Classical Mechanics

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Classical mechanics is the study of macroscopic, non-relativistic motion.

1 Drag force

It is known that the drag force acting on an object is a function of its velocity. Expanding its Taylor series,

$$F(v) = \sum_{n=0}^{\infty} \left.\frac{\mathrm{d}^n F}{\mathrm{d}v^n}\right|_{v=0} \frac{v^n}{n!}$$

Considering only the first three terms at $v = 0$,

$$F(v) \approx F(0) + \left.\frac{\mathrm{d}F}{\mathrm{d}v}\right|_{v=0} v + \frac{1}{2}\left.\frac{\mathrm{d}^2 F}{\mathrm{d}v^2}\right|_{v=0} v^2$$

Since $F(0) = 0$,

$$F(v) \approx \left.\frac{\mathrm{d}F}{\mathrm{d}v}\right|_{v=0} v + \frac{1}{2}\left.\frac{\mathrm{d}^2 F}{\mathrm{d}v^2}\right|_{v=0} v^2$$

The coefficients of the linear and quadratic terms depend on the medium, so we can absorb them as constants and write

$$F(v) \approx bv + cv^2$$

where b and c are constants. The linear term is dominant at low velocities, while the quadratic term is dominant at high velocities.

Example

Consider an object of mass m falling under the influence of gravity. The drag force acting on it can be modeled as

$$F(v) \approx bv + cv^2$$

where b and c depend on the properties of the medium (e.g., air density, shape of the object). At low velocities, the linear term bv dominates, leading to a constant acceleration. At high velocities, the quadratic term cv^2 becomes significant, resulting in a terminal velocity where the drag force balances the gravitational force.