Asymptotics of Fredholm determinants of products of spectral projections

We discuss recent results on upper bounds on the asymptotics of Fredholm determinants of products of spectral projections

$$\det\bigl(1 - 1_{(-\infty,E-\epsilon)}(H_0)1_{(E+\epsilon,\infty)}(H)1_{(-\infty,E-\epsilon)}(H_0)\bigr)$$

as $\epsilon \searrow 0$ achieved by [GKM14, GKMO14, FP15] . In the latter the operators H_0 and H are a pair of Schrödinger operators, which differ by a relative trace-class perturbation. It turns out that these bounds are given in terms of the square of the scattering phase shifts. Such asymptotics are closely related to the orthogonality of ground states of pairs of non-interacting Fermi systems. Later on we prove exact asymptotics for the special case of the Laplacian and a perturbation by a Dirac-δ perturbation.

References

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