

Lab 1 note (Thursday 9/10/20)

What is the probability of seeing a sum of 7 when rolling 2 fair dice?

$$S = \{(1,1), (1,2), (1,3), \dots, (6,6)\}$$

36 sample points

$$A = \{(1,6), (6,1), (2,5), (5,2), (3,4)\}$$

$(4,3) \}$

$$|A| = 6 \quad |S| = 36$$

$$P(A) = \frac{6}{36} = \boxed{\frac{1}{6}}$$

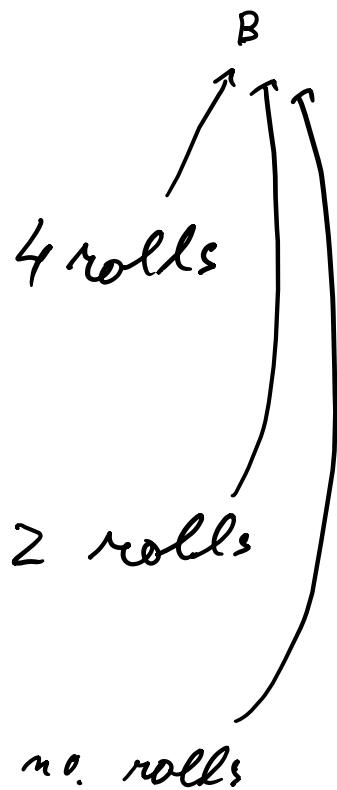
1<sup>st</sup> experiment

3, 2, 6, 8, 7  
                  
    7

2<sup>nd</sup> experiment

10, 11, 7  
                  
    7

⋮  
100,000<sup>th</sup> exp.



What is the probability of seeing sum of 7 after exactly 3 rolls?

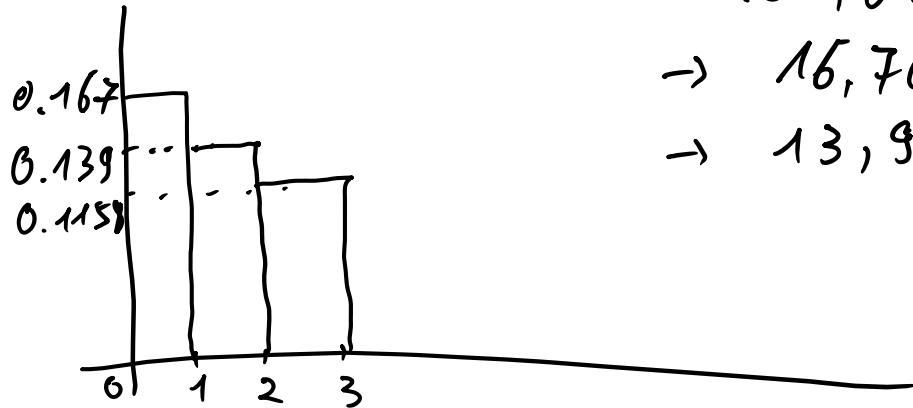
1 roll :  $\frac{1}{6} = 0.167 \rightarrow$  need 0 roll before it to see 7

2 rolls :  $\frac{5}{6} \times \frac{1}{6} = 0.139 \rightarrow$  need 1 roll before it to see 7

3 rolls :  $\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = 0.1158$

1<sup>st</sup> roll, 2<sup>nd</sup> roll, 3<sup>rd</sup> roll  
 5            5  
 6            6  
 8            8  
 and        and  
 .            .

n rolls :



100,000

$\rightarrow 16,700$  0 rolls

$\rightarrow 13,900$  1 roll

coins: 1 2 3 ... 100  
 3)  $a_1 a_2 a_3 a_4 \dots a_{100}$   
 $\downarrow \downarrow \downarrow \downarrow \quad \downarrow$   
 $2 \times 2 \times 2 \times 2 \times \dots \times 2$   
 Total possible outcomes:  $2^{100}$

Outcomes that has 35 heads:

$$C(100, 35)$$

Prob. of seeing 35 heads when tossing 100 coins:  $\frac{C(100, 35)}{2^{100}} = 8.63 \times 10^{-4}$

Method 2: binomial distribution

$$C(n, x) \times p^x \times q^{(n-x)}$$

$$\begin{array}{l} n: 100 \\ x: 35 \end{array} \rightarrow C(100, 35) \times \left(\frac{1}{2}\right)^{35} \times \left(\frac{1}{2}\right)^{65}$$

$$p: \frac{1}{2} = 8.63 \times 10^{-4}$$

$$q: \frac{1}{2}$$

random. randint(0, 1)

1<sup>st</sup> experiment:

0, 1, 0, 0, 1, ... 1  
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 coin 1 coin 2 3 4 ... 100

If you see 35 heads  $\rightarrow$  count + 1

## 2<sup>nd</sup> experiment

0, 0, 1, . . . . . 1

count/  
N (Total experi - )

4)      4 Ones      - cards poker draw  
        4 Twos  
        21 Threes  
        ⋮  
        21 Kings  
different kinds  
 $(1, 2, 3, \dots, 13)$

$$\underbrace{C(13, 1) \times C(4, 4)}_{4 \text{ cards}}$$

$$\underbrace{C(48, 2)}_{2 \text{ cards}}$$

$$C(52, 6)$$

$$= 0.0007.$$

$$\frac{C(13,1) \times C(4,4) \times C(48,1)}{C(52,5)}$$

5-card poker draw

$$\{1, 2, 3, 4\} \rightarrow \left\{ \begin{array}{l} \text{Diamond A}\diamond, \text{Club A}\diamond \\ \dots \end{array} \right\}$$

$$\{5, 6, 7, 8\} \rightarrow \left\{ \begin{array}{l} \text{Diamond two}, \text{Club two} \\ \dots \\ \dots \end{array} \right\}$$

⋮

$$\{49, 50, 51, 52\} \rightarrow \left\{ \begin{array}{l} \text{Diamond King}, \text{Club King} \\ \dots \end{array} \right\}$$

Creating a set of 6 random numbers  
in  $[1, 52]$

1, 5, 10, 12, 17, 18

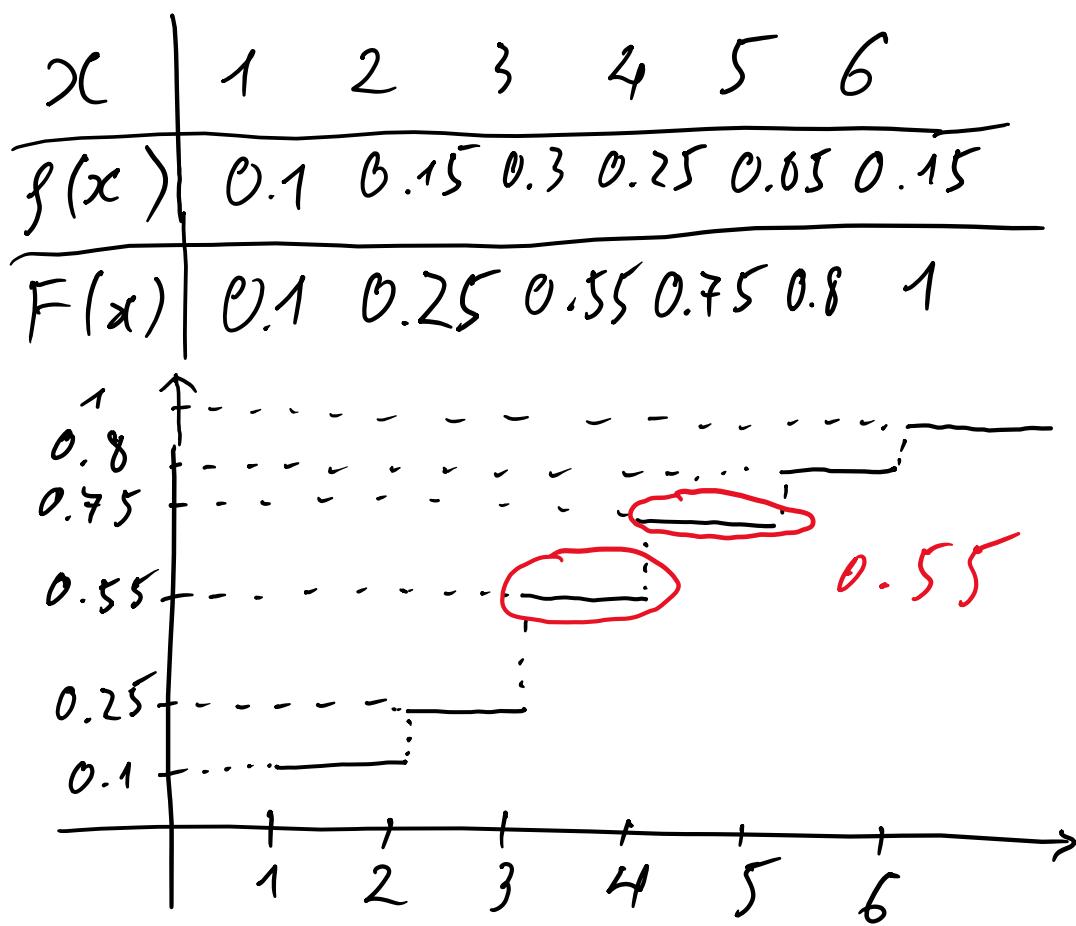
if in same kind  $\rightarrow$  count + 1

count / N

2) Toss a die 100 time:

No. 1 will occur: 10 times

2	11	:	15	"
3	11	:	30	"
4	11	:	25	"
5	11	:	5	"
6	11	:	15	"



$$r = 0.58$$

Toss the die 10,000.

$$0.55 < r < 0.9$$

$\frac{3}{3} \quad \frac{4}{4} \quad \frac{6}{6}$

Using  $r$ : a random no. betw 0 & 1 to represent the no. of the die (1, 2, 3, 4, 5, 6)

1      2      3      4      5      6

$$F = [0.1 \ 0.25 \ 0.55 \ 0.8 \ 0.85 \ 1]$$

1<sup>st</sup> toss  $F = [0 \ 0.1 \ 0.25 \ 0.55 \ 0.8 \ 0.85 \ 1]$

$r = 0.23$

$0.05 \in [0, 0.1)$   
 $\Rightarrow 1$

Check which range  $r$  is falling in

$[0.1, 0.25)$  or  $[0.25, 0.55)$

...  $[0.85, 1)$

$r$  is in  $[0.1, 0.25)$

$\downarrow$   
1      2

$s = (10000, 1)$

$s[0] = 2$

$\Rightarrow r$  represent no. 2.

$\Rightarrow$  the no. occurring on the  
1<sup>st</sup> toss is 2.

2<sup>nd</sup> toss :

$$r = 0.57$$

$$S[1] = 4$$

check the range

$$[0.55, 0.8]$$

3

4

$\Rightarrow r$  represent no. 4.

⋮  
⋮  
⋮

10000<sup>th</sup> toss .

$$S[3999] = 5$$

$$[x, y)$$

5

