Abstracts of the December 12, 2025 session of the Paris-London Analysis Seminar

Yann Brenier (CNRS et Université Paris Saclay)

Solving initial value problems by space-time convex optimization

Abstract. I will discuss a possible strategy to solve the Cauchy problem for nonlinear evolution PDEs by space-time convex optimization based on their weak formulation. One of the simplest example is the quadratic porous medium equation for which the Aronson-Benilan inequality is sharply used to prove that the strategy works for arbitrarily long time intervals. A similar result holds true for the Burgers equation. For the more challenging Euler equations, the concept of subsolution (in the sense of convex integration theory) plays a crucial role. Finally, I will mention how the Einstein equations in vacuum can be considered in that framework.

Adina Ciomaga (Université Paris Cité, Romanian Academy Iaşi Branch)

Nonlocal Hamilton Jacobi equations

Abstract. Hamilton Jacobi equations with nonlocal terms of integro - differential type naturally arise in optimal control problems, front propagation, and stochastic processes with jumps. These equations typically take the form

$$u - I[u](x) + H(x, Du) = 0, \text{ in } \mathbb{R}^d.$$

where I is a nonlocal operator associated with a Lévy measure and encodes long-range interactions or jump dynamics and H is a Hamiltonian. Firstly, I will discuss the analytical framework for viscosity solutions in the presence of nonlocal operators, emphasizing well-posedness, comparison principles, and stability under natural structural conditions on the Hamiltonian H and the nonlocal operator I. Secondly, I will address regularity of solutions for Hamilton–Jacobi equations with integro-differential terms. This is a delicate issue due to the interplay between the first-order nonlinear Hamiltonian and the nonlocal operator. Unlike the purely local case, where the coercivity of H or convexity assumptions can yield Lipschitz continuity of solutions, the presence of a nonlocal operator I introduces long-range interactions that may either enhance or obstruct regularization effects. In many cases, the nonlocal term provides a compensating effect that prevents singularity formation and yields regularity of solutions. However, optimal regularity remains mostly open and strongly depends on fine properties of the jump measure and the interaction between nonlocal term and the nonlinear Hamiltonian.

Leonid Parnovski (University College London)
Bethe-Sommerfeld Property of multi-dimensional Schrödinger operators with periodic and almost-periodic potentials

Abstract. This will be a survey talk about the Bethe-Sommerfeld property (spectrum containing a semi-axis) of periodic and almost-periodic operators. I will describe known results as well as open problems. This talk will be not very technical and will contain no proofs.

Oleg Zaboronski (University of Warwick)
Asymptotic expansions for a class of Fredholm Pfaffians and interacting particle systems

Abstract. Motivated by the phenomenon of duality for interacting particle systems we introduce two classes of Pfaffian kernels describing a number of Pfaffian point processes in the 'bulk' and at the 'edge'. Using the probabilistic method due to Mark Kac, we prove two Szegő-type asymptotic expansion theorems for the corresponding Fredholm Pfaffians.
The idea of the proof is to introduce an effective random walk with transition density determined by the Pfaffian kernel, express the logarithm of the Fredholm Pfaffian through expectations with respect to the random walk, and analyse the expectations using general results on random walks. We demonstrate the utility of the theorems by calculating asymptotics for the empty interval and non-crossing probabilities for a number of examples of Pfaffian point processes: coalescing/annihilating Brownian motions, massive coalescing Brownian motions, real zeros of Gaussian power series and Kac polynomials, and real eigenvalues for the real Ginibre ensemble. (Joint work with Will FitzGerald and Roger Tribe.)