Lecture 4 (9/15/20)

Sample point	HH	HT	TH	TT
X	2	1	1	0
	$\downarrow$ x_3	$\downarrow$ x_2	$\downarrow$ x_2	$\downarrow$ x_1

$$X = \{\underset{\substack{\downarrow\\ x_1}}{0},\ \underset{\substack{\downarrow\\ x_2}}{1},\ \underset{\substack{\downarrow\\ x_3}}{2}\}$$

$$P(X = \underset{0}{x_1}) = f(x_1)$$

f: probability function of x.

$$P(X = x_4) = 0$$

$$x_4 = 4$$

$$P(X = 0) = f(x_1)$$
$$P(X = 1) = f(x_2)$$
$$P(X = 2) = f(x_3)$$

In-class exercise:

Random variable X: represents selecting a selective course.

X	Art	Geology	Psychology
	$\downarrow$	$\downarrow$	$\downarrow$
	x_1	x_2	x_3

$f(x_1)+f(x_2)+f(x_3)=1$

$$P(Art) = P(Psychology)$$

$$P(Geology) = 2 \cdot P(Art)$$

$$P(Art) + P(Psych.) + P(Geo.) = 1$$

$$\begin{cases} P(Art) = P(Psych.) = \frac{1}{4} \\ P(Geo.) = 2 \cdot \frac{1}{4} = \frac{1}{2} \end{cases}$$

x	x_1	x_2	x_3
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$1/4$

$$F(2) = P(X \leq 2) = P(0) + P(1) + P(2) = 1$$

$$F(1) = P(X \leq 1) = P(0) + P(1)$$

$$X = \{x_1, x_2, x_3, \ldots, x_n\}$$

$x_1 \downarrow$ smallest no.

$x_n \downarrow$ largest no.

x	x_1	x_2	x_3	$\ldots$	x_n
$f(x)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$\ldots$	$f(x_n)$

$$F(x) = P(X \leq x) = \sum_{u \leq x} f(u)$$

$$F(x) = \begin{cases} 0 & -\infty < x < x_1 \\ f(x_1) & x_1 \leq x < x_2 \quad x = x_1 \\ f(x_1) + f(x_2) & x_2 \leq x < x_3 \quad x = x_2 \\ \vdots & \end{cases}$$

$$F(x_1) = P(X \leq x_1) = P(X < x_1) + P(X = x_1)$$

$$\phantom{F(x_1) = P(X \leq x_1) = {}} 0 \qquad + f(x_1)$$

$$= 0 + f(x_1)$$

$$F(x_2) = P(X \leq x_2) =$$

$$= P(X < x_1) + P(X = x_1) + P(X = x_2)$$

$$= 0 + f(x_1) + f(x_2)$$

$$F(2) = P(X \leq 2)$$
$$= P(-\infty < x < 0) + P(x = 0)$$
$$+ \cancel{P(0 < x < 1)}^{\,0} + P(x = 1)$$
$$+ \cancel{P(1 < x < 2)}_{\,0} + P(x = 2)$$

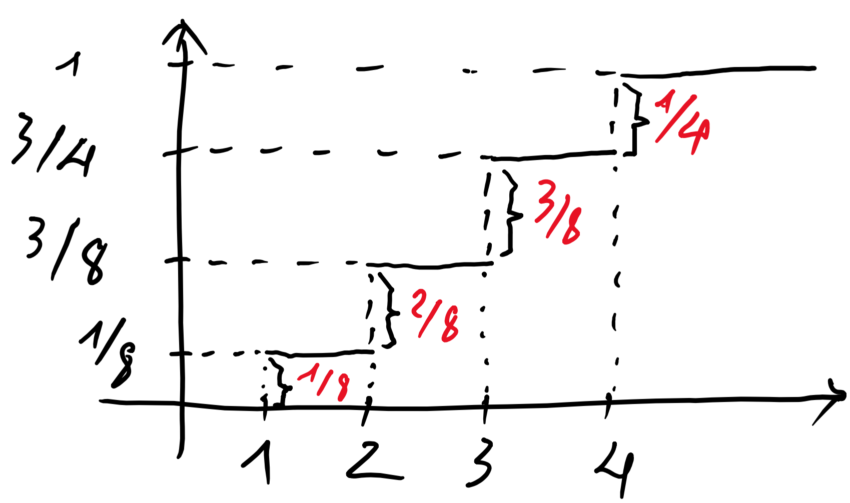
0.5 , 0.6
1.4 , 1.8

x : no. of head coming up.

In-class exercise :

F : cumulative probability function

f : probability function



a)

x	1	2	3	4
$f(x)$	1/8	1/4	3/8	1/4

b) $P(X > 2)$
$= P(3) + P(4)$
$= 3/8 + 1/4$
$= \boxed{5/8}$

Method 2 :

$$P(X > 2) = 1 - P(X \leq 2)$$
$$= 1 - F(2)$$
$$= 1 - 3/8 = \boxed{5/8}$$

$$F(x_2, y_2) = ?$$
$$= P(X \leq x_2, Y \leq y_2)$$
$$= f(x_1, y_1) + f(x_2, y_1)$$
$$+ f(x_1, y_2) + f(x_2, y_2)$$

$$F(x_2, y_1) = P(X \leq x_2, Y \leq y_1)$$
$$= f(x_1, y_1) + f(x_2, y_1)$$

In-class exercise

C : Cancer

S : Smoke

C \ S	0	1
0	40/60	10/60
1	7/60	3/60

a) $P(C=0, S=0) = \frac{40}{60}$

$P(C=1, S=1) = 3/60$

b) $P(\underset{B}{C=1} \mid \underset{A}{S=1}) = \dfrac{P(C=1, S=1)}{P(S=1)}$

$= \dfrac{3/60}{10/60 + 3/60} = \boxed{\dfrac{3}{13}}$

c) $F(C=1, S=0) = P(C \leq 1, S \leq 0)$

$= \frac{40}{60} + \frac{7}{60} = \boxed{\frac{47}{60}}$

$$P(A \cap B) = P(B|A) \cdot P(A)$$

If A & B are independent:

$$P(A \cap B) = P(B) \cdot P(A)$$

In-class exercise:

$$P(C=1, S=1) = 3/60 = 0.05$$

$$P(C=1) = \frac{10}{60}$$

$$P(S=1) = \frac{13}{60}$$

$$\Rightarrow P(C=1, S=1) \neq P(C=1) \times P(S=1)$$

$\Rightarrow$ C & S are dependent!

$$g(u) = P(U=u) = P(\phi(x) = u)$$
$$= P(X = \psi(u)) = f(\psi(u))$$