        self.split_to_optimize,
                                              self.leaf_prediction)
91                         + training_error(self.dataset.target, true_examples,
92                                          self.split_to_optimize,
                                                self.leaf_prediction))
93                  self.display(3," split on",feat.__doc__,"has error=",err,
94                               "splits
                                     into",len(true_examples),":",len(false_examples))
95                  if err < best_error:
96                      best_feat = feat
97                      best_error=err
98                      best_partition = false_examples, true_examples
99          self.display(3,"best split is on",best_feat.__doc__,
100                           "with err=",best_error)
101         return best_feat, best_partition
102
103 def partition(data_subset,feature):
104     """partitions the data_subset by the feature"""
105     true_examples = []
106     false_examples = []
107     for example in data_subset:
108         if feature(example):
```

```
109             true_examples.append(example)
110         else:
111             false_examples.append(example)
112     return false_examples, true_examples
113
114
115 def training_error(target, data_subset, eval_critereon, leaf_prediction):
116     """returns training error for dataset on the target (with no more
            splits)
117     We make a single prediction using leaf_prediction
118     It is evaluated using eval_critereon for each example
119     """
120     prediction = leaf_prediction(target(e) for e in data_subset)
121     error = sum(eval_critereon(prediction, target(e))
122                 for e in data_subset)
123     return error
```

Test cases:

learnDT.py — (continued)

```
125 from learnProblem import Data_set, Data_from_file
126
127 def testDT(data, print_tree=True, selections = Predict.all):
128     """Prints errors and the trees for various evaluation criteria and ways
            to select leaves.
129     """
130     evaluation_criteria = [squared_error, absolute_error, log_loss]
131     print("Split Choice","Leaf Choice",'\t'.join(ecrit.__doc__
132                                                 for ecrit in
                                                        evaluation_criteria),sep="\t")
133     for crit in evaluation_criteria:
134         for leaf in selections:
135             tree = DT_learner(data, split_to_optimize=crit,
                        leaf_prediction=leaf).learn()
136             print(crit.__doc__, leaf.__doc__,
137                   "\t".join("{:.7f}".format(data.evaluate_dataset(data.test,
                              tree, ecrit))
138                             for ecrit in evaluation_criteria),sep="\t")
139             if print_tree:
140                 print(tree.__doc__)
141
142 if __name__ == "__main__":
143     print("SPECT.csv"); testDT(data=Data_from_file('data/SPECT.csv',
            target_index=0), print_tree=False)
144     # print("carbool.csv"); testDT(data =
            Data_from_file('data/carbool.csv', target_index=-1))
145     # print("mail_reading.csv"); testDT(data =
            Data_from_file('data/mail_reading.csv', target_index=-1))
146     # print("holiday.csv"); testDT(data =
            Data_from_file('data/holiday.csv', num_train=19, target_index=-1))
```

Note that different runs may provide different values as they splt the training and test sets differetly. So if you have a hypothesis about what works better, make sure it is true for different runs.

Exercise 7.6 The current algorithm does not have a very sophisticated stopping criterion. What is the current stopping criterion? (Hint: you need to look at both *learn_tree* and *select_split*.)

Exercise 7.7 Extend the current algorithm to include in the stopping criterion

(a) A minimum child size; don't use a split if one of the children has fewer elements that this.

(b) A depth-bound on the depth of the tree.

(c) An improvement bound such that a split is only carried out if error with the split is better than the error without the split by at least the improvement bound.

Which values for these parameters make the prediction errors on the test set the smallest? Try it on more than one dataset.

Exercise 7.8 Without any input features, it is often better to include a pseudo-count that is added to the counts from the training data. Modify the code so that it includes a pseudo-count for the predictions. When evaluating a split, including pseudo counts can make the split worse than no split. Does pruning with an improvement bound and pseudo-counts make the algorithm work better than with an improvement bound by itself?

Exercise 7.9 Some people have suggested using information gain (which is equivalent to greedy optimization of logloss) as the measure of improvement when building the tree, even in they want to have non-probabilistic predictions in the final tree. Does this work better than myopically choosing the split that is best for the evaluation criteria we will use to judge the final prediction?

7.5 Cross Validation and Parameter Tuning

To run the cross validation demo, in folder "aipython", load "learnCrossValidation.py", using e.g., `ipython -i learnCrossValidation.py`. Run the examples at the end to produce a graph like Figure 7.15. Note that different runs will produce different graphs, so your graph will not look like the one in the textbook. To try more examples, copy and paste the commented-out commands at the bottom of that file. This requires Python 3 with `matplotlib`.

The above decision tree overfits the data. One way to determine whether the prediction is overfitting is by cross validation. The code below implements k-fold cross validation, which can be used to choose the value of parameters to best fit the training data. If we want to use parameter tuning to improve

predictions on a particular data set, we can only use the training data (and not the test data) to tune the parameter.

In k-fold cross validation, we partition the training set into k approximately equal-sized folds (each fold is an enumeration of examples). For each fold, we train on the other examples, and determine the error of the prediction on that fold. For example, if there are 10 folds, we train on 90% of the data, and then test on remaining 10% of the data. We do this 10 times, so that each example gets used as a test set once, and in the training set 9 times.

The code below creates one copy of the data, and multiple views of the data. For each fold, *fold* enumerates the examples in the fold, and *fold_complement* enumerates the examples not in the fold.

learnCrossValidation.py — Cross Validation for Parameter Tuning

```
from learnProblem import Data_set, Data_from_file, squared_error,
    absolute_error, log_loss
from learnDT import DT_learner
import matplotlib.pyplot as plt
import random

class K_fold_dataset(object):
    def __init__(self, training_set, num_folds):
        self.data = training_set.train.copy()
        self.target = training_set.target
        self.input_features = training_set.input_features
        self.num_folds = num_folds
        random.shuffle(self.data)
        self.fold_boundaries = [(len(self.data)*i)//num_folds
                                for i in range(0,num_folds+1)]

    def fold(self, fold_num):
        for i in range(self.fold_boundaries[fold_num],
                       self.fold_boundaries[fold_num+1]):
            yield self.data[i]

    def fold_complement(self, fold_num):
        for i in range(0,self.fold_boundaries[fold_num]):
            yield self.data[i]
        for i in range(self.fold_boundaries[fold_num+1],len(self.data)):
            yield self.data[i]
```

The validation error is the average error for each example, where we test on each fold, and learn on the other folds.

learnCrossValidation.py — (continued)

```
    def validation_error(self, learner, error_measure, **other_params):
        error = 0
        try:
            for i in range(self.num_folds):
                predictor = learner(self,
                    train=list(self.fold_complement(i)),
```

```
                                **other_params).learn()
                error += sum( error_measure(predictor(e), self.target(e))
                              for e in self.fold(i))
        except ValueError:
            return float("inf") #infinity
        return error/len(self.data)
```

The *plot_error* method plots the average error as a function of a the minimun number of examples in decision-tree search, both for the validation set and for the test set. The error on the validation set can be used to tune the parameter — choose the value of the parameter that minimizes the error. The error on the test set cannot be used to tune the parameters; if is were to be used this way then it cannot be used to test.

learnCrossValidation.py — (continued)

```
def plot_error(data, criterion=squared_error, num_folds=5,
    xscale='linear'):
    """Plots the error on the validation set and the test set
    with respect to settings of the minimum number of examples.
    xscale should be 'log' or 'linear'
    """
    plt.ion()
    plt.xscale(xscale) # change between log and linear scale
    plt.xlabel("minimum number of examples")
    plt.ylabel("average "+criterion.__doc__)
    folded_data = K_fold_dataset(data, num_folds)
    verrors = [] # validation errors
    terrors = [] # test set errors
    for mne in range(1,len(data.train)+2):
        verrors.append(folded_data.validation_error(DT_learner,criterion,
                                                    min_number_examples=mne))
        tree = DT_learner(data, criterion, min_number_examples=mne).learn()
        terrors.append(data.evaluate_dataset(data.test,tree,criterion))
    plt.plot(range(1,len(data.train)+2), verrors, ls='-',color='k',
                label="validation for "+criterion.__doc__)
    plt.plot(range(1,len(data.train)+2), terrors, ls='--',color='k',
                label="test set for "+criterion.__doc__)
    plt.legend()
    plt.draw()

# The following produces Figure 7.15 of Poole and Mackworth [2017]
# Different runs produce different plots, because folds change.
# data = Data_from_file('data/SPECT.csv',target_index=0)
# plot_error(data)  # warning, may take a long time depending on the
    dataset

#also try:
# data = Data_from_file('data/mail_reading.csv', target_index=-1)
# data = Data_from_file('data/carbool.csv', target_index=-1)
```

Note that different runs for the same data will have the same test error, but different validation error. If you rerun the `Data_from_file`, you will get the new test and training sets, and so the graph will change.

Exercise 7.10 Change the error plot so that it can evaluate the stopping criteria of the exercise of Section 7.6. Which criteria makes the most difference?

7.6 Linear Regression and Classification

Here we give a gradient descent searcher for linear regression and classification.

learnLinear.py — Linear Regression and Classification

```
from learnProblem import Learner
import random, math

class Linear_learner(Learner):
    def __init__(self, dataset, train=None,
                 learning_rate=0.1, max_init = 0.2,
                 squashed=True):
        """Creates a gradient descent searcher for a linear classifier.
        The main learning is carried out by learn()

        dataset provides the target and the input features
        train provides a subset of the training data to use
        number_iterations is the default number of steps of gradient descent
        learning_rate is the gradient descent step size
        max_init is the maximum absolute value of the initial weights
        squashed specifies whether the output is a squashed linear function
        """
        self.dataset = dataset
        self.target = dataset.target
        if train==None:
            self.train = self.dataset.train
        else:
            self.train = train
        self.learning_rate = learning_rate
        self.squashed = squashed
        self.input_features = [one]+dataset.input_features # one is defined
            below
        self.weights = {feat:random.uniform(-max_init,max_init)
                        for feat in self.input_features}
```

predictor predicts the value of an example from the current parameter settings. *predictor_string* gives a string representation of the predictor.

learnLinear.py — (continued)

```

    def predictor(self,e):
        """returns the prediction of the learner on example e"""
```

```
43          linpred = sum(w*f(e) for f,w in self.weights.items())
44          if self.squashed:
45              return sigmoid(linpred)
46          else:
47              return linpred
48
49      def predictor_string(self, sig_dig=3):
50          """returns the doc string for the current prediction function
51          sig_dig is the number of significant digits in the numbers"""
52          doc = "+".join(str(round(val,sig_dig))+"*"+feat.__doc__
53                         for feat,val in self.weights.items())
54          if self.squashed:
55              return "sigmoid("+ doc+")"
56          else:
57              return doc
```

learn is the main algorithm of the learner. It does *num_iter* steps of stochastic gradient descent with batch size = 1. The other parameters it gets from the class.

learnLinear.py — (continued)

```
59      def learn(self,num_iter=100):
60          for it in range(num_iter):
61              self.display(2,"prediction=",self.predictor_string())
62              for e in self.train:
63                  predicted = self.predictor(e)
64                  error = self.target(e) - predicted
65                  update = self.learning_rate*error
66                  for feat in self.weights:
67                      self.weights[feat] += update*feat(e)
68          #self.predictor.__doc__ = self.predictor_string()
69          #return self.predictor
```

one is a function that always returns 1. This is used for one of the input properties.

learnLinear.py — (continued)

```
71  def one(e):
72      "1"
73      return 1
```

$sigmoid(x)$ is the function

$$\frac{1}{1+e^{-x}}$$

The inverse of *sigmoid* is the *logit* function

learnLinear.py — (continued)

```
75  def sigmoid(x):
76      return 1/(1+math.exp(-x))
77
```

```
78 def logit(x):
79     return -math.log(1/x-1)
```

The following tests the learner on a data sets. Uncomment the other data sets for different examples.

learnLinear.py — (continued)

```
80 from learnProblem import Data_set, Data_from_file, evaluation_criteria
81 import matplotlib.pyplot as plt
82
83 def test(**args):
84     data = Data_from_file('data/SPECT.csv', target_index=0)
85     # data = Data_from_file('data/mail_reading.csv', target_index=-1)
86     # data = Data_from_file('data/carbool.csv', target_index=-1)
87     learner = Linear_learner(data,**args)
88     learner.learn()
89     print("function learned is", learner.predictor_string())
90     for ecrit in evaluation_criteria:
91         test_error = data.evaluate_dataset(data.test, learner.predictor,
               ecrit)
92         print("   Average", ecrit.__doc__, "error is", test_error)
```

The following plots the errors on the training and test sets as a function of the number of steps of gradient descent.

learnLinear.py — (continued)

```
94 def plot_steps(learner=None,
95                data = None,
96                criterion="sum-of-squares",
97                step=1,
98                num_steps=1000,
99                log_scale=True,
100               label=""):
101    """
102    plots the training and test error for a learner.
103    data is the
104    learner_class is the class of the learning algorithm
105    criterion gives the evaluation criterion plotted on the y-axis
106    step specifies how many steps are run for each point on the plot
107    num_steps is the number of points to plot
108
109    """
110    plt.ion()
111    plt.xlabel("step")
112    plt.ylabel("Average "+criterion+" error")
113    if log_scale:
114        plt.xscale('log') #plt.semilogx() #Makes a log scale
115    else:
116        plt.xscale('linear')
117    if data is None:
118        data = Data_from_file('data/holiday.csv', num_train=19,
               target_index=-1)
```

```
119         #data = Data_from_file('data/SPECT.csv', target_index=0)
120         # data = Data_from_file('data/mail_reading.csv', target_index=-1)
121         # data = Data_from_file('data/carbool.csv', target_index=-1)
122     random.seed(None)  # reset seed
123     if learner is None:
124         learner = Linear_learner(data)
125     train_errors = []
126     test_errors = []
127     for i in range(1,num_steps+1,step):
128         test_errors.append(data.evaluate_dataset(data.test,
                learner.predictor, criterion))
129         train_errors.append(data.evaluate_dataset(data.train,
                learner.predictor, criterion))
130         learner.display(2, "Train error:",train_errors[-1],
131                         "Test error:",test_errors[-1])
132         learner.learn(num_iter=step)
133     plt.plot(range(1,num_steps+1,step),train_errors,ls='-',c='k',label="training
            errors")
134     plt.plot(range(1,num_steps+1,step),test_errors,ls='--',c='k',label="test
            errors")
135     plt.legend()
136     plt.draw()
137     learner.display(1, "Train error:",train_errors[-1],
138                         "Test error:",test_errors[-1])
139
140 if __name__ == "__main__":
141     test()
142
143 # This generates the figure
144 # from learnProblem import Data_set_augmented,prod_feat
145 # data = Data_from_file('data/SPECT.csv', prob_test=0.5, target_index=0)
146 # dataplus = Data_set_augmented(data,[],[prod_feat])
147 # plot_steps(data=data,num_steps=10000)
148 # plot_steps(data=dataplus,num_steps=10000) # warning very slow
```

Exercise 7.11 The squashed learner only makes predictions in the range $(0,1)$. If the output values are $\{1,2,3,4\}$ there is no use prediction less than 1 or greater than 4. Change the squashed learner so that it can learn values in the range $(1,4)$. Test it on the file `'data/car.csv'`.

The following plots the prediction as a function of the function of the number of steps of gradient descent. We first define a version of *range* that allows for real numbers (integers and floats).

learnLinear.py — (continued)

```
149 def arange(start,stop,step):
150     """returns enumeration of values in the range [start,stop) separated by
            step.
151     like the built-in range(start,stop,step) but allows for integers and
            floats.
```

```
    Note that rounding errors are expected with real numbers. (or use
        numpy.arange)
    """
    while start<stop:
        yield start
        start += step

def plot_prediction(learner=None,
                data = None,
                minx = 0,
                maxx = 5,
                step_size = 0.01,  # for plotting
                label="function"):
    plt.ion()
    plt.xlabel("x")
    plt.ylabel("y")
    if data is None:
        data = Data_from_file('data/simp_regr.csv', prob_test=0,
                              boolean_features=False, target_index=-1)
    if learner is None:
        learner = Linear_learner(data,squashed=False)
    learner.learning_rate=0.001
    learner.learn(100)
    learner.learning_rate=0.0001
    learner.learn(1000)
    learner.learning_rate=0.00001
    learner.learn(10000)
    learner.display(1,"function learned is", learner.predictor_string(),
              "error=",data.evaluate_dataset(data.train, learner.predictor,
                  "sum-of-squares"))
    plt.plot([e[0] for e in data.train],[e[-1] for e in
        data.train],"bo",label="data")
    plt.plot(list(arange(minx,maxx,step_size)),[learner.predictor([x])
                                        for x in
                                            arange(minx,maxx,step_size)],
                                      label=label)
    plt.legend()
    plt.draw()
```

learnLinear.py — (continued)

```
from learnProblem import Data_set_augmented, power_feat
def plot_polynomials(data=None,
                learner_class = Linear_learner,
                max_degree=5,
                minx = 0,
                maxx = 5,
                num_iter = 100000,
                learning_rate = 0.0001,
                step_size = 0.01, # for plotting
                ):
```

```
197     plt.ion()
198     plt.xlabel("x")
199     plt.ylabel("y")
200     if data is None:
201         data = Data_from_file('data/simp_regr.csv', prob_test=0,
202                               boolean_features=False, target_index=-1)
203     plt.plot([e[0] for e in data.train],[e[-1] for e in
            data.train],"ko",label="data")
204     x_values = list(arange(minx,maxx,step_size))
205     line_styles = ['-','--','-.',':']
206     colors = ['0.5','k','k','k','k']
207     for degree in range(max_degree):
208         data_aug = Data_set_augmented(data,[power_feat(n) for n in
                range(1,degree+1)],
209                                       include_orig=False)
210         learner = learner_class(data_aug,squashed=False)
211         learner.learning_rate=learning_rate
212         learner.learn(num_iter)
213         learner.display(1,"For degree",degree,
214                         "function learned is", learner.predictor_string(),
215                         "error=",data.evaluate_dataset(data.train,
                            learner.predictor, "sum-of-squares"))
216         ls = line_styles[degree % len(line_styles)]
217         col = colors[degree % len(colors)]
218         plt.plot(x_values,[learner.predictor([x]) for x in x_values],
                linestyle=ls, color=col,
219                          label="degree="+str(degree))
220         plt.legend(loc='upper left')
221         plt.draw()
222
223 # Try:
224 # plot_prediction()
225 # plot_polynomials()
226 #data = Data_from_file('data/mail_reading.csv', target_index=-1)
227 #plot_prediction(data=data)
```

7.6.1 Batched Stochastic Gradient Descent

This implements batched stochastic gradient descent. If the batch size is 1, it can be simplified by not storing the differences in d, but applying them directly; this would the be equivalent to the original code!

This overrides the learner *Linear_learner*. Note that the comparison with regular gradient descent is unfair as the number of updates per step is not the same. (How could it me made more fair?)

learnLinearBSGD.py — Linear Learner with Batched Stochastic Gradient Descent

```
11 from learnLinear import Linear_learner
12 import random, math
13
```

```
class Linear_learner_bsgd(Linear_learner):
    def __init__(self, *args, batch_size=10, **kargs):
        Linear_learner.__init__(self, *args, **kargs)
        self.batch_size = batch_size

    def learn(self,num_iter=None):
        if num_iter is None:
            num_iter = self.number_iterations
        batch_size = min(self.batch_size, len(self.train))
        d = {feat:0 for feat in self.weights}
        for it in range(num_iter):
            self.display(2,"prediction=",self.predictor_string())
            for e in random.sample(self.train, batch_size):
                predicted = self.predictor(e)
                error = self.target(e) - predicted
                update = self.learning_rate*error
                for feat in self.weights:
                    d[feat] += update*feat(e)
            for feat in self.weights:
                self.weights[feat] += d[feat]
                d[feat]=0

# from learnLinear import plot_steps
# from learnProblem import Data_from_file
# data = Data_from_file('data/holiday.csv', target_index=-1)
# learner = Linear_learner_bsgd(data)
# plot_steps(learner = learner, data=data)

# to plot polynomials with batching (compare to SGD)
# from learnLinear import plot_polynomials
# plot_polynomials(learner_class = Linear_learner_bsgd)
```

7.7 Deep Neural Network Learning

This provides a modular implementation that implements the layers modularly. Layers can easily be configured in many configurations. A layer needs to implement a function to compute the output values from the inputs and a way to back-propagate the error.

learnNN.py — Neural Network Learning

```
from learnProblem import Learner, Data_set, Data_from_file
from learnLinear import sigmoid, one
import random, math

class Layer(object):
    def __init__(self,nn,num_outputs=None):
        """Given a list of inputs, outputs will produce a list of length
            num_outputs.
        nn is the neural network this is part of
```

```
19          num outputs is the number of outputs for this layer.
20          """
21          self.nn = nn
22          self.num_inputs = nn.num_outputs # output of nn is the input to
                this layer
23          if num_outputs:
24              self.num_outputs = num_outputs
25          else:
26              self.num_outputs = nn.num_outputs # same as the inputs
27
28      def output_values(self,input_values):
29          """Return the outputs for this layer for the given input values.
30          input_values is a list of the inputs to this layer (of length
                num_inputs)
31          returns a list of length self.num_outputs
32          """
33          raise NotImplementedError("output_values") # abstract method
34
35      def backprop(self,errors):
36          """Backpropagate the errors on the outputs, return the errors on
                the inputs.
37          errors is a list of errors for the outputs (of length
                self.num_outputs).
38          Return the errors for the inputs to this layer (of length
                self.num_inputs).
39          You can assume that this is only called after corresponding
                output_values,
40             and it can remember information information required for the
                   back-propagation.
41          """
42          raise NotImplementedError("backprop") # abstract method
```

A linear layer maintains an array of weights. $self.weights[o][i]$ is the weight between input i and output o. A 1 is added to the inputs.

learnNN.py — (continued)

```
44  class Linear_complete_layer(Layer):
45      """a completely connected layer"""
46      def __init__(self, nn, num_outputs, max_init=0.2):
47          """A completely connected linear layer.
48          nn is a neural network that the inputs come from
49          num_outputs is the number of outputs
50          max_init is the maximum value for random initialization of
                parameters
51          """
52          Layer.__init__(self, nn, num_outputs)
53          # self.weights[o][i] is the weight between input i and output o
54          self.weights = [[random.uniform(-max_init, max_init)
55                           for inf in range(self.num_inputs+1)]
56                          for outf in range(self.num_outputs)]
57
```

```
58      def output_values(self,input_values):
59          """Returns the outputs for the input values.
60          It remembers the values for the backprop.
61
62          Note in self.weights there is a weight list for every output,
63          so wts in self.weights effectively loops over the outputs.
64          """
65          self.inputs = input_values + [1]
66          return [sum(w*val for (w,val) in zip(wts,self.inputs))
67                      for wts in self.weights]
68
69      def backprop(self,errors):
70          """Backpropagate the errors, updating the weights and returning the
                error in its inputs.
71          """
72          input_errors = [0]*(self.num_inputs+1)
73          for out in range(self.num_outputs):
74              for inp in range(self.num_inputs+1):
75                  input_errors[inp] += self.weights[out][inp] * errors[out]
76                  self.weights[out][inp] += self.nn.learning_rate *
                        self.inputs[inp] * errors[out]
77          return input_errors[:-1] # remove the error for the "1"
```

learnNN.py — (continued)

```
79  class Sigmoid_layer(Layer):
80      """sigmoids of the inputs.
81      The number of outputs is equal to the number of inputs.
82      Each output is the sigmoid of its corresponding input.
83      """
84      def __init__(self, nn):
85          Layer.__init__(self, nn)
86
87      def output_values(self,input_values):
88          """Returns the outputs for the input values.
89          It remembers the output values for the backprop.
90          """
91          self.outputs= [sigmoid(inp) for inp in input_values]
92          return self.outputs
93
94      def backprop(self,errors):
95          """Returns the derivative of the errors"""
96          return [e*out*(1-out) for e,out in zip(errors, self.outputs)]
```

learnNN.py — (continued)

```
98  class ReLU_layer(Layer):
99      """Rectified linear unit (ReLU) f(z) = max(0, z).
100     The number of outputs is equal to the number of inputs.
101     """
102     def __init__(self, nn):
103         Layer.__init__(self, nn)
```

```
104
105     def output_values(self,input_values):
106         """Returns the outputs for the input values.
107         It remembers the input values for the backprop.
108         """
109         self.input_values = input_values
110         self.outputs= [max(0,inp) for inp in input_values]
111         return self.outputs
112
113     def backprop(self,errors):
114         """Returns the derivative of the errors"""
115         return [e if inp>0 else 0 for e,inp in zip(errors,
                self.input_values)]
```

learnNN.py — (continued)

```
117 class NN(Learner):
118     def __init__(self, dataset, learning_rate=0.1):
119         self.dataset = dataset
120         self.learning_rate = learning_rate
121         self.input_features = dataset.input_features
122         self.num_outputs = len(self.input_features)
123         self.layers = []
124
125     def add_layer(self,layer):
126         """add a layer to the network.
127         Each layer gets values from the previous layer.
128         """
129         self.layers.append(layer)
130         self.num_outputs = layer.num_outputs
131
132     def predictor(self,ex):
133         """Predicts the value of the first output feature for example ex.
134         """
135         values = [f(ex) for f in self.input_features]
136         for layer in self.layers:
137             values = layer.output_values(values)
138         return values[0]
139
140     def predictor_string(self):
141         return "not implemented"
```

The *test* method learns a network and evaluates it according to various criteria.

learnNN.py — (continued)

```
143
144     def learn(self,num_iter):
145         """Learns parameters for a neural network using stochastic gradient
                decent.
146         num_iter is the number of iterations
147         """
148         for i in range(num_iter):
```

```
149             for e in
                    random.sample(self.dataset.train,len(self.dataset.train)):
150                 # compute all outputs
151                 values = [f(e) for f in self.input_features]
152                 for layer in self.layers:
153                     values = layer.output_values(values)
154                 # backpropagate
155                 errors =
                        self.sum_squares_error([self.dataset.target(e)],values)
156                 for layer in reversed(self.layers):
157                     errors = layer.backprop(errors)
158
159     def sum_squares_error(self,observed,predicted):
160         """Returns the errors for each of the target features.
161         """
162         return [obsd-pred for obsd,pred in zip(observed,predicted)]
```

This constructs a neural network consisting of neural network with one hidden layer. The hidden using used a ReLU activation function. The output layer used a sigmoid.

learnNN.py — (continued)

```
165 data = Data_from_file('data/mail_reading.csv', target_index=-1)
166 #data = Data_from_file('data/mail_reading_consis.csv', target_index=-1)
167 #data = Data_from_file('data/SPECT.csv', prob_test=0.5, target_index=0)
168 #data = Data_from_file('data/holiday.csv', target_index=-1) #,
        num_train=19)
169 nn1 = NN(data)
170 nn1.add_layer(Linear_complete_layer(nn1,3))
171 nn1.add_layer(Sigmoid_layer(nn1)) # comment this or the next
172 # nn1.add_layer(ReLU_layer(nn1))
173 nn1.add_layer(Linear_complete_layer(nn1,1))
174 nn1.add_layer(Sigmoid_layer(nn1))
175 nn1.learning_rate=0.1
176 #nn1.learn(100)
177
178 from learnLinear import plot_steps
179 import time
180 start_time = time.perf_counter()
181 plot_steps(learner = nn1, data = data, num_steps=10000)
182 for eg in data.train:
183     print(eg,nn1.predictor(eg))
184 end_time = time.perf_counter()
185 print("Time:", end_time - start_time)
```

Exercise 7.12 In the definition of $nn1$ above, for each of the following, first hypothesize what will happen, then test your hypothesis, then explain whether you testing confirms your hypothesis or not. Test it for more than one data set, and use more than one run for each data set.

(a) Which fits the data better, having a sigmoid layer or a ReLU layer after the first linear layer?

(b) Which is faster, having a sigmoid layer or a ReLU layer after the first linear layer?

(c) What happens if you have both the sigmoid layer and then a ReLU layer after the first linear layer and before the second linear layer?

(d) What happens if you have neither the sigmoid layer nor a ReLU layer after the first linear layer?

(e) What happens if you have a ReLU layer then a sigmoid layer after the first linear layer and before the second linear layer?

Exercise 7.13 Do some

It is even possible to define a perceptron layer. Warning: you may need to change the learning rate to make this work. Should I add it into the code? It doesn't follow the official line.

```
class PerceptronLayer(Layer):
    def __init__(self, nn):
        Layer.__init__(self, nn)

    def output_values(self,input_values):
        """Returns the outputs for the input values.
        """
        self.outputs= [1 if inp>0 else -1 for inp in input_values]
        return self.outputs

    def backprop(self,errors):
        """Pass the errors through"""
        return errors
```

7.8 Boosting

The following code implements functional gradient boosting for regression.

A Boosted dataset is created from a base dataset by subtracting the prediction of the offset function from each example. This does not save the new dataset, but generates it as needed. The amount of space used is constant, independent on the size of the data set.

learnBoosting.py — Functional Gradient Boosting

```
11 from learnProblem import Data_set, Learner
12
13 class Boosted_dataset(Data_set):
14     def __init__(self, base_dataset, offset_fun):
15         """new dataset which is like base_dataset,
16            but offset_fun(e) is subtracted from the target of each example e
17         """
18         self.base_dataset = base_dataset
19         self.offset_fun = offset_fun
```

```
20          Data_set.__init__(self, base_dataset.train, base_dataset.test,
21                            base_dataset.prob_test, base_dataset.target_index)
22
23      def create_features(self):
24          self.input_features = self.base_dataset.input_features
25          def newout(e):
26              return self.base_dataset.target(e) - self.offset_fun(e)
27          newout.frange = self.base_dataset.target.frange
28          self.target = newout
```

A boosting learner takes in a dataset and a base learner, and returns a new predictor. The base learner, takes a dataset, and returns a Learner object.

learnBoosting.py — (continued)

```
30  class Boosting_learner(Learner):
31      def __init__(self, dataset, base_learner_class):
32          self.dataset = dataset
33          self.base_learner_class = base_learner_class
34          mean = sum(self.dataset.target(e)
35                     for e in self.dataset.train)/len(self.dataset.train)
36          self.predictor = lambda e:mean  # function that returns mean for
                each example
37          self.predictor.__doc__ = "lambda e:"+str(mean)
38          self.offsets = [self.predictor]
39          self.errors = [data.evaluate_dataset(data.test, self.predictor,
                "sum-of-squares")]
40          self.display(1,"Predict mean test set error=", self.errors[0] )
41
42
43      def learn(self, num_ensemble=10):
44          """adds num_ensemble learners to the ensemble.
45          returns a new predictor.
46          """
47          for i in range(num_ensemble):
48              train_subset = Boosted_dataset(self.dataset, self.predictor)
49              learner = self.base_learner_class(train_subset)
50              new_offset = learner.learn()
51              self.offsets.append(new_offset)
52              def new_pred(e, old_pred=self.predictor, off=new_offset):
53                  return old_pred(e)+off(e)
54              self.predictor = new_pred
55              self.errors.append(data.evaluate_dataset(data.test,
                    self.predictor,"sum-of-squares"))
56              self.display(1,"After Iteration",len(self.offsets)-1,"test set
                    error=", self.errors[-1])
57          return self.predictor
```

For testing, *sp_DT_learner* returns a function that constructs a decision tree learner where the minimum number of examples is a proportion of the number of training examples. The value of 0.9 tends to have one split, and a value of 0.5

tends to have two splits (but test it). Thus this can be used to construct small decision trees that can be used as weak learners.

learnBoosting.py — (continued)

```
# Testing

from learnDT import DT_learner
from learnProblem import Data_set, Data_from_file

def sp_DT_learner(min_prop=0.9):
    def make_learner(dataset):
        mne = len(dataset.train)*min_prop
        return DT_learner(dataset,min_number_examples=mne)
    return make_learner

data = Data_from_file('data/carbool.csv', target_index=-1)
#data = Data_from_file('data/SPECT.csv', target_index=0)
#data = Data_from_file('data/mail_reading.csv', target_index=-1)
#data = Data_from_file('data/holiday.csv', num_train=19, target_index=-1)
learner9 = Boosting_learner(data, sp_DT_learner(0.9))
#learner7 = Boosting_learner(data, sp_DT_learner(0.7))
#learner5 = Boosting_learner(data, sp_DT_learner(0.5))
predictor9 =learner9.learn(10)
for i in learner9.offsets: print(i.__doc__)
import matplotlib.pyplot as plt

def plot_boosting(data,steps=10, thresholds=[0.5,0.1,0.01,0.001],
    markers=['-','--','-.',':'] ):
    learners = [Boosting_learner(data, sp_DT_learner(th)) for th in
        thresholds]
    predictors = [learner.learn(steps) for learner in learners]
    plt.ion()
    plt.xscale('linear') # change between log and linear scale
    plt.xlabel("number of trees")
    plt.ylabel(" error")
    for (learner,(threshold,marker)) in
        zip(learners,zip(thresholds,markers)):
        plt.plot(range(len(learner.errors)), learner.errors,
            ls=marker,c='k',
                label=str(round(threshold*100))+"% min example
                    threshold")
    plt.legend()
    plt.draw()

# plot_boosting(data)
```

Chapter 8

Reasoning Under Uncertainty

8.1 Representing Probabilistic Models

A variable consists of a name, a domain and an optional (x,y) position (for displaying). The domain of a variable is a list or a tuple, as the ordering will matter in the representation of factors.

probVariables.py — Probabilistic Variables

```
import random

class Variable(object):
    """A random variable.
    name (string) - name of the variable
    domain (list) - a list of the values for the variable.
    Variables are ordered according to their name.
    """

    def __init__(self, name, domain, position=None):
        """Variable
        name a string
        domain a list of printable values
        position of form (x,y)
        """
        self.name = name # string
        self.domain = domain # list of values
        self.position = position if position else (random.random(),
            random.random())
        self.size = len(domain)

    def __str__(self):
        return self.name

```

```
34      def __repr__(self):
35          return self.name # f"Variable({self.name})"
```

8.2 Representing Factors

A **factor** is, mathematically, a function from variables into a number; that is given a value for each of its variable, it gives a number. Factors are used for conditional probabilities, utilities in the next chapter, and are explicitly constructed by some algorithms (in particular variable elimination).

A variable assignment, or just **assignment**, is represented as a {*variable* : *value*} dictionary. A factor can be evaluated when all of its variables are assigned. The method `get_value` evaluates the factor for an assignment. The assignment can include extra variables not in the factor. This method needs to be defined for every subclass.

probFactors.py — Factors for graphical models

```
11  from display import Displayable
12  import math
13
14  class Factor(Displayable):
15      nextid=0 # each factor has a unique identifier; for printing
16
17      def __init__(self,variables):
18          self.variables = variables # ordered list of variables
19          self.id = Factor.nextid
20          self.name = f"f{self.id}"
21          Factor.nextid += 1
22
23      def can_evaluate(self,assignment):
24          """True when the factor can be evaluated in the assignment
25          assignment is a {variable:value} dict
26          """
27          return all(v in assignment for v in self.variables)
28
29      def get_value(self,assignment):
30          """Returns the value of the factor given the assignment of values
                to variables.
31          Needs to be defined for each subclass.
32          """
33          assert self.can_evaluate(assignment)
34          raise NotImplementedError("get_value") # abstract method
```

The method `__str__` returns a brief definition (like "`f7(X,Y,Z)`").The method `to_table` returns string representations of a table showing all of the assignments of values to variables, and the corresponding value.

probFactors.py — (continued)

```
36      def __str__(self):
```

```
        """returns a string representing a summary of the factor"""
        return f"{self.name}({','.join(str(var) for var in
            self.variables)})"

    def to_table(self, variables=None, given={}):
        """returns a string representation of the factor.
        Allows for an arbitrary variable ordering.
        variables is a list of the variables in the factor
        (can contain other variables)"""
        if variables==None:
            variables = [v for v in self.variables if v not in given]
        else: #enforce ordering and allow for extra variables in ordering
            variables = [v for v in variables if v in self.variables and v
                not in given]
        head = "\t".join(str(v) for v in variables)
        return head+"\n"+self.ass_to_str(variables, given, variables)

    def ass_to_str(self, vars, asst, allvars):
        #print(f"ass_to_str({vars}, {asst}, {allvars})")
        if vars:
            return "\n".join(self.ass_to_str(vars[1:], {**asst,
                vars[0]:val}, allvars)
                           for val in vars[0].domain)
        else:
            return ("\t".join(str(asst[var]) for var in allvars)
                       + "\t"+"{:.6f}".format(self.get_value(asst)) )

    __repr__ = __str__
```

8.3 Conditional Probability Distributions

A **conditional probability distribution (CPD)** is a type of factor that represents a conditional probability. A CPD representing $P(X \mid Y_1 \ldots Y_k)$ is a type of factor, where given values for X and each Y_i returns a number.

probFactors.py — (continued)

```
class CPD(Factor):
    def __init__(self, child, parents):
        """represents P(variable | parents)
        """
        self.parents = parents
        self.child = child
        Factor.__init__(self, parents+[child])

    def __str__(self):
        """A brief description of a factor using in tracing"""
        if self.parents:
            return f"P({self.child}|{','.join(str(p) for p in
                self.parents)})"
```

```
75          else:
76              return f"P({self.child})"
77
78      __repr__ = __str__
```

The simplest CPD is the constant that has probability 1 when the child has the value specified.

probFactors.py — (continued)

```
80  class ConstantCPD(CPD):
81      def __init__(self, variable, value):
82          CPD.__init__(self, variable, [])
83          self.value = value
84      def get_value(self, assignment):
85          return 1 if self.value==assignment[self.child] else 0
```

8.3.1 Logistic Regression

A **logistic regression** CPD, for Boolean variable X represents $P(X{=}True \mid Y_1 \ldots Y_k)$, using $k+1$ real-values weights so

$$P(X{=}True \mid Y_1 \ldots Y_k) = sigmoid(w_0 + \sum_i w_i Y_i)$$

where for Boolean Y_i, True is represented as 1 and False as 0.

probFactors.py — (continued)

```
87  from learnLinear import sigmoid, logit
88
89  class LogisticRegression(CPD):
90      def __init__(self, child, parents, weights):
91          """A logistic regression representation of a conditional
                probability.
92          child is the Boolean (or 0/1) variable whose CPD is being defined
93          parents is the list of parents
94          weights is list of parameters, such that weights[i+1] is the weight
                for parents[i]
95          """
96          assert len(weights) == 1+len(parents)
97          CPD.__init__(self, child, parents)
98          self.weights = weights
99
100     def get_value(self,assignment):
101         assert self.can_evaluate(assignment)
102         prob = sigmoid(self.weights[0]
103                        + sum(self.weights[i+1]*assignment[self.parents[i]]
104                              for i in range(len(self.parents))))
105         if assignment[self.child]: #child is true
106             return prob
107         else:
108             return (1-prob)
```

8.3.2 Noisy-or

A **noisy-or**, for Boolean variable X with Boolean parents $Y_1 \ldots Y_k$ is parametrized by $k+1$ parameters $p_0, p_1, \ldots, p_k$, where each $0 \leq p_i \leq 1$. The sematics is defined as though there are $k+1$ hidden variables $Z_0, Z_1 \ldots Z_k$, where $P(Z_0) = p_0$ and $P(Z_i \mid Y_i) = p_i$ for $i \geq 1$, and where X is true if and only if $Z_0 \vee Z_1 \vee \cdots \vee Z_k$ (where $\vee$ is "or"). Thus X is false if all of the Z_i are false. Intuitively, Z_0 is the probability of X when all Y_i are false and each Z_i is a noisy (probabilistic) measure that Y_i makes X true, and X only needs one to make it true.

probFactors.py — (continued)

```
110 class NoisyOR(CPD):
111     def __init__(self, child, parents, weights):
112         """A noisy representation of a conditional probability.
113         variable is the Boolean (or 0/1) child variable whose CPD is being
                defined
114         parents is the list of Boolean (or 0/1) parents
115         weights is list of parameters, such that weights[i+1] is the weight
                for parents[i]
116         """
117         assert len(weights) == 1+len(parents)
118         CPD.__init__(self, child, parents)
119         self.weights = weights
120 
121     def get_value(self,assignment):
122         assert self.can_evaluate(assignment)
123         probfalse = (1-self.weights[0])*math.prod(1-self.weights[i+1]
124                                                   for i in
                                                       range(len(self.parents))
125                                                   if
                                                       assignment[self.parents[i]])
126         if assignment[self.child]:
127             return 1-probfalse
128         else:
129             return probfalse
```

8.3.3 Tabular Factors

A **tabular factor** is a factor that represents each assignment of values to variables separately. It is represented by a Python array (or python dict). If the variables are $V_1, V_2, \ldots, V_k$, the value of $f(V_1 = v_1, V_2 = v_1, \ldots, V_k = v_k)$ is stored in $f[v_1][v_2] \ldots [v_k]$.

If the domain of V_i is $[0, \ldots, n_i - 1]$ this can be represented as an array. Otherwise we can use a dictionary. Python is nice in that it doesn't care, whether an array or dict is used **except when enumerating the values**; enumerating a dict gives the keys (the variables) but enumerating an array gives the values. So we have to be careful not to do this.

probFactors.py — (continued)

```
from functools import reduce

class TabFactor(Factor):

    def __init__(self, variables, values):
        Factor.__init__(self, variables)
        self.values = values

    def get_value(self, assignment):
        return self.get_val_rec(self.values, self.variables, assignment)

    def get_val_rec(self, value, variables, assignment):
        if variables == []:
            return value
        else:
            return self.get_val_rec(value[assignment[variables[0]]],
                                    variables[1:],assignment)
```

Prob is a factor that represents a conditional probability by enumerating all of the values.

probFactors.py — (continued)

```
class Prob(CPD,TabFactor):
    """A factor defined by a conditional probability table"""
    def __init__(self,var,pars,cpt):
        """Creates a factor from a conditional probability table, cpt
        The cpt values are assumed to be for the ordering par+[var]
        """
        TabFactor.__init__(self,pars+[var],cpt)
        self.child = var
        self.parents = pars
```

8.4 Graphical Models

A graphical model consists of a set of variables and a set of factors. A belief network is a graphical model where all of the factors represent conditional probabilities. There are some operations (such as pruning variables) which are applicable to belief networks, but are not applicable to more general models. At the moment, we will treat them as the same.

probGraphicalModels.py — Graphical Models and Belief Networks

```
from display import Displayable
from probFactors import CPD
import matplotlib.pyplot as plt

class GraphicalModel(Displayable):
    """The class of graphical models.
    A graphical model consists of a title, a set of variables and a set of
        factors.
```

```
18
19      vars is a set of variables
20      factors is a set of factors
21      """
22      def __init__(self, title, variables=None, factors=None):
23          self.title = title
24          self.variables = variables
25          self.factors = factors
```

A **belief network** (also known as a **Bayesian network**) is a graphical model where all of the factors are conditional probabilities, and every variable has a conditional probability of it given its parents. This only checks the first condition, and builds some useful data structures.

probGraphicalModels.py — (continued)

```
27  class BeliefNetwork(GraphicalModel):
28      """The class of belief networks."""
29
30      def __init__(self, title, variables, factors):
31          """vars is a set of variables
32          factors is a set of factors. All of the factors are instances of
                CPD (e.g., Prob).
33          """
34          GraphicalModel.__init__(self, title, variables, factors)
35          assert all(isinstance(f,CPD) for f in factors)
36          self.var2cpt = {f.child:f for f in factors}
37          self.var2parents = {f.child:f.parents for f in factors}
38          self.children = {n:[] for n in self.variables}
39          for v in self.var2parents:
40              for par in self.var2parents[v]:
41                  self.children[par].append(v)
42          self.topological_sort_saved = None
```

The following creates a topological sort of the nodes, where the parents of a node come before the node in the resulting order. This is based on Kahn's algorithm from 1962.

probGraphicalModels.py — (continued)

```
44      def topological_sort(self):
45          """creates a topological ordering of variables such that the
                parents of
46          a node are before the node.
47          """
48          if self.topological_sort_saved:
49              return self.topological_sort_saved
50          next_vars = {n for n in self.var2parents if not self.var2parents[n]
                }
51          self.display(3,'topological_sort: next_vars',next_vars)
52          top_order=[]
53          while next_vars:
54              var = next_vars.pop()
```

```
55            self.display(3,'select variable',var)
56            top_order.append(var)
57            next_vars |= {ch for ch in self.children[var]
58                          if all(p in top_order for p in
                                self.var2parents[ch])}
59            self.display(3,'var_with_no_parents_left',next_vars)
60        self.display(3,"top_order",top_order)
61        assert
              set(top_order)==set(self.var2parents),(top_order,self.var2parents)
62        self.topologicalsort_saved=top_order
63        return top_order
```

The **show** method uses matplotlib to show the graphical structure of a belief network.

probGraphicalModels.py — (continued)

```
65    def show(self):
66        plt.ion()  # interactive
67        ax = plt.figure().gca()
68        ax.set_axis_off()
69        plt.title(self.title)
70        bbox = dict(boxstyle="round4,pad=1.0,rounding_size=0.5")
71        for var in reversed(self.topological_sort()):
72            if self.var2parents[var]:
73                for par in self.var2parents[var]:
74                    ax.annotate(var.name, par.position, xytext=var.position,
75                                arrowprops={'arrowstyle':'<-'},bbox=bbox,
76                                ha='center')
77            else:
78                x,y = var.position
79                plt.text(x,y,var.name,bbox=bbox,ha='center')
```

8.4.1 Example Belief Networks

A Chain of 4 Variables

The first example belief network is a simple chain $A \longrightarrow B \longrightarrow C \longrightarrow D$.

Please do not change this, as it is the example used for testing.

probGraphicalModels.py — (continued)

```
81 from probVariables import Variable
82 from probFactors import Prob, LogisticRegression, NoisyOR
83
84 boolean = [False, True]
85 A = Variable("A", boolean, position=(0,0.8))
86 B = Variable("B", boolean, position=(0.333,0.6))
87 C = Variable("C", boolean, position=(0.666,0.4))
88 D = Variable("D", boolean, position=(1,0.2))
89
90 f_a = Prob(A,[],[0.4,0.6])
```

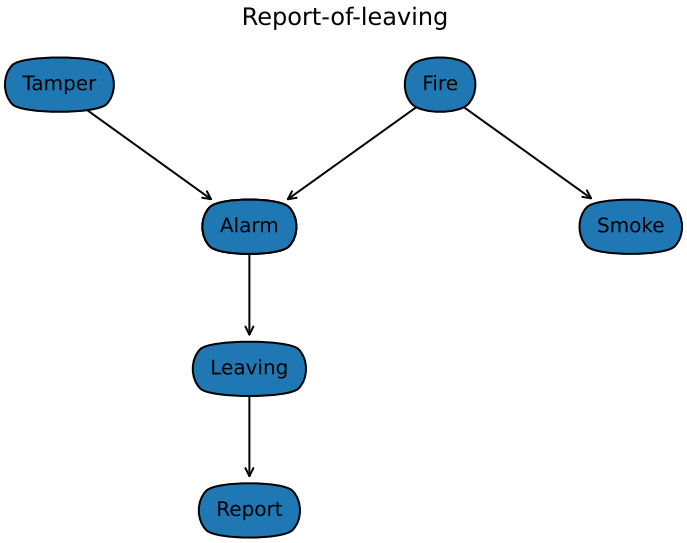


Figure 8.1: The report-of-leaving belief network

```
f_b = Prob(B,[A],[[0.9,0.1],[0.2,0.8]])
f_c = Prob(C,[B],[[0.6,0.4],[0.3,0.7]])
f_d = Prob(D,[C],[[0.1,0.9],[0.75,0.25]])

bn_4ch = BeliefNetwork("4-chain", {A,B,C,D}, {f_a,f_b,f_c,f_d})
```

Report-of-Leaving Example

The second belief network, `bn_report`, is Example 8.15 of Poole and Mackworth [2017] (`http://artint.info`). The output of `bn_report.show()` is shown in Figure 8.1 of this document.

probGraphicalModels.py — (continued)

```
# Belief network report-of-leaving example (Example 8.15 shown in Figure
    8.3) of
# Poole and Mackworth, Artificial Intelligence, 2017 http://artint.info

Alarm =  Variable("Alarm", boolean, position=(0.366,0.633))
Fire =   Variable("Fire",  boolean, position=(0.633,0.9))
Leaving = Variable("Leaving", boolean, position=(0.366,0.366))
Report = Variable("Report", boolean, position=(0.366,0.1))
Smoke =  Variable("Smoke", boolean, position=(0.9,0.633))
Tamper = Variable("Tamper", boolean, position=(0.1,0.9))

```

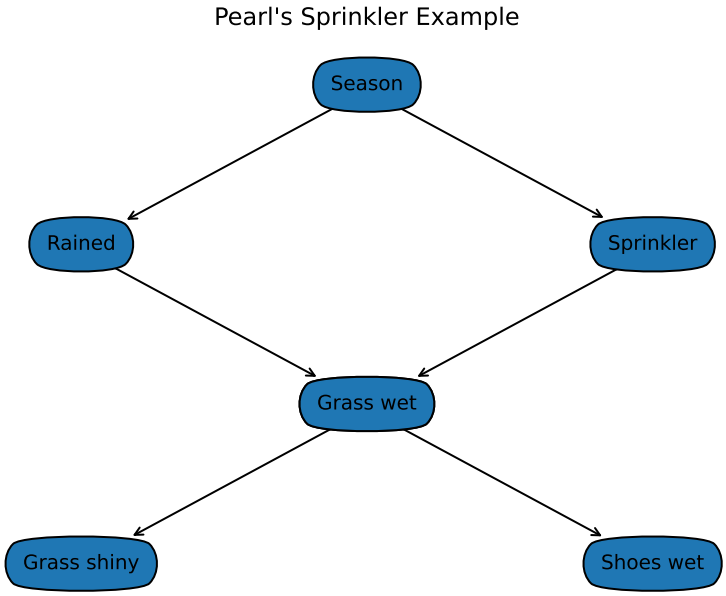


Figure 8.2: The sprinkler belief network

```
f_ta = Prob(Tamper,[],[0.98,0.02])
f_fi = Prob(Fire,[],[0.99,0.01])
f_sm = Prob(Smoke,[Fire],[[0.99,0.01],[0.1,0.9]])
f_al = Prob(Alarm,[Fire,Tamper],[[[0.9999, 0.0001], [0.15, 0.85]], [[0.01,
    0.99], [0.5, 0.5]]])
f_lv = Prob(Leaving,[Alarm],[[0.999, 0.001], [0.12, 0.88]])
f_re = Prob(Report,[Leaving],[[0.99, 0.01], [0.25, 0.75]])

bn_report = BeliefNetwork("Report-of-leaving",
    {Tamper,Fire,Smoke,Alarm,Leaving,Report},
                            {f_ta,f_fi,f_sm,f_al,f_lv,f_re})
```

Sprinkler Example

The third belief network is the sprinkler example from Pearl. The output of `bn_sprinkler.show()` is shown in Figure 8.2 of this document.

probGraphicalModels.py — (continued)

```
Season = Variable("Season", ["summer","winter"], position=(0.5,0.9))
Sprinkler = Variable("Sprinkler", ["on","off"], position=(0.9,0.6))
Rained = Variable("Rained", boolean, position=(0.1,0.6))
Grass_wet = Variable("Grass wet", boolean, position=(0.5,0.3))
Grass_shiny = Variable("Grass shiny", boolean, position=(0.1,0))
Shoes_wet = Variable("Shoes wet", boolean, position=(0.9,0))
```

```
123
124 f_season = Prob(Season,[],{'summer':0.5, 'winter':0.5})
125 f_sprinkler = Prob(Sprinkler,[Season],{'summer':{'on':0.9,'off':0.1},
126                                        'winter':{'on':0.01,'off':0.99}})
127 f_rained = Prob(Rained,[Season],{'summer':[0.9,0.1], 'winter': [0.2,0.8]})
128 f_wet = Prob(Grass_wet,[Sprinkler,Rained], {'on': [[0.1,0.9],[0.01,0.99]],
129                                             'off':[[0.99,0.01],[0.3,0.7]]})
130 f_shiny = Prob(Grass_shiny, [Grass_wet], [[0.95,0.05], [0.3,0.7]])
131 f_shoes = Prob(Shoes_wet, [Grass_wet], [[0.98,0.02], [0.35,0.65]])
132
133 bn_sprinkler = BeliefNetwork("Pearl's Sprinkler Example",
134                       {Season, Sprinkler, Rained, Grass_wet, Grass_shiny,
                              Shoes_wet},
135                       {f_season, f_sprinkler, f_rained, f_wet, f_shiny,
                              f_shoes})
136
137 bn_sprinkler_soff = BeliefNetwork("Pearl's Sprinkler Example
        (do(Sprinkler=off))",
138                       {Season, Sprinkler, Rained, Grass_wet, Grass_shiny,
                              Shoes_wet},
139                       {f_season, f_rained, f_wet, f_shiny, f_shoes,
140                            Prob(Sprinkler,[],{'on':0,'off':1})})
```

Bipartite Diagnostic Model with Noisy-or

The belief network bn_no1 is a bipartite diagnostic model, with independent diseases, and the symtoms depend on the diseases, where the CPDs are defined using noisy-or. Bipartite means it is in two parts; the diseases are only connected to the symptoms and the symptoms are only connected to the diseases. The output of bn_no1.show() is shown in Figure 8.3 of this document.

probGraphicalModels.py — (continued)

```
142 Cough = Variable("Cough", boolean, (0.1,0.1))
143 Fever = Variable("Fever", boolean, (0.5,0.1))
144 Sneeze = Variable("Sneeze", boolean, (0.9,0.1))
145 Cold = Variable("Cold",boolean, (0.1,0.9))
146 Flu = Variable("Flu",boolean, (0.5,0.9))
147 Covid = Variable("Covid",boolean, (0.9,0.9))
148
149 p_cold_no = Prob(Cold,[],[0.9,0.1])
150 p_flu_no = Prob(Flu,[],[0.95,0.05])
151 p_covid_no = Prob(Covid,[],[0.99,0.01])
152
153 p_cough_no = NoisyOR(Cough, [Cold,Flu,Covid], [0.1, 0.3, 0.2, 0.7])
154 p_fever_no = NoisyOR(Fever, [   Flu,Covid], [0.01,    0.6, 0.7])
155 p_sneeze_no = NoisyOR(Sneeze, [Cold,Flu ], [0.05, 0.5, 0.2   ])
156
157 bn_no1 = BeliefNetwork("Bipartite Diagnostic Network (noisy-or)",
```

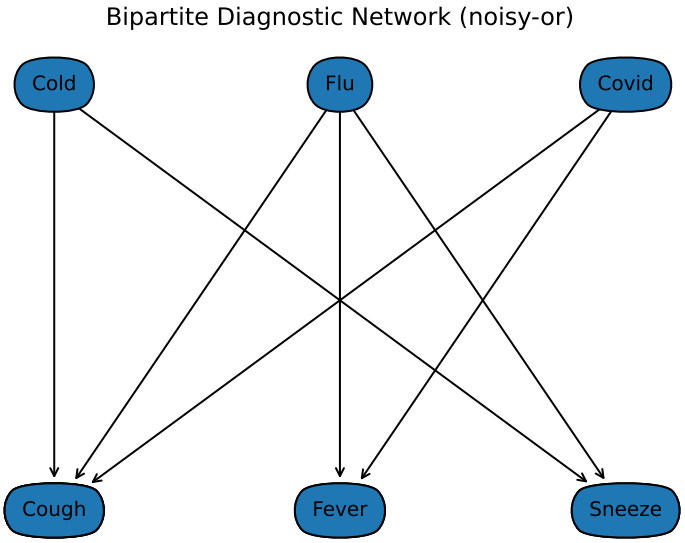


Figure 8.3: A bipartite diagnostic network

```
                        {Cough, Fever, Sneeze, Cold, Flu, Covid},
                         {p_cold_no, p_flu_no, p_covid_no, p_cough_no,
                             p_fever_no, p_sneeze_no})

# to see the conditional probability of Noisy-or do:
# print(p_cough_no.to_table())

# example from box "Noisy-or compared to logistic regression"
# X = Variable("X",boolean)
# w0 = 0.01
# print(NoisyOR(X,[A,B,C,D],[w0, 1-(1-0.05)/(1-w0), 1-(1-0.1)/(1-w0),
    1-(1-0.2)/(1-w0), 1-(1-0.2)/(1-w0), ]).to_table(given={X:True}))
```

Bipartite Diagnostic Model with Logistic Regression

The belief network `bn_lr1` is a bipartite diagnostic model, with independent diseases, and the symtoms depend on the diseases, where the CPDs are defined using logistic regression. It has the same graphical structure as the previous example (see Figure 8.3). This has the (approximately) the same conditional probabilities as the previous example when zero or one diseases are present. Note that $sigmoid(-2.2) \approx 0.1$

probGraphicalModels.py — (continued)

```
169
170 p_cold_lr = Prob(Cold,[],[0.9,0.1])
171 p_flu_lr = Prob(Flu,[],[0.95,0.05])
172 p_covid_lr = Prob(Covid,[],[0.99,0.01])
173
174 p_cough_lr = LogisticRegression(Cough, [Cold,Flu,Covid], [-2.2, 1.67,
        1.26, 3.19])
175 p_fever_lr = LogisticRegression(Fever, [  Flu,Covid], [-4.6,      5.02,
        5.46])
176 p_sneeze_lr = LogisticRegression(Sneeze, [Cold,Flu ], [-2.94, 3.04, 1.79
        ])
177
178 bn_lr1 = BeliefNetwork("Bipartite Diagnostic Network - logistic
        regression",
179                        {Cough, Fever, Sneeze, Cold, Flu, Covid},
180                         {p_cold_lr, p_flu_lr, p_covid_lr, p_cough_lr,
                                p_fever_lr, p_sneeze_lr})
181
182 # to see the conditional probability of Noisy-or do:
183 #print(p_cough_lr.to_table())
184
185 # example from box "Noisy-or compared to logistic regression"
186 # from learnLinear import sigmoid, logit
187 # w0=logit(0.01)
188 # X = Variable("X",boolean)
189 # print(LogisticRegression(X,[A,B,C,D],[w0, logit(0.05)-w0, logit(0.1)-w0,
        logit(0.2)-w0, logit(0.2)-w0]).to_table(given={X:True}))
190 # try to predict what would happen (and then test) if we had
191 # w0=logit(0.01)
```

8.5 Inference Methods

Each of the inference methods implements the query method that computes the posterior probability of a variable given a dictionary of $\{variable : value\}$ observations. The methods are Displayable because they implement the *display* method which is currently text-based.

probGraphicalModels.py — (continued)

```
193 from display import Displayable
194
195 class InferenceMethod(Displayable):
196     """The abstract class of graphical model inference methods"""
197     method_name = "unnamed" # each method should have a method name
198
199     def __init__(self,gm=None):
200         self.gm = gm
201
202     def query(self, qvar, obs={}):
203         """returns a {value:prob} dictionary for the query variable"""
```

```
204         raise NotImplementedError("InferenceMethod query") # abstract method
```

We use bn_4ch as the test case, in particular $P(B \mid D = true)$. This needs an error threshold, particularly for the approximate methods, where the default threshold is much too accurate.

probGraphicalModels.py — (continued)

```
206     def testIM(self, threshold=0.0000000001):
207         solver = self(bn_4ch)
208         res = solver.query(B,{D:True})
209         correct_answer = 0.429632380245
210         assert correct_answer-threshold < res[True] <
                correct_answer+threshold, \
211                f"value {res[True]} not in desired range for
                       {self.method_name}"
212         print(f"Unit test passed for {self.method_name}.")
```

8.6 Recursive Conditioning

An instance of a *RC* object takes in a graphical model. The query method uses recursive conditioning to compute the probability of a query variable given observations on other variables.

probRC.py — Recursive Conditioning for Graphical Models

```
11  import math
12  from probGraphicalModels import GraphicalModel, InferenceMethod
13  from probFactors import Factor
14  from utilities import dict_union
15
16  class ProbSearch(InferenceMethod):
17      """The class that queries graphical models using recursive conditioning
18
19      gm is graphical model to query
20      """
21      method_name = "recursive conditioning"
22
23      def __init__(self,gm=None):
24          InferenceMethod.__init__(self, gm)
25          ## self.max_display_level = 3
26
27      def query(self, qvar, obs={}, split_order=None):
28          """computes P(qvar | obs) where
29          qvar is the query variable
30          obs is a variable:value dictionary
31          split_order is a list of the non-observed non-query variables in gm
32          """
33          if qvar in obs:
34              return {val:(1 if val == obs[qvar] else 0) for val in
                    qvar.domain}
```

```
35        else:
36            if split_order == None:
37                split_order = [v for v in self.gm.variables if (v not in
                      obs) and v != qvar]
38            unnorm = [self.prob_search(dict_union({qvar:val},obs),
                      self.gm.factors, split_order)
39                      for val in qvar.domain]
40            p_obs = sum(unnorm)
41            return {val:pr/p_obs for val,pr in zip(qvar.domain, unnorm)}
```

The following is the naive search-based algorithm. It is exponential in the number of variables, so is not very useful. However, it is simple, and useful to understand before looking at the more complicated algorithm used in the subclass.

probRC.py — (continued)

```
43    def prob_search(self, context, factors, split_order):
44        """simple search algorithm
45        context is a variable:value dictionary
46        factors is a set of factors
47        split_order is a list of variables in factors not assigned in
                  context
48        returns sum over variable assignments to variables in split order
                  or product of factors """
49        self.display(2,"calling prob_search,",(context,factors))
50        if not factors:
51            return 1
52        elif to_eval := {fac for fac in factors if
                  fac.can_evaluate(context)}:
53            # evaluate factors when all variables are assigned
54            self.display(3,"prob_search evaluating factors",to_eval)
55            val = math.prod(fac.get_value(context) for fac in to_eval)
56            return val * self.prob_search(context, factors-to_eval,
                      split_order)
57        else:
58            total = 0
59            var = split_order[0]
60            self.display(3, "prob_search branching on", var)
61            for val in var.domain:
62                total += self.prob_search(dict_union({var:val},context),
                          factors, split_order[1:])
63            self.display(3, "prob_search branching on", var,"returning",
                      total)
64            return total
```

The **recursive conditioning** algorithm adds forgetting and caching and recognizing disconnected components. We do this by adding a cache and redefining the recursive search algorithm. In inherits the query method.

probRC.py — (continued)

```
66  class ProbRC(ProbSearch):
```

```
67      def __init__(self,gm=None):
68          self.cache = {(frozenset(), frozenset()):1}
69          ProbSearch.__init__(self,gm)
70
71      def prob_search(self, context, factors, split_order):
72          """ returns the number \sum_{split_order} \prod_{factors} given
                assignments in context
73          context is a variable:value dictionary
74          factors is a set of factors
75          split_order is a list of variables in factors that are not assigned
                in context
76          returns sum over variable assignments to variables in split_order
77                    of the product of factors
78          """
79          self.display(3,"calling rc,",(context,factors))
80          ce = (frozenset(context.items()), frozenset(factors)) # key for the
                cache entry
81          if ce in self.cache:
82              self.display(3,"rc cache lookup",(context,factors))
83              return self.cache[ce]
84  #       if not factors: # no factors; needed if you don't have forgetting
        and caching
85  #           return 1
86          elif vars_not_in_factors := {var for var in context
87                                       if not any(var in fac.variables for
                                           fac in factors)}:
88              # forget variables not in any factor
89              self.display(3,"rc forgetting variables", vars_not_in_factors)
90              return self.prob_search({key:val for (key,val) in
                    context.items()
91                                       if key not in vars_not_in_factors},
92                                      factors, split_order)
93          elif to_eval := {fac for fac in factors if
                fac.can_evaluate(context)}:
94              # evaluate factors when all variables are assigned
95              self.display(3,"rc evaluating factors",to_eval)
96              val = math.prod(fac.get_value(context) for fac in to_eval)
97              if val == 0:
98                  return 0
99              else:
100                 return val * self.prob_search(context, {fac for fac in factors
101                                                     if fac not in to_eval},
                                                        split_order)
102         elif len(comp := connected_components(context, factors,
                split_order)) > 1:
103             # there are disconnected components
104             self.display(3,"splitting into connected components",comp,"in
                    context",context)
105             return(math.prod(self.prob_search(context,f,eo) for (f,eo) in
                    comp))
```

```
106         else:
107             assert split_order, "split_order should not be empty to get
                    here"
108             total = 0
109             var = split_order[0]
110             self.display(3, "rc branching on", var)
111             for val in var.domain:
112                 total += self.prob_search(dict_union({var:val},context),
                        factors, split_order[1:])
113             self.cache[ce] = total
114             self.display(2, "rc branching on", var,"returning", total)
115             return total
```

`connected_components` returns a list of connected components, where a connected component is a set of factors and a set of variables, where the graph that connects variables and factors that involve them is connected. The connected components are built one at a time; with a current connected component. At all times `factors` is partitioned into 3 disjoint sets:

- `component_factors` containing factors in the current connected component where all factors that share a variable are already in the component
- `factors_to_check` containing factors in the current connected component where potentially some factors that share a variable are not in the component; these need to be checked
- `other_factors` the other factors that are not (yet) in the connected component

probRC.py — (continued)

```
117 def connected_components(context, factors, split_order):
118     """returns a list of (f,e) where f is a subset of factors and e is a
            subset of split_order
119     such that each element shares the same variables that are disjoint from
            other elements.
120     """
121     other_factors = set(factors) #copies factors
122     factors_to_check = {other_factors.pop()} # factors in connected
            component still to be checked
123     component_factors = set() # factors in first connected component
            already checked
124     component_variables = set() # variables in first connected component
125     while factors_to_check:
126         next_fac = factors_to_check.pop()
127         component_factors.add(next_fac)
128         new_vars = set(next_fac.variables) - component_variables -
                context.keys()
129         component_variables |= new_vars
130         for var in new_vars:
```

```
131             factors_to_check |= {f for f in other_factors if var in
                    f.variables}
132             other_factors -= factors_to_check # set difference
133         if other_factors:
134             return ( [(component_factors,[e for e in split_order if e in
                    component_variables])]
135                     + connected_components(context, other_factors, [e for e in
                        split_order
136                                                                     if e not in
                                                                         component_variables])
                                                                         )
137         else:
138             return [(component_factors, split_order)]
```

Testing:

probRC.py — (continued)

```
140 from probGraphicalModels import bn_4ch, A,B,C,D,f_a,f_b,f_c,f_d
141 bn_4chv = ProbRC(bn_4ch)
142 ## bn_4chv.query(A,{})
143 ## bn_4chv.query(D,{})
144 ## InferenceMethod.max_display_level = 3 # show more detail in displaying
145 ## InferenceMethod.max_display_level = 1 # show less detail in displaying
146 ## bn_4chv.query(A,{D:True},[C,B])
147 ## bn_4chv.query(B,{A:True,D:False})
148
149 from probGraphicalModels import
        bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
150 bn_reportRC = ProbRC(bn_report) # answers queries using recursive
        conditioning
151 ## bn_reportRC.query(Tamper,{})
152 ## InferenceMethod.max_display_level = 0 # show no detail in displaying
153 ## bn_reportRC.query(Leaving,{})
154 ## bn_reportRC.query(Tamper,{},
        split_order=[Smoke,Fire,Alarm,Leaving,Report])
155 ## bn_reportRC.query(Tamper,{Report:True})
156 ## bn_reportRC.query(Tamper,{Report:True,Smoke:False})
157 ## Note what happens to the cache when these are called in turn:
158 ## bn_reportRC.query(Tamper,{Report:True},
        split_order=[Smoke,Fire,Alarm,Leaving])
159 ## bn_reportRC.query(Smoke,{Report:True},
        split_order=[Tamper,Fire,Alarm,Leaving])
160
161 from probGraphicalModels import bn_sprinkler, Season, Sprinkler, Rained,
        Grass_wet, Grass_shiny, Shoes_wet
162 bn_sprinklerv = ProbRC(bn_sprinkler)
163 ## bn_sprinklerv.query(Shoes_wet,{})
164 ## bn_sprinklerv.query(Shoes_wet,{Rained:True})
165 ## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:True})
166 ## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:False,Rained:True})
167
```

```
168 from probGraphicalModels import bn_no1, bn_lr1, Cough, Fever, Sneeze,
        Cold, Flu, Covid
169 bn_no1v = ProbRC(bn_no1)
170 bn_lr1v = ProbRC(bn_lr1)
171 ## bn_no1v.query(Flu, {Fever:1, Sneeze:1})
172 ## bn_lr1v.query(Flu, {Fever:1, Sneeze:1})
173 ## bn_lr1v.query(Cough,{})
174 ## bn_lr1v.query(Cold,{Cough:1,Sneeze:0,Fever:1})
175 ## bn_lr1v.query(Flu,{Cough:0,Sneeze:1,Fever:1})
176 ## bn_lr1v.query(Covid,{Cough:1,Sneeze:0,Fever:1})
177 ## bn_lr1v.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:0})
178 ## bn_lr1v.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:1})
179
180 if __name__ == "__main__":
181     InferenceMethod.testIM(ProbRC)
```

8.7 Variable Elimination

An instance of a *VE* object takes in a graphical model. The query method uses variable elimination to compute the probability of a variable given observations on some other variables.

probVE.py — Variable Elimination for Graphical Models

```
11 from probFactors import Factor, FactorObserved, FactorSum, factor_times
12 from probGraphicalModels import GraphicalModel, InferenceMethod
13
14 class VE(InferenceMethod):
15     """The class that queries Graphical Models using variable elimination.
16
17     gm is graphical model to query
18     """
19     method_name = "variable elimination"
20
21     def __init__(self,gm=None):
22         InferenceMethod.__init__(self, gm)
23
24     def query(self,var,obs={},elim_order=None):
25         """computes P(var|obs) where
26         var is a variable
27         obs is a {variable:value} dictionary"""
28         if var in obs:
29             return {var:1 if val == obs[var] else 0 for val in var.domain}
30         else:
31             if elim_order == None:
32                 elim_order = self.gm.variables
33             projFactors = [self.project_observations(fact,obs)
34                            for fact in self.gm.factors]
35             for v in elim_order:
36                 if v != var and v not in obs:
```

```
37                  projFactors = self.eliminate_var(projFactors,v)
38          unnorm = factor_times(var,projFactors)
39          p_obs=sum(unnorm)
40          self.display(1,"Unnormalized probs:",unnorm,"Prob obs:",p_obs)
41          return {val:pr/p_obs for val,pr in zip(var.domain, unnorm)}
```

A *FactorObserved* is a factor that is the result of some observations on another factor. We don't store the values in a list; we just look them up as needed. The observations can include variables that are not in the list, but should have some intersection with the variables in the factor.

probFactors.py — (continued)

```
159 class FactorObserved(Factor):
160     def __init__(self,factor,obs):
161         Factor.__init__(self, [v for v in factor.variables if v not in obs])
162         self.observed = obs
163         self.orig_factor = factor
164 
165     def get_value(self,assignment):
166         ass = assignment.copy()
167         for ob in self.observed:
168             ass[ob]=self.observed[ob]
169         return self.orig_factor.get_value(ass)
```

A *FactorSum* is a factor that is the result of summing out a variable from the product of other factors. I.e., it constructs a representation of:

$$\sum_{var} \prod_{f \in factors} f.$$

We store the values in a list in a lazy manner; if they are already computed, we used the stored values. If they are not already computed we can compute and store them.

probFactors.py — (continued)

```
171 class FactorSum(Factor):
172     def __init__(self,var,factors):
173         self.var_summed_out = var
174         self.factors = factors
175         vars = []
176         for fac in factors:
177             for v in fac.variables:
178                 if v is not var and v not in vars:
179                     vars.append(v)
180         Factor.__init__(self,vars)
181         self.values = {}
182 
183     def get_value(self,assignment):
184         """lazy implementation: if not saved, compute it. Return saved
                value"""
185         asst = frozenset(assignment.items())
```

```
186         if asst in self.values:
187             return self.values[asst]
188         else:
189             total = 0
190             new_asst = assignment.copy()
191             for val in self.var_summed_out.domain:
192                 new_asst[self.var_summed_out] = val
193                 total += math.prod(fac.get_value(new_asst) for fac in
                        self.factors)
194             self.values[asst] = total
195             return total
```

The method *factor_times* multiples a set of factors that are all factors on the same variable (or on no variables). This is the last step in variable elimination before normalizing. It returns an array giving the product for each value of *variable*.

probFactors.py — (continued)

```
197 def factor_times(variable, factors):
198     """when factors are factors just on variable (or on no variables)"""
199     prods = []
200     facs = [f for f in factors if variable in f.variables]
201     for val in variable.domain:
202         ast = {variable:val}
203         prods.append(math.prod(f.get_value(ast) for f in facs))
204     return prods
```

To project observations onto a factor, for each variable that is observed in the factor, we construct a new factor that is the factor projected onto that variable. *Factor_observed* creates a new factor that is the result is assigning a value to a single variable.

probVE.py — (continued)

```
43     def project_observations(self,factor,obs):
44         """Returns the resulting factor after observing obs
45 
46         obs is a dictionary of {variable:value} pairs.
47         """
48         if any((var in obs) for var in factor.variables):
49             # a variable in factor is observed
50             return FactorObserved(factor,obs)
51         else:
52             return factor
53 
54     def eliminate_var(self,factors,var):
55         """Eliminate a variable var from a list of factors.
56         Returns a new set of factors that has var summed out.
57         """
58         self.display(2,"eliminating ",str(var))
59         contains_var = []
60         not_contains_var = []
61         for fac in factors:
```

```
            if var in fac.variables:
                contains_var.append(fac)
            else:
                not_contains_var.append(fac)
        if contains_var == []:
            return factors
        else:
            newFactor = FactorSum(var,contains_var)
            self.display(2,"Multiplying:",[str(f) for f in contains_var])
            self.display(2,"Creating factor:", newFactor)
            self.display(3, newFactor.to_table()) # factor in detail
            not_contains_var.append(newFactor)
            return not_contains_var

from probGraphicalModels import bn_4ch, A,B,C,D
bn_4chv = VE(bn_4ch)
## bn_4chv.query(A,{})
## bn_4chv.query(D,{})
## InferenceMethod.max_display_level = 3 # show more detail in displaying
## InferenceMethod.max_display_level = 1 # show less detail in displaying
## bn_4chv.query(A,{D:True})
## bn_4chv.query(B,{A:True,D:False})

from probGraphicalModels import
    bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
bn_reportv = VE(bn_report) # answers queries using variable elimination
## bn_reportv.query(Tamper,{})
## InferenceMethod.max_display_level = 0 # show no detail in displaying
## bn_reportv.query(Leaving,{})
## bn_reportv.query(Tamper,{},elim_order=[Smoke,Report,Leaving,Alarm,Fire])
## bn_reportv.query(Tamper,{Report:True})
## bn_reportv.query(Tamper,{Report:True,Smoke:False})

from probGraphicalModels import bn_sprinkler, Season, Sprinkler, Rained,
    Grass_wet, Grass_shiny, Shoes_wet
bn_sprinklerv = VE(bn_sprinkler)
## bn_sprinklerv.query(Shoes_wet,{})
## bn_sprinklerv.query(Shoes_wet,{Rained:True})
## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:True})
## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:False,Rained:True})

from probGraphicalModels import bn_lr1, Cough, Fever, Sneeze, Cold, Flu,
    Covid
vediag = VE(bn_lr1)
## vediag.query(Cough,{})
## vediag.query(Cold,{Cough:1,Sneeze:0,Fever:1})
## vediag.query(Flu,{Cough:0,Sneeze:1,Fever:1})
## vediag.query(Covid,{Cough:1,Sneeze:0,Fever:1})
## vediag.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:0})
## vediag.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:1})
```

```
109
110 if __name__ == "__main__":
111     InferenceMethod.testIM(VE)
```

8.8 Stochastic Simulation

8.8.1 Sampling from a discrete distribution

The method *sample_one* generates a single sample from a (possible unnormalized) distribution. *dist* is a $\{value : weight\}$ dictionary, where $weight \geq 0$. This returns a value with probability in proportion to its weight.

probStochSim.py — Probabilistic inference using stochastic simulation

```
11 import random
12 from probGraphicalModels import InferenceMethod
13
14 def sample_one(dist):
15     """returns the index of a single sample from normalized distribution
           dist."""
16     rand = random.random()*sum(dist.values())
17     cum = 0    # cumulative weights
18     for v in dist:
19         cum += dist[v]
20         if cum > rand:
21             return v
```

If we want to generate multiple samples, repeatedly calling *sample_one* may not be efficient. If we want to generate n samples, and the distribution is over m values, *sample_one* takes time $O(mn)$. If m and n are of the same order of magnitude, we can do better.

The method *sample_multiple* generates multiple samples from a distribution defined by *dist*, where *dist* is a $\{value : weight\}$ dictionary, where $weight \geq 0$ and the weights cannot all be zero. This returns a list of values, of length *num_samples*, where each sample is selected with a probability proportional to its weight.

The method generates all of the random numbers, sorts them, and then goes through the distribution once, saving the selected samples.

probStochSim.py — (continued)

```
23 def sample_multiple(dist, num_samples):
24     """returns a list of num_samples values selected using distribution
           dist.
25     dist is a {value:weight} dictionary that does not need to be normalized
26     """
27     total = sum(dist.values())
28     rands = sorted(random.random()*total for i in range(num_samples))
29     result = []
30     dist_items = list(dist.items())
```

```
31      cum = dist_items[0][1] # cumulative sum
32      index = 0
33      for r in rands:
34          while r>cum:
35              index += 1
36              cum += dist_items[index][1]
37          result.append(dist_items[index][0])
38      return result
```

Exercise 8.1

What is the time and space complexity the following 4 methods to generate n samples, where m is the length of *dist*:

(a) n calls to *sample_one*

(b) *sample_multiple*

(c) Create the cumulative distribution (choose how this is represented) and, for each random number, do a binary search to determine the sample associated with the random number.

(d) Choose a random number in the range $[i/n, (i+1)/n)$ for each $i \in range(n)$, where n is the number of samples. Use these as the random numbers to select the particles. (Does this give random samples?)

For each method suggest when it might be the best method.

The *test_sampling* method can be used to generate the statistics from a number of samples. It is useful to see the variability as a function of the number of samples. Try it for few samples and also for many samples.

probStochSim.py — (continued)

```
40  def test_sampling(dist, num_samples):
41      """Given a distribution, dist, draw num_samples samples
42      and return the resulting counts
43      """
44      result = {v:0 for v in dist}
45      for v in sample_multiple(dist, num_samples):
46          result[v] += 1
47      return result
48
49  # try the following queries a number of times each:
50  # test_sampling({1:1,2:2,3:3,4:4}, 100)
51  # test_sampling({1:1,2:2,3:3,4:4}, 100000)
```

8.8.2 Sampling Methods for Belief Network Inference

A *SamplingInferenceMethod* is an *InferenceMethod*, but the query method also takes arguments for the number of samples and the sample-order (which is an ordering of factors). The first methods assume a belief network (and not an undirected graphical model).

probStochSim.py — (continued)

```
53 class SamplingInferenceMethod(InferenceMethod):
54     """The abstract class of sampling-based belief network inference
           methods"""
55
56     def __init__(self,gm=None):
57         InferenceMethod.__init__(self, gm)
58
59     def query(self,qvar,obs={},number_samples=1000,sample_order=None):
60         raise NotImplementedError("SamplingInferenceMethod query") #
               abstract
```

8.8.3 Rejection Sampling

probStochSim.py — (continued)

```
62 class RejectionSampling(SamplingInferenceMethod):
63     """The class that queries Graphical Models using Rejection Sampling.
64
65     gm is a belief network to query
66     """
67     method_name = "rejection sampling"
68
69     def __init__(self, gm=None):
70         SamplingInferenceMethod.__init__(self, gm)
71
72     def query(self, qvar, obs={}, number_samples=1000, sample_order=None):
73         """computes P(qvar | obs) where
74         qvar is a variable.
75         obs is a {variable:value} dictionary.
76         sample_order is a list of variables where the parents
77           come before the variable.
78         """
79         if sample_order is None:
80             sample_order = self.gm.topological_sort()
81         self.display(2,*sample_order,sep="\t")
82         counts = {val:0 for val in qvar.domain}
83         for i in range(number_samples):
84             rejected = False
85             sample = {}
86             for nvar in sample_order:
87                 fac = self.gm.var2cpt[nvar]  #factor with nvar as child
88                 val = sample_one({v:fac.get_value({**sample, nvar:v}) for v
                        in nvar.domain})
89                 self.display(2,val,end="\t")
90                 if nvar in obs and obs[nvar] != val:
91                     rejected = True
92                     self.display(2,"Rejected")
93                     break
94                 sample[nvar] = val
```

```
95              if not rejected:
96                  counts[sample[qvar]] += 1
97                  self.display(2,"Accepted")
98          tot = sum(counts.values())
99          # As well as the distribution we also include raw counts
100         dist = {c:v/tot if tot>0 else 1/len(qvar.domain) for (c,v) in
                counts.items()}
101         dist["raw_counts"] = counts
102         return dist
```

8.8.4 Likelihood Weighting

Likelihood weighting includes a weight for each sample. Instead of rejecting samples based on observations, likelihood weighting changes the weights of the sample in proportion with the probability of the observation. The weight then becomes the probability that the variable would have been rejected.

probStochSim.py — (continued)

```
104 class LikelihoodWeighting(SamplingInferenceMethod):
105     """The class that queries Graphical Models using Likelihood weighting.
106
107     gm is a belief network to query
108     """
109     method_name = "likelihood weighting"
110
111     def __init__(self, gm=None):
112         SamplingInferenceMethod.__init__(self, gm)
113
114     def query(self,qvar,obs={},number_samples=1000,sample_order=None):
115         """computes P(qvar | obs) where
116         qvar is a variable.
117         obs is a {variable:value} dictionary.
118         sample_order is a list of factors where factors defining the parents
119           come before the factors for the child.
120         """
121         if sample_order is None:
122             sample_order = self.gm.topological_sort()
123         self.display(2,*[v for v in sample_order
124                          if v not in obs],sep="\t")
125         counts = {val:0 for val in qvar.domain}
126         for i in range(number_samples):
127             sample = {}
128             weight = 1.0
129             for nvar in sample_order:
130                 fac = self.gm.var2cpt[nvar]
131                 if nvar in obs:
132                     sample[nvar] = obs[nvar]
133                     weight *= fac.get_value(sample)
134                 else:
135                     val = sample_one({v:fac.get_value({**sample,nvar:v}) for
                            v in nvar.domain})
```

```
                self.display(2,val,end="\t")
                sample[nvar] = val
            counts[sample[qvar]] += weight
            self.display(2,weight)
        tot = sum(counts.values())
        # as well as the distribution we also include the raw counts
        dist = {c:v/tot for (c,v) in counts.items()}
        dist["raw_counts"] = counts
        return dist
```

Exercise 8.2 Change this algorithm so that it does **importance sampling** using a proposal distribution. It needs *sample_one* using a different distribution and then update the weight of the current sample. For testing, use a proposal distribution that only specifies probabilities for some of the variables (and the algorithm uses the probabilities for the network in other cases).

8.8.5 Particle Filtering

In this implementation, a particle is a $\{variable : value\}$ dictionary. Because adding a new value to dictionary involves a side effect, the dictionaries need to be copied during resampling.

probStochSim.py — (continued)

```
class ParticleFiltering(SamplingInferenceMethod):
    """The class that queries Graphical Models using Particle Filtering.

    gm is a belief network to query
    """
    method_name = "particle filtering"

    def __init__(self, gm=None):
        SamplingInferenceMethod.__init__(self, gm)

    def query(self, qvar, obs={}, number_samples=1000, sample_order=None):
        """computes P(qvar | obs) where
        qvar is a variable.
        obs is a {variable:value} dictionary.
        sample_order is a list of factors where factors defining the parents
          come before the factors for the child.
        """
        if sample_order is None:
            sample_order = self.gm.topological_sort()
        self.display(2,*[v for v in sample_order
                         if v not in obs],sep="\t")
        particles = [{} for i in range(number_samples)]
        for nvar in sample_order:
            fac = self.gm.var2cpt[nvar]
            if nvar in obs:
                weights = [fac.get_value({**part, nvar:obs[nvar]}) for part
                    in particles]
```

```
172                 particles = [{**p, nvar:obs[nvar]} for p in
                        resample(particles, weights, number_samples)]
173             else:
174                 for part in particles:
175                     part[nvar] = sample_one({v:fac.get_value({**part,
                            nvar:v}) for v in nvar.domain})
176                 self.display(2,part[nvar],end="\t")
177         counts = {val:0 for val in qvar.domain}
178         for part in particles:
179             counts[part[qvar]] += 1
180         tot = sum(counts.values())
181         # as well as the distribution we also include the raw counts
182         dist = {c:v/tot for (c,v) in counts.items()}
183         dist["raw_counts"] = counts
184         return dist
```

Resampling

Resample is based on *sample_multiple* but works with an array of particles. (Aside: Python doesn't let us use *sample_multiple* directly as it uses a dictionary, and particles, represented as dictionaries can't be the key of dictionaries).

probStochSim.py — (continued)

```
186 def resample(particles, weights, num_samples):
187     """returns num_samples copies of particles resampled according to
            weights.
188     particles is a list of particles
189     weights is a list of positive numbers, of same length as particles
190     num_samples is n integer
191     """
192     total = sum(weights)
193     rands = sorted(random.random()*total for i in range(num_samples))
194     result = []
195     cum = weights[0]   # cumulative sum
196     index = 0
197     for r in rands:
198         while r>cum:
199             index += 1
200             cum += weights[index]
201         result.append(particles[index])
202     return result
```

8.8.6 Examples

probStochSim.py — (continued)

```
204 from probGraphicalModels import bn_4ch, A,B,C,D
205 bn_4chr = RejectionSampling(bn_4ch)
206 bn_4chL = LikelihoodWeighting(bn_4ch)
```

```
## InferenceMethod.max_display_level = 2 # detailed tracing for all
    inference methods
## bn_4chr.query(A,{})
## bn_4chr.query(C,{})
## bn_4chr.query(A,{C:True})
## bn_4chr.query(B,{A:True,C:False})

from probGraphicalModels import
    bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
bn_reportr = RejectionSampling(bn_report) # answers queries using
    rejection sampling
bn_reportL = LikelihoodWeighting(bn_report) # answers queries using
    likelihood weighting
bn_reportp = ParticleFiltering(bn_report) # answers queries using particle
    filtering
## bn_reportr.query(Tamper,{})
## bn_reportr.query(Tamper,{})
## bn_reportr.query(Tamper,{Report:True})
## InferenceMethod.max_display_level = 0 # no detailed tracing for all
    inference methods
## bn_reportr.query(Tamper,{Report:True},number_samples=100000)
## bn_reportr.query(Tamper,{Report:True,Smoke:False})
## bn_reportr.query(Tamper,{Report:True,Smoke:False},number_samples=100)

## bn_reportL.query(Tamper,{Report:True,Smoke:False},number_samples=100)
## bn_reportL.query(Tamper,{Report:True,Smoke:False},number_samples=100)

from probGraphicalModels import bn_sprinkler,Season, Sprinkler
from probGraphicalModels import Rained, Grass_wet, Grass_shiny, Shoes_wet
bn_sprinklerr = RejectionSampling(bn_sprinkler) # answers queries using
    rejection sampling
bn_sprinklerL = LikelihoodWeighting(bn_sprinkler) # answers queries using
    rejection sampling
bn_sprinklerp = ParticleFiltering(bn_sprinkler) # answers queries using
    particle filtering
#bn_sprinklerr.query(Shoes_wet,{Grass_shiny:True,Rained:True})
#bn_sprinklerL.query(Shoes_wet,{Grass_shiny:True,Rained:True})
#bn_sprinklerp.query(Shoes_wet,{Grass_shiny:True,Rained:True})

if __name__ == "__main__":
    InferenceMethod.testIM(RejectionSampling, threshold=0.1)
    InferenceMethod.testIM(LikelihoodWeighting, threshold=0.1)
    InferenceMethod.testIM(ParticleFiltering, threshold=0.1)
```

Exercise 8.3 This code keeps regenerating the distribution of a variable given its parents. Implement one or both of the following, and compare them to the original. Make *cond_dist* return a slice that corresponds to the distribution, and then use the slice instead of the dictionary (a list slice does not generate new data structures). Make *cond_dist* remember values it has already computed, and only return these.

8.8.7 Gibbs Sampling

The following implements **Gibbs sampling**, a form of **Markov Chain Monte Carlo** MCMC.

probStochSim.py — (continued)

```
#import random
#from probGraphicalModels import InferenceMethod

#from probStochSim import sample_one, SamplingInferenceMethod

class GibbsSampling(SamplingInferenceMethod):
    """The class that queries Graphical Models using Gibbs Sampling.

    bn is a graphical model (e.g., a belief network) to query
    """
    method_name = "Gibbs sampling"

    def __init__(self, gm=None):
        SamplingInferenceMethod.__init__(self, gm)
        self.gm = gm

    def query(self, qvar, obs={}, number_samples=1000, burn_in=100,
              sample_order=None):
        """computes P(qvar | obs) where
        qvar is a variable.
        obs is a {variable:value} dictionary.
        sample_order is a list of non-observed variables in order, or
        if sample_order None, the variables are shuffled at each iteration.
        """
        counts = {val:0 for val in qvar.domain}
        if sample_order is not None:
            variables = sample_order
        else:
            variables = [v for v in self.gm.variables if v not in obs]
        var_to_factors = {v:set() for v in self.gm.variables}
        for fac in self.gm.factors:
            for var in fac.variables:
                var_to_factors[var].add(fac)
        sample = {var:random.choice(var.domain) for var in variables}
        self.display(2,"Sample:",sample)
        sample.update(obs)
        for i in range(burn_in + number_samples):
            if sample_order == None:
                random.shuffle(variables)
            for var in variables:
                # get unnormalized probability distribution of var given its
                    neighbours
                vardist = {val:1 for val in var.domain}
                for val in var.domain:
                    sample[var] = val
```

```
                    for fac in var_to_factors[var]: # Markov blanket
                        vardist[val] *= fac.get_value(sample)
                sample[var] = sample_one(vardist)
            if i >= burn_in:
                counts[sample[qvar]] +=1
        tot = sum(counts.values())
        # as well as the computed distribution, we also include raw counts
        dist = {c:v/tot for (c,v) in counts.items()}
        dist["raw_counts"] = counts
        return dist

#from probGraphicalModels import bn_4ch, A,B,C,D
bn_4chg = GibbsSampling(bn_4ch)
## InferenceMethod.max_display_level = 2 # detailed tracing for all
    inference methods
bn_4chg.query(A,{})
## bn_4chg.query(D,{})
## bn_4chg.query(B,{D:True})
## bn_4chg.query(B,{A:True,C:False})

from probGraphicalModels import
    bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
bn_reportg = GibbsSampling(bn_report)
## bn_reportg.query(Tamper,{Report:True},number_samples=1000)

if __name__ == "__main__":
    InferenceMethod.testIM(GibbsSampling, threshold=0.1)
```

Exercise 8.4 Change the code so that it can have multiple query variables. Make the list of query variable be an input to the algorithm, so that the default value is the list of all non-observed variables.

Exercise 8.5 In this algorithm, explain where it computes the probability of a variable given its Markov blanket. Instead of returning the average of the samples for the query variable, it is possible to return the average estimate of the probability of the query variable given its Markov blanket. Does this converge to the same answer as the given code? Does it converge faster, slower, or the same?

8.8.8 Plotting Behaviour of Stochastic Simulators

The stochastic simulation runs can give different answers each time they are run. For the algorithms that give the same answer in the limit as the number of samples approaches infinity (as do all of these algorithms), the algorithms can be compared by comparing the accuracy for multiple runs. Summary statistics like the variance may provide some information, but the assumptions behind the variance being appropriate (namely that the distribution is approximately Gaussian) may not hold for cases where the predictions are bounded and often skewed.

It is more appropriate to plot the distribution of predictions over multiple runs. The *plot_stats* method plots the prediction of a particular variable (or for the partition function) for a number of runs of the same algorithm. On the x-axis, is the prediction of the algorithm. On the y-axis is the number of runs with prediction less than or equal to the x value. Thus this is like a cumulative distribution over the predictions, but with counts on the y-axis.

Note that for runs where there are no samples that are consistent with the observations (as can happen with rejection sampling), the prediction of probability is 1.0 (as a convention for $0/0$).

That variable *what* contains the query variable, or *what* is "*prob_ev*", the probability of evidence.

probStochSim.py — (continued)

```
import matplotlib.pyplot as plt

def plot_stats(method, qvar, qval, obs, number_runs=1000, **queryargs):
    """Plots a cumulative distribution of the prediction of the model.
    method is a InferenceMethod (that implements appropriate query(.))
    plots P(qvar=qval | obs)
    qvar is the query variable, qval is corresponding value
    obs is the {variable:value} dictionary representing the observations
    number_iterations is the number of runs that are plotted
    **queryargs is the arguments to query (often number_samples for
        sampling methods)
    """
    plt.ion()
    plt.xlabel("value")
    plt.ylabel("Cumulative Number")
    method.max_display_level, prev_mdl = 0, method.max_display_level #no
        display
    answers = [method.query(qvar,obs,**queryargs)
               for i in range(number_runs)]
    values = [ans[qval] for ans in answers]
    label = f"{method.method_name} P({qvar}={qval}|{','.join(f'{var}={val}'
        for (var,val) in obs.items())})"
    values.sort()
    plt.plot(values,range(number_runs),label=label)
    plt.legend() #loc="upper left")
    plt.draw()
    method.max_display_level = prev_mdl # restore display level

# Try:
#
    plot_stats(bn_reportr,Tamper,True,{Report:True,Smoke:True},number_samples=1000,
    number_runs=1000)
#
    plot_stats(bn_reportL,Tamper,True,{Report:True,Smoke:True},number_samples=1000,
    number_runs=1000)
#
```

```
    plot_stats(bn_reportp,Tamper,True,{Report:True,Smoke:True},number_samples=1000,
    number_runs=1000)
#
    plot_stats(bn_reportr,Tamper,True,{Report:True,Smoke:True},number_samples=100,
    number_runs=1000)
#
    plot_stats(bn_reportL,Tamper,True,{Report:True,Smoke:True},number_samples=100,
    number_runs=1000)
#
    plot_stats(bn_reportg,Tamper,True,{Report:True,Smoke:True},number_samples=1000,
    number_runs=1000)

def plot_mult(methods, example, qvar, qval, obs, number_samples=1000,
    number_runs=1000):
    for method in methods:
        solver = method(example)
        if isinstance(method,SamplingInferenceMethod):
            plot_stats(solver, qvar, qval, obs, number_samples, number_runs)
        else:
            plot_stats(solver, qvar, qval, obs, number_runs)

from probRC import ProbRC
# Try following (but it takes a while..)
methods =
    [ProbRC,RejectionSampling,LikelihoodWeighting,ParticleFiltering,GibbsSampling]
#plot_mult(methods,bn_report,Tamper,True,{Report:True,Smoke:False},number_samples=100,
    number_runs=1000)
#
    plot_mult(methods,bn_report,Tamper,True,{Report:False,Smoke:True},number_samples=100,
    number_runs=1000)

# Sprinkler Example:
#
    plot_stats(bn_sprinklerr,Shoes_wet,True,{Grass_shiny:True,Rained:True},number_samples=1000)
#
    plot_stats(bn_sprinklerL,Shoes_wet,True,{Grass_shiny:True,Rained:True},number_samples=1000)
```

8.9 Hidden Markov Models

This code for hidden Markov models is independent of the graphical models code, to keep it simple. Section 8.10 gives code that models hidden Markov models, and more generally, dynamic belief networks, using the graphical models code.

This HMM code assumes there are multiple Boolean observation variables that depend on the current state and are independent of each other given the state.

probHMM.py — Hidden Markov Model

```
import random
from probStochSim import sample_one, sample_multiple

class HMM(object):
    def __init__(self, states, obsvars, pobs, trans, indist):
        """A hidden Markov model.
        states - set of states
        obsvars - set of observation variables
        pobs - probability of observations, pobs[i][s] is P(Obs_i=True |
            State=s)
        trans - transition probability - trans[i][j] gives P(State=j |
            State=i)
        indist - initial distribution - indist[s] is P(State_0 = s)
        """
        self.states = states
        self.obsvars = obsvars
        self.pobs = pobs
        self.trans = trans
        self.indist = indist
```

Consider the following example. Suppose you want to unobtrusively keep track of an animal in a triangular enclosure using sound. Suppose you have 3 microphones that provide unreliable (noisy) binary information at each time step. The animal is either close to one of the 3 points of the triangle or in the middle of the triangle.

probHMM.py — (continued)

```
# state
#       0=middle, 1,2,3 are corners
states1 = {'middle', 'c1', 'c2', 'c3'} # states
obs1 = {'m1','m2','m3'} # microphones
```

The observation model is as follows. If the animal is in a corner, it will be detected by the microphone at that corner with probability 0.6, and will be independently detected by each of the other microphones with a probability of 0.1. If the animal is in the middle, it will be detected by each microphone with a probability of 0.4.

probHMM.py — (continued)

```
# pobs gives the observation model:
#pobs[mi][state] is P(mi=on | state)
closeMic=0.6; farMic=0.1; midMic=0.4
pobs1 = {'m1':{'middle':midMic, 'c1':closeMic, 'c2':farMic, 'c3':farMic},
    # mic 1
        'm2':{'middle':midMic, 'c1':farMic, 'c2':closeMic, 'c3':farMic}, #
            mic 2
        'm3':{'middle':midMic, 'c1':farMic, 'c2':farMic, 'c3':closeMic}} #
            mic 3
```

The transition model is as follows: If the animal is in a corner it stays in the same corner with probability 0.80, goes to the middle with probability 0.1

or goes to one of the other corners with probability 0.05 each. If it is in the middle, it stays in the middle with probability 0.7, otherwise it moves to one the corners, each with probability 0.1.

probHMM.py — (continued)

```
# trans specifies the dynamics
# trans[i] is the distribution over states resulting from state i
# trans[i][j] gives P(S=j | S=i)
sm=0.7; mmc=0.1             # transition probabilities when in middle
sc=0.8; mcm=0.1; mcc=0.05   # transition probabilities when in a corner
trans1 = {'middle':{'middle':sm, 'c1':mmc, 'c2':mmc, 'c3':mmc}, # was in middle
          'c1':{'middle':mcm, 'c1':sc, 'c2':mcc, 'c3':mcc}, # was in corner 1
          'c2':{'middle':mcm, 'c1':mcc, 'c2':sc, 'c3':mcc}, # was in corner 2
          'c3':{'middle':mcm, 'c1':mcc, 'c2':mcc, 'c3':sc}} # was in corner 3
```

Initially the animal is in one of the four states, with equal probability.

probHMM.py — (continued)

```
# initially we have a uniform distribution over the animal's state
indist1 = {st:1.0/len(states1) for st in states1}

hmm1 = HMM(states1, obs1, pobs1, trans1, indist1)
```

8.9.1 Exact Filtering for HMMs

A *HMMVEfilter* has a current state distribution which can be updated by observing or by advancing to the next time.

probHMM.py — (continued)

```
from display import Displayable

class HMMVEfilter(Displayable):
    def __init__(self,hmm):
        self.hmm = hmm
        self.state_dist = hmm.indist

    def filter(self, obsseq):
        """updates and returns the state distribution following the sequence of
        observations in obsseq using variable elimination.

        Note that it first advances time.
        This is what is required if it is called sequentially.
        If that is not what is wanted initially, do an observe first.
        """
        for obs in obsseq:
```

```
72              self.advance()    # advance time
73              self.observe(obs) # observe
74          return self.state_dist
75
76      def observe(self, obs):
77          """updates state conditioned on observations.
78          obs is a list of values for each observation variable"""
79          for i in self.hmm.obsvars:
80              self.state_dist = {st:self.state_dist[st]*(self.hmm.pobs[i][st]
81                                                  if obs[i] else
                                                        (1-self.hmm.pobs[i][st]))
82                                 for st in self.hmm.states}
83          norm = sum(self.state_dist.values()) # normalizing constant
84          self.state_dist = {st:self.state_dist[st]/norm for st in
                self.hmm.states}
85          self.display(2,"After observing",obs,"state
                distribution:",self.state_dist)
86
87      def advance(self):
88          """advance to the next time"""
89          nextstate = {st:0.0 for st in self.hmm.states}  # distribution over
                next states
90          for j in self.hmm.states:     # j ranges over next states
91              for i in self.hmm.states: # i ranges over previous states
92                  nextstate[j] += self.hmm.trans[i][j]*self.state_dist[i]
93          self.state_dist = nextstate
94          self.display(2,"After advancing state
                distribution:",self.state_dist)
```

The following are some queries for *hmm*1.

probHMM.py — (continued)

```
96  hmm1f1 = HMMVEfilter(hmm1)
97  # hmm1f1.filter([{'m1':0, 'm2':1, 'm3':1}, {'m1':1, 'm2':0, 'm3':1}])
98  ## HMMVEfilter.max_display_level = 2 # show more detail in displaying
99  # hmm1f2 = HMMVEfilter(hmm1)
100 # hmm1f2.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':1, 'm3':0},
        {'m1':1, 'm2':0, 'm3':0},
101 #              {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
        {'m1':0, 'm2':0, 'm3':0},
102 #              {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':1},
        {'m1':0, 'm2':0, 'm3':1},
103 #              {'m1':0, 'm2':0, 'm3':1}])
104 # hmm1f3 = HMMVEfilter(hmm1)
105 # hmm1f3.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
        {'m1':1, 'm2':0, 'm3':0}, {'m1':1, 'm2':0, 'm3':1}])
106
107 # How do the following differ in the resulting state distribution?
108 # Note they start the same, but have different initial observations.
109 ## HMMVEfilter.max_display_level = 1 # show less detail in displaying
110 # for i in range(100): hmm1f1.advance()
```

```
# hmm1f1.state_dist
# for i in range(100): hmm1f3.advance()
# hmm1f3.state_dist
```

Exercise 8.6 The representation assumes that there are a list of Boolean observations. Extend the representation so that the each observation variable can have multiple discrete values. You need to choose a representation for the model, and change the algorithm.

8.9.2 Localization

The localization example in the book is a controlled HMM, where there is a given action at each time and the transition depends on the action. In this class, the transition is set to None initially, and needs to be provided with an action to determine the transition probability.

probLocalization.py — Controlled HMM and Localization example

```
from probHMM import HMMVEfilter, HMM
from display import Displayable
import matplotlib.pyplot as plt
from matplotlib.widgets import Button, CheckButtons

class HMM_Controlled(HMM):
    """A controlled HMM, where the transition probability depends on the
       action.
      Instead of the transition probability, it has a function act2trans
      from action to transition probability.
      Any algorithms need to select the transition probability according
          to the action.
    """
    def __init__(self, states, obsvars, pobs, act2trans, indist):
        self.act2trans = act2trans
        HMM.__init__(self, states, obsvars, pobs, None, indist)


local_states = list(range(16))
door_positions = {2,4,7,11}
def prob_door(loc): return 0.8 if loc in door_positions else 0.1
local_obs = {'door':[prob_door(i) for i in range(16)]}
act2trans = {'right': [[0.1 if next == current
                            else 0.8 if next == (current+1)%16
                            else 0.074 if next == (current+2)%16
                            else 0.002 for next in range(16)] for
                                current in range(16)],
                     'left': [[0.1 if next == current
                            else 0.8 if next == (current-1)%16
                            else 0.074 if next == (current-2)%16
                            else 0.002 for next in range(16)] for
                                current in range(16)]}
```

```
hmm_16pos = HMM_Controlled(local_states, {'door'}, local_obs, act2trans,
    [1/16 for i in range(16)])
```

To change the VE localization code to allow for controlled HMMs, notice that the action selects which transition probability to us.

probLocalization.py — (continued)

```
class HMM_Local(HMMVEfilter):
    """VE filter for controlled HMMs
    """
    def __init__(self, hmm):
        HMMVEfilter.__init__(self, hmm)

    def go(self, action):
        self.hmm.trans = self.hmm.act2trans[action]
        self.advance()

loc_filt = HMM_Local(hmm_16pos)
# loc_filt.observe({'door':True}); loc_filt.go("right");
    loc_filt.observe({'door':False}); loc_filt.go("right");
    loc_filt.observe({'door':True})
# loc_filt.state_dist
```

The following lets us interactively move the agent and provide observations. It shows the distribution over locations.

probLocalization.py — (continued)

```
class Show_Localization(Displayable):
    def __init__(self,hmm):
        self.hmm = hmm
        self.loc_filt = HMM_Local(hmm)
        fig,(self.ax) = plt.subplots()
        plt.subplots_adjust(bottom=0.2)
        left_butt = Button(plt.axes([0.05,0.02,0.1,0.05]), "left")
        left_butt.on_clicked(self.left)
        right_butt = Button(plt.axes([0.25,0.02,0.1,0.05]), "right")
        right_butt.on_clicked(self.right)
        door_butt = Button(plt.axes([0.45,0.02,0.1,0.05]), "door")
        door_butt.on_clicked(self.door)
        nodoor_butt = Button(plt.axes([0.65,0.02,0.1,0.05]), "no door")
        nodoor_butt.on_clicked(self.nodoor)
        reset_butt = Button(plt.axes([0.85,0.02,0.1,0.05]), "reset")
        reset_butt.on_clicked(self.reset)
               #this makes sure y-axis goes to 1, graph overwritten in
                   draw_dist
        self.draw_dist()
        plt.show()

    def draw_dist(self):
        self.ax.clear()
        plt.ylim(0,1)
```

```
77          self.ax.set_ylabel("Probability")
78          self.ax.set_xlabel("Location")
79          self.ax.set_title("Location Probability Distribution")
80          self.ax.set_xticks(self.hmm.states)
81          vals = [self.loc_filt.state_dist[i] for i in self.hmm.states]
82          self.bars = self.ax.bar(self.hmm.states, vals, color='black')
83          self.ax.bar_label(self.bars,["{v:.2f}".format(v=v) for v in vals],
                padding = 1)
84          plt.draw()
85
86      def left(self,event):
87          self.loc_filt.go("left")
88          self.draw_dist()
89      def right(self,event):
90          self.loc_filt.go("right")
91          self.draw_dist()
92      def door(self,event):
93          self.loc_filt.observe({'door':True})
94          self.draw_dist()
95      def nodoor(self,event):
96          self.loc_filt.observe({'door':False})
97          self.draw_dist()
98      def reset(self,event):
99          self.loc_filt.state_dist = {i:1/16 for i in range(16)}
100         self.draw_dist()
101
102 # sl = Show_Localization(hmm_16pos)
```

8.9.3 Particle Filtering for HMMs

In this implementation a particle is just a state. If you want to do some form of smoothing, a particle should probably be a history of states. This maintains, *particles*, an array of states, *weights* an array of (non-negative) real numbers, such that $weights[i]$ is the weight of $particles[i]$.

probHMM.py — (continued)

```
114 from display import Displayable
115 from probStochSim import resample
116
117 class HMMparticleFilter(Displayable):
118     def __init__(self,hmm,number_particles=1000):
119         self.hmm = hmm
120         self.particles = [sample_one(hmm.indist)
121                           for i in range(number_particles)]
122         self.weights = [1 for i in range(number_particles)]
123
124     def filter(self, obsseq):
125         """returns the state distribution following the sequence of
126         observations in obsseq using particle filtering.
```

```
127
128         Note that it first advances time.
129         This is what is required if it is called after previous filtering.
130         If that is not what is wanted initially, do an observe first.
131         """
132         for obs in obsseq:
133             self.advance()   # advance time
134             self.observe(obs) # observe
135             self.resample_particles()
136             self.display(2,"After observing", str(obs),
137                          "state distribution:",
                                 self.histogram(self.particles))
138         self.display(1,"Final state distribution:",
                self.histogram(self.particles))
139         return self.histogram(self.particles)
140
141     def advance(self):
142         """advance to the next time.
143         This assumes that all of the weights are 1."""
144         self.particles = [sample_one(self.hmm.trans[st])
145                           for st in self.particles]
146
147     def observe(self, obs):
148         """reweighs the particles to incorporate observations obs"""
149         for i in range(len(self.particles)):
150             for obv in obs:
151                 if obs[obv]:
152                     self.weights[i] *= self.hmm.pobs[obv][self.particles[i]]
153                 else:
154                     self.weights[i] *=
                            1-self.hmm.pobs[obv][self.particles[i]]
155
156     def histogram(self, particles):
157         """returns list of the probability of each state as represented by
158         the particles"""
159         tot=0
160         hist = {st: 0.0 for st in self.hmm.states}
161         for (st,wt) in zip(self.particles,self.weights):
162             hist[st]+=wt
163             tot += wt
164         return {st:hist[st]/tot for st in hist}
165
166     def resample_particles(self):
167         """resamples to give a new set of particles."""
168         self.particles = resample(self.particles, self.weights,
                len(self.particles))
169         self.weights = [1] * len(self.particles)
```

The following are some queries for *hmm*1.

probHMM.py — (continued)

```
hmm1pf1 = HMMparticleFilter(hmm1)
# HMMparticleFilter.max_display_level = 2 # show each step
# hmm1pf1.filter([{'m1':0, 'm2':1, 'm3':1}, {'m1':1, 'm2':0, 'm3':1}])
# hmm1pf2 = HMMparticleFilter(hmm1)
# hmm1pf2.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':1, 'm3':0},
    {'m1':1, 'm2':0, 'm3':0},
#               {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
    {'m1':0, 'm2':0, 'm3':0},
#               {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':1},
    {'m1':0, 'm2':0, 'm3':1},
#               {'m1':0, 'm2':0, 'm3':1}])
# hmm1pf3 = HMMparticleFilter(hmm1)
# hmm1pf3.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
    {'m1':1, 'm2':0, 'm3':0}, {'m1':1, 'm2':0, 'm3':1}])
```

Exercise 8.7 A form of importance sampling can be obtained by not resampling. Is it better or worse than particle filtering? Hint: you need to think about how they can be compared. Is the comparison different if there are more states than particles?

Exercise 8.8 Extend the particle filtering code to continuous variables and observations. In particular, suppose the state transition is a linear function with Gaussian noise of the previous state, and the observations are linear functions with Gaussian noise of the state. You may need to research how to sample from a Gaussian distribution.

8.9.4 Generating Examples

The following code is useful for generating examples.

probHMM.py — (continued)

```
def simulate(hmm,horizon):
    """returns a pair of (state sequence, observation sequence) of length
        horizon.
    for each time t, the agent is in state_sequence[t] and
    observes observation_sequence[t]
    """
    state = sample_one(hmm.indist)
    obsseq=[]
    stateseq=[]
    for time in range(horizon):
        stateseq.append(state)
        newobs =
            {obs:sample_one({0:1-hmm.pobs[obs][state],1:hmm.pobs[obs][state]})
                 for obs in hmm.obsvars}
        obsseq.append(newobs)
        state = sample_one(hmm.trans[state])
    return stateseq,obsseq

def simobs(hmm,stateseq):
```

```
199     """returns observation sequence for the state sequence"""
200     obsseq=[]
201     for state in stateseq:
202         newobs =
                {obs:sample_one({0:1-hmm.pobs[obs][state],1:hmm.pobs[obs][state]})
203                  for obs in hmm.obsvars}
204         obsseq.append(newobs)
205     return obsseq
206
207 def create_eg(hmm,n):
208     """Create an annotated example for horizon n"""
209     seq,obs = simulate(hmm,n)
210     print("True state sequence:",seq)
211     print("Sequence of observations:\n",obs)
212     hmmfilter = HMMVEfilter(hmm)
213     dist = hmmfilter.filter(obs)
214     print("Resulting distribution over states:\n",dist)
```

8.10 Dynamic Belief Networks

A **dynamic belief network (DBN)** is a belief network that extends in time.

There are a number of ways that reasoning can be carried out in a DBN, including:

- Rolling out the DBN for some time period, and using standard belief network inference. The latest time that needs to be in the rolled out network is the time of the latest observation or the time of a query (whichever is later). This allows us to observe any variables at any time and query any variables at any time. This is covered in Section 8.10.2.

- An unrolled belief network may be very large, and we might only be interested in asking about "now". In this case we can just representing the variables "now". In this approach we can observe and query the current variables. We can them move to the next time. This does not allow for arbitrary historical queries (about the past or the future), but can be much simpler. This is covered in Section 8.10.3.

8.10.1 Representing Dynamic Belief Networks

To specify a DBN, think about the distribution *now*. *Now* will be represented as time 1. Each variable will have a corresponding previous variable; these will be created together.

A dynamic belief network consists of:

- A set of features. A variable is a feature-time pair.

- An initial distribution over the features "now" (time 1). This is a belief network with all variables being time 1 variables.
- A specification of the dynamics. We define the how the variables *now* (time 1) depend on variables *now* and the previous time (time 0), in such a way that the graph is acyclic.

probDBN.py — Dynamic belief networks

```
from probVariables import Variable
from probGraphicalModels import GraphicalModel, BeliefNetwork
from probFactors import Prob, Factor, CPD
from probVE import VE
from display import Displayable
from utilities import dict_union

class DBNvariable(Variable):
    """A random variable that incorporates the stage (time)

    A variable can have both a name and an index. The index defaults to 1.
    """
    def __init__(self,name,domain=[False,True],index=1):
        Variable.__init__(self,f"{name}_{index}",domain)
        self.basename = name
        self.domain = domain
        self.index = index
        self.previous = None

    def __lt__(self,other):
        if self.name != other.name:
            return self.name<other.name
        else:
            return self.index<other.index

    def __gt__(self,other):
        return other<self

def variable_pair(name,domain=[False,True]):
    """returns a variable and its predecessor. This is used to define
        2-stage DBNs

    If the name is X, it returns the pair of variables X_prev,X_now"""
    var_now = DBNvariable(name,domain,index='now')
    var_prev = DBNvariable(name,domain,index='prev')
    var_now.previous = var_prev
    return var_prev, var_now
```

A *FactorRename* is a factor that is the result renaming the variables in the factor. It takes a factor, *fac*, and a $\{new : old\}$ dictionary, where *new* is the name of a variable in the resulting factor and *old* is the corresponding name in *fac*. This assumes that the all variables are renamed.

probDBN.py — (continued)

```
class FactorRename(Factor):
    def __init__(self,fac,renaming):
        """A renamed factor.
        fac is a factor
        renaming is a dictionary of the form {new:old} where old and new
            var variables,
           where the variables in fac appear exactly once in the renaming
        """
        Factor.__init__(self,[n for (n,o) in renaming.items() if o in
            fac.variables])
        self.orig_fac = fac
        self.renaming = renaming

    def get_value(self,assignment):
        return self.orig_fac.get_value({self.renaming[var]:val
                                        for (var,val) in assignment.items()
                                        if var in self.variables})
```

The following class renames the variables of a conditional probability distribution. It is used for template models (e.g., dynamic decision networks or relational models)

probDBN.py — (continued)

```
class CPDrename(FactorRename, CPD):
    def __init__(self, cpd, renaming):
        renaming_inverse = {old:new for (new,old) in renaming.items()}
        CPD.__init__(self,renaming_inverse[cpd.child],[renaming_inverse[p]
            for p in cpd.parents])
        self.orig_fac = cpd
        self.renaming = renaming
```

probDBN.py — (continued)

```
class DBN(Displayable):
    """The class of stationary Dynamic Belief networks.
    * name is the DBN name
    * vars_now is a list of current variables (each must have
    previous variable).
    * transition_factors is a list of factors for P(X|parents) where X
    is a current variable and parents is a list of current or previous
        variables.
    * init_factors is a list of factors for P(X|parents) where X is a
    current variable and parents can only include current variables
    The graph of transition factors + init factors must be acyclic.

    """
    def __init__(self, title, vars_now, transition_factors=None,
        init_factors=None):
        self.title = title
        self.vars_now = vars_now
```

```
        self.vars_prev = [v.previous for v in vars_now]
        self.transition_factors = transition_factors
        self.init_factors = init_factors
        self.var_index = {}    # var_index[v] is the index of variable v
        for i,v in enumerate(vars_now):
            self.var_index[v]=i
```

Here is a 3 variable DBN:

probDBN.py — (continued)

```
A0,A1 = variable_pair("A", domain=[False,True])
B0,B1 = variable_pair("B", domain=[False,True])
C0,C1 = variable_pair("C", domain=[False,True])

# dynamics
pc = Prob(C1,[B1,C0],[[[0.03,0.97],[0.38,0.62]],[[0.23,0.77],[0.78,0.22]]])
pb = Prob(B1,[A0,A1],[[[0.5,0.5],[0.77,0.23]],[[0.4,0.6],[0.83,0.17]]])
pa = Prob(A1,[A0,B0],[[[0.1,0.9],[0.65,0.35]],[[0.3,0.7],[0.8,0.2]]])

# initial distribution
pa0 = Prob(A1,[],[0.9,0.1])
pb0 = Prob(B1,[A1],[[0.3,0.7],[0.8,0.2]])
pc0 = Prob(C1,[],[0.2,0.8])

dbn1 = DBN("Simple DBN",[A1,B1,C1],[pa,pb,pc],[pa0,pb0,pc0])
```

Here is the animal example

probDBN.py — (continued)

```
from probHMM import closeMic, farMic, midMic, sm, mmc, sc, mcm, mcc

Pos_0,Pos_1 = variable_pair("Position",domain=[0,1,2,3])
Mic1_0,Mic1_1 = variable_pair("Mic1")
Mic2_0,Mic2_1 = variable_pair("Mic2")
Mic3_0,Mic3_1 = variable_pair("Mic3")

# conditional probabilities - see hmm for the values of sm,mmc, etc
ppos = Prob(Pos_1, [Pos_0],
           [[sm, mmc, mmc, mmc], #was in middle
            [mcm, sc, mcc, mcc], #was in corner 1
            [mcm, mcc, sc, mcc], #was in corner 2
            [mcm, mcc, mcc, sc]]) #was in corner 3
pm1 = Prob(Mic1_1, [Pos_1], [[1-midMic, midMic], [1-closeMic, closeMic],
                         [1-farMic, farMic], [1-farMic, farMic]])
pm2 = Prob(Mic2_1, [Pos_1], [[1-midMic, midMic], [1-farMic, farMic],
                         [1-closeMic, closeMic], [1-farMic, farMic]])
pm3 = Prob(Mic3_1, [Pos_1], [[1-midMic, midMic], [1-farMic, farMic],
                         [1-farMic, farMic], [1-closeMic, closeMic]])
ipos = Prob(Pos_1,[], [0.25, 0.25, 0.25, 0.25])
dbn_an =DBN("Animal DBN",[Pos_1,Mic1_1,Mic2_1,Mic3_1],
           [ppos, pm1, pm2, pm3],
           [ipos, pm1, pm2, pm3])
```

8.10.2 Unrolling DBNs

probDBN.py — (continued)

```
class BNfromDBN(BeliefNetwork):
    """Belief Network unrolled from a dynamic belief network
    """

    def __init__(self,dbn,horizon):
        """dbn is the dynamic belief network being unrolled
        horizon>0 is the number of steps (so there will be horizon+1
            variables for each DBN variable.
        """
        self.name2var = {var.basename:
            [DBNvariable(var.basename,var.domain,index) for index in
            range(horizon+1)]
                         for var in dbn.vars_now}
        self.display(1,f"name2var={self.name2var}")
        variables = {v for vs in self.name2var.values() for v in vs}
        self.display(1,f"variables={variables}")
        bnfactors = {CPDrename(fac,{self.name2var[var.basename][0]:var
                                    for var in fac.variables})
                     for fac in dbn.init_factors}
        bnfactors |=
            {CPDrename(fac,dict_union({self.name2var[var.basename][i]:var
                                    for var in fac.variables if
                                        var.index=='prev'}
                                , {self.name2var[var.basename][i+1]:var
                                    for var in fac.variables if
                                        var.index=='now'}))
                     for fac in dbn.transition_factors
                        for i in range(horizon)}
        self.display(1,f"bnfactors={bnfactors}")
        BeliefNetwork.__init__(self, dbn.title, variables, bnfactors)
```

Here are two examples. Note that we need to use `bn.name2var['B'][2]` to get the variable B2 (B at time 2).

probDBN.py — (continued)

```
# Try
#from probRC import ProbRC
#bn = BNfromDBN(dbn1,2) # construct belief network
#drc = ProbRC(bn)           # initialize recursive conditioning
#B2 = bn.name2var['B'][2]
#drc.query(B2) #P(B2)
#drc.query(bn.name2var['B'][1],{bn.name2var['B'][0]:True,bn.name2var['C'][1]:False})
    #P(B1|B0,C1)
```

8.10.3 DBN Filtering

If we only wanted to ask questions about the current state, we can save space by forgetting the history variables.

probDBN.py — (continued)

```
class DBNVEfilter(VE):
    def __init__(self,dbn):
        self.dbn = dbn
        self.current_factors = dbn.init_factors
        self.current_obs = {}

    def observe(self, obs):
        """updates the current observations with obs.
        obs is a variable:value dictionary where variable is a current
        variable.
        """
        assert all(self.current_obs[var]==obs[var] for var in obs
                   if var in self.current_obs),"inconsistent current
                       observations"
        self.current_obs.update(obs) # note 'update' is a dict method

    def query(self,var):
        """returns the posterior probability of current variable var"""
        return
            VE(GraphicalModel(self.dbn.title,self.dbn.vars_now,self.current_factors)).query(var,sel

    def advance(self):
        """advance to the next time"""
        prev_factors = [self.make_previous(fac) for fac in
            self.current_factors]
        prev_obs = {var.previous:val for var,val in
            self.current_obs.items()}
        two_stage_factors = prev_factors + self.dbn.transition_factors
        self.current_factors =
            self.elim_vars(two_stage_factors,self.dbn.vars_prev,prev_obs)
        self.current_obs = {}

    def make_previous(self,fac):
         """Creates new factor from fac where the current variables in fac
         are renamed to previous variables.
         """
         return FactorRename(fac, {var.previous:var for var in
             fac.variables})

    def elim_vars(self,factors, vars, obs):
        for var in vars:
            if var in obs:
                factors = [self.project_observations(fac,obs) for fac in
                    factors]
            else:
                factors = self.eliminate_var(factors, var)
        return factors
```

Example queries:

probDBN.py — (continued)

```
#df = DBNVEfilter(dbn1)
#df.observe({B1:True}); df.advance(); df.observe({C1:False})
#df.query(B1) #P(B1|B0,C1)
#df.advance(); df.query(B1)
#dfa = DBNVEfilter(dbn_an)
# dfa.observe({Mic1_1:0, Mic2_1:1, Mic3_1:1})
# dfa.advance()
# dfa.observe({Mic1_1:1, Mic2_1:0, Mic3_1:1})
# dfa.query(Pos_1)
```

8.11 Causal Models

A causal model can answer "do" questions.

The following adds the queryDo method to the InferenceMethod class, so it can be used with any inference method.

probDo.py — Probabilistic inference with the do operator

```
from probGraphicalModels import InferenceMethod, BeliefNetwork
from probFactors import CPD, ConstantCPD

def queryDo(self, qvar, obs={}, do={}):
    assert isinstance(self.gm, BeliefNetwork), "Do only applies to belief
        networks"
    if do=={}:
        return self.query(qvar, obs)
    else:
        newfacs = ({f for (ch,f) in self.gm.var2cpt.items() if ch not in
            do} |
                   {ConstantCPD(v,c) for (v,c) in do.items()})
        self.modBN = BeliefNetwork(self.gm.title+"(mod)",
            self.gm.variables, newfacs)
        oldBN, self.gm = self.gm, self.modBN
        result = self.query(qvar, obs)
        self.gm = oldBN # restore original
        return result

InferenceMethod.queryDo = queryDo
```

probDo.py — (continued)

```
from probRC import ProbRC

from probGraphicalModels import bn_sprinkler, Season, Sprinkler, Rained,
    Grass_wet, Grass_shiny, Shoes_wet, bn_sprinkler_soff
bn_sprinklerv = ProbRC(bn_sprinkler)
## bn_sprinklerv.queryDo(Shoes_wet)
## bn_sprinklerv.queryDo(Shoes_wet,obs={Sprinkler:"off"})
## bn_sprinklerv.queryDo(Shoes_wet,do={Sprinkler:"off"})
```

```
## ProbRC(bn_sprinkler_soff).query(Shoes_wet) # should be same as previous
    case
## bn_sprinklerv.queryDo(Season, obs={Sprinkler:"off"})
## bn_sprinklerv.queryDo(Season, do={Sprinkler:"off"})
```

probDo.py — (continued)

```
from probVariables import Variable
from probFactors import Prob
from probGraphicalModels import boolean

Drug_Prone = Variable("Drug_Prone", boolean, position=(0.1,0.5))
Takes_Marijuana = Variable("Takes_Marijuana", boolean, position=(0.1,0.5))
Side_Effects = Variable("Side_Effects", boolean, position=(0.1,0.5))
Takes_Hard_Drugs = Variable("Takes_Hard_Drugs", boolean,
    position=(0.9,0.5))

p_dp = Prob(Drug_Prone, [], [0.8, 0.2])
p_tm = Prob(Takes_Marijuana, [Drug_Prone], [[0.98, 0.02], [0.2, 0.8]])
p_be = Prob(Side_Effects, [Takes_Marijuana], [[1, 0], [0.4, 0.6]])
p_thd = Prob(Takes_Hard_Drugs, [Side_Effects, Drug_Prone],
               # Drug_Prone=False  Drug_Prone=True
               [[[0.999, 0.001],   [0.6, 0.4]], # Side_Effects=False
                [[0.99999, 0.00001], [0.995, 0.005]]]) # Side_Effects=True

drugs = BeliefNetwork("Gateway Drugs",
                  [Drug_Prone,Takes_Marijuana,Side_Effects,Takes_Hard_Drugs],
                  [p_dp, p_tm, p_be, p_thd])
drugsq = ProbRC(drugs)
# drugsq.queryDo(Takes_Hard_Drugs)
# drugsq.queryDo(Takes_Hard_Drugs, obs = {Takes_Marijuana: True})
# drugsq.queryDo(Takes_Hard_Drugs, obs = {Takes_Marijuana: False})
# drugsq.queryDo(Takes_Hard_Drugs, do = {Takes_Marijuana: True})
# drugsq.queryDo(Takes_Hard_Drugs, do = {Takes_Marijuana: False})
```

Chapter 9

Planning with Uncertainty

9.1 Decision Networks

The decision network code builds on the representation for belief networks of Chapter 8.

We first allow for factors that define the utility. Here the **utility** is a function of the variables in *vars*. In a **utility table** the utility is defined in terms of a, a list that enumerates the values as in Section 8.3.3.

decnNetworks.py — Representations for Decision Networks

```
from probGraphicalModels import GraphicalModel, BeliefNetwork
from probFactors import Factor, CPD, TabFactor, factor_times, Prob
from probVariables import Variable
import matplotlib.pyplot as plt

class Utility(Factor):
    """A factor defining a utility"""
    pass

class UtilityTable(TabFactor, Utility):
    """A factor defining a utility using a table"""
    def __init__(self, vars, table, position=None):
        """Creates a factor on vars from the table.
        The table is ordered according to vars.
        """
        TabFactor.__init__(self,vars,table)
        self.position = position
```

A **decision variable** is a like a random variable with a string name, and a domain, which is a list of possible values. The decision variable also includes the parents, a list of the variables whose value will be known when the decision is made. It also includes a potion, which is only used for plotting.

decnNetworks.py — (continued)

```
class DecisionVariable(Variable):
    def __init__(self, name, domain, parents, position=None):
        Variable.__init__(self, name, domain, position)
        self.parents = parents
        self.all_vars = set(parents) | {self}
```

A decision network is a graphical model where the variables can be random variables or decision variables. Among the factors we assume there is one utility factor.

decnNetworks.py — (continued)

```
class DecisionNetwork(BeliefNetwork):
    def __init__(self, title, vars, factors):
        """vars is a list of variables
        factors is a list of factors (instances of CPD and Utility)
        """
        GraphicalModel.__init__(self, title, vars, factors) # don't call
            init for BeliefNetwork
        self.var2parents = ({v : v.parents for v in vars if
            isinstance(v,DecisionVariable)}
                    | {f.child:f.parents for f in factors if
                        isinstance(f,CPD)})
        self.children = {n:[] for n in self.variables}
        for v in self.var2parents:
            for par in self.var2parents[v]:
                self.children[par].append(v)
        self.utility_factor = [f for f in factors if
            isinstance(f,Utility)][0]
        self.topological_sort_saved = None
```

The split order ensures that the parents of a decision node are split before the decision node, and no other variables (if that is possible).

decnNetworks.py — (continued)

```
    def split_order(self):
        so = []
        tops = self.topological_sort()
        for v in tops:
            if isinstance(v,DecisionVariable):
                so += [p for p in v.parents if p not in so]
                so.append(v)
        so += [v for v in tops if v not in so]
        return so
```

decnNetworks.py — (continued)

```
    def show(self):
        plt.ion()  # interactive
        ax = plt.figure().gca()
        ax.set_axis_off()
        plt.title(self.title)
```

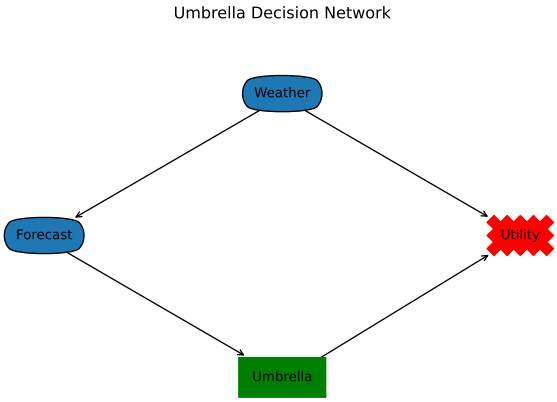


Figure 9.1: The umbrella decision network

```
65          for par in self.utility_factor.variables:
66              ax.annotate("Utility", par.position,
                    xytext=self.utility_factor.position,
67                                  arrowprops={'arrowstyle':'<-'},bbox=dict(boxstyle="sawtooth,pad=1
68                                  ha='center')
69          for var in reversed(self.topological_sort()):
70              if isinstance(var,DecisionVariable):
71                  bbox = dict(boxstyle="square,pad=1.0",color="green")
72              else:
73                 bbox = dict(boxstyle="round4,pad=1.0,rounding_size=0.5")
74              if self.var2parents[var]:
75                  for par in self.var2parents[var]:
76                      ax.annotate(var.name, par.position, xytext=var.position,
77                                  arrowprops={'arrowstyle':'<-'},bbox=bbox,
78                                  ha='center')
79              else:
80                  x,y = var.position
81                  plt.text(x,y,var.name,bbox=bbox,ha='center')
```

9.1.1 Example Decision Networks

Umbrella Decision Network

Here is a simple "umbrella" decision network. The output of `umbrella_dn.show()` is shown in Figure 9.1.

decnNetworks.py — (continued)

```
83 Weather = Variable("Weather", ["NoRain", "Rain"], position=(0.5,0.8))
84 Forecast = Variable("Forecast", ["Sunny", "Cloudy", "Rainy"],
       position=(0,0.4))
85 # Each variant uses one of the following:
```

```
Umbrella = DecisionVariable("Umbrella", ["Take", "Leave"], {Forecast},
    position=(0.5,0))

p_weather = Prob(Weather, [], [0.7, 0.3])
p_forecast = Prob(Forecast, [Weather], [[0.7, 0.2, 0.1], [0.15, 0.25,
    0.6]])
umb_utility = UtilityTable([Weather, Umbrella], [[20, 100], [70, 0]],
    position=(1,0.4))

umbrella_dn = DecisionNetwork("Umbrella Decision Network",
                              {Weather, Forecast, Umbrella},
                              {p_weather, p_forecast, umb_utility})
```

The following is a variant with the umbrella decision having 2 parents; nothing else has changed. This is interesting because one of the parents is not needed; if the agent knows the weather, it can ignore the forecast.

decnNetworks.py — (continued)

```
Umbrella2p = DecisionVariable("Umbrella", ["Take", "Leave"], {Forecast,
    Weather}, position=(0.5,0))
umb_utility2p = UtilityTable([Weather, Umbrella2p], [[20, 100], [70, 0]],
    position=(1,0.4))
umbrella_dn2p = DecisionNetwork("Umbrella Decision Network (extra arc)",
                              {Weather, Forecast, Umbrella2p},
                              {p_weather, p_forecast, umb_utility2p})
```

Fire Decision Network

The fire decision network of Figure 9.2 (showing the result of `fire_dn.show()`) is represented as:

decnNetworks.py — (continued)

```
boolean = [False, True]
Alarm = Variable("Alarm", boolean, position=(0.25,0.633))
Fire = Variable("Fire", boolean, position=(0.5,0.9))
Leaving = Variable("Leaving", boolean, position=(0.25,0.366))
Report = Variable("Report", boolean, position=(0.25,0.1))
Smoke = Variable("Smoke", boolean, position=(0.75,0.633))
Tamper = Variable("Tamper", boolean, position=(0,0.9))

See_Sm = Variable("See_Sm", boolean, position=(0.75,0.366) )
Chk_Sm = DecisionVariable("Chk_Sm", boolean, {Report}, position=(0.5,
    0.366))
Call = DecisionVariable("Call", boolean,{See_Sm,Chk_Sm,Report},
    position=(0.75,0.1))

f_ta = Prob(Tamper,[],[0.98,0.02])
f_fi = Prob(Fire,[],[0.99,0.01])
f_sm = Prob(Smoke,[Fire],[[0.99,0.01],[0.1,0.9]])
f_al = Prob(Alarm,[Fire,Tamper],[[[0.9999, 0.0001], [0.15, 0.85]], [[0.01,
    0.99], [0.5, 0.5]]])
```

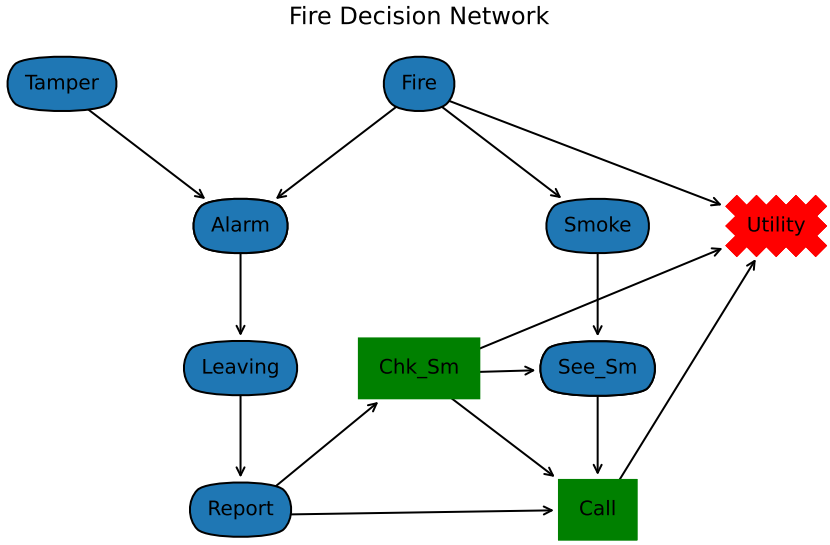


Figure 9.2: Fire Decision Network

```
f_lv = Prob(Leaving,[Alarm],[[0.999, 0.001], [0.12, 0.88]])
f_re = Prob(Report,[Leaving],[[0.99, 0.01], [0.25, 0.75]])
f_ss = Prob(See_Sm,[Chk_Sm,Smoke],[[[1,0],[1,0]],[[1,0],[0,1]]])

ut =
    UtilityTable([Chk_Sm,Fire,Call],[[[0,-200],[-5000,-200]],[[-20,-220],[-5020,-220]]],
    position=(1,0.633))

fire_dn = DecisionNetwork("Fire Decision Network",
                          {Tamper,Fire,Alarm,Leaving,Smoke,Call,See_Sm,Chk_Sm,Report},
                          {f_ta,f_fi,f_sm,f_al,f_lv,f_re,f_ss,ut})
```

Cheating Decision Network

The following is the representation of the cheating decision of Figure 9.3. Note that we keep the names of the variables short (less than 8 characters) so that the tables look good when printed.

decnNetworks.py — (continued)

```
grades = ['A','B','C','F']
Watched = Variable("Watched", boolean, position=(0,0.9))
Caught1 = Variable("Caught1", boolean, position=(0.2,0.7))
Caught2 = Variable("Caught2", boolean, position=(0.6,0.7))
```

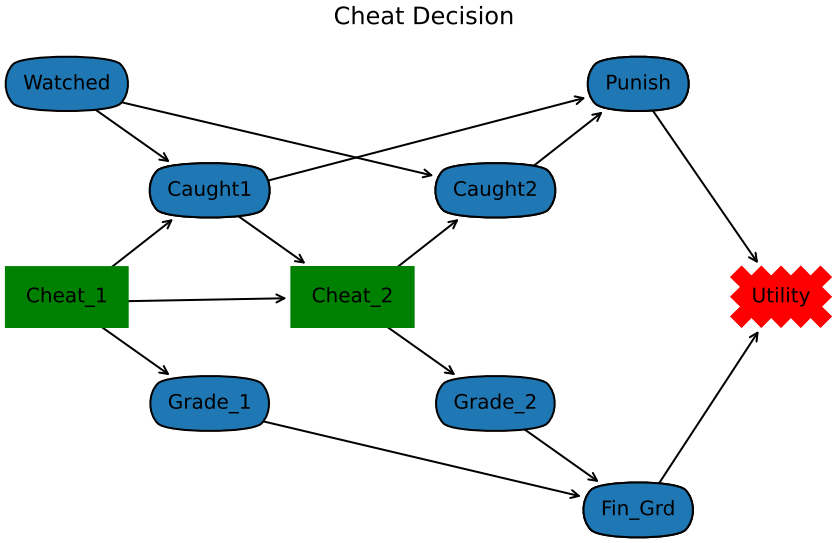


Figure 9.3: Cheating Decision Network

```
Punish = Variable("Punish", ["None","Suspension","Recorded"],
    position=(0.8,0.9))
Grade_1 = Variable("Grade_1", grades, position=(0.2,0.3))
Grade_2 = Variable("Grade_2", grades, position=(0.6,0.3))
Fin_Grd = Variable("Fin_Grd", grades, position=(0.8,0.1))
Cheat_1 = DecisionVariable("Cheat_1", boolean, set(), position=(0,0.5))
    #no parents
Cheat_2 = DecisionVariable("Cheat_2", boolean, {Cheat_1,Caught1},
    position=(0.4,0.5))

p_wa = Prob(Watched,[],[0.7, 0.3])
p_cc1 = Prob(Caught1,[Watched,Cheat_1],[[[1.0, 0.0], [0.9, 0.1]], [[1.0,
    0.0], [0.5, 0.5]]])
p_cc2 = Prob(Caught2,[Watched,Cheat_2],[[[1.0, 0.0], [0.9, 0.1]], [[1.0,
    0.0], [0.5, 0.5]]])
p_pun = Prob(Punish,[Caught1,Caught2],[[[1.0, 0.0, 0.0], [0.5, 0.4, 0.1]],
    [[0.6, 0.2, 0.2], [0.2, 0.5, 0.3]]])
p_gr1 = Prob(Grade_1,[Cheat_1], [{'A':0.2, 'B':0.3, 'C':0.3, 'D': 0.2},
    {'A':0.5, 'B':0.3, 'C':0.2, 'D':0.0}])
p_gr2 = Prob(Grade_2,[Cheat_2], [{'A':0.2, 'B':0.3, 'C':0.3, 'D': 0.2},
    {'A':0.5, 'B':0.3, 'C':0.2, 'D':0.0}])
p_fg = Prob(Fin_Grd,[Grade_1,Grade_2],
       {'A':{'A':{'A':1.0, 'B':0.0, 'C': 0.0, 'D':0.0},
             'B': {'A':0.5, 'B':0.5, 'C': 0.0, 'D':0.0},
```

```
148             'C':{'A':0.25, 'B':0.5, 'C': 0.25, 'D':0.0},
149             'D':{'A':0.25, 'B':0.25, 'C': 0.25, 'D':0.25}},
150         'B':{'A':{'A':0.5, 'B':0.5, 'C': 0.0, 'D':0.0},
151             'B': {'A':0.0, 'B':1, 'C': 0.0, 'D':0.0},
152             'C':{'A':0.0, 'B':0.5, 'C': 0.5, 'D':0.0},
153             'D':{'A':0.0, 'B':0.25, 'C': 0.5, 'D':0.25}},
154         'C':{'A':{'A':0.25, 'B':0.5, 'C': 0.25, 'D':0.0},
155             'B': {'A':0.0, 'B':0.5, 'C': 0.5, 'D':0.0},
156             'C':{'A':0.0, 'B':0.0, 'C': 1, 'D':0.0},
157             'D':{'A':0.0, 'B':0.0, 'C': 0.5, 'D':0.5}},
158         'D':{'A':{'A':0.25, 'B':0.25, 'C': 0.25, 'D':0.25},
159             'B': {'A':0.0, 'B':0.25, 'C': 0.5, 'D':0.25},
160             'C':{'A':0.0, 'B':0.0, 'C': 0.5, 'D':0.5},
161             'D':{'A':0.0, 'B':0.0, 'C': 0, 'D':1.0}}})
162
163 utc = UtilityTable([Punish,Fin_Grd],{'None':{'A':100, 'B':90, 'C': 70,
        'D':50},
164                                      'Suspension':{'A':40, 'B':20, 'C': 10,
                                             'D':0},
165                                      'Recorded':{'A':70, 'B':60, 'C': 40,
                                             'D':20}}, position=(1,0.5))
166
167 cheat_dn = DecisionNetwork("Cheat Decision",
168                            {Punish,Caught2,Watched,Fin_Grd,Grade_2,Grade_1,Cheat_2,Caught1,Cheat_1},
169                            {p_wa, p_cc1, p_cc2, p_pun, p_gr1,
                                p_gr2,p_fg,utc})
```

Chain of 3 decisions

The following example is a finite-stage fully-observable Markov decision process with a single reward (utility) at the end. It is interesting because the parents do not include all predecessors. The methods we use will work without change on this, even though the agent does not condition on all of its previous observations and actions. The output of `ch3.show()` is shown in Figure 9.4.

decnNetworks.py — (continued)

```
171 S0 = Variable('S0', boolean, position=(0,0.5))
172 D0 = DecisionVariable('D0', boolean, {S0}, position=(1/7,0.1))
173 S1 = Variable('S1', boolean, position=(2/7,0.5))
174 D1 = DecisionVariable('D1', boolean, {S1}, position=(3/7,0.1))
175 S2 = Variable('S2', boolean, position=(4/7,0.5))
176 D2 = DecisionVariable('D2', boolean, {S2}, position=(5/7,0.1))
177 S3 = Variable('S3', boolean, position=(6/7,0.5))
178
179 p_s0 = Prob(S0, [], [0.5,0.5])
180 tr = [[[0.1, 0.9], [0.9, 0.1]], [[0.2, 0.8], [0.8, 0.2]]] # 0 is flip, 1
        is keep value
181 p_s1 = Prob(S1, [D0,S0], tr)
182 p_s2 = Prob(S2, [D1,S1], tr)
183 p_s3 = Prob(S3, [D2,S2], tr)
```

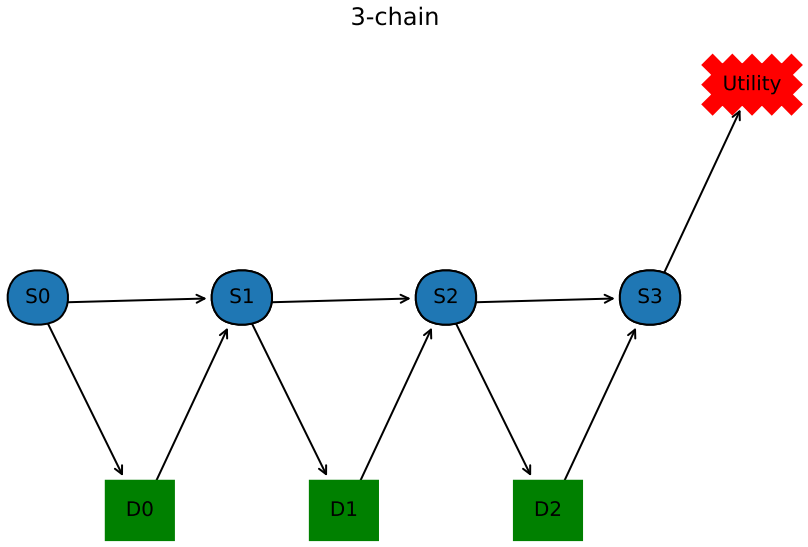


Figure 9.4: A decision network that is a chain of 3 decisions

```
184
185 ch3U = UtilityTable([S3],[0,1], position=(7/7,0.9))
186
187 ch3 = DecisionNetwork("3-chain",
        {S0,D0,S1,D1,S2,D2,S3},{p_s0,p_s1,p_s2,p_s3,ch3U})
188 #rc3 = RC_DN(ch3)
189 #rc3.optimize()
190 #rc3.opt_policy
```

9.1.2 Recursive Conditioning for decision networks

An instance of a `RC_DN` object takes in a decision network. The query method uses recursive conditioning to compute the expected utility of the optimal policy. `self.opt_policy` becomes the optimal policy.

decnNetworks.py — (continued)

```
192 import math
193 from probGraphicalModels import GraphicalModel, InferenceMethod
194 from probFactors import Factor
195 from utilities import dict_union
196 from probRC import connected_components
197
198 class RC_DN(InferenceMethod):
199     """The class that queries graphical models using recursive conditioning
```

```
200
201     gm is graphical model to query
202     """
203
204     def __init__(self,gm=None):
205         self.gm = gm
206         self.cache = {(frozenset(), frozenset()):1}
207         ## self.max_display_level = 3
208
209     def optimize(self, split_order=None):
210         """computes expected utility, and creates optimal decision
                functions, where
211         elim_order is a list of the non-observed non-query variables in gm
212         """
213         if split_order == None:
214             split_order = self.gm.split_order()
215         self.opt_policy = {}
216         return self.rc({}, self.gm.factors, split_order)
```

The following us the simplest search-based algorithm. It is exponential in the number of variables, so is not very useful. However, it is simple, and useful to understand before looking at the more complicated algorithm. Note that the above code does not call `rc0`; you will need to change the `self.rc` to `self.rc0` in above code to use it.

decnNetworks.py — (continued)

```
218     def rc0(self, context, factors, split_order):
219         """simplest search algorithm"""
220         self.display(2,"calling rc0,",(context,factors),"with
                SO",split_order)
221         if not factors:
222             return 1
223         elif to_eval := {fac for fac in factors if
                fac.can_evaluate(context)}:
224             self.display(3,"rc0 evaluating factors",to_eval)
225             val = math.prod(fac.get_value(context) for fac in to_eval)
226             return val * self.rc0(context, factors-to_eval, split_order)
227         else:
228             var = split_order[0]
229             self.display(3, "rc0 branching on", var)
230             if isinstance(var,DecisionVariable):
231                 assert set(context) <= set(var.parents), f"cannot optimize
                        {var} in context {context}"
232                 maxres = -math.inf
233                 for val in var.domain:
234                     self.display(3,"In rc0, branching on",var,"=",val)
235                     newres = self.rc0(dict_union({var:val},context),
                            factors, split_order[1:])
236                     if newres > maxres:
237                         maxres = newres
238                         theval = val
```

```
239                 self.opt_policy[frozenset(context.items())] = (var,theval)
240                 return maxres
241             else:
242                 total = 0
243                 for val in var.domain:
244                     total += self.rc0(dict_union({var:val},context),
                            factors, split_order[1:])
245                 self.display(3, "rc0 branching on", var,"returning", total)
246                 return total
```

We can combine the optimization for decision networks above, with the improvements of recursive conditioning used for graphical models (Section 8.6, page 174).

decnNetworks.py — (continued)

```
248     def rc(self, context, factors, split_order):
249         """ returns the number \sum_{split_order} \prod_{factors} given
                assignments in context
250         context is a variable:value dictionary
251         factors is a set of factors
252         split_order is a list of variables in factors that are not in
                context
253         """
254         self.display(3,"calling rc,",(context,factors))
255         ce = (frozenset(context.items()), frozenset(factors)) # key for the
                cache entry
256         if ce in self.cache:
257             self.display(2,"rc cache lookup",(context,factors))
258             return self.cache[ce]
259 #       if not factors: # no factors; needed if you don't have forgetting
        and caching
260 #           return 1
261         elif vars_not_in_factors := {var for var in context
262                                      if not any(var in fac.variables for
                                           fac in factors)}:
263             # forget variables not in any factor
264             self.display(3,"rc forgetting variables", vars_not_in_factors)
265             return self.rc({key:val for (key,val) in context.items()
266                             if key not in vars_not_in_factors},
267                            factors, split_order)
268         elif to_eval := {fac for fac in factors if
                fac.can_evaluate(context)}:
269             # evaluate factors when all variables are assigned
270             self.display(3,"rc evaluating factors",to_eval)
271             val = math.prod(fac.get_value(context) for fac in to_eval)
272             if val == 0:
273                 return 0
274             else:
275                 return val * self.rc(context, {fac for fac in factors if fac
                        not in to_eval}, split_order)
```

```
276         elif len(comp := connected_components(context, factors,
                split_order)) > 1:
277             # there are disconnected components
278             self.display(2,"splitting into connected components",comp)
279             return(math.prod(self.rc(context,f,eo) for (f,eo) in comp))
280         else:
281             assert split_order, f"split_order empty rc({context},{factors})"
282             var = split_order[0]
283             self.display(3, "rc branching on", var)
284             if isinstance(var,DecisionVariable):
285                 assert set(context) <= set(var.parents), f"cannot optimize
                        {var} in context {context}"
286                 maxres = -math.inf
287                 for val in var.domain:
288                     self.display(3,"In rc, branching on",var,"=",val)
289                     newres = self.rc(dict_union({var:val},context), factors,
                            split_order[1:])
290                     if newres > maxres:
291                         maxres = newres
292                         theval = val
293                 self.opt_policy[frozenset(context.items())] = (var,theval)
294                 self.cache[ce] = maxres
295                 return maxres
296             else:
297                 total = 0
298                 for val in var.domain:
299                     total += self.rc(dict_union({var:val},context), factors,
                            split_order[1:])
300                 self.display(3, "rc branching on", var,"returning", total)
301                 self.cache[ce] = total
302                 return total
```

Here is how to run the optimize the example decision networks:

decnNetworks.py — (continued)

```
304 # Umbrella decision network
305 #urc = RC_DN(umberella_dn)
306 #urc.optimize()
307 #urc.opt_policy
308
309 #rc_fire = RC_DN(fire_dn)
310 #rc_fire.optimize()
311 #rc_fire.opt_policy
312
313 #rc_cheat = RC_DN(cheat_dn)
314 #rc_cheat.optimize()
315 #rc_cheat.opt_policy
316
317 #rc_ch3 = RC_DN(ch3)
318 #rc_ch3.optimize()
319 #rc_ch3.opt_policy
```

9.1.3 Variable elimination for decision networks

VE_DN is variable elimination for decision networks. The method *optimize* is used to optimize all the decisions. Note that *optimize* requires a legal elimination ordering of the random and decision variables, otherwise it will give an exception. (A decision node can only be maximized if the variables that are not its parents have already been eliminated.)

decnNetworks.py — (continued)

```
321 from probVE import VE
322
323 class VE_DN(VE):
324     """Variable Elimination for Decision Networks"""
325     def __init__(self,dn=None):
326         """dn is a decision network"""
327         VE.__init__(self,dn)
328         self.dn = dn
329
330     def optimize(self,elim_order=None,obs={}):
331         if elim_order == None:
332                 elim_order = reversed(self.gm.split_order())
333         policy = []
334         proj_factors = [self.project_observations(fact,obs)
335                         for fact in self.dn.factors]
336         for v in elim_order:
337             if isinstance(v,DecisionVariable):
338                 to_max = [fac for fac in proj_factors
339                           if v in fac.variables and set(fac.variables) <=
                              v.all_vars]
340                 assert len(to_max)==1, "illegal variable order
                      "+str(elim_order)+" at "+str(v)
341                 newFac = FactorMax(v, to_max[0])
342                 policy.append(newFac.decision_fun)
343                 proj_factors = [fac for fac in proj_factors if fac is not
                      to_max[0]]+[newFac]
344                 self.display(2,"maximizing",v,"resulting
                      factor",newFac.brief() )
345                 self.display(3,newFac)
346             else:
347                 proj_factors = self.eliminate_var(proj_factors, v)
348         assert len(proj_factors)==1,"Should there be only one element of
               proj_factors?"
349         value = proj_factors[0].get_value({})
350         return value,policy
```

decnNetworks.py — (continued)

```
352 class FactorMax(Factor):
353     """A factor obtained by maximizing a variable in a factor.
354     Also builds a decision_function. This is based on FactorSum.
355     """
```

```
356
357     def __init__(self, dvar, factor):
358         """dvar is a decision variable.
359         factor is a factor that contains dvar and only parents of dvar
360         """
361         self.dvar = dvar
362         self.factor = factor
363         vars = [v for v in factor.variables if v is not dvar]
364         Factor.__init__(self,vars)
365         self.values = [None]*self.size
366         self.decision_fun = FactorDF(dvar,vars,[None]*self.size)
367
368     def get_value(self,assignment):
369         """lazy implementation: if saved, return saved value, else compute
                it"""
370         index = self.assignment_to_index(assignment)
371         if self.values[index]:
372             return self.values[index]
373         else:
374             max_val = float("-inf") # -infinity
375             new_asst = assignment.copy()
376             for elt in self.dvar.domain:
377                 new_asst[self.dvar] = elt
378                 fac_val = self.factor.get_value(new_asst)
379                 if fac_val>max_val:
380                     max_val = fac_val
381                     best_elt = elt
382             self.values[index] = max_val
383             self.decision_fun.values[index] = best_elt
384             return max_val
```

A decision function is a stored factor.

decnNetworks.py — (continued)

```
386 class FactorDF(TabFactor):
387     """A decision function"""
388     def __init__(self,dvar, vars, values):
389         TabStored.__init__(self,vars,values)
390         self.dvar = dvar
391         self.name = str(dvar) # Used in printing
```

Here are some example queries:

decnNetworks.py — (continued)

```
393 # Example queries:
394 # v,p = VE_DN(fire_dn).optimize(); print(v)
395 # for df in p: print(df,"\n")
396
397 # VE_DN.max_display_level = 3 # if you want to show lots of detail
398 # v,p = VE_DN(cheat_dn).optimize(); print(v)
399 # for df in p: print(df,"\n") # print decision functions
```

9.2 Markov Decision Processes

We will represent a **Markov decision process** (**MDP**) directly, rather than using the recursive conditioning or variable elimination code, as we did for decision networks.

mdpProblem.py — Representations for Markov Decision Processes

```
from utilities import argmaxd
import random
import matplotlib.pyplot as plt
from matplotlib.widgets import Button, CheckButtons

class MDP(object):
    """A Markov Decision Process. Must define:
    self.states the set (or list) of states
    self.actions the set (or list) of actions
    self.discount a real-valued discount
    """

    def __init__(self, states, actions, discount, init=0):
        self.states = states
        self.actions = actions
        self.discount = discount
        self.initv = self.v = {s:init for s in self.states}
        self.initq = self.q = {s: {a: init for a in self.actions} for s in
            self.states}

    def P(self,s,a):
        """Transition probability function
        returns a dictionary of {s1:p1} such that P(s1 | s,a)=p1. Other
            probabilities are zero.
        """
        raise NotImplementedError("P") # abstract method

    def R(self,s,a):
        """Reward function R(s,a)
        returns the expected reward for doing a in state s.
        """
        raise NotImplementedError("R") # abstract method
```

Two state partying example (Example 9.27 in Poole and Mackworth [2017]):

mdpExamples.py — MDP Examples

```
from mdpProblem import MDP, GridMDP

class party(MDP):
    """Simple 2-state, 2-Action Partying MDP Example"""
    def __init__(self, discount=0.9):
        states = {'healthy','sick'}
        actions = {'relax', 'party'}
        MDP.__init__(self, states, actions, discount)
```

```
19
20      def R(self,s,a):
21          "R(s,a)"
22          return { 'healthy': {'relax': 7, 'party': 10},
23                   'sick':   {'relax': 0, 'party': 2 }}[s][a]
24
25      def P(self,s,a):
26          "returns a dictionary of {s1:p1} such that P(s1 | s,a)=p1. Other
                probabilities are zero."
27          phealthy = { # P('healthy' | s, a)
28                       'healthy': {'relax': 0.95, 'party': 0.7},
29                       'sick': {'relax': 0.5, 'party': 0.1 }}[s][a]
30          return {'healthy':phealthy, 'sick':1-phealthy}
```

The next example is the tiny game from Example 12.1 and Figure 12.1 of Poole and Mackworth [2017]. The state is represented as (x, y) where x counts from zero from the left, and y counts from zero upwards, so the state $(0, 0)$ is on the bottom-left state. The actions are *upC* for up-careful, and *upR* for up-risky. (Note that `GridMDP` is just a type of MDP for which we have methods to show; you can assume it is just `MDP` here).

mdpExamples.py — (continued)

```
33  class MDPtiny(GridMDP):
34      def __init__(self, discount=0.9):
35          actions = ['right', 'upC', 'left', 'upR']
36          self.x_dim = 2 # x-dimension
37          self.y_dim = 3
38          states = [(x,y) for x in range(self.x_dim) for y in
                range(self.y_dim)]
39          # for GridMDP
40          self.xoff = {'right':0.25, 'upC':0, 'left':-0.25, 'upR':0}
41          self.yoff = {'right':0, 'upC':-0.25, 'left':0, 'upR':0.25}
42          GridMDP.__init__(self, states, actions, discount)
43
44      def P(self,s,a):
45          """return a dictionary of {s1:p1} if P(s1 | s,a)=p1. Other
                probabilities are zero.
46          """
47          (x,y) = s
48          if a == 'right':
49              return {(1,y):1}
50          elif a == 'upC':
51              return {(x,min(y+1,2)):1}
52          elif a == 'left':
53              if (x,y) == (0,2): return {(0,0):1}
54              else: return {(0,y): 1}
55          elif a == 'upR':
56              if x==0:
57                  if y<2: return {(x,y):0.1, (x+1,y):0.1, (x,y+1):0.8}
58                  else: # at (0,2)
59                      return {(0,0):0.1, (1,2): 0.1, (0,2): 0.8}
```

```
60            elif y < 2: # x==1
61                return {(0,y):0.1, (1,y):0.1, (1,y+1):0.8}
62            else: # at (1,2)
63               return {(0,2):0.1, (1,2): 0.9}
64
65    def R(self,s,a):
66        (x,y) = s
67        if a == 'right':
68            return [0,-1][x]
69        elif a == 'upC':
70            return [-1,-1,-2][y]
71        elif a == 'left':
72            if x==0:
73                return [-1, -100, 10][y]
74            else: return 0
75        elif a == 'upR':
76            return [[-0.1, -10, 0.2],[-0.1, -0.1, -0.9]][x][y]
77                # at (0,2) reward is 0.1*10+0.8*-1=0.2
```

Here is the domain of Example 9.28 of Poole and Mackworth [2017]. Here the state is represented as (x, y) where x counts from zero from the left, and y counts from zero upwards, so the state $(0, 0)$ is on the bottom-left state.

mdpExamples.py — (continued)

```
79 class grid(GridMDP):
80     """ x_dim * y_dim grid with rewarding states"""
81     def __init__(self, discount= 0.9, x_dim=10, y_dim=10):
82         self.x_dim = x_dim # size in x-direction
83         self.y_dim = y_dim # size in y-direction
84         actions = ['up', 'down', 'right', 'left']
85         states = [(x,y) for x in range(y_dim) for y in range(y_dim)]
86         self.rewarding_states = {(3,2):-10, (3,5):-5, (8,2):10, (7,7):3}
87         self.fling_states = {(8,2), (7,7)}
88         self.xoff = {'right':0.25, 'up':0, 'left':-0.25, 'down':0}
89         self.yoff = {'right':0, 'up':0.25, 'left':0, 'down':-0.25}
90         GridMDP.__init__(self, states, actions, discount)
91
92     def intended_next(self,s,a):
93         """returns the next state in the direction a.
94         This is where the agent will end up if to goes in its
               intended_direction
95              (which it does with probability 0.7).
96         """
97         (x,y) = s
98         if a=='up':
99             return (x, y+1 if y+1 < self.y_dim else y)
100        if a=='down':
101            return (x, y-1 if y > 0 else y)
102        if a=='right':
103            return (x+1 if x+1 < self.x_dim else x,y)
104        if a=='left':
```

```
105             return (x-1 if x > 0 else x,y)
106
107     def P(self,s,a):
108         """return a dictionary of {s1:p1} if P(s1 | s,a)=p1. Other
                probabilities are zero.
109         Corners are tricky because different actions result in same state.
110         """
111         if s in self.fling_states:
112             return {(0,0): 0.25, (self.x_dim-1,0):0.25,
                    (0,self.y_dim-1):0.25, (self.x_dim-1,self.y_dim-1):0.25}
113         res = dict()
114         for ai in self.actions:
115             s1 = self.intended_next(s,ai)
116             ps1 = 0.7 if ai==a else 0.1
117             if s1 in res: # occurs in corners
118                 res[s1] += ps1
119             else:
120                 res[s1] = ps1
121         return res
122
123     def R(self,s,a):
124          if s in self.rewarding_states:
125              return self.rewarding_states[s]
126          else:
127              (x,y) = s
128              rew = 0
129              # rewards from crashing:
130              if y==0: ## on bottom.
131                  rew += -0.7 if a == 'down' else -0.1
132              if y==self.y_dim-1: ## on top.
133                  rew += -0.7 if a == 'up' else -0.1
134              if x==0: ## on left
135                  rew += -0.7 if a == 'left' else -0.1
136              if x==self.x_dim-1: ## on right.
137                  rew += -0.7 if a == 'right' else -0.1
138              return rew
```

9.2.1 Value Iteration

This implements value iteration.

This uses indexes of the states and actions (not the names). The value function is represented so $v[s]$ is the value of state with index s. A Q function is represented so $q[s][a]$ is the value for doing action with index a state with index s. Similarly a policy π is represented as a list where $pi[s]$, where s is the index of a state, returns the index of the action.

mdpProblem.py — (continued)

```
42     def vi(self, n):
```

```
43          """carries out n iterations of value iteration, updating value
                function self.v
44          Returns a Q-function, value function, policy
45          """
46          print("calling vi")
47          assert n>0,"You must carry out at least one iteration of vi.
                n="+str(n)
48          #v = v0 if v0 is not None else {s:0 for s in self.states}
49          for i in range(n):
50              self.q = {s: {a: self.R(s,a)+self.discount*sum(p1*self.v[s1]
51                                                             for (s1,p1) in
                                                                 self.P(s,a).items())
52                            for a in self.actions}
53                    for s in self.states}
54              self.v = {s: max(self.q[s][a] for a in self.actions)
55                        for s in self.states}
56          self.pi = {s: argmaxd(self.q[s])
57                     for s in self.states}
58          return self.q, self.v, self.pi
```

The following shows how this can be used.

mdpExamples.py — (continued)

```
140 ## Testing value iteration
141 # Try the following:
142 # pt = party(discount=0.9)
143 # pt.vi(1)
144 # pt.vi(100)
145 # party(discount=0.99).vi(100)
146 # party(discount=0.4).vi(100)
147
148 # gr = grid()
149 # gr.show()
150 # q,v,pi = gr.vi(100)
151 # q[(7,2)]
```

9.2.2 Showing Grid MDPs

A `GridMDP` is a type of MDP where we the states are (x,y) positions. It is a special sort of MDP only because we have methods to show it.

mdpProblem.py — (continued)

```
60  class GridMDP(MDP):
61      def __init__(self, states, actions, discount):
62          MDP.__init__(self, states, actions, discount)
63
64      def show(self):
65          #plt.ion()  # interactive
66          fig,(self.ax) = plt.subplots()
67          plt.subplots_adjust(bottom=0.2)
```

```
68          stepB = Button(plt.axes([0.8,0.05,0.1,0.075]), "step")
69          stepB.on_clicked(self.on_step)
70          resetB = Button(plt.axes([0.6,0.05,0.1,0.075]), "reset")
71          resetB.on_clicked(self.on_reset)
72          self.qcheck = CheckButtons(plt.axes([0.2,0.05,0.35,0.075]),
73                                     ["show q-values","show policy"])
74          self.qcheck.on_clicked(self.show_vals)
75          self.show_vals(None)
76          plt.show()
77
78      def show_vals(self,event):
79          self.ax.cla()
80          array = [[self.v[(x,y)] for x in range(self.x_dim)]
81                                              for y in range(self.y_dim)]
82          self.ax.pcolormesh([x-0.5 for x in range(self.x_dim+1)],
83                             [x-0.5 for x in range(self.y_dim+1)],
84                             array, edgecolors='black',cmap='summer')
85              # for cmap see
                    https://matplotlib.org/stable/tutorials/colors/colormaps.html
86          if self.qcheck.get_status()[1]: # "show policy"
87                 for (x,y) in self.q:
88                    maxv = max(self.q[(x,y)][a] for a in self.actions)
89                    for a in self.actions:
90                        if self.q[(x,y)][a] == maxv:
91                           # draw arrow in appropriate direction
92                           self.ax.arrow(x,y,self.xoff[a]*2,self.yoff[a]*2,
93                                    color='red',width=0.05, head_width=0.2,
                                         length_includes_head=True)
94          if self.qcheck.get_status()[0]: # "show q-values"
95             self.show_q(event)
96          else:
97             self.show_v(event)
98          self.ax.set_xticks(range(self.x_dim))
99          self.ax.set_xticklabels(range(self.x_dim))
100         self.ax.set_yticks(range(self.y_dim))
101         self.ax.set_yticklabels(range(self.y_dim))
102         plt.draw()
103
104     def on_step(self,event):
105         self.vi(1)
106         self.show_vals(event)
107
108     def show_v(self,event):
109         """show values"""
110         for (x,y) in self.v:
111             self.ax.text(x,y,"{val:.2f}".format(val=self.v[(x,y)]),ha='center')
112
113     def show_q(self,event):
114         """show q-values"""
115         for (x,y) in self.q:
```

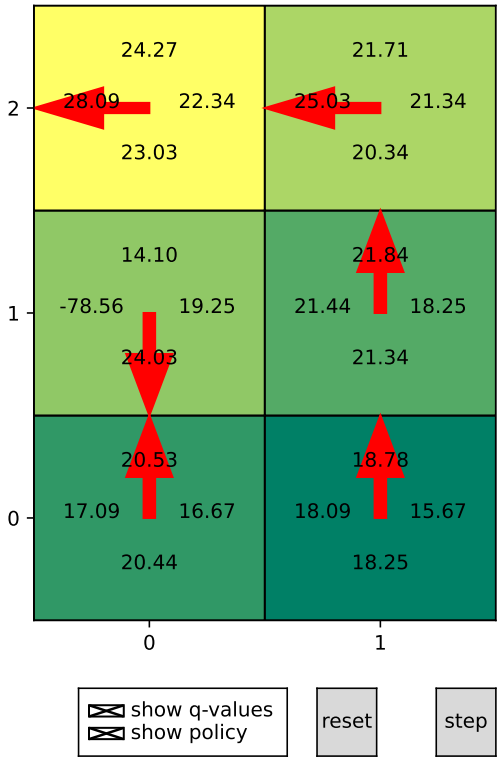


Figure 9.5: Interface for tiny example, after a number of steps. Each rectangle represents a state. In each rectangle are the 4 Q-values for the state. The leftmost number is the for the left action; the rightmost number is for the right action; the upper most is for the *upR* (up-risky) action and the lowest number is for the *upC* action. The arrow points to the action(s) with the maximum Q-value.

```
116             for a in self.actions:
117                 self.ax.text(x+self.xoff[a],y+self.yoff[a],
118                              "{val:.2f}".format(val=self.q[(x,y)][a]),ha='center')
119
120     def on_reset(self,event):
121         self.v = self.initv
122         self.q = self.initq
123         self.show_vals(event)
```

Figure 9.6 shows the user interface, which can be obtained using `tiny().show()`, resizing it, checking "show q-values" and "show policy", and clicking "step" a few times.

Figure **??** shows the user interface, which can be obtained using `grid().show()`,

resizing it, checking "show q-values" and "show policy", and clicking "step" a few times.

Exercise 9.1 Computing q before v may seem like a waste of space because we don't need to store q in order to compute value function or the policy. Change the algorithm so that it loops through the states and actions once per iteration, and only stores the value function and the policy. Note that to get the same results as before, you would need to make sure that you use the previous value of v in the computation not the current value of v. Does using the current value of v hurt the algorithm or make it better (in approaching the actual value function)?

9.2.3 Asynchronous Value Iteration

This implements asynchronous value iteration, storing Q.

A Q function is represented so $q[s][a]$ is the value for doing action with index a state with index s.

mdpProblem.py — (continued)

```
125     def avi(self,n):
126         states = list(self.states)
127         actions = list(self.actions)
128         for i in range(n):
129             s = random.choice(states)
130             a = random.choice(actions)
131             self.q[s][a] = (self.R(s,a) + self.discount *
132                             sum(p1 * max(self.q[s1][a1]
133                                          for a1 in self.actions)
134                                 for (s1,p1) in self.P(s,a).items()))
135         return Q
```

The following shows how `avi` can be used.

mdpExamples.py — (continued)

```
154 ## Testing asynchronous value iteration
155 # Try the following:
156 # pt = party(discount=0.9)
157 # pt.avi(10)
158 # pt.vi(1000)
159
160 # gr = grid()
161 # q = gr.avi(100000)
162 # q[(7,2)]
```

Exercise 9.2 Implement value iteration that stores the V-values rather than the Q-values. Does it work better than storing Q? (What might better mean?)

Exercise 9.3 In asynchronous value iteration, try a number of different ways to choose the states and actions to update (e.g., sweeping through the state-action pairs, choosing them at random). Note that the best way may be to determine

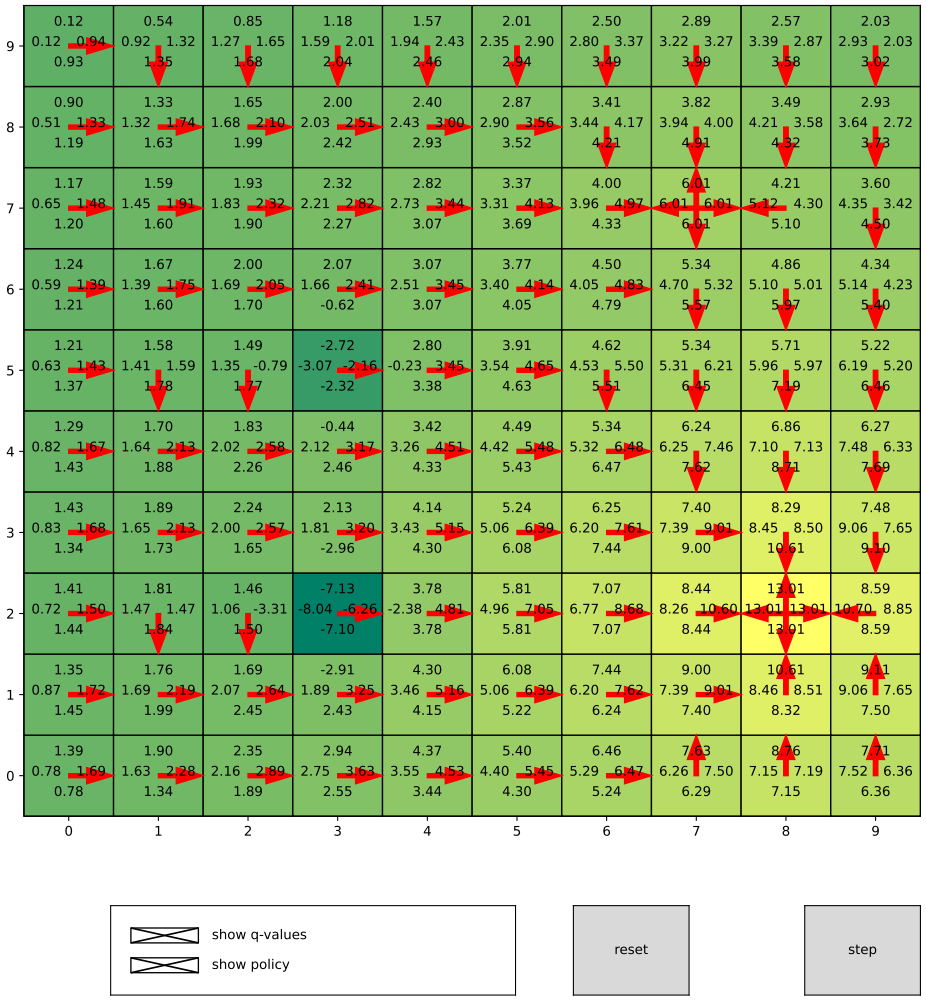


Figure 9.6: Interface for grid example, after a number of steps. Each rectangle represents a state. In each rectangle are the 4 Q-values for the state. The leftmost number is the for the left action; the rightmost number is for the right action; the upper most is for the up action and the lowest number is for the down action. The arrow points to the action(s) with the maximum Q-value.

which states have had their Q-values change the most, and then update the previous ones, but that is not so straightforward to implement, because you need to find those previous states.

Chapter 10

Learning with Uncertainty

10.1 K-means

The k-means learner maintains two lists that suffice as sufficient statistics to classify examples, and to learn the classification:

- *class_counts* is a list such that $class\_counts[c]$ is the number of examples in the training set with $class = c$.
- *feature_sum* is a list such that $feature\_sum[i][c]$ is sum of the values for the i'th feature i for members of class c. The average value of the ith feature in class i is

$$\frac{feature\_sum[i][c]}{class\_counts[c]}$$

The class is initialized by randomly assigning examples to classes, and updating the statistics for *class_counts* and *feature_sum*.

learnKMeans.py — k-means learning

```
11 from learnProblem import Data_set, Learner, Data_from_file
12 import random
13 import matplotlib.pyplot as plt
14
15 class K_means_learner(Learner):
16     def __init__(self,dataset, num_classes):
17         self.dataset = dataset
18         self.num_classes = num_classes
19         self.random_initialize()
20
21     def random_initialize(self):
```

```
22        # class_counts[c] is the number of examples with class=c
23        self.class_counts = [0]*self.num_classes
24        # feature_sum[i][c] is the sum of the values of feature i for class
              c
25        self.feature_sum = [[0]*self.num_classes
26                            for feat in self.dataset.input_features]
27        for eg in self.dataset.train:
28            cl = random.randrange(self.num_classes) # assign eg to random
                  class
29            self.class_counts[cl] += 1
30            for (ind,feat) in enumerate(self.dataset.input_features):
31                self.feature_sum[ind][cl] += feat(eg)
32        self.num_iterations = 0
33        self.display(1,"Initial class counts: ",self.class_counts)
```

The distance from (the mean of) a class to an example is the sum, over all fratures, of the sum-of-squares differences of the class mean and the example value.

learnKMeans.py — (continued)

```
35    def distance(self,cl,eg):
36        """distance of the eg from the mean of the class"""
37        return sum( (self.class_prediction(ind,cl)-feat(eg))**2
38                        for (ind,feat) in
39                            enumerate(self.dataset.input_features))
40
41    def class_prediction(self,feat_ind,cl):
42        """prediction of the class cl on the feature with index feat_ind"""
43        if self.class_counts[cl] == 0:
44            return 0 # there are no examples so we can choose any value
45        else:
46            return self.feature_sum[feat_ind][cl]/self.class_counts[cl]
47
48    def class_of_eg(self,eg):
49        """class to which eg is assigned"""
50        return (min((self.distance(cl,eg),cl)
51                        for cl in range(self.num_classes)))[1]
52              # second element of tuple, which is a class with minimum
                    distance
```

One step of k-means updates the *class_counts* and *feature_sum*. It uses the old values to determine the classes, and so the new values for *class_counts* and *feature_sum*. At the end it determines whether the values of these have changes, and then replaces the old ones with the new ones. It returns an indicator of whether the values are stable (have not changed).

learnKMeans.py — (continued)

```
53    def k_means_step(self):
54        """Updates the model with one step of k-means.
55        Returns whether the assignment is stable.
56        """
```

```
57          new_class_counts = [0]*self.num_classes
58          # feature_sum[i][c] is the sum of the values of feature i for class
                c
59          new_feature_sum = [[0]*self.num_classes
60                             for feat in self.dataset.input_features]
61          for eg in self.dataset.train:
62              cl = self.class_of_eg(eg)
63              new_class_counts[cl] += 1
64              for (ind,feat) in enumerate(self.dataset.input_features):
65                  new_feature_sum[ind][cl] += feat(eg)
66          stable = (new_class_counts == self.class_counts) and
                (self.feature_sum == new_feature_sum)
67          self.class_counts = new_class_counts
68          self.feature_sum = new_feature_sum
69          self.num_iterations += 1
70          return stable
71
72
73      def learn(self,n=100):
74          """do n steps of k-means, or until convergence"""
75          i=0
76          stable = False
77          while i<n and not stable:
78              stable = self.k_means_step()
79              i += 1
80              self.display(1,"Iteration",self.num_iterations,
81                           "class counts: ",self.class_counts,"
                                Stable=",stable)
82          return stable
83
84      def show_classes(self):
85          """sorts the data by the class and prints in order.
86          For visualizing small data sets
87          """
88          class_examples = [[] for i in range(self.num_classes)]
89          for eg in self.dataset.train:
90              class_examples[self.class_of_eg(eg)].append(eg)
91          print("Class","Example",sep='\t')
92          for cl in range(self.num_classes):
93              for eg in class_examples[cl]:
94                  print(cl,*eg,sep='\t')
95
96      def plot_error(self, maxstep=20):
97          """Plots the sum-of-suares error as a function of the number of
                steps"""
98          plt.ion()
99          plt.xlabel("step")
100         plt.ylabel("Ave sum-of-squares error")
101         train_errors = []
102         if self.dataset.test:
```

```
103             test_errors = []
104         for i in range(maxstep):
105             self.learn(1)
106             train_errors.append( sum(self.distance(self.class_of_eg(eg),eg)
107                                      for eg in self.dataset.train)
108                                  /len(self.dataset.train))
109             if self.dataset.test:
110                 test_errors.append(
                        sum(self.distance(self.class_of_eg(eg),eg)
111                                       for eg in self.dataset.test)
112                                   /len(self.dataset.test))
113         plt.plot(range(1,maxstep+1),train_errors,
114                  label=str(self.num_classes)+" classes. Training set")
115         if self.dataset.test:
116             plt.plot(range(1,maxstep+1),test_errors,
117                      label=str(self.num_classes)+" classes. Test set")
118         plt.legend()
119         plt.draw()
120
121 %data = Data_from_file('data/emdata1.csv', num_train=10,
        target_index=2000) % trivial example
122 data = Data_from_file('data/emdata2.csv', num_train=10, target_index=2000)
123 %data = Data_from_file('data/emdata0.csv', num_train=14,
        target_index=2000) % example from textbook
124 kml = K_means_learner(data,2)
125 num_iter=4
126 print("Class assignment after",num_iter,"iterations:")
127 kml.learn(num_iter); kml.show_classes()
128
129 # Plot the error
130 # km2=K_means_learner(data,2); km2.plot_error(20) # 2 classes
131 # km3=K_means_learner(data,3); km3.plot_error(20) # 3 classes
132 # km13=K_means_learner(data,13); km13.plot_error(20) # 13 classes
133
134 # data = Data_from_file('data/carbool.csv',
        target_index=2000,boolean_features=True)
135 # kml = K_means_learner(data,3)
136 # kml.learn(20); kml.show_classes()
137 # km3=K_means_learner(data,3); km3.plot_error(20) # 3 classes
138 # km3=K_means_learner(data,30); km3.plot_error(20) # 30 classes
```

Exercise 10.1 Change *boolean_features = True* flag to allow for numerical features. K-means assumes the features are numerical, so we want to make non-numerical features into numerical features (using characteristic functions) but we probably don't want to change numerical features into Boolean.

Exercise 10.2 If there are many classes, some of the classes can become empty (e.g., try 100 classes with carbool.csv). Implement a way to put some examples into a class, if possible. Two ideas are:

(a) Initialize the classes with actual examples, so that the classes will not start empty. (Do the classes become empty?)

(b) In *class_prediction*, we test whether the code is empty, and make a prediction of 0 for an empty class. It is possible to make a different prediction to "steal" an example (but you should make sure that a class has a consistent value for each feature in a loop).

Make your own suggestions, and compare it with the original, and whichever of these you think may work better.

10.2 EM

In the following definition, a class, c, is a integer in range $[0, num\_classes)$. i is an index of a feature, so $feat[i]$ is the ith feature, and a feature is a function from tuples to values. val is a value of a feature.

A model consists of 2 lists, which form the sufficient statistics:

- $class\_counts$ is a list such that $class\_counts[c]$ is the number of tuples with $class = c$, where each tuple is weighted by its probability, i.e.,

$$class\_counts[c] = \sum_{t:class(t)=c} P(t)$$

- $feature\_counts$ is a list such that $feature\_counts[i][val][c]$ is the weighted count of the number of tuples t with $feat[i](t) = val$ and $class(t) = c$, each tuple is weighted by its probability, i.e.,

$$feature\_counts[i][val][c] = \sum_{t:feat[i](t)=val \text{ and } class(t)=c} P(t)$$

learnEM.py — EM Learning

```
from learnProblem import Data_set, Learner, Data_from_file
import random
import math
import matplotlib.pyplot as plt

class EM_learner(Learner):
    def __init__(self,dataset, num_classes):
        self.dataset = dataset
        self.num_classes = num_classes
        self.class_counts = None
        self.feature_counts = None
```

The function *em_step* goes though the training examples, and updates these counts. The first time it is run, when there is no model, it uses random distributions.

learnEM.py — (continued)

```
    def em_step(self, orig_class_counts, orig_feature_counts):
        """updates the model."""
        class_counts = [0]*self.num_classes
        feature_counts = [{val:[0]*self.num_classes
                           for val in feat.frange}
                           for feat in self.dataset.input_features]
        for tple in self.dataset.train:
            if orig_class_counts: # a model exists
                tpl_class_dist = self.prob(tple, orig_class_counts,
                    orig_feature_counts)
            else:               # initially, with no model, return a random
                distribution
                tpl_class_dist = random_dist(self.num_classes)
            for cl in range(self.num_classes):
                class_counts[cl] += tpl_class_dist[cl]
                for (ind,feat) in enumerate(self.dataset.input_features):
                    feature_counts[ind][feat(tple)][cl] += tpl_class_dist[cl]
        return class_counts, feature_counts
```

prob computes the probability of a class c for a tuple *tpl*, given the current statistics.

$$\begin{aligned} P(c \mid tple) &\propto P(c) * \prod_i P(X_i{=}tple(i) \mid c) \\ &= \frac{class\_counts[c]}{len(self.dataset)} * \prod_i \frac{feature\_counts[i][feat_i(tple)][c]}{class\_counts[c]} \\ &\propto \frac{\prod_i feature\_counts[i][feat_i(tple)][c]}{class\_counts[c]^{|feats|-1}} \end{aligned}$$

The last step is because $len(self.dataset)$ is a constant (independent of c). $class\_counts[c]$ can be taken out of the product, but needs to be raised to the power of the number of features, and one of them cancels.

learnEM.py — (continued)

```
    def prob(self, tple, class_counts, feature_counts):
        """returns a distribution over the classes for tuple tple in the
            model defined by the counts
        """
        feats = self.dataset.input_features
        unnorm = [prod(feature_counts[i][feat(tple)][c]
                       for (i,feat) in enumerate(feats))
                      /(class_counts[c]**(len(feats)-1))
                  for c in range(self.num_classes)]
        thesum = sum(unnorm)
        return [un/thesum for un in unnorm]
```

learn does n steps of EM:

learnEM.py — (continued)

```
51      def learn(self,n):
52          """do n steps of em"""
53          for i in range(n):
54              self.class_counts,self.feature_counts =
                    self.em_step(self.class_counts,
55                                                          self.feature_counts)
```

The following is for visualizing the classes. It prints the dataset ordered by the probability of class c.

learnEM.py — (continued)

```
57      def show_class(self,c):
58          """sorts the data by the class and prints in order.
59          For visualizing small data sets
60          """
61          sorted_data =
                sorted((self.prob(tpl,self.class_counts,self.feature_counts)[c],
62                                ind,  # preserve ordering for equal
                                      probabilities
63                                tpl)
64                               for (ind,tpl) in enumerate(self.dataset.train))
65          for cc,r,tpl in sorted_data:
66              print(cc,*tpl,sep='\t')
```

The following are for evaluating the classes.

The probability of a tuple can be evaluated by marginalizing over the classes:

$$\begin{aligned} P(tple) &= \sum_c P(c) * \prod_i P(X_i{=}tple(i) \mid c) \\ &= \sum_c \frac{cc[c]}{len(self.dataset)} * \prod_i \frac{fc[i][feat_i(tple)][c]}{cc[c]} \end{aligned}$$

where cc is the class count and fc is feature count. $len(self.dataset)$ can be distributed out of the sum, and $cc[c]$ can be taken out of the product:

$$= \frac{1}{len(self.dataset)} \sum_c \frac{1}{cc[c]^{\#feats-1}} * \prod_i fc[i][feat_i(tple)][c]$$

Given the probability of each tuple, we can evaluate the logloss, as the negative of the log probability:

learnEM.py — (continued)

```
68      def logloss(self,tple):
69          """returns the logloss of the prediction on tple, which is
                -log(P(tple))
70          based on the current class counts and feature counts
71          """
72          feats = self.dataset.input_features
73          res = 0
74          cc = self.class_counts
75          fc = self.feature_counts
```

```
        for c in range(self.num_classes):
            res += prod(fc[i][feat(tple)][c]
                        for (i,feat) in
                            enumerate(feats))/(cc[c]**(len(feats)-1))
        if res>0:
            return -math.log2(res/len(self.dataset.train))
        else:
            return float("inf") #infinity

    def plot_error(self, maxstep=20):
        """Plots the logloss error as a function of the number of steps"""
        plt.ion()
        plt.xlabel("step")
        plt.ylabel("Ave Logloss (bits)")
        train_errors = []
        if self.dataset.test:
            test_errors = []
        for i in range(maxstep):
            self.learn(1)
            train_errors.append( sum(self.logloss(tple) for tple in
                self.dataset.train)
                                 /len(self.dataset.train))
            if self.dataset.test:
                test_errors.append( sum(self.logloss(tple) for tple in
                    self.dataset.test)
                                    /len(self.dataset.test))
        plt.plot(range(1,maxstep+1),train_errors,
                 label=str(self.num_classes)+" classes. Training set")
        if self.dataset.test:
            plt.plot(range(1,maxstep+1),test_errors,
                     label=str(self.num_classes)+" classes. Test set")
        plt.legend()
        plt.draw()

def prod(L):
    """returns the product of the elements of L"""
    res = 1
    for e in L:
        res *= e
    return res

def random_dist(k):
    """generate k random numbers that sum to 1"""
    res = [random.random() for i in range(k)]
    s = sum(res)
    return [v/s for v in res]

data = Data_from_file('data/emdata2.csv', num_train=10, target_index=2000)
eml = EM_learner(data,2)
num_iter=2
```

```
print("Class assignment after",num_iter,"iterations:")
eml.learn(num_iter); eml.show_class(0)

# Plot the error
# em2=EM_learner(data,2); em2.plot_error(40) # 2 classes
# em3=EM_learner(data,3); em3.plot_error(40) # 3 classes
# em13=EM_learner(data,13); em13.plot_error(40) # 13 classes

# data = Data_from_file('data/carbool.csv',
    target_index=2000,boolean_features=False)
# [f.frange for f in data.input_features]
# eml = EM_learner(data,3)
# eml.learn(20); eml.show_class(0)
# em3=EM_learner(data,3); em3.plot_error(60) # 3 classes
# em3=EM_learner(data,30); em3.plot_error(60) # 30 classes
```

Exercise 10.3 For the EM data, where there are naturally 2 classes, 3 classes does better on the training set after a while than 2 classes, but worse on the test set. Explain why. Hint: look what the 3 classes are. Use "em3.show_class(i)" for each of the classes $i \in [0, 3)$.

Exercise 10.4 Write code to plot the logloss as a function of the number of classes (from 1 to say 15) for a fixed number of iterations. (From the experience with the existing code, think about how many iterations is appropriate.)

Chapter 11

Multiagent Systems

11.1 Minimax

Here we consider two-player zero-sum games. Here a player only wins when another player loses. This can be modeled as where there is a single utility which one agent (the maximizing agent) is trying minimize and the other agent (the minimizing agent) is trying to minimize.

11.1.1 Creating a two-player game

masProblem.py — A Multiagent Problem

```
11 from display import Displayable
12
13 class Node(Displayable):
14     """A node in a search tree. It has a
15     name a string
16     isMax is True if it is a maximizing node, otherwise it is minimizing
           node
17     children is the list of children
18     value is what it evaluates to if it is a leaf.
19     """
20     def __init__(self, name, isMax, value, children):
21         self.name = name
22         self.isMax = isMax
23         self.value = value
24         self.allchildren = children
25
26     def isLeaf(self):
27         """returns true of this is a leaf node"""
28         return self.allchildren is None
```

```
29
30      def children(self):
31          """returns the list of all children."""
32          return self.allchildren
33
34      def evaluate(self):
35          """returns the evaluation for this node if it is a leaf"""
36          return self.value
```

The following gives the tree from Figure 11.5 of the book. Note how 888 is used as a value here, but never appears in the trace.

masProblem.py — (continued)

```
38  fig10_5 = Node("a",True,None, [
39              Node("b",False,None, [
40                  Node("d",True,None, [
41                      Node("h",False,None, [
42                          Node("h1",True,7,None),
43                          Node("h2",True,9,None)]),
44                      Node("i",False,None, [
45                          Node("i1",True,6,None),
46                          Node("i2",True,888,None)])]),
47                  Node("e",True,None, [
48                      Node("j",False,None, [
49                          Node("j1",True,11,None),
50                          Node("j2",True,12,None)]),
51                      Node("k",False,None, [
52                          Node("k1",True,888,None),
53                          Node("k2",True,888,None)])])]),
54              Node("c",False,None, [
55                  Node("f",True,None, [
56                      Node("l",False,None, [
57                          Node("l1",True,5,None),
58                          Node("l2",True,888,None)]),
59                      Node("m",False,None, [
60                          Node("m1",True,4,None),
61                          Node("m2",True,888,None)])]),
62                  Node("g",True,None, [
63                      Node("n",False,None, [
64                          Node("n1",True,888,None),
65                          Node("n2",True,888,None)]),
66                      Node("o",False,None, [
67                          Node("o1",True,888,None),
68                          Node("o2",True,888,None)])])])])
```

The following is a representation of a **magic-sum game**, where players take turns picking a number in the range $[1, 9]$, and the first player to have 3 numbers that sum to 15 wins. Note that this is a syntactic variant of **tic-tac-toe** or **naughts and crosses**. To see this, consider the numbers on a **magic square** (Figure 11.1); 3 numbers that add to 15 correspond exactly to the winning positions

6	1	8
7	5	3
2	9	4

Figure 11.1: Magic Square

of tic-tac-toe played on the magic square.

Note that we do not remove symmetries. (What are the symmetries? How do the symmetries of tic-tac-toe translate here?)

masProblem.py — (continued)

```
70
71  class Magic_sum(Node):
72      def __init__(self, xmove=True, last_move=None,
73                   available=[1,2,3,4,5,6,7,8,9], x=[], o=[]):
74          """This is a node in the search for the magic-sum game.
75          xmove is True if the next move belongs to X.
76          last_move is the number selected in the last move
77          available is the list of numbers that are available to be chosen
78          x is the list of numbers already chosen by x
79          o is the list of numbers already chosen by o
80          """
81          self.isMax = self.xmove = xmove
82          self.last_move = last_move
83          self.available = available
84          self.x = x
85          self.o = o
86          self.allchildren = None #computed on demand
87          lm = str(last_move)
88          self.name = "start" if not last_move else "o="+lm if xmove else
                "x="+lm
89
90      def children(self):
91          if self.allchildren is None:
92              if self.xmove:
93                  self.allchildren = [
94                      Magic_sum(xmove = not self.xmove,
95                                last_move = sel,
96                                available = [e for e in self.available if e is
                                    not sel],
97                                x = self.x+[sel],
98                                o = self.o)
99                               for sel in self.available]
100             else:
101                 self.allchildren = [
102                     Magic_sum(xmove = not self.xmove,
103                               last_move = sel,
104                               available = [e for e in self.available if e is
                                    not sel],
```

```
                            x = self.x,
                            o = self.o+[sel])
                           for sel in self.available]
        return self.allchildren

    def isLeaf(self):
        """A leaf has no numbers available or is a win for one of the
            players.
        We only need to check for a win for o if it is currently x's turn,
        and only check for a win for x if it is o's turn (otherwise it would
        have been a win earlier).
        """
        return (self.available == [] or
                (sum_to_15(self.last_move,self.o)
                 if self.xmove
                 else sum_to_15(self.last_move,self.x)))

    def evaluate(self):
        if self.xmove and sum_to_15(self.last_move,self.o):
            return -1
        elif not self.xmove and sum_to_15(self.last_move,self.x):
            return 1
        else:
            return 0

def sum_to_15(last,selected):
    """is true if last, toegether with two other elements of selected sum
        to 15.
    """
    return any(last+a+b == 15
               for a in selected if a != last
               for b in selected if b != last and b != a)
```

11.1.2 Minimax and α-β Pruning

This is a naive depth-first **minimax algorithm**:

masMiniMax.py — Minimax search with alpha-beta pruning

```
11 def minimax(node,depth):
12     """returns the value of node, and a best path for the agents
13     """
14     if node.isLeaf():
15         return node.evaluate(),None
16     elif node.isMax:
17         max_score = float("-inf")
18         max_path = None
19         for C in node.children():
20             score,path = minimax(C,depth+1)
21             if score > max_score:
22                 max_score = score
23                 max_path = C.name,path
24         return max_score,max_path
25     else:
26         min_score = float("inf")
27         min_path = None
28         for C in node.children():
29             score,path = minimax(C,depth+1)
30             if score < min_score:
31                 min_score = score
32                 min_path = C.name,path
33         return min_score,min_path
```

The following is a depth-first minimax with α-β **pruning**. It returns the value for a node as well as a best path for the agents.

masMiniMax.py — (continued)

```
35 def minimax_alpha_beta(node,alpha,beta,depth=0):
36     """node is a Node, alpha and beta are cutoffs, depth is the depth
37     returns value, path
38     where path is a sequence of nodes that results in the value
39     """
40     node.display(2," "*depth,"minimax_alpha_beta(",node.name,", ",alpha, ",
           ", beta,")")
41     best=None    # only used if it will be pruned
42     if node.isLeaf():
43         node.display(2," "*depth,"returning leaf value",node.evaluate())
44         return node.evaluate(),None
45     elif node.isMax:
46         for C in node.children():
47             score,path = minimax_alpha_beta(C,alpha,beta,depth+1)
48             if score >= beta:  # beta pruning
49                 node.display(2," "*depth,"pruned due to
                       beta=",beta,"C=",C.name)
50                 return score, None
51             if score > alpha:
52                 alpha = score
53                 best = C.name, path
54         node.display(2," "*depth,"returning max alpha",alpha,"best",best)
55         return alpha,best
56     else:
57         for C in node.children():
58             score,path = minimax_alpha_beta(C,alpha,beta,depth+1)
59             if score <= alpha:   # alpha pruning
60                 node.display(2," "*depth,"pruned due to
                       alpha=",alpha,"C=",C.name)
61                 return score, None
62             if score < beta:
63                 beta=score
64                 best = C.name,path
65         node.display(2," "*depth,"returning min beta",beta,"best=",best)
66         return beta,best
```

Testing:

masMiniMax.py — (continued)

```
68 from masProblem import fig10_5, Magic_sum, Node
69
70 # Node.max_display_level=2 # print detailed trace
71 # minimax_alpha_beta(fig10_5, -9999, 9999,0)
72 # minimax_alpha_beta(Magic_sum(), -9999, 9999,0)
73
74 #To see how much time alpha-beta pruning can save over minimax, uncomment
       the following:
```

```
75 ## import timeit
76 ## timeit.Timer("minimax(Magic_sum(),0)",setup="from __main__ import
       minimax, Magic_sum"
77 ##             ).timeit(number=1)
78 ## trace=False
79 ## timeit.Timer("minimax_alpha_beta(Magic_sum(), -9999, 9999,0)",
80 ##             setup="from __main__ import minimax_alpha_beta, Magic_sum"
81 ##             ).timeit(number=1)
```

Chapter 12

Reinforcement Learning

12.1 Representing Agents and Environments

When the learning agent does an action in the environment, it observes a $(state, reward)$ pair from the environment. The *state* is the world state; this is the fully observable assumption.

An RL environment implements a $do(action)$ method that returns a $(state, reward)$ pair.

rlProblem.py — Representations for Reinforcement Learning

```
11 import random
12 from display import Displayable
13 from utilities import flip
14
15 class RL_env(Displayable):
16     def __init__(self,actions,state):
17         self.actions = actions # set of actions
18         self.state = state     # initial state
19
20     def do(self, action):
21         """do action
22         returns state,reward
23         """
24         raise NotImplementedError("RL_env.do") # abstract method
```

Here is the definition of the simple 2-state, 2-action party/relax decision.

rlProblem.py — (continued)

```
26 class Healthy_env(RL_env):
27     def __init__(self):
28         RL_env.__init__(self,["party","relax"], "healthy")
29
```

```
30      def do(self, action):
31          """updates the state based on the agent doing action.
32          returns state,reward
33          """
34          if self.state=="healthy":
35              if action=="party":
36                  self.state = "healthy" if flip(0.7) else "sick"
37                  reward = 10
38              else: # action=="relax"
39                  self.state = "healthy" if flip(0.95) else "sick"
40                  reward = 7
41          else: # self.state=="sick"
42              if action=="party":
43                  self.state = "healthy" if flip(0.1) else "sick"
44                  reward = 2
45              else:
46                  self.state = "healthy" if flip(0.5) else "sick"
47                  reward = 0
48          return self.state,reward
```

12.1.1 Simulating an environment from an MDP

Given the definition for an MDP (page 224), *Env_from_MDP* takes in an MDP and simulates the environment with those dynamics.

Note that the MDP does not contain enough information to simulate a system, because it loses any dependency between the rewards and the resulting state; here we assume the agent always received the average reward for the state and action.

rlProblem.py — (continued)

```
50  class Env_from_MDP(RL_env):
51      def __init__(self, mdp):
52          initial_state = mdp.states[0]
53          RL_env.__init__(self,mdp.actions, initial_state)
54          self.mdp = mdp
55          self.action_index = {action:index for (index,action) in
                enumerate(mdp.actions)}
56          self.state_index = {state:index for (index,state) in
                enumerate(mdp.states)}
57
58      def do(self, action):
59          """updates the state based on the agent doing action.
60          returns state,reward
61          """
62          action_ind = self.action_index[action]
63          state_ind = self.state_index[self.state]
64          self.state = pick_from_dist(self.mdp.trans[state_ind][action_ind],
                self.mdp.states)
65          reward = self.mdp.reward[state_ind][action_ind]
```

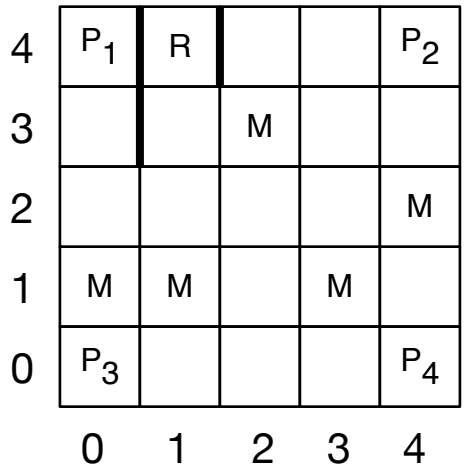


Figure 12.1: Monster game

```
66          return self.state, reward
67
68  def pick_from_dist(dist,values):
69      """
70      e.g. pick_from_dist([0.3,0.5,0.2],['a','b','c']) should pick 'a' with
            probability 0.3, etc.
71      """
72      ran = random.random()
73      i=0
74      while ran>dist[i]:
75          ran -= dist[i]
76          i += 1
77      return values[i]
```

12.1.2 Simple Game

This is for the game depicted in Figure 12.1.

rlSimpleEnv.py — Simple game

```
11  import random
12  from utilities import flip
13  from rlProblem import RL_env
14
15  class Simple_game_env(RL_env):
16      xdim = 5
17      ydim = 5
18
19      vwalls = [(0,3), (0,4), (1,4)] # vertical walls right of these locations
20      hwalls = [] # not implemented
21      crashed_reward = -1
```

```
22
23      prize_locs = [(0,0), (0,4), (4,0), (4,4)]
24      prize_apears_prob = 0.3
25      prize_reward = 10
26
27      monster_locs = [(0,1), (1,1), (2,3), (3,1), (4,2)]
28      monster_appears_prob = 0.4
29      monster_reward_when_damaged = -10
30      repair_stations = [(1,4)]
31
32      actions = ["up","down","left","right"]
33
34      def __init__(self):
35          # State:
36          self.x = 2
37          self.y = 2
38          self.damaged = False
39          self.prize = None
40          # Statistics
41          self.number_steps = 0
42          self.total_reward = 0
43          self.min_reward = 0
44          self.min_step = 0
45          self.zero_crossing = 0
46          RL_env.__init__(self, Simple_game_env.actions,
47                          (self.x, self.y, self.damaged, self.prize))
48          self.display(2,"","Step","Tot Rew","Ave Rew",sep="\t")
49
50      def do(self,action):
51          """updates the state based on the agent doing action.
52          returns state,reward
53          """
54          reward = 0.0
55          # A prize can appear:
56          if self.prize is None and flip(self.prize_apears_prob):
57                 self.prize = random.choice(self.prize_locs)
58          # Actions can be noisy
59          if flip(0.4):
60              actual_direction = random.choice(self.actions)
61          else:
62              actual_direction = action
63          # Modeling the actions given the actual direction
64          if actual_direction == "right":
65              if self.x==self.xdim-1 or (self.x,self.y) in self.vwalls:
66                  reward += self.crashed_reward
67              else:
68                  self.x += 1
69          elif actual_direction == "left":
70              if self.x==0 or (self.x-1,self.y) in self.vwalls:
71                  reward += self.crashed_reward
```

```
            else:
                self.x += -1
        elif actual_direction == "up":
            if self.y==self.ydim-1:
                reward += self.crashed_reward
            else:
                self.y += 1
        elif actual_direction == "down":
            if self.y==0:
                reward += self.crashed_reward
            else:
                self.y += -1
        else:
            raise RuntimeError("unknown_direction "+str(direction))

        # Monsters
        if (self.x,self.y) in self.monster_locs and
            flip(self.monster_appears_prob):
            if self.damaged:
                reward += self.monster_reward_when_damaged
            else:
                self.damaged = True
        if (self.x,self.y) in self.repair_stations:
            self.damaged = False

        # Prizes
        if (self.x,self.y) == self.prize:
            reward += self.prize_reward
            self.prize = None

        # Statistics
        self.number_steps += 1
        self.total_reward += reward
        if self.total_reward < self.min_reward:
            self.min_reward = self.total_reward
            self.min_step = self.number_steps
        if self.total_reward>0 and reward>self.total_reward:
            self.zero_crossing = self.number_steps
        self.display(2,"",self.number_steps,self.total_reward,
                     self.total_reward/self.number_steps,sep="\t")

        return (self.x, self.y, self.damaged, self.prize), reward
```

12.1.3 Evaluation and Plotting

rlPlot.py — RL Plotter

```
import matplotlib.pyplot as plt

def plot_rl(ag, label=None, yplot='Total', step_size=None,
```

```
        steps_explore=1000, steps_exploit=1000, xscale='linear'):
    """
    plots the agent ag
    label is the label for the plot
    yplot is 'Average' or 'Total'
    step_size is the number of steps between each point plotted
    steps_explore is the number of steps the agent spends exploring
    steps_exploit is the number of steps the agent spends exploiting
    xscale is 'log' or 'linear'

    returns total reward when exploring, total reward when exploiting
    """
    assert yplot in ['Average','Total']
    if step_size is None:
        step_size = max(1,(steps_explore+steps_exploit)//500)
    if label is None:
        label = ag.label
    ag.max_display_level,old_mdl = 1,ag.max_display_level
    plt.ion()
    plt.xscale(xscale)
    plt.xlabel("step")
    plt.ylabel(yplot+" reward")
    steps = []       # steps
    rewards = []     # return
    ag.restart()
    step = 0
    while step < steps_explore:
        ag.do(step_size)
        step += step_size
        steps.append(step)
        if yplot == "Average":
            rewards.append(ag.acc_rewards/step)
        else:
            rewards.append(ag.acc_rewards)
    acc_rewards_exploring = ag.acc_rewards
    ag.explore,explore_save = 0,ag.explore
    while step < steps_explore+steps_exploit:
        ag.do(step_size)
        step += step_size
        steps.append(step)
        if yplot == "Average":
            rewards.append(ag.acc_rewards/step)
        else:
            rewards.append(ag.acc_rewards)
    plt.plot(steps,rewards,label=label)
    plt.legend(loc="upper left")
    plt.draw()
    ag.max_display_level = old_mdl
    ag.explore=explore_save
    return acc_rewards_exploring, ag.acc_rewards-acc_rewards_exploring
```

12.2 Q Learning

To run the Q-learning demo, in folder "aipython", load "rlQTest.py", and copy and paste the example queries at the bottom of that file. This assumes Python 3.

rlQLearner.py — Q Learning

```
import random
from display import Displayable
from utilities import argmaxe, flip

class RL_agent(Displayable):
    """An RL_Agent
    has percepts (s, r) for some state s and real reward r
    """
```

rlQLearner.py — (continued)

```
class Q_learner(RL_agent):
    """A Q-learning agent has
    belief-state consisting of
        state is the previous state
        q is a {(state,action):value} dict
        visits is a {(state,action):n} dict. n is how many times action was
            done in state
        acc_rewards is the accumulated reward

    it observes (s, r) for some world-state s and real reward r
    """
```

rlQLearner.py — (continued)

```
    def __init__(self, env, discount, explore=0.1, fixed_alpha=True,
        alpha=0.2,
                alpha_fun=lambda k:1/k,
                qinit=0, label="Q_learner"):
        """env is the environment to interact with.
        discount is the discount factor
        explore is the proportion of time the agent will explore
        fixed_alpha specifies whether alpha is fixed or varies with the
            number of visits
        alpha is the weight of new experiences compared to old experiences
        alpha_fun is a function that computes alpha from the number of
            visits
        qinit is the initial value of the Q's
        label is the label for plotting
        """
        RL_agent.__init__(self)
        self.env = env
        self.actions = env.actions
```

```
46          self.discount = discount
47          self.explore = explore
48          self.fixed_alpha = fixed_alpha
49          self.alpha = alpha
50          self.alpha_fun = alpha_fun
51          self.qinit = qinit
52          self.label = label
53          self.restart()
```

restart is used to make the learner relearn everything. This is used by the plotter to create new plots.

rlQLearner.py — (continued)

```
55      def restart(self):
56          """make the agent relearn, and reset the accumulated rewards
57          """
58          self.acc_rewards = 0
59          self.state = self.env.state
60          self.q = {}
61          self.visits = {}
```

do takes in the number of steps.

rlQLearner.py — (continued)

```
63      def do(self,num_steps=100):
64          """do num_steps of interaction with the environment"""
65          self.display(2,"s\ta\tr\ts'\tQ")
66          alpha = self.alpha
67          for i in range(num_steps):
68              action = self.select_action(self.state)
69              next_state,reward = self.env.do(action)
70              if not self.fixed_alpha:
71                  k = self.visits[(self.state, action)] =
                        self.visits.get((self.state, action),0)+1
72                  alpha = self.alpha_fun(k)
73              self.q[(self.state, action)] = (
74                  (1-alpha) * self.q.get((self.state, action),self.qinit)
75                  + alpha * (reward + self.discount
76                                    * max(self.q.get((next_state,
                                            next_act),self.qinit)
77                                          for next_act in self.actions)))
78              self.display(2,self.state, action, reward, next_state,
79                           self.q[(self.state, action)], sep='\t')
80              self.state = next_state
81              self.acc_rewards += reward
```

select_action us used to select the next action to perform. This can be reimplemented to give a different exploration strategy.

rlQLearner.py — (continued)

```
83      def select_action(self, state):
84          """returns an action to carry out for the current agent
```

```
85          given the state, and the q-function
86          """
87          if flip(self.explore):
88              return random.choice(self.actions)
89          else:
90              return argmaxe((next_act, self.q.get((state,
                    next_act),self.qinit))
91                             for next_act in self.actions)
```

Exercise 12.1 Implement a soft-max action selection. Choose a temperature that works well for the domain. Explain how you picked this temperature. Compare the epsilon-greedy, soft-max and optimism in the face of uncertainty.

Exercise 12.2 Implement SARSA. Hint: it does not do a *max* in *do*. Instead it needs to choose *next_act* before it does the update.

12.2.1 Testing Q-learning

The first tests are for the 2-action 2-state

rlQTest.py — RL Q Tester

```
11 from rlProblem import Healthy_env
12 from rlQLearner import Q_learner
13 from rlPlot import plot_rl
14
15 env = Healthy_env()
16 ag = Q_learner(env, 0.7)
17 ag_opt = Q_learner(env, 0.7, qinit=100, label="optimistic" ) # optimistic
       agent
18 ag_exp_l = Q_learner(env, 0.7, explore=0.01, label="less explore")
19 ag_exp_m = Q_learner(env, 0.7, explore=0.5, label="more explore")
20 ag_disc = Q_learner(env, 0.9, qinit=100, label="disc 0.9")
21 ag_va = Q_learner(env, 0.7, qinit=100,fixed_alpha=False,alpha_fun=lambda
       k:10/(9+k),label="alpha=10/(9+k)")
22
23 # ag.max_display_level = 2
24 # ag.do(20)
25 # ag.q   # get the learned q-values
26 # ag.max_display_level = 1
27 # ag.do(1000)
28 # ag.q   # get the learned q-values
29 # plot_rl(ag,yplot="Average")
30 # plot_rl(ag_opt,yplot="Average")
31 # plot_rl(ag_exp_l,yplot="Average")
32 # plot_rl(ag_exp_m,yplot="Average")
33 # plot_rl(ag_disc,yplot="Average")
34 # plot_rl(ag_va,yplot="Average")
35
36 from mdpExamples import MDPtiny
37 from rlProblem import Env_from_MDP
38 envt = Env_from_MDP(MDPtiny())
```

```
agt = Q_learner(envt, 0.8)
# agt.do(20)

from rlSimpleEnv import Simple_game_env
senv = Simple_game_env()
sag1 = Q_learner(senv,0.9,explore=0.2,fixed_alpha=True,alpha=0.1)
#
    plot_rl(sag1,steps_explore=100000,steps_exploit=100000,label="alpha="+str(sag1.alpha))
sag2 = Q_learner(senv,0.9,explore=0.2,fixed_alpha=False)
# plot_rl(sag2,steps_explore=100000,steps_exploit=100000,label="alpha=1/k")
sag3 = Q_learner(senv,0.9,explore=0.2,fixed_alpha=False,alpha_fun=lambda
    k:10/(9+k))
#
    plot_rl(sag3,steps_explore=100000,steps_exploit=100000,label="alpha=10/(9+k)")
```

12.3 Q-leaning with Experience Replay

Warning: not properly dubugged

rlQExperienceReplay.py — Linear Reinforcement Learner with Experience Replay

```
from rlQLearner import Q_learner
from utilities import flip
import random

class BoundedBuffer(object):
    def __init__(self, buffer_size=1000):
        self.buffer_size = buffer_size
        self.buffer = [0]*buffer_size
        self.number_added = 0

    def add(self,experience):
        if self.number_added < self.buffer_size:
            self.buffer[self.number_added] = experience
        else:
            if flip(self.buffer_size/self.number_added):
                position = random.randrange(self.buffer_size)
                self.buffer[position] = experience
        self.number_added += 1

    def get(self):
        return self.buffer[random.randrange(min(self.number_added,
            self.buffer_size))]

class Q_AR_learner(Q_learner):
    def __init__(self, env, discount, explore=0.1, fixed_alpha=True,
        alpha=0.2,
                alpha_fun=lambda k:1/k, qinit=0, label="Q_AR_learner",
                    max_buffer_size=5000,
                num_updates_per_action=5, burn_in=1000 ):
```

```
        Q_learner.__init__(self, env, discount, explore, fixed_alpha, alpha,
                alpha_fun, qinit, label)
        self.experience_buffer = BoundedBuffer(max_buffer_size)
        self.num_updates_per_action = num_updates_per_action
        self.burn_in = burn_in


    def do(self,num_steps=100):
        """do num_steps of interaction with the environment"""
        self.display(2,"s\ta\tr\ts'\tQ")
        alpha = self.alpha
        for i in range(num_steps):
            action = self.select_action(self.state)
            next_state,reward = self.env.do(action)
            self.experience_buffer.add((self.state,action,reward,next_state))
                #remember experience
            if not self.fixed_alpha:
                k = self.visits[(self.state, action)] =
                    self.visits.get((self.state, action),0)+1
                alpha = self.alpha_fun(k)
            self.q[(self.state, action)] = (
                (1-alpha) * self.q.get((self.state, action),self.qinit)
                + alpha * (reward + self.discount
                                * max(self.q.get((next_state,
                                    next_act),self.qinit)
                                    for next_act in self.actions)))
            self.display(2,self.state, action, reward, next_state,
                        self.q[(self.state, action)], sep='\t')
            self.state = next_state
            self.acc_rewards += reward
            # do some updates from experince buffer
            if self.experience_buffer.number_added > self.burn_in:
              for i in range(self.num_updates_per_action):
                (s,a,r,ns) = self.experience_buffer.get()
                if not self.fixed_alpha:
                    k = self.visits[(s,a)]
                    alpha = self.alpha_fun(k)
                self.q[(s,a)] = (
                    (1-alpha) * self.q[(s,a)]
                    + alpha * (reward + self.discount
                                * max(self.q.get((ns,na),self.qinit)
                                       for na in self.actions)))
```

rlQExperienceReplay.py — (continued)

```
from rlSimpleEnv import Simple_game_env
from rlQTest import sag1, sag2, sag3
from rlPlot import plot_rl

senv = Simple_game_env()
sag1ar = Q_AR_learner(senv,0.9,explore=0.2,fixed_alpha=True,alpha=0.1)
```

```
83 # plot_rl(sag1ar,steps_explore=100000,steps_exploit=100000,label="AR
       alpha="+str(sag1ar.alpha))
84 sag2ar = Q_AR_learner(senv,0.9,explore=0.2,fixed_alpha=False)
85 # plot_rl(sag2ar,steps_explore=100000,steps_exploit=100000,label="AR
       alpha=1/k")
86 sag3ar =
       Q_AR_learner(senv,0.9,explore=0.2,fixed_alpha=False,alpha_fun=lambda
       k:10/(9+k))
87 # plot_rl(sag3ar,steps_explore=100000,steps_exploit=100000,label="AR
       alpha=10/(9+k)")
```

12.4 Model-based Reinforcement Learner

To run the demo, in folder "aipython", load "rlModelLearner.py", and copy and paste the example queries at the bottom of that file. This assumes Python 3.

A model-based reinforcement learner builds a Markov decision process model of the domain, simultaneously learns the model and plans with that model.

The model-based reinforcement learner used the following data structures:

- $q[s,a]$ is dictionary that, given a (s,a) pair returns the Q-value, the estimate of the future (discounted) value of being in state s and doing action a.
- $r[s,a]$ is dictionary that, given a (s,a) pair returns the average reward from doing a in state s.
- $t[s,a,s']$ is dictionary that, given a (s,a,s') tuple returns the number of times a was done in state s, with the result being state s'.
- $visits[s,a]$ is dictionary that, given a (s,a) pair returns the number of times action a was carried out in state s.
- $res\_states[s,a]$ is dictionary that, given a (s,a) pair returns the list of resulting states that have occurred when action a was carried out in state s. This is used in the asynchronous value iteration to determine the s' states to sum over.
- *visits_list* is a list of (s,a) pair that have been carried out. This is used to ensure there is no divide-by zero in the asynchronous value iteration. Note that this could be constructed from r, *visits* or *res_states* by enumerating the keys, but needs to be a list for *random.choice*, and we don't want to keep recreating it.

rlModelLearner.py — Model-based Reinforcement Learner

```
import random
from rlQLearner import RL_agent
from display import Displayable
from utilities import argmaxe, flip

class Model_based_reinforcement_learner(RL_agent):
    """A Model-based reinforcement learner
    """

    def __init__(self, env, discount, explore=0.1, qinit=0,
                 updates_per_step=10, label="MBR_learner"):
        """env is the environment to interact with.
        discount is the discount factor
        explore is the proportion of time the agent will explore
        qinit is the initial value of the Q's
        updates_per_step is the number of AVI updates per action
        label is the label for plotting
        """
        RL_agent.__init__(self)
        self.env = env
        self.actions = env.actions
        self.discount = discount
        self.explore = explore
        self.qinit = qinit
        self.updates_per_step = updates_per_step
        self.label = label
        self.restart()
```

rlModelLearner.py — (continued)

```
    def restart(self):
        """make the agent relearn, and reset the accumulated rewards
        """
        self.acc_rewards = 0
        self.state = self.env.state
        self.q = {}          # {(st,action):q_value} map
        self.r = {}          # {(st,action):reward} map
        self.t = {}          # {(st,action,st_next):count} map
        self.visits = {}     # {(st,action):count} map
        self.res_states = {}  # {(st,action):set_of_states} map
        self.visits_list = [] # list of (st,action)
        self.previous_action = None
```

rlModelLearner.py — (continued)

```
    def do(self,num_steps=100):
        """do num_steps of interaction with the environment
        for each action, do updates_per_step iterations of asynchronous
            value iteration
        """
        for step in range(num_steps):
```

```
57            pst = self.state   # previous state
58            action = self.select_action(pst)
59            self.state,reward = self.env.do(action)
60            self.acc_rewards += reward
61            self.t[(pst,action,self.state)] = self.t.get((pst,
                  action,self.state),0)+1
62            if (pst,action) in self.visits:
63                self.visits[(pst,action)] += 1
64                self.r[(pst,action)] +=
                      (reward-self.r[(pst,action)])/self.visits[(pst,action)]
65                self.res_states[(pst,action)].add(self.state)
66            else:
67                self.visits[(pst,action)] = 1
68                self.r[(pst,action)] = reward
69                self.res_states[(pst,action)] = {self.state}
70                self.visits_list.append((pst,action))
71            st,act = pst,action   #initial state-action pair for AVI
72            for update in range(self.updates_per_step):
73                self.q[(st,act)] = self.r[(st,act)]+self.discount*(
74                    sum(self.t[st,act,rst]/self.visits[st,act]*
75                        max(self.q.get((rst,nact),self.qinit) for nact in
                                self.actions)
76                        for rst in self.res_states[(st,act)]))
77                st,act = random.choice(self.visits_list)
```

rlModelLearner.py — (continued)

```
79    def select_action(self, state):
80        """returns an action to carry out for the current agent
81        given the state, and the q-function
82        """
83        if flip(self.explore):
84            return random.choice(self.actions)
85        else:
86            return argmaxe((next_act, self.q.get((state,
                  next_act),self.qinit))
87                           for next_act in self.actions)
```

rlModelLearner.py — (continued)

```
89 from rlQTest import senv # simple game environment
90 mbl1 = Model_based_reinforcement_learner(senv,0.9,updates_per_step=10)
91 #
       plot_rl(mbl1,steps_explore=100000,steps_exploit=100000,label="model-based(10)")
92 mbl2 = Model_based_reinforcement_learner(senv,0.9,updates_per_step=1)
93 #
       plot_rl(mbl2,steps_explore=100000,steps_exploit=100000,label="model-based(1)")
```

Exercise 12.3 If there was only one update per step, the algorithm can be made simpler and use less space. Explain how. Does it make it more efficient? Is it worthwhile having more than one update per step for the games implemented here?

Exercise 12.4 It is possible to implement the model-based reinforcement learner by replacing q, r, *visits*, *res_states* with a single dictionary that returns a tuple (q, r, v, tm) where q, r and v are numbers, and tm is a map from resulting states into counts. Does this make the algorithm easier to understand? Does this make the algorithm more efficient?

Exercise 12.5 If the states and the actions were mapped into integers, the dictionaries could be implemented more efficiently as arrays. This entails an extra step in specifying problems. Implement this for the simple game. Is it more efficient?

12.5 Reinforcement Learning with Features

To run the demo, in folder "aipython", load "rlFeatures.py", and copy and paste the example queries at the bottom of that file. This assumes Python 3.

12.5.1 Representing Features

A feature is a function from state and action. To construct the features for a domain, we construct a function that takes a state and an action and returns the list of all feature values for that state and action. This feature set is redesigned for each problem.

get_features$(state, action)$ returns the feature values appropriate for the simple game.

rlSimpleGameFeatures.py — Feature-based Reinforcement Learner

```
from rlSimpleEnv import Simple_game_env
from rlProblem import RL_env

def get_features(state,action):
    """returns the list of feature values for the state-action pair
    """
    assert action in Simple_game_env.actions
    (x,y,d,p) = state
    # f1: would go to a monster
    f1 = monster_ahead(x,y,action)
    # f2: would crash into wall
    f2 = wall_ahead(x,y,action)
    # f3: action is towards a prize
    f3 = towards_prize(x,y,action,p)
    # f4: damaged and action is toward repair station
    f4 = towards_repair(x,y,action) if d else 0
    # f5: damaged and towards monster
    f5 = 1 if d and f1 else 0
    # f6: damaged
    f6 = 1 if d else 0
    # f7: not damaged
```

```
    f7 = 1-f6
    # f8: damaged and prize ahead
    f8 = 1 if d and f3 else 0
    # f9: not damaged and prize ahead
    f9 = 1 if not d and f3 else 0
    features = [1,f1,f2,f3,f4,f5,f6,f7,f8,f9]
    # the next 20 features are for 5 prize locations
    # and 4 distances from outside in all directions
    for pr in Simple_game_env.prize_locs+[None]:
        if p==pr:
            features += [x, 4-x, y, 4-y]
        else:
            features += [0, 0, 0, 0]
    # fp04 feature for y when prize is at 0,4
    # this knows about the wall to the right of the prize
    if p==(0,4):
        if x==0:
            fp04 = y
        elif y<3:
            fp04 = y
        else:
            fp04 = 4-y
    else:
        fp04 = 0
    features.append(fp04)
    return features

def monster_ahead(x,y,action):
    """returns 1 if the location expected to get to by doing
    action from (x,y) can contain a monster.
    """
    if action == "right" and (x+1,y) in Simple_game_env.monster_locs:
        return 1
    elif action == "left" and (x-1,y) in Simple_game_env.monster_locs:
        return 1
    elif action == "up" and (x,y+1) in Simple_game_env.monster_locs:
        return 1
    elif action == "down" and (x,y-1) in Simple_game_env.monster_locs:
        return 1
    else:
        return 0

def wall_ahead(x,y,action):
    """returns 1 if there is a wall in the direction of action from (x,y).
    This is complicated by the internal walls.
    """
    if action == "right" and (x==Simple_game_env.xdim-1 or (x,y) in
        Simple_game_env.vwalls):
        return 1
    elif action == "left" and (x==0 or (x-1,y) in Simple_game_env.vwalls):
```

```
        return 1
    elif action == "up" and y==Simple_game_env.ydim-1:
        return 1
    elif action == "down" and y==0:
        return 1
    else:
        return 0

def towards_prize(x,y,action,p):
    """action goes in the direction of the prize from (x,y)"""
    if p is None:
        return 0
    elif p==(0,4): # take into account the wall near the top-left prize
        if action == "left" and (x>1 or x==1 and y<3):
            return 1
        elif action == "down" and (x>0 and y>2):
            return 1
        elif action == "up" and (x==0 or y<2):
            return 1
        else:
            return 0
    else:
        px,py = p
        if p==(4,4) and x==0:
            if (action=="right" and y<3) or (action=="down" and y>2) or
                (action=="up" and y<2):
                return 1
            else:
                return 0
        if (action == "up" and y<py) or (action == "down" and py<y):
            return 1
        elif (action == "left" and px<x) or (action == "right" and x<px):
            return 1
        else:
            return 0

def towards_repair(x,y,action):
    """returns 1 if action is towards the repair station.
    """
    if action == "up" and (x>0 and y<4 or x==0 and y<2):
        return 1
    elif action == "left" and x>1:
        return 1
    elif action == "right" and x==0 and y<3:
        return 1
    elif action == "down" and x==0 and y>2:
        return 1
    else:
        return 0

```

```
130 def simp_features(state,action):
131     """returns a list of feature values for the state-action pair
132     """
133     assert action in Simple_game_env.actions
134     (x,y,d,p) = state
135     # f1: would go to a monster
136     f1 = monster_ahead(x,y,action)
137     # f2: would crash into wall
138     f2 = wall_ahead(x,y,action)
139     # f3: action is towards a prize
140     f3 = towards_prize(x,y,action,p)
141     return [1,f1,f2,f3]
```

12.5.2 Feature-based RL learner

This learns a linear function approximation of the Q-values. It requires the function *get_features* that given a state and an action returns a list of values for all of the features. Each environment requires this function to be provided.

rlFeatures.py — Feature-based Reinforcement Learner

```
11 import random
12 from rlQLearner import RL_agent
13 from display import Displayable
14 from utilities import argmaxe, flip
15
16 class SARSA_LFA_learner(RL_agent):
17     """A SARSA_LFA learning agent has
18     belief-state consisting of
19         state is the previous state
20         q is a {(state,action):value} dict
21         visits is a {(state,action):n} dict. n is how many times action was
               done in state
22         acc_rewards is the accumulated reward
23
24     it observes (s, r) for some world-state s and real reward r
25     """
26     def __init__(self, env, get_features, discount, explore=0.2,
           step_size=0.01,
27                  winit=0, label="SARSA_LFA"):
28         """env is the feature environment to interact with
29         get_features is a function get_features(state,action) that returns
               the list of feature values
30         discount is the discount factor
31         explore is the proportion of time the agent will explore
32         step_size is gradient descent step size
33         winit is the initial value of the weights
34         label is the label for plotting
35         """
36         RL_agent.__init__(self)
37         self.env = env
```

```
38          self.get_features = get_features
39          self.actions = env.actions
40          self.discount = discount
41          self.explore = explore
42          self.step_size = step_size
43          self.winit = winit
44          self.label = label
45          self.restart()
```

restart() is used to make the learner relearn everything. This is used by the plotter to create new plots.

rlFeatures.py — (continued)

```
47      def restart(self):
48          """make the agent relearn, and reset the accumulated rewards
49          """
50          self.acc_rewards = 0
51          self.state = self.env.state
52          self.features = self.get_features(self.state,
                list(self.env.actions)[0])
53          self.weights = [self.winit for f in self.features]
54          self.action = self.select_action(self.state)
```

do takes in the number of steps.

rlFeatures.py — (continued)

```
56      def do(self,num_steps=100):
57          """do num_steps of interaction with the environment"""
58          self.display(2,"s\ta\tr\ts'\tQ\tdelta")
59          for i in range(num_steps):
60              next_state,reward = self.env.do(self.action)
61              self.acc_rewards += reward
62              next_action = self.select_action(next_state)
63              feature_values = self.get_features(self.state,self.action)
64              oldQ = dot_product(self.weights, feature_values)
65              nextQ = dot_product(self.weights,
                    self.get_features(next_state,next_action))
66              delta = reward + self.discount * nextQ - oldQ
67              for i in range(len(self.weights)):
68                  self.weights[i] += self.step_size * delta * feature_values[i]
69              self.display(2,self.state, self.action, reward, next_state,
70                           dot_product(self.weights, feature_values), delta,
                                sep='\t')
71              self.state = next_state
72              self.action = next_action
73
74      def select_action(self, state):
75          """returns an action to carry out for the current agent
76          given the state, and the q-function.
77          This implements an epsilon-greedy approach
78          where self.explore is the probability of exploring.
79          """
```

```
80          if flip(self.explore):
81              return random.choice(self.actions)
82          else:
83              return argmaxe((next_act, dot_product(self.weights,
84                                                    self.get_features(state,next_act)))
85                             for next_act in self.actions)
86
87      def show_actions(self,state=None):
88          """prints the value for each action in a state.
89          This may be useful for debugging.
90          """
91          if state is None:
92              state = self.state
93          for next_act in self.actions:
94              print(next_act,dot_product(self.weights,
                      self.get_features(state,next_act)))
95
96  def dot_product(l1,l2):
97      return sum(e1*e2 for (e1,e2) in zip(l1,l2))
```

Test code:

rlFeatures.py — (continued)

```
100 from rlQTest import senv # simple game environment
101 from rlSimpleGameFeatures import get_features, simp_features
102 from rlPlot import plot_rl
103
104 fa1 = SARSA_LFA_learner(senv, get_features, 0.9, step_size=0.01)
105 #fa1.max_display_level = 2
106 #fa1.do(20)
107 #plot_rl(fa1,steps_explore=10000,steps_exploit=10000,label="SARSA_LFA(0.01)")
108 fas1 = SARSA_LFA_learner(senv, simp_features, 0.9, step_size=0.01)
109 #plot_rl(fas1,steps_explore=10000,steps_exploit=10000,label="SARSA_LFA(simp)")
```

Exercise 12.6 How does the step-size affect performance? Try different step sizes (e.g., 0.1, 0.001, other sizes in between). Explain the behaviour you observe. Which step size works best for this example. Explain what evidence you are basing your prediction on.

Exercise 12.7 Does having extra features always help? Does it sometime help? Does whether it helps depend on the step size? Give evidence for your claims.

Exercise 12.8 For each of the following first predict, then plot, then explain the behavour you observed:

(a) SARSA_LFA, Model-based learning (with 1 update per step) and Q-learning for 10,000 steps 20% exploring followed by 10,000 steps 100% exploiting

(b) SARSA_LFA, model-based learning and Q-learning for

i) 100,000 steps 20% exploring followed by 100,000 steps 100% exploit

ii) 10,000 steps 20% exploring followed by 190,000 steps 100% exploit

(c) Suppose your goal was to have the best accumulated reward after 200,000 steps. You are allowed to change the exploration rate at a fixed number of steps. For each of the methods, which is the best position to start exploiting more? Which method is better? What if you wanted to have the best reward after 10,000 or 1,000 steps?

Based on this evidence, explain when it is preferable to use SARSA_LFA, Model-based learner, or Q-learning.

Important: you need to run each algorithm more than once. Your explanation should include the variability as well as the typical behavior.

12.5.3 Experience Replay

Here we consider experience replay with a bounded replay buffer for SARSA_LFA. Warning: does not work properly yet.

Should self.env return (reward,state) to be consistent with (S,A,R,S)?

rlLinExperienceReplay.py — Linear Reinforcement Learner with Experience Replay

```
from rlFeatures import SARSA_LFA_learner, dot_product
from utilities import flip
import random

class SARSA_LFA_AR_learner(SARSA_LFA_learner):

    def __init__(self, env, get_features, discount, explore=0.2,
        step_size=0.01,
                 winit=0, label="SARSA_LFA-AR", max_buffer_size=500,
                 num_updates_per_action=5, burn_in=100 ):
        SARSA_LFA_learner.__init__(self, env, get_features, discount,
            explore, step_size,
                                   winit, label)
        self. max_buffer_size = max_buffer_size
        self.action_buffer = [0]*max_buffer_size
        self.number_added = 0
        self.num_updates_per_action = num_updates_per_action
        self.burn_in = burn_in

    def add_to_buffer(self,experience):
        if self.number_added < self.max_buffer_size:
            self.action_buffer[self.number_added] = experience
        else:
            if flip(self.max_buffer_size/self.number_added):
                position = random.randrange(self.max_buffer_size)
                self.action_buffer[position] = experience
        self.number_added += 1

    def do(self,num_steps=100):
        """do num_steps of interaction with the environment"""
        self.display(2,"s\ta\tr\ts'\tQ\tdelta")
        for i in range(num_steps):
```

```
41            next_state,reward = self.env.do(self.action)
42            self.add_to_buffer((self.state,self.action,reward,next_state))
                  #remember experience
43            self.acc_rewards += reward
44            next_action = self.select_action(next_state)
45            feature_values = self.get_features(self.state,self.action)
46            oldQ = dot_product(self.weights, feature_values)
47            nextQ = dot_product(self.weights,
                  self.get_features(next_state,next_action))
48            delta = reward + self.discount * nextQ - oldQ
49            for i in range(len(self.weights)):
50               self.weights[i] += self.step_size * delta * feature_values[i]
51            self.display(2,self.state, self.action, reward, next_state,
52                         dot_product(self.weights, feature_values), delta,
                                sep='\t')
53            self.state = next_state
54            self.action = next_action
55            if self.number_added > self.burn_in:
56              for i in range(self.num_updates_per_action):
57                (s,a,r,ns) =
                      self.action_buffer[random.randrange(min(self.number_added,
58                                                              self.max_buffer_size))]
59                na = self.select_action(ns)
60                feature_values = self.get_features(s,a)
61                oldQ = dot_product(self.weights, feature_values)
62                nextQ = dot_product(self.weights, self.get_features(ns,na))
63                delta = reward + self.discount * nextQ - oldQ
64                for i in range(len(self.weights)):
65                   self.weights[i] += self.step_size * delta *
                          feature_values[i]
```

Test code:

rlLinExperienceReplay.py — (continued)

```
67 from rlQTest import senv # simple game environment
68 from rlSimpleGameFeatures import get_features, simp_features
69 from rlPlot import plot_rl
70
71 fa1 = SARSA_LFA_AR_learner(senv, get_features, 0.9, step_size=0.01)
72 #fa1.max_display_level = 2
73 #fa1.do(20)
74 #plot_rl(fa1,steps_explore=10000,steps_exploit=10000,label="SARSA_LFA_AR(0.01)")
75 fas1 = SARSA_LFA_AR_learner(senv, simp_features, 0.9, step_size=0.01)
76 #plot_rl(fas1,steps_explore=10000,steps_exploit=10000,label="SARSA_LFA_AR(simp)")
```

12.6 Multiagent Learning

The next code of for multiple agnets that learn when interacting with other agents. This code is designed to be extended, and as such is restricted to being

two agents, a single state, and the only observation is the reward. Coordinating agents can't easily implement that agent architecture. However, in that architecture, an agent calls the environment. That architecture was chosen because it was simple. However, it does not really work when there are multiple agents, instead we have a controller that tells the egents the percepts (here the percepts are just the reward).

masLearn.py — Simulations of agents learning

```
from display import Displayable
import utilities # argmaxall for (element,value) pairs
import matplotlib.pyplot as plt
import random

class GameAgent(Displayable):
    next_id=0
    def __init__(self, actions):
        """
        Actions is the set of actions the agent can do. It needs to be told
            that!
        """
        self.actions = actions
        self.id = GameAgent.next_id
        GameAgent.next_id += 1
        self.display(2,f"Agent {self.id} has actions {actions}")
        self.dist = {act:1 for act in actions} # unnormalized distibution
        self.total_score = 0

    def init_action(self):
        """ The initial action.
        Act randomly initially
        Could be overridden (but I'm not sure why you would).
        """
        self.act = random.choice(self.actions)
        return self.act

    def select_action(self, reward):
        """
        Select the action given the reward.
        This implements "Act randomly" and should be overridden!
        """
        self.total_score += reward
        self.act = random.choice(self.actions)
        return self.act
```

masLearn.py — (continued)

```
class SimpleCountingAgent(GameAgent):
    """This agent just counts the number of times (it thinks) it has won
        and does the
    actions it thinks is most likely to win.
    """
```

```
50      def __init__(self, actions, prior_count=1):
51          """
52          Actions is the set of actions the agent can do. It needs to be told
                that!
53          """
54          GameAgent.__init__(self, actions)
55          self.prior_count = prior_count
56          self.dist = {a: prior_count for a in self.actions} # unnormalized
                distibution
57          self.averew = 0
58          self.num_steps = 0
59
60      def select_action(self, reward):
61          self.total_score += reward
62          self.num_steps += 1
63          self.display(2,f"The reward for agent {self.id} was {reward}")
64          self.averew = self.averew+(reward-self.averew)/self.num_steps
65          if reward>self.averew:
66              self.dist[self.act] += 1
67          else:
68              for otheract in self.actions:
69                  if otheract != self.act:
70                      self.dist[otheract] += 1/(len(self.actions))
71          self.display(2,f"Distribution for agent {self.id} is
                {normalize(self.dist)}")
72          self.act = select_from_dist(self.dist)
73          self.display(2,f"Agent {self.id} did {self.act}")
74          return self.act
```

masLearn.py — (continued)

```
76  class SimpleQAgent(GameAgent):
77      """This agent maintains the Q-function for each state.
78      (Or just the average reward as the future state is all the same).
79      Chooses the best action using
80      """
81      def __init__(self, actions, q_init=100, alpha=0.1,
            prob_step_size=0.001, min_prob=0.01):
82          """
83          Actions is the set of actions the agent can do. It needs to be told
                that!
84          q_init is the initial q-values
85          alpha is the step size for action estimate
86          prob_step_size is the step size for probability change
87          min_prob is the minimum a probability should become
88          """
89          GameAgent.__init__(self, actions)
90          self.Q = {a:q_init for a in self.actions}
91          self.dist = normalize({a:0.7+random.random() for a in
                self.actions}) # start with random dist but not too close to
                zero
```

```
92          self.alpha = alpha
93          self.prob_step_size = prob_step_size
94          self.min_prob = min_prob
95          self.num_steps = 1 # (1 because it isonly used after initial step)
96
97      def select_action(self, reward):
98          self.total_score += reward
99          self.display(2,f"The reward for agent {self.id} was {reward}")
100         self.Q[self.act] += self.alpha*(reward-self.Q[self.act])
101         a_best = utilities.argmaxall(self.Q.items())
102         for a in self.actions:
103             if a in a_best:
104                 self.dist[a] += self.prob_step_size
105             else:
106                 self.dist[a] -= min(self.dist[a], self.prob_step_size)
107                 self.dist[a] = max(self.dist[a],self.min_prob)
108         self.dist = normalize(self.dist)
109         self.display(2,f"Distribution for agent {self.id} is {self.dist}")
110         self.act = select_from_dist(self.dist)
111         self.display(2,f"Agent {self.id} did {self.act}")
112         return self.act
113
114 def normalize(dist):
115     """unnorm dict is a {value:number} dictionary, where the numbers are
            all non-negative
116     returns dict where the numbers sum to one
117     """
118     tot = sum(dist.values())
119     return {var:val/tot for (var,val) in dist.items()}
120
121 def select_from_dist(dist):
122     rand = random.random()
123     for (act,prob) in normalize(dist).items():
124         rand -= prob
125         if rand < 0:
126             return act
```

The simulator takes a game and simulates the game:

masLearn.py — (continued)

```
128 class SimulateGame(Displayable):
129     def __init__(self, game, agents):
130         self.game = game
131         self.agents = agents # list of agents
132         self.action_history = []
133         self.reward_history = []
134         self.dist_history = []
135         self.actions = tuple(ag.init_action() for ag in self.agents)
136         self.num_steps = 0
137
138     def go(self, steps):
```

```
        for i in range(steps):
            self.num_steps += 1
            self.rewards = self.game.play(self.actions)
            self.reward_history.append(self.rewards)
            self.actions =
                tuple(self.agents[i].select_action(self.rewards[i])
                                for i in range(self.game.num_agents))
            self.action_history.append(self.actions)
            self.dist_history.append([normalize(ag.dist) for ag in
                self.agents])
        print("Scores:", ' '.join(f"Agent {ag.id} average
            reward={ag.total_score/self.num_steps}" for ag in self.agents))
        #return self.reward_history, self.action_history

    def action_dist(self,which_actions=[1,1]):
        """ which actions is [a0,a1]
        returns the empirical disctribition of actions for agents,
           where ai specifies the index of the actions for agent i
        """
        return [sum(1 for a in sim.action_history
                        if
                            a[i]==gm.actions[i][which_actions[i]])/len(sim.action_history)
                    for i in range(2)]
```

masLearn.py — (continued)

```

    def plot_dynamics(self, x_action=0, y_action=0):
        plt.ion() # make it interactive
        agents = self.agents
        x_act = self.game.actions[0][x_action]
        y_act = self.game.actions[1][y_action]
        plt.xlabel(f"Action {self.agents[0].actions[x_action]} for Agent
            {agents[0].id}")
        plt.ylabel(f"Action {self.agents[1].actions[y_action]} for Agent
            {agents[1].id}")
        plt.plot([self.dist_history[t][0][x_act] for t in
            range(len(self.dist_history))],
                [self.dist_history[t][1][y_act] for t in
                    range(len(self.dist_history))])
        #plt.legend()
```

The following are some games from Poole and Mackworth [2017].

masLearn.py — (continued)

```
class ShoppingGame(Displayable):
    def __init__(self):
        self.num_agents = 2
        self.actions = [['shopping', 'football']]*2

    def play(self, actions):
        return {('football', 'football'): (2,1),
```

```
            ('football', 'shopping'): (0,0),
            ('shopping', 'football'): (0,0),
            ('shopping', 'shopping'): (1,2)}[actions]


class SoccerGame(Displayable):
    def __init__(self):
        self.num_agents = 2
        self.actions = [['left', 'right']]*2

    def play(self, actions):
        return {('left', 'left'): (0.6, 0.4),
                ('left', 'right'): (0.2, 0.8),
                ('right', 'left'): (0.3, 0.7),
                ('right', 'right'): (0.9,0.1)
               }[actions]

class GameShow(Displayable):
    def __init__(self):
        self.num_agents = 2
        self.actions = [['take', 'give']]*2

    def play(self, actions):
        return {('take', 'take'): (100, 100),
                ('take', 'give'): (1100, 0),
                ('give', 'take'): (0, 1100),
                ('give', 'give'): (1000,1000)
               }[actions]


class UniqueNEGameExample(Displayable):
    def __init__(self):
        self.num_agents = 2
        self.actions = [['a1', 'b1', 'c1'],['d2', 'e2', 'f2']]

    def play(self, actions):
        return {('a1', 'd2'): (3, 5),
                ('a1', 'e2'): (5, 1),
                ('a1', 'f2'): (1, 2),
                ('b1', 'd2'): (1, 1),
                ('b1', 'e2'): (2, 9),
                ('b1', 'f2'): (6, 4),
                ('c1', 'd2'): (2, 6),
                ('c1', 'e2'): (4, 7),
                ('c1', 'f2'): (0, 8)
               }[actions]

# Choose one:
# gm = ShoppingGame()
# gm = SoccerGame()
```

```
# gm = GameShow()
# gm = UniqueNEGameExample()

# Choose one:
# sim=SimulateGame(gm,[SimpleQAgent(gm.actions[0]),
    SimpleQAgent(gm.actions[1])]); sim.go(10000)
# sim= SimulateGame(gm,[SimpleCountingAgent(gm.actions[0]),
    SimpleCountingAgent(gm.actions[1])]); sim.go(10000)
# sim=SimulateGame(gm,[SimpleCountingAgent(gm.actions[0]),
    SimpleQAgent(gm.actions[1])]); sim.go(10000)


# sim.plot_dynamics()

# empirical proportion that agents did their action at index 1:
# sim.action_dist([1,1])

# learned distribution for agent 0
# sim.agents[0].dist
```

Chapter 13

Relational Learning

13.1 Collaborative Filtering

Based on gradient descent algorithm of Koren, Y., Bell, R. and Volinsky, C., Matrix Factorization Techniques for Recommender Systems, IEEE Computer 2009.

This assumes the form of the dataset from movielens (`http://grouplens.org/datasets/movielens/`). The rating are a set of $(user, item, rating, timestamp)$ tuples.

relnCollFilt.py — Latent Property-based Collaborative Filtering

```
11 import random
12 import matplotlib.pyplot as plt
13 import urllib.request
14 from learnProblem import Learner
15 from display import Displayable
16
17 class CF_learner(Learner):
18     def __init__(self,
19                  rating_set,         # a Rating_set object
20                  rating_subset = None, # subset of ratings to be used as
                         training ratings
21                  test_subset = None, # subset of ratings to be used as test
                         ratings
22                  step_size = 0.01,   # gradient descent step size
23                  reglz = 1.0,        # the weight for the regularization
                         terms
24                  num_properties = 10,  # number of hidden properties
25                  property_range = 0.02 # properties are initialized to be
                         between
26                                      # -property_range and property_range
```

```
            ):
        self.rating_set = rating_set
        self.ratings = rating_subset or rating_set.training_ratings #
            whichever is not empty
        if test_subset is None:
            self.test_ratings = self.rating_set.test_ratings
        else:
            self.test_ratings = test_subset
        self.step_size = step_size
        self.reglz = reglz
        self.num_properties = num_properties
        self.num_ratings = len(self.ratings)
        self.ave_rating = (sum(r for (u,i,r,t) in self.ratings)
                           /self.num_ratings)
        self.users = {u for (u,i,r,t) in self.ratings}
        self.items = {i for (u,i,r,t) in self.ratings}
        self.user_bias = {u:0 for u in self.users}
        self.item_bias = {i:0 for i in self.items}
        self.user_prop = {u:[random.uniform(-property_range,property_range)
                             for p in range(num_properties)]
                           for u in self.users}
        self.item_prop = {i:[random.uniform(-property_range,property_range)
                             for p in range(num_properties)]
                           for i in self.items}
        self.zeros = [0 for p in range(num_properties)]
        self.iter=0

    def stats(self):
        self.display(1,"ave sumsq error of mean for training=",
                 sum((self.ave_rating-rating)**2 for
                     (user,item,rating,timestamp)
                    in self.ratings)/len(self.ratings))
        self.display(1,"ave sumsq error of mean for test=",
                 sum((self.ave_rating-rating)**2 for
                     (user,item,rating,timestamp)
                    in self.test_ratings)/len(self.test_ratings))
        self.display(1,"error on training set",
                    self.evaluate(self.ratings))
        self.display(1,"error on test set",
                    self.evaluate(self.test_ratings))
```

learn carries out *num_iter* steps of gradient descent.

relnCollFilt.py — (continued)

```
    def prediction(self,user,item):
        """Returns prediction for this user on this item.
        The use of .get() is to handle users or items not in the training
            set.
        """
        return (self.ave_rating
               +  self.user_bias.get(user,0) #self.user_bias[user]
```

```
71              + self.item_bias.get(item,0) #self.item_bias[item]
72              +
                    sum([self.user_prop.get(user,self.zeros)[p]*self.item_prop.get(item,self.zeros)
73                     for p in range(self.num_properties)]))
74
75      def learn(self, num_iter = 50):
76          """ do num_iter iterations of gradient descent."""
77          for i in range(num_iter):
78              self.iter += 1
79              abs_error=0
80              sumsq_error=0
81              for (user,item,rating,timestamp) in
                    random.sample(self.ratings,len(self.ratings)):
82                  error = self.prediction(user,item) - rating
83                  abs_error += abs(error)
84                  sumsq_error += error * error
85                  self.user_bias[user] -= self.step_size*error
86                  self.item_bias[item] -= self.step_size*error
87                  for p in range(self.num_properties):
88                      self.user_prop[user][p] -=
                            self.step_size*error*self.item_prop[item][p]
89                      self.item_prop[item][p] -=
                            self.step_size*error*self.user_prop[user][p]
90              for user in self.users:
91                  self.user_bias[user] -= self.step_size*self.reglz*
                        self.user_bias[user]
92                  for p in range(self.num_properties):
93                      self.user_prop[user][p] -=
                            self.step_size*self.reglz*self.user_prop[user][p]
94              for item in self.items:
95                  self.item_bias[item] -=
                        self.step_size*self.reglz*self.item_bias[item]
96                  for p in range(self.num_properties):
97                      self.item_prop[item][p] -=
                            self.step_size*self.reglz*self.item_prop[item][p]
98              self.display(1,"Iteration",self.iter,
99                   "(Ave Abs,AveSumSq) training
                         =",self.evaluate(self.ratings),
100                  "test =",self.evaluate(self.test_ratings))
```

evaluate evaluates current predictions on the rating set:

reInCollFilt.py — (continued)

```
102     def evaluate(self,ratings):
103         """returns (avergage_absolute_error, average_sum_squares_error) for
                ratings
104         """
105         abs_error = 0
106         sumsq_error = 0
107         if not ratings: return (0,0)
108         for (user,item,rating,timestamp) in ratings:
```

```
109             error = self.prediction(user,item) - rating
110             abs_error += abs(error)
111             sumsq_error += error * error
112         return abs_error/len(ratings), sumsq_error/len(ratings)
```

13.1.1 Alternative Formulation

An alternative formulation is to regularize after each update.

13.1.2 Plotting

relnCollFilt.py — (continued)

```
114     def plot_predictions(self, examples="test"):
115         """
116         examples is either "test" or "training" or the actual examples
117         """
118         if examples == "test":
119             examples = self.test_ratings
120         elif examples == "training":
121             examples = self.ratings
122         plt.ion()
123         plt.xlabel("prediction")
124         plt.ylabel("cumulative proportion")
125         self.actuals = [[] for r in range(0,6)]
126         for (user,item,rating,timestamp) in examples:
127             self.actuals[rating].append(self.prediction(user,item))
128         for rating in range(1,6):
129             self.actuals[rating].sort()
130             numrat=len(self.actuals[rating])
131             yvals = [i/numrat for i in range(numrat)]
132             plt.plot(self.actuals[rating], yvals,
                        label="rating="+str(rating))
133         plt.legend()
134         plt.draw()
```

This plots a single property. Each $(user, item, rating)$ is plotted where the x-value is the value of the property for the user, the y-value is the value of the property for the item, and the rating is plotted at this (x, y) position. That is, *rating* is plotted at the (x, y) position $(p(user), p(item))$.

relnCollFilt.py — (continued)

```
136     def plot_property(self,
137                       p,              # property
138                       plot_all=False, # true if all points should be plotted
139                       num_points=200 # number of random points plotted if not
                              all
140                       ):
141         """plot some of the user-movie ratings,
```

```
        if plot_all is true
        num_points is the number of points selected at random plotted.

        the plot has the users on the x-axis sorted by their value on
            property p and
        with the items on the y-axis sorted by their value on property p and
        the ratings plotted at the corresponding x-y position.
        """
        plt.ion()
        plt.xlabel("users")
        plt.ylabel("items")
        user_vals = [self.user_prop[u][p]
                     for u in self.users]
        item_vals = [self.item_prop[i][p]
                     for i in self.items]
        plt.axis([min(user_vals)-0.02,
                  max(user_vals)+0.05,
                  min(item_vals)-0.02,
                  max(item_vals)+0.05])
        if plot_all:
            for (u,i,r,t) in self.ratings:
                plt.text(self.user_prop[u][p],
                         self.item_prop[i][p],
                         str(r))
        else:
            for i in range(num_points):
                (u,i,r,t) = random.choice(self.ratings)
                plt.text(self.user_prop[u][p],
                         self.item_prop[i][p],
                         str(r))
        plt.show()
```

13.1.3 Creating Rating Sets

A rating set can be read from the Internet or read from a local file. The default is to read the Movielens 100K dataset from the Internet. It would be more efficient to save the dataset as a local file, and then set *local_file = True*, as then it will not need to download the dataset every time the program is run.

relnCollFilt.py — (continued)

```
class Rating_set(Displayable):
    def __init__(self,
                 date_split=892000000,
                 local_file=False,
                 url="http://files.grouplens.org/datasets/movielens/ml-100k/u.data",
                 file_name="u.data"):
        self.display(1,"reading...")
        if local_file:
            lines = open(file_name,'r')
        else:
```

```
183             lines = (line.decode('utf-8') for line in
                    urllib.request.urlopen(url))
184         all_ratings = (tuple(int(e) for e in line.strip().split('\t'))
185                        for line in lines)
186         self.training_ratings = []
187         self.training_stats = {1:0, 2:0, 3:0, 4:0 ,5:0}
188         self.test_ratings = []
189         self.test_stats = {1:0, 2:0, 3:0, 4:0 ,5:0}
190         for rate in all_ratings:
191             if rate[3] < date_split: # rate[3] is timestamp
192                 self.training_ratings.append(rate)
193                 self.training_stats[rate[2]] += 1
194             else:
195                 self.test_ratings.append(rate)
196                 self.test_stats[rate[2]] += 1
197         self.display(1,"...read:", len(self.training_ratings),"training
                ratings and",
198                      len(self.test_ratings),"test ratings")
199         tr_users = {user for (user,item,rating,timestamp) in
                self.training_ratings}
200         test_users = {user for (user,item,rating,timestamp) in
                self.test_ratings}
201         self.display(1,"users:",len(tr_users),"training,",len(test_users),"test,",
202                      len(tr_users & test_users),"in common")
203         tr_items = {item for (user,item,rating,timestamp) in
                self.training_ratings}
204         test_items = {item for (user,item,rating,timestamp) in
                self.test_ratings}
205         self.display(1,"items:",len(tr_items),"training,",len(test_items),"test,",
206                      len(tr_items & test_items),"in common")
207         self.display(1,"Rating statistics for training set:
                ",self.training_stats)
208         self.display(1,"Rating statistics for test set: ",self.test_stats)
```

Sometimes it is useful to plot a property for all $(user, item, rating)$ triples. There are too many such triples in the data set. The method *create_top_subset* creates a much smaller dataset where this makes sense. It picks the most rated items, then picks the users who have the most ratings on these items. It is designed for depicting the meaning of properties, and may not be useful for other purposes.

relnCollFilt.py — (continued)

```
210     def create_top_subset(self, num_items = 30, num_users = 30):
211         """Returns a subset of the ratings by picking the most rated items,
212         and then the users that have most ratings on these, and then all of
                the
213         ratings that involve these users and items.
214         """
215         items = {item for (user,item,rating,timestamp) in
                self.training_ratings}
216
```

```
        item_counts = {i:0 for i in items}
        for (user,item,rating,timestamp) in self.training_ratings:
            item_counts[item] += 1

        items_sorted = sorted((item_counts[i],i) for i in items)
        top_items = items_sorted[-num_items:]
        set_top_items = set(item for (count, item) in top_items)

        users = {user for (user,item,rating,timestamp) in
            self.training_ratings}
        user_counts = {u:0 for u in users}
        for (user,item,rating,timestamp) in self.training_ratings:
            if item in set_top_items:
                user_counts[user] += 1

        users_sorted = sorted((user_counts[u],u)
                              for u in users)
        top_users = users_sorted[-num_users:]
        set_top_users = set(user for (count, user) in top_users)
        used_ratings = [ (user,item,rating,timestamp)
                        for (user,item,rating,timestamp) in
                            self.training_ratings
                        if user in set_top_users and item in set_top_items]
        return used_ratings

movielens = Rating_set()
learner1 = CF_learner(movielens, num_properties = 1)
#learner1.learn(50)
# learner1.plot_predictions(examples = "training")
# learner1.plot_predictions(examples = "test")
#learner1.plot_property(0)
#movielens_subset = movielens.create_top_subset(num_items = 20, num_users
    = 20)
#learner_s = CF_learner(movielens, rating_subset=movielens_subset,
    test_subset=[], num_properties=1)
#learner_s.learn(1000)
#learner_s.plot_property(0,plot_all=True)
```

Chapter 14

Version History

- 2021-07-08 Version 0.9.1 updated the CSP code to have the same representation of variables as used by the probability code
- 2021-05-13 Version 0.9.0 Major revisions to chapters 8 and 9. Introduced recursive conditioning, simplified much code. New section on multiagent reinforcement learning.
- 2020-11-04 Version 0.8.6 simplified value iteration for MDPs.
- 2020-10-20 Version 0.8.4 planning simplified, and gives error if goal not part of state (by design). Fixed arc costs.
- 2020-07-21 Version 0.8.2 added positions and string to constraints
- 2019-09-17 Version 0.8.0 rerepresented blocks world (Section 6.1.2) due to bug found by Donato Meoli.

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