Data Science Cheatsheets

List of Data Science Cheatsheets to rule the world.

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README.md

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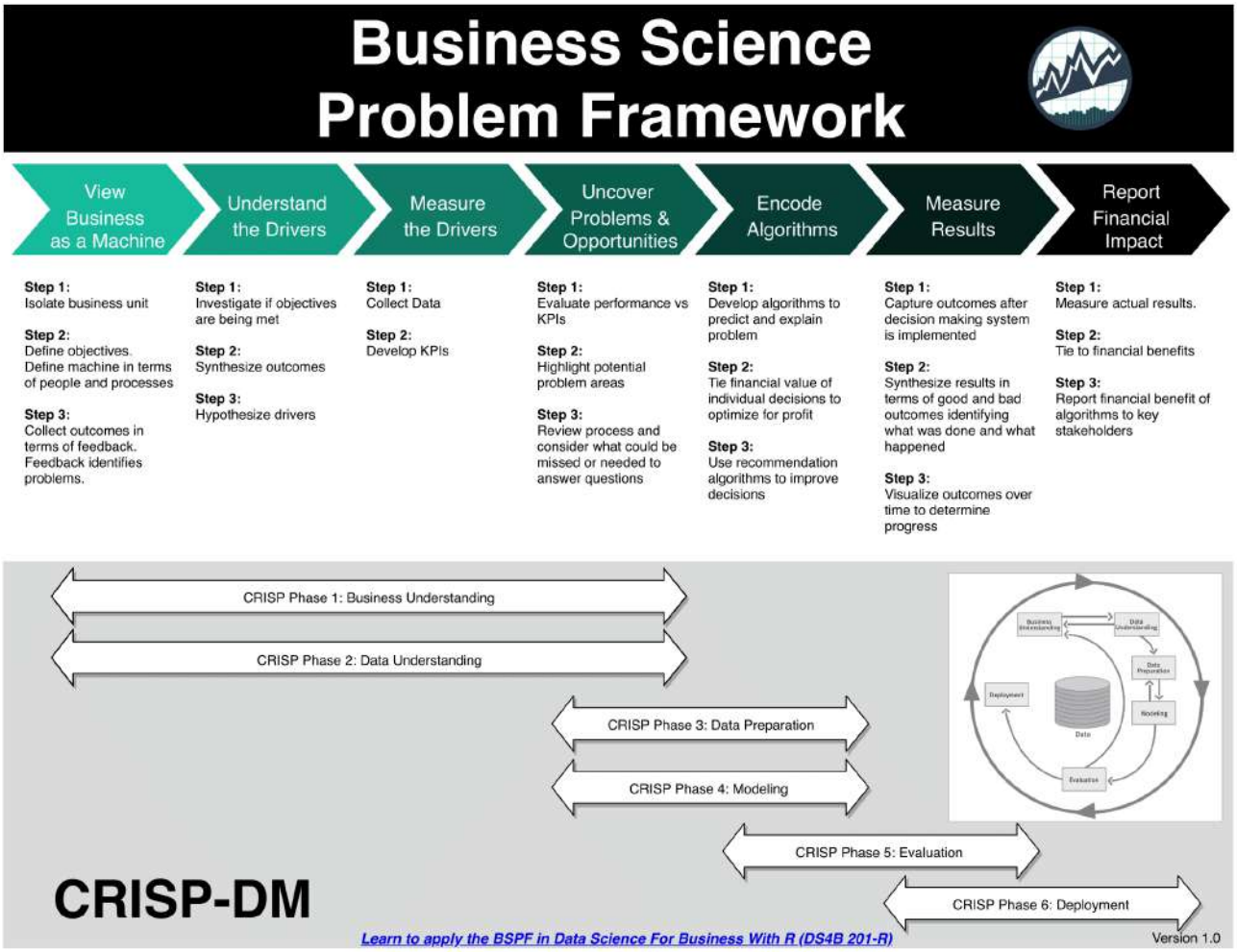
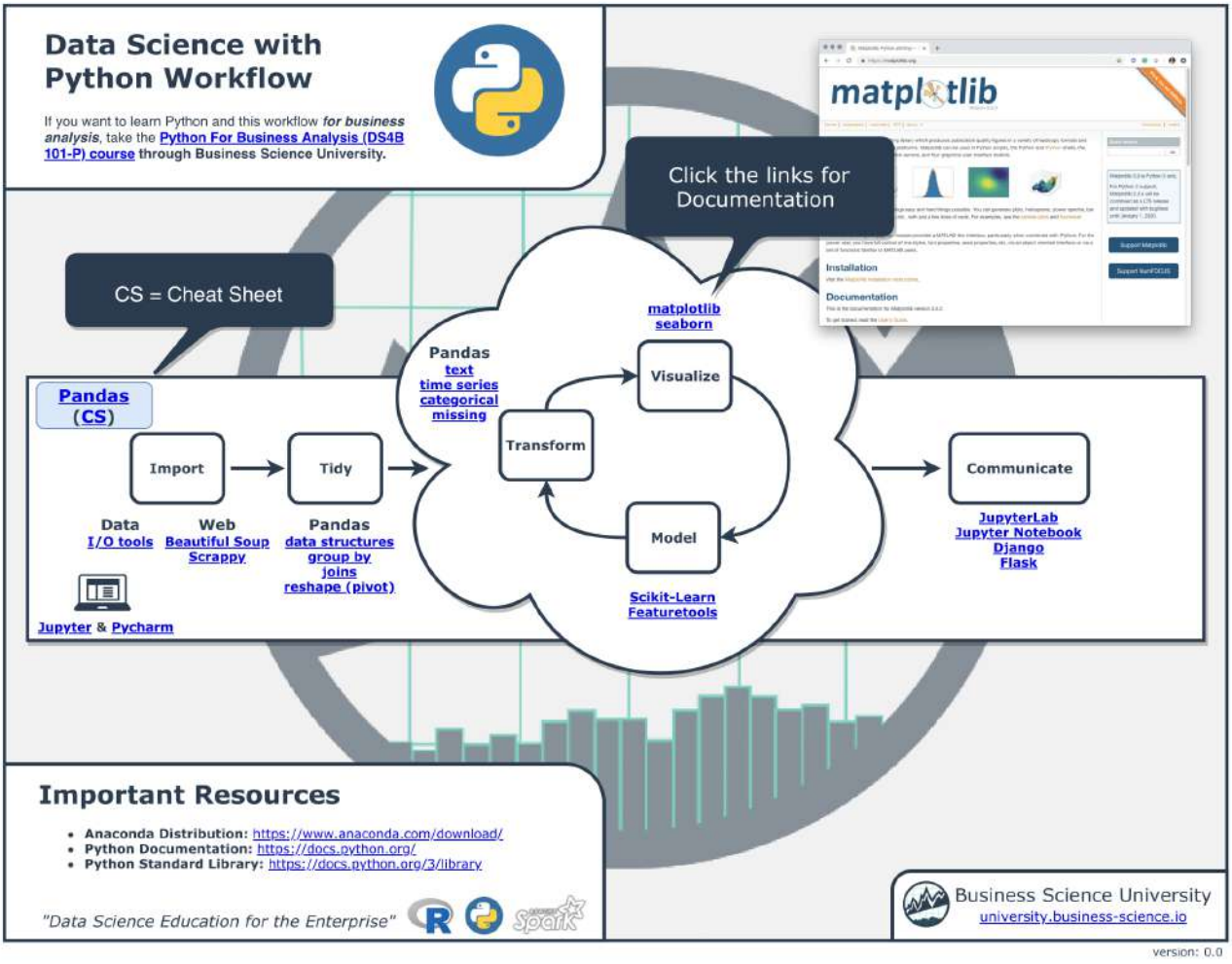
Business Science

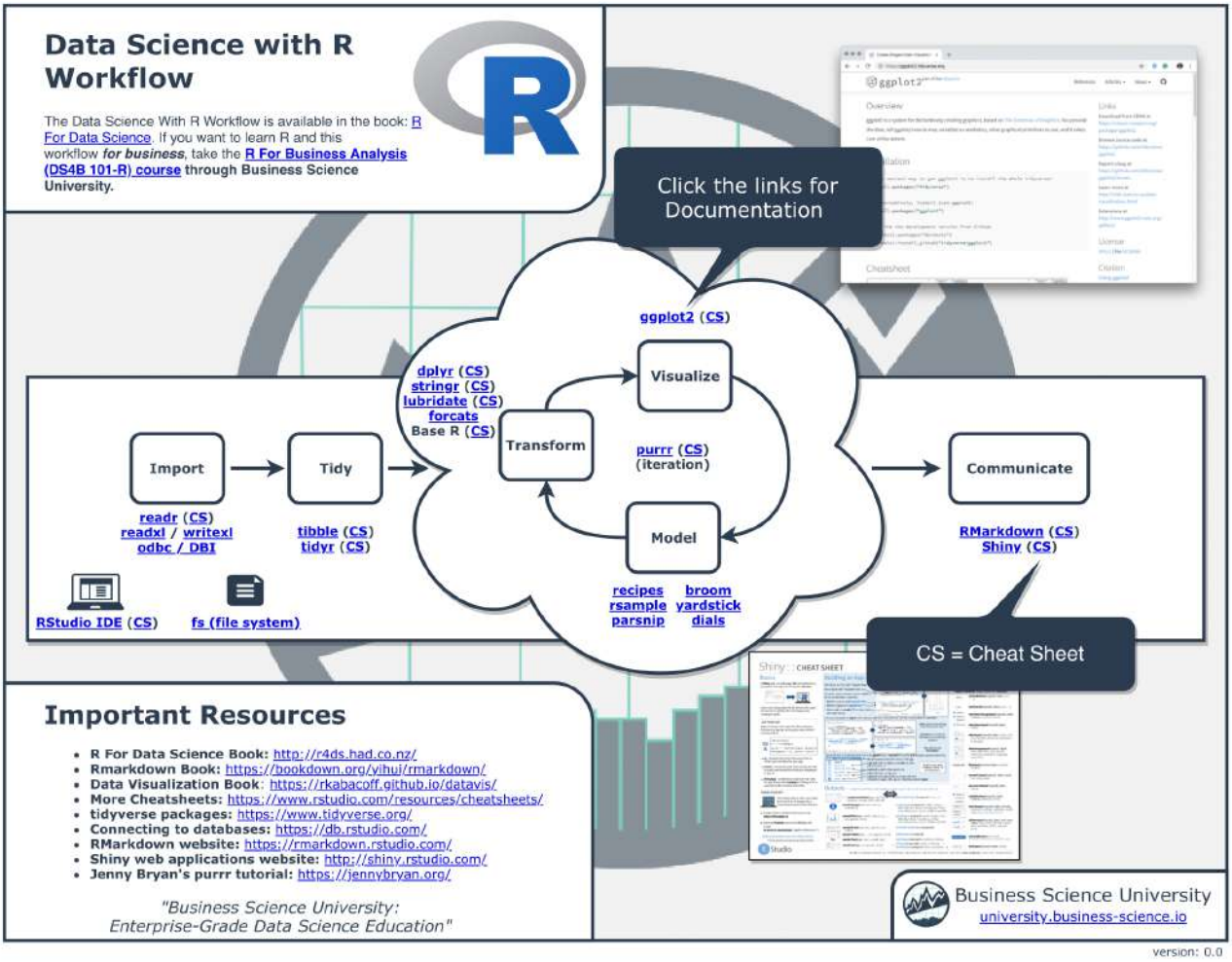
Business Science Problem Framework (PDF)



Data Science with Python Workflow (PDF)



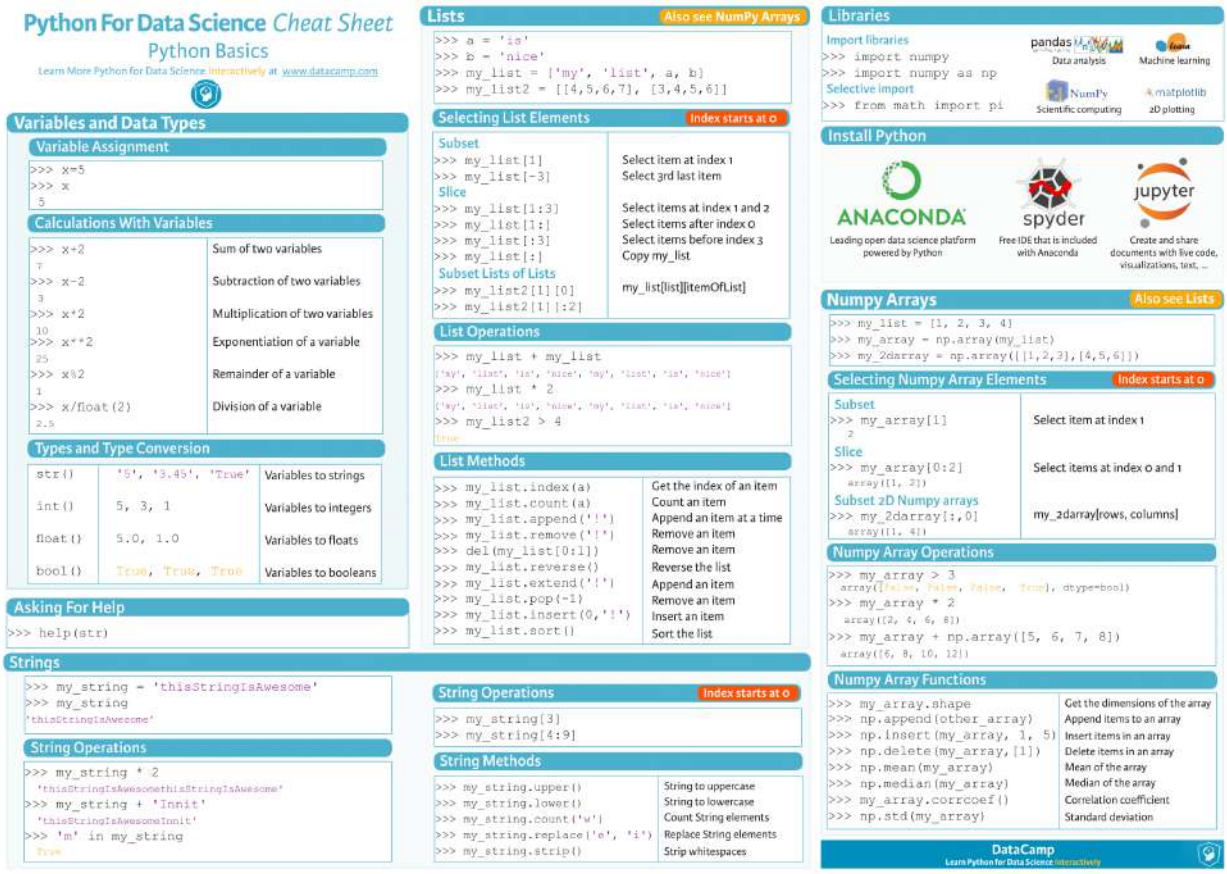
Data Science with R Workflow (PDF)



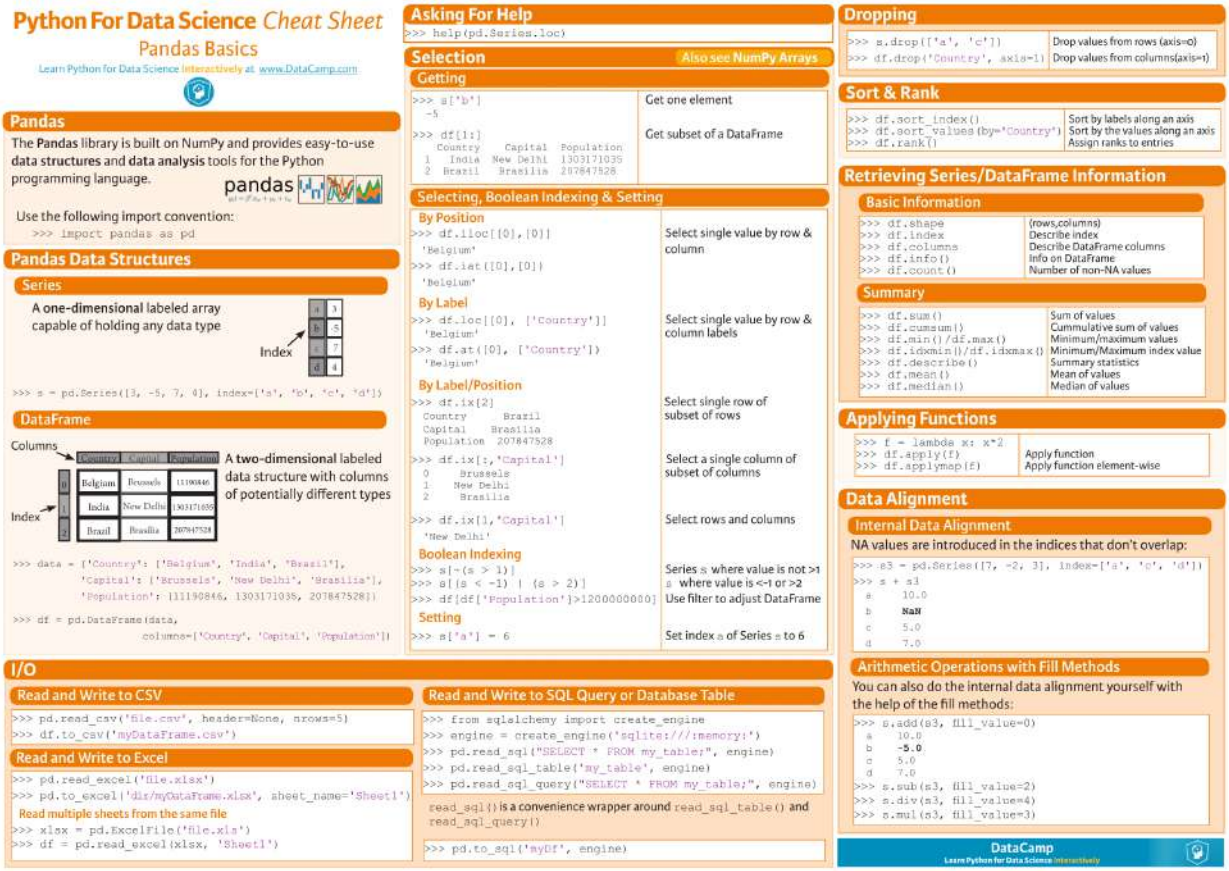
Python

Datacamp

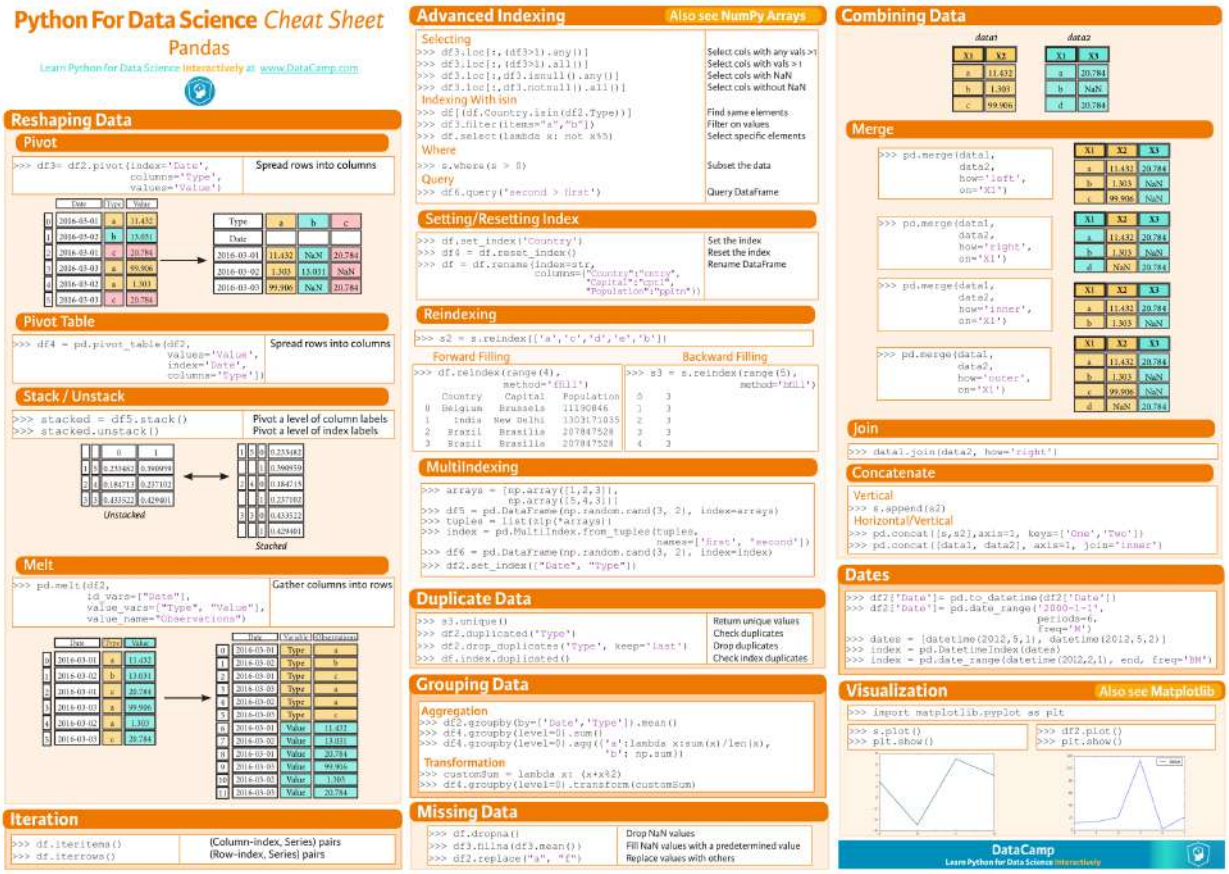
- Python Basics (PDF)



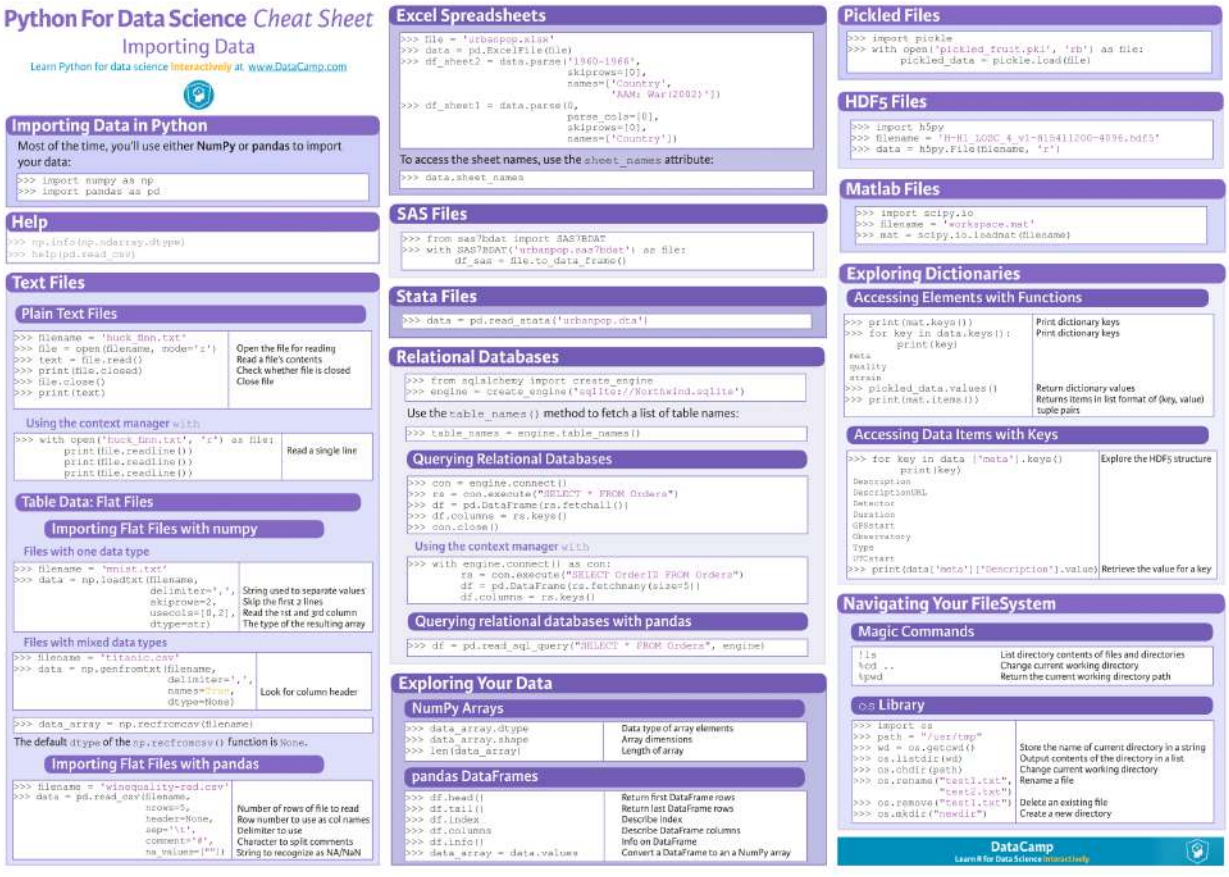
- Pandas Basics (PDF)



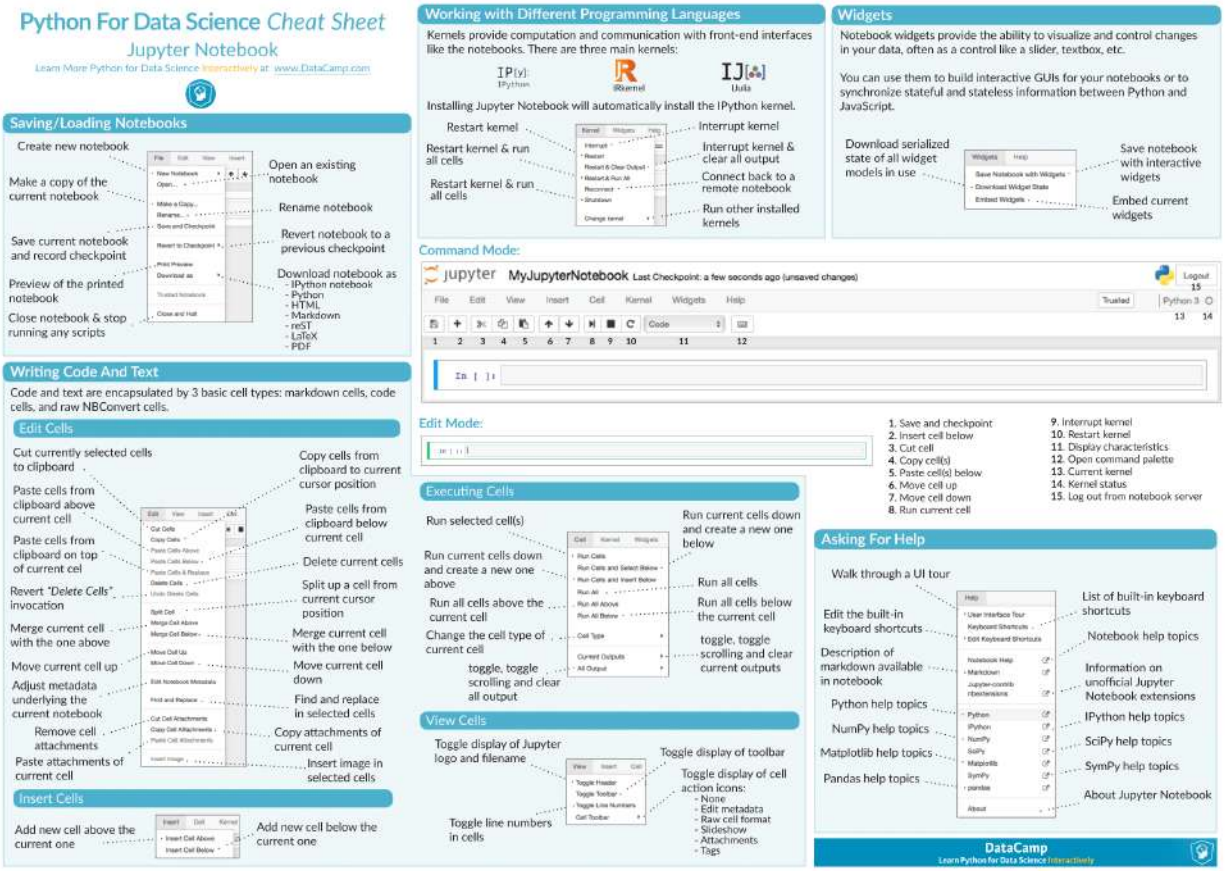
- Pandas (PDF)



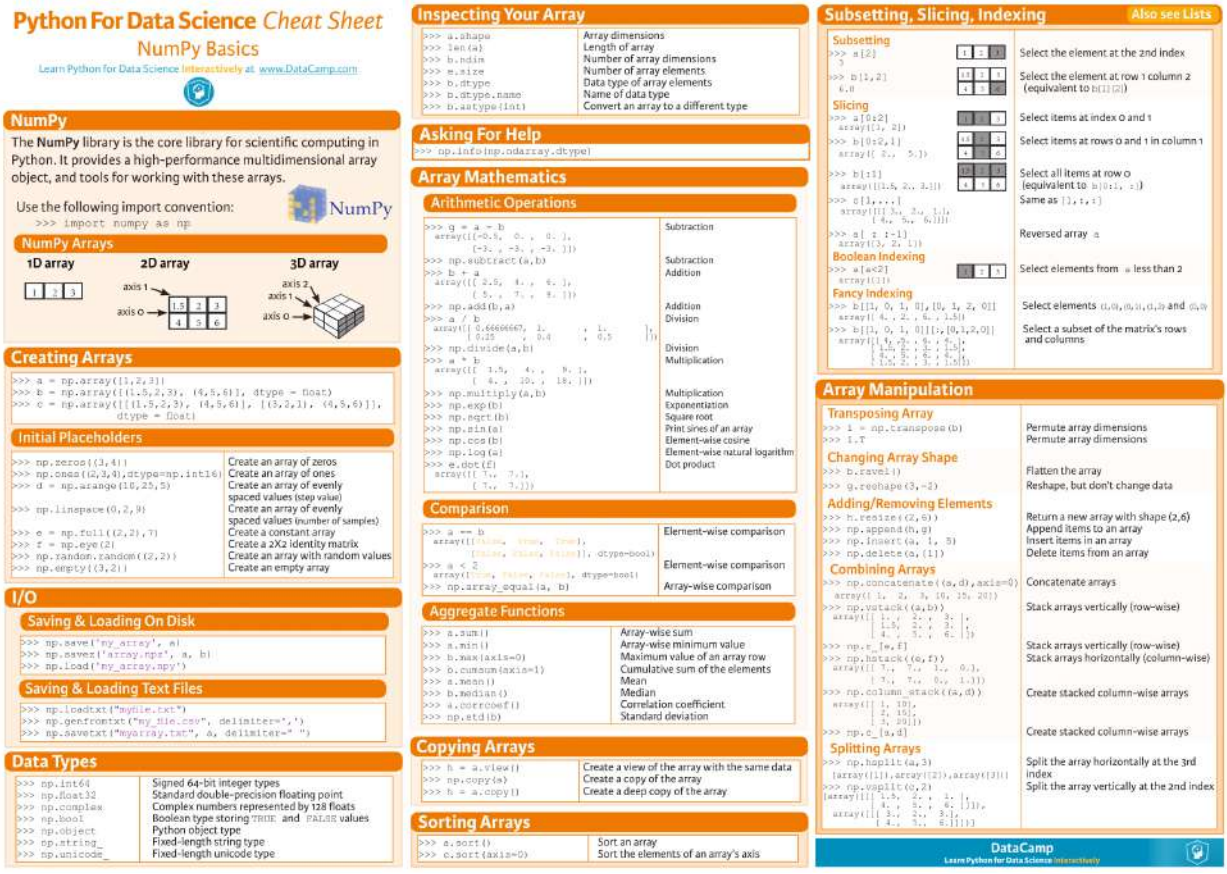
- Importing Data (PDF)



- Jupyter (PDF)

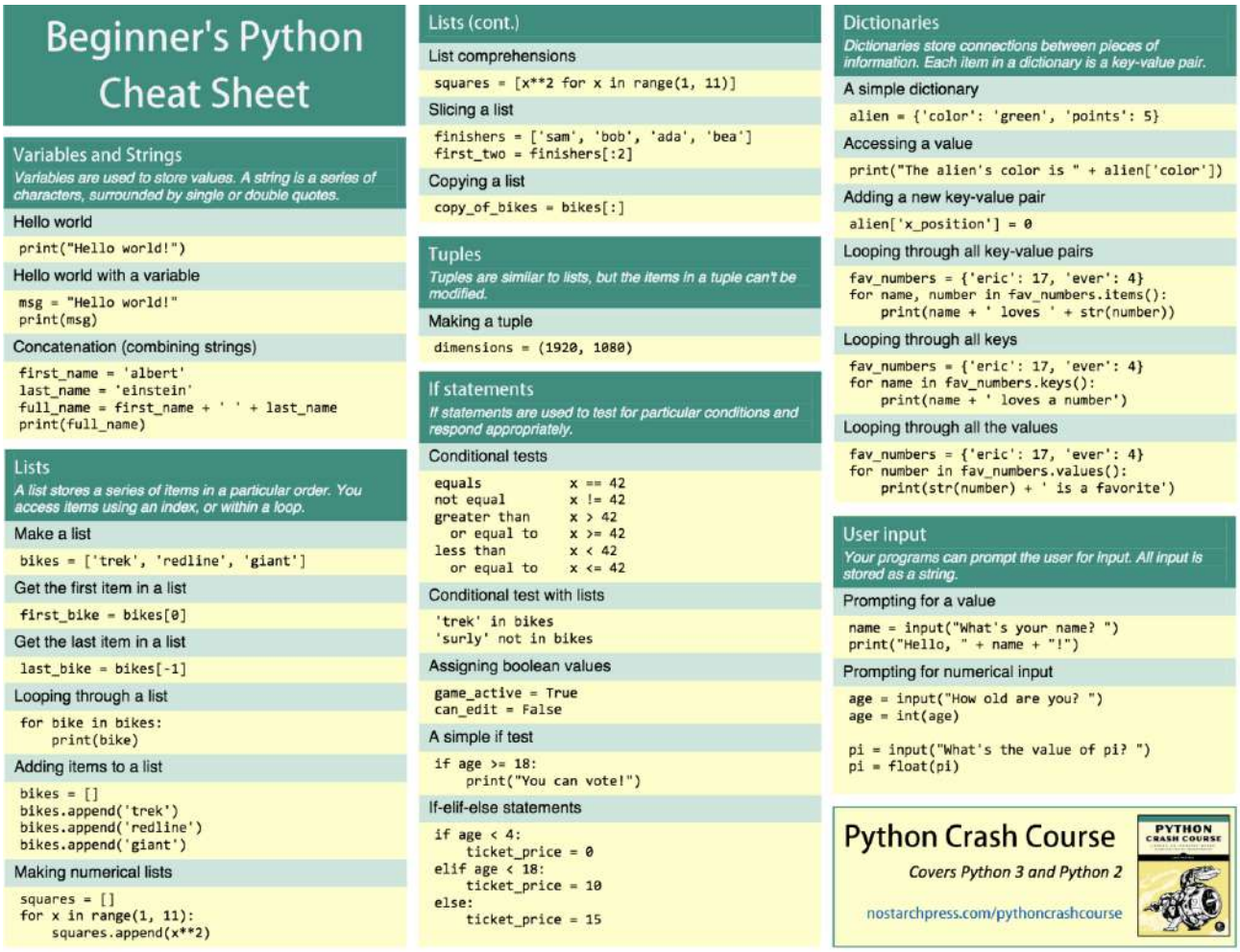


- Numpy Basics (PDF)



Python Crash Course

- Beginners Python Cheat Sheet (Long PDF)



Beginner's Python Cheat Sheet

Variables and Strings

Variables are used to store values. A string is a series of characters, surrounded by single or double quotes.

Hello world

```
print("Hello world!")
```

Hello world with a variable

```
msg = "Hello world!"
print(msg)
```

Concatenation (combining strings)

```
first_name = 'albert'
last_name = 'einstein'
full_name = first_name + ' ' + last_name
print(full_name)
```

Lists

A list stores a series of items in a particular order. You access items using an index, or within a loop.

Make a list

```
bikes = ['trek', 'redline', 'giant']
```

Get the first item in a list

```
first_bike = bikes[0]
```

Get the last item in a list

```
last_bike = bikes[-1]
```

Looping through a list

```
for bike in bikes:
    print(bike)
```

Adding items to a list

```
bikes = []
bikes.append('trek')
bikes.append('redline')
bikes.append('giant')
```

Making numerical lists

```
squares = []
for x in range(1, 11):
    squares.append(x**2)
```

Lists (cont.)

List comprehensions

```
squares = [x**2 for x in range(1, 11)]
```

Slicing a list

```
finishers = ['sam', 'bob', 'ada', 'bea']
first_two = finishers[:2]
```

Copying a list

```
copy_of_bikes = bikes[:]
```

Tuples

Tuples are similar to lists, but the items in a tuple can't be modified.

Making a tuple

```
dimensions = (1920, 1080)
```

If statements

If statements are used to test for particular conditions and respond appropriately.

Conditional tests

```
equals            x == 42
not equal         x != 42
greater than      x > 42
  or equal to     x >= 42
less than         x < 42
  or equal to     x <= 42
```

Conditional test with lists

```
'trek' in bikes
'surly' not in bikes
```

Assigning boolean values

```
game_active = True
can_edit = False
```

A simple if test

```
if age >= 18:
    print("You can vote!")
```

If-elif-else statements

```
if age < 4:
    ticket_price = 0
elif age < 18:
    ticket_price = 10
else:
    ticket_price = 15
```

Dictionaries

Dictionaries store connections between pieces of information. Each item in a dictionary is a key-value pair.

A simple dictionary

```
alien = {'color': 'green', 'points': 5}
```

Accessing a value

```
print("The alien's color is " + alien['color'])
```

Adding a new key-value pair

```
alien['x_position'] = 0
```

Looping through all key-value pairs

```
fav_numbers = {'eric': 17, 'ever': 4}
for name, number in fav_numbers.items():
    print(name + ' loves ' + str(number))
```

Looping through all keys

```
fav_numbers = {'eric': 17, 'ever': 4}
for name in fav_numbers.keys():
    print(name + ' loves a number')
```

Looping through all the values

```
fav_numbers = {'eric': 17, 'ever': 4}
for number in fav_numbers.values():
    print(str(number) + ' is a favorite')
```

User input

Your programs can prompt the user for input. All input is stored as a string.

Prompting for a value

```
name = input("What's your name? ")
print("Hello, " + name + "!")
```

Prompting for numerical input

```
age = input("How old are you? ")
age = int(age)

pi = input("What's the value of pi? ")
pi = float(pi)
```

Python Crash Course

Covers Python 3 and Python 2

nostarchpress.com/pythoncrashcourse

Dataquest

- Intermediate Python (PDF)

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Data Science Cheat Sheet

Python - Intermediate

KEY BASICS, PRINTING AND GETTING HELP

This cheat sheet assumes you are familiar with the content of our Python Basics Cheat Sheet

`s` - A Python string variable
`i` - A Python integer variable
`f` - A Python float variable
`l` - A Python list variable
`d` - A Python dictionary variable

LISTS

`l.pop(3)` - Returns the fourth item from `l` and deletes it from the list
`l.remove(x)` - Removes the first item in `l` that is equal to `x`
`l.reverse()` - Reverses the order of the items in `l`
`l[1::2]` - Returns every second item from `l`, commencing from the 1st item
`l[-5:]` - Returns the last 5 items from `l` specific axis

STRINGS

`s.lower()` - Returns a lowercase version of `s`
`s.title()` - Returns `s` with the first letter of every word capitalized
`"23".zfill(4)` - Returns `"0023"` by left-filling the string with `0`'s to make it's length `4`.
`s.splitlines()` - Returns a list by splitting the string on any newline characters.

Python strings share some common methods with lists

`s[:5]` - Returns the first 5 characters of `s`
`"fri" + "end"` - Returns `"friend"`
`"end" in s` - Returns `True` if the substring `"end"` is found in `s`

RANGE

Range objects are useful for creating sequences of integers for looping.

`range(5)` - Returns a sequence from `0` to `4`
`range(2000,2018)` - Returns a sequence from `2000` to `2017`
`range(0,11,2)` - Returns a sequence from `0` to `10`, with each item incrementing by `2`
`range(0,-10,-1)` - Returns a sequence from `0` to `-9`
`list(range(5))` - Returns a list from `0` to `4`

DICTIONARIES

`max(d, key=d.get)` - Return the key that corresponds to the largest value in `d`
`min(d, key=d.get)` - Return the key that corresponds to the smallest value in `d`

SETS

`my_set = set(l)` - Return a `set` object containing the unique values from `l`
`len(my_set)` - Returns the number of objects in `my_set` (or, the number of unique values from `l`)
`a in my_set` - Returns `True` if the value `a` exists in `my_set`

REGULAR EXPRESSIONS

`import re` - Import the Regular Expressions module
`re.search("abc",s)` - Returns a `match` object if the regex `"abc"` is found in `s`, otherwise `None`
`re.sub("abc","xyz",s)` - Returns a string where all instances matching regex `"abc"` are replaced by `"xyz"`

LIST COMPREHENSION

A one-line expression of a for loop

`[i ** 2 for i in range(10)]` - Returns a list of the squares of values from `0` to `9`
`[s.lower() for s in l_strings]` - Returns the list `l_strings`, with each item having had the `.lower()` method applied
`[i for i in l_floats if i < 0.5]` - Returns the items from `l_floats` that are less than `0.5`

FUNCTIONS FOR LOOPING

```
for i, value in enumerate(l):
  print("The value of item {} is {}".
  format(i,value))
```

- Iterate over the list `l`, printing the index location of each item and its value

```
for one, two in zip(l_one,l_two):
  print("one: {}, two: {}".format(one,two))
```

- Iterate over two lists, `l_one` and `l_two` and print each value

```
while x < 10:
  x += 1
```

- Run the code in the body of the loop until the value of `x` is no longer less than `10`

DATETIME

`import datetime as dt` - Import the `datetime` module
`now = dt.datetime.now()` - Assign `datetime` object representing the current time to `now`
`wks4 = dt.datetime.timedelta(weeks=4)` - Assign a `timedelta` object representing a timespan of 4 weeks to `wks4`
`now - wks4` - Return a `datetime` object representing the time 4 weeks prior to `now`
`newyear_2020 = dt.datetime(year=2020, month=12, day=31)` - Assign a `datetime` object representing December 25, 2020 to `newyear_2020`
`newyear_2020.strftime("%A, %b %d, %Y")` - Returns `"Thursday, Dec 31, 2020"`
`dt.datetime.strptime('Dec 31, 2020',"%b %d, %Y")` - Return a `datetime` object representing December 31, 2020

RANDOM

`import random` - Import the `random` module
`random.random()` - Returns a random float between `0.0` and `1.0`
`random.randint(0,10)` - Returns a random integer between `0` and `10`
`random.choice(l)` - Returns a random item from the list `l`

COUNTER

`from collections import Counter` - Import the `Counter` class
`c = Counter(l)` - Assign a `Counter` (dict-like) object with the counts of each unique item from `l`, to `c`
`c.most_common(3)` - Return the 3 most common items from `l`

TRY/EXCEPT

Catch and deal with Errors

`l_ints = [1, 2, 3, "", 5]` - Assign a list of integers with one missing value to `l_ints`

```
l_floats = []
for i in l_ints:
  try:
    l_floats.append(float(i))
  except:
    l_floats.append(i)
```

- Convert each value of `l_ints` to a float, catching and handling `ValueError: could not convert string to float:` where values are missing.

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- Python REGEX (PDF)

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Data Science Cheat Sheet

Python Regular Expressions

SPECIAL CHARACTERS

^ | Matches the expression to its right at the start of a string. It matches every such instance before each \n in the string.

$ | Matches the expression to its left at the end of a string. It matches every such instance before each \n in the string.

. | Matches any character except line terminators like \n.

\ | Escapes special characters or denotes character classes.

A|B | Matches expression A or B. If A is matched first, B is left untried.

+ | Greedily matches the expression to its left 1 or more times.

* | Greedily matches the expression to its left 0 or more times.

? | Greedily matches the expression to its left 0 or 1 times. But if ? is added to qualifiers (+, *, and ? itself) it will perform matches in a non-greedy manner.

{m} | Matches the expression to its left m times, and not less.

{m,n} | Matches the expression to its left m to n times, and not less.

{m,n}? | Matches the expression to its left m times, and ignores n. See ? above.

CHARACTER CLASSES [A.K.A. SPECIAL SEQUENCES]

\w | Matches alphanumeric characters, which means a-z, A-Z, and 0-9. It also matches the underscore, _.

\d | Matches digits, which means 0-9.

\D | Matches any non-digits.

\s | Matches whitespace characters, which include the \t, \n, \r, and space characters.

\S | Matches non-whitespace characters.

\b | Matches the boundary (or empty string) at the start and end of a word, that is, between \w and \W.

\B | Matches where \b does not, that is, the boundary of \w characters.

\A | Matches the expression to its right at the absolute start of a string whether in single or multi-line mode.

\Z | Matches the expression to its left at the absolute end of a string whether in single or multi-line mode.

SETS

[ ] | Contains a set of characters to match.

[amk] | Matches either a, m, or k. It does not match amk.

[a-z] | Matches any alphabet from a to z.

[a\-z] | Matches a, -, or z. It matches - because \ escapes it.

[a-] | Matches a or -, because - is not being used to indicate a series of characters.

[-a] | As above, matches a or -.

[a-z0-9] | Matches characters from a to z and also from 0 to 9.

[(+*)] | Special characters become literal inside a set, so this matches (, +, *, and).

[^ab5] | Adding ^ excludes any character in the set. Here, it matches characters that are not a, b, or 5.

GROUPS

( ) | Matches the expression inside the parentheses and groups it.

(?) | Inside parentheses like this, ? acts as an extension notation. Its meaning depends on the character immediately to its right.

(?PAB) | Matches the expression AB, and it can be accessed with the group name.

(?aiLmsux) | Here, a, i, L, m, s, u, and x are flags:
a — Matches ASCII only
i — Ignore case
L — Locale dependent
m — Multi-line
s — Matches all
u — Matches unicode
x — Verbose

(?:A) | Matches the expression as represented by A, but unlike (?PAB), it cannot be retrieved afterwards.

(?#...) | A comment. Contents are for us to read, not for matching.

A(?=B) | Lookahead assertion. This matches the expression A only if it is followed by B.

A(?!B) | Negative lookahead assertion. This matches the expression A only if it is not followed by B.

(?<=B)A | Positive lookbehind assertion. This matches the expression A only if B is immediately to its left. This can only matched fixed length expressions.

(?<!B)A | Negative lookbehind assertion. This matches the expression A only if B is not immediately to its left. This can only matched fixed length expressions.

(?P=name) | Matches the expression matched by an earlier group named "name".

(...)\1 | The number 1 corresponds to the first group to be matched. If we want to match more instances of the same expression, simply use its number instead of writing out the whole expression again. We can use from 1 up to 99 such groups and their corresponding numbers.

POPULAR PYTHON RE MODULE FUNCTIONS

re.findall(A, B) | Matches all instances of an expression A in a string B and returns them in a list.

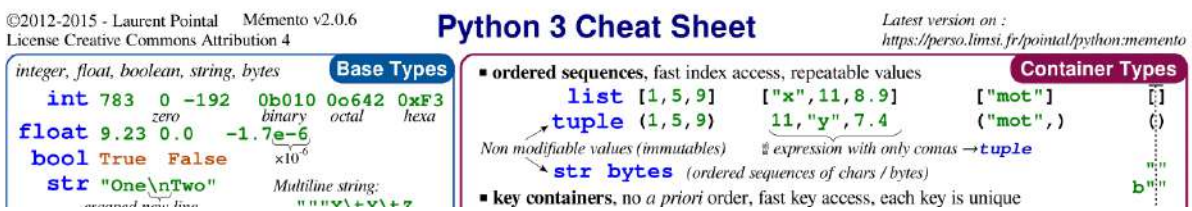
re.search(A, B) | Matches the first instance of an expression A in a string B, and returns it as a re match object.

re.split(A, B) | Split a string B into a list using the delimiter A.

re.sub(A, B, C) | Replace A with B in the string C.

Others

- Python 3 Memento (PDF)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Python 3 Cheat Sheet
Latest version on :
https://perso.limsi.fr/pointal/python:memento
Base Types
integer, float, boolean, string, bytes
int 783 0 -192 0b010 0o642 0xF3
zero binary octal hexa
float 9.23 0.0 -1.7e-6
bool True False
str "One\nTwo"
Container Types
ordered sequences, fast index access, repeatable values
list [1,5,9] ["x",11,8.9] ["mot"] []
tuple (1,5,9) 11,"y",7.4 ("mot",) ()
Non modifiable values (immutables)
expression with only comas → tuple
str bytes (ordered sequences of chars / bytes)
b""
key containers, no a priori order, fast key access, each key is unique

escaped new line
'I\'m'
escaped '
bytes b"toto\xfe\775"
hexadecimal octal
"""X\t1\t2
1\t2\t3"""
escaped tab
immutables

dictionary dict {"key":"value"} dict(a=3,b=4,k="v") {}
(key/value associations) {1:"one",3:"three",2:"two",3.14:"π"}
collection set {"key1","key2"} {1,9,3,0} set()
keys=hashable values (base types, immutables…) frozenset immutable set empty

Identifiers

for variables, functions, modules, classes… names

a…zA…Z_ followed by a…zA…Z_0…9
- diacritics allowed but should be avoided
- language keywords forbidden
- lower/UPPER case discrimination

☺ a toto x7 y_max BigOne
☹ 8y and for

Variables assignment

= assignment ⇔ binding of a name with a value
1) evaluation of right side expression value
2) assignment in order with left side names

```
x=1.2+8+sin(y)
a=b=c=0        assignment to same value
y,z,r=9.2,-7.6,0   multiple assignments
a,b=b,a        values swap
a,*b=seq       unpacking of sequence in
*a,b=seq       item and list
x+=3           increment ⇔ x=x+3      and *= /= %= …
x-=2           decrement ⇔ x=x-2
x=None         « undefined » constant value
del x          remove name x
```

Conversions

type(expression)

```
int("15") → 15
int("3f",16) → 63          can specify integer number base in 2nd parameter
int(15.56) → 15            truncate decimal part
float("-11.24e8") → -1124000000.0
round(15.56,1)→15.6        rounding to 1 decimal (0 decimal → integer number)
bool(x) False for null x, empty container x , None or False x ; True for other x
str(x)→ "…"   representation string of x for display (cf. formatting on the back)
chr(64)→'@'  ord('@')→64   code ↔ char
repr(x)→ "…"  literal representation string of x
bytes([72,9,64]) → b'H\t@'
list("abc") → ['a','b','c']
dict([(3,"three"),(1,"one")]) → {1:'one',3:'three'}
set(["one","two"]) → {'one','two'}
separator str and sequence of str → assembled str
   ':'.join(['toto','12','pswd']) → 'toto:12:pswd'
str splitted on whitespaces → list of str
   "words with  spaces".split() → ['words','with','spaces']
str splitted on separator str → list of str
   "1,4,8,2".split(",") → ['1','4','8','2']
sequence of one type → list of another type (via list comprehension)
   [int(x) for x in ('1','29','-3')] → [1,29,-3]
```

Sequence Containers Indexing

for lists, tuples, strings, bytes…

```
negative index  -5  -4  -3  -2  -1
positive index   0   1   2   3   4
lst=[10, 20, 30, 40, 50]
positive slice   0   1   2   3   4   5
negative slice  -5  -4  -3  -2  -1
```

Items count: `len(lst)→5`

index from 0 (here from 0 to 4)

Individual access to items via `lst[index]`:
`lst[0]→10` ⇒ first one; `lst[1]→20`
`lst[-1]→50` ⇒ last one; `lst[-2]→40`

On mutable sequences (`list`), remove with `del lst[3]` and modify with assignment `lst[4]=25`

Access to sub-sequences via `lst[start slice:end slice:step]`

```
lst[:-1]→[10,20,30,40]   lst[::-1]→[50,40,30,20,10]   lst[1:3]→[20,30]   lst[:3]→[10,20,30]
lst[1:-1]→[20,30,40]     lst[::-2]→[50,30,10]         lst[-3:-1]→[30,40] lst[3:]→[40,50]
lst[::2]→[10,30,50]      lst[:]→[10,20,30,40,50]  shallow copy of sequence
```

Missing slice indication → from start / up to end.
On mutable sequences (`list`), remove with `del lst[3:5]` and modify with assignment `lst[1:4]=[15,25]`

Boolean Logic

Comparisons: < > <= >= == != (boolean results) ≤ ≥ = ≠

- a and b — logical and — both simultaneously
- a or b — logical or — one or other or both

pitfall: and and or return value of a or of b (under shortcut evaluation).
⇒ ensure that a and b are booleans.

- not a — logical not
- True, False — True and False constants

Statements Blocks

```
parent statement:
    statement block 1…
parent statement:
    statement block2…
next statement after block 1
```

indentation !

configure editor to insert 4 spaces in place of an indentation tab.

Modules/Names Imports

module `truc` ⇔ file `truc.py`

```
from monmod import nom1,nom2 as fct
```
→ direct access to names, renaming with as

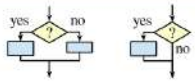
```
import monmod
```
→ access via `monmod.nom1` …

modules and packages searched in python path (cf `sys.path`)

Conditional Statement

statement block executed only if a condition is true

```
if logical condition:
    statements block
```



Can go with several elif, elif… and only one final else. Only the block of first true condition is executed.

```
if age<=18:
    state="Kid"
elif age>65:
    state="Retired"
else:
    state="Active"
```

with a var x:
`if bool(x)==True:` ⇔ `if x:`
`if bool(x)==False:` ⇔ `if not x:`

Maths

floating numbers… approximated values — angles in radians

Operators: + - * / // % **
Priority (…) × ÷ ↑ ↑ a^b
integer ÷ ÷ remainder
@ → matrix × python3.5+numpy

```
(1+5.3)*2→12.6
abs(-3.2)→3.2
round(3.57,1)→3.6
pow(4,3)→64.0
```

usual order of operations

```
from math import sin,pi…
sin(pi/4)→0.707…
cos(2*pi/3)→-0.4999…
sqrt(81)→9.0 ✓
log(e**2)→2.0
ceil(12.5)→13
floor(12.5)→12
```

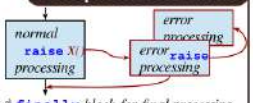
modules math, statistics, random, decimal, fractions, numpy, etc. (cf. doc)

Exceptions on Errors

Signaling an error:
`raise ExcClass(…)`

Errors processing:

```
try:
    normal procesising block
except Exception as e:
    error processing block
```



finally block for final processing in all cases.

Conditional Loop Statement

statements block executed as long as condition is true

```
while logical condition:
    statements block
```


```
s = 0
i = 1
```
initializations before the loop condition with a least one variable value (here i)

```
while i <= 100:
    s = s + i**2
    i = i + 1
print("sum:",s)
```

make condition variable change !

Algo: $s = \sum_{i=1}^{i=100} i^2$

Loop Control

- break — immediate exit
- continue — next iteration

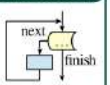
else block for normal loop exit.

beware of infinite loops!

Iterative Loop Statement

statements block executed for each item of a container or iterator

```
for var in sequence:
    statements block
```



Go over sequence's values

```
s = "Some text"
cnt = 0
for c in s:
    if c == "e":
        cnt = cnt + 1
print("found",cnt,"'e'")
```

initializations before the loop
loop variable, assignment managed by for statement

Algo: count number of e in the string.

loop on dict/set ⇔ loop on keys sequences
use slices to loop on a subset of a sequence

Go over sequence's index
- modify item at index
- access items around index (before / after)

```
lst = [11,18,9,12,23,4,17]
lost = []
```

Algo: limit values greater

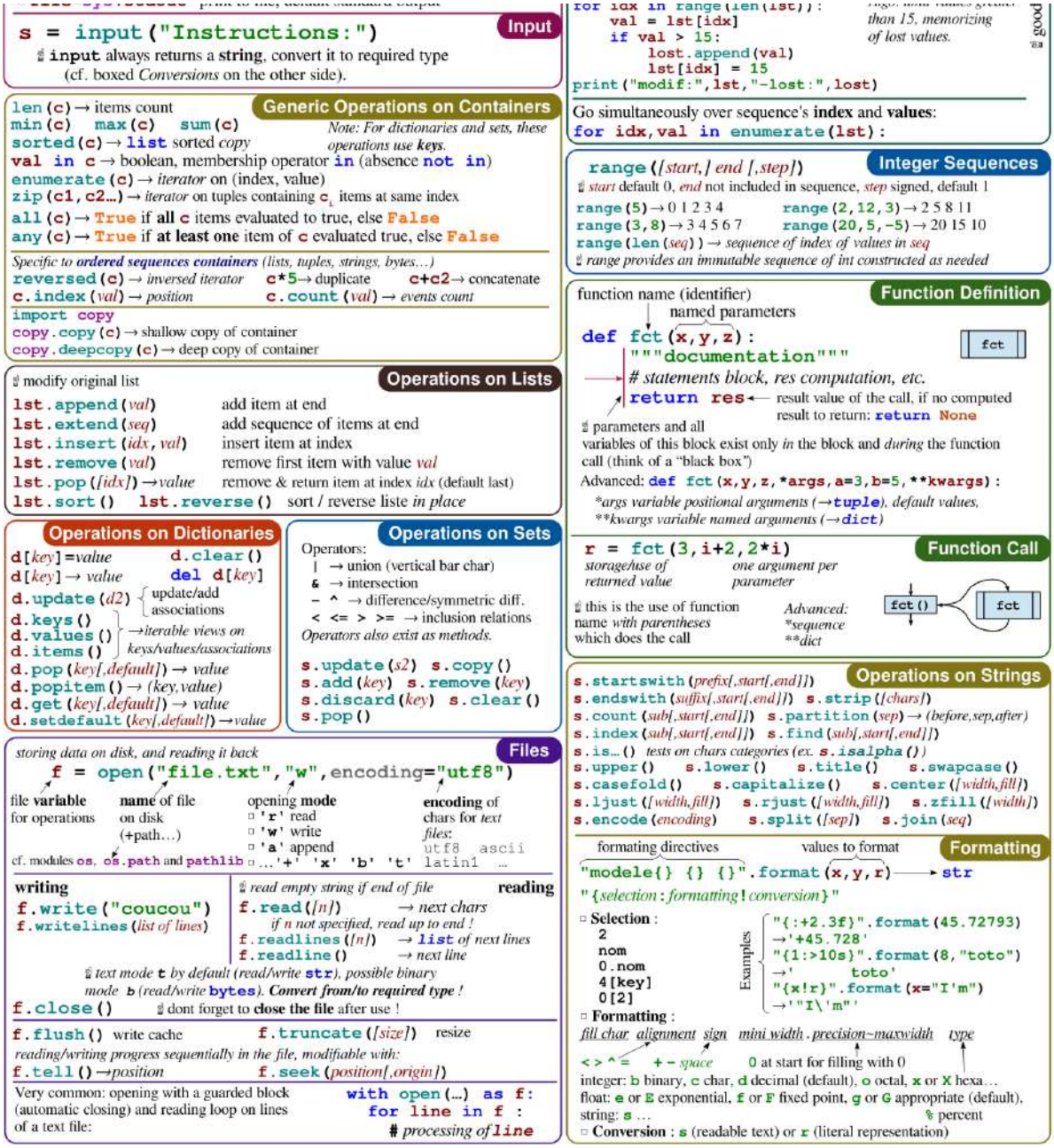
habit : don't modify loop variable

Display

```
print("v=",3,"cm :",x,",",y+4)
```

items to display : literal values, variables, expressions

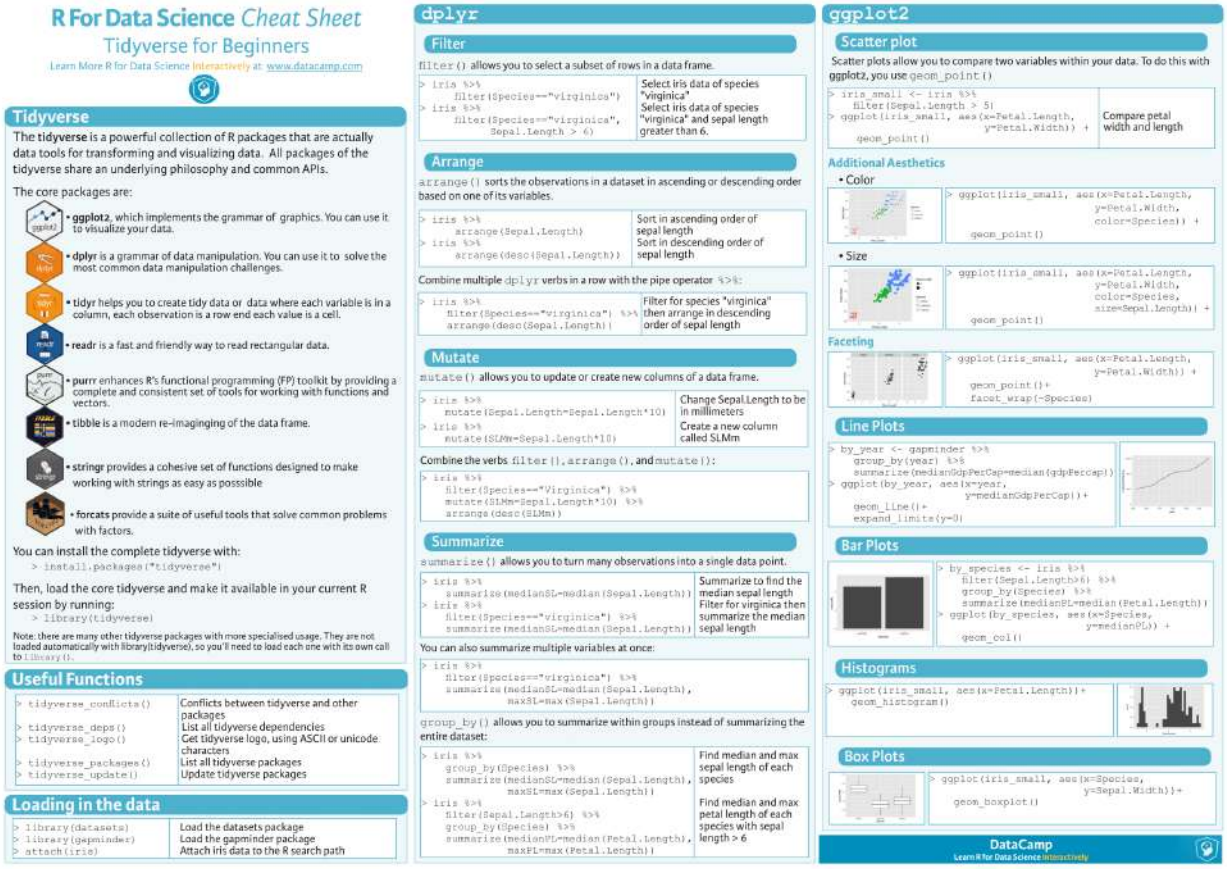
print options:
- sep=" " — items separator, default space
- end="\n" — end of print, default new line
- file=sys.stdout — print to file, default standard output


Input
s = input("Instructions:")
input always returns a string, convert it to required type
(cf. boxed Conversions on the other side).
Generic Operations on Containers
len(c) → items count
min(c) max(c) sum(c)
sorted(c) → list sorted copy
val in c → boolean, membership operator in (absence not in)
enumerate(c) → iterator on (index, value)
zip(c1,c2…) → iterator on tuples containing ci items at same index
all(c) → True if all c items evaluated to true, else False
any(c) → True if at least one item of c evaluated true, else False
Note: For dictionaries and sets, these operations use keys.
Specific to ordered sequences containers (lists, tuples, strings, bytes…)
reversed(c) → inversed iterator
c*5 → duplicate
c+c2 → concatenate
c.index(val) → position
c.count(val) → events count
import copy
copy.copy(c) → shallow copy of container
copy.deepcopy(c) → deep copy of container
Operations on Lists
modify original list
lst.append(val) add item at end
lst.extend(seq) add sequence of items at end
lst.insert(idx,val) insert item at index
lst.remove(val) remove first item with value val
lst.pop([idx]) → value remove & return item at index idx (default last)
lst.sort() lst.reverse() sort / reverse liste in place
Operations on Dictionaries
d[key]=value
d.clear()
d[key] → value
del d[key]
d.update(d2) update/add associations
d.keys()
d.values()
d.items()
→ iterable views on keys/values/associations
d.pop(key[,default]) → value
d.popitem() → (key,value)
d.get(key[,default]) → value
d.setdefault(key[,default]) → value
Operations on Sets
Operators:
| → union (vertical bar char)
& → intersection
- ^ → difference/symmetric diff.
< <= > >= → inclusion relations
Operators also exist as methods.
s.update(s2) s.copy()
s.add(key) s.remove(key)
s.discard(key) s.clear()
s.pop()
Files
storing data on disk, and reading it back
f = open("file.txt","w",encoding="utf8")
file variable for operations
name of file on disk (+path…)
opening mode
'r' read
'w' write
'a' append
…'+' 'x' 'b' 't'
encoding of chars for text files:
utf8 ascii
latin1 …
cf. modules os, os.path and pathlib
writing
f.write("coucou")
f.writelines(list of lines)
read empty string if end of file
reading
f.read([n]) → next chars
if n not specified, read up to end !
f.readlines([n]) → list of next lines
f.readline() → next line
text mode t by default (read/write str), possible binary mode b (read/write bytes). Convert from/to required type !
f.close()
dont forget to close the file after use !
f.flush() write cache
f.truncate([size]) resize
reading/writing progress sequentially in the file, modifiable with:
f.tell() → position
f.seek(position[,origin])
Very common: opening with a guarded block (automatic closing) and reading loop on lines of a text file:
with open(…) as f:
for line in f :
# processing of line
for idx in range(len(lst)):
val = lst[idx]
if val > 15:
lost.append(val)
lst[idx] = 15
print("modif:",lst,"-lost:",lost)
than 15, memorizing of lost values.
Go simultaneously over sequence's index and values:
for idx,val in enumerate(lst):
Integer Sequences
range([start,] end [,step])
start default 0, end not included in sequence, step signed, default 1
range(5) → 0 1 2 3 4
range(3,8) → 3 4 5 6 7
range(len(seq)) → sequence of index of values in seq
range(2,12,3) → 2 5 8 11
range(20,5,-5) → 20 15 10
range provides an immutable sequence of int constructed as needed
Function Definition
function name (identifier)
named parameters
def fct(x,y,z):
"""documentation"""
statements block, res computation, etc.
return res
result value of the call, if no computed result to return: return None
fct
parameters and all variables of this block exist only in the block and during the function call (think of a "black box")
Advanced: def fct(x,y,z,*args,a=3,b=5,**kwargs):
*args variable positional arguments (→tuple), default values,
**kwargs variable named arguments (→dict)
Function Call
r = fct(3,i+2,2*i)
storage/use of returned value
one argument per parameter
this is the use of function name with parentheses which does the call
Advanced: *sequence **dict
fct()
fct
Operations on Strings
s.startswith(prefix[,start[,end]])
s.endswith(suffix[,start[,end]])
s.strip([chars])
s.count(sub[,start[,end]])
s.partition(sep) → (before,sep,after)
s.index(sub[,start[,end]])
s.find(sub[,start[,end]])
s.is…() tests on chars categories (ex. s.isalpha())
s.upper() s.lower() s.title() s.swapcase()
s.casefold() s.capitalize() s.center([width,fill])
s.ljust([width,fill]) s.rjust([width,fill]) s.zfill([width])
s.encode(encoding) s.split([sep]) s.join(seq)
Formatting
formating directives
values to format
"modele{} {} {}".format(x,y,r) → str
"{selection:formatting!conversion}"
Selection :
2
nom
0.nom
4[key]
0[2]
Examples
"{:+2.3f}".format(45.72793) → '+45.728'
"{1:>10s}".format(8,"toto") → ' toto'
"{x!r}".format(x="I'm") → '"I\'m"'
Formatting :
fill char alignment sign mini width.precision~maxwidth type
< > ^ = + - space
0 at start for filling with 0
integer: b binary, c char, d decimal (default), o octal, x or X hexa…
float: e or E exponential, f or F fixed point, g or G appropriate (default),
string: s …
% percent
Conversion : s (readable text) or r (literal representation)

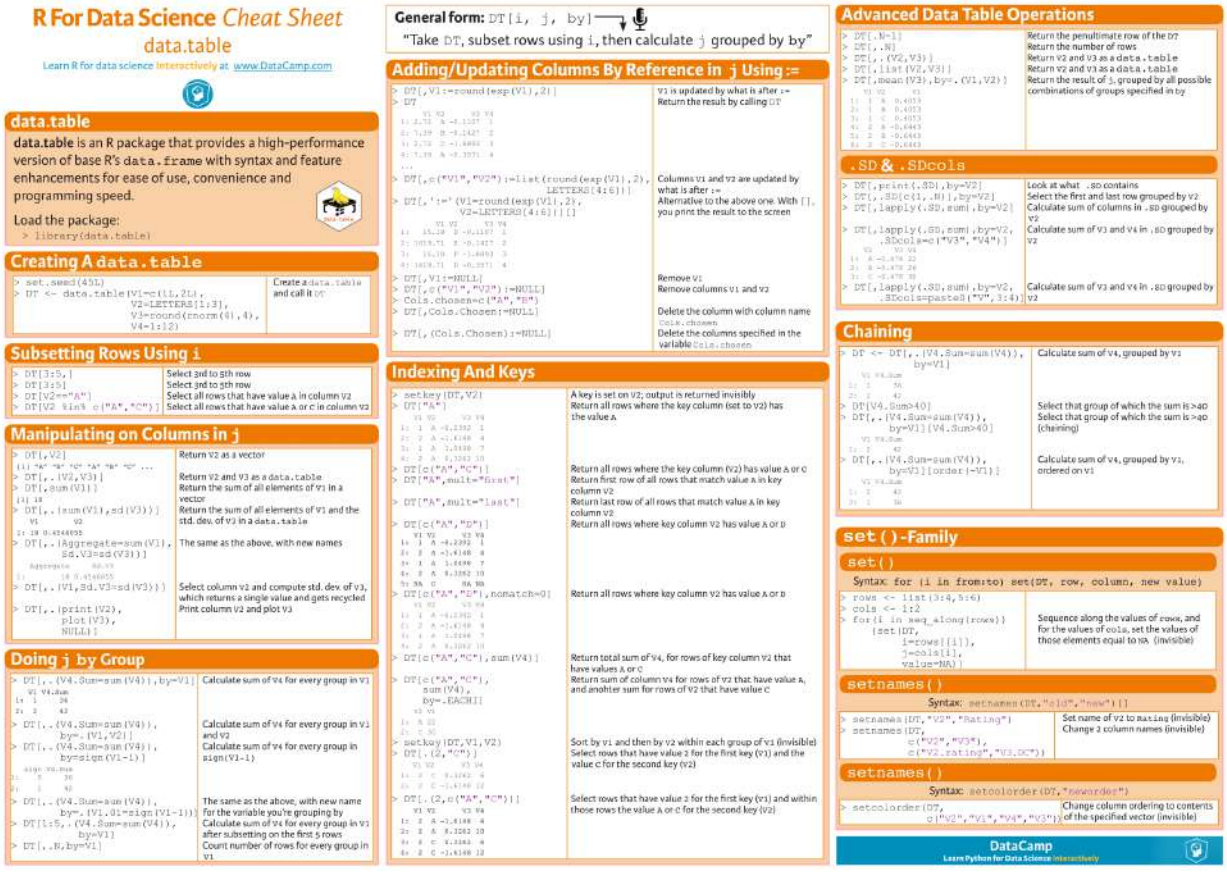
R

Datacamp

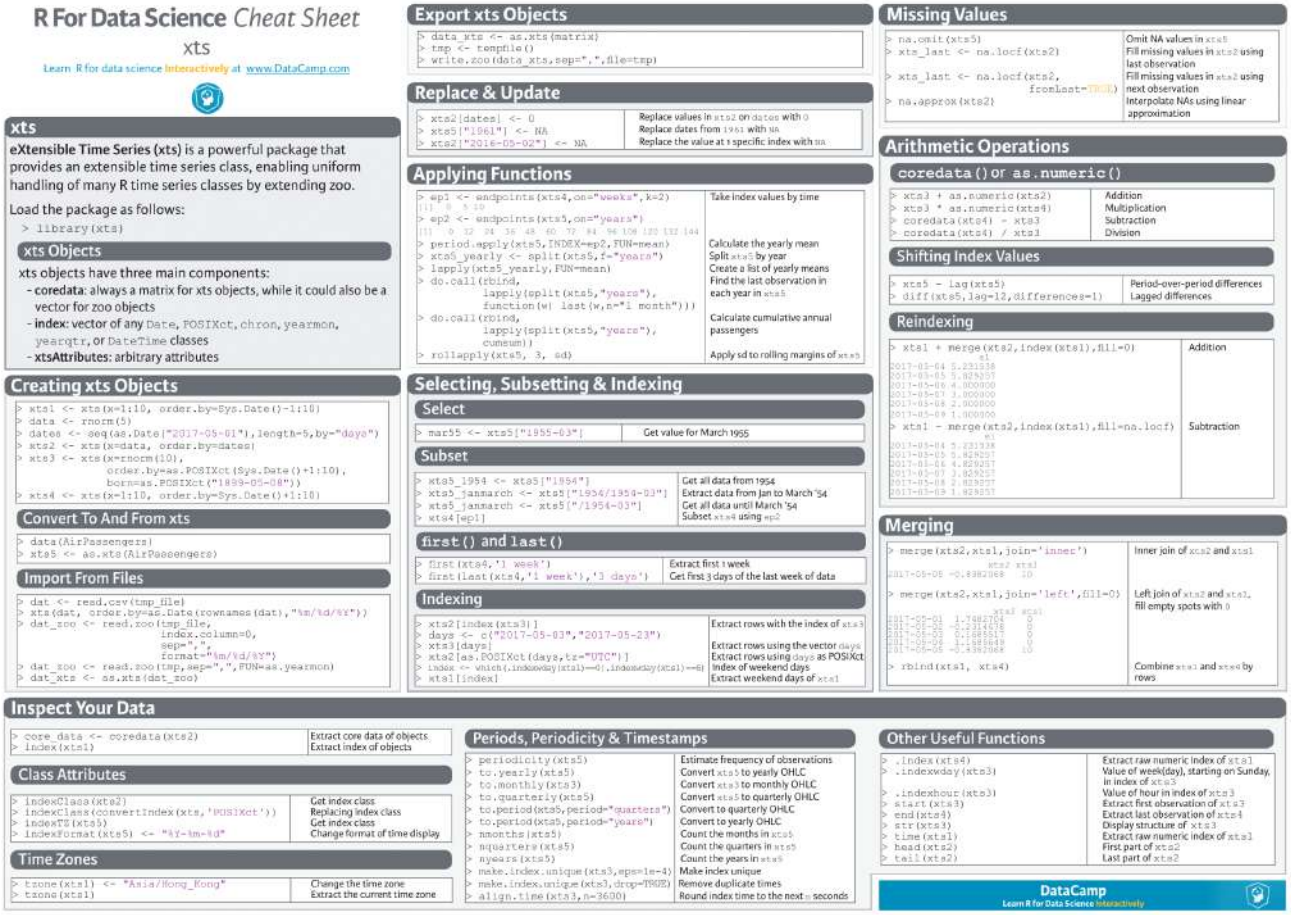
- Tidiverse (PDF)



- data.table (PDF)



-xts (PDF)



RStudio

- R Studio IDE (PDF)
- Base R (PDF)
- Data Import with readr (PDF)
- Data Transformation with Dplyr (PDF)
- Apply Functions with purrr (PDF)
- Data transformation with data.table (PDF)
- Dates and Times with lubridate (PDF)

- Randomizr (PDF)
- Regular Expressions (PDF)
- Work with Strings with stringr (PDF)
- Tidy Evaluation with rlang (PDF)
- Xplain (PDF)
- Sintax Comparison (PDF)
- Data and Variable Transformation with sjmisc (PDF)
- R Markdown (PDF)
- Package Development with devtools (PDF)

Math and Calculus

From @afshinea, @stat110 and @wzchen:

- Refresher Algebra and Calculus (PDF)

VIP Refresher: Linear Algebra and Calculus

Afshine Amidi and Shervine Amidi

October 6, 2018

General notations

Vector – We note $x \in \mathbb{R}^n$ a vector with n entries, where $x_i \in \mathbb{R}$ is the i^{th} entry:

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$$

Matrix – We note $A \in \mathbb{R}^{m \times n}$ a matrix with m rows and n columns, where $A_{i,j} \in \mathbb{R}$ is the entry located in the i^{th} row and j^{th} column:

$$A = \begin{pmatrix} A_{1,1} & \cdots & A_{1,n} \\ \vdots & & \vdots \\ A_{m,1} & \cdots & A_{m,n} \end{pmatrix} \in \mathbb{R}^{m \times n}$$

Remark: the vector x defined above can be viewed as a $n \times 1$ matrix and is more particularly called a column-vector.

Identity matrix – The identity matrix $I \in \mathbb{R}^{n \times n}$ is a square matrix with ones in its diagonal and zero everywhere else:

$$I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix}$$

Remark: for all matrices $A \in \mathbb{R}^{n \times n}$, we have $A \times I = I \times A = A$.

Diagonal matrix – A diagonal matrix $D \in \mathbb{R}^{n \times n}$ is a square matrix with nonzero values in its diagonal and zero everywhere else:

$$D = \begin{pmatrix} d_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & d_n \end{pmatrix}$$

Remark: we also note D as $diag(d_1, ..., d_n)$.

Matrix operations

Vector-vector multiplication – There are two types of vector-vector products:

- inner product: for $x, y \in \mathbb{R}^n$, we have:

$$\boxed{x^T y = \sum_{i=1}^{n} x_i y_i \in \mathbb{R}}$$

- outer product: for $x \in \mathbb{R}^m, y \in \mathbb{R}^n$, we have:

$$\boxed{xy^T = \begin{pmatrix} x_1 y_1 & \cdots & x_1 y_n \\ \vdots & & \vdots \\ x_m y_1 & \cdots & x_m y_n \end{pmatrix} \in \mathbb{R}^{m \times n}}$$

Matrix-vector multiplication – The product of matrix $A \in \mathbb{R}^{m \times n}$ and vector $x \in \mathbb{R}^n$ is a vector of size $\mathbb{R}^m$, such that:

$$\boxed{Ax = \begin{pmatrix} a_{r,1}^T x \\ \vdots \\ a_{r,m}^T x \end{pmatrix} = \sum_{i=1}^{n} a_{c,i} x_i \in \mathbb{R}^m}$$

where $a_{r,i}^T$ are the vector rows and $a_{c,j}$ are the vector columns of A, and x_i are the entries of x.

Matrix-matrix multiplication – The product of matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$ is a matrix of size $\mathbb{R}^{n \times p}$, such that:

$$\boxed{AB = \begin{pmatrix} a_{r,1}^T b_{c,1} & \cdots & a_{r,1}^T b_{c,p} \\ \vdots & & \vdots \\ a_{r,m}^T b_{c,1} & \cdots & a_{r,m}^T b_{c,p} \end{pmatrix} = \sum_{i=1}^{n} a_{c,i} b_{r,i}^T \in \mathbb{R}^{n \times p}}$$

where $a_{r,i}^T, b_{r,i}^T$ are the vector rows and $a_{c,j}, b_{c,j}$ are the vector columns of A and B respectively.

Transpose – The transpose of a matrix $A \in \mathbb{R}^{m \times n}$, noted A^T, is such that its entries are flipped:

$$\boxed{\forall i,j, \quad A_{i,j}^T = A_{j,i}}$$

Remark: for matrices A, B, we have $(AB)^T = B^T A^T$.

Inverse – The inverse of an invertible square matrix A is noted A^{-1} and is the only matrix such that:

$$\boxed{AA^{-1} = A^{-1}A = I}$$

Remark: not all square matrices are invertible. Also, for matrices A, B, we have $(AB)^{-1} = B^{-1}A^{-1}$.

Trace – The trace of a square matrix A, noted $\mathrm{tr}(A)$, is the sum of its diagonal entries:

$$\boxed{\mathrm{tr}(A) = \sum_{i=1}^{n} A_{i,i}}$$

Remark: for matrices A, B, we have $tr(A^T) = tr(A)$ and $tr(AB) = tr(BA)$.

Determinant – The determinant of a square matrix $A \in \mathbb{R}^{n \times n}$, noted $|A|$ or $\det(A)$ is expressed recursively in terms of $A_{\setminus i, \setminus j}$, which is the matrix A without its i^{th} row and j^{th} column, as follows:

$$\boxed{\det(A) = |A| = \sum_{j=1}^{n} (-1)^{i+j} A_{i,j} |A_{\setminus i, \setminus j}|}$$

Remark: A is invertible if and only if $|A| \neq 0$. Also, $|AB| = |A||B|$ and $|A^T| = |A|$.

Matrix properties

Symmetric decomposition – A given matrix A can be expressed in terms of its symmetric and antisymmetric parts as follows:

$$A = \underbrace{\frac{A + A^T}{2}}_{\text{Symmetric}} + \underbrace{\frac{A - A^T}{2}}_{\text{Antisymmetric}}$$

Norm – A norm is a function $N : V \longrightarrow [0, +\infty[$ where V is a vector space, and such that for all $x, y \in V$, we have:

- $N(x + y) \leqslant N(x) + N(y)$
- $N(ax) = |a|N(x)$ for a scalar
- if $N(x) = 0$, then $x = 0$

For $x \in V$, the most commonly used norms are summed up in the table below:

Norm	Notation	Definition	Use case
Manhattan, L^1	$\|x\|_1$	$\sum_{i=1}^{n} \|x_i\|$	LASSO regularization
Euclidean, L^2	$\|x\|_2$	$\sqrt{\sum_{i=1}^{n} x_i^2}$	Ridge regularization
p-norm, L^p	$\|x\|_p$	$\left(\sum_{i=1}^{n} x_i^p\right)^{\frac{1}{p}}$	Hölder inequality
Infinity, L^∞	$\|x\|_\infty$	$\max_i \|x_i\|$	Uniform convergence

Linearly dependence – A set of vectors is said to be linearly dependent if one of the vectors in the set can be defined as a linear combination of the others.
Remark: if no vector can be written this way, then the vectors are said to be linearly independent.

Matrix rank – The rank of a given matrix A is noted $\mathrm{rank}(A)$ and is the dimension of the vector space generated by its columns. This is equivalent to the maximum number of linearly independent columns of A.

Positive semi-definite matrix – A matrix $A \in \mathbb{R}^{n \times n}$ is positive semi-definite (PSD) and is noted $A \succeq 0$ if we have:

$$\boxed{A = A^T} \quad \text{and} \quad \boxed{\forall x \in \mathbb{R}^n, \quad x^T A x \geqslant 0}$$

Remark: similarly, a matrix A is said to be positive definite, and is noted $A \succ 0$, if it is a PSD matrix which satisfies for all non-zero vector x, $x^T A x > 0$.

Eigenvalue, eigenvector – Given a matrix $A \in \mathbb{R}^{n \times n}$, λ is said to be an eigenvalue of A if there exists a vector $z \in \mathbb{R}^n \setminus \{0\}$, called eigenvector, such that we have:

$$\boxed{Az = \lambda z}$$

Spectral theorem – Let $A \in \mathbb{R}^{n \times n}$. If A is symmetric, then A is diagonalizable by a real orthogonal matrix $U \in \mathbb{R}^{n \times n}$. By noting $\Lambda = \mathrm{diag}(\lambda_1, ..., \lambda_n)$, we have:

$$\boxed{\exists \Lambda \text{ diagonal}, \quad A = U \Lambda U^T}$$

Singular-value decomposition – For a given matrix A of dimensions $m \times n$, the singular-value decomposition (SVD) is a factorization technique that guarantees the existence of U $m \times m$ unitary, Σ $m \times n$ diagonal and V $n \times n$ unitary matrices, such that:

$$\boxed{A = U \Sigma V^T}$$

Matrix calculus

Gradient – Let $f : \mathbb{R}^{m \times n} \to \mathbb{R}$ be a function and $A \in \mathbb{R}^{m \times n}$ be a matrix. The gradient of f with respect to A is a $m \times n$ matrix, noted $\nabla_A f(A)$, such that:

$$\boxed{\left(\nabla_A f(A)\right)_{i,j} = \frac{\partial f(A)}{\partial A_{i,j}}}$$

Remark: the gradient of f is only defined when f is a function that returns a scalar.

Hessian – Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function and $x \in \mathbb{R}^n$ be a vector. The hessian of f with respect to x is a $n \times n$ symmetric matrix, noted $\nabla_x^2 f(x)$, such that:

$$\boxed{\left(\nabla_x^2 f(x)\right)_{i,j} = \frac{\partial^2 f(x)}{\partial x_i \partial x_j}}$$

Remark: the hessian of f is only defined when f is a function that returns a scalar.

Gradient operations – For matrices A, B, C, the following gradient properties are worth having in mind:

$$\boxed{\nabla_A \mathrm{tr}(AB) = B^T} \quad \boxed{\nabla_A f(A) = (\nabla_A f(A))^T}$$

$$\boxed{\nabla_A \mathrm{tr}(ABA^T C) = CAB + C^T AB^T} \quad \boxed{\nabla_A |A| = |A|(A^{-1})^T}$$

- Refresher Probabilities and Statistics (PDF)

VIP Refresher: Probabilities and Statistics

Afshine Amidi and Shervine Amidi

August 6, 2018

Introduction to Probability and Combinatorics

❒ **Sample space** – The set of all possible outcomes of an experiment is known as the sample space of the experiment and is denoted by S.

❒ **Event** – Any subset E of the sample space is known as an event. That is, an event is a set consisting of possible outcomes of the experiment. If the outcome of the experiment is contained in E, then we say that E has occurred.

❒ **Axioms of probability** – For each event E, we denote $P(E)$ as the probability of event E occuring. By noting $E_1,...,E_n$ mutually exclusive events, we have the 3 following axioms:

$$(1) \quad \boxed{0 \leqslant P(E) \leqslant 1} \quad (2) \quad \boxed{P(S) = 1} \quad (3) \quad \boxed{P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i)}$$

❒ **Permutation** – A permutation is an arrangement of r objects from a pool of n objects, in a given order. The number of such arrangements is given by $P(n,r)$, defined as:

$$\boxed{P(n,r) = \frac{n!}{(n-r)!}}$$

❒ **Combination** – A combination is an arrangement of r objects from a pool of n objects, where the order does not matter. The number of such arrangements is given by $C(n,r)$, defined as:

$$\boxed{C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}}$$

Remark: we note that for $0 \leqslant r \leqslant n$, we have $P(n,r) \geqslant C(n,r)$.

Conditional Probability

❒ **Bayes' rule** – For events A and B such that $P(B) > 0$, we have:

$$\boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}}$$

Remark: we have $P(A \cap B) = P(A)P(B|A) = P(A|B)P(B)$.

❒ **Partition** – Let $\{A_i, i \in [\![1,n]\!]\}$ be such that for all i, $A_i \neq \varnothing$. We say that $\{A_i\}$ is a partition if we have:

$$\boxed{\forall i \neq j, A_i \cap A_j = \emptyset \quad \text{and} \quad \bigcup_{i=1}^{n} A_i = S}$$

Remark: for any event B in the sample space, we have $P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$.

❒ **Extended form of Bayes' rule** – Let $\{A_i, i \in [\![1,n]\!]\}$ be a partition of the sample space. We have:

$$\boxed{P(A_k|B) = \frac{P(B|A_k)P(A_k)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}}$$

❒ **Independence** – Two events A and B are independent if and only if we have:

$$\boxed{P(A \cap B) = P(A)P(B)}$$

Random Variables

❒ **Random variable** – A random variable, often noted X, is a function that maps every element in a sample space to a real line.

❒ **Cumulative distribution function (CDF)** – The cumulative distribution function F, which is monotonically non-decreasing and is such that $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to +\infty} F(x) = 1$, is defined as:

$$\boxed{F(x) = P(X \leqslant x)}$$

Remark: we have $P(a < X \leqslant B) = F(b) - F(a)$.

❒ **Probability density function (PDF)** – The probability density function f is the probability that X takes on values between two adjacent realizations of the random variable.

❒ **Relationships involving the PDF and CDF** – Here are the important properties to know in the discrete (D) and the continuous (C) cases.

Case	CDF F	PDF f	Properties of PDF
(D)	$F(x) = \sum_{x_i \leqslant x} P(X = x_i)$	$f(x_j) = P(X = x_j)$	$0 \leqslant f(x_j) \leqslant 1$ and $\sum_j f(x_j) = 1$
(C)	$F(x) = \int_{-\infty}^{x} f(y)dy$	$f(x) = \frac{dF}{dx}$	$f(x) \geqslant 0$ and $\int_{-\infty}^{+\infty} f(x)dx = 1$

❒ **Variance** – The variance of a random variable, often noted $\text{Var}(X)$ or σ^2, is a measure of the spread of its distribution function. It is determined as follows:

$$\boxed{\text{Var}(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2}$$

❒ **Standard deviation** – The standard deviation of a random variable, often noted σ, is a measure of the spread of its distribution function which is compatible with the units of the actual random variable. It is determined as follows:

$$\boxed{\sigma = \sqrt{\text{Var}(X)}}$$

❒ **Expectation and Moments of the Distribution** – Here are the expressions of the expected value $E[X]$, generalized expected value $E[g(X)]$, k^{th} moment $E[X^k]$ and characteristic function $\psi(\omega)$ for the discrete and continuous cases:

Case	$E[X]$	$E[g(X)]$	$E[X^k]$	$\psi(\omega)$
(D)	$\sum_{i=1}^{n} x_i f(x_i)$	$\sum_{i=1}^{n} g(x_i) f(x_i)$	$\sum_{i=1}^{n} x_i^k f(x_i)$	$\sum_{i=1}^{n} f(x_i) e^{i\omega x_i}$
(C)	$\int_{-\infty}^{+\infty} x f(x) dx$	$\int_{-\infty}^{+\infty} g(x) f(x) dx$	$\int_{-\infty}^{+\infty} x^k f(x) dx$	$\int_{-\infty}^{+\infty} f(x) e^{i\omega x} dx$

Remark: we have $e^{i\omega x} = \cos(\omega x) + i\sin(\omega x)$.

❒ **Revisiting the k^{th} moment** – The k^{th} moment can also be computed with the characteristic function as follows:

$$\boxed{E[X^k] = \frac{1}{i^k}\left[\frac{\partial^k \psi}{\partial \omega^k}\right]_{\omega=0}}$$

❒ **Transformation of random variables** – Let the variables X and Y be linked by some function. By noting f_X and f_Y the distribution function of X and Y respectively, we have:

$$\boxed{f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|}$$

❒ **Leibniz integral rule** – Let g be a function of x and potentially c, and a, b boundaries that may depend on c. We have:

$$\boxed{\frac{\partial}{\partial c}\left(\int_a^b g(x)dx\right) = \frac{\partial b}{\partial c} \cdot g(b) - \frac{\partial a}{\partial c} \cdot g(a) + \int_a^b \frac{\partial g}{\partial c}(x)dx}$$

❒ **Chebyshev's inequality** – Let X be a random variable with expected value μ and standard deviation σ. For $k, \sigma > 0$, we have the following inequality:

$$\boxed{P(|X-\mu| \geqslant k\sigma) \leqslant \frac{1}{k^2}}$$

Jointly Distributed Random Variables

❒ **Conditional density** – The conditional density of X with respect to Y, often noted $f_{X|Y}$, is defined as follows:

$$\boxed{f_{X|Y}(x) = \frac{f_{XY}(x,y)}{f_Y(y)}}$$

❒ **Independence** – Two random variables X and Y are said to be independent if we have:

$$\boxed{f_{XY}(x,y) = f_X(x) f_Y(y)}$$

❒ **Marginal density and cumulative distribution** – From the joint density probability function f_{XY}, we have:

Case	Marginal density	Cumulative function
(D)	$f_X(x_i) = \sum_j f_{XY}(x_i, y_j)$	$F_{XY}(x,y) = \sum_{x_i \leqslant x}\sum_{y_j \leqslant y} f_{XY}(x_i, y_j)$
(C)	$f_X(x) = \int_{-\infty}^{+\infty} f_{XY}(x,y)dy$	$F_{XY}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{XY}(x',y')dx'dy'$

❒ **Distribution of a sum of independent random variables** – Let $Y = X_1 + ... + X_n$ with $X_1, ..., X_n$ independent. We have:

$$\boxed{\psi_Y(\omega) = \prod_{k=1}^{n} \psi_{X_k}(\omega)}$$

❒ **Covariance** – We define the covariance of two random variables X and Y, that we note σ_{XY}^2 or more commonly $\text{Cov}(X,Y)$, as follows:

$$\boxed{\text{Cov}(X,Y) \underset{\text{def}}{=} \sigma_{XY}^2 = E[(X-\mu_X)(Y-\mu_Y)] = E[XY] - \mu_X\mu_Y}$$

❒ **Correlation** – By noting σ_X, σ_Y the standard deviations of X and Y, we define the correlation between the random variables X and Y, noted ρ_{XY}, as follows:

$$\boxed{\rho_{XY} = \frac{\sigma_{XY}^2}{\sigma_X \sigma_Y}}$$

Remarks: For any X, Y, we have $\rho_{XY} \in [-1,1]$. If X and Y are independent, then $\rho_{XY} = 0$.

❒ **Main distributions** – Here are the main distributions to have in mind:

Type	Distribution	PDF	$\psi(\omega)$	$E[X]$	$\text{Var}(X)$
(D)	$X \sim \mathcal{B}(n,p)$ Binomial	$P(X=x) = \binom{n}{x} p^x q^{n-x}$ $x \in [\![0,n]\!]$	$(pe^{i\omega} + q)^n$	np	npq
	$X \sim \text{Po}(\mu)$ Poisson	$P(X=x) = \frac{\mu^x}{x!}e^{-\mu}$ $x \in \mathbb{N}$	$e^{\mu(e^{i\omega}-1)}$	μ	μ
(C)	$X \sim \mathcal{U}(a,b)$ Uniform	$f(x) = \frac{1}{b-a}$ $x \in [a,b]$	$\frac{e^{i\omega b} - e^{i\omega a}}{(b-a)i\omega}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
	$X \sim \mathcal{N}(\mu,\sigma)$ Gaussian	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $x \in \mathbb{R}$	$e^{i\omega\mu - \frac{1}{2}\omega^2\sigma^2}$	μ	σ^2
	$X \sim \text{Exp}(\lambda)$ Exponential	$f(x) = \lambda e^{-\lambda x}$ $x \in \mathbb{R}_+$	$\frac{1}{1 - \frac{i\omega}{\lambda}}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

Parameter estimation

❒ **Random sample** – A random sample is a collection of n random variables $X_1, ..., X_n$ that are independent and identically distributed with X.

❒ **Estimator** – An estimator $\hat{\theta}$ is a function of the data that is used to infer the value of an unknown parameter θ in a statistical model.

❒ **Bias** – The bias of an estimator $\hat{\theta}$ is defined as being the difference between the expected value of the distribution of $\hat{\theta}$ and the true value, i.e.:

$$\boxed{\text{Bias}(\hat{\theta}) = E[\hat{\theta}] - \theta}$$

Remark: an estimator is said to be unbiased when we have $E[\hat{\theta}] = \theta$.

❒ **Sample mean and variance** – The sample mean and the sample variance of a random sample are used to estimate the true mean μ and the true variance σ^2 of a distribution, are noted $\overline{X}$ and s^2 respectively, and are such that:

$$\boxed{\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i} \quad \text{and} \quad \boxed{s^2 = \hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2}$$

❒ **Central Limit Theorem** – Let us have a random sample $X_1, ..., X_n$ following a given distribution with mean μ and variance σ^2, then we have:

$$\boxed{\overline{X} \underset{n \to +\infty}{\sim} \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right)}$$

- Fundamentals of Probabilities (PDF)

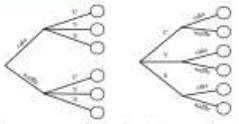
Probability Cheatsheet v2.0

Compiled by William Chen (http://wzchen.com) and Joe Blitzstein, with contributions from Sebastian Chiu, Yuan Jiang, Yuqi Hou, and Jessy Hwang. Material based on Joe Blitzstein's (@stat110) lectures (http://stat110.net) and Blitzstein/Hwang's Introduction to Probability textbook (http://bit.ly/introprobability). Licensed under CC BY-NC-SA 4.0. Please share comments, suggestions, and errors at http://github.com/wzchen/probability_cheatsheet.

Last Updated September 4, 2015

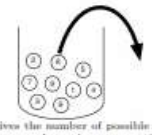
Counting

Multiplication Rule



Let's say we have a compound experiment (an experiment with multiple components). If the 1st component has n_1 possible outcomes, the 2nd component has n_2 possible outcomes, ..., and the rth component has n_r possible outcomes, then overall there are $n_1 n_2 \ldots n_r$ possibilities for the whole experiment.

Sampling Table



The sampling table gives the number of possible samples of size k out of a population of size n, under various assumptions about how the sample is collected.

	Order Matters	Not Matter
With Replacement	n^k	$\binom{n+k-1}{k}$
Without Replacement	$\frac{n!}{(n-k)!}$	$\binom{n}{k}$

Naive Definition of Probability

If all outcomes are equally likely, the probability of an event A happening is:

$$P_{\text{naive}}(A) = \frac{\text{number of outcomes favorable to } A}{\text{number of outcomes}}$$

Thinking Conditionally

Independence

Independent Events A and B are independent if knowing whether A occurred gives no information about whether B occurred. More formally, A and B (which have nonzero probability) are independent if and only if one of the following equivalent statements holds:

$$P(A \cap B) = P(A)P(B)$$
$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$

Conditional Independence A and B are conditionally independent given C if $P(A \cap B|C) = P(A|C)P(B|C)$. Conditional independence does not imply independence, and independence does not imply conditional independence.

Unions, Intersections, and Complements

De Morgan's Laws A useful identity that can make calculating probabilities of unions easier by relating them to intersections, and vice versa. Analogous results hold with more than two sets.

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

Joint, Marginal, and Conditional

Joint Probability $P(A \cap B)$ or $P(A, B)$ – Probability of A and B.

Marginal (Unconditional) Probability $P(A)$ – Probability of A.

Conditional Probability $P(A|B) = P(A, B)/P(B)$ – Probability of A, given that B occurred.

Conditional Probability is Probability $P(A|B)$ is a probability function for any fixed B. Any theorem that holds for probability also holds for conditional probability.

Probability of an Intersection or Union

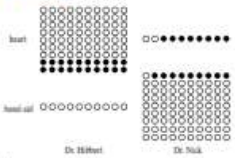
Intersections via Conditioning

$$P(A, B) = P(A)P(B|A)$$
$$P(A, B, C) = P(A)P(B|A)P(C|A, B)$$

Unions via Inclusion-Exclusion

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Simpson's Paradox



It is possible to have

$$P(A \mid B, C) < P(A \mid B^c, C) \text{ and } P(A \mid B, C^c) < P(A \mid B^c, C^c)$$

yet also $P(A \mid B) > P(A \mid B^c)$.

Law of Total Probability (LOTP)

Let $B_1, B_2, B_3, \ldots, B_n$ be a *partition* of the sample space (i.e., they are disjoint and their union is the entire sample space).

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \cdots + P(A|B_n)P(B_n)$$
$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_n)$$

For **LOTP with extra conditioning**, just add in another event C!

$$P(A|C) = P(A|B_1, C)P(B_1|C) + \cdots + P(A|B_n, C)P(B_n|C)$$
$$P(A|C) = P(A \cap B_1|C) + P(A \cap B_2|C) + \cdots + P(A \cap B_n|C)$$

Special case of LOTP with B and B^c as partition:

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$
$$P(A) = P(A \cap B) + P(A \cap B^c)$$

Bayes' Rule

Bayes' Rule, and with extra conditioning (just add in C!)

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$
$$P(A|B, C) = \frac{P(B|A, C)P(A|C)}{P(B|C)}$$

We can also write

$$P(A|B, C) = \frac{P(A, B, C)}{P(B, C)} = \frac{P(B, C|A)P(A)}{P(B, C)}$$

Odds Form of Bayes' Rule

$$\frac{P(A|B)}{P(A^c|B)} = \frac{P(B|A)}{P(B|A^c)} \frac{P(A)}{P(A^c)}$$

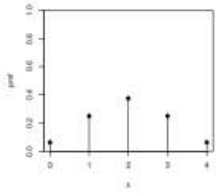
The *posterior odds* of A are the *likelihood ratio* times the *prior odds*.

Random Variables and their Distributions

PMF, CDF, and Independence

Probability Mass Function (PMF) Gives the probability that a discrete random variable takes on the value x.

$$p_X(x) = P(X = x)$$

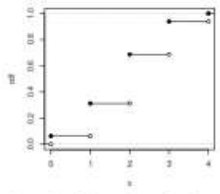


The PMF satisfies

$$p_X(x) \geq 0 \text{ and } \sum_x p_X(x) = 1$$

Cumulative Distribution Function (CDF) Gives the probability that a random variable is less than or equal to x.

$$F_X(x) = P(X \leq x)$$



The CDF is an increasing, right-continuous function with

$$F_X(x) \to 0 \text{ as } x \to -\infty \text{ and } F_X(x) \to 1 \text{ as } x \to \infty$$

Independence Intuitively, two random variables are independent if knowing the value of one gives no information about the other. Discrete r.v.s X and Y are independent if for *all* values of x and y

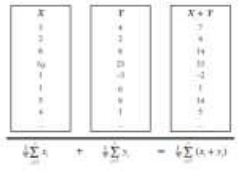
$$P(X = x, Y = y) = P(X = x)P(Y = y)$$

Expected Value and Indicators

Expected Value and Linearity

Expected Value (a.k.a. *mean, expectation,* or *average*) is a weighted average of the possible outcomes of our random variable. Mathematically, if $x_1, x_2, x_3, \ldots$ are all of the distinct possible values that X can take, the expected value of X is

$$E(X) = \sum_x x P(X = x)$$



Linearity For any r.v.s X and Y, and constants a, b, c,

$$E(aX + bY + c) = aE(X) + bE(Y) + c$$

Same distribution implies same mean If X and Y have the same distribution, then $E(X) = E(Y)$ and, more generally,

$$E(g(X)) = E(g(Y))$$

Conditional Expected Value is defined like expectation, only conditioned on any event A.

$$E(X|A) = \sum_x x P(X = x|A)$$

Indicator Random Variables

Indicator Random Variable is a random variable that takes on the value 1 or 0. It is always an indicator of some event: if the event occurs, the indicator is 1; otherwise it is 0. They are useful for many problems about counting how many events of some kind occur. Write

$$I_A = \begin{cases} 1 & \text{if } A \text{ occurs,} \\ 0 & \text{if } A \text{ does not occur.} \end{cases}$$

Note that $I_A^k = I_A I_B = I_{A \cap B}$, and $I_{A \cup B} = I_A + I_B - I_A I_B$.

Distribution $I_A \sim \text{Bern}(p)$ where $p = P(A)$.

Fundamental Bridge The expectation of the indicator for event A is the probability of event A: $E(I_A) = P(A)$.

Variance and Standard Deviation

$$\text{Var}(X) = E(X - E(X))^2 = E(X^2) - (E(X))^2$$
$$\text{SD}(X) = \sqrt{\text{Var}(X)}$$

Continuous RVs, LOTUS, UoU

Continuous Random Variables (CRVs)

What's the probability that a CRV is in an interval? Take the difference in CDF values (or use the PDF as described later).

$$P(a \leq X \leq b) = P(X \leq b) - P(X \leq a) = F_X(b) - F_X(a)$$

For $X \sim \mathcal{N}(\mu, \sigma^2)$, this becomes

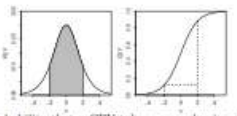
$$P(a \leq X \leq b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

What is the Probability Density Function (PDF)? The PDF f is the derivative of the CDF F.

$$F'(x) = f(x)$$

A PDF is nonnegative and integrates to 1. By the fundamental theorem of calculus, to get from PDF back to CDF we can integrate:

$$F(x) = \int_{-\infty}^{x} f(t)dt$$



To find the probability that a CRV takes on a value in an interval, integrate the PDF over that interval.

$$F(b) - F(a) = \int_a^b f(x)dx$$

How do I find the expected value of a CRV? Analogous to the discrete case, where you sum x times the PMF, for CRVs you integrate x times the PDF.

$$E(X) = \int_{-\infty}^{\infty} x f(x)dx$$

LOTUS

Expected value of a function of an r.v. The expected value of X is defined this way:

$$E(X) = \sum_x x P(X = x) \text{ (for discrete } X)$$

$$E(X) = \int_{-\infty}^{\infty} x f(x)dx \text{ (for continuous } X)$$

The **Law of the Unconscious Statistician (LOTUS)** states that you can find the expected value of a *function of a random variable*, $g(X)$, in a similar way, by replacing the x in front of the PMF/PDF by $g(x)$ but still working with the PMF/PDF of X:

$$E(g(X)) = \sum_x g(x) P(X = x) \text{ (for discrete } X)$$

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f(x)dx \text{ (for continuous } X)$$

What's a function of a random variable? A function of a random variable is also a random variable. For example, if X is the number of bikes you see in an hour, then $g(X) = 2X$ is the number of bike wheels you see in that hour and $h(X) = \binom{X}{2} = \frac{X(X-1)}{2}$ is the number of pairs of bikes such that you see both of those bikes in that hour.

What's the point? You don't need to know the PMF/PDF of $g(X)$ to find its expected value. All you need is the PMF/PDF of X.

Universality of Uniform (UoU)

When you plug any CRV into its own CDF, you get a Uniform(0,1) random variable. When you plug a Uniform(0,1) r.v. into an inverse CDF, you get an r.v. with that CDF. For example, let's say that a random variable X has CDF

$$F(x) = 1 - e^{-x}, \text{ for } x > 0$$

By UoU, if we plug X into this function then we get a uniformly distributed random variable.

$$F(X) = 1 - e^{-X} \sim \text{Unif}(0, 1)$$

Similarly, if $U \sim \text{Unif}(0, 1)$ then $F^{-1}(U)$ has CDF F. The key point is that for any continuous random variable X, we can transform it into a Uniform random variable and back by using its CDF.

Moments and MGFs

Moments

Moments describe the shape of a distribution. Let X have mean μ and standard deviation σ, and $Z = (X - \mu)/\sigma$ be the *standardized* version of X. The kth moment of X is $\mu_k = E(X^k)$ and the kth standardized moment of X is $m_k = E(Z^k)$. The mean, variance, skewness, and kurtosis are important summaries of the shape of a distribution.

Mean $E(X) = \mu_1$

Variance $\text{Var}(X) = \mu_2 - \mu_1^2$

Skewness $\text{Skew}(X) = m_3$

Kurtosis $\text{Kurt}(X) = m_4 - 3$

Moment Generating Functions

MGF For any random variable X, the function

$$M_X(t) = E(e^{tX})$$

is the **moment generating function (MGF)** of X, if it exists for all t in some open interval containing 0. The variable t could just as well have been called u or v. It's a bookkeeping device that lets us work with the *function* M_X rather than the *sequence* of moments.

Why is it called the Moment Generating Function? Because the kth derivative of the moment generating function, evaluated at 0, is the kth moment of X.

$$\mu_k = E(X^k) = M_X^{(k)}(0)$$

This is true by Taylor expansion of e^{tX} since

$$M_X(t) = E(e^{tX}) = \sum_{k=0}^{\infty} \frac{E(X^k)t^k}{k!} = \sum_{k=0}^{\infty} \frac{\mu_k t^k}{k!}$$

MGF of linear functions If we have $Y = aX + b$, then

$$M_Y(t) = E(e^{t(aX+b)}) = e^{bt}E(e^{(at)X}) = e^{bt}M_X(at)$$

Uniqueness *If it exists, the MGF uniquely determines the distribution.* This means that for any two random variables X and Y, they are distributed the same (their PMFs/PDFs are equal) if and only if their MGFs are equal.

Summing Independent RVs by Multiplying MGFs. If X and Y are independent, then

$$M_{X+Y}(t) = E(e^{t(X+Y)}) = E(e^{tX})E(e^{tY}) = M_X(t) \cdot M_Y(t)$$

The MGF of the sum of two random variables is the product of the MGFs of those two random variables.

Joint PDFs and CDFs

Joint Distributions

The **joint CDF** of X and Y is

$$F(x, y) = P(X \leq x, Y \leq y)$$

In the discrete case, X and Y have a **joint PMF**

$$p_{X,Y}(x, y) = P(X = x, Y = y).$$

In the continuous case, they have a **joint PDF**

$$f_{X,Y}(x, y) = \frac{\partial^2}{\partial x \partial y} F_{X,Y}(x, y).$$

The joint PMF/PDF must be nonnegative and sum/integrate to 1.

Conditional Distributions

Conditioning and Bayes' rule for discrete r.v.s

$$P(Y = y|X = x) = \frac{P(X = x, Y = y)}{P(X = x)} = \frac{P(X = x|Y = y)P(Y = y)}{P(X = x)}$$

Conditioning and Bayes' rule for continuous r.v.s

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x, y)}{f_X(x)} = \frac{f_{X|Y}(x|y)f_Y(y)}{f_X(x)}$$

Hybrid Bayes' rule

$$f_X(x|A) = \frac{P(A|X = x)f_X(x)}{P(A)}$$

Poisson Process

Definition We have a **Poisson process** of rate λ arrivals per unit time if the following conditions hold:

1. The number of arrivals in a time interval of length t is $\text{Pois}(\lambda t)$.
2. Numbers of arrivals in disjoint time intervals are independent.

For example, the numbers of arrivals in the time intervals $[0, 5]$, $(5, 12)$, and $(13, 23)$ are independent with $\text{Pois}(5\lambda)$, $\text{Pois}(7\lambda)$, $\text{Pois}(10\lambda)$ distributions, respectively.

Count-Time Duality Consider a Poisson process of emails arriving in an inbox at rate λ emails per hour. Let T_n be the time of arrival of the nth email (relative to some starting time 0) and N_t be the number of emails that arrive in $[0, t]$. Let's find the distribution of T_1. The event $T_1 > t$, the event that you have to wait more than t hours to get the first email, is the same as the event $N_t = 0$, which is the event that there are no emails in the first t hours. So

$$P(T_1 > t) = P(N_t = 0) = e^{-\lambda t} \longrightarrow P(T_1 \leq t) = 1 - e^{-\lambda t}$$

Thus we have $T_1 \sim \text{Expo}(\lambda)$. By the memoryless property and similar reasoning, the interarrival times between emails are i.i.d. $\text{Expo}(\lambda)$, i.e., the differences $T_n - T_{n-1}$ are i.i.d. $\text{Expo}(\lambda)$.

Order Statistics

Definition Let's say you have n i.i.d. r.v.s $X_1, X_2, \ldots, X_n$. If you arrange them from smallest to largest, the ith element in that list is the ith order statistic, denoted $X_{(i)}$. So $X_{(1)}$ is the smallest in the list and $X_{(n)}$ is the largest in the list.

Note that the order statistics are *dependent*, e.g., learning $X_{(4)} = 42$ gives us the information that $X_{(1)}, X_{(2)}, X_{(3)}$ are ≤ 42 and $X_{(5)}, X_{(6)}, \ldots, X_{(n)}$ are ≥ 42.

Distribution Taking n i.i.d. random variables $X_1, X_2, \ldots, X_n$ with CDF $F(x)$ and PDF $f(x)$, the CDF and PDF of $X_{(i)}$ are:

$$F_{X_{(i)}}(x) = P(X_{(i)} \leq x) = \sum_{k=i}^{n} \binom{n}{k} F(x)^k (1 - F(x))^{n-k}$$

$$f_{X_{(i)}}(x) = n\binom{n-1}{i-1} F(x)^{i-1}(1 - F(x))^{n-i} f(x)$$

Uniform Order Statistics The jth order statistic of i.i.d. $U_1, \ldots, U_n \sim \text{Unif}(0, 1)$ is $U_{(j)} \sim \text{Beta}(j, n - j + 1)$.

Conditional Expectation

Conditioning on an Event We can find $E(Y|A)$, the expected value of Y given that event A occurred. A very important case is when A is the event $X = x$. Note that $E(Y|A)$ is a *number*. For example:

- The expected value of a fair die roll, given that it is prime, is $\frac{1}{3} \cdot 2 + \frac{1}{3} \cdot 3 + \frac{1}{3} \cdot 5 = \frac{10}{3}$.
- Let Y be the number of successes in 10 independent Bernoulli trials with probability p of success. Let A be the event that the first 3 trials are all successes. Then

$$E(Y|A) = 3 + 7p$$

since the number of successes among the last 7 trials is $\text{Bin}(7, p)$.

Marginal Distributions

To find the distribution of one (or more) random variables from a joint PMF/PDF, sum/integrate over the unwanted random variables.

Marginal PMF from joint PMF

$$P(X = x) = \sum_y P(X = x, Y = y)$$

Marginal PDF from joint PDF

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)dy$$

Independence of Random Variables

Random variables X and Y are independent if and only if any of the following conditions holds:

- Joint CDF is the product of the marginal CDFs
- Joint PMF/PDF is the product of the marginal PMFs/PDFs
- Conditional distribution of Y given X is the marginal distribution of Y

Write $X \perp\!\!\!\perp Y$ to denote that X and Y are independent.

Multivariate LOTUS

LOTUS in more than one dimension is analogous to the 1D LOTUS. For discrete random variables:

$$E(g(X, Y)) = \sum_x \sum_y g(x, y)P(X = x, Y = y)$$

For continuous random variables:

$$E(g(X, Y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f_{X,Y}(x, y)dxdy$$

Covariance and Transformations

Covariance and Correlation

Covariance is the analog of variance for two random variables.

$$\text{Cov}(X, Y) = E\left((X - E(X))(Y - E(Y))\right) = E(XY) - E(X)E(Y)$$

Note that

$$\text{Cov}(X, X) = E(X^2) - (E(X))^2 = \text{Var}(X)$$

Correlation is a standardized version of covariance that is always between -1 and 1.

$$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$$

Covariance and Independence If two random variables are independent, then they are uncorrelated. The converse is not necessarily true (e.g., consider $X \sim \mathcal{N}(0, 1)$ and $Y = X^2$).

$$X \perp\!\!\!\perp Y \longrightarrow \text{Cov}(X, Y) = 0 \longrightarrow E(XY) = E(X)E(Y)$$

Covariance and Variance The variance of a sum can be found by

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y)$$

$$\text{Var}(X_1 + X_2 + \cdots + X_n) = \sum_{i=1}^{n} \text{Var}(X_i) + 2\sum_{i<j} \text{Cov}(X_i, X_j)$$

If X and Y are independent then they have covariance 0, so

$$X \perp\!\!\!\perp Y \Longrightarrow \text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$$

If $X_1, X_2, \ldots, X_n$ are identically distributed and have the same covariance relationships (often by **symmetry**), then

$$\text{Var}(X_1 + X_2 + \cdots + X_n) = n\text{Var}(X_1) + 2\binom{n}{2}\text{Cov}(X_1, X_2)$$

- Let $T \sim \text{Expo}(1/10)$ be how long you have to wait until the shuttle comes. Given that you have already waited t minutes, the expected additional waiting time is 10 more minutes, by the memoryless property. That is, $E(T|T > t) = t + 10$.

Discrete Y	Continuous Y
$E(Y) = \sum_y yP(Y = y)$	$E(Y) = \int_{-\infty}^{\infty} yf_Y(y)dy$
$E(Y\|A) = \sum_y yP(Y = y\|A)$	$E(Y\|A) = \int_{-\infty}^{\infty} yf(y\|A)dy$

Conditioning on a Random Variable We can also find $E(Y|X)$, the expected value of Y given the random variable X. This is a *function of the random variable* X. It is not a number except in certain special cases such as if $X \perp\!\!\!\perp Y$. To find $E(Y|X)$, find $E(Y|X = x)$ and then plug in X for x. For example:

- If $E(Y|X = x) = x^3 + 5x$, then $E(Y|X) = X^3 + 5X$.
- Let Y be the number of successes in 10 independent Bernoulli trials with probability p of success and X be the number of successes among the first 3 trials. Then $E(Y|X) = X + 7p$.
- Let $X \sim \mathcal{N}(0, 1)$ and $Y = X^2$. Then $E(Y|X = x) = x^2$ since if we know $X = x$ then we know $Y = x^2$. And $E(X|Y = y) = 0$ since if we know $Y = y$ then we know $X = \pm\sqrt{y}$, with equal probabilities (by symmetry). So $E(Y|X) = X^2$, $E(X|Y) = 0$.

Properties of Conditional Expectation

1. $E(Y|X) = E(Y)$ if $X \perp\!\!\!\perp Y$
2. $E(h(X)W|X) = h(X)E(W|X)$ (**taking out what's known**) In particular, $E(h(X)|X) = h(X)$.
3. $E(E(Y|X)) = E(Y)$ (**Adam's Law**, a.k.a. Law of Total Expectation)

Adam's Law (a.k.a. Law of Total Expectation) can also be written in a way that looks analogous to LOTP. For any events $A_1, A_2, \ldots, A_n$ that partition the sample space,

$$E(Y) = E(Y|A_1)P(A_1) + \cdots + E(Y|A_n)P(A_n)$$

For the special case where the partition is A, A^c, this says

$$E(Y) = E(Y|A)P(A) + E(Y|A^c)P(A^c)$$

Eve's Law (a.k.a. Law of Total Variance)

$$\text{Var}(Y) = E(\text{Var}(Y|X)) + \text{Var}(E(Y|X))$$

MVN, LLN, CLT

Law of Large Numbers (LLN)

Let $X_1, X_2, X_3, \ldots$ be i.i.d. with mean μ. The **sample mean** is

$$X_n = \frac{X_1 + X_2 + X_3 + \cdots + X_n}{n}$$

The **Law of Large Numbers** states that as $n \to \infty$, $X_n \to \mu$ with probability 1. For example, in flips of a coin with probability p of Heads, let X_j be the indicator of the jth flip being Heads. Then LLN says the proportion of Heads converges to p (with probability 1).

Covariance Properties For random variables W, X, Y, Z and constants a, b:

$$\begin{aligned}\text{Cov}(X, Y) &= \text{Cov}(Y, X)\\ \text{Cov}(X + a, Y + b) &= \text{Cov}(X, Y)\\ \text{Cov}(aX, bY) &= ab\text{Cov}(X, Y)\\ \text{Cov}(W + X, Y + Z) &= \text{Cov}(W, Y) + \text{Cov}(W, Z) + \text{Cov}(X, Y)\\ &\quad + \text{Cov}(X, Z)\end{aligned}$$

Correlation is location-invariant and scale-invariant For any constants a, b, c, d with a and c nonzero,

$$\text{Corr}(aX + b, cY + d) = \text{Corr}(X, Y)$$

Transformations

One Variable Transformations Let's say that we have a random variable X with PDF $f_X(x)$, but we are also interested in some function of X. We call this function $Y = g(X)$. Also let $y = g(x)$. If g is differentiable and strictly increasing (or strictly decreasing), then the PDF of Y is

$$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right| = f_X(g^{-1}(y))\left|\frac{d}{dy}g^{-1}(y)\right|$$

The derivative of the inverse transformation is called the **Jacobian**.

Two Variable Transformations Similarly, let's say we know the joint PDF of U and V but are also interested in the random vector (X, Y) defined by $(X, Y) = g(U, V)$. Let

$$\frac{\partial(u, v)}{\partial(x, y)} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}$$

be the **Jacobian matrix**. If the entries in this matrix exist and are continuous, and the determinant of the matrix is never 0, then

$$f_{X,Y}(x, y) = f_{U,V}(u, v)\left|\left|\frac{\partial(u, v)}{\partial(x, y)}\right|\right|$$

The inner bars tells us to take the matrix's determinant, and the outer bars tell us to take the absolute value. In a 2×2 matrix,

$$\left\|\begin{matrix} a & b \\ c & d \end{matrix}\right\| = |ad - bc|$$

Convolutions

Convolution Integral If you want to find the PDF of the sum of two independent CRVs X and Y, you can do the following integral:

$$f_{X+Y}(t) = \int_{-\infty}^{\infty} f_X(x)f_Y(t - x)dx$$

Example Let $X, Y \sim \mathcal{N}(0, 1)$ be i.i.d. Then for each fixed t,

$$f_{X+Y}(t) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \frac{1}{\sqrt{2\pi}} e^{-(t-x)^2/2} dx$$

By completing the square and using the fact that a Normal PDF integrates to 1, this works out to $f_{X+Y}(t)$ being the $\mathcal{N}(0, 2)$ PDF.

Central Limit Theorem (CLT)

Approximation using CLT

We use $\sim$ to denote *is approximately distributed*. We can use the **Central Limit Theorem** to approximate the distribution of a random variable $Y = X_1 + X_2 + \cdots + X_n$ that is a sum of n i.i.d. random variables X_i. Let $E(Y) = \mu_Y$ and $\text{Var}(Y) = \sigma_Y^2$. The CLT says

$$Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$$

If the X_i are i.i.d. with mean μ_X and variance σ_X^2, then $\mu_Y = n\mu_X$ and $\sigma_Y^2 = n\sigma_X^2$. For the sample mean X_n, the CLT says

$$X_n = \frac{1}{n}(X_1 + X_2 + \cdots + X_n) \sim \mathcal{N}(\mu_X, \sigma_X^2/n)$$

Asymptotic Distributions using CLT

We use $\xrightarrow{D}$ to denote *converges in distribution to as* $n \to \infty$. The CLT says that if we standardize the sum $X_1 + \cdots + X_n$ then the distribution of the sum converges to $\mathcal{N}(0, 1)$ as $n \to \infty$:

$$\frac{1}{\sigma\sqrt{n}}(X_1 + \cdots + X_n - n\mu_X) \xrightarrow{D} \mathcal{N}(0, 1)$$

In other words, the CDF of the left-hand side goes to the standard Normal CDF, Φ. In terms of the sample mean, the CLT says

$$\frac{\sqrt{n}(X_n - \mu_X)}{\sigma_X} \xrightarrow{D} \mathcal{N}(0, 1)$$

Markov Chains

Definition

A Markov chain is a random walk in a **state space**, which we will assume is finite, say $\{1, 2, \ldots, M\}$. We let X_t denote which element of the state space the walk is visiting at time t. The Markov chain is the sequence of random variables tracking where the walk is at all points in time, $X_0, X_1, X_2, \ldots$. By definition, a Markov chain must satisfy the **Markov property**, which says that if you want to predict where the chain will be at a future time, if we know the present state then the entire past history is irrelevant. *Given the present, the past and future are conditionally independent.* In symbols,

$$P(X_{n+1} = j|X_0 = i_0, X_1 = i_1, \ldots, X_n = i) = P(X_{n+1} = j|X_n = i)$$

State Properties

A state is either recurrent or transient.

- If you start at a **recurrent state**, then you will always return back to that state at some point in the future. *You can check-out any time you like, but you can never leave.*
- Otherwise you are at a **transient state**. There is some positive probability that once you leave you will never return. *You don't have to go home, but you can't stay here.*

A state is either periodic or aperiodic.

- If you start at a **periodic state** of period k, then the GCD of the possible numbers of steps it would take to return back is $k > 1$.
- Otherwise you are at an **aperiodic state**. The GCD of the possible numbers of steps it would take to return back is 1.

Transition Matrix

Let the state space be $\{1, 2, \ldots, M\}$. The transition matrix Q is the $M \times M$ matrix where element q_{ij} is the probability that the chain goes from state i to state j in one step:

$$q_{ij} = P(X_{n+1} = j | X_n = i)$$

To find the probability that the chain goes from state i to state j in exactly m steps, take the (i, j) element of Q^m.

$$q_{ij}^{(m)} = P(X_{n+m} = j | X_n = i)$$

If X_0 is distributed according to the row vector PMF $\vec{p}$, i.e., $p_j = P(X_0 = j)$, then the PMF of X_n is $\vec{p}Q^n$.

Chain Properties

A chain is **irreducible** if you can get from anywhere to anywhere. If a chain (on a finite state space) is irreducible, then all of its states are recurrent. A chain is **periodic** if any of its states are periodic, and is **aperiodic** if none of its states are periodic. In an irreducible chain, all states have the same period.

A chain is **reversible** with respect to $\vec{s}$ if $s_i q_{ij} = s_j q_{ji}$ for all i, j. Examples of reversible chains include any chain with $q_{ij} = q_{ji}$, with $\vec{s} = (\frac{1}{M}, \frac{1}{M}, \ldots, \frac{1}{M})$, and random walk on an undirected network.

Stationary Distribution

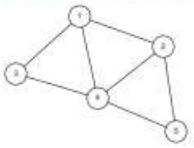
Let us say that the vector $\vec{s} = (s_1, s_2, \ldots, s_M)$ be a PMF (written as a row vector). We will call $\vec{s}$ the **stationary distribution** for the chain if $\vec{s}Q = \vec{s}$. As a consequence, if X_n has the stationary distribution, then all future $X_{n+1}, X_{n+2}, \ldots$ also have the stationary distribution.

For irreducible, aperiodic chains, the stationary distribution exists, is unique, and s_i is the long-run probability of a chain being at state i. The expected number of steps to return to i starting from i is $1/s_i$.

To find the stationary distribution, you can solve the matrix equation $(Q^t - I)\vec{s}^t = 0$. The stationary distribution is uniform if the columns of Q sum to 1.

Reversibility Condition Implies Stationarity If you have a PMF $\vec{s}$ and a Markov chain with transition matrix Q, then $s_i q_{ij} = s_j q_{ji}$ for all states i, j implies that $\vec{s}$ is stationary.

Random Walk on an Undirected Network



If you have a collection of **nodes**, pairs of which can be connected by undirected **edges**, and a Markov chain is run by going from the current node to a uniformly random node that is connected to it by an edge, then this is a random walk on an undirected network. The stationary distribution of this chain is proportional to the **degree sequence** (this is the sequence of degrees, where the degree of a node is how many edges are attached to it). For example, the stationary distribution of random walk on the network shown above is proportional to $(3, 3, 2, 4, 2)$, so it's $(\frac{3}{14}, \frac{3}{14}, \frac{2}{14}, \frac{4}{14}, \frac{2}{14})$.

Continuous Distributions

Uniform Distribution

Let us say that U is distributed Unif(a, b). We know the following:

Properties of the Uniform For a Uniform distribution, the probability of a draw from any interval within the support is proportional to the length of the interval. See *Universality of Uniform* and *Order Statistics* for other properties.

Example William throws darts really badly, so his darts are uniform over the whole room because they're equally likely to appear anywhere. William's darts have a Uniform distribution on the surface of the room. The Uniform is the only distribution where the probability of hitting in any specific region is proportional to the length/area/volume of that region, and where the density of occurrence in any one specific spot is constant throughout the whole support.

Normal Distribution

Let us say that X is distributed $\mathcal{N}(\mu, \sigma^2)$. We know the following:

Central Limit Theorem The Normal distribution is ubiquitous because of the Central Limit Theorem, which states that the sample mean of i.i.d. r.v.s will approach a Normal distribution as the sample size grows, regardless of the initial distribution.

Location-Scale Transformation Every time we shift a Normal r.v. (by adding a constant) or rescale a Normal (by multiplying by a constant), we change it to another Normal r.v. For any Normal $X \sim \mathcal{N}(\mu, \sigma^2)$, we can transform it to the standard $\mathcal{N}(0, 1)$ by the following transformation:

$$Z = \frac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1)$$

Standard Normal The Standard Normal, $Z \sim \mathcal{N}(0, 1)$, has mean 0 and variance 1. Its CDF is denoted by Φ.

Exponential Distribution

Let us say that X is distributed Expo(λ). We know the following:

Story You're sitting on an open meadow right before the break of dawn, wishing that airplanes in the night sky were shooting stars, because you could really use a wish right now. You know that shooting stars come on average every 15 minutes, but a shooting star is not "due" to come just because you've waited as long. Your waiting time is memoryless; the additional time until the next shooting star comes does not depend on how long you've waited already.

Example The waiting time until the next shooting star is distributed Expo(4) hours. Here $\lambda = 4$ is the **rate parameter**, since shooting stars arrive at a rate of 1 per 1/4 hour on average. The expected time until the next shooting star is $1/\lambda = 1/4$ hour.

Expos as a rescaled Expo(1)

$$Y \sim \text{Expo}(\lambda) \to X = \lambda Y \sim \text{Expo}(1)$$

Memorylessness The Exponential Distribution is the only continuous memoryless distribution. The memoryless property says that for $X \sim \text{Expo}(\lambda)$ and any positive numbers s and t,

$$P(X > s + t | X > s) = P(X > t)$$

Equivalently,

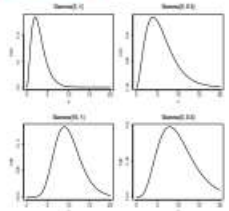
$$X - a | (X > a) \sim \text{Expo}(\lambda)$$

For example, a product with an Expo(λ) lifetime is always "as good as new" (it doesn't experience wear and tear). Given that the product has survived a years, the additional time that it will last is still Expo(λ).

Min of Expos If we have independent $X_i \sim \text{Expo}(\lambda_i)$, then $\min(X_1, \ldots, X_k) \sim \text{Expo}(\lambda_1 + \lambda_2 + \cdots + \lambda_k)$.

Max of Expos If we have i.i.d. $X_i \sim \text{Expo}(\lambda)$, then $\max(X_1, \ldots, X_k)$ has the same distribution as $Y_1 + Y_2 + \cdots + Y_k$, where $Y_j \sim \text{Expo}(j\lambda)$ and the Y_j are independent.

Gamma Distribution

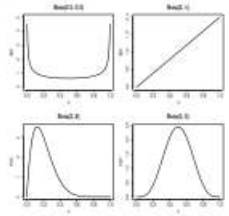


Let us say that X is distributed Gamma(a, λ). We know the following:

Story You sit waiting for shooting stars, where the waiting time for a star is distributed Expo(λ). You want to see n shooting stars before you go home. The total waiting time for the nth shooting star is Gamma(n, λ).

Example You are at a bank, and there are 3 people ahead of you. The serving time for each person is Exponential with mean 2 minutes. Only one person at a time can be served. The distribution of your waiting time until it's your turn to be served is Gamma$(3, \frac{1}{2})$.

Beta Distribution



Conjugate Prior of the Binomial In the Bayesian approach to statistics, parameters are viewed as random variables, to reflect our uncertainty. The *prior* for a parameter is its distribution before observing data. The *posterior* is the distribution for the parameter after observing data. Beta is the conjugate prior of the Binomial because if you have a Beta-distributed prior on p in a Binomial, then the posterior distribution on p given the Binomial data is also Beta-distributed. Consider the following two-level model:

$$X|p \sim \text{Bin}(n, p)$$
$$p \sim \text{Beta}(a, b)$$

Then after observing $X = x$, we get the posterior distribution

$$p|(X = x) \sim \text{Beta}(a + x, b + n - x)$$

Order statistics of the Uniform See *Order Statistics*.

Beta-Gamma relationship If $X \sim \text{Gamma}(a, \lambda)$, $Y \sim \text{Gamma}(b, \lambda)$, with $X \perp\!\!\!\perp Y$ then

- $\frac{X}{X+Y} \sim \text{Beta}(a, b)$
- $X + Y \perp\!\!\!\perp \frac{X}{X+Y}$

This is known as the **bank–post office result**.

χ^2 (Chi-Square) Distribution

Let us say that X is distributed χ^2_n. We know the following:

Story A Chi-Square(n) is the sum of the squares of n independent standard Normal r.v.s.

Properties and Representations

X is distributed as $Z_1^2 + Z_2^2 + \cdots + Z_n^2$ for i.i.d. $Z_i \sim \mathcal{N}(0, 1)$

$$X \sim \text{Gamma}(n/2, 1/2)$$

Discrete Distributions

Distributions for four sampling schemes

	Replace	No Replace
Fixed # trials (n)	Binomial (Bern if $n = 1$)	HGeom
Draw until r success	NBin (Geom if $r = 1$)	NHGeom

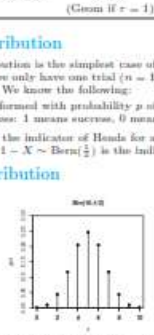
Bernoulli Distribution

The Bernoulli distribution is the simplest case of the Binomial distribution, where we only have one trial ($n = 1$). Let us say that X is distributed Bern(p). We know the following:

Story A trial is performed with probability p of "success", and X is the indicator of success: 1 means success, 0 means failure.

Example Let X be the indicator of Heads for a fair coin toss. Then $X \sim \text{Bern}(\frac{1}{2})$. Also, $1 - X \sim \text{Bern}(\frac{1}{2})$ is the indicator of Tails.

Binomial Distribution



Let us say that X is distributed Bin(n, p). We know the following:

Story X is the number of "successes" that we will achieve in n independent trials, where each trial is either a success or a failure, each with the same probability p of success. We can also write X as a sum of multiple independent Bern(p) random variables. Let $X \sim \text{Bin}(n, p)$ and $X_j \sim \text{Bern}(p)$, where all of the Bernoullis are independent. Then

$$X = X_1 + X_2 + X_3 + \cdots + X_n$$

Example If Jeremy Lin makes 10 free throws and each one independently has a $\frac{7}{8}$ chance of getting in, then the number of free throws he makes is distributed Bin$(10, \frac{7}{8})$.

Properties Let $X \sim \text{Bin}(n, p), Y \sim \text{Bin}(m, p)$ with $X \perp\!\!\!\perp Y$.

- **Redefine success** $n - X \sim \text{Bin}(n, 1 - p)$
- **Sum** $X + Y \sim \text{Bin}(n + m, p)$
- **Conditional** $X|(X + Y = r) \sim \text{HGeom}(n, m, r)$
- **Binomial-Poisson Relationship** Bin(n, p) is approximately Pois(λ) if p is small.
- **Binomial-Normal Relationship** Bin(n, p) is approximately $\mathcal{N}(np, np(1-p))$ if n is large and p is not near 0 or 1.

Geometric Distribution

Let us say that X is distributed Geom(p). We know the following:

Story X is the number of "failures" that we will achieve before we achieve our first success. Our successes have probability p.

Example If each pokeball we throw has probability $\frac{1}{10}$ to catch Mew, the number of failed pokeballs will be distributed Geom$(\frac{1}{10})$.

First Success Distribution

Equivalent to the Geometric distribution, except that it includes the first success in the count. This is 1 more than the number of failures. If $X \sim \text{FS}(p)$ then $E(X) = 1/p$.

Negative Binomial Distribution

Let us say that X is distributed NBin(r, p). We know the following:

Story X is the number of "failures" that we will have before we achieve our rth success. Our successes have probability p.

Example Thundershock has 60% accuracy and can faint a wild Raticate in 3 hits. The number of misses before Pikachu faints Raticate with Thundershock is distributed NBin$(3, 0.6)$.

Hypergeometric Distribution

Let us say that X is distributed HGeom(w, b, n). We know the following:

Story In a population of w desired objects and b undesired objects, X is the number of "successes" we will have in a draw of n objects, without replacement. The draw of n objects is assumed to be a **simple random sample** (all sets of n objects are equally likely).

Examples Here are some HGeom examples.

- Let's say that we have only 5 Weedles (failure) and w Pikachus (success) in Viridian Forest. We encounter n Pokemon in the forest, and X is the number of Pikachus in our encounters.
- The number of Aces in a 5 card hand.
- You have w white balls and b black balls, and you draw n balls. You will draw X white balls.
- You have w white balls and b black balls, and you draw n balls without replacement. The number of white balls in your sample is HGeom(w, b, n); the number of black balls is HGeom(b, w, n).
- **Capture-recapture** A forest has N elk, you capture n of them, tag them, and release them. Then you recapture a new sample of size m. How many tagged elk are now in the new sample? HGeom$(n, N - n, m)$

Poisson Distribution

Let us say that X is distributed Pois(λ). We know the following:

Story There are rare events (low probability events) that occur many different ways (high possibilities of occurrence) at an average rate of λ occurrences per unit space or time. The number of events that occur in that unit of space or time is X.

Example A certain busy intersection has an average of 2 accidents per month. Since an accident is a low probability event that can happen many different ways, it is reasonable to model the number of accidents in a month at that intersection as Pois(2). Then the number of accidents that happen in two months at that intersection is distributed Pois(4).

Properties Let $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$, with $X \perp\!\!\!\perp Y$.

1. **Sum** $X + Y \sim \text{Pois}(\lambda_1 + \lambda_2)$
2. **Conditional** $X|(X + Y = n) \sim \text{Bin}\left(n, \frac{\lambda_1}{\lambda_1 + \lambda_2}\right)$
3. **Chicken-egg** If there are $Z \sim \text{Pois}(\lambda)$ items and we randomly and independently "accept" each item with probability p, then the number of accepted items $Z_1 \sim \text{Pois}(\lambda p)$, and the number of rejected items $Z_2 \sim \text{Pois}(\lambda(1-p))$, and $Z_1 \perp\!\!\!\perp Z_2$.

Multivariate Distributions

Multinomial Distribution

Let us say that the vector $\vec{X} = (X_1, X_2, X_3, \ldots, X_k) \sim \text{Mult}_k(n, \vec{p})$ where $\vec{p} = (p_1, p_2, \ldots, p_k)$.

Story We have n items, which can fall into any one of the k buckets independently with the probabilities $\vec{p} = (p_1, p_2, \ldots, p_k)$.

Example Let us assume that every year, 100 students in the Harry Potter Universe are randomly and independently sorted into one of four houses with equal probability. The number of people in each of the houses is distributed $\text{Mult}_4(100, \vec{p})$, where $\vec{p} = (0.25, 0.25, 0.25, 0.25)$. Note that $X_1 + X_2 + \cdots + X_4 = 100$, and they are dependent.

Joint PMF For $n = n_1 + n_2 + \cdots + n_k$,

$$P(\vec{X} = \vec{n}) = \frac{n!}{n_1! n_2! \ldots n_k!} p_1^{n_1} p_2^{n_2} \ldots p_k^{n_k}$$

Marginal PMF, Lumping, and Conditionals Marginally, $X_i \sim \text{Bin}(n, p_i)$ since we can define "success" to mean category i. If you lump together multiple categories in a Multinomial, then it is still Multinomial. For example, $X_i + X_j \sim \text{Bin}(n, p_i + p_j)$ for $i \neq j$ since we can define "success" to mean being in category i or j. Similarly, if $k = 6$ and we lump categories 1-2 and lump categories 3-5, then $(X_1 + X_2, X_3 + X_4 + X_5, X_6) \sim \text{Mult}_3(n, (p_1 + p_2, p_3 + p_4 + p_5, p_6))$. Conditioning on some X_j also still gives a Multinomial:

$$X_1, \ldots, X_{k-1} | X_k = n_k \sim \text{Mult}_{k-1}\left(n - n_k, \left(\frac{p_1}{1 - p_k}, \ldots, \frac{p_{k-1}}{1 - p_k}\right)\right)$$

Variances and Covariances We have $X_i \sim \text{Bin}(n, p_i)$ marginally, so $\text{Var}(X_i) = np_i(1 - p_i)$. Also, $\text{Cov}(X_i, X_j) = -np_i p_j$ for $i \neq j$.

Multivariate Uniform Distribution

See the univariate Uniform for stories and examples. For the 2D Uniform on some region, probability is proportional to area. Every point in the support has equal density, of value $\frac{1}{\text{area of region}}$. For the 3D Uniform, probability is proportional to volume.

Multivariate Normal (MVN) Distribution

A vector $\vec{X} = (X_1, X_2, \ldots, X_k)$ is Multivariate Normal if every linear combination is Normally distributed, i.e., $t_1 X_1 + t_2 X_2 + \cdots + t_k X_k$ is Normal for any constants $t_1, t_2, \ldots, t_k$. The parameters of the Multivariate Normal are the **mean vector** $\vec{\mu} = (\mu_1, \mu_2, \ldots, \mu_k)$ and the **covariance matrix** where the (i, j) entry is $\text{Cov}(X_i, X_j)$.

Properties The Multivariate Normal has the following properties.

- Any subvector is also MVN.
- If any two elements within an MVN are uncorrelated, then they are independent.
- The joint PDF of a Bivariate Normal (X, Y) with $\mathcal{N}(0, 1)$ marginal distributions and correlation $\rho \in (-1, 1)$ is

$$f_{X,Y}(x, y) = \frac{1}{2\pi\tau} \exp\left(-\frac{1}{2\tau^2}(x^2 + y^2 - 2\rho xy)\right),$$

with $\tau = \sqrt{1 - \rho^2}$.

Distribution Properties

Important CDFs

Standard Normal Φ

Exponential(λ) $F(x) = 1 - e^{-\lambda x}$, for $x \in (0, \infty)$

Uniform(0,1) $F(x) = x$, for $x \in (0, 1)$

Convolutions of Random Variables

A convolution of n random variables is simply their sum. For the following results, let X and Y be *independent*.

1. $X \sim \text{Pois}(\lambda_1)$, $Y \sim \text{Pois}(\lambda_2) \longrightarrow X + Y \sim \text{Pois}(\lambda_1 + \lambda_2)$
2. $X \sim \text{Bin}(n_1, p)$, $Y \sim \text{Bin}(n_2, p) \longrightarrow X + Y \sim \text{Bin}(n_1 + n_2, p)$. $\text{Bin}(n, p)$ can be thought of as a sum of i.i.d. $\text{Bern}(p)$ r.v.s.
3. $X \sim \text{Gamma}(a_1, \lambda)$, $Y \sim \text{Gamma}(a_2, \lambda)$ $\longrightarrow X + Y \sim \text{Gamma}(a_1 + a_2, \lambda)$. $\text{Gamma}(n, \lambda)$ with n an integer can be thought of as a sum of i.i.d. $\text{Expo}(\lambda)$ r.v.s.
4. $X \sim \text{NBin}(r_1, p)$, $Y \sim \text{NBin}(r_2, p)$ $\longrightarrow X + Y \sim \text{NBin}(r_1 + r_2, p)$. $\text{NBin}(r, p)$ can be thought of as a sum of i.i.d. $\text{Geom}(p)$ r.v.s.
5. $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$, $Y \sim \mathcal{N}(\mu_2, \sigma_2^2)$ $\longrightarrow X + Y \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

Special Cases of Distributions

1. $\text{Bin}(1, p) \sim \text{Bern}(p)$
2. $\text{Beta}(1, 1) \sim \text{Unif}(0, 1)$
3. $\text{Gamma}(1, \lambda) \sim \text{Expo}(\lambda)$
4. $\chi_n^2 \sim \text{Gamma}\left(\frac{n}{2}, \frac{1}{2}\right)$
5. $\text{NBin}(1, p) \sim \text{Geom}(p)$

Inequalities

1. **Cauchy-Schwarz** $|E(XY)| \leq \sqrt{E(X^2)E(Y^2)}$
2. **Markov** $P(X \geq a) \leq \frac{E|X|}{a}$ for $a > 0$
3. **Chebyshev** $P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$ for $E(X) = \mu, \text{Var}(X) = \sigma^2$
4. **Jensen** $E(g(X)) \geq g(E(X))$ for g convex; reverse if g is concave

Formulas

Geometric Series

$$1 + r + r^2 + \cdots + r^{n-1} = \sum_{k=0}^{n-1} r^k = \frac{1 - r^n}{1 - r}$$

$$1 + r + r^2 + \cdots = \frac{1}{1 - r} \text{ if } |r| < 1$$

Exponential Function (e^x)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n$$

Gamma and Beta Integrals

You can sometimes solve complicated-looking integrals by pattern-matching to a gamma or beta integral:

$$\int_0^\infty x^{t-1} e^{-x}\, dx = \Gamma(t) \qquad \int_0^1 x^{a-1}(1-x)^{b-1}\, dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Also, $\Gamma(a+1) = a\Gamma(a)$, and $\Gamma(n) = (n-1)!$ if n is a positive integer.

Euler's Approximation for Harmonic Sums

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \approx \log n + 0.577\ldots$$

Stirling's Approximation for Factorials

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

Miscellaneous Definitions

Medians and Quantiles Let X have CDF F. Then X has median m if $F(m) \geq 0.5$ and $P(X \geq m) \geq 0.5$. For X continuous, m satisfies $F(m) = 1/2$. In general, the ath quantile of X is $\min\{x : F(x) \geq a\}$; the median is the case $a = 1/2$.

log Statisticians generally use log to refer to natural log (i.e., base e).

i.i.d r.v.s Independent, identically-distributed random variables.

Example Problems

Contributions from Sebastian Chiu

Calculating Probability

A textbook has n typos, which are randomly scattered amongst its n pages, independently. You pick a random page. What is the probability that it has no typos? **Answer:** There is a $\left(1 - \frac{1}{n}\right)$ probability that any specific typo isn't on your page, and thus a $\boxed{\left(1 - \frac{1}{n}\right)^n}$ probability that there are no typos on your page. For n large, this is approximately $e^{-1} = 1/e$.

Linearity and Indicators (1)

In a group of n people, what is the expected number of distinct birthdays (month and day)? What is the expected number of birthday matches? **Answer:** Let X be the number of distinct birthdays and I_j be the indicator for the jth day being represented.

$$E(I_j) = 1 - P(\text{no one born on day } j) = 1 - (364/365)^n$$

By linearity, $\boxed{E(X) = 365\left(1 - (364/365)^n\right)}$. Now let Y be the number of birthday matches and J_i be the indicator that the ith pair of people have the same birthday. The probability that any two specific people share a birthday is $1/365$, so $\boxed{E(Y) = \binom{n}{2}/365}$

Linearity and Indicators (2)

This problem is commonly known as the ***hat-matching problem.*** There are n people at a party, each with hat. At the end of the party, they each leave with a random hat. What is the expected number of people who leave with the right hat? **Answer:** Each hat has a $1/n$ chance of going to the right person. By linearity, the average number of hats that go to their owners is $\boxed{n(1/n) = 1}$.

Linearity and First Success

This problem is commonly known as the ***coupon collector problem.*** There are n coupon types. At each draw, you get a uniformly random coupon type. What is the expected number of coupons needed until you have a complete set? **Answer:** Let N be the number of coupons needed; we want $E(N)$. Let $N = N_1 + \cdots + N_n$, where N_1 is the draws to get our first new coupon, N_2 is the additional draws needed to draw our second new coupon and so on. By the story of the First Success, $N_2 \sim \text{FS}((n-1)/n)$ (after collecting first coupon type, there's $(n-1)/n$ chance you'll get something new). Similarly, $N_3 \sim \text{FS}((n-2)/n)$, and $N_j \sim \text{FS}((n-j+1)/n)$. By linearity,

$$E(N) = E(N_1) + \cdots + E(N_n) = \frac{n}{n} + \frac{n}{n-1} + \cdots + \frac{n}{1} = \boxed{n\sum_{j=1}^{n} \frac{1}{j}}$$

This is approximately $n(\log(n) + 0.577)$ by Euler's approximation.

Orderings of i.i.d. random variables

I call 2 UberX's and 3 Lyfts at the same time. If the time it takes for the rides to reach me are i.i.d., what is the probability that all the Lyfts will arrive first? **Answer:** Since the arrival times of the five cars are i.i.d., all 5! orderings of the arrivals are equally likely. There are $3!2!$ orderings that involve the Lyfts arriving first, so the probability that the Lyfts arrive first is $\boxed{\frac{3!2!}{5!} = 1/10}$. Alternatively, there are $\binom{5}{3}$ ways to choose 3 of the 5 slots for the Lyfts to occupy, where each of the choices are equally likely. One of these choices has all 3 of the Lyfts arriving first, so the probability is $\boxed{1/\binom{5}{3} = 1/10}$.

Expectation of Negative Hypergeometric

What is the expected number of cards that you draw before you pick your first Ace in a shuffled deck (not counting the Ace)? **Answer:** Consider a non-Ace. Denote this to be card j. Let I_j be the indicator that card j will be drawn before the first Ace. Note that $I_j = 1$ says that j is before all 4 of the Aces in the deck. The probability that this occurs is $1/5$ by symmetry. Let X be the number of cards drawn before the first Ace. Then $X = I_1 + I_2 + \ldots + I_{48}$, where each indicator corresponds to one of the 48 non-Aces. Thus,

$$E(X) = E(I_1) + E(I_2) + \ldots + E(I_{48}) = 48/5 = \boxed{9.6}$$

Minimum and Maximum of RVs

What is the CDF of the maximum of n independent Unif(0,1) random variables? **Answer:** Note that for r.v.s $X_1, X_2, \ldots, X_n$,

$$P(\min(X_1, X_2, \ldots, X_n) \geq a) = P(X_1 \geq a, X_2 \geq a, \ldots, X_n \geq a)$$

Similarly,

$$P(\max(X_1, X_2, \ldots, X_n) \leq a) = P(X_1 \leq a, X_2 \leq a, \ldots, X_n \leq a)$$

We will use this principle to find the CDF of $U_{(n)}$, where $U_{(n)} = \max(U_1, U_2, \ldots, U_n)$ and $U_i \sim \text{Unif}(0,1)$ are i.i.d.

$$\begin{aligned} P(\max(U_1, U_2, \ldots, U_n) \leq a) &= P(U_1 \leq a, U_2 \leq a, \ldots, U_n \leq a) \\ &= P(U_1 \leq a)P(U_2 \leq a)\ldots P(U_n \leq a) \\ &= \boxed{a^n} \end{aligned}$$

for $0 < a < 1$ (and the CDF is 0 for $a \leq 0$ and 1 for $a \geq 1$).

Pattern-matching with e^x Taylor series

For $X \sim \text{Pois}(\lambda)$, find $E\left(\frac{1}{X+1}\right)$. **Answer:** By LOTUS,

$$E\left(\frac{1}{X+1}\right) = \sum_{k=0}^{\infty} \frac{1}{k+1} \frac{e^{-\lambda}\lambda^k}{k!} = \frac{e^{-\lambda}}{\lambda} \sum_{k=0}^{\infty} \frac{\lambda^{k+1}}{(k+1)!} = \boxed{\frac{e^{-\lambda}}{\lambda}(e^{\lambda} - 1)}$$

Adam's Law and Eve's Law

William really likes speedsolving Rubik's Cubes. But he's pretty bad at it, so sometimes he fails. On any given day, William will attempt $N \sim \text{Geom}(s)$ Rubik's Cubes. Suppose each time, he has probability p of solving the cube, independently. Let T be the number of Rubik's Cubes he solves during a day. Find the mean and variance of T. **Answer:** Note that $T|N \sim \text{Bin}(N, p)$. So by Adam's Law,

$$E(T) = E(E(T|N)) = E(Np) = \boxed{\frac{p(1-s)}{s}}$$

Similarly, by Eve's Law, we have that

$$\begin{aligned} \text{Var}(T) &= E(\text{Var}(T|N)) + \text{Var}(E(T|N)) = E(Np(1-p)) + \text{Var}(Np) \\ &= \frac{p(1-p)(1-s)}{s} + \frac{p^2(1-s)}{s^2} = \boxed{\frac{p(1-s)(p+s(1-p))}{s^2}} \end{aligned}$$

MGF – Finding Moments

Find $E(X^3)$ for $X \sim \text{Expo}(\lambda)$ using the MGF of X. **Answer:** The MGF of an $\text{Expo}(\lambda)$ is $M(t) = \frac{\lambda}{\lambda - t}$. To get the third moment, we can take the third derivative of the MGF and evaluate at $t = 0$:

$$\boxed{E(X^3) = \frac{6}{\lambda^3}}$$

But a much nicer way to use the MGF here is via pattern recognition: note that $M(t)$ looks like it came from a geometric series:

$$\frac{1}{1 - \frac{t}{\lambda}} = \sum_{n=0}^{\infty} \left(\frac{t}{\lambda}\right)^n = \sum_{n=0}^{\infty} \frac{n!}{\lambda^n} \frac{t^n}{n!}$$

The coefficient of $\frac{t^n}{n!}$ here is the nth moment of X, so we have $E(X^n) = \frac{n!}{\lambda^n}$ for all nonnegative integers n.

Markov chains (1)

Suppose X_n is a two-state Markov chain with transition matrix

$$Q = \begin{matrix} \\ 0 \\ 1 \end{matrix} \begin{matrix} \begin{matrix} 0 & \quad 1 \end{matrix} \\ \begin{pmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{pmatrix} \end{matrix}$$

Find the stationary distribution $\vec{s} = (s_0, s_1)$ of X_n by solving $\vec{s}Q = \vec{s}$, and show that the chain is reversible with respect to $\vec{s}$. **Answer:** The equation $\vec{s}Q = \vec{s}$ says that

$$s_0 = s_0(1-\alpha) + s_1\beta \text{ and } s_1 = s_0(\alpha) + s_1(1-\beta)$$

By solving this system of linear equations, we have

$$\boxed{\vec{s} = \left(\frac{\beta}{\alpha+\beta}, \frac{\alpha}{\alpha+\beta}\right)}$$

To show that the chain is reversible with respect to $\vec{s}$, we must show $s_i q_{ij} = s_j q_{ji}$ for all i, j. This is done if we can show $s_0 q_{01} = s_1 q_{10}$. And indeed,

$$s_0 q_{01} = \frac{\alpha\beta}{\alpha+\beta} = s_1 q_{10}$$

Markov chains (2)

William and Sebastian play a modified game of Settlers of Catan, where every turn they randomly move the robber (which starts on the center tile) to one of the adjacent hexagons.

(a) Is this Markov chain irreducible? Is it aperiodic? **Answer:** $\boxed{\text{Yes to both.}}$ The Markov chain is irreducible because it can get from anywhere to anywhere else. The Markov chain is aperiodic because the robber can return back to a square in $2, 3, 4, 5, \ldots$ moves, and the GCD of those numbers is 1.

(b) What is the stationary distribution of this Markov chain? **Answer:** Since this is a random walk on an undirected graph, the stationary distribution is proportional to the degree sequence. The degree for the corner pieces is 3, the degree for the ridge pieces is 4, and the degree for the center pieces is 6. To normalize this degree sequence, we divide by its sum. The sum of the degrees is $6(3) + 6(4) + 7(6) = 84$. Thus the stationary probability of being on a corner is $3/84 = 1/28$, on an edge is $4/84 = 1/21$, and in the center is $6/84 = 1/14$.

(c) What fraction of the time will the robber be in the center tile in this game, in the long run? **Answer:** By the above, $\boxed{1/14}$

(d) What is the expected amount of moves it will take for the robber to return to the center tile? **Answer:** Since this chain is irreducible and aperiodic, to get the expected time to return we can just invert the stationary probability. Thus on average it will take $\boxed{14}$ turns for the robber to return to the center tile.

Problem-Solving Strategies

Contributions from Jessy Hwang, Yuan Jiang, Yuqi Hou

1. **Getting started.** Start by *defining relevant events and random variables.* ("Let A be the event that I pick the fair coin"; "Let X be the number of successes.") Clear notion is important for clear thinking! Then decide what it is that you're supposed to be finding, in terms of your notation ("I want to find $P(X = 3|A)$"). Think about what type of object your answer should be (a number? A random variable? A PMF? A PDF?) and what it should be in terms of.
   Try simple and extreme cases. To make an abstract experiment more concrete, try *drawing a picture* or making up numbers that could have happened. Pattern recognition: does the structure of the problem resemble something we've seen before?
2. **Calculating probability of an event.** Use counting principles if the naive definition of probability applies. Is the probability of the complement easier to find? Look for symmetries. Look for something to condition on, then apply Bayes' Rule or the Law of Total Probability.
3. **Finding the distribution of a random variable.** First make sure you need the full distribution not just the mean (see next item). Check the *support* of the random variable: what values can it take on? Use this to rule out distributions that don't fit. Is there a *story* for one of the named distributions that fits the problem at hand? Can you write the random variable as a function of an r.v. with a known distribution, say $Y = g(X)$?
4. **Calculating expectation.** If it has a named distribution, check out the table of distributions. If it's a function of an r.v. with a named distribution, try LOTUS. If it's a count of something, try breaking it up into indicator r.v.s. If you can condition on something natural, consider using Adam's law.
5. **Calculating variance.** Consider independence, named distributions, and LOTUS. If it's a count of something, break it up into a sum of indicator r.v.s. If it's a sum, use properties of covariance. If you can condition on something natural, consider using Eve's Law.
6. **Calculating $E(X^2)$.** Do you already know $E(X)$ or $\text{Var}(X)$? Recall that $\text{Var}(X) = E(X^2) - (E(X))^2$. Otherwise try LOTUS.
7. **Calculating covariance.** Use the properties of covariance. If you're trying to find the covariance between two components of a Multinomial distribution, X_i, X_j, then the covariance is $-np_ip_j$ for $i \neq j$.
8. **Symmetry.** If $X_1, \ldots, X_n$ are i.i.d., consider using symmetry.
9. **Calculating probabilities of orderings.** Remember that all $n!$ orderings of i.i.d. continuous random variables $X_1, \ldots, X_n$ are equally likely.
10. **Determining independence.** There are several equivalent definitions. Think about simple and extreme cases to see if you can find a counterexample.
11. **Do a painful integral.** If your integral looks painful, see if you can write your integral in terms of a known PDF (like Gamma or Beta), and use the fact that PDFs integrate to 1?
12. **Before moving on.** Check some simple and extreme cases, check whether the answer seems plausible, check for biohazards.

Biohazards

Contributions from Jessy Hwang

1. **Don't misuse the naive definition of probability.** When answering "What is the probability that in a group of 3 people, no two have the same birth month?", it is not correct to treat the people as distinguishable balls being placed into 12 bins, since that assumes the list of birth months {January, January, January} is just as likely as the list {January, April, June}, even though the latter is six times more likely.
2. **Don't confuse unconditional, conditional, and joint probabilities.** In applying $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$, it is not correct to say "$P(B) = 1$ because we know B happened"; $P(B)$ is the *prior* probability of B. Don't confuse $P(A|B)$ with $P(A, B)$.
3. **Don't assume independence without justification.** In the matching problem, the probability that card 1 is a match and card 2 is a match is not $1/n^2$. Binomial and Hypergeometric are often confused; the trials are independent in the Binomial story and dependent in the Hypergeometric story.
4. **Don't forget to do sanity checks.** Probabilities must be between 0 and 1. Variances must be ≥ 0. Supports must make sense. PMFs must sum to 1. PDFs must integrate to 1.
5. **Don't confuse random variables, numbers, and events.** Let X be an r.v. Then $g(X)$ is an r.v. for any function g. In particular, X^2, $|X|$, $F(X)$, and $I_{X>3}$ are r.v.s. $P(X^3 < X|X \geq 0)$, $E(X)$, $\text{Var}(X)$, and $g(E(X))$ are numbers. $X = 2$ and $F(X) \geq -1$ are events. It does not make sense to write $\int_{-\infty}^{\infty} F(X)dx$, because $F(X)$ is a random variable. It does not make sense to write $P(X)$, because X is not an event.

6. **Don't confuse a random variable with its distribution.** To get the PDF of X^2, you can't just square the PDF of X. The right way is to use transformations. To get the PDF of $X+Y$, you can't just add the PDF of X and the PDF of Y. The right way is to compute the convolution.

7. **Don't pull non-linear functions out of expectations.** $E(g(X))$ does not equal $g(E(X))$ in general. The St. Petersburg paradox is an extreme example. See also Jensen's inequality. The right way to find $E(g(X))$ is with LOTUS.

Recommended Resources

- Introduction to Probability Book (http://bit.ly/introprobability)
- Stat 110 Online (http://stat110.net)
- Stat 110 Quora Blog (https://stat110.quora.com/)
- Quora Probability FAQ (http://bit.ly/probabilityfaq)
- R Studio (https://www.rstudio.com)
- LaTeX File (github.com/wzchen/probability_cheatsheet)

Please share this cheatsheet with friends!
http://wzchen.com/probability-cheatsheet

Distributions in R

Command	What it does
help(Distributions)	shows documentation on distributions
dbinom(k,n,p)	PMF $P(X = k)$ for $X \sim \text{Bin}(n, p)$
pbinom(x,n,p)	CDF $P(X \leq x)$ for $X \sim \text{Bin}(n, p)$
qbinom(a,n,p)	ath quantile for $X \sim \text{Bin}(n, p)$
rbinom(r,n,p)	vector of r i.i.d. $\text{Bin}(n, p)$ r.v.s
dgeom(k,p)	PMF $P(X = k)$ for $X \sim \text{Geom}(p)$
dhyper(k,w,b,n)	PMF $P(X = k)$ for $X \sim \text{HGeom}(w, b, n)$
dnbinom(k,r,p)	PMF $P(X = k)$ for $X \sim \text{NBin}(r, p)$
dpois(k,r)	PMF $P(X = k)$ for $X \sim \text{Pois}(r)$
dbeta(x,a,b)	PDF $f(x)$ for $X \sim \text{Beta}(a, b)$
dchisq(x,n)	PDF $f(x)$ for $X \sim \chi^2_n$
dexp(x,b)	PDF $f(x)$ for $X \sim \text{Expo}(b)$
dgamma(x,a,r)	PDF $f(x)$ for $X \sim \text{Gamma}(a, r)$
dlnorm(x,m,s)	PDF $f(x)$ for $X \sim \mathcal{LN}(m, s^2)$
dnorm(x,m,s)	PDF $f(x)$ for $X \sim \mathcal{N}(m, s^2)$
dt(x,n)	PDF $f(x)$ for $X \sim t_n$
dunif(x,a,b)	PDF $f(x)$ for $X \sim \text{Unif}(a, b)$

The table above gives R commands for working with various named distributions. Commands analogous to **pbinom**, **qbinom**, and **rbinom** work for the other distributions in the table. For example, **pnorm**, **qnorm**, and **rnorm** can be used to get the CDF, quantiles, and random generation for the Normal. For the Multinomial, **dmultinom** can be used for calculating the joint PMF and **rmultinom** can be used for generating random vectors. For the Multivariate Normal, after installing and loading the **mvtnorm** package **dmvnorm** can be used for calculating the joint PDF and **rmvnorm** can be used for generating random vectors.

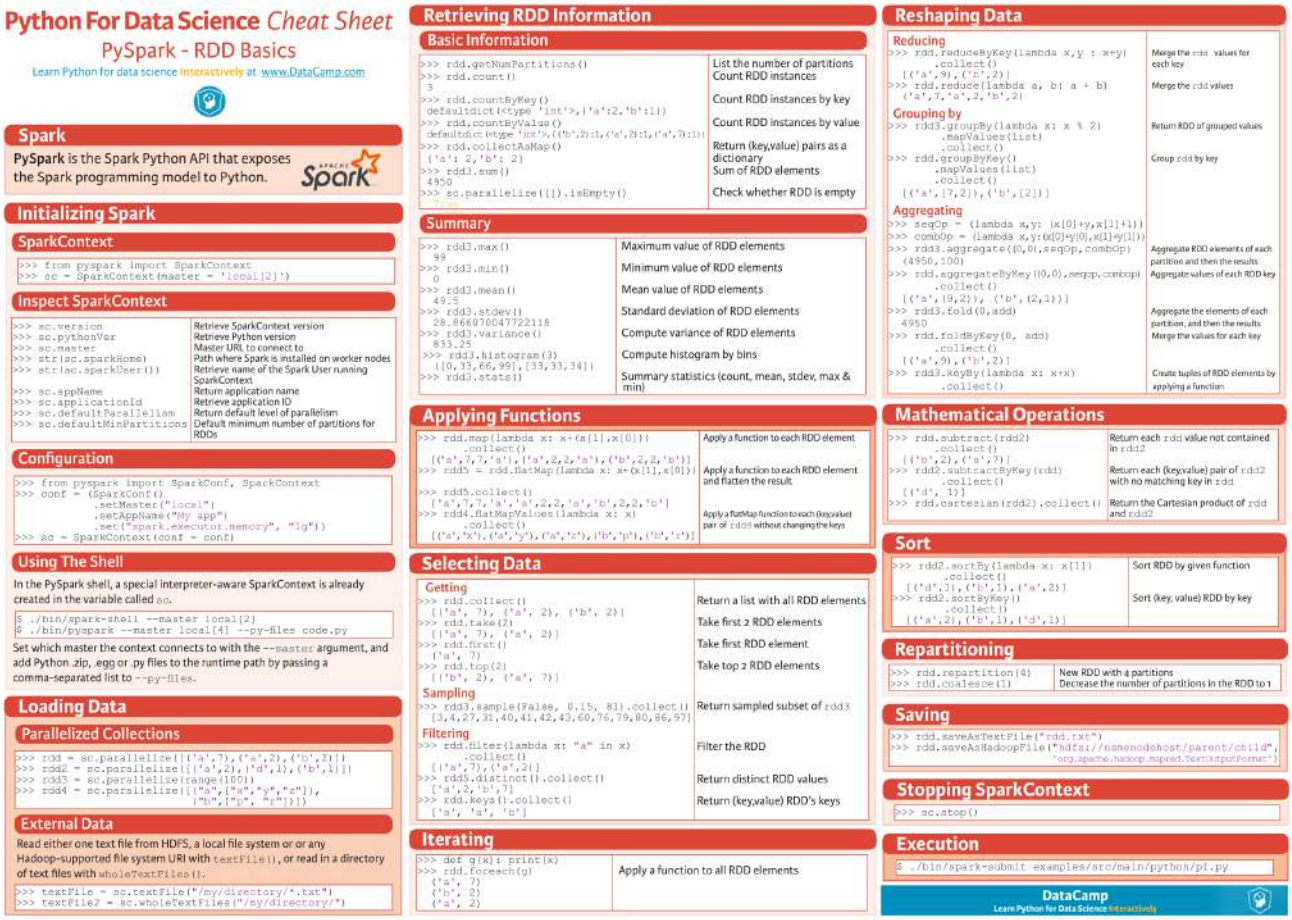
Table of Distributions

Distribution	PMF/PDF and Support	Expected Value	Variance	MGF
Bernoulli $\text{Bern}(p)$	$P(X = 1) = p$ $P(X = 0) = q = 1 - p$	p	pq	$q + pe^t$
Binomial $\text{Bin}(n, p)$	$P(X = k) = \binom{n}{k} p^k q^{n-k}$ $k \in \{0, 1, 2, \dots n\}$	np	npq	$(q + pe^t)^n$
Geometric $\text{Geom}(p)$	$P(X = k) = q^k p$ $k \in \{0, 1, 2, \dots\}$	q/p	q/p^2	$\frac{p}{1-qe^t}$, $qe^t < 1$
Negative Binomial $\text{NBin}(r, p)$	$P(X = n) = \binom{r+n-1}{r-1} p^r q^n$ $n \in \{0, 1, 2, \dots\}$	rq/p	rq/p^2	$(\frac{p}{1-qe^t})^r$, $qe^t < 1$
Hypergeometric $\text{HGeom}(w, b, n)$	$P(X = k) = \binom{w}{k}\binom{b}{n-k} / \binom{w+b}{n}$ $k \in \{0, 1, 2, \dots, n\}$	$\mu = \frac{nw}{b+w}$	$\left(\frac{w+b-n}{w+b-1}\right) n \frac{\mu}{n}(1 - \frac{\mu}{n})$	messy
Poisson $\text{Pois}(\lambda)$	$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$ $k \in \{0, 1, 2, \dots\}$	λ	λ	$e^{\lambda(e^t-1)}$
Uniform $\text{Unif}(a, b)$	$f(x) = \frac{1}{b-a}$ $x \in (a, b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Normal $\mathcal{N}(\mu, \sigma^2)$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$ $x \in (-\infty, \infty)$	μ	σ^2	$e^{t\mu + \frac{\sigma^2 t^2}{2}}$
Exponential $\text{Expo}(\lambda)$	$f(x) = \lambda e^{-\lambda x}$ $x \in (0, \infty)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}$, $t < \lambda$
Gamma $\text{Gamma}(a, \lambda)$	$f(x) = \frac{1}{\Gamma(a)}(\lambda x)^a e^{-\lambda x} \frac{1}{x}$ $x \in (0, \infty)$	$\frac{a}{\lambda}$	$\frac{a}{\lambda^2}$	$\left(\frac{\lambda}{\lambda - t}\right)^a$, $t < \lambda$
Beta $\text{Beta}(a, b)$	$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$ $x \in (0, 1)$	$\mu = \frac{a}{a+b}$	$\frac{\mu(1-\mu)}{(a+b+1)}$	messy
Log-Normal $\mathcal{LN}(\mu, \sigma^2)$	$\frac{1}{x\sigma\sqrt{2\pi}} e^{-(\log x - \mu)^2/(2\sigma^2)}$ $x \in (0, \infty)$	$\theta = e^{\mu + \sigma^2/2}$	$\theta^2(e^{\sigma^2} - 1)$	doesn't exist
Chi-Square χ^2_n	$\frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2}$ $x \in (0, \infty)$	n	$2n$	$(1 - 2t)^{-n/2}$, $t < 1/2$
Student-t t_n	$\frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\,\Gamma(n/2)} (1 + x^2/n)^{-(n+1)/2}$ $x \in (-\infty, \infty)$	0 if $n > 1$	$\frac{n}{n-2}$ if $n > 2$	doesn't exist

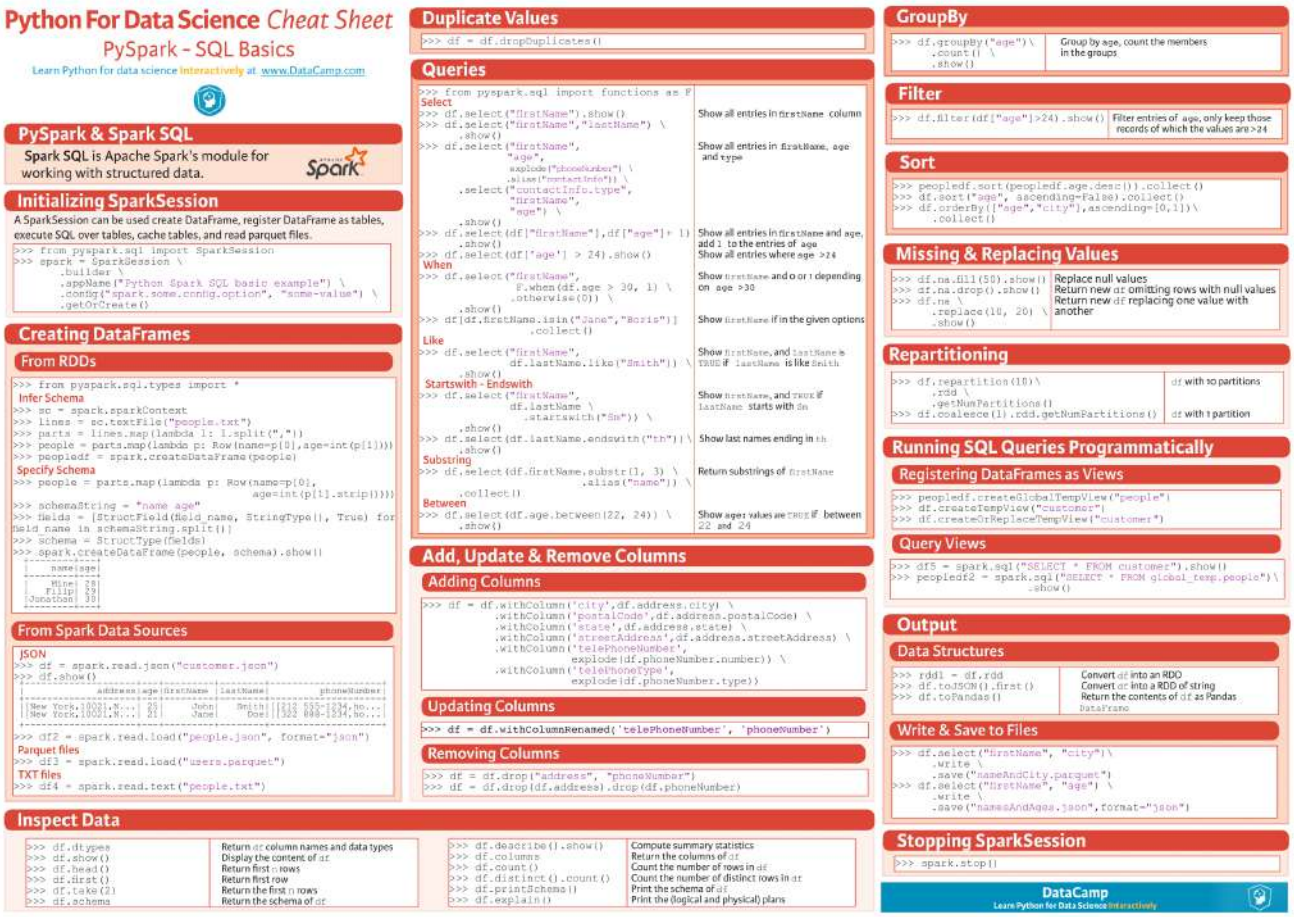
Big Data

Python

- Pyspark RDD (PDF)



- Pyspark DF (PDF)


Python For Data Science Cheat Sheet
PySpark - SQL Basics
Learn Python for data science Interactively at www.DataCamp.com
PySpark & Spark SQL
Spark SQL is Apache Spark's module for working with structured data.
Initializing SparkSession
A SparkSession can be used create DataFrame, register DataFrame as tables, execute SQL over tables, cache tables, and read parquet files.
Creating DataFrames
From RDDs
Infer Schema
Specify Schema
From Spark Data Sources
JSON
Parquet files
TXT files
Duplicate Values
Queries
Select
Like
When
Startswith - Endswith
Substring
Between
Add, Update & Remove Columns
Adding Columns
Updating Columns
Removing Columns
GroupBy
Group by age, count the members in the groups
Filter
Filter entries of age, only keep those records of which the values are >24
Sort
Missing & Replacing Values
Replace null values
Return new df omitting rows with null values
Return new df replacing one value with another
Repartitioning
df with 10 partitions
df with 1 partition
Running SQL Queries Programmatically
Registering DataFrames as Views
Query Views
Output
Data Structures
Convert df into an RDD
Convert df into a RDD of string
Return the contents of df as Pandas DataFrame
Write & Save to Files
Stopping SparkSession
DataCamp
Learn Python for Data Science Interactively
Inspect Data
Return df column names and data types
Display the content of df
Return first n rows
Return first row
Return the first n rows
Return the schema of df
Compute summary statistics
Return the columns of df
Count the number of rows in df
Count the number of distinct rows in df
Print the schema of df
Print the (logical and physical) plans

- Dask (PDF)

DASK FOR PARALLEL COMPUTING CHEAT SHEET

See full Dask documentation at: http://dask.pydata.org/

These instructions use the conda environment manager. Get yours at http://bit.ly/getconda

DASK QUICK INSTALL	
Install Dask with conda	`conda install dask`
Install Dask with pip	`pip install dask[complete]`
DASK COLLECTIONS	**EASY TO USE BIG DATA COLLECTIONS**
DASK DATAFRAMES	**PARALLEL PANDAS DATAFRAMES FOR LARGE DATA**
Import	`import dask.dataframe as dd`
Read CSV data	`df = dd.read_csv('my-data.*.csv')`
Read Parquet data	`df = dd.read_parquet('my-data.parquet')`
Filter and manipulate data with Pandas syntax	`df['z'] = df.x + df.y`
Standard groupby aggregations, joins, etc.	`result = df.groupby(df.z).y.mean()`
Compute result as a Pandas dataframe	`out = result.compute()`
Or store to CSV, Parquet, or other formats	`result.to_parquet('my-output.parquet')`

EXAMPLE	`df = dd.read_csv('filenames.*.csv')`
	`df.groupby(df.timestamp.day)\`
	`  .value.mean().compute()`

DASK ARRAYS	**PARALLEL NUMPY ARRAYS FOR LARGE DATA**
Import	`import dask.array as da`
Create from any array-like object	`import h5py` `dataset = h5py.File('my-data.hdf5')['/group/dataset']`
Including HFD5, NetCDF, or other on-disk formats.	`x = da.from_array(dataset, chunks=(1000, 1000))`
Alternatively generate an array from a random	`da.random.uniform(shape=(1e4, 1e4), chunks=(100, 100))`

distribution.	
Perform operations with NumPy syntax	`y = x.dot(x.T - 1) - x.mean(axis=0)`
Compute result as a NumPy array	`result = y.compute()`
Or store to HDF5, NetCDF or other on-disk format	`out = f.create_dataset(...)` `x.store(out)`

EXAMPLE	`with h5py.File('my-data.hdf5') as f:`
	`  x = da.from_array(f['/path'], chunks=(1000, 1000))`
	`  x -= x.mean(axis=0)`
	`  out = f.create_dataset(...)`
	`  x.store(out)`

DASK BAGS	**PARELLEL LISTS FOR UNSTRUCTURED DATA**
Import	`import dask.bag as db`
Create Dask Bag from a sequence	`b = db.from_sequence(seq, npartitions)`
Or read from text formats	`b = db.read_text('my-data.*.json')`

Map and filter results	`import json`
	`records = b.map(json.loads)`
	`  .filter(lambda d: d["name"] == "Alice")`

Compute aggregations like mean, count, sum	`records.pluck('key-name').mean().compute()`
Or store results back to text formats	`records.to_textfiles('output.*.json')`

EXAMPLE	`db.read_text('s3://bucket/my-data.*.json')`
	`  .map(json.loads)`
	`  .filter(lambda d: d["name"] == "Alice")`
	`  .to_textfiles('s3://bucket/output.*.json')`

ANACONDA.

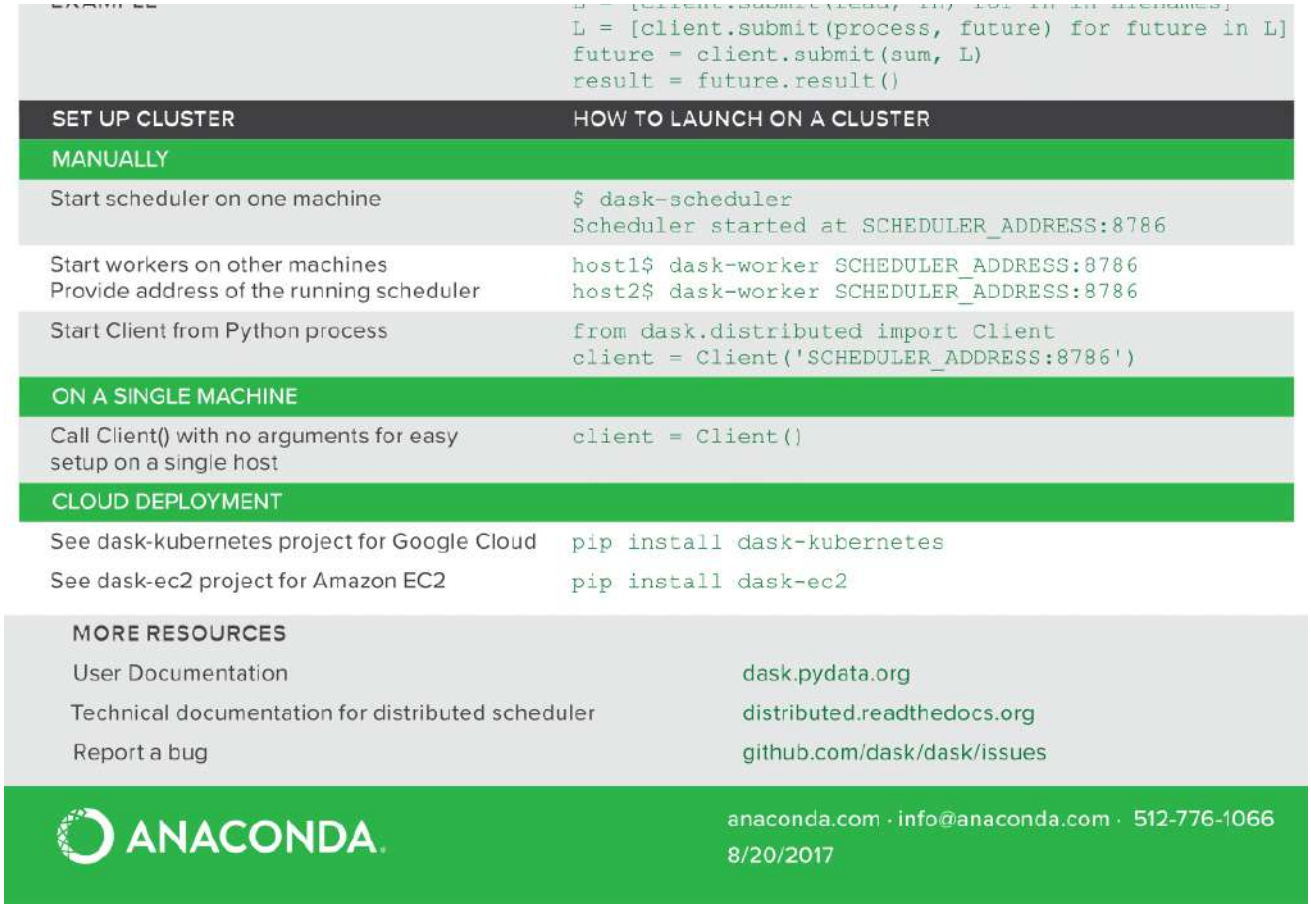
CONTINUED ON BACK →

DASK COLLECTIONS (CONTINUED)	
ADVANCED	
Read from distributed file systems or cloud storage Prepend prefixes like hdfs://, s3://, or gcs:// to paths	`df = dd.read_parquet('s3://bucket/myfile.parquet')` `b = db.read_text('hdfs:///path/to/my-data.*.json')`
Persist lazy computations in memory	`df = df.persist()`
Compute multiple outputs at once	`dask.compute(x.min(), x.max())`
CUSTOM COMPUTATIONS	**FOR CUSTOM CODE AND COMPLEX ALGORITHMS**
DASK DELAYED	**LAZY PARALLELISM FOR CUSTOM CODE**
Import	`import dask`

Wrap custom functions with the @dask.delayed annotation	`@dask.delayed`
	`def load(filename):`
	`  ...`

Delayed functions operate lazily, producing a task graph rather than executing immediately	`@dask.delayed`
	`def process(data):`
	`  ...`

Passing delayed results to other delayed functions creates dependencies between tasks	`load = dask.delayed(load)` `process = dask.delayed(process)`
Call functions in normal code	`data = [load(fn) for fn in filenames]` `results = [process(d) for d in data]`
Compute results to execute in parallel	`dask.compute(results)`
CONCURRENT.FUTURES	**ASYNCHRONOUS REAL-TIME PARALLELISM**
Import	`from dask.distributed import Client`
Start local Dask Client	`client = Client()`
Submit individual task asynchronously	`future = client.submit(func, *args, **kwargs)`
Block and gather individual result	`result = future.result()`
Process results as they arrive	`for future in as_completed(futures):` `  ...`
EXAMPLE	`L = [client.submit(read, fn) for fn in filenames]`



```
L = [client.submit(process, future) for future in L]
future = client.submit(sum, L)
result = future.result()
```

SET UP CLUSTER	HOW TO LAUNCH ON A CLUSTER
MANUALLY	
Start scheduler on one machine	`$ dask-scheduler` `Scheduler started at SCHEDULER_ADDRESS:8786`
Start workers on other machines Provide address of the running scheduler	`host1$ dask-worker SCHEDULER_ADDRESS:8786` `host2$ dask-worker SCHEDULER_ADDRESS:8786`
Start Client from Python process	`from dask.distributed import Client` `client = Client('SCHEDULER_ADDRESS:8786')`
ON A SINGLE MACHINE	
Call Client() with no arguments for easy setup on a single host	`client = Client()`
CLOUD DEPLOYMENT	
See dask-kubernetes project for Google Cloud	`pip install dask-kubernetes`
See dask-ec2 project for Amazon EC2	`pip install dask-ec2`

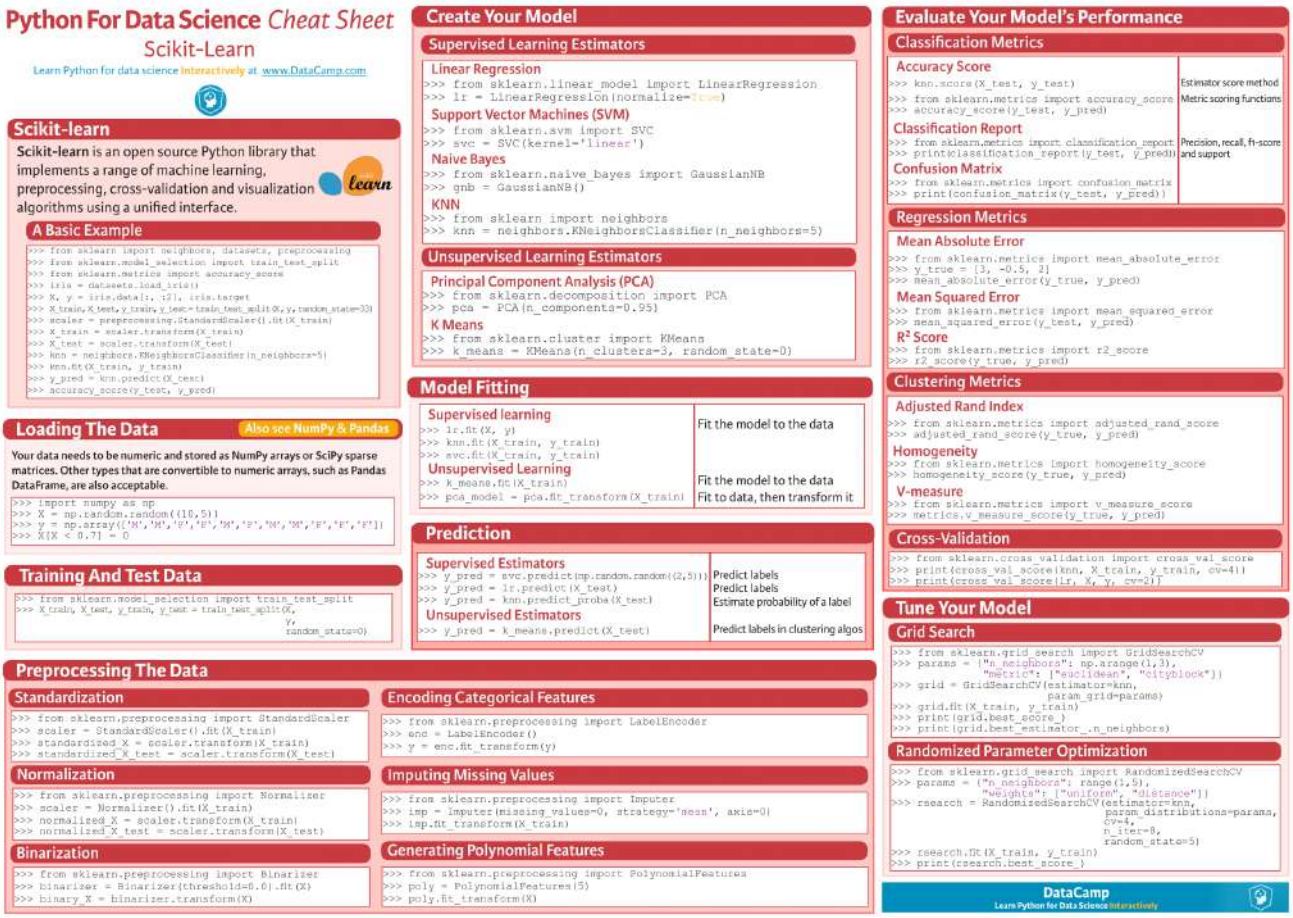
R

- Sparklyr (PDF)

Machine Learning

Python

- Scitk-Learn (PDF)



R

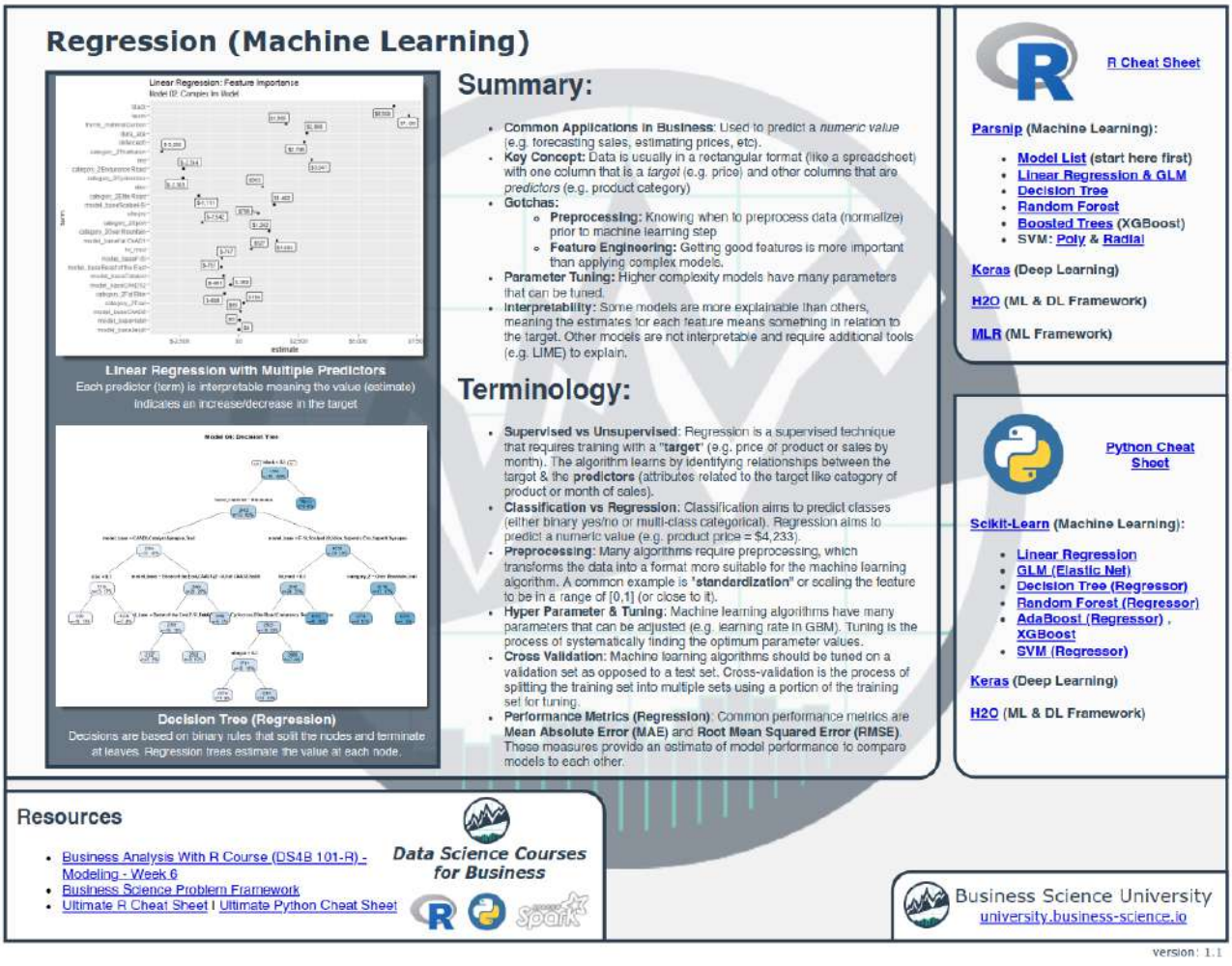
- Machine Learning Modelling in R (PDF)
- Caret (PDF)
- Estimatr (PDF)

_ H2O (PDF)

- mlr (PDF)

Supervised Learning

- Regression (PDF)



From @afshinea:

- VIP Supervised Learning (PDF)

VIP Cheatsheet: Supervised Learning

Afshine Amidi and Shervine Amidi

September 9, 2018

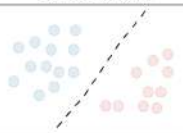
Introduction to Supervised Learning

Given a set of data points $\{x^{(1)},...,x^{(m)}\}$ associated to a set of outcomes $\{y^{(1)},...,y^{(m)}\}$, we want to build a classifier that learns how to predict y from x.

❒ **Type of prediction** – The different types of predictive models are summed up in the table below:

	Regression	Classifier
Outcome	Continuous	Class
Examples	Linear regression	Logistic regression, SVM, Naive Bayes

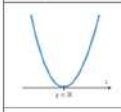
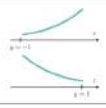
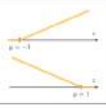
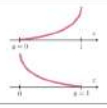
❒ **Type of model** – The different models are summed up in the table below:

	Discriminative model	Generative model
Goal	Directly estimate $P(y\|x)$	Estimate $P(x\|y)$ to deduce $P(y\|x)$
What's learned	Decision boundary	Probability distributions of the data
Illustration		
Examples	Regressions, SVMs	GDA, Naive Bayes

Notations and general concepts

❒ **Hypothesis** – The hypothesis is noted h_θ and is the model that we choose. For a given input data $x^{(i)}$, the model prediction output is $h_\theta(x^{(i)})$.

❒ **Loss function** – A loss function is a function $L:(z,y)\in\mathbb{R}\times Y\longmapsto L(z,y)\in\mathbb{R}$ that takes as inputs the predicted value z corresponding to the real data value y and outputs how different they are. The common loss functions are summed up in the table below:

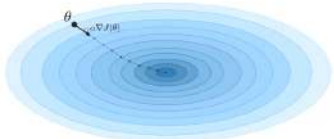
Least squared	Logistic	Hinge	Cross-entropy
$\frac{1}{2}(y-z)^2$	$\log(1+\exp(-yz))$	$\max(0,1-yz)$	$-\left[y\log(z)+(1-y)\log(1-z)\right]$
			
Linear regression	Logistic regression	SVM	Neural Network

❒ **Cost function** – The cost function J is commonly used to assess the performance of a model, and is defined with the loss function L as follows:

$$\boxed{J(\theta)=\sum_{i=1}^{m}L(h_\theta(x^{(i)}),y^{(i)})}$$

❒ **Gradient descent** – By noting $\alpha\in\mathbb{R}$ the learning rate, the update rule for gradient descent is expressed with the learning rate and the cost function J as follows:

$$\boxed{\theta\longleftarrow\theta-\alpha\nabla J(\theta)}$$



Remark: Stochastic gradient descent (SGD) is updating the parameter based on each training example, and batch gradient descent is on a batch of training examples.

❒ **Likelihood** – The likelihood of a model $L(\theta)$ given parameters θ is used to find the optimal parameters θ through maximizing the likelihood. In practice, we use the log-likelihood $\ell(\theta)=\log(L(\theta))$ which is easier to optimize. We have:

$$\boxed{\theta^{\text{opt}}=\arg\max_\theta L(\theta)}$$

❒ **Newton's algorithm** – The Newton's algorithm is a numerical method that finds θ such that $\ell'(\theta)=0$. Its update rule is as follows:

$$\boxed{\theta\leftarrow\theta-\frac{\ell'(\theta)}{\ell''(\theta)}}$$

Remark: the multidimensional generalization, also known as the Newton-Raphson method, has the following update rule:

$$\theta\leftarrow\theta-\left(\nabla^2_\theta\ell(\theta)\right)^{-1}\nabla_\theta\ell(\theta)$$

Linear regression

We assume here that $y|x;\theta\sim\mathcal{N}(\mu,\sigma^2)$

❒ **Normal equations** – By noting X the matrix design, the value of θ that minimizes the cost function is a closed-form solution such that:

$$\boxed{\theta=(X^TX)^{-1}X^Ty}$$

❒ **LMS algorithm** – By noting α the learning rate, the update rule of the Least Mean Squares (LMS) algorithm for a training set of m data points, which is also known as the Widrow-Hoff learning rule, is as follows:

$$\boxed{\forall j,\quad\theta_j\leftarrow\theta_j+\alpha\sum_{i=1}^{m}\left[y^{(i)}-h_\theta(x^{(i)})\right]x_j^{(i)}}$$

Remark: the update rule is a particular case of the gradient ascent.

❒ **LWR** – Locally Weighted Regression, also known as LWR, is a variant of linear regression that weights each training example in its cost function by $w^{(i)}(x)$, which is defined with parameter $\tau\in\mathbb{R}$ as:

$$\boxed{w^{(i)}(x)=\exp\left(-\frac{(x^{(i)}-x)^2}{2\tau^2}\right)}$$

Classification and logistic regression

❒ **Sigmoid function** – The sigmoid function g, also known as the logistic function, is defined as follows:

$$\forall z\in\mathbb{R},\quad\boxed{g(z)=\frac{1}{1+e^{-z}}\in]0,1[}$$

❒ **Logistic regression** – We assume here that $y|x;\theta\sim\text{Bernoulli}(\phi)$. We have the following form:

$$\boxed{\phi=p(y=1|x;\theta)=\frac{1}{1+\exp(-\theta^Tx)}=g(\theta^Tx)}$$

Remark: there is no closed form solution for the case of logistic regressions.

❒ **Softmax regression** – A softmax regression, also called a multiclass logistic regression, is used to generalize logistic regression when there are more than 2 outcome classes. By convention, we set $\theta_K=0$, which makes the Bernoulli parameter ϕ_i of each class i equal to:

$$\boxed{\phi_i=\frac{\exp(\theta_i^Tx)}{\sum_{j=1}^{K}\exp(\theta_j^Tx)}}$$

Generalized Linear Models

❒ **Exponential family** – A class of distributions is said to be in the exponential family if it can be written in terms of a natural parameter, also called the canonical parameter or link function, η, a sufficient statistic $T(y)$ and a log-partition function $a(\eta)$ as follows:

$$\boxed{p(y;\eta)=b(y)\exp(\eta T(y)-a(\eta))}$$

Remark: we will often have $T(y)=y$. Also, $\exp(-a(\eta))$ can be seen as a normalization parameter that will make sure that the probabilities sum to one.

Here are the most common exponential distributions summed up in the following table:

Distribution	η	$T(y)$	$a(\eta)$	$b(y)$
Bernoulli	$\log\left(\frac{\phi}{1-\phi}\right)$	y	$\log(1+\exp(\eta))$	1
Gaussian	μ	y	$\frac{\eta^2}{2}$	$\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{y^2}{2}\right)$
Poisson	$\log(\lambda)$	y	e^η	$\frac{1}{y!}$
Geometric	$\log(1-\phi)$	y	$\log\left(\frac{e^\eta}{1-e^\eta}\right)$	1

❒ **Assumptions of GLMs** – Generalized Linear Models (GLM) aim at predicting a random variable y as a function fo $x\in\mathbb{R}^{n+1}$ and rely on the following 3 assumptions:

(1) $\boxed{y|x;\theta\sim\text{ExpFamily}(\eta)}$ (2) $\boxed{h_\theta(x)=E[y|x;\theta]}$ (3) $\boxed{\eta=\theta^Tx}$

Remark: ordinary least squares and logistic regression are special cases of generalized linear models.

Support Vector Machines

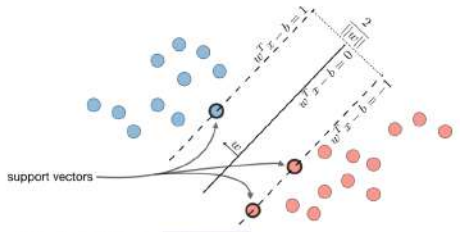
The goal of support vector machines is to find the line that maximizes the minimum distance to the line.

❒ **Optimal margin classifier** – The optimal margin classifier h is such that:

$$\boxed{h(x)=\text{sign}(w^Tx-b)}$$

where $(w,b)\in\mathbb{R}^n\times\mathbb{R}$ is the solution of the following optimization problem:

$$\boxed{\min\frac{1}{2}||w||^2}\quad\text{such that}\quad\boxed{y^{(i)}(w^Tx^{(i)}-b)\geqslant1}$$



Remark: the line is defined as $\boxed{w^T x - b = 0}$.

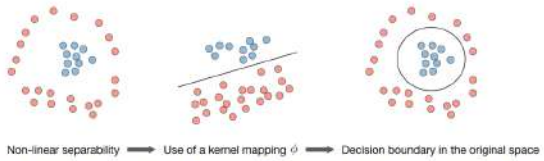
❒ **Hinge loss** – The hinge loss is used in the setting of SVMs and is defined as follows:

$$\boxed{L(z,y) = [1-yz]_+ = \max(0,1-yz)}$$

❒ **Kernel** – Given a feature mapping ϕ, we define the kernel K to be defined as:

$$\boxed{K(x,z) = \phi(x)^T \phi(z)}$$

In practice, the kernel K defined by $K(x,z) = \exp\left(-\frac{\|x-z\|^2}{2\sigma^2}\right)$ is called the Gaussian kernel and is commonly used.



Remark: we say that we use the "kernel trick" to compute the cost function using the kernel because we actually don't need to know the explicit mapping ϕ, which is often very complicated. Instead, only the values $K(x,z)$ are needed.

❒ **Lagrangian** – We define the Lagrangian $\mathcal{L}(w,b)$ as follows:

$$\boxed{\mathcal{L}(w,b) = f(w) + \sum_{i=1}^{l} \beta_i h_i(w)}$$

Remark: the coefficients β_i are called the Lagrange multipliers.

Generative Learning

A generative model first tries to learn how the data is generated by estimating $P(x|y)$, which we can then use to estimate $P(y|x)$ by using Bayes' rule.

Gaussian Discriminant Analysis

❒ **Setting** – The Gaussian Discriminant Analysis assumes that y and $x|y=0$ and $x|y=1$ are such that:

$$\boxed{y \sim \text{Bernoulli}(\phi)}$$

$$\boxed{x|y=0 \sim \mathcal{N}(\mu_0, \Sigma)} \quad \text{and} \quad \boxed{x|y=1 \sim \mathcal{N}(\mu_1, \Sigma)}$$

❒ **Estimation** – The following table sums up the estimates that we find when maximizing the likelihood:

$\widehat{\phi}$	$\widehat{\mu_j}$ $(j=0,1)$	$\widehat{\Sigma}$
$\frac{1}{m}\sum_{i=1}^{m} 1_{\{y^{(i)}=1\}}$	$\frac{\sum_{i=1}^{m} 1_{\{y^{(i)}=j\}} x^{(i)}}{\sum_{i=1}^{m} 1_{\{y^{(i)}=j\}}}$	$\frac{1}{m}\sum_{i=1}^{m} (x^{(i)} - \mu_{y^{(i)}})(x^{(i)} - \mu_{y^{(i)}})^T$

Naive Bayes

❒ **Assumption** – The Naive Bayes model supposes that the features of each data point are all independent:

$$\boxed{P(x|y) = P(x_1, x_2, ...|y) = P(x_1|y)P(x_2|y)... = \prod_{i=1}^{n} P(x_i|y)}$$

❒ **Solutions** – Maximizing the log-likelihood gives the following solutions, with $k \in \{0,1\}$, $l \in [\![1,L]\!]$

$$\boxed{P(y=k) = \frac{1}{m} \times \#\{j|y^{(j)}=k\}} \quad \text{and} \quad \boxed{P(x_i=l|y=k) = \frac{\#\{j|y^{(j)}=k \text{ and } x_i^{(j)}=l\}}{\#\{j|y^{(j)}=k\}}}$$

Remark: Naive Bayes is widely used for text classification and spam detection.

Tree-based and ensemble methods

These methods can be used for both regression and classification problems.

❒ **CART** – Classification and Regression Trees (CART), commonly known as decision trees, can be represented as binary trees. They have the advantage to be very interpretable.

❒ **Random forest** – It is a tree-based technique that uses a high number of decision trees built out of randomly selected sets of features. Contrary to the simple decision tree, it is highly uninterpretable but its generally good performance makes it a popular algorithm.

Remark: random forests are a type of ensemble methods.

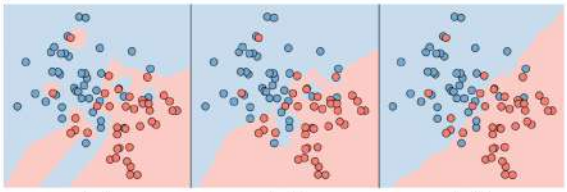
❒ **Boosting** – The idea of boosting methods is to combine several weak learners to form a stronger one. The main ones are summed up in the table below:

Adaptive boosting	Gradient boosting
- High weights are put on errors to improve at the next boosting step - Known as Adaboost	- Weak learners trained on remaining errors

Other non-parametric approaches

❒ **k-nearest neighbors** – The k-nearest neighbors algorithm, commonly known as k-NN, is a non-parametric approach where the response of a data point is determined by the nature of its k neighbors from the training set. It can be used in both classification and regression settings.

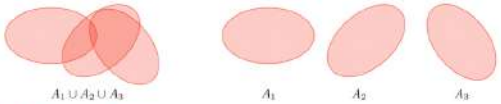
Remark: The higher the parameter k, the higher the bias, and the lower the parameter k, the higher the variance.



Learning Theory

❒ **Union bound** – Let $A_1, ..., A_k$ be k events. We have:

$$\boxed{P(A_1 \cup ... \cup A_k) \leqslant P(A_1) + ... + P(A_k)}$$



❒ **Hoeffding inequality** – Let $Z_1, .., Z_m$ be m iid variables drawn from a Bernoulli distribution of parameter ϕ. Let $\widehat{\phi}$ be their sample mean and $\gamma > 0$ fixed. We have:

$$\boxed{P(|\phi - \widehat{\phi}| > \gamma) \leqslant 2\exp(-2\gamma^2 m)}$$

Remark: this inequality is also known as the Chernoff bound.

❒ **Training error** – For a given classifier h, we define the training error $\widehat{\epsilon}(h)$, also known as the empirical risk or empirical error, to be as follows:

$$\boxed{\widehat{\epsilon}(h) = \frac{1}{m}\sum_{i=1}^{m} 1_{\{h(x^{(i)}) \neq y^{(i)}\}}}$$

❒ **Probably Approximately Correct (PAC)** – PAC is a framework under which numerous results on learning theory were proved, and has the following set of assumptions:

- the training and testing sets follow the same distribution
- the training examples are drawn independently

❒ **Shattering** – Given a set $S = \{x^{(1)}, ..., x^{(d)}\}$, and a set of classifiers $\mathcal{H}$, we say that $\mathcal{H}$ shatters S if for any set of labels $\{y^{(1)}, ..., y^{(d)}\}$, we have:

$$\boxed{\exists h \in \mathcal{H}, \quad \forall i \in [\![1,d]\!], \quad h(x^{(i)}) = y^{(i)}}$$

❒ **Upper bound theorem** – Let $\mathcal{H}$ be a finite hypothesis class such that $|\mathcal{H}| = k$ and let δ and the sample size m be fixed. Then, with probability of at least $1-\delta$, we have:

$$\boxed{\epsilon(\widehat{h}) \leqslant \left(\min_{h \in \mathcal{H}} \epsilon(h)\right) + 2\sqrt{\frac{1}{2m}\log\left(\frac{2k}{\delta}\right)}}$$

❒ **VC dimension** – The Vapnik-Chervonenkis (VC) dimension of a given infinite hypothesis class $\mathcal{H}$, noted $\text{VC}(\mathcal{H})$ is the size of the largest set that is shattered by $\mathcal{H}$.

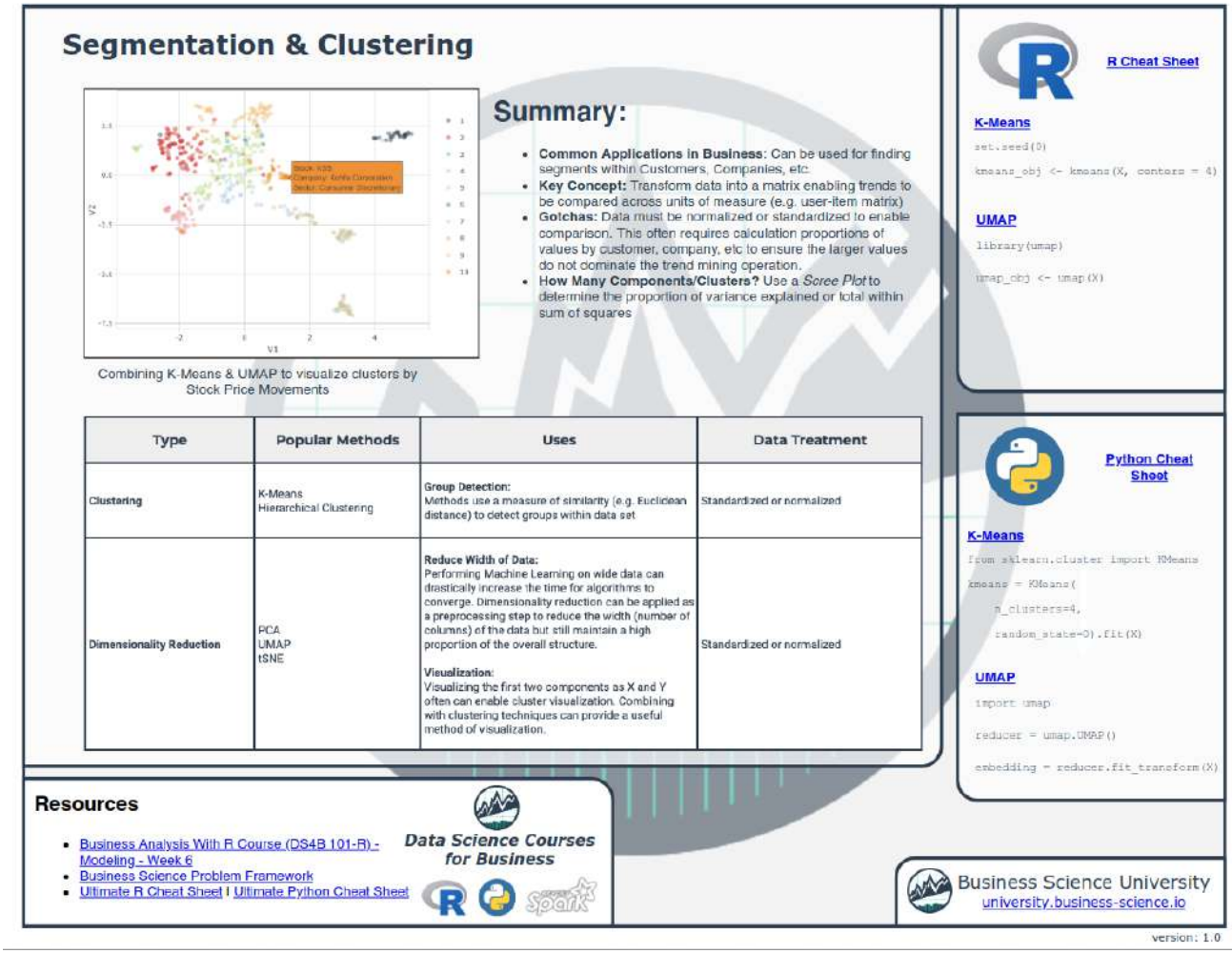
Remark: the VC dimension of $\mathcal{H} =$ {set of linear classifiers in 2 dimensions} is 3.

❒ **Theorem (Vapnik)** – Let $\mathcal{H}$ be given, with $\text{VC}(\mathcal{H}) = d$ and m the number of training examples. With probability at least $1-\delta$, we have:

$$\boxed{\epsilon(\widehat{h}) \leqslant \left(\min_{h \in \mathcal{H}} \epsilon(h)\right) + O\left(\sqrt{\frac{d}{m}\log\left(\frac{m}{d}\right) + \frac{1}{m}\log\left(\frac{1}{\delta}\right)}\right)}$$

Unsupervised Learning

- Segmentation and Clustering (PDF)



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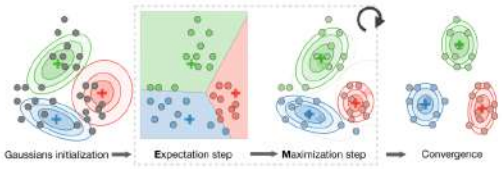
- VIP Unsupervised Learning (PDF)

VIP Cheatsheet: Unsupervised Learning

Afshine Amidi and Shervine Amidi

September 9, 2018



Introduction to Unsupervised Learning

❒ **Motivation** – The goal of unsupervised learning is to find hidden patterns in unlabeled data $\{x^{(1)},...,x^{(m)}\}$.

❒ **Jensen's inequality** – Let f be a convex function and X a random variable. We have the following inequality:

$$\boxed{E[f(X)] \geqslant f(E[X])}$$

Expectation-Maximization

❒ **Latent variables** – Latent variables are hidden/unobserved variables that make estimation problems difficult, and are often denoted z. Here are the most common settings where there are latent variables:

Setting	Latent variable z	$x\|z$	Comments
Mixture of k Gaussians	Multinomial(ϕ)	$\mathcal{N}(\mu_j,\Sigma_j)$	$\mu_j \in \mathbb{R}^n, \phi \in \mathbb{R}^k$
Factor analysis	$\mathcal{N}(0,I)$	$\mathcal{N}(\mu+\Lambda z,\psi)$	$\mu_j \in \mathbb{R}^n$

❒ **Algorithm** – The Expectation-Maximization (EM) algorithm gives an efficient method at estimating the parameter θ through maximum likelihood estimation by repeatedly constructing a lower-bound on the likelihood (E-step) and optimizing that lower bound (M-step) as follows:

- E-step: Evaluate the posterior probability $Q_i(z^{(i)})$ that each data point $x^{(i)}$ came from a particular cluster $z^{(i)}$ as follows:

$$\boxed{Q_i(z^{(i)}) = P(z^{(i)}|x^{(i)};\theta)}$$

- M-step: Use the posterior probabilities $Q_i(z^{(i)})$ as cluster specific weights on data points $x^{(i)}$ to separately re-estimate each cluster model as follows:

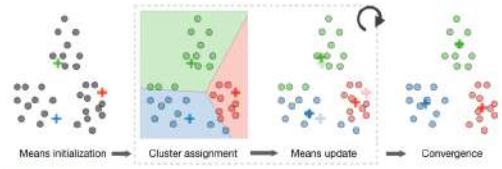
$$\boxed{\theta_i = \underset{\theta}{\text{argmax}} \sum_i \int_{z^{(i)}} Q_i(z^{(i)}) \log\left(\frac{P(x^{(i)},z^{(i)};\theta)}{Q_i(z^{(i)})}\right) dz^{(i)}}$$

k-means clustering

We note $c^{(i)}$ the cluster of data point i and μ_j the center of cluster j.

❒ **Algorithm** – After randomly initializing the cluster centroids $\mu_1,\mu_2,...,\mu_k \in \mathbb{R}^n$, the k-means algorithm repeats the following step until convergence:

$$\boxed{c^{(i)} = \arg\min_j ||x^{(i)} - \mu_j||^2} \quad \text{and} \quad \boxed{\mu_j = \frac{\sum_{i=1}^m 1_{\{c^{(i)}=j\}} x^{(i)}}{\sum_{i=1}^m 1_{\{c^{(i)}=j\}}}}$$



❒ **Distortion function** – In order to see if the algorithm converges, we look at the distortion function defined as follows:

$$\boxed{J(c,\mu) = \sum_{i=1}^m ||x^{(i)} - \mu_{c^{(i)}}||^2}$$

Hierarchical clustering

❒ **Algorithm** – It is a clustering algorithm with an agglomerative hierarchical approach that build nested clusters in a successive manner.

❒ **Types** – There are different sorts of hierarchical clustering algorithms that aims at optimizing different objective functions, which is summed up in the table below:

Ward linkage	Average linkage	Complete linkage
Minimize within cluster distance	Minimize average distance between cluster pairs	Minimize maximum distance of between cluster pairs

Clustering assessment metrics

In an unsupervised learning setting, it is often hard to assess the performance of a model since we don't have the ground truth labels as was the case in the supervised learning setting.

❒ **Silhouette coefficient** – By noting a and b the mean distance between a sample and all other points in the same class, and between a sample and all other points in the next nearest cluster, the silhouette coefficient s for a single sample is defined as follows:

$$\boxed{s = \frac{b-a}{\max(a,b)}}$$

❒ **Calinski-Harabaz index** – By noting k the number of clusters, B_k and W_k the between and within-clustering dispersion matrices respectively defined as

$$B_k = \sum_{j=1}^k n_{c(i)}(\mu_{c(i)} - \mu)(\mu_{c(i)} - \mu)^T, \qquad W_k = \sum_{i=1}^m (x^{(i)} - \mu_{c(i)})(x^{(i)} - \mu_{c(i)})^T$$

the Calinski-Harabaz index $s(k)$ indicates how well a clustering model defines its clusters, such that the higher the score, the more dense and well separated the clusters are. It is defined as follows:

$$\boxed{s(k) = \frac{\text{Tr}(B_k)}{\text{Tr}(W_k)} \times \frac{N-k}{k-1}}$$

Principal component analysis

It is a dimension reduction technique that finds the variance maximizing directions onto which to project the data.

❒ **Eigenvalue, eigenvector** – Given a matrix $A \in \mathbb{R}^{n\times n}$, λ is said to be an eigenvalue of A if there exists a vector $z \in \mathbb{R}^n \backslash \{0\}$, called eigenvector, such that we have:

$$\boxed{Az = \lambda z}$$

❒ **Spectral theorem** – Let $A \in \mathbb{R}^{n\times n}$. If A is symmetric, then A is diagonalizable by a real orthogonal matrix $U \in \mathbb{R}^{n\times n}$. By noting $\Lambda = \text{diag}(\lambda_1,...,\lambda_n)$, we have:

$$\boxed{\exists \Lambda \text{ diagonal}, \quad A = U\Lambda U^T}$$

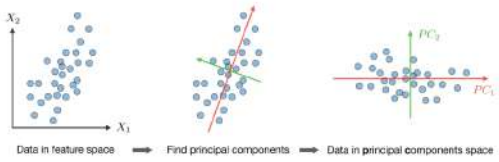
Remark: the eigenvector associated with the largest eigenvalue is called principal eigenvector of matrix A.

❒ **Algorithm** – The Principal Component Analysis (PCA) procedure is a dimension reduction technique that projects the data on k dimensions by maximizing the variance of the data as follows:

- Step 1: Normalize the data to have a mean of 0 and standard deviation of 1.

$$\boxed{x_j^{(i)} \leftarrow \frac{x_j^{(i)} - \mu_j}{\sigma_j}} \quad \text{where} \quad \boxed{\mu_j = \frac{1}{m}\sum_{i=1}^m x_j^{(i)}} \quad \text{and} \quad \boxed{\sigma_j^2 = \frac{1}{m}\sum_{i=1}^m (x_j^{(i)} - \mu_j)^2}$$

- Step 2: Compute $\Sigma = \frac{1}{m}\sum_{i=1}^m x^{(i)}x^{(i)T} \in \mathbb{R}^{n\times n}$, which is symmetric with real eigenvalues.
- Step 3: Compute $u_1,...,u_k \in \mathbb{R}^n$ the k orthogonal principal eigenvectors of Σ, i.e. the orthogonal eigenvectors of the k largest eigenvalues.
- Step 4: Project the data on $\text{span}_{\mathbb{R}}(u_1,...,u_k)$. This procedure maximizes the variance among all k-dimensional spaces.



Independent component analysis

It is a technique meant to find the underlying generating sources.

❒ **Assumptions** – We assume that our data x has been generated by the n-dimensional source vector $s = (s_1,...,s_n)$, where s_i are independent random variables, via a mixing and non-singular matrix A as follows:

$$\boxed{x = As}$$

The goal is to find the unmixing matrix $W = A^{-1}$ by an update rule.

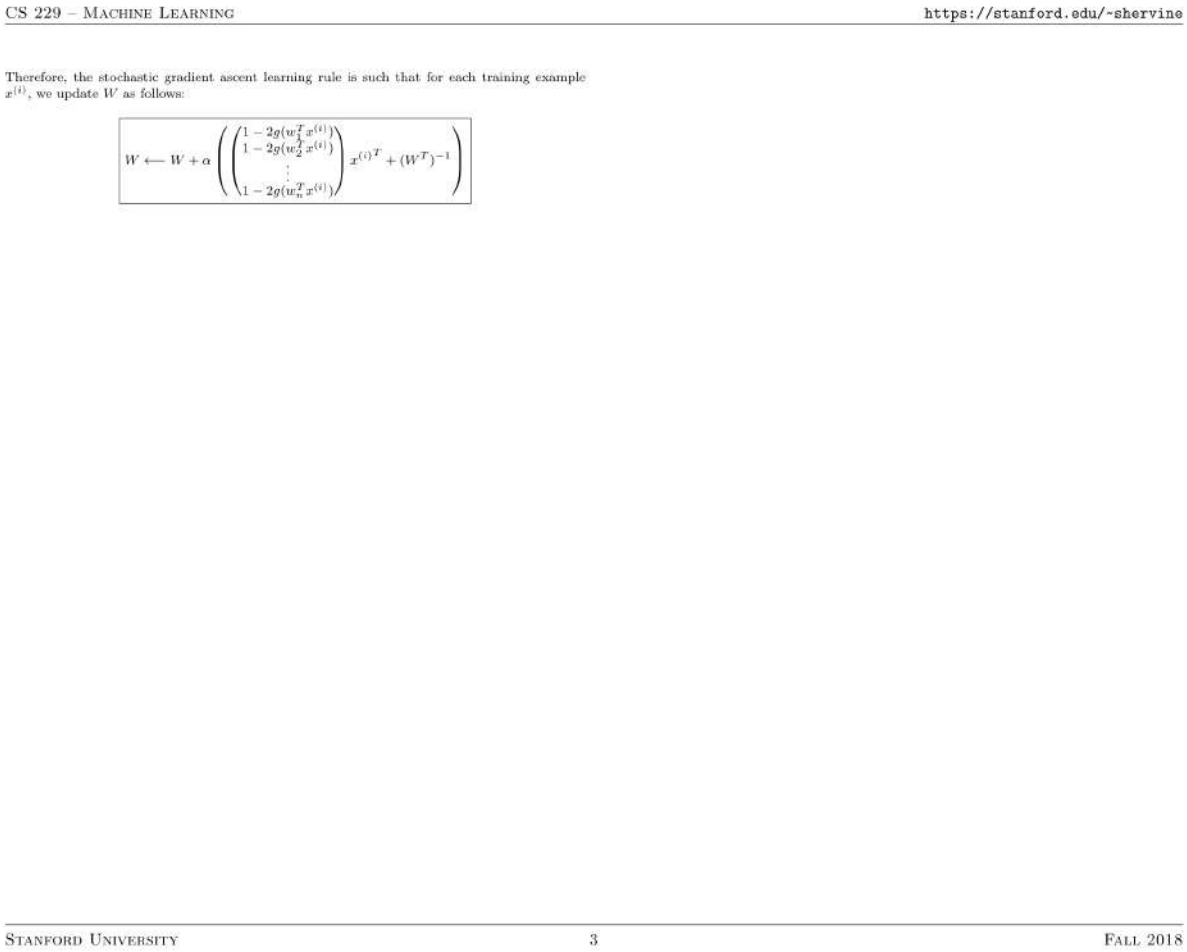
❒ **Bell and Sejnowski ICA algorithm** – This algorithm finds the unmixing matrix W by following the steps below:

- Write the probability of $x = As = W^{-1}s$ as:

$$p(x) = \prod_{i=1}^n p_s(w_i^T x) \cdot |W|$$

- Write the log likelihood given our training data $\{x^{(i)}, i \in [\![1,m]\!]\}$ and by noting g the sigmoid function as:

$$l(W) = \sum_{i=1}^m \left(\sum_{j=1}^n \log\left(g'(w_j^T x^{(i)})\right) + \log|W|\right)$$



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Therefore, the stochastic gradient ascent learning rule is such that for each training example $x^{(i)}$, we update W as follows:

$$W \longleftarrow W + \alpha \left(\begin{pmatrix} 1 - 2g(w_1^T x^{(i)}) \\ 1 - 2g(w_2^T x^{(i)}) \\ \vdots \\ 1 - 2g(w_n^T x^{(i)}) \end{pmatrix} x^{(i)T} + (W^T)^{-1} \right)$$

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Hacks, tricks and tips

From @afshinea:

- VIP Machine Learning Tips and Tricks (PDF)

VIP Cheatsheet: Machine Learning Tips

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September 9, 2018

Metrics

Given a set of data points $\{x^{(1)},...,x^{(m)}\}$, where each $x^{(i)}$ has n features, associated to a set of outcomes $\{y^{(1)},...,y^{(m)}\}$, we want to assess a given classifier that learns how to predict y from x.

Classification

In a context of a binary classification, here are the main metrics that are important to track to assess the performance of the model.

❒ **Confusion matrix** – The confusion matrix is used to have a more complete picture when assessing the performance of a model. It is defined as follows:

Actual class \ Predicted class	+	−
+	**TP** True Positives	**FN** False Negatives Type II error
−	**FP** False Positives Type I error	**TN** True Negatives

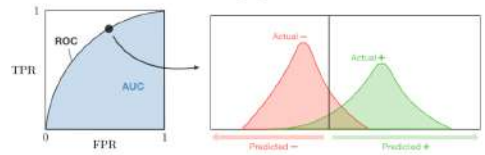
❒ **Main metrics** – The following metrics are commonly used to assess the performance of classification models:

Metric	Formula	Interpretation
Accuracy	$\frac{\text{TP}+\text{TN}}{\text{TP}+\text{TN}+\text{FP}+\text{FN}}$	Overall performance of model
Precision	$\frac{\text{TP}}{\text{TP}+\text{FP}}$	How accurate the positive predictions are
Recall Sensitivity	$\frac{\text{TP}}{\text{TP}+\text{FN}}$	Coverage of actual positive sample
Specificity	$\frac{\text{TN}}{\text{TN}+\text{FP}}$	Coverage of actual negative sample
F1 score	$\frac{2\text{TP}}{2\text{TP}+\text{FP}+\text{FN}}$	Hybrid metric useful for unbalanced classes

❒ **ROC** – The receiver operating curve, also noted ROC, is the plot of TPR versus FPR by varying the threshold. These metrics are are summed up in the table below:

Metric	Formula	Equivalent
True Positive Rate TPR	$\frac{\text{TP}}{\text{TP}+\text{FN}}$	Recall, sensitivity
False Positive Rate FPR	$\frac{\text{FP}}{\text{TN}+\text{FP}}$	1-specificity

❒ **AUC** – The area under the receiving operating curve, also noted AUC or AUROC, is the area below the ROC as shown in the following figure:



Regression

❒ **Basic metrics** – Given a regression model f, the following metrics are commonly used to assess the performance of the model:

Total sum of squares	Explained sum of squares	Residual sum of squares
$\text{SS}_{\text{tot}}=\sum_{i=1}^{m}(y_i-\overline{y})^2$	$\text{SS}_{\text{reg}}=\sum_{i=1}^{m}(f(x_i)-\overline{y})^2$	$\text{SS}_{\text{res}}=\sum_{i=1}^{m}(y_i-f(x_i))^2$

❒ **Coefficient of determination** – The coefficient of determination, often noted R^2 or r^2, provides a measure of how well the observed outcomes are replicated by the model and is defined as follows:

$$R^2=1-\frac{\text{SS}_{\text{res}}}{\text{SS}_{\text{tot}}}$$

❒ **Main metrics** – The following metrics are commonly used to assess the performance of regression models, by taking into account the number of variables n that they take into consideration:

Mallow's Cp	AIC	BIC	Adjusted R^2
$\frac{\text{SS}_{\text{res}}+2(n+1)\widetilde{\sigma}^2}{m}$	$2\left[(n+2)-\log(L)\right]$	$\log(m)(n+2)-2\log(L)$	$1-\frac{(1-R^2)(m-1)}{m-n-1}$

where L is the likelihood and $\widetilde{\sigma}^2$ is an estimate of the variance associated with each response.

Model selection

❒ **Vocabulary** – When selecting a model, we distinguish 3 different parts of the data that we have as follows:

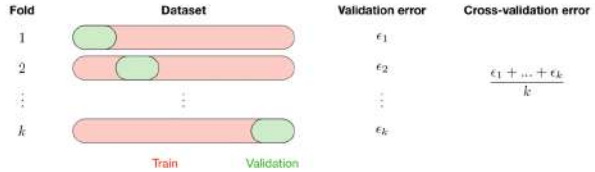
Training set	Validation set	Testing set
- Model is trained - Usually 80% of the dataset	- Model is assessed - Usually 20% of the dataset - Also called hold-out or development set	- Model gives predictions - Unseen data

Once the model has been chosen, it is trained on the entire dataset and tested on the unseen test set. These are represented in the figure below:

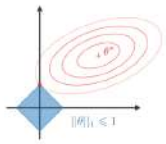
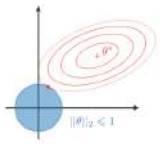
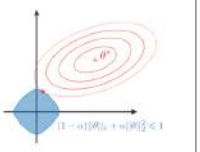
❒ **Cross-validation** – Cross-validation, also noted CV, is a method that is used to select a model that does not rely too much on the initial training set. The different types are summed up in the table below:

k-fold	Leave-p-out
- Training on $k-1$ folds and assessment on the remaining one - Generally $k=5$ or 10	- Training on $n-p$ observations and assessment on the p remaining ones - Case $p=1$ is called leave-one-out

The most commonly used method is called k-fold cross-validation and splits the training data into k folds to validate the model on one fold while training the model on the $k-1$ other folds, all of this k times. The error is then averaged over the k folds and is named cross-validation error.



❒ **Regularization** – The regularization procedure aims at avoiding the model to overfit the data and thus deals with high variance issues. The following table sums up the different types of commonly used regularization techniques:

LASSO	Ridge	Elastic Net
- Shrinks coefficients to 0 - Good for variable selection	Makes coefficients smaller	Tradeoff between variable selection and small coefficients
		
$...+\lambda\|\theta\|_1$ $\lambda\in\mathbb{R}$	$...+\lambda\|\theta\|_2^2$ $\lambda\in\mathbb{R}$	$...+\lambda\left[(1-\alpha)\|\theta\|_1+\alpha\|\theta\|_2^2\right]$ $\lambda\in\mathbb{R},\quad\alpha\in[0,1]$

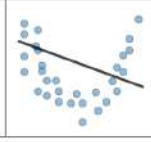
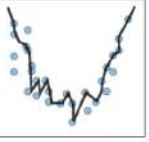
❒ **Model selection** – Train model on training set, then evaluate on the development set, then pick best performance model on the development set, and retrain all of that model on the whole training set.

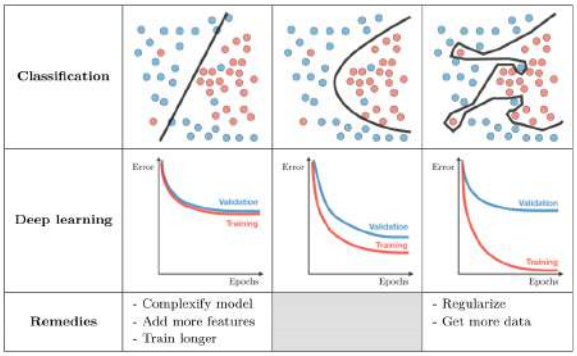
Diagnostics

❒ **Bias** – The bias of a model is the difference between the expected prediction and the correct model that we try to predict for given data points.

❒ **Variance** – The variance of a model is the variability of the model prediction for given data points.

❒ **Bias/variance tradeoff** – The simpler the model, the higher the bias, and the more complex the model, the higher the variance.

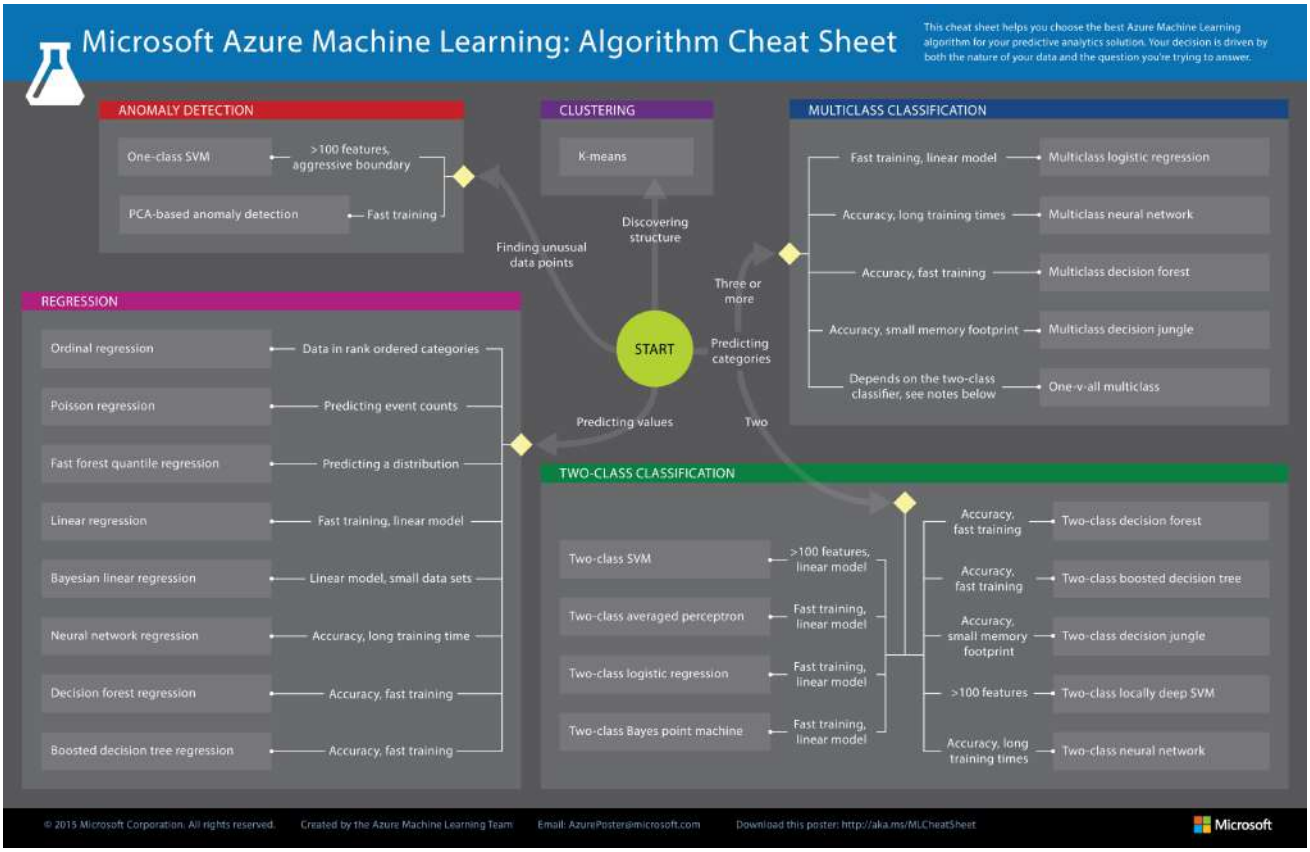
	Underfitting	Just right	Overfitting
Symptoms	- High training error - Training error close to test error - High bias	- Training error slightly lower than test error	- Low training error - Training error much lower than test error - High variance
Regression			

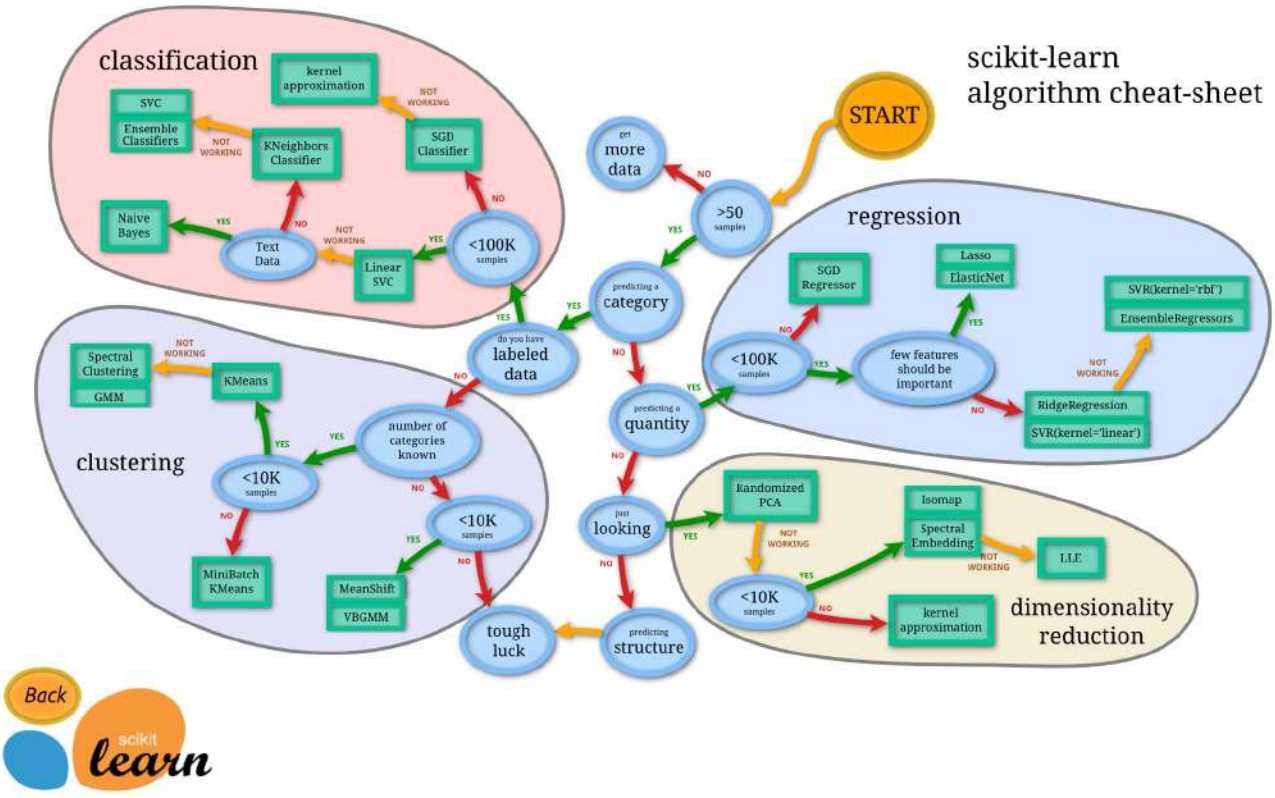


❒ **Error analysis** – Error analysis is analyzing the root cause of the difference in performance between the current and the perfect models.

❒ **Ablative analysis** – Ablative analysis is analyzing the root cause of the difference in performance between the current and the baseline models.

Choosing the right model





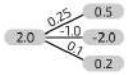
Deep Learning

Neural Nets

An informative chart to build

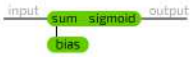
Neural Network Cells

Fixed Weight
(fixed at 1)

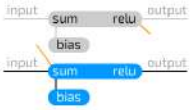


Weights

Recurrent Weights
(grouped by colour)

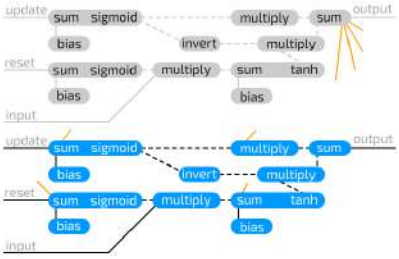


Feed Forward Cell
(basic cell)



Recurrent Cell
(previous iteration)

Recurrent Cell



GRU Cell
(previous iteration)

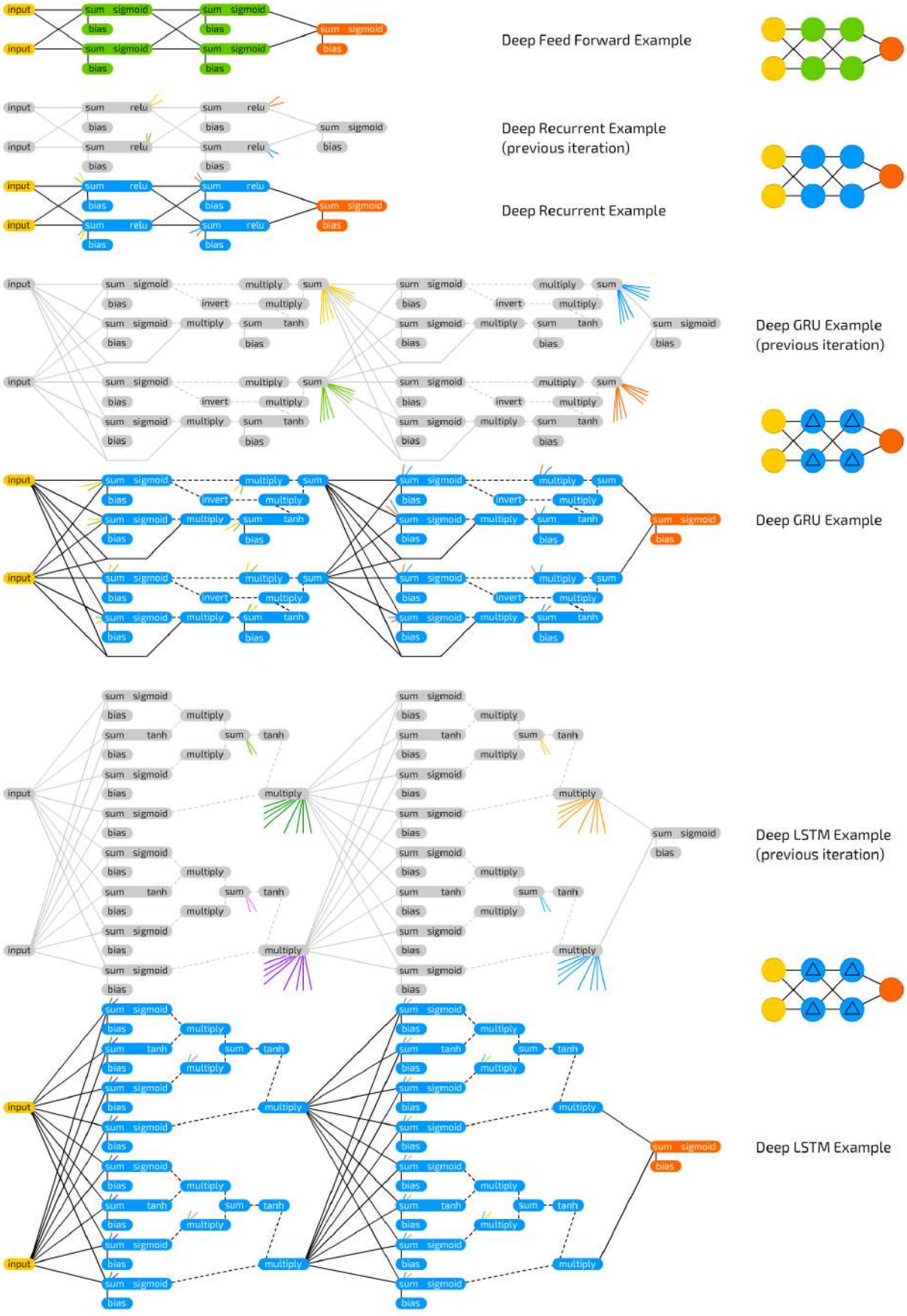
GRU Cell

LSTM Cell
(previous iteration)

LSTM Cell

An informative chart to build

Neural Network Graphs

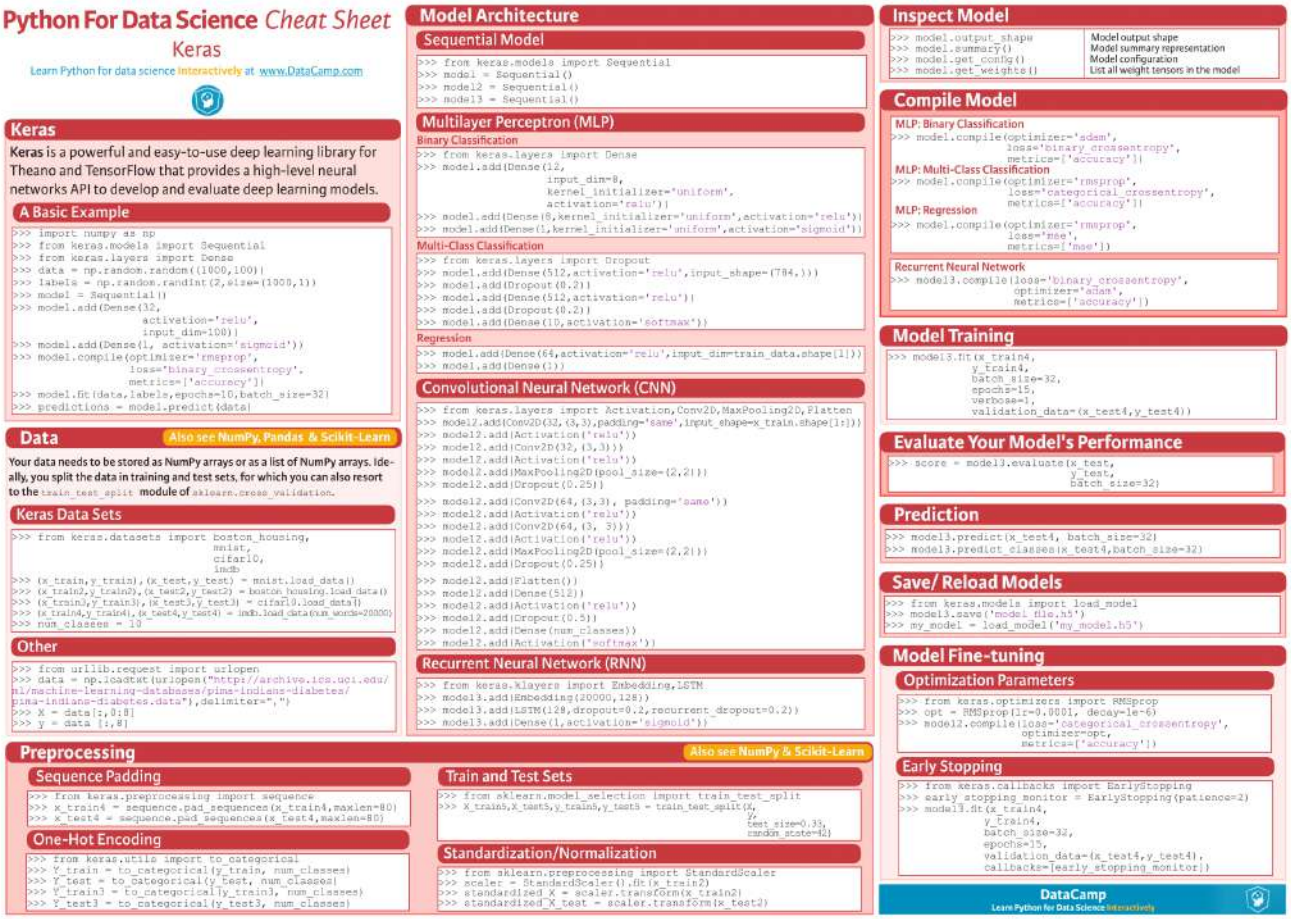


R

- Keras RStudio (PDF)

Python

- Keras (PDF)



From @afshinea:

- Deep Learning Basics (PDF)

VIP Cheatsheet: Deep Learning

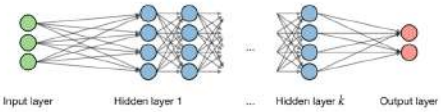
Afshine Amidi and Shervine Amidi

September 15, 2018

Neural Networks

Neural networks are a class of models that are built with layers. Commonly used types of neural networks include convolutional and recurrent neural networks.

Architecture – The vocabulary around neural networks architectures is described in the figure below:

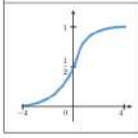
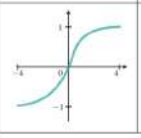
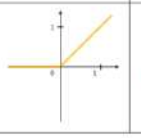
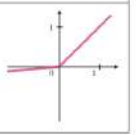


By noting i the i^{th} layer of the network and j the j^{th} hidden unit of the layer, we have:

$$\boxed{z_j^{[i]} = w_j^{[i]T} x + b_j^{[i]}}$$

where we note w, b, z the weight, bias and output respectively.

Activation function – Activation functions are used at the end of a hidden unit to introduce non-linear complexities to the model. Here are the most common ones:

Sigmoid	Tanh	ReLU	Leaky ReLU
$g(z) = \frac{1}{1+e^{-z}}$	$g(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$	$g(z) = \max(0,z)$	$g(z) = \max(\epsilon z, z)$ with $\epsilon \ll 1$
			

Cross-entropy loss – In the context of neural networks, the cross-entropy loss $L(z,y)$ is commonly used and is defined as follows:

$$\boxed{L(z,y) = -\Big[y\log(z) + (1-y)\log(1-z)\Big]}$$

Learning rate – The learning rate, often noted η, indicates at which pace the weights get updated. This can be fixed or adaptively changed. The current most popular method is called Adam, which is a method that adapts the learning rate.

Backpropagation – Backpropagation is a method to update the weights in the neural network by taking into account the actual output and the desired output. The derivative with respect to weight w is computed using chain rule and is of the following form:

$$\boxed{\frac{\partial L(z,y)}{\partial w} = \frac{\partial L(z,y)}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial w}}$$

As a result, the weight is updated as follows:

$$\boxed{w \longleftarrow w - \eta \frac{\partial L(z,y)}{\partial w}}$$

Updating weights – In a neural network, weights are updated as follows:

- Step 1: Take a batch of training data.
- Step 2: Perform forward propagation to obtain the corresponding loss.
- Step 3: Backpropagate the loss to get the gradients.
- Step 4: Use the gradients to update the weights of the network.

Dropout – Dropout is a technique meant at preventing overfitting the training data by dropping out units in a neural network. In practice, neurons are either dropped with probability p or kept with probability $1-p$.

Convolutional Neural Networks

Convolutional layer requirement – By noting W the input volume size, F the size of the convolutional layer neurons, P the amount of zero padding, then the number of neurons N that fit in a given volume is such that:

$$\boxed{N = \frac{W - F + 2P}{S} + 1}$$

Batch normalization – It is a step of hyperparameter γ, β that normalizes the batch $\{x_i\}$. By noting μ_B, σ_B^2 the mean and variance of that we want to correct to the batch, it is done as follows:

$$\boxed{x_i \longleftarrow \gamma \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}} + \beta}$$

It is usually done after a fully connected/convolutional layer and before a non-linearity layer and aims at allowing higher learning rates and reducing the strong dependence on initialization.

Recurrent Neural Networks

Types of gates – Here are the different types of gates that we encounter in a typical recurrent neural network:

Input gate	Forget gate	Output gate	Gate
Write to cell or not?	Erase a cell or not?	Reveal a cell or not?	How much writing?

LSTM – A long short-term memory (LSTM) network is a type of RNN model that avoids the vanishing gradient problem by adding 'forget' gates.

Reinforcement Learning and Control

The goal of reinforcement learning is for an agent to learn how to evolve in an environment.

Markov decision processes – A Markov decision process (MDP) is a 5-tuple $(S, A, \{P_{sa}\}, \gamma, R)$ where:

- $\mathcal{S}$ is the set of states
- $\mathcal{A}$ is the set of actions
- $\{P_{sa}\}$ are the state transition probabilities for $s \in \mathcal{S}$ and $a \in \mathcal{A}$
- $\gamma \in [0,1[$ is the discount factor
- $R : \mathcal{S} \times \mathcal{A} \longrightarrow \mathbb{R}$ or $R : \mathcal{S} \longrightarrow \mathbb{R}$ is the reward function that the algorithm wants to maximize

Policy – A policy π is a function $\pi : \mathcal{S} \longrightarrow \mathcal{A}$ that maps states to actions.

Remark: we say that we execute a given policy π if given a state s we take the action $a = \pi(s)$.

Value function – For a given policy π and a given state s, we define the value function V^π as follows:

$$\boxed{V^\pi(s) = E\Big[R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + ...|s_0 = s, \pi\Big]}$$

Bellman equation – The optimal Bellman equations characterizes the value function V^{π^*} of the optimal policy π^*:

$$\boxed{V^{\pi^*}(s) = R(s) + \max_{a \in \mathcal{A}} \gamma \sum_{s' \in S} P_{sa}(s') V^{\pi^*}(s')}$$

Remark: we note that the optimal policy π^* for a given state s is such that:

$$\boxed{\pi^*(s) = \underset{a \in \mathcal{A}}{\text{argmax}} \sum_{s' \in \mathcal{S}} P_{sa}(s') V^*(s')}$$

Value iteration algorithm – The value iteration algorithm is in two steps:

- We initialize the value:

$$\boxed{V_0(s) = 0}$$

- We iterate the value based on the values before:

$$\boxed{V_{i+1}(s) = R(s) + \max_{a \in \mathcal{A}} \left[\sum_{s' \in \mathcal{S}} \gamma P_{sa}(s') V_i(s')\right]}$$

Maximum likelihood estimate – The maximum likelihood estimates for the state transition probabilities are as follows:

$$\boxed{P_{sa}(s') = \frac{\#\text{times took action } a \text{ in state } s \text{ and got to } s'}{\#\text{times took action } a \text{ in state } s}}$$

Q-learning – Q-learning is a model-free estimation of Q, which is done as follows:

$$\boxed{Q(s,a) \leftarrow Q(s,a) + \alpha\Big[R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)\Big]}$$

- Convolutional Neural Networkds (PDF)

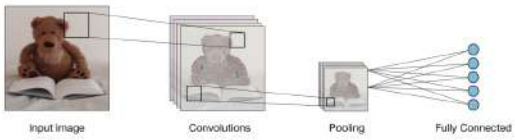
VIP Cheatsheet: Convolutional Neural Networks

Afshine Amidi and Shervine Amidi

November 26, 2018

Overview

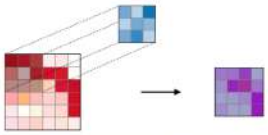
❒ **Architecture of a traditional CNN** – Convolutional neural networks, also known as CNNs, are a specific type of neural networks that are generally composed of the following layers:



The convolution layer and the pooling layer can be fine-tuned with respect to hyperparameters that are described in the next sections.

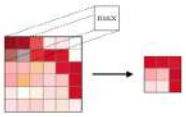
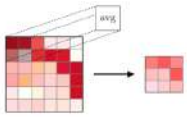
Types of layer

❒ **Convolutional layer (CONV)** – The convolution layer (CONV) uses filters that perform convolution operations as it is scanning the input I with respect to its dimensions. Its hyperparameters include the filter size F and stride S. The resulting output O is called *feature map* or *activation map*.

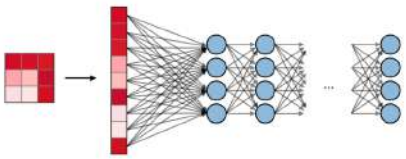


Remark: the convolution step can be generalized to the 1D and 3D cases as well.

❒ **Pooling (POOL)** – The pooling layer (POOL) is a downsampling operation, typically applied after a convolution layer, which does some spatial invariance. In particular, max and average pooling are special kinds of pooling where the maximum and average value is taken, respectively.

	Max pooling	Average pooling
Purpose	Each pooling operation selects the maximum value of the current view	Each pooling operation averages the values of the current view
Illustration		
Comments	- Preserves detected features - Most commonly used	- Downsamples feature map - Used in LeNet

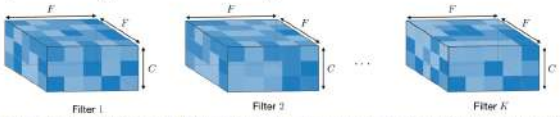
❒ **Fully Connected (FC)** – The fully connected layer (FC) operates on a flattened input where each input is connected to all neurons. If present, FC layers are usually found towards the end of CNN architectures and can be used to optimize objectives such as class scores.



Filter hyperparameters

The convolution layer contains filters for which it is important to know the meaning behind its hyperparameters.

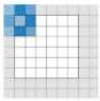
❒ **Dimensions of a filter** – A filter of size $F \times F$ applied to an input containing C channels is a $F \times F \times C$ volume that performs convolutions on an input of size $I \times I \times C$ and produces an output feature map (also called activation map) of size $O \times O \times 1$.



Remark: the application of K filters of size $F \times F$ results in an output feature map of size $O \times O \times K$.

❒ **Stride** – For a convolutional or a pooling operation, the stride S denotes the number of pixels by which the window moves after each operation.

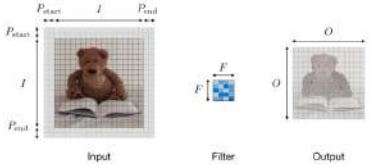
❒ **Zero-padding** – Zero-padding denotes the process of adding P zeroes to each side of the boundaries of the input. This value can either be manually specified or automatically set through one of the three modes detailed below:

	Valid	Same	Full
Value	$P = 0$	$P_{\text{start}} = \left\lfloor \frac{S\lceil \frac{I}{S} \rceil - I + F - S}{2} \right\rfloor$ $P_{\text{end}} = \left\lceil \frac{S\lceil \frac{I}{S} \rceil - I + F - S}{2} \right\rceil$	$P_{\text{start}} \in [\![0, F-1]\!]$ $P_{\text{end}} = F - 1$
Illustration			
Purpose	- No padding - Drops last convolution if dimensions do not match	- Padding such that feature map size has size $\left\lceil \frac{I}{S} \right\rceil$ - Output size is mathematically convenient - Also called 'half' padding	- Maximum padding such that end convolutions are applied on the limits of the input - Filter 'sees' the input end-to-end

Tuning hyperparameters

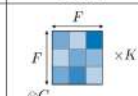
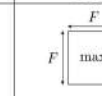
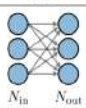
❒ **Parameter compatibility in convolution layer** – By noting I the length of the input volume size, F the length of the filter, P the amount of zero padding, S the stride, then the output size O of the feature map along that dimension is given by:

$$O = \frac{I - F + P_{\text{start}} + P_{\text{end}}}{S} + 1$$



Remark: often times, $P_{start} = P_{end} \triangleq P$, in which case we can replace $P_{start} + P_{end}$ by $2P$ in the formula above.

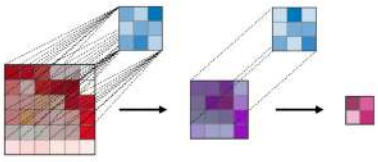
❒ **Understanding the complexity of the model** – In order to assess the complexity of a model, it is often useful to determine the number of parameters that its architecture will have. In a given layer of a convolutional neural network, it is done as follows:

	CONV	POOL	FC
Illustration			
Input size	$I \times I \times C$	$I \times I \times C$	N_{in}
Output size	$O \times O \times K$	$O \times O \times C$	N_{out}
Number of parameters	$(F \times F \times C + 1) \cdot K$	0	$(N_{\text{in}} + 1) \times N_{\text{out}}$
Remarks	- One bias parameter per filter - In most cases, $S < F$ - A common choice for K is $2C$	- Pooling operation done channel-wise - In most cases, $S = F$	- Input is flattened - One bias parameter per neuron - The number of FC neurons is free of structural constraints

❒ **Receptive field** – The receptive field at layer k is the area denoted $R_k \times R_k$ of the input that each pixel of the k-th activation map can 'see'. By calling F_j the filter size of layer j and S_i the stride value of layer i and with the convention $S_0 = 1$, the receptive field at layer k can be computed with the formula:

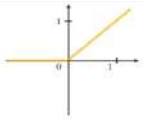
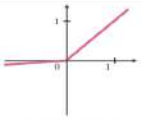
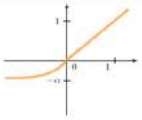
$$R_k = 1 + \sum_{j=1}^{k} (F_j - 1) \prod_{i=0}^{j-1} S_i$$

In the example below, we have $F_1 = F_2 = 3$ and $S_1 = S_2 = 1$, which gives $R_2 = 1+2 \cdot 1+2 \cdot 1 = 5$.



Commonly used activation functions

❒ **Rectified Linear Unit** – The rectified linear unit layer (ReLU) is an activation function g that is used on all elements of the volume. It aims at introducing non-linearities to the network. Its variants are summarized in the table below:

ReLU	Leaky ReLU	ELU
$g(z) = \max(0,z)$	$g(z) = \max(\epsilon z, z)$ with $\epsilon \ll 1$	$g(z) = \max(\alpha(e^z - 1), z)$ with $\alpha \ll 1$
		
Non-linearity complexities biologically interpretable	Addresses dying ReLU issue for negative values	Differentiable everywhere

❒ **Softmax** – The softmax step can be seen as a generalized logistic function that takes as input a vector of scores $x \in \mathbb{R}^n$ and outputs a vector of output probability $p \in \mathbb{R}^n$ through a softmax function at the end of the architecture. It is defined as follows:

$$p = \begin{pmatrix} p_1 \\ \vdots \\ p_n \end{pmatrix} \quad \text{where} \quad p_i = \frac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}$$

Object detection

❒ **Types of models** – There are 3 main types of object recognition algorithms, for which the nature of what is predicted is different. They are described in the table below:

Image classification	Classification w. localization	Detection
		
- Classifies a picture - Predicts probability of object	- Detects object in a picture - Predicts probability of object and where it is located	- Detects up to several objects in a picture - Predicts probabilities of objects and where they are located
Traditional CNN	Simplified YOLO, R-CNN	YOLO, R-CNN

❒ **Detection** – In the context of object detection, different methods are used depending on whether we just want to locate the object or detect a more complex shape in the image. The two main ones are summed up in the table below:

Bounding box detection	Landmark detection
Detects the part of the image where the object is located	- Detects a shape or characteristics of an object (e.g. eyes) - More granular
	
Box of center (b_x, b_y), height b_h and width b_w	Reference points (l_{1x}, l_{1y}), ..., (l_{nx}, l_{ny})

❒ **Intersection over Union** – Intersection over Union, also known as IoU, is a function that quantifies how correctly positioned a predicted bounding box B_p is over the actual bounding box B_a. It is defined as:

$$\text{IoU}(B_p, B_a) = \frac{B_p \cap B_a}{B_p \cup B_a}$$

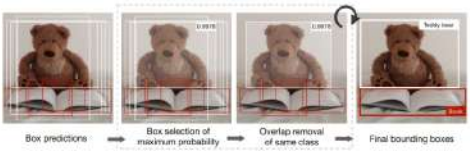
$\text{IoU}(B_p, B_a) = 0.1$ $\text{IoU}(B_p, B_a) = 0.5$ $\text{IoU}(B_p, B_a) = 0.9$

Remark: we always have $IoU \in [0,1]$. By convention, a predicted bounding box B_p is considered as being reasonably good if $IoU(B_p, B_a) \geqslant 0.5$.

❒ **Anchor boxes** – Anchor boxing is a technique used to predict overlapping bounding boxes. In practice, the network is allowed to predict more than one box simultaneously, where each box prediction is constrained to have a given set of geometrical properties. For instance, the first prediction can potentially be a rectangular box of a given form, while the second will be another rectangular box of a different geometrical form.

❒ **Non-max suppression** – The non-max suppression technique aims at removing duplicate overlapping bounding boxes of a same object by selecting the most representative ones. After having removed all boxes having a probability prediction lower than 0.6, the following steps are repeated while there are boxes remaining:

- Step 1: Pick the box with the largest prediction probability.
- Step 2: Discard any box having an IoU $\geqslant 0.5$ with the previous box.



Box predictions → Box selection of maximum probability → Overlap removal of same class → Final bounding boxes

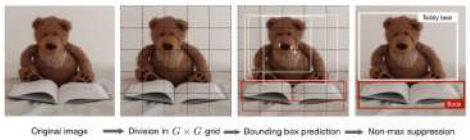
❒ **YOLO** – You Only Look Once (YOLO) is an object detection algorithm that performs the following steps:

- Step 1: Divide the input image into a $G \times G$ grid.
- Step 2: For each grid cell, run a CNN that predicts y of the following form:

$$y = \Big[\underbrace{p_c, b_x, b_y, b_h, b_w, c_1, c_2, \ldots, c_p}_{\text{repeated } k \text{ times}}, \ldots\Big]^T \in \mathbb{R}^{G \times G \times k \times (5+p)}$$

where p_c is the probability of detecting an object, b_x, b_y, b_h, b_w are the properties of the detected bouding box, $c_1, \ldots, c_p$ is a one-hot representation of which of the p classes were detected, and k is the number of anchor boxes.

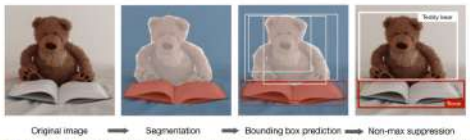
- Step 3: Run the non-max suppression algorithm to remove any potential duplicate overlapping bounding boxes.



Original image → Division in $G \times G$ grid → Bounding box prediction → Non-max suppression

Remark: when $p_c = 0$, then the network does not detect any object. In that case, the corresponding predictions $b_x, \ldots, c_p$ have to be ignored.

❒ **R-CNN** – Region with Convolutional Neural Networks (R-CNN) is an object detection algorithm that first segments the image to find potential relevant bounding boxes and then run the detection algorithm to find most probable objects in those bounding boxes.

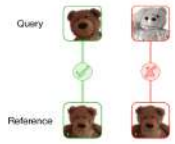
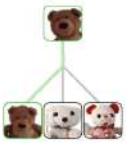


Original image → Segmentation → Bounding box prediction → Non-max suppression

Remark: although the original algorithm is computationally expensive and slow, newer architectures enabled the algorithm to run faster, such as Fast R-CNN and Faster R-CNN.

Face verification and recognition

❒ **Types of models** – Two main types of model are summed up in table below:

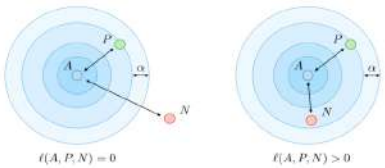
Face verification	Face recognition
- Is this the correct person? - One-to-one lookup	- Is this one of the K persons in the database? - One-to-many lookup
	

❒ **One Shot Learning** – One Shot Learning is a face verification algorithm that uses a limited training set to learn a similarity function that quantifies how different two given images are. The similarity function applied to two images is often noted $d(\text{image 1}, \text{image 2})$.

❒ **Siamese Network** – Siamese Networks aim at learning how to encode images to then quantify how different two images are. For a given input image $x^{(i)}$, the encoded output is often noted as $f(x^{(i)})$.

❒ **Triplet loss** – The triplet loss ℓ is a loss function computed on the embedding representation of a triplet of images A (anchor), P (positive) and N (negative). The anchor and the positive example belong to a same class, while the negative example to another one. By calling $\alpha \in \mathbb{R}^+$ the margin parameter, this loss is defined as follows:

$$\ell(A,P,N) = \max\left(d(A,P) - d(A,N) + \alpha, 0\right)$$



$\ell(A,P,N) = 0$ $\ell(A,P,N) > 0$

Neural style transfer

❒ **Motivation** – The goal of neural style transfer is to generate an image G based on a given content C and a given style S.

❒ **Activation** – In a given layer l, the activation is noted $a^{[l]}$ and is of dimensions $n_H \times n_w \times n_c$

❒ **Content cost function** – The content cost function $J_{\text{content}}(C,G)$ is used to determine how the generated image G differs from the original content image C. It is defined as follows:

$$J_{\text{content}}(C,G) = \frac{1}{2}||a^{[l](C)} - a^{[l](G)}||^2$$

❒ **Style matrix** – The style matrix $G^{[l]}$ of a given layer l is a Gram matrix where each of its elements $G^{[l]}_{kk'}$ quantifies how correlated the channels k and k' are. It is defined with respect to activations $a^{[l]}$ as follows:

$$G^{[l]}_{kk'} = \sum_{i=1}^{n_H^{[l]}} \sum_{j=1}^{n_w^{[l]}} a^{[l]}_{ijk} a^{[l]}_{ijk'}$$

Remark: the style matrix for the style image and the generated image are noted $G^{[l](S)}$ and $G^{[l](G)}$ respectively.

❒ **Style cost function** – The style cost function $J_{\text{style}}(S,G)$ is used to determine how the generated image G differs from the style S. It is defined as follows:

$$J^{[l]}_{\text{style}}(S,G) = \frac{1}{(2n_H n_w n_c)^2}||G^{[l](S)} - G^{[l](G)}||^2_F = \frac{1}{(2n_H n_w n_c)^2} \sum_{k,k'=1}^{n_c} \left(G^{[l](S)}_{kk'} - G^{[l](G)}_{kk'}\right)^2$$

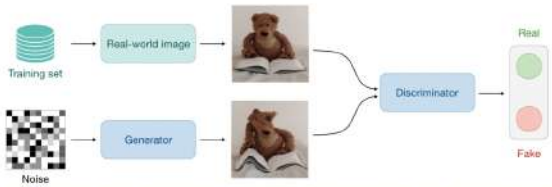
❒ **Overall cost function** – The overall cost function is defined as being a combination of the content and style cost functions, weighted by parameters α, β, as follows:

$$J(G) = \alpha J_{\text{content}}(C,G) + \beta J_{\text{style}}(S,G)$$

Remark: a higher value of α will make the model care more about the content while a higher value of β will make it care more about the style.

Architectures using computational tricks

❒ **Generative Adversarial Network** – Generative adversarial networks, also known as GANs, are composed of a generative and a discriminative model, where the generative model aims at generating the most truthful output that will be fed into the discriminative which aims at differentiating the generated and true image.



Remark: use cases using variants of GANs include text to image, music generation and synthesis.

❒ **ResNet** – The Residual Network architecture (also called ResNet) uses residual blocks with a high number of layers meant to decrease the training error. The residual block has the following characterizing equation:

$$a^{[l+2]} = g(a^{[l]} + z^{[l+2]})$$

❒ **Inception Network** – This architecture uses inception modules and aims at giving a try at different convolutions in order to increase its performance. In particular, it uses the 1×1 convolution trick to lower the burden of computation.

⋆ ⋆ ⋆

- Recurrent Neural Networks (PDF)

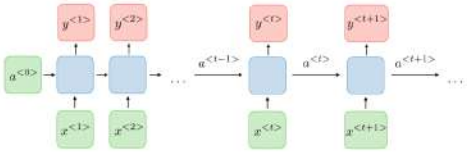
VIP Cheatsheet: Recurrent Neural Networks

Afshine Amidi and Shervine Amidi

November 26, 2018

Overview

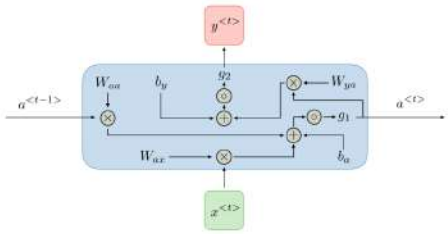
❒ **Architecture of a traditional RNN** – Recurrent neural networks, also known as RNNs, are a class of neural networks that allow previous outputs to be used as inputs while having hidden states. They are typically as follows:



For each timestep t, the activation $a^{<t>}$ and the output $y^{<t>}$ are expressed as follows:

$$a^{<t>} = g_1(W_{aa}a^{<t-1>} + W_{ax}x^{<t>} + b_a) \quad \text{and} \quad y^{<t>} = g_2(W_{ya}a^{<t>} + b_y)$$

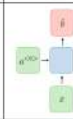
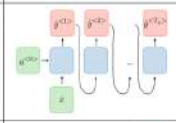
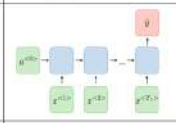
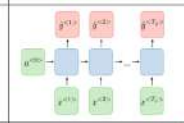
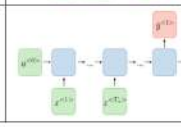
where $W_{ax}, W_{aa}, W_{ya}, b_a, b_y$ are coefficients that are shared temporally and g_1, g_2 activation functions



The pros and cons of a typical RNN architecture are summed up in the table below:

Advantages	Drawbacks
- Possibility of processing input of any length - Model size not increasing with size of input - Computation takes into account historical information - Weights are shared across time	- Computation being slow - Difficulty of accessing information from a long time ago - Cannot consider any future input for the current state

❒ **Applications of RNNs** – RNN models are mostly used in the fields of natural language processing and speech recognition. The different applications are summed up in the table below:

Type of RNN	Illustration	Example
One-to-one $T_x = T_y = 1$		Traditional neural network
One-to-many $T_x = 1, T_y > 1$		Music generation
Many-to-one $T_x > 1, T_y = 1$		Sentiment classification
Many-to-many $T_x = T_y$		Name entity recognition
Many-to-many $T_x \neq T_y$		Machine translation

❒ **Loss function** – In the case of a recurrent neural network, the loss function $\mathcal{L}$ of all time

steps is defined based on the loss at every time step as follows:

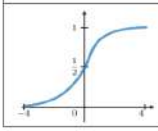
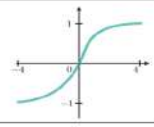
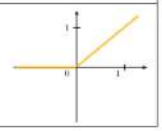
$$\mathcal{L}(\widehat{y}, y) = \sum_{t=1}^{T_y} \mathcal{L}(\widehat{y}^{<t>}, y^{<t>})$$

❒ **Backpropagation through time** – Backpropagation is done at each point in time. At timestep T, the derivative of the loss $\mathcal{L}$ with respect to weight matrix W is expressed as follows:

$$\frac{\partial \mathcal{L}^{(T)}}{\partial W} = \sum_{t=1}^{T} \frac{\partial \mathcal{L}^{(T)}}{\partial W}\bigg|_{(t)}$$

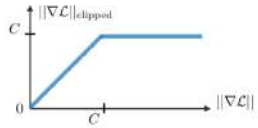
Handling long term dependencies

❒ **Commonly used activation functions** – The most common activation functions used in RNN modules are described below:

Sigmoid	Tanh	RELU
$g(z) = \frac{1}{1+e^{-z}}$	$g(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$	$g(z) = \max(0, z)$
		

❒ **Vanishing/exploding gradient** – The vanishing and exploding gradient phenomena are often encountered in the context of RNNs. The reason why they happen is that it is difficult to capture long term dependencies because of multiplicative gradient that can be exponentially decreasing/increasing with respect to the number of layers.

❒ **Gradient clipping** – It is a technique used to cope with the exploding gradient problem sometimes encountered when performing backpropagation. By capping the maximum value for the gradient, this phenomenon is controlled in practice.



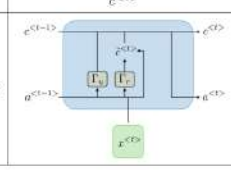
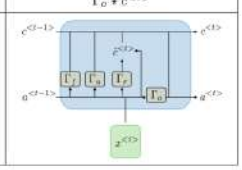
❒ **Types of gates** – In order to remedy the vanishing gradient problem, specific gates are used in some types of RNNs and usually have a well-defined purpose. They are usually noted Γ and are equal to:

$$\Gamma = \sigma(Wx^{<t>} + Ua^{<t-1>} + b)$$

where W, U, b are coefficients specific to the gate and σ is the sigmoid function. The main ones are summed up in the table below:

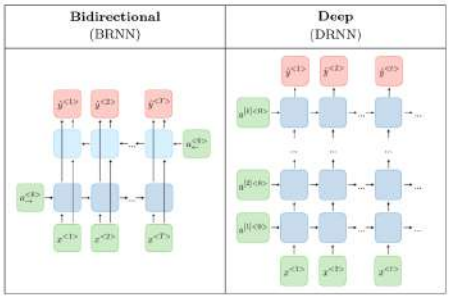
Type of gate	Role	Used in
Update gate Γ_u	How much past should matter now?	GRU, LSTM
Relevance gate Γ_r	Drop previous information?	GRU, LSTM
Forget gate Γ_f	Erase a cell or not?	LSTM
Output gate Γ_o	How much to reveal of a cell?	LSTM

❒ **GRU/LSTM** – Gated Recurrent Unit (GRU) and Long Short-Term Memory units (LSTM) deal with the vanishing gradient problem encountered by traditional RNNs, with LSTM being a generalization of GRU. Below is a table summing up the characterizing equations of each architecture:

	Gated Recurrent Unit (GRU)	Long Short-Term Memory (LSTM)
$\tilde{c}^{<t>}$	$\tanh(W_c[\Gamma_r \star a^{<t-1>}, x^{<t>}] + b_c)$	$\tanh(W_c[\Gamma_r \star a^{<t-1>}, x^{<t>}] + b_c)$
$c^{<t>}$	$\Gamma_u \star \tilde{c}^{<t>} + (1 - \Gamma_u) \star c^{<t-1>}$	$\Gamma_u \star \tilde{c}^{<t>} + \Gamma_f \star c^{<t-1>}$
$a^{<t>}$	$c^{<t>}$	$\Gamma_o \star c^{<t>}$
Dependencies		

Remark: the sign $\star$ denotes the element-wise multiplication between two vectors.

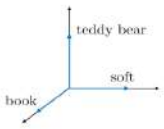
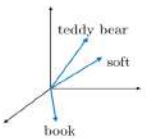
❒ **Variants of RNNs** – The table below sums up the other commonly used RNN architectures:



Learning word representation

In this section, we note V the vocabulary and $|V|$ its size.

❒ **Representation techniques** – The two main ways of representing words are summed up in the table below:

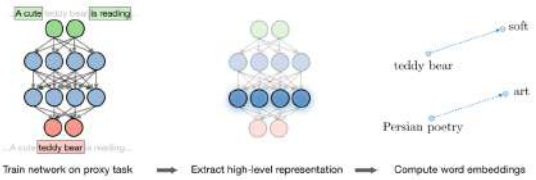
1-hot representation	Word embedding
	
- Noted o_w - Naive approach, no similarity information	- Noted e_w - Takes into account words similarity

❒ **Embedding matrix** – For a given word w, the embedding matrix E is a matrix that maps its 1-hot representation o_w to its embedding e_w as follows:

$$\boxed{e_w = Eo_w}$$

Remark: learning the embedding matrix can be done using target/context likelihood models.

❒ **Word2vec** – Word2vec is a framework aimed at learning word embeddings by estimating the likelihood that a given word is surrounded by other words. Popular models include skip-gram, negative sampling and CBOW.



❒ **Skip-gram** – The skip-gram word2vec model is a supervised learning task that learns word embeddings by assessing the likelihood of any given target word t happening with a context word c. By noting θ_t a parameter associated with t, the probability $P(t|c)$ is given by:

$$\boxed{P(t|c) = \frac{\exp(\theta_t^T e_c)}{\sum_{j=1}^{|V|} \exp(\theta_j^T e_c)}}$$

Remark: summing over the whole vocabulary in the denominator of the softmax part makes this model computationally expensive. CBOW is another word2vec model using the surrounding words to predict a given word.

❒ **Negative sampling** – It is a set of binary classifiers using logistic regressions that aim at assessing how a given context and a given target words are likely to appear simultaneously, with the models being trained on sets of k negative examples and 1 positive example. Given a context word c and a target word t, the prediction is expressed by:

$$\boxed{P(y = 1|c,t) = \sigma(\theta_t^T e_c)}$$

Remark: this method is less computationally expensive than the skip-gram model.

❒ **GloVe** – The GloVe model, short for global vectors for word representation, is a word embedding technique that uses a co-occurence matrix X where each $X_{i,j}$ denotes the number of times that a target i occurred with a context j. Its cost function J is as follows:

$$\boxed{J(\theta) = \frac{1}{2}\sum_{i,j=1}^{|V|} f(X_{ij})(\theta_i^T e_j + b_i + b'_j - \log(X_{ij}))^2}$$

here f is a weighting function such that $X_{i,j} = 0 \Longrightarrow f(X_{i,j}) = 0$.

Given the symmetry that e and θ play in this model, the final word embedding $e_w^{(\text{final})}$ is given by:

$$\boxed{e_w^{(\text{final})} = \frac{e_w + \theta_w}{2}}$$

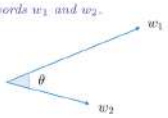
Remark: the individual components of the learned word embeddings are not necessarily interpretable.

Comparing words

❒ **Cosine similarity** – The cosine similarity between words w_1 and w_2 is expressed as follows:

$$\boxed{\text{similarity} = \frac{w_1 \cdot w_2}{||w_1||\ ||w_2||} = \cos(\theta)}$$

Remark: θ is the angle between words w_1 and w_2.



❒ **t-SNE** – t-SNE (t-distributed Stochastic Neighbor Embedding) is a technique aimed at reducing high-dimensional embeddings into a lower dimensional space. In practice, it is commonly used to visualize word vectors in the 2D space.

Language model

❒ **Overview** – A language model aims at estimating the probability of a sentence $P(y)$.

❒ **n-gram model** – This model is a naive approach aiming at quantifying the probability that an expression appears in a corpus by counting its number of appearance in the training data.

❒ **Perplexity** – Language models are commonly assessed using the perplexity metric, also known as PP, which can be interpreted as the inverse probability of the dataset normalized by the number of words T. The perplexity is such that the lower, the better and is defined as follows:

$$\text{PP} = \prod_{t=1}^{T}\left(\frac{1}{\sum_{j=1}^{|V|} y_j^{(t)} \cdot \hat{y}_j^{(t)}}\right)^{\frac{1}{T}}$$

Remark: PP is commonly used in t-SNE.

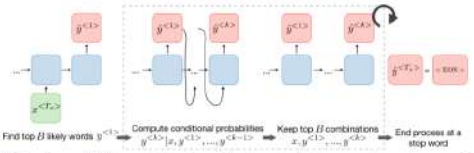
Machine translation

❒ **Overview** – A machine translation model is similar to a language model except it has an encoder network placed before. For this reason, it is sometimes referred as a conditional language model. The goal is to find a sentence y such that:

$$y = \underset{y^{<1>},\ldots,y^{<T_y>}}{\arg\max}\ P(y^{<1>},\ldots,y^{<T_y>}|x)$$

❒ **Beam search** – It is a heuristic search algorithm used in machine translation and speech recognition to find the likeliest sentence y given an input x.

- Step 1: Find top B likely words $y^{<1>}$
- Step 2: Compute conditional probabilities $y^{<k>}|x,y^{<1>},\ldots,y^{<k-1>}$
- Step 3: Keep top B combinations $x,y^{<1>},\ldots,y^{<k>}$



Remark: if the beam width is set to 1, then this is equivalent to a naive greedy search.

❒ **Beam width** – The beam width B is a parameter for beam search. Large values of B yield to better result but with slower performance and increased memory. Small values of B lead to worse results but is less computationally intensive. A standard value for B is around 10.

❒ **Length normalization** – In order to improve numerical stability, beam search is usually applied on the following normalized objective, often called the normalized log-likelihood objective, defined as:

$$\text{Objective} = \frac{1}{T_y^\alpha}\sum_{t=1}^{T_y}\log\left[p(y^{<t>}|x,y^{<1>},\ldots,y^{<t-1>})\right]$$

Remark: the parameter α can be seen as a softener, and its value is usually between 0.5 and 1.

❒ **Error analysis** – When obtaining a predicted translation $\hat{y}$ that is bad, one can wonder why we did not get a good translation y^* by performing the following error analysis:

Case	$P(y^*\|x) > P(\hat{y}\|x)$	$P(y^*\|x) \leqslant P(\hat{y}\|x)$
Root cause	Beam search faulty	RNN faulty
Remedies	Increase beam width	- Try different architecture - Regularize - Get more data

❒ **Bleu score** – The bilingual evaluation understudy (bleu) score quantifies how good a machine translation is by computing a similarity score based on n-gram precision. It is defined as follows:

$$\text{bleu score} = \exp\left(\frac{1}{n}\sum_{k=1}^{n} p_k\right)$$

where p_n is the bleu score on n-gram only defined as follows:

$$p_n = \frac{\sum_{\text{n-gram} \in \widehat{y}} \text{count}_{\text{clip}}(\text{n-gram})}{\sum_{\text{n-gram} \in \widehat{y}} \text{count}(\text{n-gram})}$$

Remark: a brevity penalty may be applied to short predicted translations to prevent an artificially inflated bleu score.

Attention

❐ **Attention model** – This model allows an RNN to pay attention to specific parts of the input that is considered as being important, which improves the performance of the resulting model in practice. By noting $\alpha^{<t,t'>}$ the amount of attention that the output $y^{<t>}$ should pay to the activation $a^{<t'>}$ and $c^{<t>}$ the context at time t, we have:

$$\boxed{c^{<t>} = \sum_{t'} \alpha^{<t,t'>} a^{<t'>}} \quad \text{with} \quad \sum_{t'} \alpha^{<t,t'>} = 1$$

Remark: the attention scores are commonly used in image captioning and machine translation.

A cute teddy bear is reading Persian literature

A cute teddy bear is reading Persian literature

❐ **Attention weight** – The amount of attention that the output $y^{<t>}$ should pay to the activation $a^{<t'>}$ is given by $\alpha^{<t,t'>}$ computed as follows:

$$\boxed{\alpha^{<t,t'>} = \frac{\exp(e^{<t,t'>})}{\sum_{t''=1}^{T_x} \exp(e^{<t,t''>})}}$$

Remark: computation complexity is quadratic with respect to T_x.

* * *

- Tips and Tricks (PDF)

VIP Cheatsheet: Tips and Tricks

Afshine Amidi and Shervine Amidi

November 26, 2018

Data processing

Data augmentation – Deep learning models usually need a lot of data to be properly trained. It is often useful to get more data from the existing ones using data augmentation techniques. The main ones are summed up in the table below. More precisely, given the following input image, here are the techniques that we can apply:

Original	Flip	Rotation	Random crop
			
- Image without any modification	- Flipped with respect to an axis for which the meaning of the image is preserved	- Rotation with a slight angle - Simulates incorrect horizon calibration	- Random focus on one part of the image - Several random crops can be done in a row

Color shift	Noise addition	Information loss	Contrast change
			
- Nuances of RGB is slightly changed - Captures noise that can occur with light exposure	- Addition of noise - More tolerance to quality variation of inputs	- Parts of image ignored - Mimics potential loss of parts of image	- Luminosity changes - Controls difference in exposition due to time of day

Batch normalization – It is a step of hyperparameter γ, β that normalizes the batch $\{x_i\}$. By noting μ_B, σ_B^2 the mean and variance of that we want to correct to the batch, it is done as follows:

$$x_i \longleftarrow \gamma\frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}} + \beta$$

It is usually done after a fully connected/convolutional layer and before a non-linearity layer and aims at allowing higher learning rates and reducing the strong dependence on initialization.

Training a neural network

Epoch – In the context of training a model, epoch is a term used to refer to one iteration where the model sees the whole training set to update its weights.

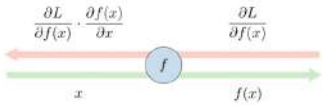
Mini-batch gradient descent – During the training phase, updating weights is usually not based on the whole training set at once due to computation complexities or one data point due to noise issues. Instead, the update step is done on mini-batches, where the number of data points in a batch is a hyperparameter that we can tune.

Loss function – In order to quantify how a given model performs, the loss function L is usually used to evaluate to what extent the actual outputs y are correctly predicted by the model outputs z.

Cross-entropy loss – In the context of binary classification in neural networks, the cross-entropy loss $L(z,y)$ is commonly used and is defined as follows:

$$L(z,y) = -\Big[y\log(z) + (1-y)\log(1-z)\Big]$$

Backpropagation – Backpropagation is a method to update the weights in the neural network by taking into account the actual output and the desired output. The derivative with respect to each weight w is computed using the chain rule.

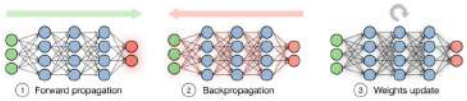


Using this method, each weight is updated with the rule:

$$w \longleftarrow w - \alpha\frac{\partial L(z,y)}{\partial w}$$

Updating weights – In a neural network, weights are updated as follows:

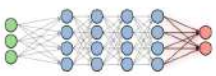
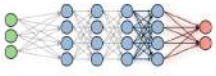
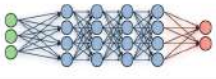
- Step 1: Take a batch of training data and perform forward propagation to compute the loss.
- Step 2: Backpropagate the loss to get the gradient of the loss with respect to each weight.
- Step 3: Use the gradients to update the weights of the network.



Parameter tuning

Xavier initialization – Instead of initializing the weights in a purely random manner, Xavier initialization enables to have initial weights that take into account characteristics that are unique to the architecture.

Transfer learning – Training a deep learning model requires a lot of data and more importantly a lot of time. It is often useful to take advantage of pre-trained weights on huge datasets that took days/weeks to train, and leverage it towards our use case. Depending on how much data we have at hand, here are the different ways to leverage this:

Training size	Illustration	Explanation
Small		Freezes all layers, trains weights on softmax
Medium		Freezes most layers, trains weights on last layers and softmax
Large		Trains weights on layers and softmax by initializing weights on pre-trained ones

Learning rate – The learning rate, often noted α or sometimes η, indicates at which pace the weights get updated. It can be fixed or adaptively changed. The current most popular method is called Adam, which is a method that adapts the learning rate.

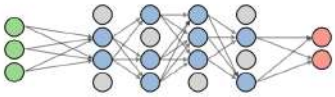
Adaptive learning rates – Letting the learning rate vary when training a model can reduce the training time and improve the numerical optimal solution. While Adam optimizer is the most commonly used technique, others can also be useful. They are summed up in the table below:

Method	Explanation	Update of w	Update of b
Momentum	- Dampens oscillations - Improvement to SGD - 2 parameters to tune	$w - \alpha v_{dw}$	$b - \alpha v_{db}$
RMSprop	- Root Mean Square propagation - Speeds up learning algorithm by controlling oscillations	$w - \alpha\frac{dw}{\sqrt{s_{dw}}}$	$b \longleftarrow b - \alpha\frac{db}{\sqrt{s_{db}}}$
Adam	- Adaptive Moment estimation - Most popular method - 4 parameters to tune	$w - \alpha\frac{v_{dw}}{\sqrt{s_{dw}} + \epsilon}$	$b \longleftarrow b - \alpha\frac{v_{db}}{\sqrt{s_{db}} + \epsilon}$

Remark: other methods include Adadelta, Adagrad and SGD.

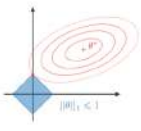
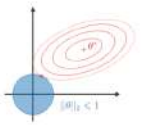
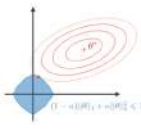
Regularization

Dropout – Dropout is a technique used in neural networks to prevent overfitting the training data by dropping out neurons with probability $p > 0$. It forces the model to avoid relying too much on particular sets of features.

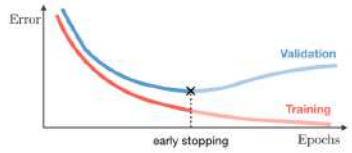


Remark: most deep learning frameworks parametrize dropout through the 'keep' parameter $1-p$.

Weight regularization – In order to make sure that the weights are not too large and that the model is not overfitting the training set, regularization techniques are usually performed on the model weights. The main ones are summed up in the table below:

LASSO	Ridge	Elastic Net
- Shrinks coefficients to 0 - Good for variable selection	Makes coefficients smaller	Tradeoff between variable selection and small coefficients
		
$\ldots + \lambda\|\theta\|_1$ $\lambda \in \mathbb{R}$	$\ldots + \lambda\|\theta\|_2^2$ $\lambda \in \mathbb{R}$	$\ldots + \lambda\Big[(1-\alpha)\|\theta\|_1 + \alpha\|\theta\|_2^2\Big]$ $\lambda \in \mathbb{R}, \alpha \in [0,1]$

❒ **Early stopping** – This regularization technique stops the training process as soon as the validation loss reaches a plateau or starts to increase.



Good practices

❒ **Overfitting small batch** – When debugging a model, it is often useful to make quick tests to see if there is any major issue with the architecture of the model itself. In particular, in order to make sure that the model can be properly trained, a mini-batch is passed inside the network to see if it can overfit on it. If it cannot, it means that the model is either too complex or not complex enough to even overfit on a small batch, let alone a normal-sized training set.

❒ **Gradient checking** – Gradient checking is a method used during the implementation of the backward pass of a neural network. It compares the value of the analytical gradient to the numerical gradient at given points and plays the role of a sanity-check for correctness.

	Numerical gradient	Analytical gradient
Formula	$\frac{df}{dx}(x) \approx \frac{f(x+h)-f(x-h)}{2h}$	$\frac{df}{dx}(x) = f'(x)$
Comments	- Expensive; loss has to be computed two times per dimension - Used to verify correctness of analytical implementation -Trade-off in choosing h not too small (numerical instability) nor too large (poor gradient approx.)	- 'Exact' result - Direct computation - Used in the final implementation

⋆ ⋆ ⋆

SQL

- SQL cheatsheet by sqltutorial (PDF)

SQL CHEAT SHEET http://www.sqltutorial.org

QUERYING DATA FROM A TABLE

```
SELECT c1, c2 FROM t;
```
Query data in columns c1, c2 from a table

```
SELECT * FROM t;
```
Query all rows and columns from a table

```
SELECT c1, c2 FROM t
WHERE condition;
```
Query data and filter rows with a condition

```
SELECT DISTINCT c1 FROM t
WHERE condition;
```
Query distinct rows from a table

```
SELECT c1, c2 FROM t
ORDER BY c1 ASC [DESC];
```
Sort the result set in ascending or descending order

```
SELECT c1, c2 FROM t
ORDER BY c1
LIMIT n OFFSET offset;
```
Skip *offset* of rows and return the next n rows

```
SELECT c1, aggregate(c2)
FROM t
GROUP BY c1;
```
Group rows using an aggregate function

```
SELECT c1, aggregate(c2)
FROM t
GROUP BY c1
HAVING condition;
```
Filter groups using HAVING clause

QUERYING FROM MULTIPLE TABLES

```
SELECT c1, c2
FROM t1
INNER JOIN t2 ON condition;
```
Inner join t1 and t2

```
SELECT c1, c2
FROM t1
LEFT JOIN t2 ON condition;
```
Left join t1 and t1

```
SELECT c1, c2
FROM t1
RIGHT JOIN t2 ON condition;
```
Right join t1 and t2

```
SELECT c1, c2
FROM t1
FULL OUTER JOIN t2 ON condition;
```
Perform full outer join

```
SELECT c1, c2
FROM t1
CROSS JOIN t2;
```
Produce a Cartesian product of rows in tables

```
SELECT c1, c2
FROM t1, t2;
```
Another way to perform cross join

```
SELECT c1, c2
FROM t1 A
INNER JOIN t2 B ON condition;
```
Join t1 to itself using INNER JOIN clause

USING SQL OPERATORS

```
SELECT c1, c2 FROM t1
UNION [ALL]
SELECT c1, c2 FROM t2;
```
Combine rows from two queries

```
SELECT c1, c2 FROM t1
INTERSECT
SELECT c1, c2 FROM t2;
```
Return the intersection of two queries

```
SELECT c1, c2 FROM t1
MINUS
SELECT c1, c2 FROM t2;
```
Subtract a result set from another result set

```
SELECT c1, c2 FROM t1
WHERE c1 [NOT] LIKE pattern;
```
Query rows using pattern matching %, _

```
SELECT c1, c2 FROM t
WHERE c1 [NOT] IN value_list;
```
Query rows in a list

```
SELECT c1, c2 FROM t
WHERE  c1 BETWEEN low AND high;
```
Query rows between two values

```
SELECT c1, c2 FROM t
WHERE  c1 IS [NOT] NULL;
```
Check if values in a table is NULL or not

SQL CHEAT SHEET http://www.sqltutorial.org

MANAGING TABLES

```
CREATE TABLE t (
    id INT PRIMARY KEY,
    name VARCHAR NOT NULL,
    price INT DEFAULT 0
);
```
Create a new table with three columns

```
DROP TABLE t ;
```
Delete the table from the database

```
ALTER TABLE t ADD column;
```
Add a new column to the table

```
ALTER TABLE t DROP COLUMN c ;
```
Drop column c from the table

```
ALTER TABLE t ADD constraint;
```
Add a constraint

```
ALTER TABLE t DROP constraint;
```
Drop a constraint

```
ALTER TABLE t1 RENAME TO t2;
```
Rename a table from t1 to t2

```
ALTER TABLE t1 RENAME c1 TO c2 ;
```
Rename column c1 to c2

```
TRUNCATE TABLE t;
```
Remove all data in a table

USING SQL CONSTRAINTS

```
CREATE TABLE t(
   c1 INT, c2 INT, c3 VARCHAR,
   PRIMARY KEY (c1,c2)
);
```
Set c1 and c2 as a primary key

```
CREATE TABLE t1(
   c1 INT PRIMARY KEY,
   c2 INT,
   FOREIGN KEY (c2) REFERENCES t2(c2)
);
```
Set c2 column as a foreign key

```
CREATE TABLE t(
   c1 INT, c1 INT,
   UNIQUE(c2,c3)
);
```
Make the values in c1 and c2 unique

```
CREATE TABLE t(
  c1 INT, c2 INT,
  CHECK(c1> 0 AND c1 >= c2)
);
```
Ensure c1 > 0 and values in c1 >= c2

```
CREATE TABLE t(
    c1 INT PRIMARY KEY,
    c2 VARCHAR NOT NULL
);
```
Set values in c2 column not NULL

MODIFYING DATA

```
INSERT INTO t(column_list)
VALUES(value_list);
```
Insert one row into a table

```
INSERT INTO t(column_list)
VALUES (value_list),
       (value_list), ....;
```
Insert multiple rows into a table

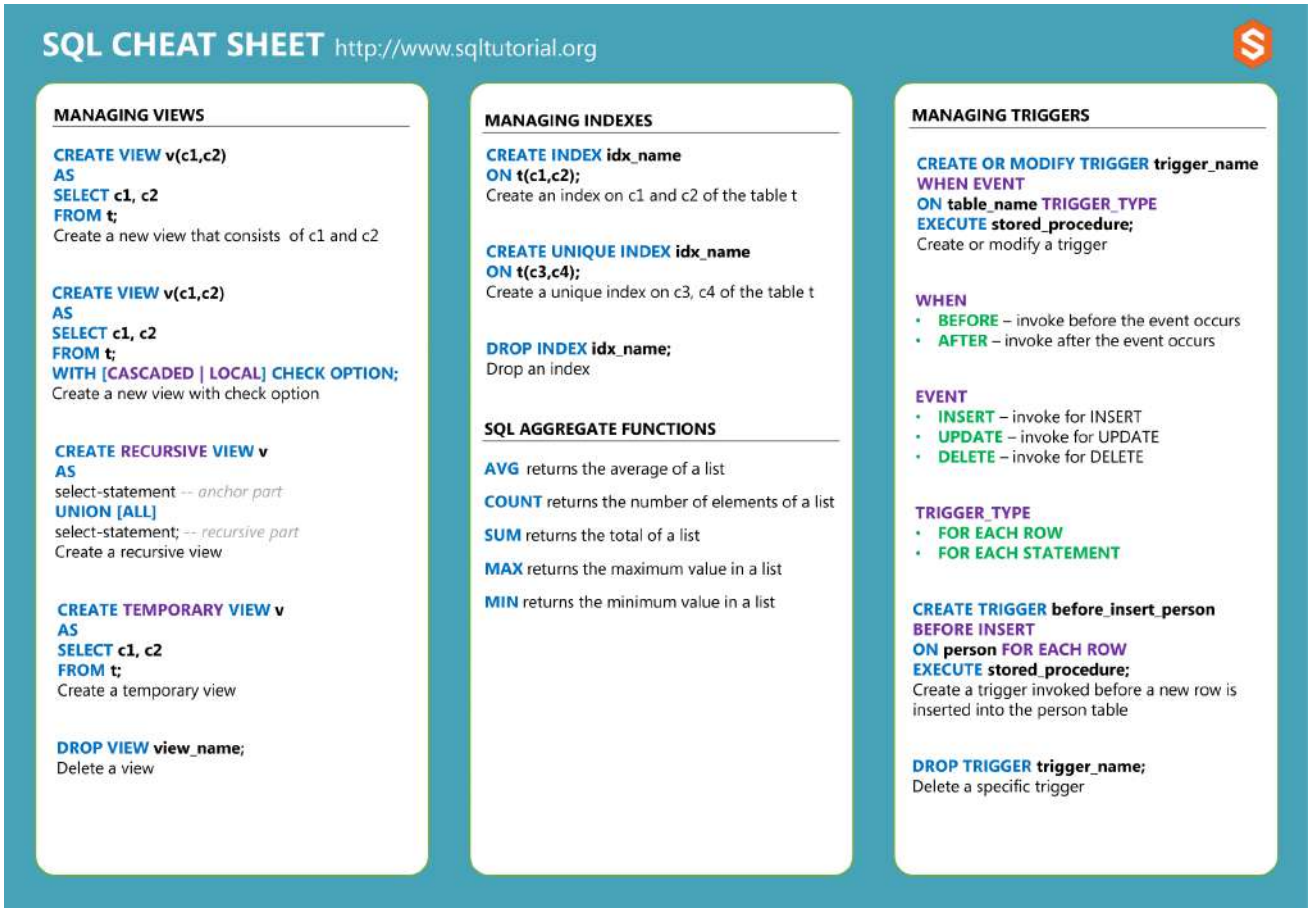
```
INSERT INTO t1(column_list)
SELECT column_list
FROM t2;
```
Insert rows from t2 into t1

```
UPDATE t
SET c1 = new_value;
```
Update new value in the column c1 for all rows

```
UPDATE t
SET c1 = new_value,
    c2 = new_value
WHERE condition;
```
Update values in the column c1, c2 that match the condition

```
DELETE FROM t;
```
Delete all data in a table

```
DELETE FROM t
WHERE condition;
```
Delete subset of rows in a table



SQL CHEAT SHEET http://www.sqltutorial.org

MANAGING VIEWS

CREATE VIEW v(c1,c2)
AS
SELECT c1, c2
FROM t;
Create a new view that consists of c1 and c2

CREATE VIEW v(c1,c2)
AS
SELECT c1, c2
FROM t;
WITH [CASCADED | LOCAL] CHECK OPTION;
Create a new view with check option

CREATE RECURSIVE VIEW v
AS
select-statement -- *anchor part*
UNION [ALL]
select-statement; -- *recursive part*
Create a recursive view

CREATE TEMPORARY VIEW v
AS
SELECT c1, c2
FROM t;
Create a temporary view

DROP VIEW view_name;
Delete a view

MANAGING INDEXES

CREATE INDEX idx_name
ON t(c1,c2);
Create an index on c1 and c2 of the table t

CREATE UNIQUE INDEX idx_name
ON t(c3,c4);
Create a unique index on c3, c4 of the table t

DROP INDEX idx_name;
Drop an index

SQL AGGREGATE FUNCTIONS

AVG returns the average of a list

COUNT returns the number of elements of a list

SUM returns the total of a list

MAX returns the maximum value in a list

MIN returns the minimum value in a list

MANAGING TRIGGERS

CREATE OR MODIFY TRIGGER trigger_name
WHEN EVENT
ON table_name TRIGGER_TYPE
EXECUTE stored_procedure;
Create or modify a trigger

WHEN
- **BEFORE** – invoke before the event occurs
- **AFTER** – invoke after the event occurs

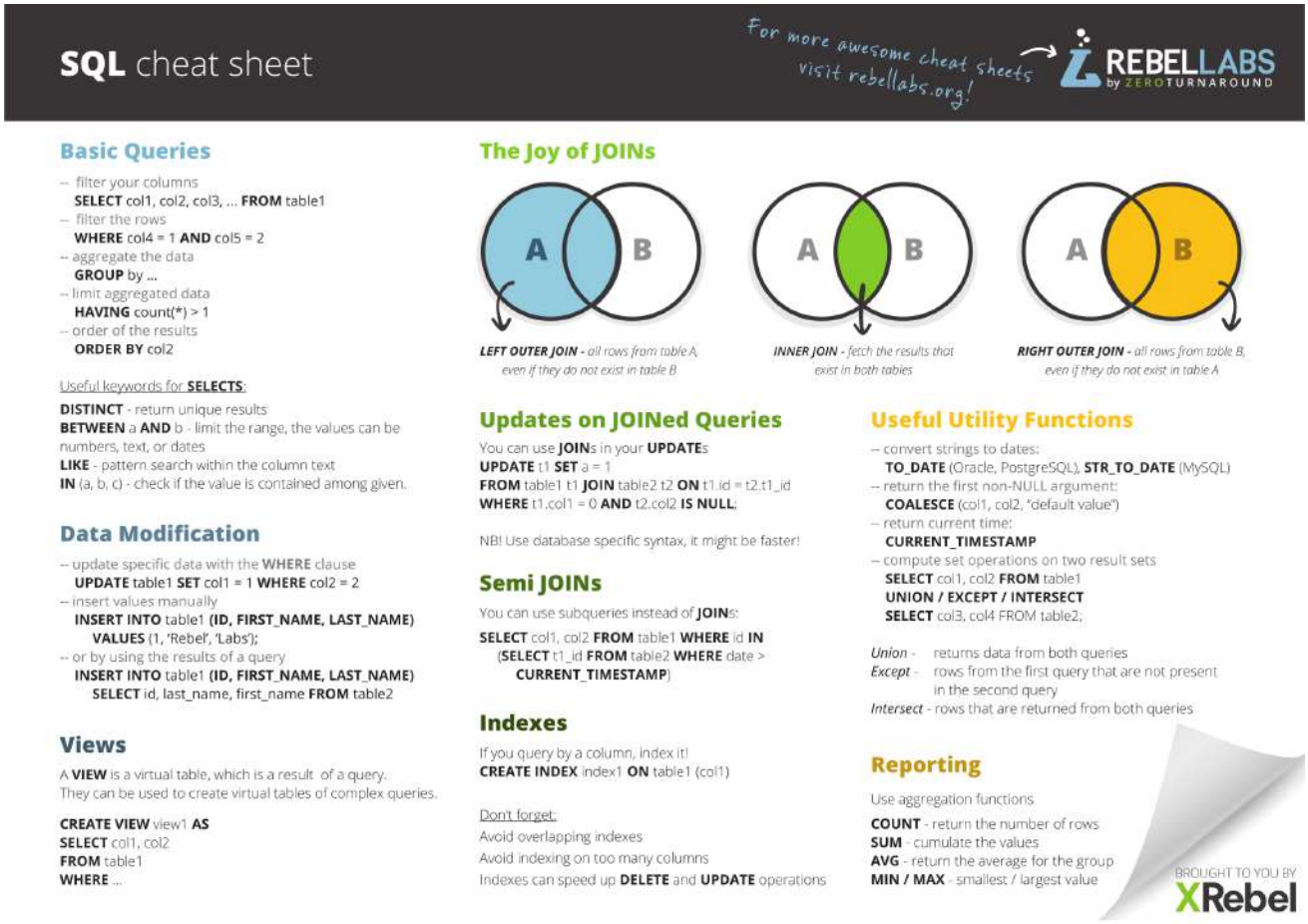
EVENT
- **INSERT** – invoke for INSERT
- **UPDATE** – invoke for UPDATE
- **DELETE** – invoke for DELETE

TRIGGER_TYPE
- **FOR EACH ROW**
- **FOR EACH STATEMENT**

CREATE TRIGGER before_insert_person
BEFORE INSERT
ON person FOR EACH ROW
EXECUTE stored_procedure;
Create a trigger invoked before a new row is inserted into the person table

DROP TRIGGER trigger_name;
Delete a specific trigger

- SQL cheatsheet by Rebel Labs



SQL cheat sheet

For more awesome cheat sheets visit rebellabs.org!

REBELLABS by ZEROTURNAROUND

Basic Queries

```
-- filter your columns
   SELECT col1, col2, col3, ... FROM table1
-- filter the rows
   WHERE col4 = 1 AND col5 = 2
-- aggregate the data
   GROUP by ...
-- limit aggregated data
   HAVING count(*) > 1
-- order of the results
   ORDER BY col2
```

Useful keywords for **SELECTS**:

DISTINCT - return unique results
BETWEEN a **AND** b - limit the range, the values can be numbers, text, or dates
LIKE - pattern search within the column text
IN (a, b, c) - check if the value is contained among given.

Data Modification

```
-- update specific data with the WHERE clause
   UPDATE table1 SET col1 = 1 WHERE col2 = 2
-- insert values manually
   INSERT INTO table1 (ID, FIRST_NAME, LAST_NAME)
      VALUES (1, 'Rebel', 'Labs');
-- or by using the results of a query
   INSERT INTO table1 (ID, FIRST_NAME, LAST_NAME)
      SELECT id, last_name, first_name FROM table2
```

Views

A **VIEW** is a virtual table, which is a result of a query.
They can be used to create virtual tables of complex queries.

```
CREATE VIEW view1 AS
SELECT col1, col2
FROM table1
WHERE ...
```

The Joy of JOINs

LEFT OUTER JOIN - *all rows from table A, even if they do not exist in table B*

INNER JOIN - *fetch the results that exist in both tables*

RIGHT OUTER JOIN - *all rows from table B, even if they do not exist in table A*

Updates on JOINed Queries

You can use **JOIN**s in your **UPDATE**s

```
UPDATE t1 SET a = 1
FROM table1 t1 JOIN table2 t2 ON t1.id = t2.t1_id
WHERE t1.col1 = 0 AND t2.col2 IS NULL;
```

NB! Use database specific syntax, it might be faster!

Semi JOINs

You can use subqueries instead of **JOIN**s:

```
SELECT col1, col2 FROM table1 WHERE id IN
   (SELECT t1_id FROM table2 WHERE date >
      CURRENT_TIMESTAMP)
```

Indexes

If you query by a column, index it!

```
CREATE INDEX index1 ON table1 (col1)
```

Don't forget:
Avoid overlapping indexes
Avoid indexing on too many columns
Indexes can speed up **DELETE** and **UPDATE** operations

Useful Utility Functions

```
-- convert strings to dates:
   TO_DATE (Oracle, PostgreSQL), STR_TO_DATE (MySQL)
-- return the first non-NULL argument:
   COALESCE (col1, col2, "default value")
-- return current time:
   CURRENT_TIMESTAMP
-- compute set operations on two result sets
   SELECT col1, col2 FROM table1
   UNION / EXCEPT / INTERSECT
   SELECT col3, col4 FROM table2;
```

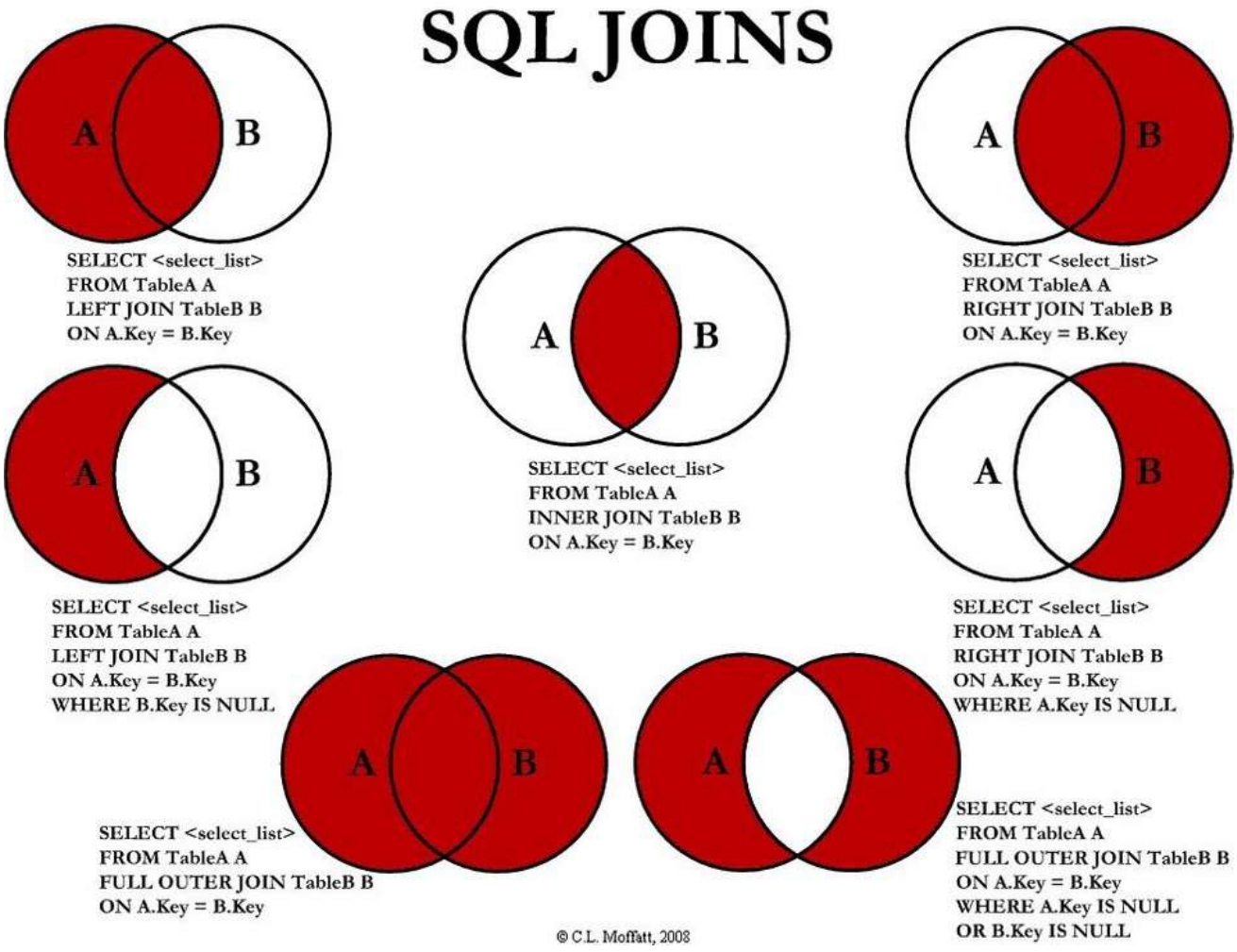
Union - returns data from both queries
Except - rows from the first query that are not present in the second query
Intersect - rows that are returned from both queries

Reporting

Use aggregation functions

COUNT - return the number of rows
SUM - cumulate the values
AVG - return the average for the group
MIN / MAX - smallest / largest value

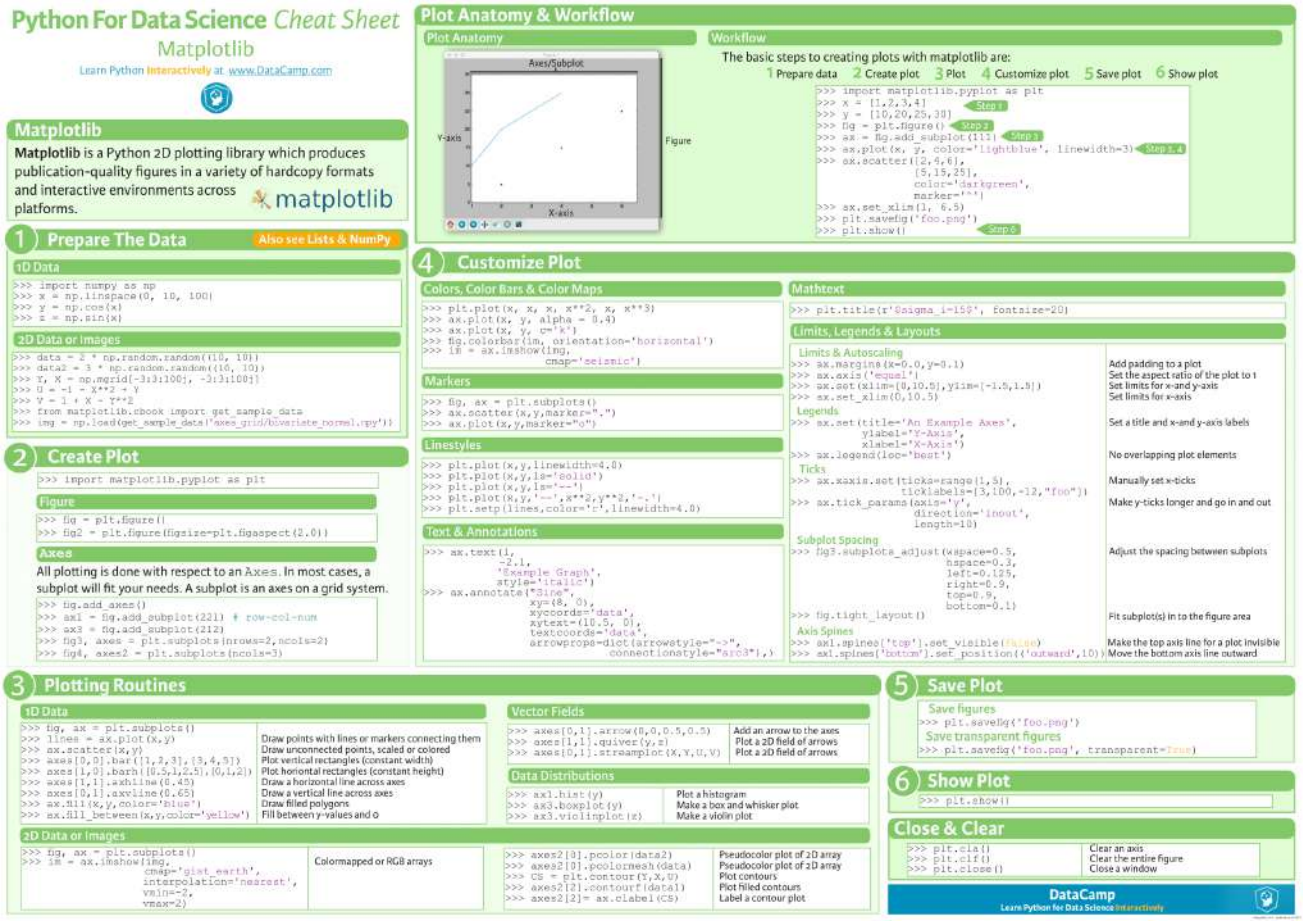
BROUGHT TO YOU BY XRebel



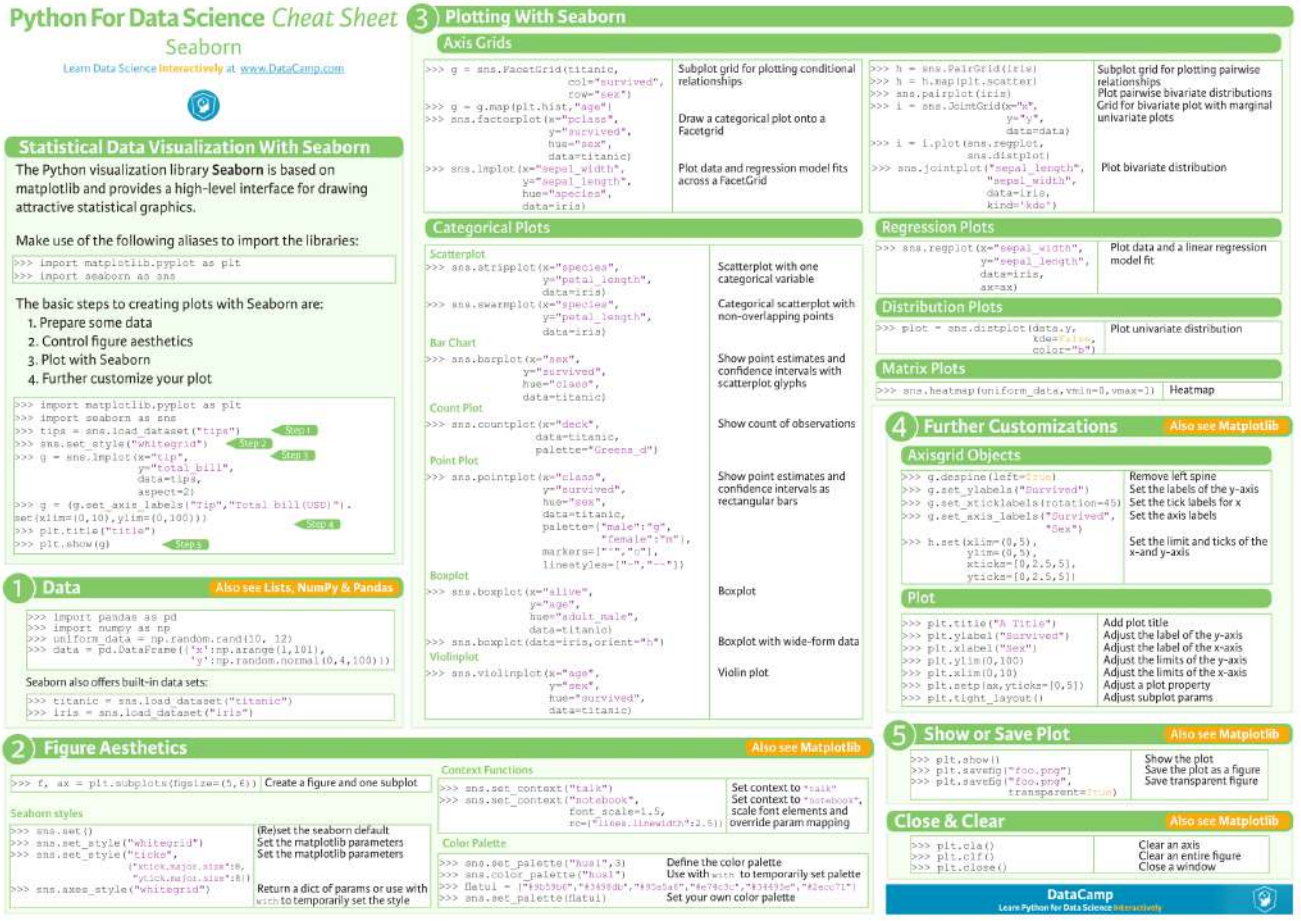
Data Visualization

Python

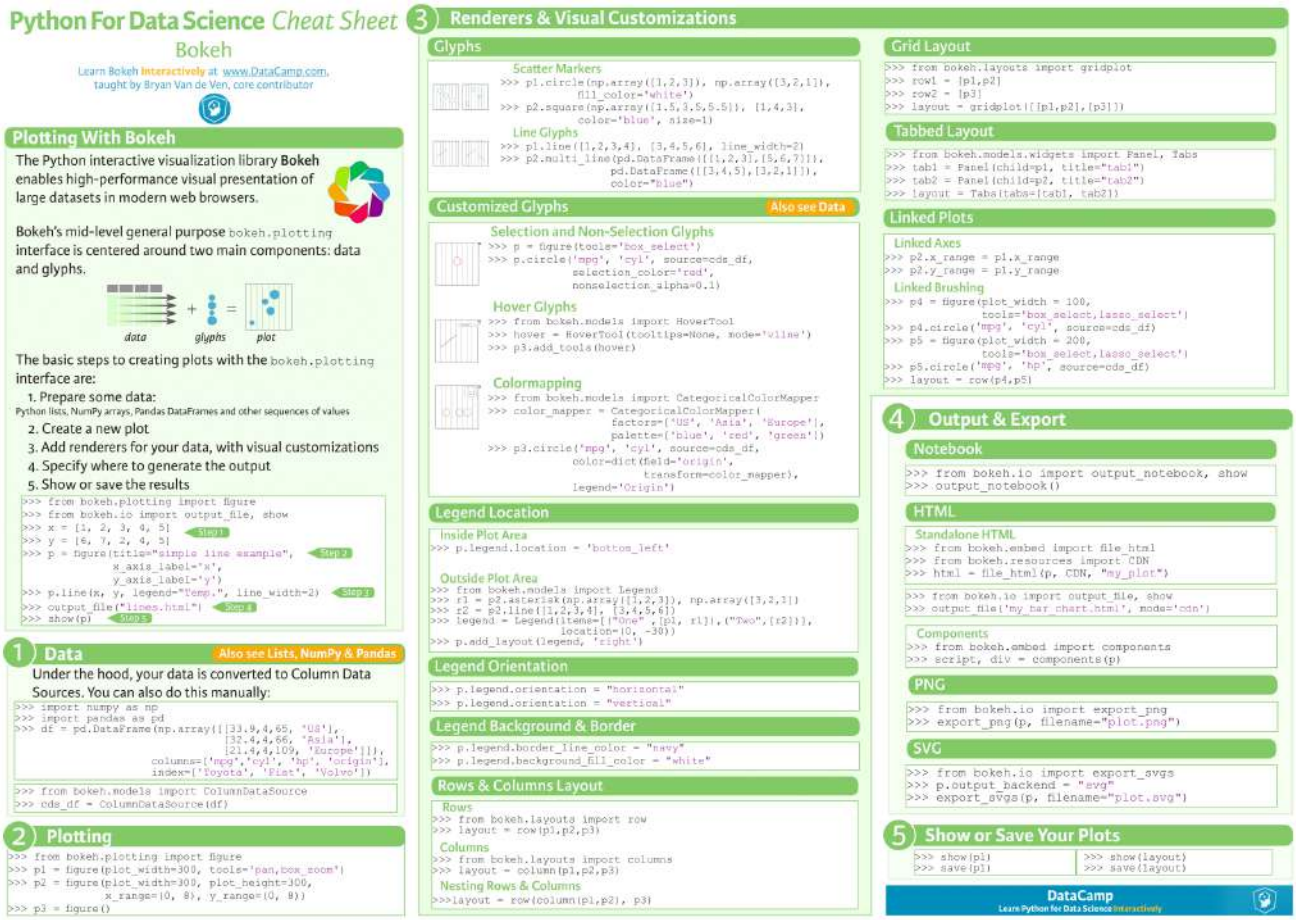
- Matplotlib (PDF)



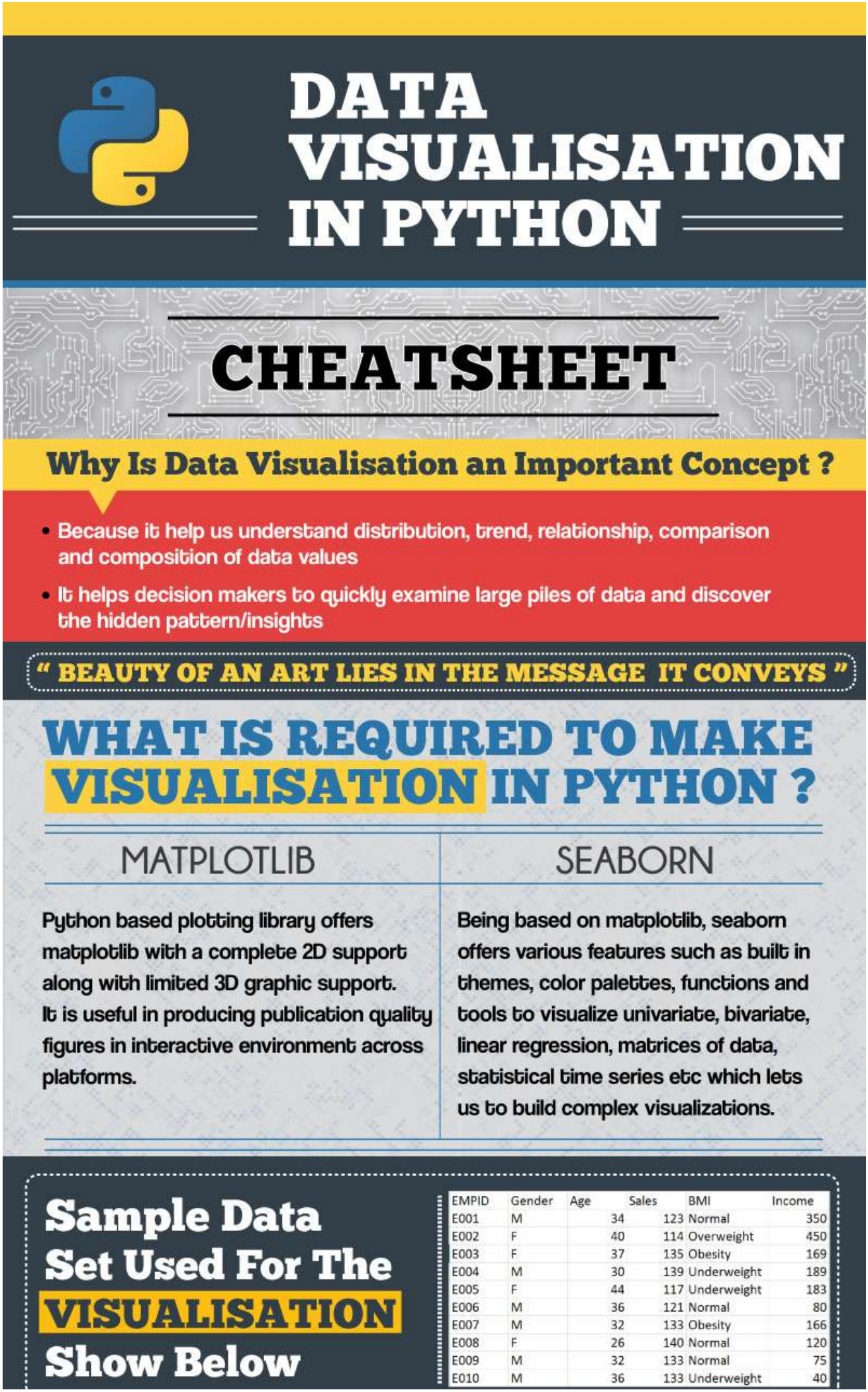
- Seaborn (PDF)



- Bokeh (PDF)



- Comprehensive Guide to Data Visualization in Python



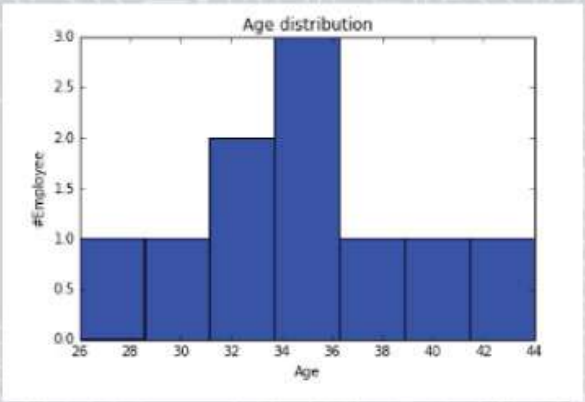
EMPID	Gender	Age	Sales	BMI	Income
E001	M	34	123	Normal	350
E002	F	40	114	Overweight	450
E003	F	37	135	Obesity	169
E004	M	30	139	Underweight	189
E005	F	44	117	Underweight	183
E006	M	36	121	Normal	80
E007	M	32	133	Obesity	166
E008	F	26	140	Normal	120
E009	M	32	133	Normal	75
E010	M	36	133	Underweight	40

Import Data Set:

```
import matplotlib.pyplot as plt
import pandas as pd
df=pd.read_excel("E:/First.xlsx", "Sheet1")
```

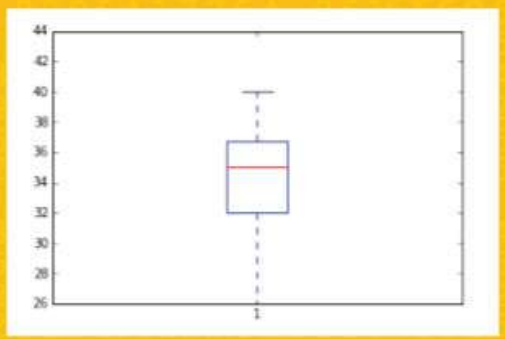
Histogram

```
fig=plt.figure()
ax = fig.add_subplot(1,1,1)
ax.hist(df['Age'],bins = 7) # Here you can
play with number of bins Labels and Tit
plt.title('Age distribution')
plt.xlabel('Age')
plt.ylabel('#Employee')
plt.show()
```



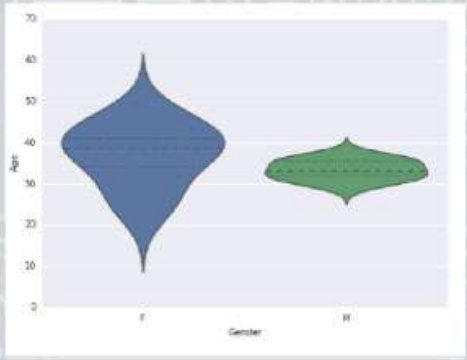
Box Plot

```
importmatplotlib.pyplot as plt
import pandas as pd
fig=plt.figure()
ax = fig.add_subplot(1,1,1)
x.boxplot(df['Age'])
plt.show()
```



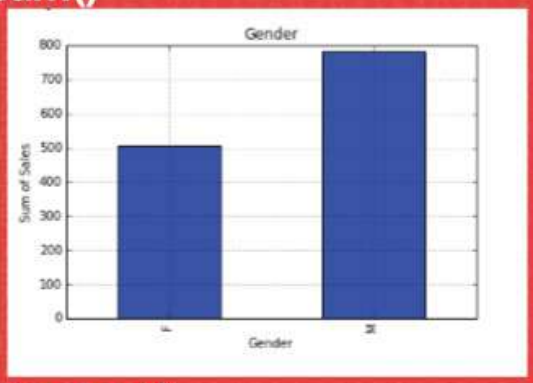
Violin Plot

```
import seaborn as sns
sns.violinplot(df['Age'], df['Gender'])
#Variable Plot
sns.despine()
```



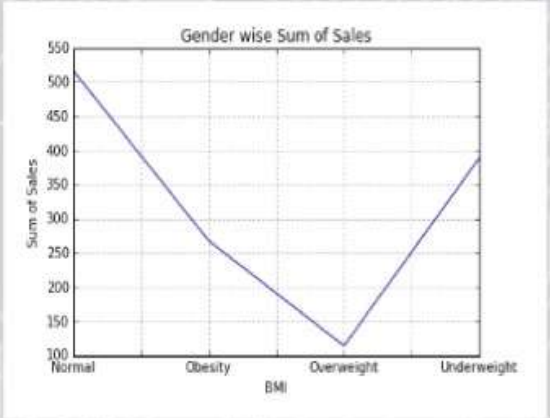
Bar Chart

```
var = df.groupby('Gender').Sales.sum()
#grouped sum of sales at
Gender level
fig = plt.figure()
ax1 = fig.add_subplot(1,1,1)
ax1.set_xlabel('Gender')
ax1.set_ylabel('Sum of Sales')
ax1.set_title("Gender wise Sum of Sales")
var.plot(kind='bar')
```



Line Chart

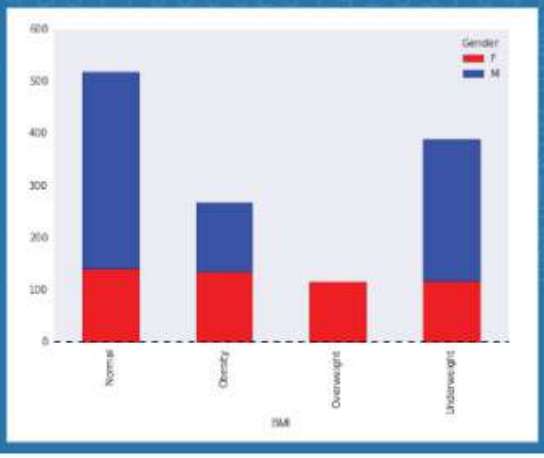
```
var = df.groupby('BMI').Sales.sum()
fig = plt.figure()
ax1 = fig.add_subplot(1,1,1)
ax1.set_xlabel('BMI')
ax1.set_ylabel('Sum of Sales')
ax1.set_title("BMI wise Sum
of Sales")
var.plot(kind='line')
```



Stacked Column Chart

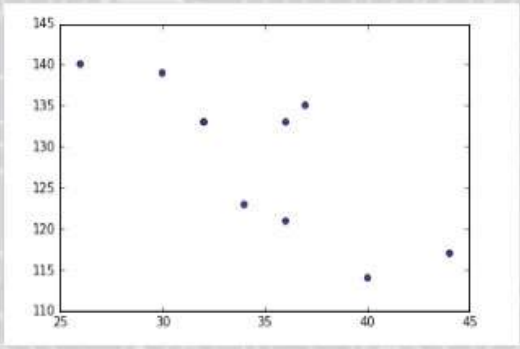
```
var = df.groupby(['BMI','Gender']).Sales.sum()
```

```
var.unstack().plot(kind='bar',stacked=True,  color=['red','blue'], grid=False)
```



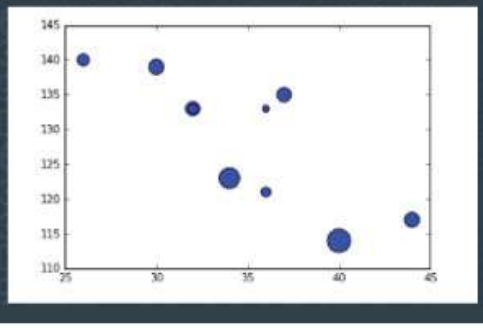
Scatter Plot

```
fig = plt.figure()
ax = fig.add_subplot(1,1,1)
ax.scatter(df['Age'],df['Sales'])
plt.show()
```



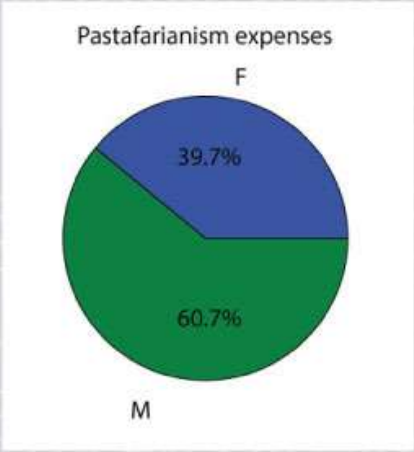
Bubble Plot

```
fig = plt.figure()
ax = fig.add_subplot(1,1,1)
ax.scatter(df['Age'],df['Sales'], s=df['Income'])
plt.show()
```



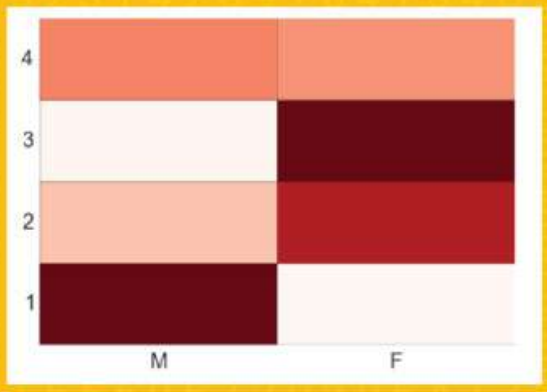
Pie Chart

```
var=df.groupby(['Gender']).sum().stack()
temp=var.unstack()
type(temp)
x_list = temp['Sales']
label_list = temp.index
pyplot.axis("equal") #The pie chart
is oval by default. To make it a
circle use pyplot.axis("equal")
plt.pie(x_list,labels=label_list,autopct="%1.1f%%")
plt.title("Pastafarianism expenses")
plt.show()
```



Heat Map

```
import numpy as np
data = np.random.rand(4,2)
rows = list('1234') #rows
categories columns =
list('MF') #column categories
fig,ax=plt.subplots()
ax.pcolor(data,cmap=plt.cm.Reds,edgecolors='k')
ax.set_xticks(np.arange(0,2)+0.5)
ax.set_yticks(np.arange(0,4)+0.5)
ax.xaxis.tick_bottom()
ax.yaxis.tick_left()
ax.set_xticklabels(columns,minor=False,fontsize=20)
ax.set_yticklabels(rows,minor=False,fontsize=20)
plt.show())
```



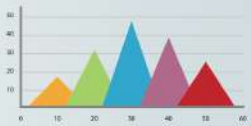
To view the complete guide on data visualisation in python

visit here : http://bit.ly/1FjTkRF

R

- Ggplot2 (PDF)
- Leaflet (PDF)
- Cartography (PDF)
- Comprehensive Guide to Data Visualization in R

Data Visualization in R CHEATSHEET



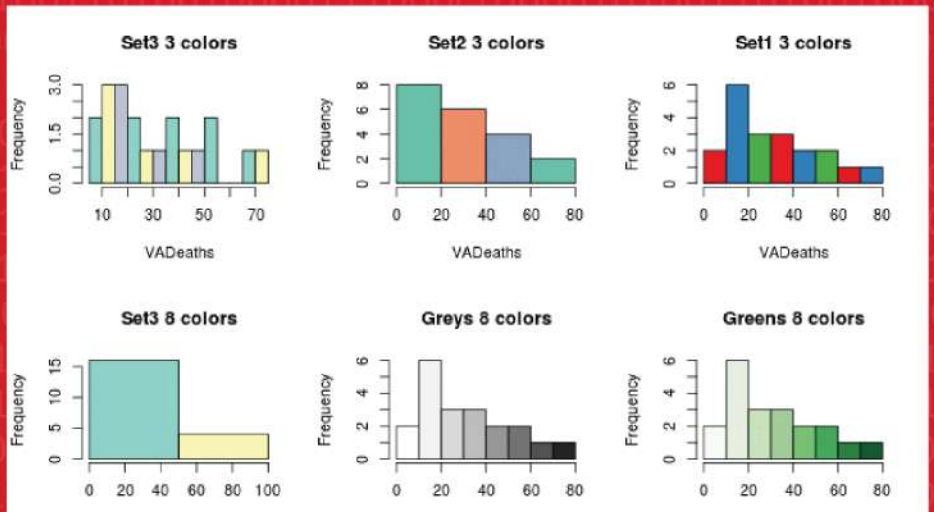
R Programming offers a set of inbuilt functions & libraries (such as ggplot2, leaflet, lattice) to create visualizations & present data stories. Below is a guide to create basic and advanced visualizations in R

BASIC VISUALIZATIONS

Histogram

```
library(RColorBrewer)

data(VADeaths)
par(mfrow=c(2,3))
hist(VADeaths,breaks=10, col=brewer.pal(3,"Set3"),main="Set3 3 colors")
```

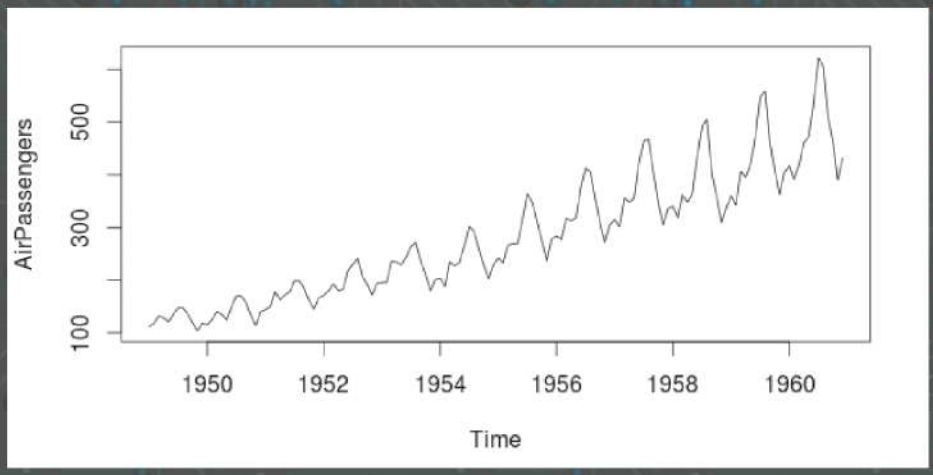


VADeaths VADeaths VADeaths

Bar/ Line Chart

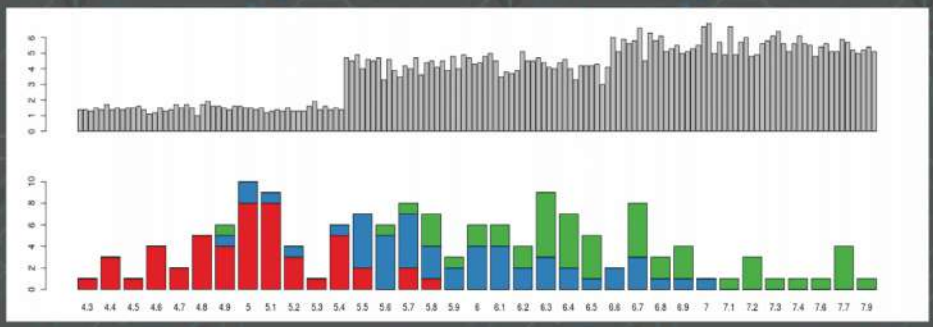
Line Chart

```
plot(AirPassengers,type="l")  #Simple Line Plot
```



Bar Chart

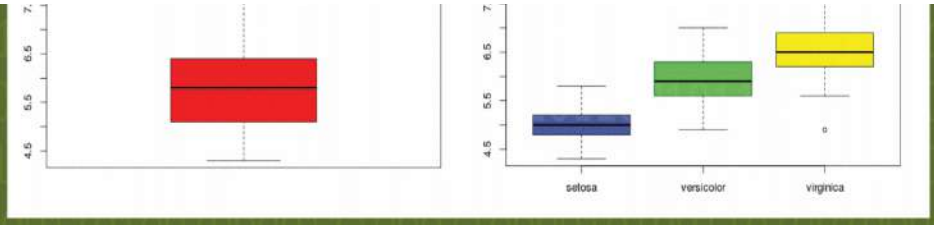
```
barplot(iris$Petal.Length) #Creating simple Bar Graph
barplot(table(iris$Species,iris$Sepal.Length),col  = brewer.pal(3,"Set1"))
```



Box Plot (including group-by option)

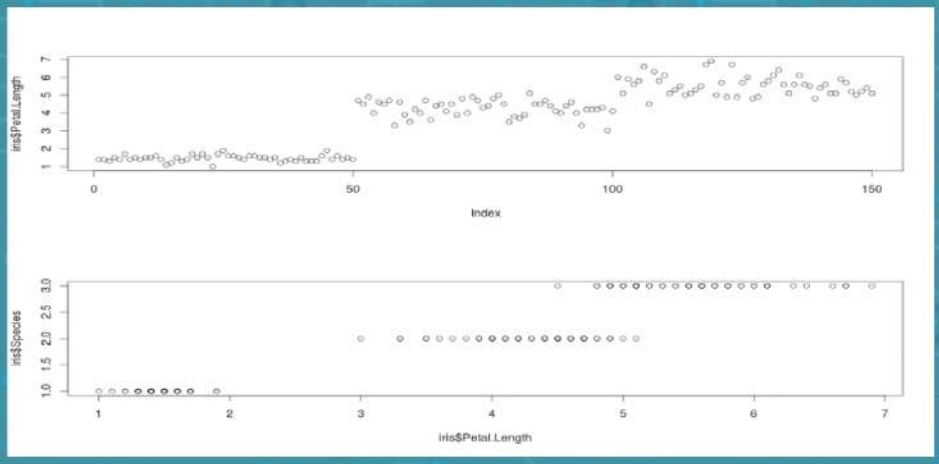
```
data(iris)
par(mfrow=c(2,2))
boxplot(iris$Sepal.Length,col="red")

boxplot(iris$Sepal.Length~iris$Species,col=topo.colors(3))
```

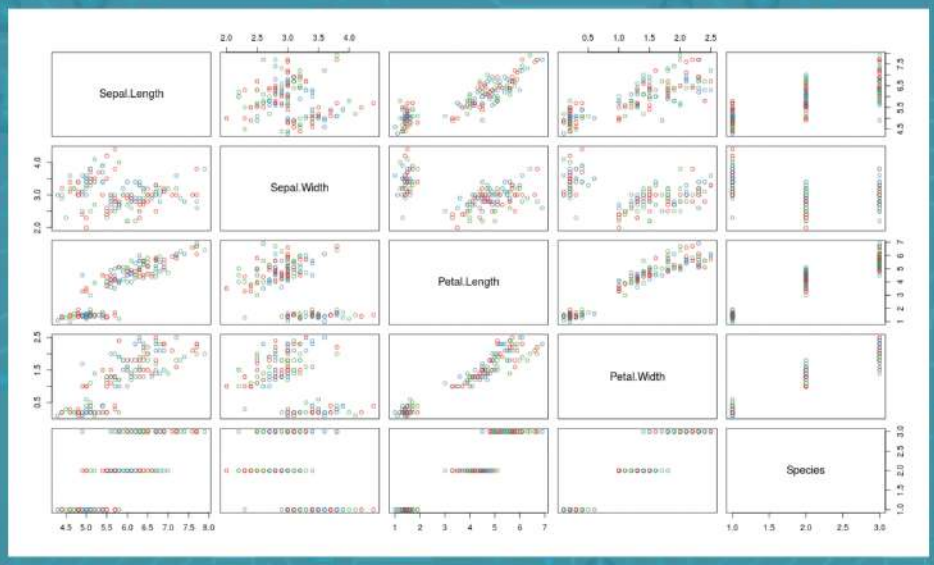


Scatter Plot (including 3D and other features)

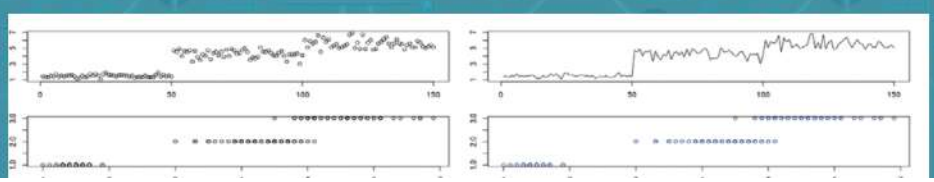
```
#Simple Scatter Plot
plot(x=iris$Petal.Length)
#Multivariate Scatter Plot
plot(x=iris$Petal.Length,y=iris$Species)
```

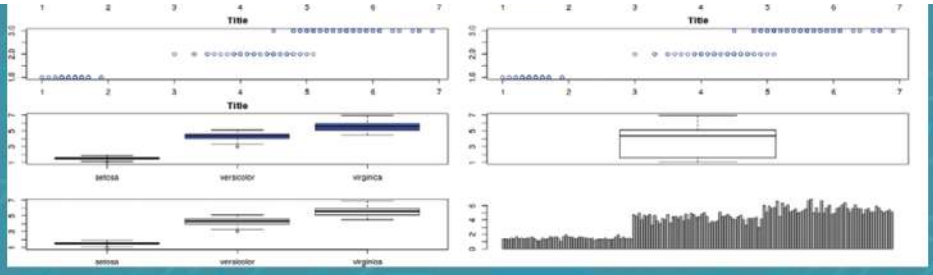



```
plot(iris,col=brewer.pal(3,"Set1"))
```




```
pie(table(iris$Species))
```

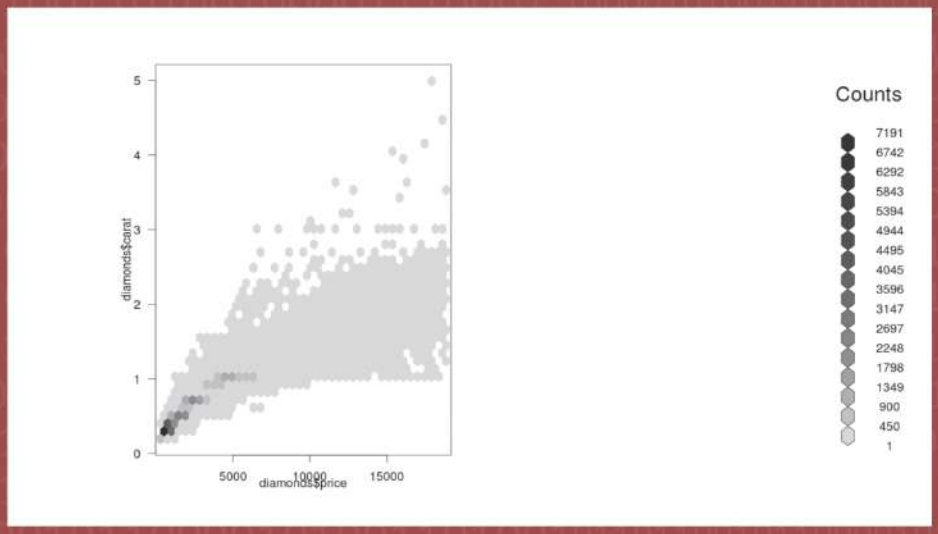




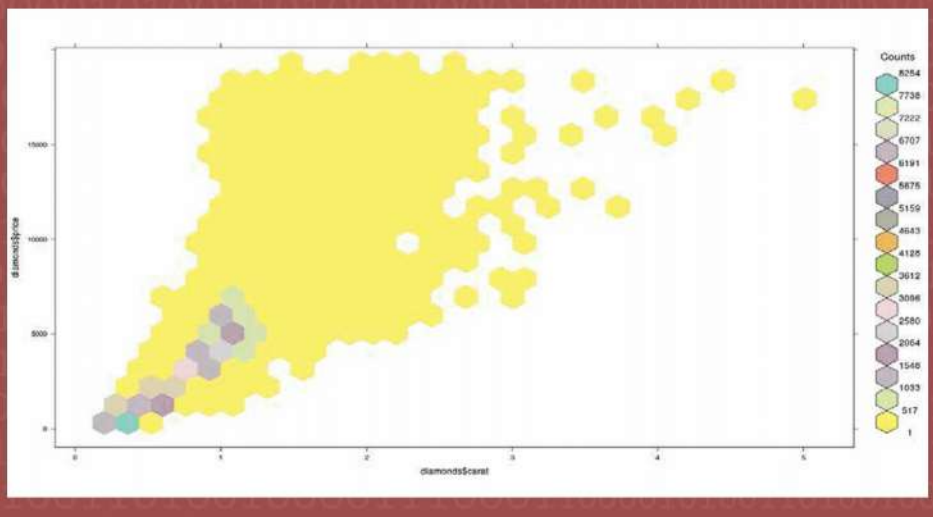
Advanced Visualizations

Hexbin Binning

```
>library(hexbin)
>a=hexbin(diamonds$price,diamonds$carat,xbins=40)
>library(RColorBrewer)
>plot(a)
```

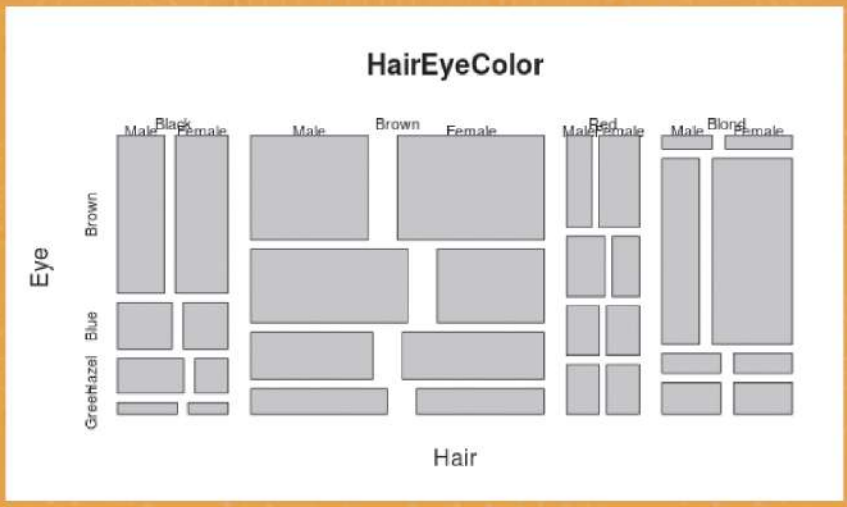



```
>library(RColorBrewer)
>rf <- colorRampPalette(rev(brewer.pal(40,'Set3')))
>hexbinplot(diamonds$price~diamonds$carat, data=diamonds, colramp=rf)
```



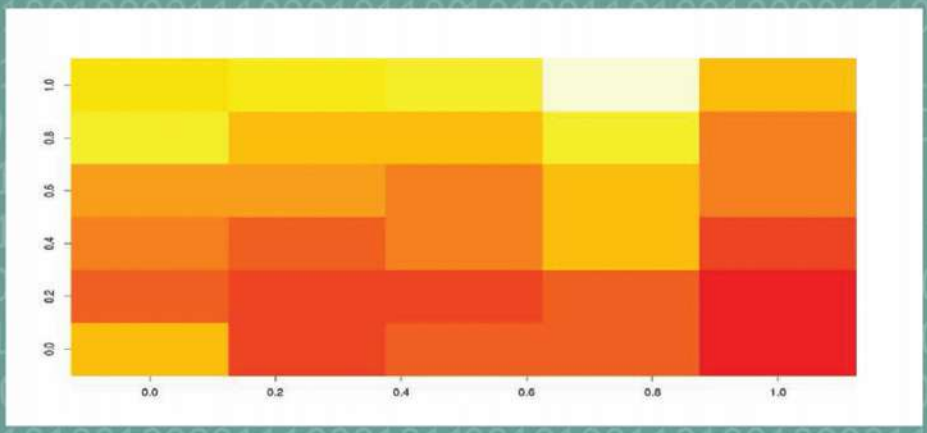
Mosaic Plot

```
> data(HairEyeColor)
> mosaicplot(HairEyeColor)
```



Heat Map

```
> heatmap(as.matrix(mtcars))
> image(as.matrix(b[2:7]))
```



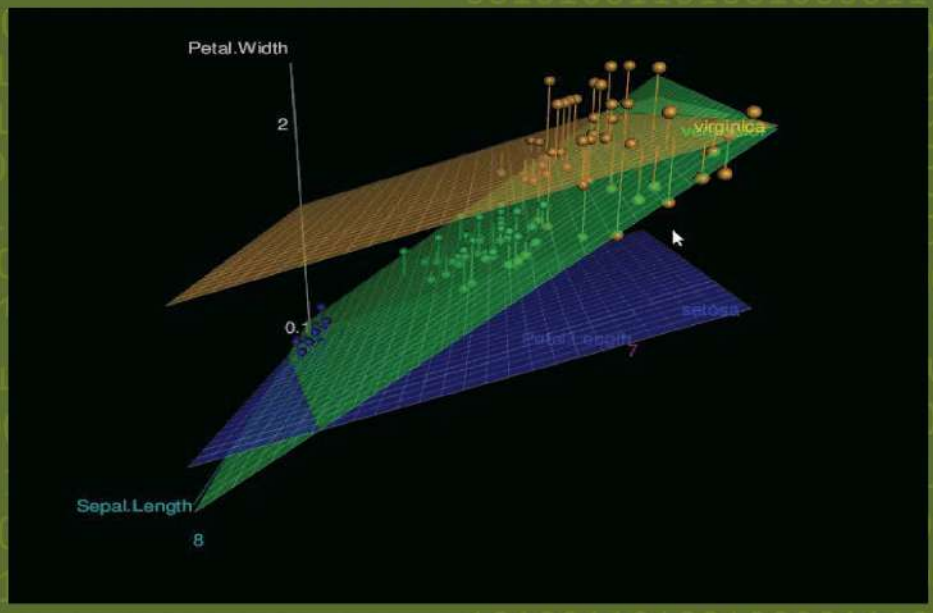
Map Visualization

```
devtools::install_github("rstudio/leaflet")
```

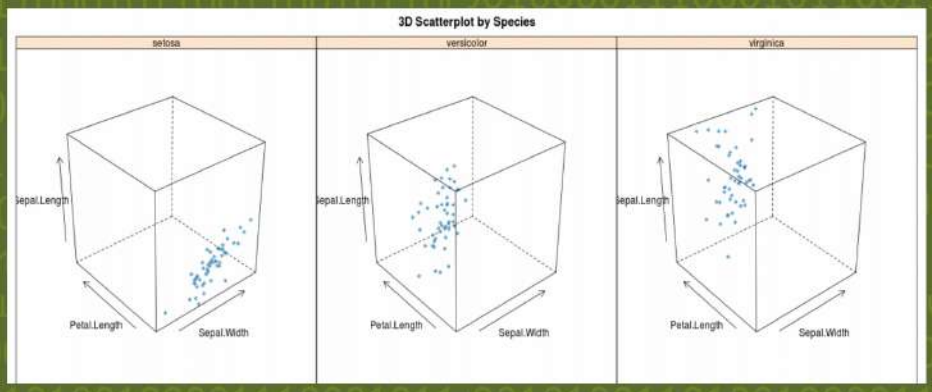
```
library(magrittr)
library(leaflet)
m <- leaflet() %>%
addTiles() %>%  # Add default OpenStreetMap map tiles
addMarkers(lng=77.2310, lat=28.6560, popup="food of chandni chowk")
m  # Print the map
```

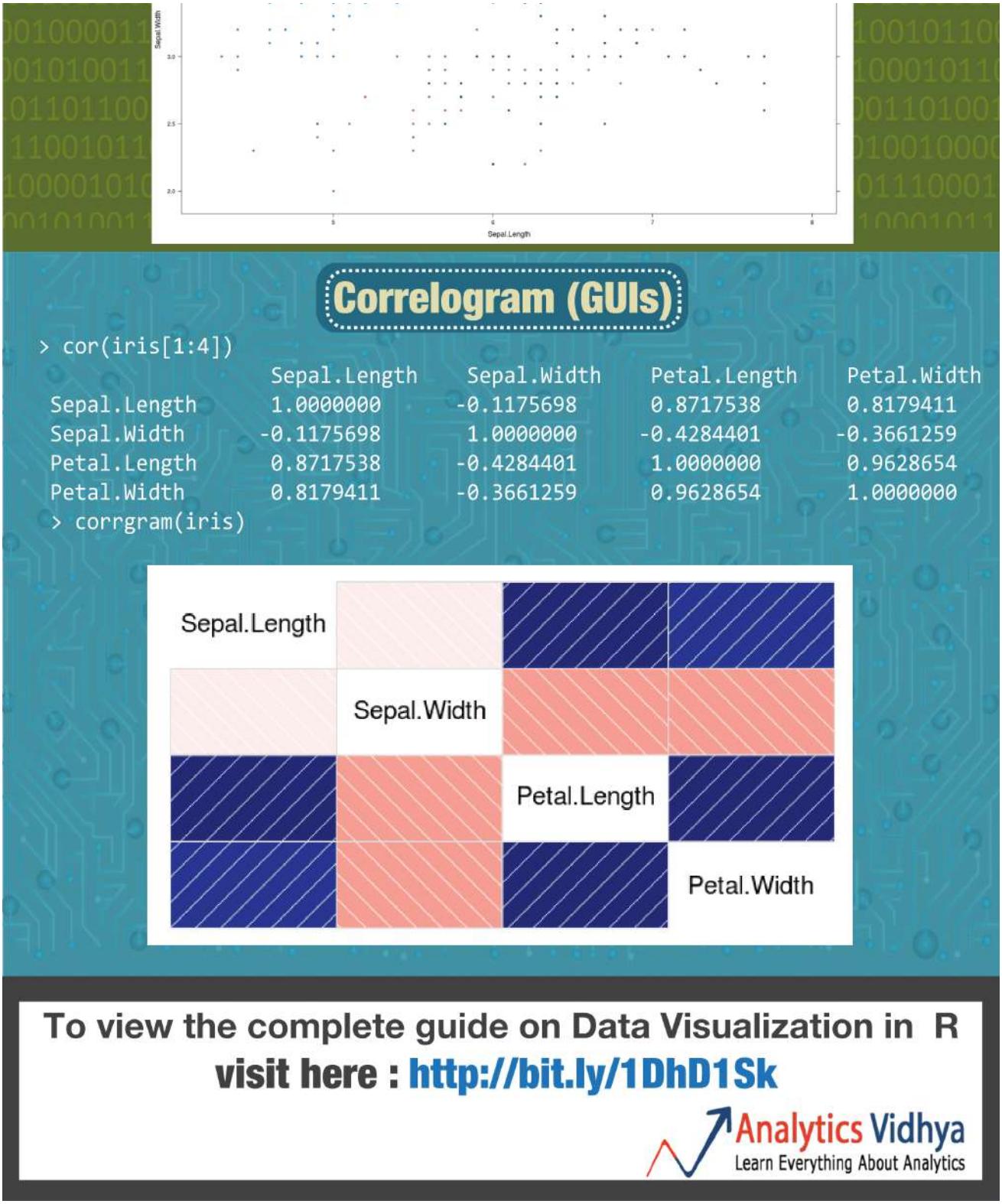
3D Graphs

```
>data(iris, package="datasets")
>scatter3d(Petal.Width~Petal.Length+Sepal.Length|Species, data=iris, fit="linear"
>residuals=TRUE, parallel=FALSE, bg="black", axis.scales=TRUE, grid=TRUE, ellipsoid=FALSE)
```




```
>attach(iris)# 3d scatterplot by factor level
>cloud(Sepal.Length~Sepal.Width*Petal.Length|Species, main="3D Scatterplot by Species")
>xyplot(Sepal.Width ~ Sepal.Length, iris, groups = iris$Species, pch= 20)
```





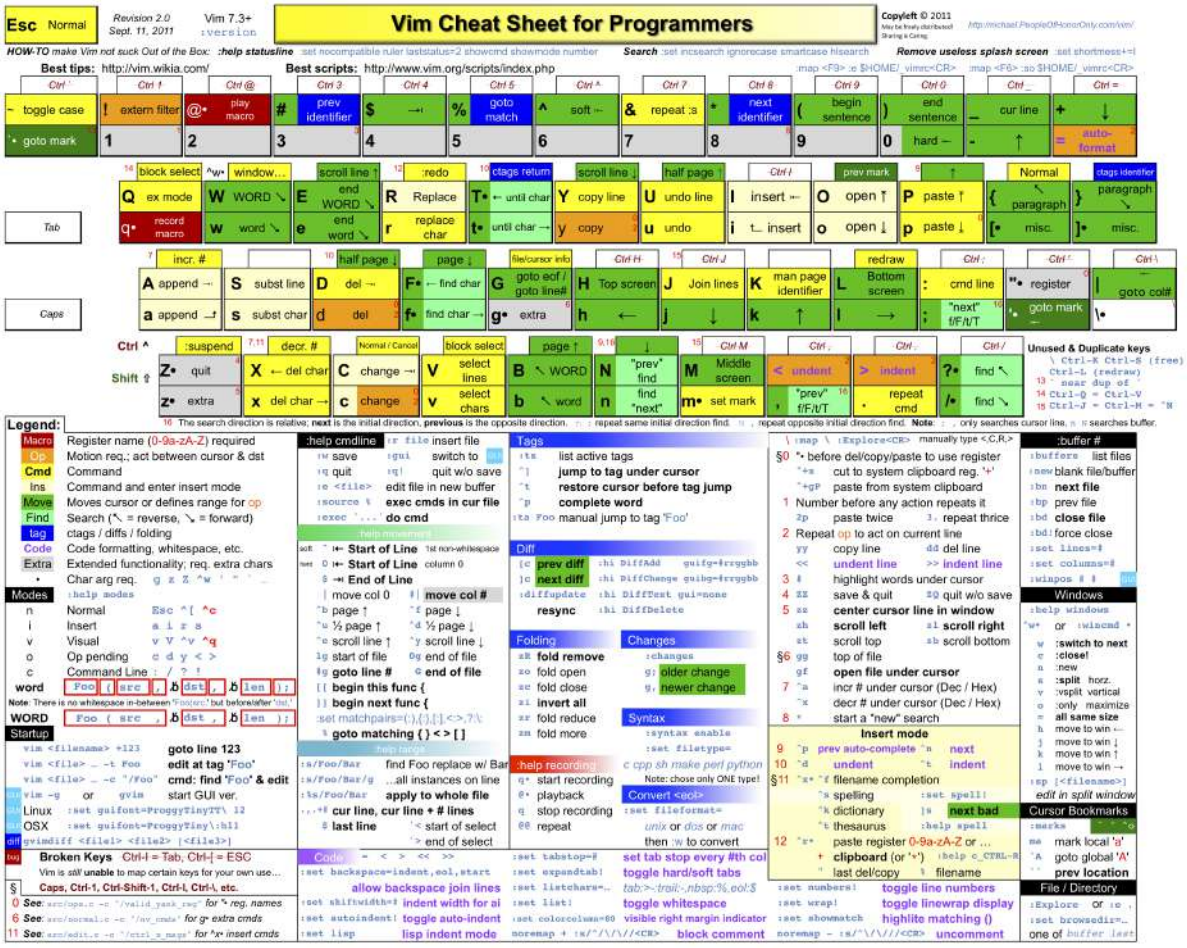
- Simple Features sf (PDF)

- survminer (PDF)

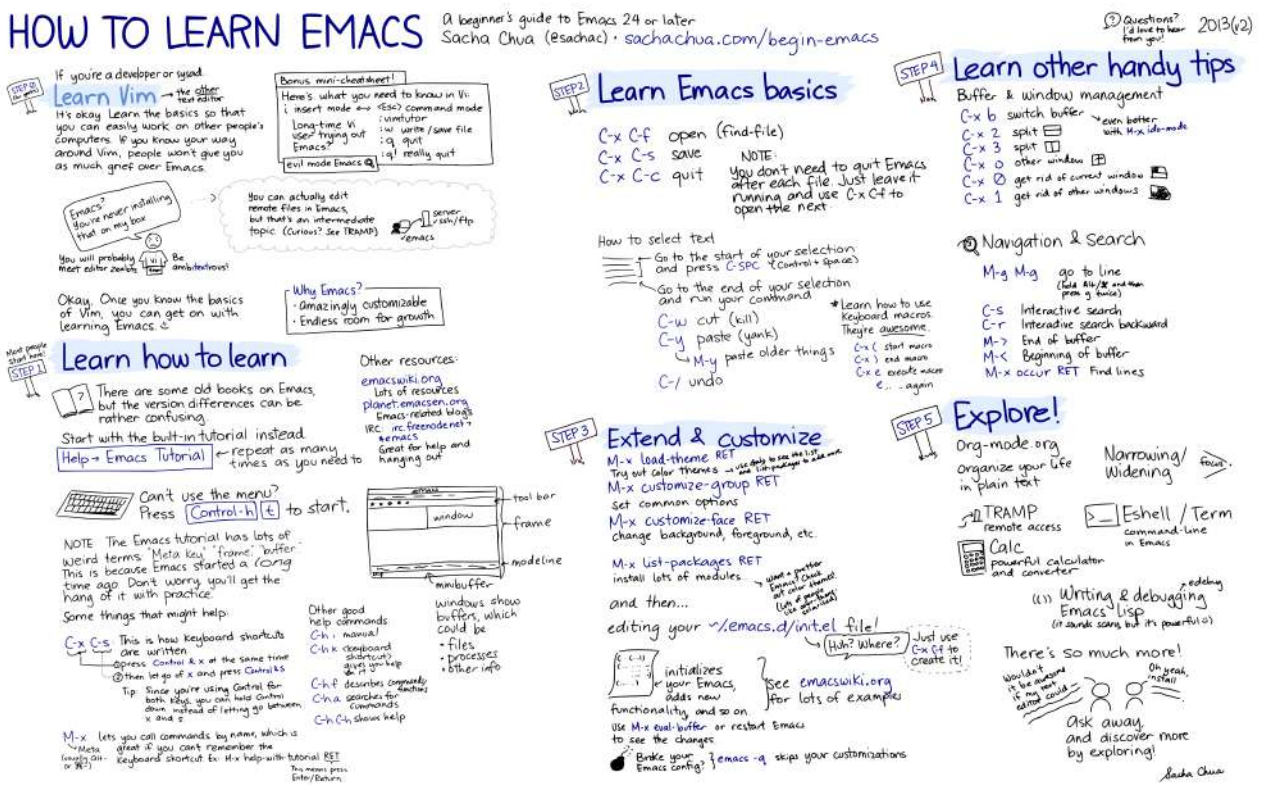
Data Science in General and Others

- Shiny (PDF)

- Vim (PDF)



- Emacs (PDF)



- Git - Atlassian's Cheatsheet (PDF)

Git Cheat Sheet

Git Basics

`git init <directory>`	Create empty Git repo in specified directory. Run with no arguments to initialize the current directory as a git repository.
`git clone <repo>`	Clone repo located at `<repo>` onto local machine. Original repo can be located on the local filesystem or on a remote machine via HTTP or SSH.
`git config user.name <name>`	Define author name to be used for all commits in current repo. Devs commonly use `--global` flag to set config options for current user.
`git add <directory>`	Stage all changes in `<directory>` for the next commit. Replace `<directory>` with a `<file>` to change a specific file.
`git commit -m "<message>"`	Commit the staged snapshot, but instead of launching a text editor, use `<message>` as the commit message.
`git status`	List which files are staged, unstaged, and untracked.
`git log`	Display the entire commit history using the default format. For customization see additional options.
`git diff`	Show unstaged changes between your index and working directory.

Undoing Changes

`git revert <commit>`	Create new commit that undoes all of the changes made in `<commit>`, then apply it to the current branch.
`git reset <file>`	Remove `<file>` from the staging area, but leave the working directory unchanged. This unstages a file without overwriting any changes.
`git clean -n`	Shows which files would be removed from working directory. Use the `-f` flag in place of the `-n` flag to execute the clean.

Rewriting Git History

`git commit --amend`	Replace the last commit with the staged changes and last commit combined. Use with nothing staged to edit the last commit's message.
`git rebase <base>`	Rebase the current branch onto `<base>`. `<base>` can be a commit ID, a branch name, a tag, or a relative reference to HEAD.
`git reflog`	Show a log of changes to the local repository's HEAD. Add `--relative-date` flag to show date info or `--all` to show all refs.

Git Branches

`git branch`	List all of the branches in your repo. Add a `<branch>` argument to create a new branch with the name `<branch>`.
`git checkout -b <branch>`	Create and check out a new branch named `<branch>`. Drop the `-b` flag to checkout an existing branch.
`git merge <branch>`	Merge `<branch>` into the current branch.

Remote Repositories

`git remote add <name> <url>`	Create a new connection to a remote repo. After adding a remote, you can use `<name>` as a shortcut for `<url>` in other commands.
`git fetch <remote> <branch>`	Fetches a specific `<branch>`, from the repo. Leave off `<branch>` to fetch all remote refs.
`git pull <remote>`	Fetch the specified remote's copy of current branch and immediately merge it into the local copy.
`git push <remote> <branch>`	Push the branch to `<remote>`, along with necessary commits and objects. Creates named branch in the remote repo if it doesn't exist.

Additional Options +

git config

`git config --global user.name <name>`	Define the author name to be used for all commits by the current user.
`git config --global user.email <email>`	Define the author email to be used for all commits by the current user.
`git config --global alias. <alias-name> <git-command>`	Create shortcut for a Git command. E.g. `alias.glog log --graph --oneline` will set `git glog` equivalent to `git log --graph --oneline`.
`git config --system core.editor <editor>`	Set text editor used by commands for all users on the machine. `<editor>` arg should be the command that launches the desired editor (e.g., vi).
`git config --global --edit`	Open the global configuration file in a text editor for manual editing.

git log

`git log -<limit>`	Limit number of commits by `<limit>`. E.g. `git log -5` will limit to 5 commits.
`git log --oneline`	Condense each commit to a single line.
`git log -p`	Display the full diff of each commit.
`git log --stat`	Include which files were altered and the relative number of lines that were added or deleted from each of them.
`git log --author="<pattern>"`	Search for commits by a particular author.
`git log --grep="<pattern>"`	Search for commits with a commit message that matches `<pattern>`.
`git log <since>..<until>`	Show commits that occur between `<since>` and `<until>`. Args can be a commit ID, branch name, `HEAD`, or any other kind of revision reference.
`git log -- <file>`	Only display commits that have the specified file.
`git log --graph --decorate`	`--graph` flag draws a text based graph of commits on left side of commit msgs. `--decorate` adds names of branches or tags of commits shown.

git diff

`git diff HEAD`	Show difference between working directory and last commit.
`git diff --cached`	Show difference between staged changes and last commit

git reset

`git reset`	Reset staging area to match most recent commit, but leave the working directory unchanged.
`git reset --hard`	Reset staging area and working directory to match most recent commit and **overwrites all changes** in the working directory.
`git reset <commit>`	Move the current branch tip backward to `<commit>`, reset the staging area to match, but leave the working directory alone.
`git reset --hard <commit>`	Same as previous, but resets both the staging area & working directory to match. **Deletes** uncommitted changes, and **all commits after** `<commit>`.

git rebase

`git rebase -i <base>`	Interactively rebase current branch onto `<base>`. Launches editor to enter commands for how each commit will be transferred to the new base.

git pull

`git pull --rebase <remote>`	Fetch the remote's copy of current branch and rebases it into the local copy. Uses git rebase instead of merge to integrate the branches.

git push

`git push <remote> --force`	Forces the `git push` even if it results in a non-fast-forward merge. Do not use the `--force` flag unless you're absolutely sure you know what you're doing.
`git push <remote> --all`	Push all of your local branches to the specified remote.
`git push <remote> --tags`	Tags aren't automatically pushed when you push a branch or use the `--all` flag. The `--tags` flag sends all of your local tags to the remote repo.

- DVC (PDF)

- BASH (PDF)

Linux Bash Shell Cheat Sheet

Basic Commands

Basic Terminal Shortcuts

```
CTRL L = Clear the terminal
CTRL D = Logout
SHIFT Page Up/Down = Go up/down the terminal
CTRL A = Cursor to start of line
CTRL E = Cursor the end of line
CTRL U = Delete left of the cursor
CTRL K = Delete right of the cursor
CTRL W = Delete word on the left
CTRL Y = Paste (after CTRL U,K or W)
TAB = auto completion of file or command
CTRL R = reverse search history
!! = repeat last command
CTRL Z = stops the current command (resume with fg in foreground or bg in background)
```

Basic Terminal Navigation

```
ls -a = list all files and folders
ls <folderName> = list files in folder
ls -lh = Detailed list, Human readable
ls -l *.jpg = list jpeg files only
ls -lh <fileName> = Result for file only

cd <folderName> = change directory
      if folder name has spaces use " "
cd / = go to root
cd .. = go up one folder, tip: ../../../

du -h: Disk usage of folders, human readable
du -ah: "     "   "  files & folders, Human readable
du -sh: only show disc usage of folders

pwd = print working directory

man <command> = shows manual (RTFM)
```

Basic file manipulation

```
cat <fileName> = show content of file
            (less, more)
head = from the top
      -n <#oflines> <fileName>

tail = from the bottom
      -n <#oflines> <fileName>

mkdir = create new folder
mkdir myStuff ..
mkdir myStuff/pictures/ ..

cp image.jpg newimage.jpg = copy and rename a file
cp image.jpg <folderName>/ = copy to folder
cp image.jpg folder/sameImageNewName.jpg
cp -R stuff otherStuff = copy and rename a folder
cp *.txt stuff/ = copy all of *<file type> to folder

mv file.txt Documents/ = move file to a folder
mv <folderName> <folderName2> = move folder in folder
mv filename.txt filename2.txt = rename file
mv <fileName> stuff/newfileName
mv <folderName>/ .. = move folder up in hierarchy

rm <fileName> .. = delete file (s)
rm -i <fileName> .. = ask for confirmation each file
rm -f <fileName> = force deletion of a file
rm -r <foldername>/ = delete folder

touch <fileName> = create or update a file

ln file1 file2 = physical link
ln -s file1 file2 = symbolic link
```

Linux Bash Shell Cheat Sheet

Basic Commands

Researching Files

The slow method (sometimes very slow):

```
locate <text> = search the content of all the files
locate <fileName> = search for a file
sudo updatedb = update database of files

find = the best file search tool (fast)
find -name "<fileName>"
find -name "text" = search for files who start with the word text
find -name "*text" = "      "   "   "   end   "   "   "   "
```

Advanced Search:

```
Search from file Size (in ~)
      find ~ -size +10M = search files bigger than.. (M,K,G)

Search from last access
      find -name "<filetype>" -atime -5
            ('-' = less than, '+' = more than and nothing = exactly)

Search only files or directory's
      find -type d --> ex: find /var/log -name "syslog" -type d
      find -type f = files

More info: man find, man locate
```

Extract, sort and filter data

```
grep <someText> <fileName> = search for text in file
      -i = Doesn't consider uppercase words
      -I = exclude binary files
grep -r <text> <folderName>/ = search for file names
      with occurrence of the text
```

With regular expressions:

```
grep -E ^<text> <fileName> = search start of lines
with the word text
grep -E <0-4> <fileName> =shows lines containing numbers 0-4
grep -E <a-zA-Z> <fileName> = retrieve all lines
with alphabetical letters

sort = sort the content of files
sort <fileName> = sort alphabetically
sort -o <file> <outputFile> = write result to a file
sort -r <fileName> = sort in reverse
sort -R <fileName> = sort randomly
sort -n <fileName> = sort numbers

wc = word count
wc <fileName> = nbr of line, nbr of words, byte size
      -l (lines), -w (words), -c (byte size), -m
      (number of characters)

cut = cut a part of a file
-c --> ex: cut -c 2-5 names.txt
      (cut the characters 2 to 5 of each line)
-d (delimiter)          (-d & -f good for .csv files)
-f (# of field to cut)

more info: man cut, man sort, man grep
```

Linux Bash Shell Cheat Sheet

Basic Commands

Time settings

```
date = view & modify time (on your computer)

View:
      date "+%H" --> If it's 9 am, then it will show 09
      date "+%H:%M:%Ss" = (hours, minutes, seconds)
      %Y = years
Modify:
                MMDDhhmmYYYY
      Month | Day | Hours | Minutes | Year

sudo date 031423421997 = March 14th 1997, 23:42
```

Execute programs at another time

```
use 'at' to execute programs in the future

Step 1, write in the terminal: at <timeOfExecution> ENTER
ex --> at 16:45 or at 13:43 7/23/11 (to be more precise)
or after a certain delay:
      at now +5 minutes (hours, days, weeks, months, years)
Step 2: <ENTER COMMAND> ENTER
      repeat step 2 as many times you need
Step 3: CTRL D to close input

atq = show a list of jobs waiting to be executed

atrm = delete a job n°<x>
ex (delete job #42) --> atrm 42

sleep = pause between commands
      with ';' you can chain commands, ex: touch file; rm file
you can make a pause between commands (minutes, hours, days)
ex --> touch file; sleep 10; rm file <-- 10 seconds
```

(continued)

```
crontab = execute a command regularly
      -e = modify the crontab
      -l = view current crontab
      -r = delete you crontab
In crontab the syntax is
<Minutes> <Hours> <Day of month> <Day of week (0-6,
0 = Sunday)> <COMMAND>

ex, create the file movies.txt every day at 15:47:
47 15 * * * touch /home/bob/movies.txt
* * * * * --> every minute
at 5:30 in the morning, from the 1st to 15th each month:
30 5 1-15 * *
at midnight on Mondays, Wednesdays and Thursdays:
0 0 * * 1,3,4
every two hours:
0 */2 * * *
every 10 minutes Monday to Friday:
*/10 * * * 1-5
```

Execute programs in the background

```
Add a '&' at the end of a command
      ex --> cp bigMovieFile.mp4 &

nohup: ignores the HUP signal when closing the console
(process will still run if the terminal is closed)
      ex --> nohup cp bigMovieFile.mp4

jobs = know what is running in the background

fg = put a background process to foreground
      ex: fg (process 1), f%2 (process 2) f%3, ...
```

Linux Bash Shell Cheat Sheet

Basic Commands

Process Management

```
w = who is logged on and what they are doing

tload = graphic representation of system load average
      (quit with CTRL C)

ps = Static process list
      -ef --> ex: ps -ef | less
      -ejH --> show process hierarchy
      -u --> process's from current user

top = Dynamic process list
While in top:
         • q to close top
         • h to show the help
         • k to kill a process

CTRL C to top a current terminal process

kill = kill a process
      You need the PID # of the process
            ps -u <AccountName> | grep <Application>
      Then
            kill <PID> .. .. ..
kill -9 <PID> = violent kill

killall = kill multiple process's
      ex --> killall locate

extras:
      sudo halt <-- to close computer
      sudo reboot <-- to reboot
```

Create and modify user accounts

```
sudo adduser bob = root creates new user
sudo passwd <AccountName> = change a user's password
sudo deluser <AccountName> = delete an account

addgroup friends = create a new user group
delgroup friends = delete a user group

usermod -g friends <Account> = add user to a group
usermod -g bob boby = change account name
usermod -aG friends bob = add groups to a user with-
out loosing the ones he's already in
```

File Permissions

```
chown = change the owner of a file
      ex --> chown bob hello.txt
chown user:bob report.txt = changes the user owning
report.txt to 'user' and the group owning it to 'bob'
-R = recursively affect all the sub folders
      ex --> chown -R bob:bob /home/Daniel

chmod = modify user access/permission - simple way
      u = user
      g = group
      o = other

      d = directory (if element is a directory)
      l = link (if element is a file link)
      r = read (read permissions)
      w = write (write permissions)
      x = eXecute (only useful for scripts and
      programs)
```

Linux Bash Shell Cheat Sheet

Basic Commands

File Permissions (continued)

```
'+' means add a right
'-' means delete a right
'=' means affect a right

ex --> chmod g+w someFile.txt
      (add to current group the right to modify someFile.txt)

more info: man chmod
```

Flow redirection

```
Redirect results of commands:

'>' at the end of a command to redirect the result to a file
      ex --> ps -ejH > process.txt
'>>' to redirect the result to the end of a file
```

Redirect errors:

```
'2>' at the end of the command to redirect the result to a file
      ex --> cut -d , -f 1 file.csv > file 2> errors.log
'2>&1' to redirect the errors the same way as the standard output

Read progressively from the keyboard

<Command> << <wordToTerminateInput>
      ex --> sort << END <-- This can be anything you want
             > Hello
             > Alex
             > Cinema
             > Game
             > Code
             > Ubuntu
             > END
```

Flow Redirection (continued)

```
terminal output:
Alex
Cinema
Code
Game
Ubuntu

      Another example --> wc -m << END
```

Chain commands

```
'|' at the end of a command to enter another one
      ex --> du | sort -nr | less
```

Archive and compress data

```
Archive and compress data the long way:

Step 1, put all the files you want to compress in
the same folder: ex --> mv *.txt folder/

Step 2, Create the tar file:
tar -cvf my_archive.tar folder/
      -c : creates a .tar archive
      -v : tells you what is happening (verbose)
      -f : assembles the archive into one file

      Step 3.1, create gzip file (most current):
gzip my_archive.tar
      to decompress: gunzip my_archive.tar.gz

Step 3.2, or create a bzip2 file (more powerful but slow):
bzip2 my_archive.tar
      to decompress: bunzip2 my_archive.tar.bz2
```

Linux Bash Shell Cheat Sheet

Basic Commands

Archive and compress data (continued)

```
step 4, to decompress the .tar file:
       tar -xvf archive.tar archive.tar

Archive and compress data the fast way:

gzip: tar -zcvf my_archive.tar.gz folder/
      decompress: tar -zcvf my_archive.tar.gz Documents/

bzip2: tar -jcvf  my_archive.tar.gz folder/
      decompress: tar -jxvf archive.tar.bz2 Documents/

Show the content of .tar, .gz or .bz2  without decompressing it:

gzip:
      gzip -ztf archive.tar.gz
bzip2:
      bzip2 -jtf archive.tar.bz2
tar:
      tar -tf archive.tar

tar extra:
      tar -rvf archive.tar file.txt = add a file to the .tar

You can also directly compress a single file and view the file
without decompressing:

Step 1, use gzip or bzip2 to compress the file:
      gzip numbers.txt

Step 2, view the file without decompressing it:
      zcat = view the entire file in the console (same as cat)
      zmore = view one screen at a time the content of the file (same as more)
      zless = view one line of the file at a time (same as less)
```

Installing software

```
When software is available in the repositories:
sudo apt-get install <nameOfSoftware>
      ex--> sudo apt-get install aptitude

If you download it from the Internets in .gz format
(or bz2) - "Compiling from source"
Step 1, create a folder to place the file:
      mkdir /home/username/src <-- then cd to it

Step 2, with 'ls' verify that the file is there
(if not, mv ../file.tar.gz /home/username/src/)

Step 3, decompress the file (if .zip: unzip <file>)
<--
Step 4, use 'ls', you should see a new directory
Step 5, cd to the new directory
Step 6.1, use ls to verify you have an INSTALL file,
then: more INSTALL
If you don't have an INSTALL file:
Step 6.2, execute ./configure <-- creates a makefile
Step 6.2.1, run make <-- builds application binaries
Step 6.2.2 : switch to root --> su
Step 6.2.3 : make install <-- installs the software
Step 7, read the readme file
```

By @ml874

- Data Science Cheatsheet (PDF)

Data Science Cheatsheet

Compiled by Maverick Lin (http://mavericklin.com)

Last Updated August 13, 2018

What is Data Science?

Multi-disciplinary field that brings together concepts from computer science, statistics/machine learning, and data analysis to understand and extract insights from the ever-increasing amounts of data.

Two paradigms of data research.

1. **Hypothesis-Driven:** Given a problem, what kind of data do we need to help solve it?
2. **Data-Driven:** Given some data, what interesting problems can be solved with it?

The heart of data science is to always ask questions. Always be curious about the world.

1. What can we learn from this data?
2. What actions can we take once we find whatever it is we are looking for?

Types of Data

Structured: Data that has predefined structures. e.g. tables, spreadsheets, or relational databases.
Unstructured Data: Data with no predefined structure, comes in any size or form, cannot be easily stored in tables. e.g. blobs of text, images, audio
Quantitative Data: Numerical. e.g. height, weight
Categorical Data: Data that can be labeled or divided into groups. e.g. race, sex, hair color.
Big Data: Massive datasets, or data that contains greater *variety* arriving in increasing *volumes* and with ever-higher *velocity* (3 Vs). Cannot fit in the memory of a single machine.

Data Sources/Fomats

Most Common Data Formats CSV, XML, SQL, JSON, Protocol Buffers
Data Sources Companies/Proprietary Data, APIs, Government, Academic, Web Scraping/Crawling

Main Types of Problems

Two problems arise repeatedly in data science.
Classification: Assigning something to a discrete set of possibilities. e.g. spam or non-spam, Democrat or Republican, blood type (A, B, AB, O)
Regression: Predicting a numerical value. e.g. someone's income, next year GDP, stock price

Probability Overview

Probability theory provides a framework for reasoning about likelihood of events.

Terminology

Experiment: procedure that yields one of a possible set of outcomes e.g. repeatedly tossing a die or coin
Sample Space S: set of possible outcomes of an experiment e.g. if tossing a die, S = (1,2,3,4,5,6
Event E: set of outcomes of an experiment e.g. event that a roll is 5, or the event that sum of 2 rolls is 7
Probability of an Outcome s or P(s): number that satisfies 2 properties

1. for each outcome s, $0 \leq P(s) \leq 1$
2. $\sum p(s) = 1$

Probability of Event E: sum of the probabilities of the outcomes of the experiment: $p(E) = \sum_{s \subset E} p(s)$
Random Variable V: numerical function on the outcomes of a probability space
Expected Value of Random Variable V: $E(V) = \sum_{s \subset S} p(s) * V(s)$

Independence, Conditional, Compound

Independent Events: A and B are independent iff:

$$P(A \cap B) = P(A)P(B)$$
$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$

Conditional Probability: $P(A|B) = P(A,B)/P(B)$
Bayes Theorem: $P(A|B) = P(B|A)P(A)/P(B)$
Joint Probability: $P(A,B) = P(B|A)P(A)$
Marginal Probability: $P(A)$

Probability Distributions

Probability Density Function (PDF) Gives the probability that a rv takes on the value x: $p_X(x) = P(X = x)$
Cumulative Density Function (CDF) Gives the probability that a random variable is less than or equal to x: $F_X(x) = P(X \leq x)$
Note: The PDF and the CDF of a given random variable contain exactly the same information.

Descriptive Statistics

Provides a way of capturing a given data set or sample. There are two main types: **centrality** and **variability** measures.

Centrality

Arithmetic Mean Useful to characterize symmetric distributions without outliers $\mu_X = \frac{1}{n}\sum x$
Geometric Mean Useful for averaging ratios. Always less than arithmetic mean $= \sqrt[n]{a_1 a_2 ... a_3}$
Median Exact middle value among a dataset. Useful for skewed distribution or data with outliers.
Mode Most frequent element in a dataset.

Variability

Standard Deviation Measures the squares differences between the individual elements and the mean

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

Variance $V = \sigma^2$

Interpreting Variance

Variance is an inherent part of the universe. It is impossible to obtain the same results after repeated observations of the same event due to random noise/error. Variance can be explained away by attributing to sampling or measurement errors. Other times, the variance is due to the random fluctuations of the universe.

Correlation Analysis

Correlation coefficients r(X,Y) is a statistic that measures the degree that Y is a function of X and vice versa. Correlation values range from -1 to 1, where 1 means fully correlated, -1 means negatively-correlated, and 0 means no correlation.
Pearson Coefficient Measures the degree of the relationship between linearly related variables

$$r = \frac{Cov(X,Y)}{\sigma(X)\sigma(Y)}$$

Spearman Rank Coefficient Computed on ranks and depicts monotonic relationships

Note: Correlation does not imply causation!

Data Cleaning

Data Cleaning is the process of turning raw data into a clean and analyzable data set. "Garbage in, garbage out." Make sure garbage doesn't get put in.

Errors vs. Artifacts

1. **Errors:** information that is lost during acquisition and can never be recovered e.g. power outage, crashed servers
2. **Artifacts:** systematic problems that arise from the data cleaning process. these problems can be corrected but we must first discover them

Data Compatibility

Data compatibility problems arise when merging datasets. Make sure you are comparing "apples to apples" and not "apples to oranges". Main types of conversions/unifications:

- **units** (metric vs. imperial)
- **numbers** (decimals vs. integers),
- **names** (John Smith vs. Smith, John),
- **time/dates** (UNIX vs. UTC vs. GMT),
- **currency** (currency type, inflation-adjusted, dividends)

Data Imputation

Process of dealing with missing values. The proper methods depend on the type of data we are working with. General methods include:

- Drop all records containing missing data
- Heuristic-Based: make a reasonable guess based on knowledge of the underlying domain
- Mean Value: fill in missing data with the mean
- Random Value
- Nearest Neighbor: fill in missing data using similar data points
- Interpolation: use a method like linear regression to predict the value of the missing data

Outlier Detection

Outliers can interfere with analysis and often arise from mistakes during data collection. It makes sense to run a "sanity check".

Miscellaneous

Lowercasing, removing non-alphanumeric, repairing, unidecode, removing unknown characters

Note: When cleaning data, always maintain both the raw data and the cleaned version(s). The raw data should be kept intact and preserved for future use. Any type of data cleaning/analysis should be done on a copy of the raw data.

Feature Engineering

Feature engineering is the process of using domain knowledge to create features or input variables that help machine learning algorithms perform better. Done correctly, it can help increase the predictive power of your models. Feature engineering is more of an art than science. FE is one of the most important steps in creating a good model. As Andrew Ng puts it:

"Coming up with features is difficult, time-consuming, requires expert knowledge. 'Applied machine learning' is basically feature engineering."

Continuous Data

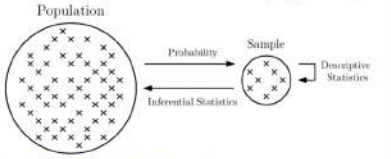
Raw Measures: data that hasn't been transformed yet
Rounding: sometimes precision is noise; round to nearest integer, decimal etc..
Scaling: log, z-score, minmax scale
Imputation: fill in missing values using mean, median, model output, etc..
Binning: transforming numeric features into categorical ones (or binned) e.g. values between 1-10 belong to A, between 10-20 belong to B, etc.
Interactions: interactions between features: e.g. subtraction, addition, multiplication, statistical test
Statistical: log/power transform (helps turn skewed distributions more normal), Box-Cox
Row Statistics: number of NaN's, 0's, negative values, max, min, etc
Dimensionality Reduction: using PCA, clustering, factor analysis etc

Discrete Data

Encoding: since some ML algorithms cannot work on categorical data, we need to turn categorical data into numerical data or vectors
Ordinal Values: convert each distinct feature into a random number (e.g. [r,g,b] becomes [1,2,3])
One-Hot Encoding: each of the m features becomes a vector of length m with containing only one 1 (e.g. [r, g, b] becomes [[1,0,0],[0,1,0],[0,0,1]])
Feature Hashing Scheme: turns arbitrary features into indices in a vector or matrix
Embeddings: if using words, convert words to vectors (word embeddings)

Statistical Analysis

Process of statistical reasoning: there is an underlying population of possible things we can potentially observe and only a small subset of them are actually sampled (ideally at random). Probability theory describes what properties our sample should have given the properties of the population, but ***statistical inference*** allows us to deduce what the full population is like after analyzing the sample.



Sampling From Distributions

Inverse Transform Sampling Sampling points from a given probability distribution is sometimes necessary to run simulations or whether your data fits a particular distribution. The general technique is called *inverse transform sampling* or Smirnov transform. First draw a random number p between [0,1]. Compute value x such that the CDF equals p: $F_X(x) = p$. Use x as the value to be the random value drawn from the distribution described by $F_X(x)$.

Monte Carlo Sampling In higher dimensions, correctly sampling from a given distribution becomes more tricky. Generally want to use Monte Carlo methods, which typically follow these rules: define a domain of possible inputs, generate random inputs from a probability distribution over the domain, perform a deterministic calculation, and analyze the results.

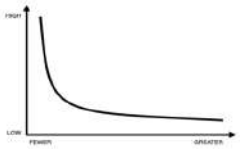
Classic Statistical Distributions

Binomial Distribution (Discrete)
Assume X is distributed Bin(n,p). X is the number of "successes" that we will achieve in n independent trials, where each trial is either a success or failure and each success occurs with the same probability p and each failure occurs with probability q=1-p.
PDF: $P(X = x) = \binom{n}{k} p^x (1-p)^{n-x}$
EV: $\mu = np$ Variance = npq

Normal/Gaussian Distribution (Continuous)
Assume X in distributed $\mathcal{N}(\mu, \sigma^2)$. It is a bell-shaped and symmetric distribution. Bulk of the values lie close to the mean and no value is too extreme. Generalization of the binomial distribution as $n \to \infty$.
PDF: $P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$
EV: μ Variance: σ^2
Implications: 68%-95%-99% rule. 68% of probability mass fall within 1σ of the mean, 95% within 2σ, and 99.7% within 3σ.

Poisson Distribution (Discrete)
Assume X is distributed Pois(λ). Poisson expresses the probability of a given number of events occurring in a fixed interval of time/space if these events occur independently and with a known constant rate λ.
PDF: $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ EV: λ Variance = λ

Power Law Distributions (Discrete)
Many data distributions have much longer tails than the normal or Poisson distributions. In other words, the change in one quantity varies as a *power* of another quantity. It helps measure the inequality in the world. e.g. wealth, word frequency and Pareto Principle (80/20 Rule)
PDF: $P(X=x) = cx^{-\alpha}$, where α is the law's exponent and c is the normalizing constant



Modeling- Overview

Modeling is the process of incorporating information into a tool which can forecast and make predictions. Usually, we are dealing with statistical modeling where we want to analyze relationships between variables. Formally, we want to estimate a function $f(X)$ such that:

$$Y = f(X) + \epsilon$$

where $X = (X_1, X_2, ...X_p)$ represents the input variables, Y represents the output variable, and ϵ represents random error.

Statistical learning is set of approaches for estimating this $f(X)$.

Why Estimate f(X)?

Prediction: once we have a good estimate $\hat{f}(X)$, we can use it to make predictions on new data. We treat $\hat{f}$ as a black box, since we only care about the accuracy of the predictions, not why or how it works.
Inference: we want to understand the relationship between X and Y. We can no longer treat $\hat{f}$ as a black box since we want to understand how Y changes with respect to $X = (X_1, X_2, ...X_p)$

More About ϵ

The error term ϵ is composed of the reducible and irreducible error, which will prevent us from ever obtaining a perfect $\hat{f}$ estimate.

- **Reducible**: error that can potentially be reduced by using the most appropriate statistical learning technique to estimate f. The goal is to minimize the reducible error.
- **Irreducible**: error that cannot be reduced no matter how well we estimate f. Irreducible error is unknown and unmeasurable and will always be an upper bound for ϵ.

Note: There will always be trade-offs between model flexibility (prediction) and model interpretability (inference). This is just another case of the bias-variance trade-off. Typically, as flexibility increases, interpretability decreases. Much of statistical learning/modeling is finding a way to balance the two.

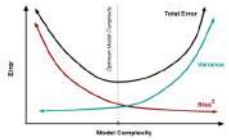
Modeling- Philosophies

Modeling is the process of incorporating information into a tool which can forecast and make predictions. Designing and validating models is important, as well as evaluating the performance of models. Note that the best forecasting model may not be the most accurate one.

Philosophies of Modeling

Occam's Razor Philosophical principle that the simplest explanation is the best explanation. In modeling, if we are given two models that predicts equally well, we should choose the simpler one. Choosing the more complex one can often result in overfitting.
Bias Variance Trade-Off Inherent part of predictive modeling, where models with lower bias will have higher variance and vice versa. Goal is to achieve low bias and low variance.

- ***Bias***: error from incorrect assumptions to make target function easier to learn (high bias → missing relevant relations or underfitting)
- ***Variance***: error from sensitivity to fluctuations in the dataset, or how much the target estimate would differ if different training data was used (high variance → modeling noise or overfitting)



No Free Lunch Theorem No single machine learning algorithm is better than all the others on all problems. It is common to try multiple models and find one that works best for a particular problem.

Thinking Like Nate Silver

1. Think Probabilistically Probabilistic forecasts are more meaningful than concrete statements and should be reported as probability distributions (including σ along with mean prediction μ.
2. Incorporate New Information Use live models, which continually updates using new information. To update, use Bayesian reasoning to calculate how probabilities change in response to new evidence.
3. Look For Consensus Forecast Use multiple distinct sources of evidence. Ssome models operate this way, such as boosting and bagging, which uses large number of weak classifiers to produce a strong one.

Modeling- Taxonomy

There are many different types of models. It is important to understand the trade-offs and when to use a certain type of model.

Parametric vs. Nonparametric

- **Parametric**: models that first make an assumption about a function form, or shape, of f (linear). Then fits the model. This reduces estimating f to just estimating set of parameters, but if our assumption was wrong, will lead to bad results.
- **Non-Parametric**: models that don't make any assumptions about f, which allows them to fit a wider range of shapes; but may lead to overfitting

Supervised vs. Unsupervised

- **Supervised**: models that fit input variables $x_i = (x_1, x_2, ...x_n)$ to a known output variables $y_i = (y_1, y_2, ...y_n)$
- **Unsupervised**: models that take in input variables $x_i = (x_1, x_2, ...x_n)$, but they do not have an associated output to supervise the training. The goal is understand relationships between the variables or observations.

Blackbox vs. Descriptive

- **Blackbox**: models that make decisions, but we do not know what happens "under the hood" e.g. deep learning, neural networks
- **Descriptive**: models that provide insight into *why* they make their decisions e.g. linear regression, decision trees

First-Principle vs. Data-Driven

- **First-Principle**: models based on a prior belief of how the system under investigation works, incorporates domain knowledge (ad-hoc)
- **Data-Driven**: models based on observed correlations between input and output variables

Deterministic vs. Stochastic

- **Deterministic**: models that produce a single "prediction" e.g. yes or no, true or false
- **Stochastic**: models that produce probability distributions over possible events

Flat vs. Hierarchical

- **Flat**: models that solve problems on a single level, no notion of subproblems
- **Hierarchical**: models that solve several different nested subproblems

Modeling- Evaluation Metrics

Need to determine how good our model is. Best way to assess models is out-of-sample predictions (data points your model has never seen).

Classification

	Predicted Yes	Predicted No
Actual Yes	True Positives (TP)	False Negatives (FN)
Actual No	False Positives (FP)	True Negatives (TN)

Accuracy: ratio of correct predictions over total predictions. Misleading when class sizes are substantially different. $accuracy = \frac{TP+TN}{TP+TN+FN+FP}$
Precision: how often the classifier is correct when it predicts positive: $precision = \frac{TP}{TP+FP}$
Recall: how often the classifier is correct for all positive instances: $recall = \frac{TP}{TP+FN}$
F-Score: single measurement to describe performance: $F = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$
ROC Curves: plots true positive rates and false positive rates for various thresholds, or where the model determines if a data point is positive or negative (e.g. if >0.8, classify as positive). Best possible area under the ROC curve (AUC) is 1, while random is 0.5, or the main diagonal line.

Regression

Errors are defined as the difference between a prediction $y\prime$ and the actual result y.
Absolute Error: $\Delta = y\prime - y$
Squared Error: $\Delta^2 = (y\prime - y)^2$
Mean-Squared Error: $MSE = \frac{1}{n}\sum_{i=1}^{n}(y\prime_i - y_i)^2$
Root Mean-Squared Error: $\text{RMSD} = \sqrt{MSE}$
Absolute Error Distribution: Plot absolute error distribution: should be symmetric, centered around 0, bell-shaped, and contain rare extreme outliers.

Modeling- Evaluation Environment

Evaluation metrics provides use with the tools to estimate errors, but what should be the process to obtain the best estimate? Resampling involves repeatedly drawing samples from a training set and refitting a model to each sample, which provides us with additional information compared to fitting the model once, such as obtaining a better estimate for the test error.

Key Concepts

Training Data: data used to fit your models or the set used for learning
Validation Data: data used to tune the parameters of a model
Test Data: data used to evaluate how good your model is. Ideally your model should never touch this data until final testing/evaluation

Cross Validation

Class of methods that estimate test error by holding out a subset of training data from the fitting process.
Validation Set: split data into training set and validation set. Train model on training and estimate test error using validation. e.g. 80-20 split
Leave-One-Out CV (LOOCV): split data into training set and validation set, but the validation set consists of 1 observation. Then repeat n-1 times until all observations have been used as validation. Test erro is the average of these n test error estimates.
k-Fold CV: randomly divide data into k groups (folds) of approximately equal size. First fold is used as validation and the rest as training. Then repeat k times and find average of the k estimates.

Bootstrapping

Methods that rely on random sampling with replacement. Bootstrapping helps with quantifying uncertainty associated with a given estimate or model.

Amplifying Small Data Sets

What can we do it we don't have enough data?

- **Create Negative Examples**- e.g. classifying presidential candidates, most people would be unqualified so label most as unqualified
- **Synthetic Data**- create additional data by adding noise to the real data

Linear Regression

Linear regression is a simple and useful tool for predicting a quantitative response. The relationship between input variables $X = (X_1, X_2, ...X_p)$ and output variable Y takes the form:

$$Y \approx \beta_0 + \beta_1 X_1 + ... + \beta_p X_p + \epsilon$$

$\beta_0 ... \beta_p$ are the unknown coefficients (parameters) which we are trying to determine. The best coefficients will lead us to the best "fit", which can be found by minimizing the *residual sum squares* (RSS), or the sum of the differences between the actual ith value and the predicted ith value. $\text{RSS} = \sum_{i=1}^{n} e_{ii}$, where $e_i = y_i - \hat{y}_i$

How to find best fit?

Matrix Form: We can solve the closed-form equation for coefficient vector w: $w = (X^T X)^{-1} X^T Y$. X represents the input data and Y represents the output data. This method is used for smaller matrices, since inverting a matrix is computationally expensive.

Gradient Descent: First-order optimization algorithm. We can find the minimum of a *convex* function by starting at an arbitrary point and repeatedly take steps in the downward direction, which can be found by taking the negative direction of the gradient. After several iterations, we will eventually converge to the minimum. In our case, the minimum corresponds to the coefficients with the minimum error, or the best line of fit. The learning rate α determines the size of the steps we take in the downward direction.

Gradient descent algorithm in two dimensions. Repeat until convergence.

1. $w_0^{t+1} := w_0^t - \alpha \frac{\partial}{\partial w_0} J(w_0, w_1)$
2. $w_1^{t+1} := w_1^t - \alpha \frac{\partial}{\partial w_1} J(w_0, w_1)$

For non-convex functions, gradient descent no longer guarantees an optimal solutions since there may be local minimas. Instead, we should run the algorithm from different starting points and use the best local minima we find for the solution.

Stochastic Gradient Descent: instead of taking a step after sampling the *entire* training set, we take a small batch of training data at random to determine our next step. Computationally more efficient and may lead to faster convergence.

Linear Regression II

Improving Linear Regression

Subset/Feature Selection: approach involves identifying a subset of the p predictors that we believe to be best related to the response. Then we fit model using the reduced set of variables.

- Best, Forward, and Backward Subset Selection

Shrinkage/Regularization: all variables are used, but estimated coefficients are shrunken towards zero relative to the least squares estimate. λ represents the tuning parameter- as λ increases, flexibility decreases $\rightarrow$ decreased variance but increased bias. The tuning parameter is key in determining the sweet spot between under and over-fitting. In addition, while Ridge will always produce a model with p variables, Lasso can force coefficients to be equal to zero.

- Lasso (L1): min RSS $+ \lambda \sum_{j=1}^{p} |\beta_j|$
- Ridge (L2): min RSS $+ \lambda \sum_{j=1}^{p} \beta_j^2$

Dimension Reduction: projecting p predictors into a M-dimensional subspace, where $M < p$. This is achieved by computing M different linear combinations of the variables. Can use PCA.

Miscellaneous: Removing outliers, feature scaling, removing multicollinearity (correlated variables)

Evaluating Model Accuracy

Residual Standard Error (RSE): $\text{RSE} = \sqrt{\frac{1}{n-2} RSS}$. Generally, the smaller the better.

R^2: Measure of fit that represents the proportion of variance explained, or the *variability in Y that can be explained using X*. It takes on a value between 0 and 1. Generally the higher the better. $R^2 = 1 - \frac{RSS}{TSS}$, where Total Sum of Squares (TSS) $= \sum (y_i - \bar{y})^2$

Evaluating Coefficient Estimates

Standard Error (SE) of the coefficients can be used to perform hypothesis tests on the coefficients:

H_0: No relationship between X and Y, H_a: Some relationship exists. A p-value can be obtained and can be interpreted as follows: a small p-value indicates that a relationship between the predictor (X) and the response (Y) exists. Typical p-value cutoffs are around 5 or 1 %.

Logistic Regression

Logistic regression is used for classification, where the response variable is categorical rather than numerical.

The model works by predicting the probability that Y belongs to a particular category by first fitting the data to a linear regression model, which is then passed to the logistic function (below). The logistic function will always produce a S-shaped curve, so regardless of X, we can always obtain a sensible answer (between 0 and 1). If the probability is above a certain predetermined threshold (e.g. P(Yes) > 0.5), then the model will predict Yes.

$$p(X) = \frac{e^{\beta_0 + \beta_1 X_1 + ... + \beta_p X_p}}{1 + e^{\beta_0 + \beta_1 X_1 + ... + \beta_p X_p}}$$

How to find best coefficients?

Maximum Likelihood: The coefficients $\beta_0 ... \beta_p$ are unknown and must be estimated from the training data. We seek estimates for $\beta_0 ... \beta_p$ such that the predicted probability $\hat{p}(x_i)$ of each observation is a number close to one if its observed in a certain class and close to zero otherwise. This is done by maximizing the likelihood function:

$$l(\beta_0, \beta_1) = \prod_{i:y_i=1} p(x_i) \prod_{i':y_{i'}=1} (1 - p(x_i))$$

Potential Issues

Imbalanced Classes: imbalance in classes in training data lead to poor classifiers. It can result in a lot of false positives and also lead to few training data. Solutions include forcing balanced data by removing observations from the larger class, replicate data from the smaller class, or heavily weigh the training examples toward instances of the larger class.

Multi-Class Classification: the more classes you try to predict, the harder it will be for the the classifier to be effective. It is possible with logistic regression, but another approach, such as Linear Discriminant Analysis (LDA), may prove better.

Distance/Network Methods

Interpreting examples as points in space provides a way to find natural groupings or clusters among data e.g. which stars are the closest to our sun? Networks can also be built from point sets (vertices) by connecting related points.

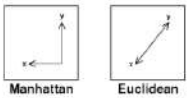
Measuring Distances/Similarity Measure

There are several ways of measuring distances between points a and b in d dimensions- with closer distances implying similarity.

Minkowski Distance Metric: $d_k(a,b) = \sqrt[k]{\sum_{i=1}^{d} |a_i - b_i|^k}$

The parameter k provides a way to tradeoff between the largest and the total dimensional difference. In other words, larger values of k place more emphasis on large differences between feature values than smaller values. Selecting the right k can significantly impact the the meaningfulness of your distance function. The most popular values are 1 and 2.

- Manhattan (k=1): city block distance, or the sum of the absolute difference between two points
- Euclidean (k=2): straight line distance



Weighted Minkowski: $d_k(a,b) = \sqrt[k]{\sum_{i=1}^{d} w_i |a_i - b_i|^k}$, in some scenarios, not all dimensions are equal. Can convey this idea using w_i. Generally not a good idea- should normalize data by Z-scores before computing distances.

Cosine Similarity: $cos(a,b) = \frac{a \cdot b}{|a||b|}$, calculates the similarity between 2 non-zero vectors, where $a \cdot b$ is the dot product (normalized between 0 and 1), higher values imply more similar vectors

Kullback-Leibler Divergence: $KL(A||B) = \sum_{i=i}^{d} a_i log_2 \frac{a_i}{b_i}$

KL divergence measures the distances between probability distributions by measuring the uncertainty gained or uncertainty lost when replacing distribution A with distribution B. However, this is not a metric but forms the basis for the Jensen-Shannon Divergence Metric.

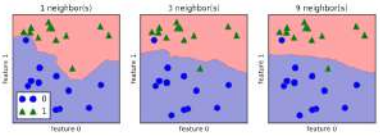
Jensen-Shannon: $JS(A,B) = \frac{1}{2} KL(A||M) + \frac{1}{2} KL(M||B)$, where M is the average of A and B. The JS function is the right metric for calculating distances between probability distributions

Nearest Neighbor Classification

Distance functions allow us to identify the points closest to a given target, or the *nearest neighbors (NN)* to a given point. The advantages of NN include simplicity, interpretability and non-linearity.

k-Nearest Neighbors

Given a positive integer k and a point x_0, the KNN classifier first identifies k points in the training data most similar to x_0, then estimates the conditional probability of x_0 being in class j as the fraction of the k points whose values belong to j. The optimal value for k can be found using cross validation.



KNN Algorithm

1. Compute distance D(a,b) from point b to all points
2. Select k closest points and their labels
3. Output class with most frequent labels in k points

Optimizing KNN

Comparing a query point a in d dimensions against n training examples computes with a runtime of $O(nd)$, which can cause lag as points reach millions or billions. Popular choices to speed up KNN include:

- **Vernoi Diagrams**: partitioning plane into regions based on distance to points in a specific subset of the plane
- **Grid Indexes**: carve up space into d-dimensional boxes or grids and calculate the NN in the same cell as the point
- **Locality Sensitive Hashing (LSH)**: abandons the idea of finding the exact nearest neighbors. Instead, batch up nearby points to quickly find the most appropriate bucket B for our query point. LSH is defined by a hash function $h(p)$ that takes a point/vector as input and produces a number/ code as output, such that it is likely that h(a) = h(b) if a and b are close to each other, and h(a)!= h(b) if they are far apart.

Clustering

Clustering is the problem of grouping points by similarity using distance metrics, which ideally reflect the similarities you are looking for. Often items come from logical "sources" and clustering is a good way to reveal those origins. Perhaps the first thing to do with any data set. Possible applications include: hypothesis development, modeling over smaller subsets of data, data reduction, outlier detection.

K-Means Clustering

Simple and elegant algorithm to partition a dataset into K distinct, non-overlapping clusters.

1. Choose a K. Randomly assign a number between 1 and K to each observation. These serve as initial cluster assignments
2. Iterate until cluster assignments stop changing
   (a) For each of the K clusters, compute the cluster centroid. The kth cluster centroid is the vector of the p feature means for the observations in the kth cluster.
   (b) Assign each observation to the cluster whose centroid is closest (where closest is defined using distance metric).

Since the results of the algorithm depends on the initial random assignments, it is a good idea to repeat the algorithm from different random initializations to obtain the best overall results. Can use MSE to determine which cluster assignment is better.

Hierarchical Clustering

Alternative clustering algorithm that does not require us to commit to a particular K. Another advantage is that it results in a nice visualization called a **dendrogram**. Observations that fuse at bottom are similar, where those at the top are quite different- we draw conclusions based on the location on the vertical rather than horizontal axis.

1. Begin with n observations and a measure of all the $\frac{(n)n-1}{2}$ pairwise dissimilarities. Treat each observation as its own cluster.
2. For i = n, n-1, ...2
   (a) Examine all pairwise inter-cluster dissimilarities among the i clusters and identify the pair of clusters that are least dissimilar (most similar). Fuse these two clusters. The dissimilarity between these two clusters indicates height in dendrogram where fusion should be placed.
   (b) Assign each observation to the cluster whose centroid is closest (where closest is defined using distance metric).

Linkage: Complete (max dissimilarity), Single (min), Average, Centroid (between centroids of cluster A and B)

Machine Learning Part I

Comparing ML Algorithms

Power and Expressibility: ML methods differ in terms of complexity. Linear regression fits linear functions while NN define piecewise-linear separation boundaries. More complex models can provide more accurate models, but at the risk of overfitting.
Interpretability: some models are more transparent and understandable than others (white box vs. black box models)
Ease of Use: some models feature few parameters/decisions (linear regression/NN), while others require more decision making to optimize (SVMs)
Training Speed: models differ in how fast they fit the necessary parameters
Prediction Speed: models differ in how fast they make predictions given a query

Method	Power of Expression	Ease of Interpretation	Ease of Use	Training Speed	Prediction Speed
Linear Regression	5	9	9	9	9
Nearest Neighbor	5	9	8	10	2
Naive Bayes	4	8	7	9	8
Decision Trees	8	8	7	7	9
Support Vector Machines	8	6	6	7	7
Boosting	9	6	6	6	6
Graphical Models	9	8	5	4	4
Deep Learning	10	3	4	3	7

Naive Bayes

Naive Bayes methods are a set of supervised learning algorithms based on applying Bayes' theorem with the "naive" assumption of independence between every pair of features.

Problem: Suppose we need to classify vector $X = x_1...x_n$ into m classes, $C_1...C_m$. We need to compute the probability of each possible class given X, so we can assign X the label of the class with highest probability. We can calculate a probability using the Bayes' Theorem:

$$P(C_i|X) = \frac{P(X|C_i)P(C_i)}{P(X)}$$

Where:

1. $P(C_i)$: the prior probability of belonging to class i
2. $P(X)$: normalizing constant, or probability of seeing the given input vector over all possible input vectors
3. $P(X|C_i)$: the conditional probability of seeing input vector X given we know the class is C_i

The prediction model will formally look like:

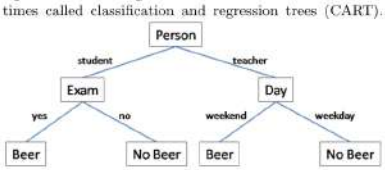
$$C(X) = argmax_{i \in classes(1)} \frac{P(X|C_i)P(C_i)}{P(X)}$$

where C(X) is the prediction returned for input X.

Machine Learning Part II

Decision Trees

Binary branching structure used to classify an arbitrary input vector X. Each node in the tree contains a simple feature comparison against some field ($x_i > 42$?). Result of each comparison is either true or false, which determines if we should proceed along to the left or right child of the given node. Also known as sometimes called classification and regression trees (CART).



Advantages: Non-linearity, support for categorical variables, easy to interpret, application to regression.
Disadvantages: Prone to overfitting, instable (not robust to noise), high variance, low bias

Note: rarely do models just use one decision tree. Instead, we aggregate many decision trees using methods like ensembling, bagging, and boosting.

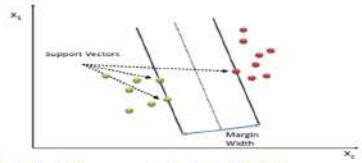
Ensembles, Bagging, Random Forests, Boosting

Ensemble learning is the strategy of combining many different classifiers/models into one predictive model. It revolves around the idea of voting: a so-called "wisdom of crowds" approach. The most predicted class will be the final prediction.
Bagging: ensemble method that works by taking B bootstrapped subsamples of the training data and constructing B trees, each tree training on a distinct subsample as
Random Forests: builds on bagging by decorrelating the trees. We do everything the same like in bagging, but when we build the trees, everytime we consider a split, a random sample of the p predictors is chosen as split candidates, not the full set (typically $m \approx \sqrt{p}$). When m = p, then we are just doing bagging.
Boosting: the main idea is to improve our model where it is not performing well by using information from previously constructed classifiers. Slow learner. Has 3 tuning parameters: number of classifiers B, learning parameter λ, interaction depth d (controls interaction order of model).

Machine Learning Part III

Support Vector Machines

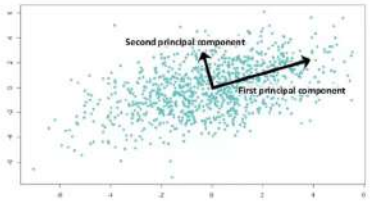
Work by constructing a hyperplane that separates points between two classes. The hyperplane is determined using the maximal margin hyperplane, which is the hyperplane that is the maximum distance from the training observations. This distance is called the margin. Points that fall on one side of the hyperplane are classified as -1 and the other +1.



Principal Component Analysis (PCA)

Principal components allow us to summarize a set of correlated variables with a smaller set of variables that collectively explain most of the variability in the original set. Essentially, we are "dropping" the least important feature variables.

Principal Component Analysis is the process by which principal components are calculated and the use of them to analyzing and understanding the data. PCA is an unsupervised approach and is used for dimensionality reduction, feature extraction, and data visualization. Variables after performing PCA are independent. Scaling variables is also important while performing PCA.



Machine Learning Part IV

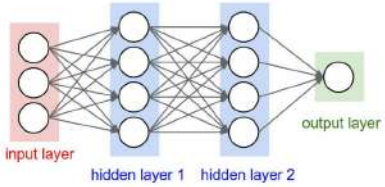
ML Terminology and Concepts

Features: input data/variables used by the ML model
Feature Engineering: transforming input features to be more useful for the models. e.g. mapping categories to buckets, normalizing between -1 and 1, removing null
Train/Eval/Test: training is data used to optimize the model, evaluation is used to asses the model on new data during training, test is used to provide the final result
Classification/Regression: regression is prediction a number (e.g. housing price), classification is prediction from a set of categories(e.g. predicting red/blue/green)
Linear Regression: predicts an output by multiplying and summing input features with weights and biases
Logistic Regression: similar to linear regression but predicts a probability
Overfitting: model performs great on the input data but poorly on the test data (combat by dropout, early stopping, or reduce # of nodes or layers)
Bias/Variance: how much output is determined by the features. more variance often can mean overfitting, more bias can mean a bad model
Regularization: variety of approaches to reduce overfitting, including adding the weights to the loss function, randomly dropping layers (dropout)
Ensemble Learning: training multiple models with different parameters to solve the same problem
A/B testing: statistical way of comparing 2+ techniques to determine which technique performs better and also if difference is statistically significant
Baseline Model: simple model/heuristic used as reference point for comparing how well a model is performing
Bias: prejudice or favoritism towards some things, people, or groups over others that can affect collection/sampling and interpretation of data, the design of a system, and how users interact with a system
Dynamic Model: model that is trained online in a continuously updating fashion
Static Model: model that is trained offline
Normalization: process of converting an actual range of values into a standard range of values, typically -1 to +1
Independently and Identically Distributed (i.i.d): data drawn from a distribution that doesn't change, and where each value drawn doesn't depend on previously drawn values; ideal but rarely found in real life
Hyperparameters: the "knobs" that you tweak during successive runs of training a model
Generalization: refers to a model's ability to make correct predictions on new, previously unseen data as opposed to the data used to train the model
Cross-Entropy: quantifies the difference between two probability distributions

Deep Learning Part I

What is Deep Learning?

Deep learning is a subset of machine learning. One popular DL technique is based on Neural Networks (NN), which loosely mimic the human brain and the code structures are arranged in layers. Each layer's input is the previous layer's output, which yields progressively higher-level features and defines a hierarchy. A Deep Neural Network is just a NN that has more than 1 hidden layer.



Recall that statistical learning is all about approximating $f(X)$. Neural networks are known as **universal approximators**, meaning no matter how complex a function is, there exists a NN that can (approximately) do the job. We can increase the approximation (or complexity) by adding more hidden layers and neurons.

Popular Architectures

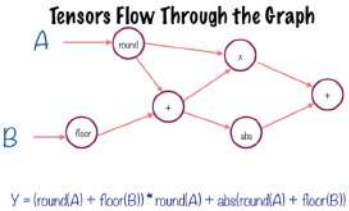
There are different kinds of NNs that are suitable for certain problems, which depend on the NN's architecture.

Linear Classifier: takes input features and combines them with weights and biases to predict output value
DNN: deep neural net, contains intermediate layers of nodes that represent "hidden features" and activation functions to represent non-linearity
CNN: convolutional NN, has a combination of convolutional, pooling, dense layers. popular for image classification.
Transfer Learning: use existing trained models as starting points and add additional layers for the specific use case. idea is that highly trained existing models know general features that serve as a good starting point for training a small network on specific examples
RNN: recurrent NN, designed for handling a sequence of inputs that have "memory" of the sequence. LSTMs are a fancy version of RNNs, popular for NLP
GAN: general adversarial NN, one model creates fake examples, and another model is served both fake example and real examples and is asked to distinguish
Wide and Deep: combines linear classifiers with deep neural net classifiers, "wide" linear parts represent memorizing specific examples and "deep" parts represent understanding high level features

Deep Learning Part II

Tensorflow

Tensorflow is an open source software library for numerical computation using data flow graphs. Everything in TF is a graph, where nodes represent operations on data and edges represent the data. Phase 1 of TF is building up a computation graph and phase 2 is executing it. It is also distributed, meaning it can run on either a cluster of machines or just a single machine.
TF is extremely popular/suitable for working with Neural Networks, since the way TF sets up the computational graph pretty much resembles a NN.



Y = (round(A) + floor(B)) * round(A) + abs(round(A) + floor(B))

Tensors

In a graph, tensors are the edges and are multidimensional data arrays that flow through the graph. Central unit of data in TF and consists of a set of primitive values shaped into an array of any number of dimensions.
A tensor is characterized by its rank (# dimensions in tensor), shape (# of dimensions and size of each dimension), data type (data type of each element in tensor).

Placeholders and Variables

Variables: best way to represent shared, persistent state manipulated by your program. These are the parameters of the ML model are altered/trained during the training process. Training variables.
Placeholders: way to specify inputs into a graph that hold the place for a Tensor that will be fed at runtime. They are assigned once, do not change after. Input nodes

Deep Learning Part III

Deep Learning Terminology and Concepts

Neuron: node in a NN, typically taking in multiple input values and generating one output value, calculates the output value by applying an activation function (nonlinear transformation) to a weighted sum of input values
Weights: edges in a NN, the goal of training is to determine the optimal weight for each feature; if weight = 0, corresponding feature does not contribute
Neural Network: composed of neurons (simple building blocks that actually "learn"), contains activation functions that makes it possible to predict non-linear outputs
Activation Functions: mathematical functions that introduce non-linearity to a network e.g. RELU, tanh
Sigmoid Function: function that maps very negative numbers to a number very close to 0, huge numbers close to 1, and 0 to .5. Useful for predicting probabilities
Gradient Descent/Backpropagation: fundamental loss optimizer algorithms, of which the other optimizers are usually based. Backpropagation is similar to gradient descent but for neural nets
Optimizer: operation that changes the weights and biases to reduce loss e.g. Adagrad or Adam
Weights / Biases: weights are values that the input features are multiplied by to predict an output value. Biases are the value of the output given a weight of 0.
Converge: algorithm that converges will eventually reach an optimal answer, even if very slowly. An algorithm that doesn't converge may never reach an optimal answer.
Learning Rate: rate at which optimizers change weights and biases. High learning rate generally trains faster but risks not converging, whereas a lower rate trains slower
Numerical Instability: issues with very large/small values due to limits of floating point numbers in computers
Embeddings: mapping from discrete objects, such as words, to vectors of real numbers. useful because classifiers/neural networks work well on vectors of real numbers
Convolutional Layer: series of convolutional operations, each acting on a different slice of the input matrix
Dropout: method for regularization in training NNs, works by removing a random selection of some units in a network layer for a single gradient step
Early Stopping: method for regularization that involves ending model training early
Gradient Descent: technique to minimize loss by computing the gradients of loss with respect to the model's parameters, conditioned on training data
Pooling: Reducing a matrix (or matrices) created by an earlier convolutional layer to a smaller matrix. Pooling usually involves taking either the maximum or average value across the pooled area

Big Data- Hadoop Overview

Data can no longer fit in memory on one machine (monolithic), so a new way of computing was devised using a group of computers to process this "big data" (distributed). Such a group is called a cluster, which makes up server farms. All of these servers have to be coordinated in the following ways: partition data, coordinate computing tasks, handle fault tolerance/recovery, and allocate capacity to process.

Hadoop

Hadoop is an open source *distributed* processing framework that manages data processing and storage for big data applications running in clustered systems. It is comprised of 3 main components:

- **Hadoop Distributed File System (HDFS)**: a distributed file system that provides high-throughput access to application data by partitioning data across many machines
- **YARN**: framework for job scheduling and cluster resource management (task coordination)
- **MapReduce**: YARN-based system for parallel processing of large data sets on multiple machines

HDFS

Each disk on a different machine in a cluster is comprised of 1 master node and the rest are workers/data nodes. The **master node** manages the overall file system by storing the directory structure and the metadata of the files. The **data nodes** physically store the data. Large files are broken up and distributed across multiple machines, which are also replicated across multiple machines to provide fault tolerance.

MapReduce

Parallel programming paradigm which allows for processing of huge amounts of data by running processes on multiple machines. Defining a MapReduce job requires two stages: map and reduce.

- **Map**: operation to be performed in parallel on small portions of the dataset. the output is a key-value pair $< K, V >$
- **Reduce**: operation to combine the results of Map

YARN- Yet Another Resource Negotiator

Coordinates tasks running on the cluster and assigns new nodes in case of failure. Comprised of 2 subcomponents: the resource manager and the node manager. The **resource manager** runs on a single master node and schedules tasks across nodes. The **node manager** runs on all other nodes and manages tasks on the individual node.

Big Data- Hadoop Ecosystem

An entire ecosystem of tools have emerged around Hadoop, which are based on interacting with HDFS. Below are some popular ones:

Hive: data warehouse software built o top of Hadoop that facilitates reading, writing, and managing large datasets residing in distributed storage using SQL-like queries (HiveQL). Hive abstracts away underlying MapReduce jobs and returns HDFS in the form of tables (not HDFS).
Pig: high level scripting language (Pig Latin) that enables writing complex data transformations. It pulls unstructured/incomplete data from sources, cleans it, and places it in a database/data warehouses. Pig performs ETL into data warehouse while Hive queries from data warehouse to perform analysis (GCP: DataFlow).
Spark: framework for writing fast, distributed programs for data processing and analysis. Spark solves similar problems as Hadoop MapReduce but with a fast in-memory approach. It is an unified engine that supports SQL queries, streaming data, machine learning and graph processing. Can operate separately from Hadoop but integrates well with Hadoop. Data is processed using Resilient Distributed Datasets (RDDs), which are immutable, lazily evaluated, and tracks lineage.
Hbase: non-relational, NoSQL, column-oriented database management system that runs on top of HDFS. Well suited for sparse data sets (GCP: BigTable)
Flink/Kafka: stream processing framework. Batch streaming is for bounded, finite datasets, with periodic updates, and delayed processing. Stream processing is for unbounded datasets, with continuous updates, and immediate processing. Stream data and stream processing must be decoupled via a message queue. Can group streaming data (windows) using tumbling (non-overlapping time), sliding (overlapping time), or session (session gap) windows.
Beam: programming model to define and execute data processing pipelines, including ETL, batch and stream (continuous) processing. After building the pipeline, it is executed by one of Beam's distributed processing back-ends (Apache Apex, Apache Flink, Apache Spark, and Google Cloud Dataflow). Modeled as a Directed Acyclic Graph (DAG).
Oozie: workflow scheduler system to manage Hadoop jobs
Sqoop: transferring framework to transfer large amounts of data into HDFS from relational databases (MySQL)

SQL Part I

Structured Query Language (SQL) is a declarative language used to access & manipulate data in databases. Usually the database is a Relational Database Management System (RDBMS), which stores data arranged in relational database tables. A table is arranged in columns and rows, where columns represent characteristics of stored data and rows represent actual data entries.

Basic Queries

- filter columns: **SELECT** col1, col3... **FROM** table1
- filter the rows: **WHERE** col4 = 1 **AND** col5 = 2
- aggregate the data: **GROUP BY**...
- limit aggregated data: **HAVING** count(*) > 1
- order of the results: **ORDER BY** col2

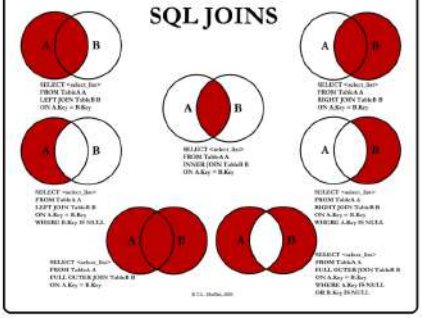
Useful Keywords for **SELECT**
DISTINCT- return unique results
BETWEEN a AND b- limit the range, the values can be numbers, text, or dates
LIKE- pattern search within the column text
IN (a, b, c) - check if the value is contained among given

Data Modification

- update specific data with the **WHERE** clause:
UPDATE table1 **SET** col1 = 1 **WHERE** col2 = 2
- insert values manually
INSERT INTO table1 (col1,col3) **VALUES** (val1,val3);
- by using the results of a query
INSERT INTO table1 (col1,col3) **SELECT** col,col2 FROM table2;

Joins

The JOIN clause is used to combine rows from two or more tables, based on a related column between them.



Python- Data Structures

Data structures are a way of storing and manipulating data and each data structure has its own strengths and weaknesses. Combined with algorithms, data structures allow us to efficiently solve problems. It is important to know the main types of data structures that you will need to efficiently solve problems.

Lists: or arrays, ordered sequences of objects, mutable

```
>>> l = [42, 3.14, "hello","world"]
```

Tuples: like lists, but immutable

```
>>> t = (42, 3.14, "hello","world")
```

Dictionaries: hash tables, key-value pairs, unsorted

```
>>> d = {"life": 42, "pi": 3.14}
```

Sets: mutable, unordered sequence of unique elements. frozensets are just immutable sets

```
>>> s = set([42, 3.14, "hello","world"])
```

Collections Module

deque: double-ended queue, generalization of stacks and queues; supports append, appendLeft, pop, rotate, etc

```
>>> s = deque([42, 3.14, "hello","world"])
```

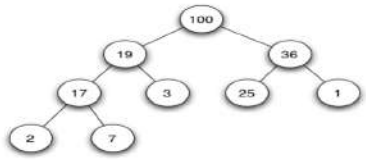
Counter: dict subclass, unordered collection where elements are stored as keys and counts stored as values

```
>>> c = Counter('apple')
>>> print(c)
Counter({'p': 2, 'a': 1, 'l': 1, 'e': 1})
```

heqpq Module

Heap Queue: priority queue, heaps are binary trees for which every parent node has a value greater than or equal to any of its children (max-heap), order is important; supports push, pop, pushpop, heapify, replace functionality

```
>>> heap = []
>>> for n in data:
...     heappush(heap, n)
>>> heap
[0, 1, 3, 6, 2, 8, 4, 7, 9, 5]
```



Recommended Resources

- Data Science Design Manual (`www.springer.com/us/book/9783319554433`)
- Introduction to Statistical Learning (`www-bcf.usc.edu/~gareth/ISL/`)
- Probability Cheatsheet (`/www.wzchen.com/probability-cheatsheet/`)
- Google's Machine Learning Crash Course (`developers.google.com/machine-learning/crash-course/`)

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Releases

No releases published

Packages

No packages published

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