ASYMPTOTIC BEHAVIOR OF ORTHOGONAL POLYNOMIALS WITHOUT THE CARLEMAN CONDITION

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Abstract. Our goal is to find an asymptotic behavior as $n \to \infty$ of orthogonal polynomials $P_n(z)$ defined by the Jacobi recurrence coefficients a_n, b_n. We suppose that the off-diagonal coefficients a_n grow so rapidly that the series $\sum a_n^{-1}$ converges, that is, the Carleman condition is violated. With respect to diagonal coefficients b_n we assume that $-b_n(a_n a_{n-1})^{-1/2} \to 2\beta_\infty$ for some $\beta_\infty \neq \pm 1$. The asymptotic formulas obtained for $P_n(z)$ are quite different from the case $\sum a_n^{-1} = \infty$ when the Carleman condition is satisfied. In particular, if $\sum a_n^{-1} < \infty$, then the phase factors in these formulas do not depend on the spectral parameter $z \in \mathbb{C}$. The asymptotic formulas obtained in the cases $|\beta_\infty| < 1$ and $|\beta_\infty| > 1$ are also qualitatively different from each other. As an application of these results, we find necessary and sufficient conditions for the essential self-adjointness of the corresponding minimal Jacobi operator.

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2000 *Mathematics Subject Classification.* 33C45, 39A70, 47A40, 47B39.