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Fourier transform versus Hilbert transform

We present several results in which the interplay between the Fourier transform and the Hilbert transform is of special form and importance.

1. In 50-s (Kahane, Izumi-Tsuchikura, Boas, etc.), the following problem in Fourier Analysis attracted much attention: Let $\{a_k\}$, $k = 0, 1, 2...$, be the sequence of the Fourier coefficients of the absolutely convergent sine (cosine) Fourier series of a function $f : \mathbb{T} = [-\pi, \pi) \to \mathbb{C}$, that is $\sum |a_k| < \infty$. Under which conditions on $\{a_k\}$ the re-expansion of $f(t)$ ($f(t) - f(0)$, respectively) in the cosine (sine) Fourier series will also be absolutely convergent?

We solve a similar problem for functions on the whole axis and their Fourier transforms. Generally, the re-expansion of a function with integrable cosine (sine) Fourier transform in the sine (cosine) Fourier transform is integrable if and only if not only the initial Fourier transform is integrable but also the Hilbert transform of the initial Fourier transform is integrable.

2. The following result is due to Hardy and Littlewood: If a (periodic) function f and its conjugate $\widetilde{f}$ are both of bounded variation, their Fourier series converge absolutely.

We generalize the Hardy-Littlewood theorem (joint work with U. Stadtmüller) to the Fourier transform of a function on the real axis and its modified Hilbert transform. The initial Hardy-Littlewood theorem is a partial case of this extension, when the function is taken to be with compact support.

3. These and other problems are integrated parts of harmonic analysis of functions of bounded variation. We have found the maximal space for the integrability of the Fourier transform of a function of bounded variation. Along with those known earlier, various interesting new spaces appear in this study. Their inter-relations lead, in particular, to improvements of Hardy's inequality.

There are multidimensional generalizations of these results.