A Rice type formula for fixed points of stochastic flows and the McKean-Singer formula

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Abstract: I will describe a Rice type formula for the expected number of fixed points of a class of stochastic flows at a given time.

It is closely related to the McKean-Singer formula for the Euler characteristic of a compact Riemannian manifold: this would give the algebraic number of fixed points of our flow. The original proof of this by McKean and Singer was operator theoretic. It was shown by Kusuoka in 1987,[3], to be a consequence of an infinite dimensional degree theorem. To do that he had to use stochastic analysis to overcome several technical difficulties because of the lack of differentiabilty of Brownian paths.

These difficulties do not arise if one uses processes with C^1 paths instead of Brownian motion. For a class of such processes the Rice formula can be proved and a similar formula to McKean-Singer's for the Euler-characteristic arises in terms of a super trace. It is not clear how the latter could be proved analytically. If one approximates the Brownian motion by paths of Ornstein -Uhlenbeck processes, generated by hypo-elliptic Laplacians in the terminology of Bismut, [2], one gets a result for such operators to which Bismut's techniques can be applied in order to recover the usual formula.

References

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