CAPACITARY WIDTH AND ITS APPLICATIONS TO PARABOLIC AND ELLIPTIC BOUNDARY BEHAVIOR

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ABSTRACT. In this talk we introduce the notion of capacitary width for arbitrary open sets. The capacitary width is useful to estimate the first eigenvalue of the Dirichlet Laplacian and the sup norm of the torsion function, i.e., the integral of the Dirichlet Green function over the domain. Note that the torsion function v is the solution to the Poisson equation $-\Delta v = 1$ in the domain with $v = 0$ on the boundary.

The capacitary width controls the boundary behavior of the Dirichlet Green function, harmonic measure and survival probability, i.e., the integral of the Dirichlet heat kernel over the domain. Note that the survival probability u is the solution to the heat equation with $u = 0$ on the lateral boundary and the initial condition $u = 1$ for $t = 0$. These controls give minimal smoothness of the domain for which the boundary Harnack principle and the intrinsic ultracontractivity hold.

We also derive the global integrability of positive harmonic functions over a Lipschitz domain and that of positive caloric functions over a Lipschitz cylinder.

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