EE 381 HW #1

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1. Given:

You drew 3 cards

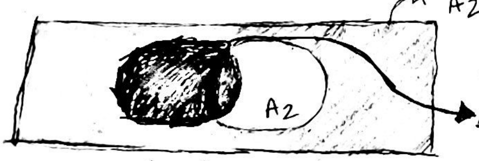
Thm 1.4:
If A' is the compliment of A, then $P(A') = 1 - P(A)$

$A1$ = "king on first draw"

$A2$ = "king on second draw"

$A3$ = "king on third draw"

a) $P(A_1 \overset{\text{and}}{\cap} A_2')$



Legend
- A2'
- A1 ∩ A2'

$P(A1 \cap A2') = P(A1) - P(A_2)$

What is the probability of getting a king on the first draw and not getting a king on the second draw?

b) $P(A1' \overset{\text{or}}{\cup} A2')$

Legend
- A1' dark shade
- A2' light shade
- A1' ∪ A2'

$(A1' \cup A2')$
$= (A_1 \cap A_2)'$
[Demorgan's law]

What is the probability of not getting a king on the first draw or not getting a kind on the second draw?

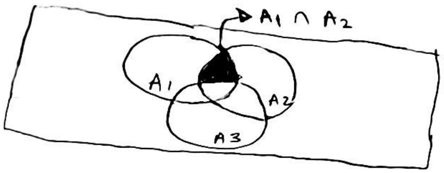
1. c) $P[(A_1 \cap A_2) \cup (A_2' \cap A_3)]$

union

intersection

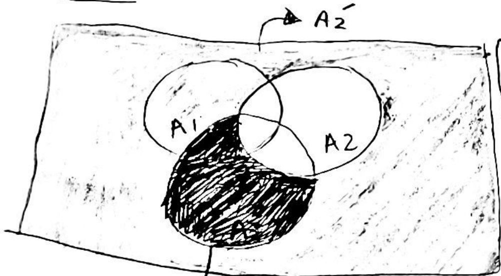
intersection

Step 1:



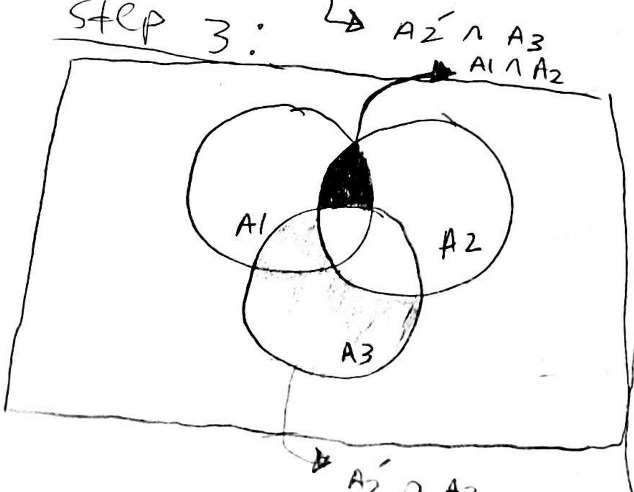
Legend
■ $A_1 \cap A_2$

Step 2:



Legend
■ A_2'
■ $A_2' \cap A_3$

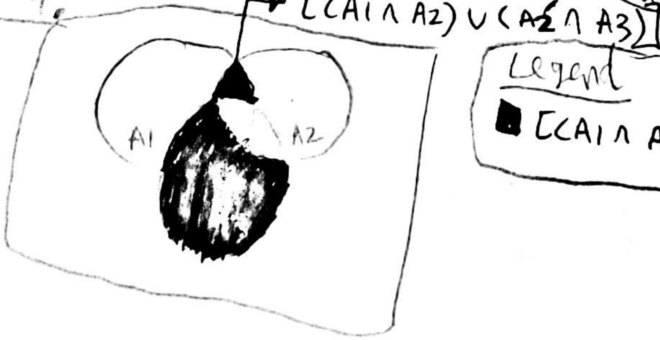
Step 3:



Legend
■ $A_1 \cap A_2$
□ $A_2' \cap A_3$
■ $(A_1 \cap A_2) \cup (A_2' \cap A_3)$

What's the probabity of (drawing a king on the first draw and a king on on the second draw) or (not drawing a king on the second draw and drawing a king on the third draw?

Step 4:



Legend
■ $[(A_1 \cap A_2) \cup (A_2' \cap A_3)]$

2. Normally when tossing a coin once, it will either result in head or tails. The probability of getting heads is 50%. Tails is also 50% b/c there are two sides of a coin, and only lands on one side.

In this case: The coin is abnormal; thus, heads will have the advantage over tails.

Given: heads will appear 2x more likely as tails.

sample space outcomes = {H, T}

let H = heads

let T = tails

$1 = P(H) + P(T)$; $H = 2(T)$ //substitute H as 2T

Thm 1.5: If $A = A_1 \cup A_2 \cup \cdots \cup A_n$, where $A_1, A_2, \ldots, A_n$ are mutually exclusive, then $P(A) = P(A_1) + P(A_2) + \cdots P(A_n)$.

$1 = P(H) + P(T)$ → $1 = P(H) + P(T)$

$1 = 2T + T$ $1 = P(H) + 1/3$

$\frac{1}{3} = \frac{3T}{3}$

substitute

$T = 1/3$

$P(T) = 1/3$

$1 = P(H) + 1/3$

$-1/3 \quad -1/3$

$1 - 1/3 = P(H)$

$3/3 - 1/3 = P(H)$

$\boxed{2/3 = P(H)}$

Heads probability = 2/3

Tails probability = 1/3

3. Default Dice sample space $= \{1,2,3,4,5,6\}$

Normally, the probability of rolling a number on a dice are equally likely: i.e: $P(1) = P(2) = P(\ldots) = P(6) = \frac{1}{6}$.

But, that's not the case.

we have a biased dice, meaning that the dice is is abnormal.

Thm 1.5: If $A = A_1 \cup A_2 \cup \cdots A_n$, where $A_1, A_2, \ldots, A_n$ are mutually exclusive, then $P(A) = P(A_1) + P(A_2) + \cdots + P(A_n)$.

Given:

The number 3 is odd

3 appears twice as often as any other number on the dice. All other number outcomes are equally likely.

sample space AppearsTwice $= \{3\}$

Equally likely sample space $= \{1,2,4,5,6\}$

with the 3 ocurring twice as likely with the remaining numbers: $P(3) = \frac{1}{6} \cdot 2 = \frac{2}{6} = \frac{1}{3}$

$P(3)' = 1 - P(3) = 1 - \frac{2}{6} = \frac{6}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$

Thm 1.5

$1 = P(3) + [P(1) + P(2) + P(4) + P(5) + P(6)]$

$1 = 1/3 + [P(1) + P(2) + P(4) + P(5) + P(6)]$

B/c there are 5 remaining numbers that are equally likely, we divide it by 5.

$\frac{2}{3} / 5 = \frac{2}{3} \cdot \frac{1}{5} = \frac{2}{15}$ $\quad P(1) = P(2) = P(4) = P(5) = P(6) = \frac{2}{15}$

$P(\text{odd}) = P(1) + P(3) + P(5) = \frac{2}{15} + \frac{1}{3} + \frac{2}{15}$

LCM: 15

$= \frac{2}{15} + \frac{5}{15} + \frac{2}{15} = \frac{9}{15} = \frac{3}{5}$ chance of rolling an odd: P(odd)

1. A card is drawn at random from an ordinary deck of 52 playing cards.

sample space = {AS, 1S, 2S, 3S, 4S, 5S, 6S, 7S, 8S, 9S, 10S, JS,
QS, KS ...
AC, 1C, 2C, 3C, 4C, 5C, 6C, 7C, 8C, 9C, 10C, JC,
QC, KC ...
AD, 1D, 2D, 3D, 4D, 5D, 6D, 7D, 8D, 9D, 10D, JD,
QD, KD ...
AH, 1H, 2H, 3H, 4H, 5H, 6H, 7H, 8H, 9H, 10H, JH,
QH, KH}

a) A three of clubs or a six of diamonds

$P(A) = P(3 \cap C + 6 \cap D)$

$= P(3 \cap C) + P(6 \cap D)$

$= \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \boxed{\frac{1}{26}}$

Thm 1.4: $P(A') = 1 - P(A)$

b) Any suit except Hearts

$P(H) = \frac{13}{52} = \frac{1}{4}$

*NOTE: there are 13 cards that are hearts in the deck, so the remaining 39 cards cannot be hearts.

$P(H)' = 1 - P(H) = 1 - \frac{1}{4} = \frac{4}{4} - \frac{1}{4} = \boxed{\frac{3}{4}}$

NOTE: There is 1 four per suit and there's four suits; thus there are four fours.

c) neither a four nor a club

$P(\text{four}) = P(4 \cap S + 4 \cap C + 4 \cap D + 4 \cap H) =$

$P(4 \cap S) + P(4 \cap C) + P(4 \cap D) + P(4 \cap H) = \frac{1}{52} + \frac{1}{52} + \frac{1}{52} + \frac{1}{52} = \frac{4}{52} = \frac{1}{13}$

LASTS ALL YEAR. GUARANTEED!*

4. c) (continued!)

$P(\text{club}) = \frac{13}{52} = \frac{1}{4}$ NOTE: there are only 13 clubs in the deck

$P(\text{neither a four nor a club}) = 1-\left(\frac{1}{13}+\frac{1}{4}\right)$

$= 1-\left(\frac{4}{52}+\frac{13}{52}\right) = 1-\frac{17}{52} = \frac{52}{52}-\frac{17}{52} = \boxed{\frac{35}{52}}$

5. A ball is drawn at random from a box containing 6 red balls, 4 white balls, and 5 blue balls.

Sample space = {6 red balls, 4 white balls, and 5 blue balls}

Total # balls = 15

a) white

$P(\text{white}) = \frac{\text{\# of white balls}}{\text{total \# of balls}} = \boxed{\frac{4}{15}}$

b) P(not red)

$P(\text{red}) = \frac{\text{\# of red balls}}{\text{total \# of balls}} = \frac{6}{15} = \frac{2}{5}$

$P(\text{not red}) = P(\text{red})' = 1-P(\text{red}) = 1-\frac{6}{15} = \frac{15}{15}-\frac{6}{15} = \frac{9}{15} = \boxed{\frac{3}{5}}$

c) P(red or white)

$P(\text{red} \cup \text{white}) = P(\text{red})+P(\text{white})$

$P(\text{red}) = \frac{6}{15} = \frac{2}{5}$

$= \frac{6}{15}+\frac{4}{15} = \frac{10}{15} = \boxed{\frac{2}{3}}$

$P(\text{white}) = \frac{4}{15}$