Homework 10 Solution

1) Population: $\{3, 7, 11, 15\}$

a) Population mean $= \mu = \frac{36}{4} = \boxed{9}$

Population variance $= \sigma^2 = \frac{(3-9)^2+(7-9)^2+(11-9)^2+(15-9)^2}{4}$

$= 20$

$\Rightarrow \quad \sigma = \boxed{\sqrt{20}}$ (population standard deviation)

b) Sampling with replacement: $4 \cdot 4 = 16$

(3,3) (3,7) (3,11) (3,15) (7,3) (7,7)

(7,11) (7,15) (11,3) (11,7) (11,11) (11,15)

(15,3) (15,7) (15,11) (15,15)

Means: 3, 5, 7, 9, 5, 7, 9, 11, 7, 9

11, 13, 9, 11, 13, 15

Sample mean $= \mu_{\bar{x}} = \frac{3+5+7+\dots+13+15}{16} = \boxed{9}$

Sample variance $= \sigma_{\bar{x}}^2 = \frac{(3-9)^2+(5-9)^2+\dots+(15-9)^2}{16} = 9.98$

Sample variance $= \sqrt{9.98} = \boxed{3.16}$

b) Sampling without replacement : $C(4,2) = 6$

(3, 7) (3, 11) (3, 15) (7, 11) (7, 15)

(11, 15)

Means : 5, 7, 9, 9, 11, 13

Sample mean : $\frac{5+7+9+9+11+13}{6} = \boxed{9}$

Sample variance : $\frac{(5-9)^2+(7-9)^2+\dots+(13-9)^2}{6} = 6.66$

Sample standard deviation : $\boxed{2.58}$

2) $\mu = 22.4$ $\sigma = 0.048$ $n = 36$ $N = 1500$

a) Sampling with replacement :

$\mu_{\bar{x}} = \mu = \boxed{22.4}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.048}{\sqrt{36}} = \boxed{0.008}$

b) Sampling without replacement

$$\mu_{\bar{x}} = \mu = \boxed{22.4}$$

$$\sigma_{\bar{x}} = \sigma \sqrt{\frac{1500-36}{36(1500-1)}} = \boxed{0.0079}$$

3) a) $22.39 \leq X \leq 22.41$

$$22.39 \to \frac{22.39 - 22.4}{0.008} = -1.25$$

$$22.41 \to \frac{22.41 - 22.4}{0.008} = 1.25$$

$$P(-1.25 < z < 1.25) = 0.3944 \times 2$$
$$= 0.7888$$

$$0.7888 \times 300 = 236.64 \approx \boxed{237}$$

b) $X > 22.42$

$$22.42 \to \frac{22.42 - 22.4}{0.008} = 2.5$$

$$P(z > 2.5) = 0.5 - 0.4938 = 0.0062$$

$$0.0062 \times 300 = 1.86 \approx \boxed{2}$$

c) $X < 22.37$

$$22.37 \rightarrow \frac{22.37 - 22.4}{0.008} = -3.75$$

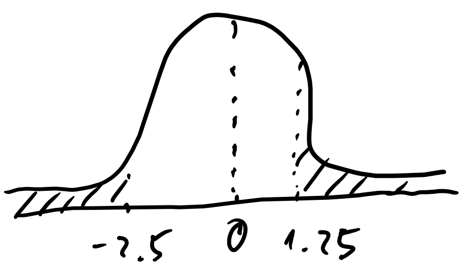
$$P(z < -3.75) = 0.5 - 0.4999 = 0.0001$$

$$0.0001 \times 300 = 0.03 \approx \boxed{0}$$

d) $X < 22.38$ or $X > 22.41$

$$22.38 \rightarrow \frac{22.38 - 22.4}{0.008} = -2.5$$

$$22.41 \rightarrow \frac{22.41 - 22.4}{0.008} = 1.25$$



$$\text{Area} = (0.5 - 0.4938) + (0.5 - 0.3944)$$
$$= 0.0062 + 0.1056$$
$$= 0.1118$$

$$0.1118 \times 300 = 33.54 \approx \boxed{34}$$

4) $\mu_{defective} = 0.05$ so $\mu_{good} = 0.95 \times 100$
$= 95$

$$\sigma = \sqrt{npq} = \sqrt{100 \times 0.05 \times 0.95}$$
$$= 2.179$$

a) $X \leqslant 90$

$$\frac{90.5 - 95}{2.179} = -2.06$$

$$P(z < -2.06) = 0.5 - 0.4803 = 0.0197$$

$$0.0197 \times 1000 = 19.7 \approx \boxed{20}$$

b) $X \geqslant 98$

$$\frac{97.5 - 95}{2.179} = 1.15$$

$$P(z > 1.14) = 0.5 - 0.3749 = 0.1251$$

$$0.1251 \times 1000 = 125.1 \approx \boxed{125}$$

5) $\mu_A = 4000 \quad \sigma_A = 300 \quad n_A = 100$

$\mu_B = 4500 \quad \sigma_B = 200 \quad n_B = 50$

$$\mu_{\bar{X}_B - \bar{X}_A} = \mu_A - \mu_B = 4500 - 4000 = 500$$

$$\sigma_{\bar{X}_B - \bar{X}_A} = \sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}} = \sqrt{\frac{300^2}{100} + \frac{200^2}{50}}$$

$$= 41.23$$

a) $X \geq 600$

$$\frac{600 - 500}{41.23} = 2.43$$

$$P(z > 2.43) = 0.5 - 0.4925$$
$$= \boxed{0.0075}$$

b) $X \geq 450$

$$\frac{450 - 500}{41.23} = -1.21$$

$$P(z > -1.22) = 0.5 + 0.3869$$
$$= \boxed{0.8869}$$

6) a) upper limit of the 5^{th} class:

$\boxed{799}$

b) Lower limit of the 8^{th} class:

$\boxed{1000}$

c) Class mark of the 7^{th} class:

$\boxed{949.5}$

d) Class boundaries of the last class:

$\boxed{1099.5 \text{ and } 1199.5}$

e) Class interval size: $\boxed{100}$

f) Frequency of the 4^{th} class: $\boxed{76}$

g) Relative frequency of the 6^{th} class: $\boxed{\frac{62}{400} = 0.155}$

h) Percentage of tubes whose lifetime do not exceed 600 hours:

$$\frac{14+46+58}{400} = 0.295 \text{ or } 29.5\%$$

i) Percentage of tubes with lifetime greater than or equal to 900 hours:

$$\frac{48+22+6}{400} = 0.19 \text{ or } 19\%.$$

7) a) The long method

x	$x-\bar{x}$	$(x-\bar{x})^2$	f	$f.(x-\bar{x})^2$
9.35	-1.74	3.0276	2	6.0552
10	-1.09	1.1881	5	5.9405
10.5	-0.59	0.3481	12	4.1772
11	-0.09	0.0081	17	0.1377
11.5	0.41	0.1681	14	2.3534
12	0.91	0.8281	6	4.9686
12.5	1.41	1.9881	3	5.9643
13	1.91	3.6481	1	3.6481

$$\bar{x} = \frac{(9.35\times 2)+(10\times 5)+(10.5\times 12)+(11\times 17)+\ldots+(13\times 1)}{60}$$

$$= \boxed{11.09}$$

b) We are unable to apply the coding method because the width of the class interval does not equal for all class intervals.

8) $n = \sum f = 60$

$\sum f(x-\bar{x})^2 = 33.245$

$$s = \sqrt{\frac{33.245}{60}} = \boxed{0.744}$$