EE381 homework #2 solution

1) P(1, 6)=1/7*1/7=1/49
   P(2, 5)=1/7*1/7=1/49
   P(3, 4)=1/7*1/7=1/49
   P(4, 3)=2/7*2/7=4/49
   P(5, 2)=1/7*1/7=1/49
   P(6, 1)=1/7*1/7=1/49
   Adding those probabilities, the probability of rolling a 7 as a sum is 9/49.
2) Let E be the event that a bit string of length four contains at least two consecutive 0s.
   Let F be the event that the first bit of a string of length four is a 0.
   The probability that a bit string of length four has at least two consecutive 0s, given that its first bit is a o, equals:
   $$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{5/16}{1/2} = \frac{5}{8}$$
   Where $P(E \cap F) = \frac{5}{16}$, and P(F)=8/16=1/2.
3) Let A_1 be the event "4,5, or 6 on first toss"
   Let A_2 be the event "1,2,3, or 4 on second toss".
   Then we are looking for $P(A_1 \cap A_2)$.
   $$P(A_1 \cap A_2) = P(A_1) * P(A_2|A_1) = P(A_1) * P(A_2) = \frac{3}{4} * \frac{4}{6} = \frac{1}{3}$$
   Note: the second toss is independent from the first toss.
4) There are eight bit strings of length four that begin with a one: 1000, 1001, 1010, 1011, 1100, 1101, 1110, and 1111.
   There are also eight bit strings of length four that contain an even number of ones: 0000, 0011, 0101, 0110, 1001, 1010, 1100, 1111. Because there are 16 bit strings of length four, it follows that:
   $$P(E) = P(F) = \frac{8}{16} = \frac{1}{2}$$
   $(E \cap F) = \{1111, 1100, 1010, 1001\}$ so $P(E \cap F) = \frac{4}{16} = \frac{1}{4}$
   $$P(E \cap F) = \frac{1}{4} = \frac{1}{2} * \frac{1}{2} = P(E) * P(F)$$
   We conclude that E and F are independent.
5) Let A1 be the event "ace on first draw" and A2 be the event "ace on second draw".
   a) $P(A_1 \cap A_2) = P(A_1) * P(A_2|A_1) = \frac{4}{52} * \frac{4}{52} = \frac{1}{169}$
   b) $P(A_1 \cap A_2) = P(A_1) * P(A_2|A_1) = \frac{4}{52} * \frac{3}{51} = \frac{1}{221}$
6)
   a) Let R denote the event "a red marble is chosen" while I and II denote the events that Box I and Box II are chosen, respectively.

   The probability of choosing a red marble is:
   $$P(R) = P(I)P(R|I) + P(II)P(R|II) = \frac{1}{2}\left(\frac{3}{3+2}\right) + \frac{1}{2}\left(\frac{2}{2+8}\right) = \frac{2}{5}$$
   b)

$$P(I \mid R) = \frac{P(I)\,P(R \mid I)}{P(I)\,P(R \mid I) + P(II)\,P(R \mid II)} = \frac{\left(\frac{1}{2}\right)\left(\frac{3}{3+2}\right)}{\left(\frac{1}{2}\right)\left(\frac{3}{3+2}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{2+8}\right)} = \frac{3}{4}$$

7) 1*26*26=676
8)
   a) 990
   b) 500
   c) 27
9) $26^2 * 10^4 + 10^2 * 26^4 = 52{,}457{,}600$