When does the norm of a Fourier multiplier dominate its L^∞ norm?

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It is well known that the L^∞ norm of a Fourier multiplier on $L^p(\mathbb{R}^n)$, $1 \le p \le \infty$ is less than or equal to its norm. The standard proof of this fact extends with almost no change to weighted L^p spaces provided the weight w is such that $w(x) = w(-x)$ for all $x \in \mathbb{R}^n$. It is natural to ask whether the norm of a Fourier multiplier on a weighted L^p space still dominates its L^∞ norm if the weight does not satisfy the above condition.
If w satisfies the Muckenhoupt A_p condition, then the L^∞ norm of a Fourier multiplier on $L^p(\mathbb{R}, w)$, $1 < p < \infty$ is less than or equal to its norm times a constant that depends only on p and w. This result first appeared in 1998 in a paper by E. Berkson and T.A. Gillespie where it was attributed to J. Bourgain. It was extended to more general function spaces over $\mathbb{R}$ by A. Karlovich (2015). We prove that the above estimsate holds with the constant equal to 1 for function spaces over $\mathbb{R}^n$ under considerably weaker restrictions. We also show that our result is in a sense optimal and that there exist weighted L^p spaces with many unbounded Fourier multipliers.

The talk is based on a joint work with Alexei Karlovich (Lisbon).