# Schur multipliers and closability properties.


**Abstract**

Let $(X, \mu)$ and $(Y, \nu)$ be measure spaces, and let $H_1 = L_2(X, \mu)$, $H_2 = L_2(Y, \nu)$. There is a method (due mainly to Birman and Solomyak) to relate to some bounded functions $\varphi$ on $X \times Y$ linear transformations $S_\varphi$ on the space $B(H_1, H_2)$ (these transformations are called *Schur multipliers* or, in a more general setting of spectral measures $\mu, \nu$, *double operator integrals*). Namely one defines firstly a map $S_\varphi$ on Hilbert Schmidt operators multiplying their integral kernels by $\varphi$; if this map turns out to be bounded in operator norm, extend it to the space $K(H_1, H_2)$ of all compact operators by continuity. Then $S_\varphi$ is defined on $B(H_1, H_2)$ as the second adjoint of the constructed map of $K(H_1, H_2)$. A characterisation of all such multipliers was first established by Peller: Schur multipliers are percisely the functions of the form

$$\varphi(x, y) = \sum_k a_k(x) b_k(y)$$

such that $(\text{esssup} \sum |a_k(x)|^2)(\text{esssup} \sum |b_k(x)|^2) < \infty$.

We shall discuss the question for which $\varphi$ the map $S_\varphi$ is closable in the operator norm or in the weak$^*$ topology of $B(H_1, H_2)$. If $\varphi$ is of Toeplitz type, i.e. $\varphi(x, y) = f(yx^{-1})$, $x, y \in G$, where $G$ is a locally compact group then the question is related to certain questions about the Fourier algebra $A(G)$; if $\varphi(x, y)$ is of the form $(f(x) - f(y))/(x - y)$ then the property is related to "operator smoothness" of $f$. This is a joint work with V.Shulman and I.Todorov.