Killip-Simon problem and Jacobi flow on GSMP matrices

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One of the first and therefore most important theorems in perturbation theory claims that for an arbitrary self-adjoint operator A there exists a perturbation B of Hilbert-Schmidt class with arbitrary small operator norm, which destroys completely the absolutely continuos (a.c.) spectrum of the initial operator A (von Neumann). However, if A is the discrete free 1-D Schrödinger operator and B is an arbitrary Jacobi matrix (of Hilbert-Schmidt class) the a.c. spectrum remains perfectly the same, that is, the interval $[-2, 2]$. Moreover, Killip and Simon described explicitly the spectral properties for such $A+B$ [Annals, 2003]. Jointly with Damanik they generalized this result to the case of perturbations of periodic Jacobi matrices in the non-degenerated case [Annals, 2010]. Recall that the spectrum of a periodic Jacobi matrix is a system of intervals of a very specific nature. Christiansen, Simon and Zinchenko posed in a review dedicated to F. Gesztesy (2013) the following question: "is there an extension of the Damanik-Killip-Simon theorem to the general finite system of intervals case?" We solved this problem completely. Our method deals with the Jacobi flow on GSMP matrices. GSMP matrices are probably a new object in the spectral theory. They form a certain Generalization of matrices related to the Strong Moment Problem, the latter ones are a very close relative of Jacobi and CMV matrices. The Jacobi flow on them is also a probably new member of the rich family of integrable systems. Finally, related to Jacobi matrices of Killip-Simon class, analytic vector bundles and their curvature play a certain role in our construction and, at least on the level of ideology, this role is quite essential.