Sums of Projections with Random Coefficients

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Abstract

We study the spectral properties of infinite sums $A_\kappa = \sum_{n\in\mathbb{Z}} \kappa_n P_n$, where $\{P_n\}_{n\in\mathbb{Z}}$ are rank-one projections in a Hilbert space (not necessarily orthogonal to one another), and $\{\kappa_n\}_{n\in\mathbb{Z}}$ are independent identically distributed positive random variables. Inspired by the spectral theory of ergodic, including random, Schrödinger operators, we define the Integrated Density of States (IDS) measure for A_κ and establish results on its continuity, including Wegner-type estimates, and the Lifshitz tail asymptotic behaviour near the spectral edges. In the asymptotic regime of nearly-orthogonal projections we prove that the spectrum of A_κ is pure point almost surely (the Anderson-type localisation). This is recent joint work with Alexander Pushnitski.