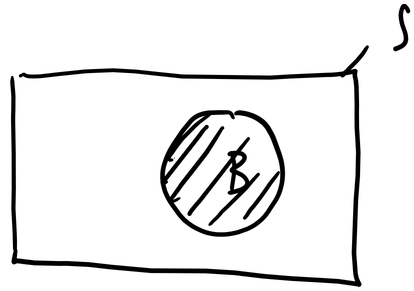
Lecture 2 (Tuesday 9/1/20)

B Sample space S



$B \subset S$

$$P(B) = \frac{|B|}{|S|}$$

$$P(B|S) = \frac{|B|}{|S|} = \frac{|B \cap S|}{|S|} = \frac{P(B \cap S)}{P(S)}$$

$$= \frac{P(B)}{1} = P(B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(A) \times P(B|A)$$

Example: $S = \{1, 2, 3, 4, 5, 6\}$

a) $B = \{1, 2, 3\}$ $\quad P(B) = P(1 \cup 2 \cup 3)$

$$= P(1) + P(2) + P(3)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

b) $A = \{1, 3, 5\} \Rightarrow P(A) = \frac{1}{2}$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/3}{1/2} = \frac{2}{3}$$

$$P(A \cap B) = \frac{|A \cap B|}{|S|} = \frac{2}{6} = \frac{1}{3}$$

$$P(A \cap B) = P(B \cap A)$$

$$P(B|A) \neq P(A|B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

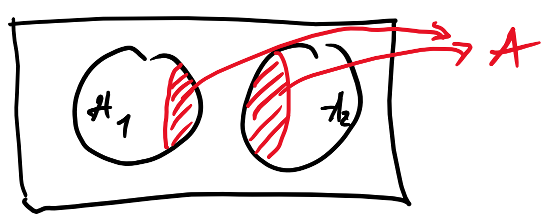
Theorem 1-9:

$$RHS = P(\cancel{A_1}) \cdot \frac{P(\cancel{A_1} \cap A_2)}{P(\cancel{A_1})} \cdot \frac{P(A_1 \cap A_2 \cap A_3)}{P(\cancel{A_1 \cap A_2})}$$

$$= P(A_1 \cap A_2 \cap A_3) = LHS \quad !!!$$

$$P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1) \cdot P(A_2|A_1) \cdot$$
$$\cdot P(A_3 | A_1 \cap A_2)$$
$$\cdot P(A_4 | A_1 \cap A_2 \cap A_3)$$

Theorem 1-10:



$$P(A) = P(A \cap A_1) + P(A \cap A_2) + \dots + P(A \cap A_n)$$
$$= P(A_1) \cdot P(A|A_1) + P(A_2) \cdot P(A|A_2) + \dots +$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$
$$= P(A) \cdot P(B)$$

$$P(A_k|A) = \frac{P(A_k)\,P(A|A_k)}{\sum_{j=1}^{n} P(A_j)\,P(A|A_j)}$$

$$P(A) = \sum_{j=1}^{n} P(A_j) \cdot P(A|A_j)$$

$$P(A_k|A) = \frac{P(A \cap A_k)}{P(A)} = \frac{P(A_k) \cdot P(A|A_k)}{\sum_{j=1}^{n} P(A_j) \cdot P(A|A_j)}$$

In-class exercise:

e) $$Pr(S_1|R_0) = \frac{Pr(R_0|S_1)\,Pr(S_1)}{Pr(R_0)} = \frac{0.1 \times 0.4}{0.58} = \boxed{0.069}$$

f) $$Pr(S_0|R_1) = \frac{Pr(R_1|S_0)\,Pr(S_0)}{Pr(R_1)} = \frac{0.1 \times 0.6}{0.42} = \boxed{0.143}$$

or $$Pr(S_0|R_1) = 1 - Pr(S_1|R_1) = 1 - 0.857 = 0.143$$

g) $\boxed{0.1}$

$$Pr(\text{error}) = Pr(R_0|S_1) \cdot Pr(S_1) + Pr(R_1|S_0) \cdot Pr(S_0) = 0.1 \times 0.4 + 0.1 \times 0.6 = \boxed{0.1}$$

$0, 1$

$a_1 a_2 a_3 \quad 2.2.2 = 2^3$

$\downarrow \downarrow \downarrow$

$2 \; 2 \; 2$

$\underbrace{\{0, 1, 2, 3 \dots 9\}}_{10}$

$|A \cup B| = |A| + |B| + |A \cap B|$

Sum rule : $|A \cup B| = |A| + |B|$

subtraction rule : $|A \cup B| = |A| + |B| + |A \cap B|$

Inclass exercise : $a_1 a_2 a_3$

a) $10^3 = 1000$

Strings contain same digit 3 times : $\underbrace{000, 111, \dots 999}_{10}$

$1000 - 10 = \boxed{990}$

b) $a_1 \; a_2 \; a_3$

{ Position that a digit is not 4 : 3
Way to choose that digit : 9 } $\Rightarrow 3 . 9 = \boxed{27}$

$a_1 \; a_2 \; a_3 \; a_4 \; a_5 \; a_6$

$X \; X \; X \; X \; X \; X$

$\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$

$6 . 5 . 4 . 3 . 2 . 1$

$6!$

$\frac{6!}{4!2!} = 15$

$a_1 \; a_2 \; a_3 \; a_4$

$X \; X \; X \; X$

$4 \; 3 \; 2 \; 1 \nearrow 4!$

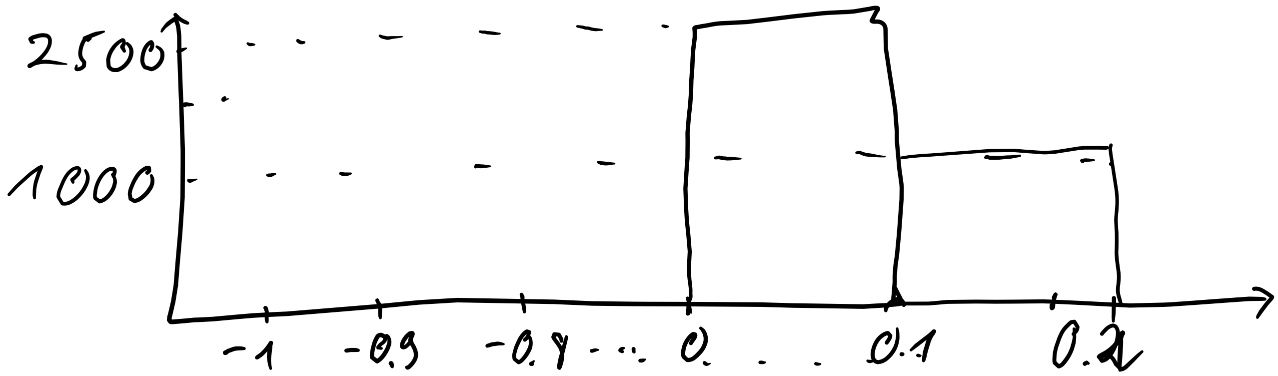
$X \; X$

$2 \; 1 \nearrow 2!$

~~65~~ number btw

0 $[-1, -0.9)$

2 500 number $\in [0, 0.1)$



1^{st} experiment :

2, 8, 9, 10, 12, 7 !

$\boxed{5}$ rolls to see "7" occur.

2^{nd} experiment

3, 9, 5, 7 !

$\boxed{3}$ rolls to see "7" occur.

⋮

100,000 experiment

$d_1 = np.zeros(3,1)$
$= [0 \; 0 \; 0]$

$d_1 = [0 \; 3 \; 0]$

$i = 1$

$d[1,:] = 3$