EE 381 HW #10

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1. A population consists of the four numbers 3, 7, 11, 15. Consider all possible samples of size two that can be drawn w/replacement from this population.

a) Population mean and standard deviation

- If $x_1, x_2, \dots, x_n$ denote values obtained in a particular sample size of n, then sample mean is

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

n = number of data points $= 4$

$$\bar{x} = \frac{3+7+11+15}{4} = \frac{36}{4} = \boxed{9 = \bar{x} = \mu}$$

Std. dev: we must find the variance first and then sqrt it.

Sample variance: $s^2 = \frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + \dots + (x_n-\bar{x})^2}{n}$

$$s^2 = \frac{(3-9)^2 + (7-9)^2 + (11-9)^2 + (15-9)^2}{4}$$

$$\boxed{\begin{array}{c}\text{Std. dev: pop: } s = \sqrt{s^2} \\ s = \sqrt{20} = 4.472\end{array}}$$

$$= \frac{(-6)^2 + (-2)^2 + (2)^2 + (36)^2}{4} = \frac{36+4+4+36}{4} = 20 = s^2$$

1b) The mean and standard deviation of the sampling distribution of means. (w/replacement)

// we need to list out the possibilities of points

Data points = 3, 7, 11, 15

(3, 3); (3, 7); (3, 11); (3, 15)
(7, 3); (7, 7); (7, 11); (7, 15) $n = 16$
(11, 3); (11, 7); (11, 11); (11, 15)
(15, 3); (15, 7); (15, 11); (15, 15)

Thm 5.1: The mean of the sampling distribution of means, denoted by $\mu_{\bar{x}}$, is $E(\bar{x}) = \mu_{\bar{x}} = \mu$

Sample means:

$E(3,3) = \frac{3+3}{2} = \frac{6}{2} = 3$

$E(3,7) = \frac{3+7}{2} = \frac{10}{2} = 5$

$E(3,11) = \frac{3+11}{2} = \frac{14}{2} = 7$

$E(3,15) = \frac{3+15}{2} = \frac{18}{2} = 9$

$E(7,3) = \frac{7+3}{2} = \frac{10}{2} = 5$

$E(7,7) = \frac{7+7}{2} = \frac{14}{2} = 7$

$E(7,11) = \frac{7+11}{2} = \frac{18}{2} = 9$

$E(7,15) = \frac{7+15}{2} = \frac{22}{2} = 11$

$E(11,3) = \frac{11+3}{2} = \frac{14}{2} = 7$

$E(11,7) = \frac{11+7}{2} = \frac{18}{2} = 9$

$E(11,11) = \frac{11+11}{2} = \frac{22}{2} = 11$

$E(11,15) = \frac{11+15}{2} = \frac{26}{2} = 13$

$E(15,3) = \frac{15+3}{2} = \frac{18}{2} = 9$

$E(15,7) = \frac{15+7}{2} = \frac{22}{2} = 11$

$E(15,11) = \frac{15+11}{2} = \frac{26}{2} = 13$

$E(15,15) = \frac{15+15}{2} = \frac{30}{2} = 15$

16) (continued!)

$$\mu_{\bar{x}} = \frac{\mu_1 + \mu_2 + \cdots + \mu_n}{n}$$

$n = 16$

$$\mu_{\bar{x}} = \frac{3+5+7+9+5+7+9+11+7+9+11+13+9+11+13+15}{16}$$

$$= \frac{144}{16} = \boxed{9 = \mu_{\bar{x}}}$$

Standard deviation: sqrt of variance

Thm 5.2: If a population is infinite and the sampling is random or if the population is finite and sampling is with replacement, then the variance of the sampling distribution of means, denoted by $\sigma_{\bar{x}}^2$ is: $E[(\bar{X}-\mu)^2] = \sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$

$$E[(\bar{X}-\mu)^2] = \frac{\begin{gathered}(3-9)^2+(5-9)^2+(7-9)^2+(9-9)^2+\\(5-9)^2+(7-9)^2+(9-9)^2+(11-9)^2+\\(7-9)^2+(9-9)^2+(11-9)^2+(13-9)^2+\\(9-9)^2+(11-9)^2+(13-9)^2+(15-9)^2\end{gathered}}{16} =$$

$$\frac{160}{16} = 10 = \sigma_{\bar{x}}^2 \Rightarrow \text{std dev} = \sqrt{\sigma_{\bar{x}}^2} = \sqrt{10} = \boxed{3.162 = \sigma_{\bar{x}} \text{ of sampling dist. of means}}$$

Thm 5.4: If the population from which samples are taken is normally distributed with mean μ and variance σ^2, then the sample mean is normally distributed w/ mean μ and variance $\frac{\sigma^2}{n}$.

1c) Repeat quest (a) and quest (B) w/o replacement.

a) population mean and standard deviation

still the same

population mean $= \mu = \bar{X} = 9$

variance $= \sigma^2 = 20$

standard deviation $= \sigma = \sqrt{\sigma^2} = 4.472$ of population

b) Mean and sampling distibutions of the mean

//List out all the possible points w/o repition

3: (3, 3); (3, 7); (3, 11); ~~(3, 15)~~

7: ~~(7, 3)~~; (7, 7); ~~(7, 11)~~; ~~(7, 15)~~

11: ~~(11, 3)~~; ~~(11, 7)~~; (11, 11); ~~(11, 15)~~

15: ~~(15, 3)~~; ~~(15, 7)~~; ~~(15, 11)~~; (15, 15)

$n = 2$

$N = 5$

NOTE: (3, 7) = (7, 3)

$$N = n = \binom{4}{2} = C(\overset{n}{4}, \overset{r}{2}) = \frac{n!}{(n-r)!\,r!} = \frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} =$$

$$\frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}\,2!} = \frac{12}{2!} = 6 \text{ possible points}$$

sample means:

$E(3,3) = \frac{3+3}{2} = \frac{6}{2} = 3$ $\quad E(7,7) = \frac{7+7}{2} = \frac{14}{2} = 7$

$E(3,7) = \frac{3+7}{2} = \frac{10}{2} = 5$ $\quad E(11,11) = \frac{11+11}{2} = \frac{22}{2} = 11$

$E(3,11) = \frac{3+11}{2} = \frac{14}{2} = 7$ $\quad E(15,15) = \frac{15+15}{2} = \frac{30}{2} = 15$

1c) (continued!)

$$\mu_{\bar{x}} = \frac{\mu_1 + \mu_2 + \cdots + \mu_n}{C(4,2)} = \frac{3+5+7+7+11+15}{6} = \frac{48}{6} =$$

$$\boxed{8 = \mu_{\bar{x}} \text{ w/o replacement}}$$

Thm 5.3: If population is of size N, if sampling is w/o replacement and if sample size is $n \leq N$, then thm. 5-2's formula is replaced by:

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$$

$N = 4$
$n = 2$

$$= \frac{20}{2}\left(\frac{4-2}{4-1}\right) = 10 \cdot \frac{2}{3} = \frac{20}{3} = \boxed{6.67}$$

2) The weights of 1500 ball bearings are normally distributed w/ a mean of 22.40 oz and a standard deviation of 0.048 oz. If 300 random samples of size 36 are drawn from this population, determine the expected mean and standard deviation of the samping distribution of means if sampling is done.

a) with replacement

Thm 5.2: If a population is infinite and the sampling is random or if the popvlation is finite and sampling is w/ replacement, then the variance of the sampling distribution of means, denoted by $\sigma_{\bar{x}}^2$, is given by:

$$E[(\bar{x}-\mu)^2] = \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$ whereas σ^2 is the variance of the population.

Given: $\mu_{\bar{x}} = E(\bar{x}) = 22.40$ oz $N = 300$ = total popvlation

standard deviation = 0.048 oz = σ

$n = 36$ = sample size (part of a popvlation)

can be chosen more than once

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \rightarrow \sqrt{\sigma_{\bar{x}}^2} = \sigma_{\bar{x}} = \frac{\sqrt{\sigma^2}}{\sqrt{n}} = \frac{\sigma}{\sqrt{36}} = \frac{0.048}{6} \boxed{= 0.008 \text{ in.} = \sigma_{\bar{x}} \text{ w/ replacement}}$$

2b) without replacement

thm. 5.3 : If the population is of size N, if sampling is w/o replacement, and if the sample size is $n \le N$, then thm 5-2 formula is replaced by:

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$$

Given: $n = 36 =$ sample size (part of population)

$N = 300 =$ total population

$\mu_{\bar{x}} = E(\bar{x}) = 22.40$ oz.

$\sigma = 0.048$ oz $=$ standard deviation

$$\sigma_{\bar{x}} = \sqrt{\frac{\sigma^2}{n}} \cdot \sqrt{\left(\frac{(N-n)}{(N-1)}\right)} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{(N-n)}{(N-1)}}$$

$$= \frac{0.048}{\sqrt{36}} \cdot \sqrt{\frac{(300-36)}{(300-1)}} = \frac{0.048}{6} \cdot \sqrt{\frac{264}{299}} =$$

$$0.008 \cdot \sqrt{\frac{264}{293}} = \boxed{0.0075 = \sigma_{\bar{x}} \text{ w/o replacement}}$$

3) From question 2, how many of the random samples would have their means:

a) Between 22.39 and 22.41 oz

Given: $\mu = 22.40$

sample size $= n = 36$

population $= N = 300 =$ total num. samples

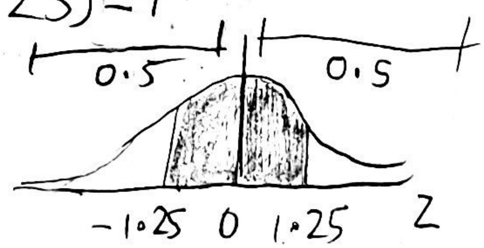
$\sigma =$ standard deviation $= 0.048$ $\qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.048}{\sqrt{36}}$

$\bar{x} = 22.39,\ 22.41$ $\qquad = 0.008$

By the definition of central limit thm,

Thm 5.5 states that the population from which samples are taken has a probability distribution w/ mean μ and variance σ^2 that is not necessarily a normal distribution. Then the standarized variable associated w/ $\bar{x}$, given by: $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$

$P(22.39 < \bar{x} < 22.41) = \left(\frac{22.39 - 22.40}{0.008} < \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} < \frac{22.41 - 22.40}{0.008}\right) = (-1.25 < z < 1.25) = P$



$P(-1.25 < z < 1.25)$

$=$ (Area between $z = -1.25$ and $z = 0$) $+$ (Area between $z = 0$ and $z = 1.25$) $= 0.3944 + 0.3944 =$

$0.7888 = P(22.39 < \bar{x} < 22.41)$

We now multiply $P(22.39 < \bar{x} < 22.41)$ by $N = 300 \cdot 0.7888 = 236.64 \approx 237$

3) (continued!)

b) Greater than 22.42 oz

Given:

$\mu = 22.40$ $\quad Z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}}$ (thm 5-5)

$\bar{x} = 22.42$

$\sigma = 0.048$

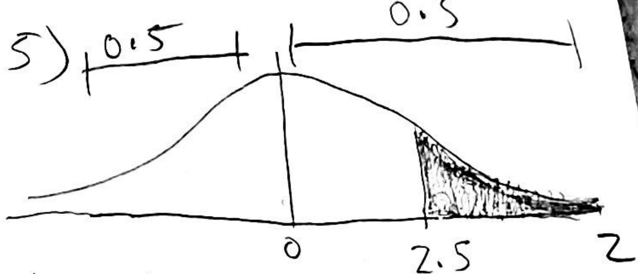
N = total num. samples = 300

sample size = n = 36

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.048}{\sqrt{36}} = \frac{0.048}{6} = 0.008$$

$$P(\bar{x} > 22.42) = \left(\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} > \frac{22.42 - 22.40}{0.008}\right) =$$

$$\left(Z > \frac{0.02}{0.008}\right) = (Z > 2.5)$$



Area between $z = 0$ and $z = 2.5$
$= 0.4938 = (0 < Z < 2.5)$

$P(Z > 2.5) = 0.5 - 0.4938$
$= 0.0062 = P(Z > 2.5)$

multiply N by $P(\bar{x} > 22.42) = 300 * 0.0062$
$= 1.86 \approx \boxed{2 = P(\bar{X} > 22.42) \text{ w/ means greater than } 22.42}$

3) (continued!)

c) less than 22.37 oz

Given:

$N = 300$ = total num. samples

$n = 36$ = sample size

$\mu = 22.40$

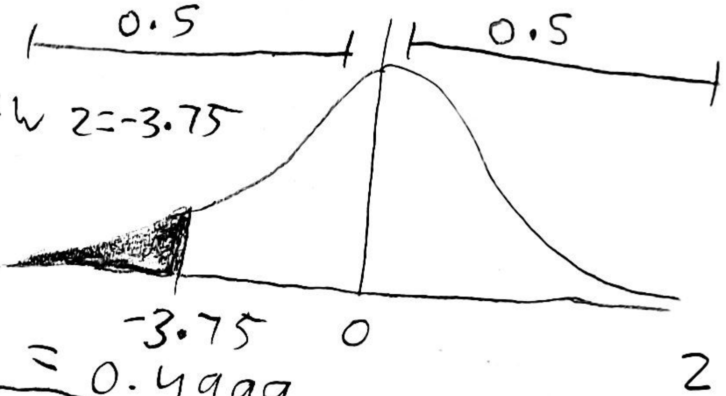
$\sigma = 0.048$ $\qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.048}{\sqrt{36}} = \frac{0.048}{6} = 0.008$

$\bar{x} = 22.37$ $\qquad z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$ (thm 5-5)

$P(\bar{x} < 22.37) = \left(\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} < \frac{22.37 - 22.40}{0.008} \right)$

$= (z < -3.75)$

$(z < -3.75) = 0.5$ - area btw $z = -3.75$ and $z = 0$



area btw $z = -3.75$ and $z = 0$ $= 0.4999$

$\boxed{(z < -3.75) = 0.5 - 0.4999 = 0.0001}$

multiply N by $(z < -3.75) = 300 * 0.0001 = 0.03$

$\approx$ $\boxed{\text{0 where the mean is less than 22.37 oz}}$

3) (continued!)

d) less than 22.38 or more than 22.41 oz

Given; less 22.38 oz

$\bar{x} = 22.38$

$n = 36$ = sample size

$N = 300$ = total num. samples

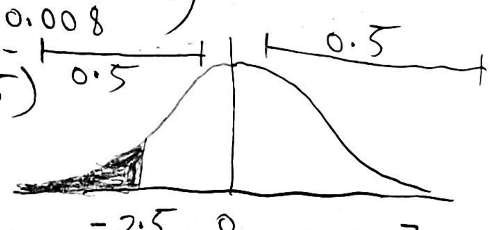
$\sigma = 0.048$

$\mu = 22.40$ $\qquad Z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}}$ (thm 5-5)

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.048}{\sqrt{36}} = \frac{0.048}{6} = 0.008$$

$$P(\bar{x} < 22.38) = \left(\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} < \frac{22.38 - 22.40}{0.008} \right)$$

$$= \left(Z < \frac{-0.02}{0.008} \right) = (Z < -2.5)$$



$(Z < -2.5)$ = area between $z = -2.5$ and $z = 0$

$= 0.5 - 0.4938 = 0.0062 = P(Z < -2.5)$

multiply N by $P(Z < -2.5) = 300 * 0.0062 =$

$1.86 \approx$ 2 where the less than 22.38

3) (continued!)

d) (continued!)

Given: Greater than 22.41 02

$N = 300$ = total num. samples

$n = 36$ = sample size

$\mu = 22.40$

$\sigma = 0.048$

$\bar{x} = 22.41$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \text{ (thm 5-5)}$$

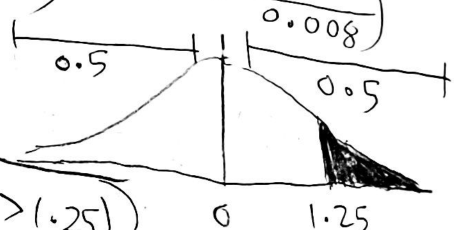
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.048}{\sqrt{36}} = \frac{0.048}{6} = 0.008$$

$$P(\bar{x} > 22.41) = \left(\frac{\bar{x} - \mu}{\sigma_{x}} > \frac{22.41 - 22.40}{0.008}\right) = \left(z > \frac{0.01}{0.008}\right)$$

$= (z > 1.25)$

$= 0.5 -$ Area btw $z = 0$ and $z = 1.25$

$= 0.5 - 0.3944 = 0.1056 = P(z > 1.25)$



multiply N by $P(z > 1.25) = 300 * 0.1056 = 31.68 \approx$

32 where the mean is > 22.41

$P(\bar{x} < 22.38)$ OR $P(\bar{x} > 22.41) = \text{num}(\bar{x} < 22.38) \cup \text{num}(\bar{x} > 22.41)$

$= \boxed{2 + 32 = 34 \text{ where the mean is } < 22.38 \text{ OR mean is } > 22.41}$

4) A manufacturer sends out 1000 lots, each consisting of 100 electric bulbs. If 5% of the bulbs are normally defective, in how many of the lots should we expect

a) Fewer than 90 good bulbs?

Given: (Binomial distributed)

Num lots = 1000

$\mu p = P = 95\% = 0.95$ $\qquad \sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{p(1-p)}{n}}$

$q = 1 - p = 1 - 0.95 = 0.05$ //not using correction factor

$n = 100$ bulbs

$P(90 \text{ good bulbs}) = \frac{\text{num good bulbs}}{n} = \frac{90}{100} = 0.9 = \bar{X}$

as $n \geq 30$; $100 \geq 30$ ✓, we use the normal distribution!

$\sigma_p = \sqrt{\frac{0.95 * 0.05}{100}} = \sqrt{\frac{0.00475}{100}} = 0.0218$

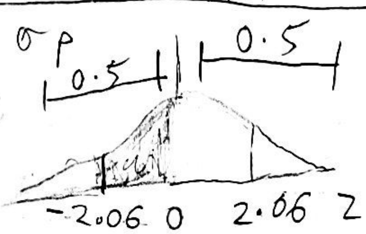
Correction Formula: $\frac{1}{2n} = \frac{1}{2(100)} = \frac{1}{200} = 0.005$

//apply correction factor

$P(\bar{x} < 90) = x + \text{corr. factor}$

$z = \frac{X - \text{correction factor} - \mu p}{\sigma p} = \frac{0.9 + 0.005 - 0.95}{0.0218} = -2.06 = z$

$= (z < 2.06)$



$z = 2.06 = 0.4803$

Area between $z = -2.06$ and $z = 0 = 0.4803$

$= 0.5 - 0.4803 = 0.0197 = (z < 2.06)$

Num lots X $(z < 2.06) = 1000 \times 0.0197 = 19.7 \approx 20$

$= \boxed{20}$ Lots : Fewer than 90 good bulbs

4) (continued!)

b) 98 or more good bulbs

Given: Binomially distributed

$Z = \frac{X - \mu}{\sigma}$

Num lots = 1000

$n = 100$ bulbs

$\mu_P = P = 98\% = 0.95$

$q = 1 - P = 1 - 0.95 = 0.05$

$\sigma_P = \sqrt{\frac{pq}{n}} = \sqrt{\frac{P(1-P)}{n}}$

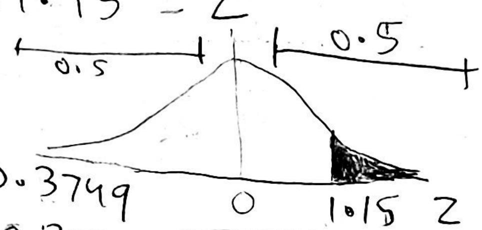
$\bar{x} = P(98$ or more bulbs$) = \frac{\text{num good bulbs}}{n} = \frac{98}{100} = 0.98$

Correction factor: $\frac{1}{2n} = \frac{1}{2(100)} = \frac{1}{200} = 0.005 = 5 \times 10^{-3})$

$P(\bar{x} \geq 98) = P(X > n - 0.5)$ // idea

$\sigma_P = \sqrt{\frac{0.95 * 0.05}{100}} = 0.0218$ $\quad Z = \frac{X - \mu}{\sigma}$

$P(\bar{x} \geq 98) = \frac{0.98 - 0.005 - 0.95}{0.0218} \approx 1.15 = Z$



Area between $z = 0$ and $1.79 = 0.3749$

$P(\bar{x} \geq 98) = P(Z > 1.15) = 0.5 - 0.3749 = \boxed{0.1251 = P(\bar{x} \geq 98)}$

Num lots $* P(\bar{x} \geq 98) = 1000 * 0.1251 = 125.1$

$\approx$ $\boxed{\text{125 lots w/ 98 or more good bulbs}}$

5th class upper limit = 799

b) Lower limit of the 8th class

8th class lower limit = 1000

5) A and B manufacture 2 types of cables, having mean breaking strengths of 4000 and 4500 lbs and standard deviation of 300 and 200 lb, respectively. If 100 cables of brand A and 50 cables of brand B are tested, what is the probability that the mean breaking

Given:

Brand A: μ strength$_A = 4000$ lbs $= \mu_{\bar{X}_A}$

std dev$_A = \sigma_{\bar{X}_A} = 300$ lbs

$n_A = 100$ cables

// standarized units

$P(\bar{x} > 450) = Z = \dfrac{(\bar{X}_B - \bar{X}_A) - (\mu_{\bar{X}_A - \bar{X}_B})}{\sigma_{\bar{X}_A} - \sigma_{\bar{X}_B}}$

Brand B: μ strength $B = 4500$ lbs $= \mu_{\bar{X}_B}$

std dev$_B = \sigma_{\bar{X}_B} = 200$ lbs

$n_B = 50$ cables

$P(600 > A) = (\bar{X}_B - \bar{X}_A) = A = P(A - B < -600)$

a) At least 600 lb more than A

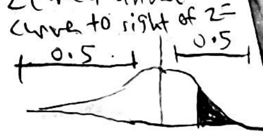
$= \dfrac{600 - (-500)}{41.23} = \dfrac{100}{41.23}$

$= (Z > 2.42)$

$\mu_{S_1 - S_2} = \mu_{S_1} - \mu_{S_2} = \mu_{S_1} - \mu_{S_2}$

$\sigma_{S_1 - S_2} = \sigma_{S_1} - \sigma_{S_2} = \sqrt{\sigma_{S_1}^2 + \sigma_{S_2}^2}$

$= Z = 2.42$ (area under normal curve to right of $Z = 2.42$)



$= 0.5 - 0.4922 = \boxed{0.0078}$ at least 600 lbs greater than A

$\mu_{\bar{X}_A - \bar{X}_B} = \mu_{\bar{X}_A} - \mu_{\bar{X}_A} = 4000 - 4500 = -500$ lbs

$\sigma_{\bar{X}_A - \bar{X}_B} = \sigma_{\bar{X}_A} - \sigma_{\bar{X}_B} = \sqrt{\dfrac{\sigma^2_A}{n_A} + \dfrac{\sigma^2_B}{n_B}} = \sqrt{\dfrac{(300)^2}{100} + \dfrac{(200)^2}{50}}$

$= \sqrt{\dfrac{9000\cancel{0}}{1\cancel{00}} + \dfrac{4000\cancel{0}}{5\cancel{0}}} = \sqrt{900 + 800} = \sqrt{1700} \approx \boxed{41.23 = \sigma_{\bar{X}_A} - \sigma_{\bar{X}_B}}$

5) (continued!)

b) At least 450 lbs more than A

Given:

$(\bar{x}_B - \bar{x}_A) = 450$

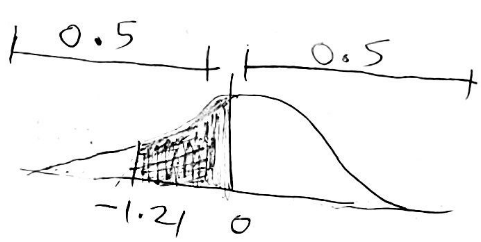
$\sigma_{\bar{x}_A - \bar{x}_B} = 41.23$

$\mu_{\bar{x}_A - \bar{x}_B} = -500$

$$P(\bar{x} > 450) = Z = \frac{(\bar{x}_B - \bar{x}_A) - (\mu_{\bar{x}_A - \bar{x}_B})}{\sigma_{\bar{x}_A - \bar{x}_B}} = \frac{450 - 500}{41.23} = \frac{-50}{41.23}$$

$\approx -1.21 = z$

$= (z > -1.21)$



$-1.21 = z \to z = 1.21 = 0.3869$

(Area under a normal curve to the right of $z = -1.21$)

$= 0.3869 + 0.5 = \boxed{0.8869}$ when there is at least 450 lbs more than A

6. Table shows a frequency distribution of the lifetimes of 400 radio tubes tested at the L&M Company.

	Life time (hours)	Number of Tubes (Frequency!)
1	300-399	14
2	400-499 <600 lifetime	46
3	500-599	58
4	600-699	76
5	700-799	68
6	800-899	62
7	900-999 ≥900 hours	48
8	1000-1099	22
9 last class	1100-1199	6
	TOTAL	400

a) upper limit of the 5th class

$\boxed{\text{5th class upper limit} = 799}$

b) Lower limit of the 8th class

$\boxed{\text{8th class lower limit} = 1000}$

c) class mark of the 7th class

$$\frac{900+999}{2} = \frac{1899}{2}$$

$\boxed{= 949.5 = \text{class mark of 7th class}}$

6. (continued!)

d) class boundaries of the 1st class

[1100 − 0.5, 1199 + 0.5]

subtract 0.5 from 1st num   add 0.5 from 2nd num

$\boxed{= [1099.5,\ 1199.5] = \text{class interval for 1st class}}$

e) class interval size

* class interval size c_j (width): difference between upper and lower class boundaries

$\boxed{\text{100 hours (e.g. 500-599 hours) = class interval size}}$

f) Frequency of the 4th class

$\boxed{\text{Frequency = number of tubes of 7th class} = 76}$

g) relative frequency of the 6th class

$$\text{Relative Frequency} = \frac{\text{Frequency of a class}}{\text{total num students}}$$

num students = 400

$$= \frac{62 \text{ (6th class Frequency)}}{400} = 0.155 = 15.5\%$$

15.5% relative frequency of 6th class

6. (continued!)

h) Percentages of tubes whose lifeline who do not exceed 600 hours
num students

$$= \frac{\sum \text{frequencies} < 600 \text{ hours}}{\text{num students}} = \frac{14+46+58}{400}$$

$$= \frac{118}{400} = 0.295 = \boxed{29.5\%}$$

i) percentages of tubes w/ lifeline's $\geq$ 900 hours

$$= \frac{\sum \text{frequencies} \geq 900 \text{ hours}}{\text{num students}} = \frac{48+22+6}{400}$$

$$= \frac{76}{400} = 0.19 = \boxed{19\%}$$

7. Determine the mean maximum loading

NOTE: 1 ton = 2000 lbs

	Maximum Load (short tons)	Number of cables
1	9 - 9.7	2
2	9.8 - 10.2	5
3	10.3 - 10.7	12
4	10.8 - 11.2	17
5	11.3 - 11.7	14
6	11.8 - 12.2	6
7	12.3 - 12.7	3
8	12.8 - 13.2	1
	Total	60

class mark: $\frac{LL + UL}{2}$

fx: class mark * Frequency

a) Long method

Maximum Load (short tons)	class mark (x)	Number of cables (f)	fx
9 - 9.7	9.35	2	18.7
9.8 - 10.2	10	5	50
10.3 - 10.7	10.5	12	126
10.8 - 11.2	11	17	187
11.3 - 11.7	11.5	14	161
11.8 - 12.2	12	6	72
12.3 - 12.7	12.5	3	37.5
12.8 - 13.2	13	1	13
		$n = \Sigma f = 60$	$\Sigma fx = 665.2$

7. (continued!)

a) (continued!)

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{665.2}{60} = \boxed{11.0867 \text{ tons}}$$

b) "coding method"

f: frequency per class

$f*u$

class mark(s) x	$u = \frac{x-11.5}{0.5}$	f	fu	fu^2
9.35	-4.3	2	-8.6	73.96
10	-3	5	-15	225
10.5	2	12	24	576
11	-1	17	-17	289
a=11.5	0	14	0	0
12	1	6	6	36
12.5	2	3	6	36
13	3	1	3	9

$\boxed{n = \Sigma f = 60}$ $\boxed{\Sigma fu = -49.6}$ $\boxed{\Sigma fu^2 = 1244.96}$

$$\bar{x} = a + \frac{c}{n} = \Sigma fu = a + c\bar{u} \quad \rightarrow -0.4133$$

$$\bar{x} = a + \left(\frac{\Sigma fu}{\Sigma f}\right) * c = \underbrace{11.5 + \left(\frac{-49.6}{60}\right) * 0.5}_{\text{PEMDAS}} =$$

$$= 11.5 - 0.4133 = \boxed{11.0867 \text{ tons}}$$

8. Find the standard deviation of the distributions of Q7.

$c = 0.5$
$n = 60$ cables

$$s^2 = \sigma = c^2\left[\frac{\Sigma fv^2}{n} - \left(\frac{\Sigma fv}{n}\right)^2\right]$$

$$= (0.5)^2 - \left[\frac{1244.96}{60} - \left(\frac{-49.6}{60}\right)^2\right]$$

$$= 0.25 - [20.749 - (-0.827)^2]$$

$$= 0.25 - [20.749 - (0.684)]$$

$$= 0.25 - [20.065] = -19.815$$

= std. deviation of distributions