Linear Algebra II

lonelyneutrino

August 2025

1 Basics

Example

For a set $\mathcal{S}$, let $\mathbb{F}^{\mathcal{S}}$ be the set of all functions from $\mathcal{S}$ to $\mathbb{F}$. Then, defined over canonical addition and scalar multiplications, $\mathbb{F}^{\mathcal{S}}$ is a vector space. The additive identity is the zero function 0, defined as $0(x) = 0$. The additive inverse can be defined as $-f : \mathcal{S} \to \mathbb{F}$ defined as $-f(x) = -(f(x)) \forall x \in \mathcal{S}$.

Note that $\mathbb{F}^n$ and $\mathbb{F}^\infty$ are special cases of $\mathbb{F}^{\mathcal{S}}$, where $\mathcal{S}$ is a finite set of size n or an infinite set, respectively.