EE 381 Hw #11

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1. Sample weights = {8.3, 10.6, 9.7, 8.8, 10.2, and 9.4} lbs.

a) The population mean

Thm 5.1: The mean of the sampling distribution of means equal to the mean of the population.

$E(\bar{x}) = \mu_{\bar{x}} = \mu$

$n = 6$

$\bar{x} = \frac{\Sigma x}{n} = \frac{8.3+10.6+9.7+8.8+10.2+9.4}{6}$

$= \frac{57}{6} = \boxed{9.5 = \bar{x}}$

b) The population variance

$\hat{s}^2 = \frac{n}{n-1} s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$

$= \frac{(8.3-9.5)^2+(10.6-9.5)^2+(9.7-9.5)^2+(8.8-9.5)^2+(10.2-9.5)^2+(9.4-9.5)^2}{6-1}$

$= \boxed{0.736 = \hat{s}^2}$

c) compare the sample standard deviation w/the estimated population standard deviation

1. c (continued!) - Estimated population standard deviation

$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{N}}$$

$N = 6$
$\mu = 9.5$

$$= \sqrt{\frac{(8.3-9.5)^2+(10.6-9.5)^2+(9.7-9.5)^2+(8.8-9.5)^2+(10.2-9.5)^2+(9.4-9.5)^2}{6}}$$

$$= \boxed{0.783 \text{ lbs}}$$

- sample standard deviation

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N-1}}$$

$$= \sqrt{\frac{(8.3-9.5)^2+(10.6-9.5)^2+(9.7-9.5)^2+(8.8-9.5)^2+(10.2-9.5)^2+(9.4-9.5)^2}{6-1}}$$

$$= \boxed{0.8579 \text{ lbs}}$$

2) Given: $n = 60$ cables

$\bar{x} = 11.09$ tons // Normal distribution

a) $\sigma = 0.73$ tons

at 95% confidence levels

unknown: confidence levels at 95%

Formula: B/c $n = 60 \geq 30$ ✓,

If the statistic S is the sample mean $\bar{x}$, the confidence limits for the population mean are given by:

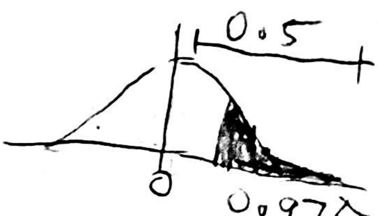
$$\bar{x} + z_c \frac{\sigma}{\sqrt{n}}$$

Solve: (confidence %)

$z_c = 1 - 95\% = 1 - 0.95 =$

$z_c = 0.05$; $z_c/2 = 0.025 = a/2$

$z_{1-a/2} = z_{1-0.025} = z_{\underset{\text{area}}{0.975}} =$



$z_{0.475} = 1.96 = z_c$

$0.975 > 0.5$ ✓

$0.975 - 0.5 = 0.475$

$11.09 \pm 1.96\left(\frac{0.73}{\sqrt{60}}\right) = 11.09 \pm 0.18$

$\boxed{= (10.91, 11.27)}$

2.b) <u>Given:</u> confidence level = 99%

$\sigma = 0.73$ tons

$n = 60$ cables is $n \geq 30 \Rightarrow 60 \geq 30$ ✓

$\bar{x} = 11.09$ tons

<u>unknown:</u> confidence intervals

<u>Formula:</u> B/c $n = 60 \geq 30$ ✓, we use

$$\bar{x} \pm z_c \frac{\sigma}{\sqrt{n}}$$

<u>Solve:</u>

(confidence levels)

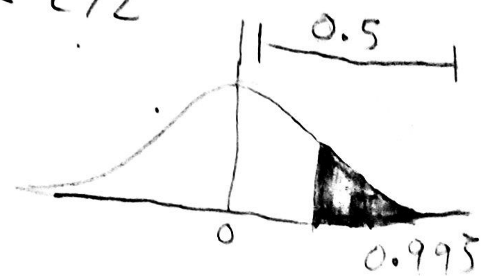
$z_c = 1 - 0.99 = 0.01$

$a/2 = z_c/2 = \frac{0.01}{2} = 0.005 = a/2$

$z_{1-a/2} = z_{1-0.005} = z_{\underbrace{0.995}_{\text{area}}}$

$= z_{0.495} = \boxed{2.57 = z_c}$

$\| z = 2.57 = 0.4949 \approx 0.495$



Is $0.995 > 0.5$ ✓

$0.995 - 0.5$

$= 0.495$

$$11.09 \pm 2.57\left(\frac{0.73}{\sqrt{60}}\right)$$

$$= 11.09 \pm 0.24 = \boxed{(10.85, 11.33)}$$

3. <u>Given:</u> $n = 12$ degree of freedom $= v = n-1$
$= 12 - 1 = 11$

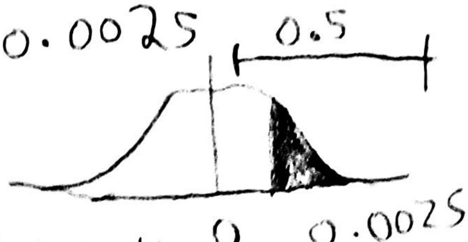
a) $\bar{x} = 7.38$ oz Is $n \geq 30$
$\hat{s} = \sigma = 1.24$ oz $12 \geq 30$ X
[student t distribution]

confidence levels $= 95\%$

<u>unknown:</u> confidence limits at 95% (confidence levels)

<u>Formula:</u> $\bar{x} \pm t_c \dfrac{\hat{s}}{\sqrt{n}}$

<u>Solve:</u> $t_c = 1 -$ confidence levels
$= 1 - 0.95 = 0.05/2 = 0.0025$
$= 1 - \frac{t_c}{2} = 1 - 0.0025$
$= 0.975 \rightarrow t_{0.975}$ and $v = 11$



$t_c = 2.20$

$7.38 \pm 2.20 \left(\dfrac{1.24}{\sqrt{12}}\right)$

$= 7.38 \pm 0.79 = \boxed{(6.59, 8.17)}$

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- sample standard deviation

3. b) <u>Given:</u> $n = 12$

degree of freedom $= v = n-1 = 12-1 = 11$

$\bar{x} = 7.38$ oz

$\hat{s} = \sigma = 1.24$ oz

Is $n \geq 30$

$12 \geq 30$ x

→ [use student t distribution]

confidence levels = 99%

<u>unknown:</u> confidence limits when confidence levels are at 99%

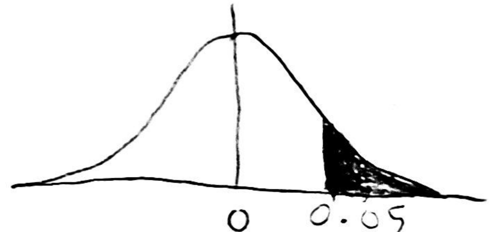
<u>Formula:</u> $\bar{x} + t_c \dfrac{\hat{s}}{\sqrt{n}}$

<u>Solve:</u>

(confidence levels)

$t_c = 1 - 0.99 = 0.01$

$\dfrac{t_c}{2} = \dfrac{0.01}{2} = 0.005$



$1 - \dfrac{t_c}{2} = 1 - 0.005 = 0.995$

$= t_{0.995}$ and $v = 11$ $= t_c = 3.11$

$7.38 \pm 3.11 \underbrace{\left(\dfrac{1.24}{\sqrt{12}}\right)}_{1.11} = 7.38 \pm 1.11 = \boxed{(6.27, 8.49)}$

4. An urn contains red and white marbles in an unknown proportion. A random sample of 60 marbles selected w/ replacement from the urn showed that 70% were red.

Find a) 95%

Given: $p = 70\% = 0.7$

$n = 60 \quad z_c = 1.96$

confidence level	99.73%	99%	98%	96%	95.45%	95%
z_c		2.58				1.96

$$p \pm 1.96\sigma_p = p \pm 1.96\sqrt{\frac{p(1-p)}{n}} = 0.7 \pm 1.96\sqrt{\frac{0.7(1-0.7)}{60}}$$

$$= \boxed{0.7 \pm 0.116}$$

b) 99% confidence levels

Given: $p = 70\% = 0.7$

$n = 60$

$z_c = 2.58$

$$p \pm 2.58\sigma_p = p \pm 2.58\sqrt{\frac{p(1-p)}{n}} =$$

$$0.7 \pm 2.58\sqrt{\frac{0.7(1-0.7)}{60}} = \boxed{0.7 \pm 0.1526}$$

5. suppose that n observations $x_1, x_2, \ldots, x_n$ are made from a poisson distribution w/ unknown parameter λ. determine maximum likelihood estimate of λ.

Poisson Distribution:

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

we present that → $L(\lambda) = \prod_{i=1}^{n} f(x_i) =$

$$\prod^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} = \frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}}{\prod_{i=1}^{n} x_i!}$$

and then we say that →

$$\boxed{\ln L(\lambda) = \ln \lambda \times \sum_{i=1}^{n} (x_i) - n\lambda - \sum_{i=1}^{n} \ln(x_i!)}$$

6. A population has a density function given by

$$k = \frac{n}{\sum_{i=1}^{n} \ln x_i} - 1$$

$$f(x) = \begin{cases} (k+1)x^k & 0 \le x \le 1 \\ 0 & \text{Otherwise} \end{cases}$$

For n observations $x_1, x_2, \dots, x_n$ made from this population, find the maximum likelihood estimate of k.

$$f(x,k) = f(x_1,k)\, f(x_2,k)\, f(x_3,k) \cdots f(x_n,k)$$

$$L = (k+1)x_1^k \times (k+1)x_2^k \times (k+1)x_3^k \times (k+1)x_n^k$$

$$L = (k+1)^n [x_1^k \cdot x_2^k \cdot x_3^k \cdots x_k^n]$$

$$\ln L = \ln([k+1)^n (x_1^k \cdot x_2^k \cdot x_3^k \cdots x_n^k)]$$

$$= \ln(k+1)^n + \ln(x_1^k \cdot x_2^k \cdot x_3^k \cdots x_n^k)$$

$$= n\ln(k+1) + [\ln x_1^k + \ln x_2^k + \ln x_3^k + \dots \ln x^k_n]$$

$$\ln L = n\ln(k+1) + [k\ln x_1 + k\ln x_2 + k\ln x_3 + \dots + k\ln x_n^k]$$

$$= n\ln(k+1) + k(\ln x_1 + \ln x_2 + \ln x_3 + \dots + \ln x_k)$$

$$\ln L = n\ln(k+1) + k(\sum_{i=1}^{n} \ln x_i) \Rightarrow \frac{n}{k+1} + \sum_{i=1}^{n} \ln x_i = 0$$

$$\frac{d\ln L}{dk} = \frac{n}{k+1} + \sum_{i=1}^{n} \ln x_i \Rightarrow \frac{d\ln L}{dk} = 0 \quad \frac{n}{k+1} = -\sum_{i=1}^{n} \ln x_i$$