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EE 381 HW #4

1. Let S be the sample space $S = \{0, 1, 2, \dots\}$
Determine if the following function is the probability function of the sample space S:

$f(x) = (1-r)^x r$ where r belongs to (0, 1) [range] and $x = 0, 1, 2 \dots$

$f(x)$ is a probability function if $f(x) \geq 0$ and $\sum_x f(x) = 1$

$f(x) \geq 0$ means that we cannot result in a negative number and $\sum_x f(x) = 1$ means the summation all values is equal to 1.

$f(x) = (1-r)$ // anything to the 0 power is 1

$$\sum_{x=0} f(x) = (1-r)^{(0)} r = (1-r)^0 r = r$$

$$\sum_{x=1} f(x) = (1-r)^{(1)} r = (1-r) r$$ *r is a constant

$$\sum_{x=2} f(x) = (1-r)^{(2)} r = (1-r)^2 r$$

$$\sum_x f(x) = f(0) + f(1) + f(2) + \dots + f(n)$$

$$\sum_x f(x) = r + (1-r) r + (1-r)^2 r + \dots$$

$$= \sum_{n=1}^{\infty} (1-r)^n r = r \sum_{n=1}^{\infty} (1-r)^n$$

let $r = (1-r)$ $$= r \sum_{n=1}^{\infty} (x)^n = \frac{1}{1-r} = \frac{1}{1-(1-r)} = \frac{r}{r} = \boxed{1}$$

$\boxed{f(x) \text{ is probability function!}}$

7.

a)

=P(

2. Let x be a random variable giving the number of
b) aces in a random draw of 4 cards from an
ordinary deck of 52 cards. //let n be the number of cards in the deck

possible ways of drawing an ace: //let r be the number of aces that you pull,

let (X = 0) be the instance where you draw no aces at all
A) and draw something else

let (x = 1) be the instance where you draw 1 ace and the 3 other
B) cards aren't aces.

let (x=2) be the instance where you draw 2 aces and the other
C) two cards are something else.

let (x=3) be the instance where you draw 3 aces and the other one card
D) is something else.

let (x=4) be in the instance where you draw 4 aces and no card is
E) something else.

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35 2. (continued!)

The total ways of drawing 4-cards from 52 is

$^{52}C_4 = C(\overset{n}{52}, \overset{r}{4}) = \frac{52!}{(52-4)!4!} = \frac{52!}{48!4!}$

$= \frac{52\cdot 51\cdot 50\cdot 49\cdot \cancel{48!}}{\cancel{48!}4!} = 270{,}725$ ways of drawing 4 cards from a deck of 52 cards.

$(X=0) = \underline{P(A)}$ (no aces) (4 non-aces)

C(no. of no Aces that can be drawn from 4 aces) $= {}^4C_0$

// combination b/c the way that you draw any ace doesn't matter because we are disregarding the suit rankings!

$C(\overset{n}{4}, \overset{r}{0}) = \frac{4!}{(4-0)!0!} = \frac{\cancel{4!}}{\cancel{4!}0!} = 1 = C(4,0)$

x

C(no. of ways where 4 non-ace cards can be drawn from a selected 48-nonaces!) $= {}^4C_{48}$

x

$C(\overset{n}{48}, \overset{r}{4}) = \frac{48!}{(48-4)!4!} = \frac{48!}{44!4!} = \frac{48\cdot 47\cdot 46\cdot 45\cdot \cancel{44!}}{\cancel{44!}4!} = 194{,}580 = C(48,4)$

$P(A) = \frac{\text{no. of no aces could be drawn for 4 aces} \cdot \text{no. 4-non ace cards could be drawn from 48 non-aces}}{\text{total ways of drawing 4-cards from the deck}}$

$= P(A) = \frac{{}^4C_0 \cdot {}^{48}C_4}{{}^{52}C_4} = \frac{194{,}580 \cdot 1}{270{,}725} \approx 0.718$

syntax: ${}^{n}C_{r} = C(n, r)$

2. (continued!)

$(X = 1) = P(B)$ (Draws one ace) (3 non-aces)

C(no. of 1 ace could be drawn from 4 aces) $= {}^{4}C_{1}$

// combinations b/c we ignore the ranking of the aces

$$C(4,1) = \frac{4!}{(4-1)!1!} = \frac{4!}{3!1!} = \frac{4 \cdot \cancel{3!}}{\cancel{3!}1!} = 4 = C(4,1)$$

C(no. of ways to select 3 non-ace cards out of 48-non ace cards)

$$= {}^{48}C_{3} = C(48,3) = \frac{48!}{(48-3)!3!} = \frac{48!}{45!3!} =$$

$$\frac{48 \cdot 47 \cdot 46 \cdot \cancel{45!}}{\cancel{45!}3!} = 17,296 = C(48,3)$$

P(B) = $\dfrac{\text{no. of 1 ace could be drawn from 4 aces} \cdot \text{no. ways to select 3 non-ace cards out of 48 non-ace cards}}{\text{total ways of drawing 4-cards from the deck}}$

$$= \frac{{}^{4}C_{1} \cdot {}^{48}C_{3}}{270,725} = \frac{4 \cdot 17,296}{270,725} = \frac{69,184}{270,725} \approx 0.255$$

(continued!)

$(x=2) = P(C) = $ (Draws 2 Aces) (2 non-ace)

C(no. of 2 aces could be drawn from 4 aces) $= {}^4C_2$

$${}^4C_2 = C(4,2) = \frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} = \frac{4\cdot 3\cdot \cancel{2!}}{\cancel{2!}\,2!} = 6 = C(4,2)$$

C(no. ways to select 2 non-ace cards out of 48 non-ace cards) $= {}^{48}C_2$

$$= C(48,2) = \frac{48!}{(48-2)!\,2!} = \frac{48!}{46!\,2!} = \frac{48\cdot 47\cdot \cancel{46!}}{\cancel{46!}\,2!} = 1128 = C(48,2)$$

$P(C) = \dfrac{\text{no. of 2 aces could be drawn from 4 aces} \cdot \text{no. of ways to select 2 non-ace cards out of 48 non-ace cards}}{\text{total ways of drawing 4-cards from the deck}}$

$$= \frac{{}^4C_2 \cdot {}^{48}C_2}{270{,}725} = \frac{6\cdot 1125}{270{,}725} \approx 0.025$$

$(x=3) = P(D) = $ (Draws 3 aces) (1 non-ace)

C(no. of 3 aces that could be drawn from 4 aces) $= {}^4C_3$

$$= C(4,3) = \frac{4!}{(4-3)\,3!} = \frac{4!}{1!\,3!} = \frac{4\cdot \cancel{3!}}{1!\,\cancel{3!}} = 4 = C(4,3)$$

C(no. of draw 1 non-ace from 48 non-ace cards) $= {}^{48}C_1$

$$= C(48,1) = \frac{48!}{(48-1)!\,1!} = \frac{48!}{47!\,1!} = \frac{48\cdot \cancel{47!}}{\cancel{47!}\,1!} = 48 = C(48,1)$$

$P(D) = \dfrac{\text{no. of 3 aces could be drawn from 4 aces} \cdot \text{no. of draw 1 non-ace from 48 non-ace cards}}{\text{total ways of drawing 4-cards from the deck}}$

$$= \frac{{}^4C_3 \cdot {}^{48}C_1}{270{,}725} = \frac{4\cdot 48}{270{,}725} = 7.092\times 10^{-4}$$

2. (continued)

$(X=4) = P(E)$ (Draws all 4 aces)(no non-ace cards)

C(no. of 4-aces could be drawn from 4 aces) $= {}^4C_4$

$= C(\overset{n}{4}, \overset{r}{4}) = \dfrac{4!}{(4-4)!\,4!} = \dfrac{\cancel{4!}}{0!\,\cancel{4!}} = \dfrac{1}{0!} = 1 = C(4,4)$

C(no. of 0 non-ace cards be drawn from 48-non-aces) $= {}^{48}C_0$

$= C(\overset{n}{48}, \overset{r}{0}) = \dfrac{48!}{(48-0)!\,0!} = \dfrac{\cancel{48!}}{\cancel{48!}\,0!} = \dfrac{1}{0!} = 1 = C(48,0)$

$=$

P(E) = $\dfrac{\text{no. of 4-aces could be drawn from 4 aces} \cdot \text{no. of 0 non-ace cards could be drawn from 48 non-ace cards}}{\text{total ways to draw 4-cards from the deck}} =$

$\dfrac{{}^4C_4 \cdot {}^{48}C_0}{270{,}725} = \dfrac{1 \cdot 1}{270{,}725} \approx 3.69 \times 10^{-6} = P(E)$

probability Function Table:

X	0	1	2	3	4
P(X)	0.718	0.255	0.025	7.092×10^{-4}	3.69×10^{-6}

3. The probability function for a random variable X is provided in the following table:

X	1	2	3
f(x)	1/2	1/3	1/6

// we are given the probability function!

Find the distribution function F(x) for the random variable X and plot it.

X	1	2	3
f(x)	1/2	1/3	1/6

Def. of distribution function:

cumulative distribution function for a random variable x is defined by:

$F(x) = P(X \leq x)$ where $X \leq x$ includes everything within its range.

where as x is any real numbers, i.e: $-\infty \leq X \leq \infty$.

$F(1) = P(X \leq 1)$ (everything from 1 and below) $= f(1) = \boxed{\frac{1}{2}}$

$F(2) = P(X \leq 2)$ (everything from 2 and below) $= f(2) + f(1) = \frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \boxed{\frac{5}{6}}$

$F(3) = P(X \leq 3)$ (everything from 3 and below) $= f(3) + f(2) + f(1) = \frac{1}{6} + \frac{1}{3} + \frac{1}{2}$

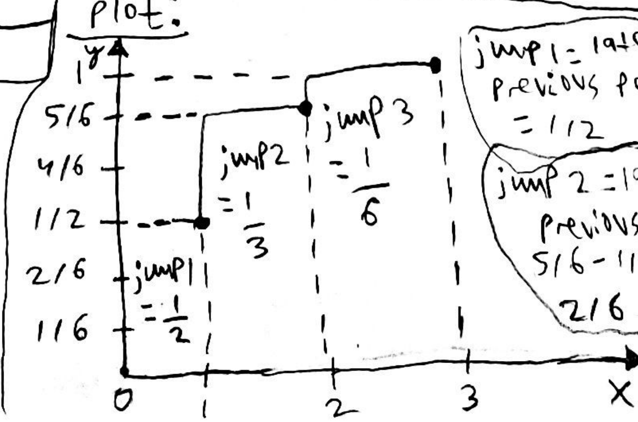
$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} = \frac{6}{6} = \boxed{1}$

New Table w/ Distribution Function:

X	1	2	3
F(X)	1/2	5/6	1

$$F(x) = \begin{cases} 0 & -\infty < 0 < \infty \\ 1/2 & 1 \leq x < 2 \\ 5/6 & 2 \leq x < 3 \\ 1 & 3 \leq x < \infty \end{cases}$$

Plot:



jump 1 = latest pt 1 − previous point = 1/2 − 0 = 1/2

jump 2 = latest pt 2 − previous point = 5/6 − 1/2 = 5/6 − 3/6 = 2/6 = 1/3

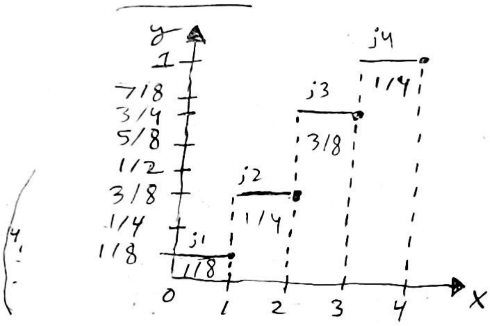
jump 3 = latest pt 3 − previous point = $1 - \frac{5}{6}$ = 1/6

4. The following table shows the distribution of a random variable X.

X	1	2	3	4
F(x)	1/8	3/8	3/4	1

a) //we are given the distribution function, so we can immediately plot the points on a graph.
//NOTE: your plotting is based ON DISTRIBUTION FUNCTION!

Plot:



j1 = jump 1 = 1st pt 1 − previous point = $1/8 - 0 = \boxed{1/8 = f(1)}$

j2 = jump 2 = 1st pt 2 − previous point = $3/8 - 1/8 = 2/8 = \boxed{1/4 = f(2)}$

j3 = jump 3 = 1st pt 3 − previous point = $3/4 - 3/8 = 6/8 - 3/8 = \boxed{3/8 = f(3)}$

j4 = jump 4 = 1st pt 4 − previous point = $1 - 3/4 = 4/4 - 3/4 = \boxed{1/4 = f(4)}$

New table w/ probability function:

X	1	2	3	4
f(x)	1/8	1/4	3/8	1/4

b) $P(1 \leq X \leq 3)$

$1 \leq x \leq 3$ can also be interpreted as x includes the range from 1 through 3 inclusively, so we add the probability of P(1), P(2), and P(3); thus, $P(1)+P(2)+P(3) = \frac{1}{8} + \frac{1}{4} + \frac{3}{8} = \frac{1}{8} + \frac{2}{8} + \frac{3}{8} = \frac{6}{8} = \boxed{\frac{3}{4} = P(1 \leq X \leq 3)}$

c) $P(X \geq 2)$ //we include 2 and everything else after 2. So, we add

$P(2)+P(3)+P(4) = \frac{1}{4} + \frac{3}{8} + \frac{1}{4} = \frac{2}{8} + \frac{3}{8} + \frac{2}{8} = \boxed{\frac{7}{8} = P(X \geq 2)}$

$\sum_{x \geq 2} f(x_i) =$

4. (continued!)

d) $P(X<3)$

// we exclude 3 and include everything that is below 3.

$P(X<3) = P(1) + P(2)$

$P(X<3) = \frac{1}{8} + \frac{1}{4} = \frac{1}{8} + \frac{2}{8} = \boxed{\frac{3}{8} = P(X<3)}$

e) $P(X>1.4)$

// we exclude anything that is 1.4 and below, includes anything greater than 1.4. Because we do not have a 1.5 or 1.4 for our x-values, we exclude the 1.4 b/c they are non-integer values. (exclude $X=1$)

So, we add $P(2)+P(3)+P(4)$ [everything after 1]

$P(X>1.4) = P(2)+P(3)+P(4) = \frac{1}{4} + \frac{3}{8} + \frac{1}{4} = \frac{2}{8} + \frac{3}{8} + \frac{2}{8} = \boxed{\frac{7}{8}}$

$\boxed{\frac{7}{8} = P(X>1.4)}$

5. the joint probability function of two random discrete variables X and Y is given by

$f(x,y) = cxy$ for $x = 1,2,3$; $y = 1,2,3$

equals 0 otherwise.

X \ Y	1	2	3	totals
1	c	2c	3c	6c
2	2c	4c	6c	12c
3	3c	6c	9c	18c
Totals	6c	12c	18c	36c

grand total is $\boxed{36c}$

Plugging in values into the equation $f(x) = cxy$:

$f(1,1) = c(1)(1) = c$

$f(1,2) = c(1)(2) = 2c$

$f(1,3) = c(1)(3) = 3c$

$f(2,1) = c(2)(1) = 2c$

$f(2,2) = c(2)(2) = 4c$

$f(2,3) = c(2)(3) = 6c$

$f(3,1) = c(3)(1) = 3c$

$f(3,2) = c(3)(2) = 6c$

$f(3,3) = c(3)(3) = 9c$

a) The constant c

Because the grand total is 36c and the sum of all rows and tables is equal to 1,

$$\sum_{j=1}^{m} f_1(x_j) = 1 \quad \sum_{k=1}^{m} f_2(y_k) = 1$$

$$= \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_j, y_k) = 1$$

$$\frac{1}{36} = \frac{\cancel{36}c}{\cancel{36}}; \rightarrow \boxed{c = \frac{1}{36}}$$

5. (continued!) ($c = 1/36$ from part A)

X \ Y	1	2	3	Total
1	c	2c	3c	6c
2	2c	4c	6c	12c
3	3c	6c	9c	18c
Total	6c	12c	18c	36c

$\rightarrow (X=2, Y=3)$

b) $P(X=2, Y=3)$

it means to find the probability of the spot that has the x coordinate of 2 and Y coordinate of 3.

$(X=2, Y=3) = 6c$

$P(X=2, Y=3) = 6c$ // plug in 1/36 from part A.

$P(X=2, Y=3) = 6\left(\frac{1}{36}\right) = \boxed{\frac{1}{6} = P(X=2, Y=3)}$

c) $P(1 \leq X \leq 2, Y \leq 2)$

X \ Y	1	2	3	Total
1	c	2c	3c	6c
2	2c	4c	6c	12c
3	3c	6c	9c	18c
Total	6c	12c	18c	36c

$1 \leq X \leq 2$ means everything within 1 and 2 for X.

$Y \leq 2$ includes 2 and everything less than 2

$(1 \leq X \leq 2, Y \leq 2)$

$P(1 \leq X \leq 2, Y \leq 2) = \sum_{1 \leq X \leq 2} \sum_{Y \leq 2} f(x,y) =$

$= P(1 \leq X \leq 2, Y \leq 2) = c + 2c + 2c + 4c = 3c + 9c = 9c = 9\left(\frac{1}{36}\right) = \boxed{\frac{1}{4}}$

// plug in 1/36 for c found from part a

$\boxed{1/4 = P(1 \leq X \leq 2, Y \leq 2)}$

5. (continued!)

d) $P(X \geq 2)$

X \ Y	1	2	3	Total
1	c	2c	3c	6c
2	2c	4c	6c	12c
3	3c	6c	9c	18c
Total	6c	12c	18c	36c

$(X \geq 2)$

$P(X \geq 2)$ means to include everything that has an x-value of 2 or greater

// plug in 1/36 for c found in part a

$$P(X \geq 2) = \sum_{x \geq 2} f(x) = 2c + 4c + 6c + 3c + 6c + 9c = 30c = \cancel{30}^{5}\left(\frac{1}{\cancel{36}_{6}}\right) =$$

$$\boxed{\frac{5}{6} = P(X \geq 2)}$$

e) $P(Y=3)$

X \ Y	1	2	3	Total
1	c	2c	3c	6c
2	2c	4c	6c	12c
3	3c	6c	9c	18C
Total	6c	12c	18C	

$P(Y=3)$ means to include everything that has a Y-value of 3.

$$P(Y=3) = \sum_{Y=3} f(y) = 3c + 6c + 9c = 18c = 18\left(\frac{1}{\cancel{36}_{2}}\right) = \boxed{\frac{1}{2} = P(Y=3)}$$

// plug in 1/36 for c found in part a

...mine marginal probability function of random variables X and Y in question 5. Determine if X and Y are independent.

X \ Y	1	2	3	Total
1	c	2c	3c	6c
2	2c	4c	6c	12c
3	3c	6c	9c	18c
Total	6c	12c	18c	36c

Definition of Discrete probability Functions:

X is a discrete random variable, and suppose that the possible values that it can assume are given by $x_1, x_2, x_3, \ldots$, arranged in some order.

The probability these assumed values are given by:

$$P(X = x_k) = f(x_k) \quad k = 1, 2, \ldots$$

probability Function AKA "marginal probability distribution" given by

$$P(X = x) = f(x)$$

General: 1) If $f(x) \geq 0$

2) $\sum_x f(x) = 1$ (all sum of possible values of X = 1)

PART A: For our X-values, we have set X as $x=1$, $x=2$, and $x=3$ (marginal probability function of x), (all values added up as a row),

when $X=1$: we got 6c

$X=2$: we got 12c; $X=3$: we got 18c.

PROOF Probability Function:

$$P(X=x) = f_1(x) = \begin{cases} 6c = 1/6 & x=1 \\ 12c = 1/3 & x=2 \\ 18c = 1/2 & x=3 \\ 0 & \text{otherwise} \end{cases}$$

Check: $\frac{1}{6} + \frac{1}{3} + \frac{1}{2} = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} = \frac{6}{6} = 1$ ✓

Function!

6. (continued!)

PART B: Marginal probability function of Y. For our Y-values, we have y set as Y=1; Y=2; and Y=3. When Y=1, we got 6c (all values of the column added up into the total section). Y=2, we got 12c. Y=3, we got 18c.

Proof of probability Function

$$P(Y=y) = f_2(y) = \begin{cases} 6c = 1/6 & y=1 \\ 12c = 1/3 & y=2 \\ 18c = 1/2 & y=3 \\ & 0 \text{ otherwise} \end{cases}$$

check: $\frac{1}{6}+\frac{1}{3}+\frac{1}{2} = \frac{1}{6}+\frac{2}{6}+\frac{3}{6} = \frac{6}{6} = 1$ ✓

$$P(Y=1) = \sum_{Y=1} f(y_1) = c+2c+3c = 6c = 6 \cdot \frac{1}{36} = \frac{1}{6} = P(Y=1)$$

+1/6

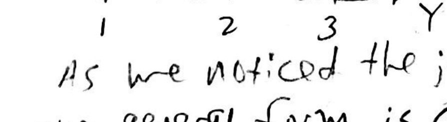
$$P(Y=2) = \sum_{Y=2} f(y_2) = 2c+4c+6c = 12c = 12 \cdot \frac{1}{36} = \frac{1}{3} = P(Y=2)$$

2+1/6

$$P(Y=3) = \sum_{Y=3} f(y_3) = 3c+6c+9c = 18c = 18 \cdot \frac{1}{36} = \frac{1}{2} = P(Y=3)$$

+1/6

Plot:



j1 = jump1 = cur. pt 1 - previous pt.
= 1/6 - 0 = 1/6

j2 = jump 2 = curr. pt. 2 - previous pt.
= 1/3 - 1/6 = 2/6 - 1/6 = 1/6

j3 = jump 3 = curr. pt. 3 - previous pt.
= 1/2 - 1/3 = 3/6 - 2/6 = 1/6

As we noticed the jumps between each point, there's a 1/6 difference. So, our general form is $\frac{y}{6}$.

check! $\sum_{y=1}^{3} \frac{1}{6} y = \frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(3) = \frac{1}{6}+\frac{1}{3}+\frac{1}{2} = \frac{1}{6}+\frac{2}{6}+\frac{3}{6} = \frac{6}{6} = 1$ ✓

$$= \sum_{Y=1}^{3} f(y) = \frac{1}{6} y \text{ for } Y = 1, 2, 3$$

6. (continued!)

part A (continued!):

In order to find the general probability function of x, we list out the summation of the total probabilities of X and then plot it to determine the jump from each point.

(c = 1/36 from part A)

$$P(X=1) = \sum_{x=1} f_1(x) = c+2c+3c = 6c = \cancel{6}\left(\frac{1}{\cancel{36}_6}\right) = \frac{1}{6} = P(X=1)$$

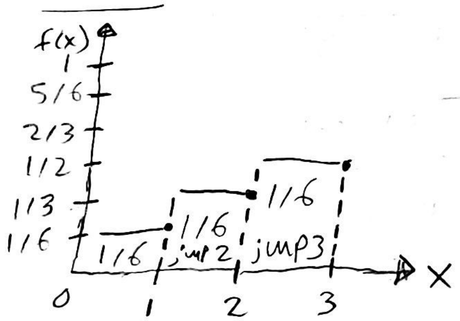
↑ 1/6

$$P(X=2) = \sum_{x=2} f_2(x) = 2c+4c+6c = 12c = \cancel{12}\left(\frac{1}{\cancel{36}_3}\right) = \frac{1}{3} = P(X=2)$$

↑ 1/6

$$P(X=3) = \sum_{x=3} f_3(x) = 3c+6c+9c = 18c = \cancel{18}\left(\frac{1}{\cancel{36}_2}\right) = \frac{1}{2} = P(X=3)$$

plot:



j1 = jump1 = current point 1 − previous point
= 1/6 − 0 = 1/6

j2 = jump2 = current point 2 − previous point
= 1/3 − 1/6 = 2/6 − 1/6 = 1/6

j3 = jump3 = current point 3 − previous point
= 1/2 − 1/3 = 3/6 − 2/6 = 1/6

we noticed that the graph has a 1/6 gap between each jump; thus, the general form is $\frac{x}{6}$.

$$\boxed{\sum_{x=1}^{3} f(y) = \frac{1}{6}x \text{ for } x = 1,2,3}$$

check:

$$\sum_{x=1}^{3} f(x) = \frac{1}{6}x = \frac{1}{6}(1) + \frac{1}{\cancel{6}_3}(2) + \frac{1}{\cancel{6}_2}(\cancel{3})$$
$$= \frac{1}{6} + \frac{1}{3} + \frac{1}{2} = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} = \frac{6}{6} = 1 ✓$$

6. (continued!)

Determine if X and Y are independent

X \ Y	1	2	3	Totals
1	c	2c	3c	6c
2	2c	4c	6c	12c
3	3c	6c	P(X=3,Y=3) 9c	18c
Totals	6c	12c	18c	36c

c = 1/36 from part A

Formula:

$$P(X=x, Y=y) \overset{?}{=} P(X=x) \cdot P(Y=y)$$

we will testing w/ X=3 and Y=3

$$P(X=3, Y=3) \overset{?}{=} P(X=3) \cdot P(Y=3)$$

P(X=3, Y=3) means the probability of getting the spot that has the x-coordinate of 3 and y-coordinate of 3

$$P(X=3, Y=3) = 9c = \cancel{9}\left(\frac{1}{\cancel{36}_4}\right) = \boxed{\frac{1}{4} = P(X=3, Y=3)}$$

P(X=3) = to add everything up where it has an x-value of 3

$$= \sum_{x=3} f_3(x) = 3c + 6c + 9c = 18c = \cancel{18}\left(\frac{1}{\cancel{36}_2}\right) = \boxed{\frac{1}{2} = P(X=3)}$$

P(Y=3) = to add up everything where it has a Y-value of 3

$$= \sum_{y=3} f_3(y) = 3c + 6c + 9c = 18c = \cancel{18}\left(\frac{1}{\cancel{36}_2}\right) = \boxed{\frac{1}{2} = P(Y=3)}$$

$$P(X=3, Y=3) \overset{?}{=} P(X=3) \cdot P(Y=3)$$

$$= \frac{1}{4} \overset{?}{=} \frac{1}{2} \cdot \frac{1}{2}$$

$$\boxed{= \frac{1}{4} \overset{\checkmark}{=} \frac{1}{4}}$$

B/C P(X=3, Y=3) is equal to P(X=3) · P(Y=3), X and Y are independent.

7. For the distribution of question 5, find the conditional probability function of:

	Y			Total
X \ Y	1	2	3	
1	c	2c	3c	6c
2	2c	4c	6c	12c
3	3c	6c	9c	18c
Totals	6c	12c	18c	36c

Conditional probability formula:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

Conditional Distributions: If X and Y are discrete random variables and we have the events (A: X = x), (B: Y = y), then

$$f(y \mid x) = P(Y = y \mid X = x) = \frac{f(x, y)}{f_1(x)}$$

$$\text{or } f(x \mid y) = P(X = x \mid Y = y) = \frac{f(x, y)}{f_2(y)}$$

a) X given Y

$$P(X \mid Y) = P(X = x \mid Y = y) = \frac{f(x, y)}{f_2(y)} = \frac{f(x, y)}{y/6} =$$

$f_2(y) = \frac{y}{6}$ or $\frac{1}{6}y$ from problem 6

$f_1(x) = \frac{x}{6}$ or $\frac{1}{6}x$ from problem 6

$= \boxed{1}$

function

7. (continued!)

a) continued! //Plug in the joint probability eq. $f(x,y) = cxy$ from problem 5

$c = 1/36$ from part 6A

$$\frac{f(x,y)}{y/6} = \frac{cxy}{y/6} = \frac{\frac{1}{36}xy}{y/6} = \frac{1/36\,xy}{y/6} = \frac{\frac{1}{\cancel{36}_6}x\cancel{y}}{1} \cdot \frac{\cancel{6}}{\cancel{y}}$$

$$= \boxed{P(X|Y) = \frac{x}{6} \text{ OR } \frac{1}{6}x}$$

$f_1(x) = \frac{x}{6}$ OR $\frac{1}{6}x$

$f_2(y) = \frac{y}{6}$ OR $\frac{1}{6}y$

Plug in cxy from problem #5

b) Y given X

Plug in 1/36 for c found in part a

$$P(Y|X) = \frac{P(Y=y \mid X=x)}{f_1(x)} = \frac{f(x,y)}{f_1(x)} = \frac{cxy}{x/6} = \frac{1/36\,xy}{x/6}$$

$$= \frac{1/36\,xy}{1} \cdot \frac{6}{x} = \frac{\frac{1}{6\cancel{36}}\cancel{x}y}{1} \cdot \frac{\cancel{6}}{\cancel{x}} = \boxed{\frac{y}{6} \text{ OR } \frac{1}{6}y = P(Y|X)}$$