MULTI-PERIOD LOT SIZING MODEL

- PRODUCTION PLANNING PROBLEM
- MAKE-TO-STOCK MANUFACTURING PLANT
- EOQ MODEL : ASSUMPTION DEMAND CONSTANT OVER TIME

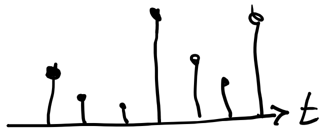
NO TREND
NO SEASONALITIES
HOURS

- TIME IS DISCRETIZED (IN WEEKS / MONTHS)

$t = 1, 2, \dots, T$ ↙ TIME UNITS

- DATA :

d_t , $t = 1, \dots, T$
DEMAND



$d_1 \; d_2 \; d_3 \; d_4$

f_t FIXED COST PAID IF PRODUCTION TAKES PLACE IN $t = 1, \dots, T$

C_t VARIABLE UNIT PRODUCTION COST

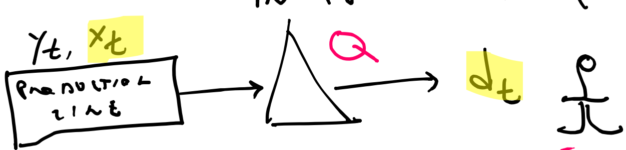
1 ITEM $\begin{cases} \text{BIKE} \\ \text{kW} \end{cases}$ $t = 1, \dots, T$

↑
TIME OF DAY,
TIME OF YEAR

ELECTRIC POWER SYSTEMS

h_t UNIT INVENTORY COST

"€ | £ SPEND TO STORE A UNIT OF PRODUCT (PALLET, E.G.) IN PERIOD t ($t = 1, 2, \dots, T$)"



I_t WAREHOUSE CAPACITY

DECISION VARIABLES

$$y_t = \begin{cases} 1 & \text{IFF PRODUCTION LINE SET-UP IN PERIOD } t \\ 0 & \text{OTHERWISE} \end{cases} \quad t = 1, \dots, T$$

$x_t \geq 0$ AMOUNT OF PRODUCT PRODUCED AT TIME t $\quad t = 1, \dots, T$

PRODUCTION PLAN

$I_t \geq 0$ INVENTORY LEVEL AT (END) OF PERIOD t $\quad t = 1, \dots, T$

OBJECTIVE FUNCTION

$$z = \text{TOTAL COST OVER PLANNING HORIZON} = \underbrace{\sum_{t=1}^{T} f_t y_t}_{\text{TOTAL P. FIXED COST}} + \underbrace{\sum_{t=1}^{T} c_t x_t}_{\text{TOTAL P. VARIABLE COST}} + \underbrace{\sum_{t=1}^{T} h_t I_t}_{\text{TOTAL INVENTORY COST}}$$

CONSTRAINTS

INITIAL INVENTORY LEVEL : $I_0 \longrightarrow$ DATA

FLOW CONSERVATION :

$$I_{t-1} + x_t - d_t = I_t \qquad t = 1, \dots, T$$

WAREHOUSE CAPACITY :

$$I_t \leq Q \qquad t = 1, \dots, T$$

LINKING x_t AND y_t VARIABLES

$$x_t \leq \Big(\sum_{i=t}^{T} d_i\Big) y_t \qquad t = 1, \dots, T$$

$\sum_{i=t}^{T} d_i \rightarrow d_7 + d_8 + d_9 + \dots d_T$

$x_t \rightarrow 7$ $\qquad$ $t \rightarrow 7$

MAXIMUM REMAINING DEMAND IN PLANNING HORIZON

$$y_t = \begin{cases} 1 & x_t \leq \sum_{i=t}^{T} d_i \quad \text{REDUNDANT CONSTRAINT} \\ 0 & \rightarrow x_t \leq 0 \rightarrow x_t = 0 \end{cases}$$

$t \rightarrow 7$: $\sum_{i=t}^{T} d_i \rightarrow d_7 + d_8 + \dots d_T$

$$\left.\begin{array}{l} I_0 = 0 \\ I_T = 0 \end{array}\right\} \text{FOR INSTANCE}$$

VARIANT

1 BACKLOG ALLOWED (AT A UNIT COST b_t)

$$I_t = I_t^+ - I_t^- \gtrless 0$$

INVENTORY LEVEL ≥ 0 BACKLOG ≥ 0

$\boxed{\text{CAN ALWAY ASSUME } I_t^+ \cdot I_t^- = 0}$

OBJECTIVE FUNCTION

$$z = \text{TOTAL COST OVER PLANNING HORIZON} = \underbrace{\sum_{t=1}^{T} f_t y_t}_{\text{TOTAL P. FIXED COST}} + \underbrace{\sum_{t=1}^{T} c_t x_t}_{\text{TOTAL P. VARIABLE COST}} + \underbrace{\sum_{t=1}^{T} h_t I_t^+}_{\text{TOTAL INVENTORY COST}} + \underbrace{\sum_{t=1}^{T} b_t I_t^-}_{\text{TOTAL BACKLOG COST}}$$

CONSTRAINTS

INITIAL INVENTORY LEVEL : $I_0 \longrightarrow$ DATA

FLOW CONSERVATION :

$$I_{t-1}^+ - I_{t-1}^- + x_t - d_t = I_t^+ - I_t^- \qquad t = 1, \dots, T$$

WAREHOUSE CAPACITY :

$$I_t^+ \leq Q \qquad t = 1, \dots, T$$

LINKING x_t AND y_t VARIABLES

$$x_t \leq \left(\sum_{i=t}^{T} d_i \right) y_t \qquad t = 1, \dots, T$$

$$\left. \begin{array}{ll} I_0^+ = 540, & I_0^- = 0 \\ I_T^+ = 540, & I_0^- = 0 \end{array} \right\} \text{FOR INSTANCE}$$

$$x_t, I_t^+, I_t^- \geq 0 \qquad t = 1, \dots, T$$

$$y_t = 0 \backslash 1 \qquad t = 1, \dots, T$$

SIMPLIFIED BEER GAME:

VARIANT x_t = AMOUNT ORDERED AT TIME t AND RECEIVED AT TIME $t+\ell$

2 LEAD TIME $\ell \in \{0, 1, 2, \dots\}$

OBJECTIVE FUNCTION

$$z = \text{TOTAL COST OVER PLANNING HORIZON} = \underbrace{\sum_{t=1}^{T} f_t y_t}_{\text{TOTAL P. FIXED COST}} + \underbrace{\sum_{t=1}^{T} c_t x_t}_{\text{TOTAL P. VARIABLE COST}} + \underbrace{\sum_{t=1}^{T} h_t I_t^+}_{\text{TOTAL INVENTORY COST}} + \underbrace{\sum_{t=1}^{T} b_t I_t^-}_{\text{TOTAL BACKLOG COST}}$$

CONSTRAINTS

INITIAL INVENTORY LEVEL : $I_0 \longrightarrow$ DATA

FLOW CONSERVATION :

$$I_{t-1}^+ - I_{t-1}^- - d_t = I_t^+ - I_t^- \qquad t = 1, \dots, \ell$$

$$I_{t-1}^+ - I_{t-1}^- + x_{t-\ell} - d_t = I_t^+ - I_t^- \qquad t = \ell+1, \dots, T$$

WAREHOUSE CAPACITY :

$$I_t^+ \leq Q \qquad t = 1, \dots, T$$

LINKING x_t AND y_t VARIABLES

$$x_t \leq \left(\sum_{i=t}^{T} d_i \right) y_t \qquad t = 1, \dots, T$$

$$\left.\begin{aligned} I_0^+ = 540, \quad I_0^- = 0 \\ I_T^+ = 540, \quad I_T^- = 0 \end{aligned}\right\} \text{FOR INSTANCE}$$

$$x_t \geq 0 \qquad t = 1, \dots, T-\ell$$

$$I_t^+, I_t^- \geq 0 \qquad t = 1, \dots, T$$

$$y_t = 0 \backslash 1 \qquad t = 1, \dots, T$$

- AT TIME t THE AMOUNT SENT TO MARKET

$$a_t = \min\left(I_t^{+} + x_{t-\ell},\ I_t^{-} + d_t \right)$$

MULTICOMMODITY LOT SIZING

- MULTIPLE PRODUCTS SHARING THE SAME RESOURCES (WAREHOUSE CAPACITY, ...)

$k = 1, 2, \ldots k$ DIFFERENT PRODUCT

DATA $\quad d_t^{k} \quad \ldots$

VARS

$y_t^{k},\ x_t^{k}$