Speaker: Johannes Sjöstrand, IMB Université de Bourgogne Franche-Comté.

Title: Adiabatic evolution and long time asymptotics.

Abstract: Joint work in progress with A. Mantile and M. Hitrik. Motivated by non-linear mesoscopic Schrödinger evolution problems, we consider the adiabatic problem

$$(\varepsilon D_t + P(t))u(t) = 0, \tag{1}$$

where $P(t)$ is a Schrödinger operator with time dependent potential, presenting a potential well in an island. Typically $P(t)$ has shape resonances $\lambda = \lambda(t)$ with $-\Im\lambda \asymp h^{\mathrm{Const.}} e^{-2S(t)/h}$ where $S(t)$ is the Agmon distance from the well to the sea at energy $\Re\lambda(t)$. A natural choice in (1) is then $\varepsilon \asymp -\Im\lambda$. The construction of formal adiabatic solutions is well-known, but for the applications we need to find corresponding exact solutions. Our project is to show that there is an exact solution, matching the formal one on time intervals of length ε^{-N} for arbitrarily large N. One of the main steps was to show that we have a very sharp lower bound on $-\Im P(t)$ when $P(t)$ acts in a space adapted to the resonances, defined by Helffer-Sjöstrand in 1986. (Complex dilations seem insufficient here.) Other steps need some more checking and will be presented as conjectures, proven to $90 - 95\%$.