EE381 homework #3 solution

1) Number of arrangements of 10 people taken 4 at a time: 10*9*8*7=5040
2) a) 336
   b) 15
   c) 6
   d) 35
   e) 1
3) a) Mathematics books arrangement: P(4,4)=4!
   Physics books arrangements: P(6,6)=6!
   Chemistry books arrangements: P(2,2)=2!
   Three kinds of books arrangement: P(3,3)=3!
   Total number of arrangements: 4!*6!*2!*3!=207,360
4) One can select either 1 out of 5 greens, 2 out of 5 greens,…, 5 out of 5 greens. The required number of salads is:

   C(5,1)+C(5,2)+C(5,3)+C(5,4)+C(5,5)=31
5) P(6,6)=6!=720
6) a) C(10,4)=210
   b) C(10,4)+ C(10,3)+ C(10,2)+ C(10,1)+ C(10,0)=386
   c) C(10,4)+ C(10,5)+ C(10,6)+ C(10,7)+ C(10,8)+C(10,9)+ C(10,10)=848
   d) C(10,5)=252
7) a) Total possible outcomes: 2^10=1024.
   b) Outcome has exactly 2 heads: C(10,2)=45.
   c) Outcome that contain at most 3 tails: C(10,3)+ C(10,2)+ C(10,1)+ C(10,0)= 176
8) First position men relative to each other. There are 8 men, so there are P(8,8) ways to do this. This creates nine slots where a woman (but not more than 1 woman) may stand: in front of the first man, between the 1^{st} and 2^{nd} men,…, between the 7^{th} and 8^{th} men, and behind the 8^{th} man. We need to choose 5 of these positions. This can be done in P(9,5) ways.
   The answer is: P(8,8)*P(9,5)=609,638,400.
9) Applying binomial expansion:

$$\left(x^2+\frac{1}{x}\right)^{12} = \sum_{k=0}^{12}\binom{12}{k}(x^2)^k\left(\frac{1}{x}\right)^{12-k} = \sum_{k=0}^{12}\binom{12}{k}x^{3k-12}.$$

   The constant term corresponds to the one for which 3k-12=0, i.e., k=4, and it is:

$$\binom{12}{4} = \frac{12\cdot 11\cdot 10\cdot 9}{4\cdot 3\cdot 2\cdot 1} = 495$$