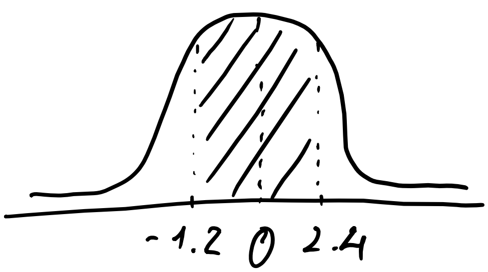
Homework 9 Solution

1) $$Z_1 = \frac{70-\mu}{\sigma} = -0.6 \rightarrow 0.6\sigma - \mu = -70$$

$$Z_2 = \frac{88-\mu}{\sigma} = 1.4 \rightarrow 1.4\sigma + \mu = 88$$

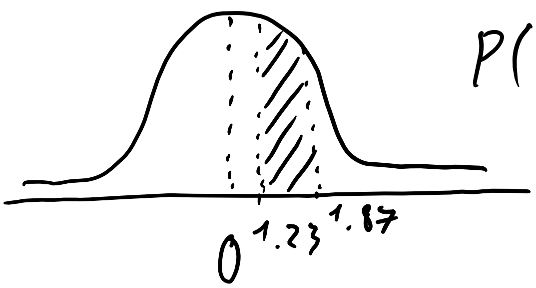
So $\boxed{\sigma = 9}$ and $\boxed{\mu = 75.4}$

2) a)



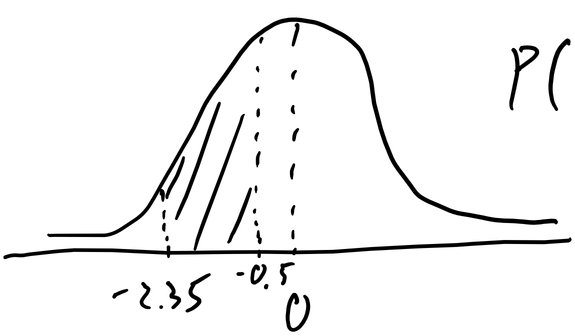
$P(-1.2 < z < 2.4) = \boxed{0.8767}$

b)



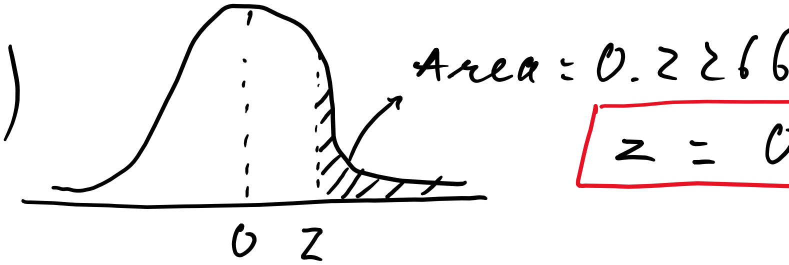
$P(1.23 < z < 1.87) = \boxed{0.0786}$

c)



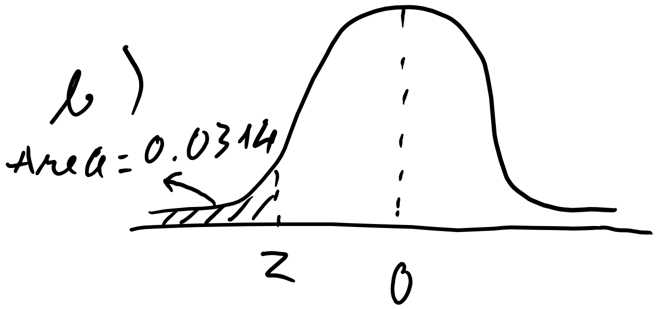
$P(-2.35 < z < -0.5) = \boxed{0.2991}$

3) a)



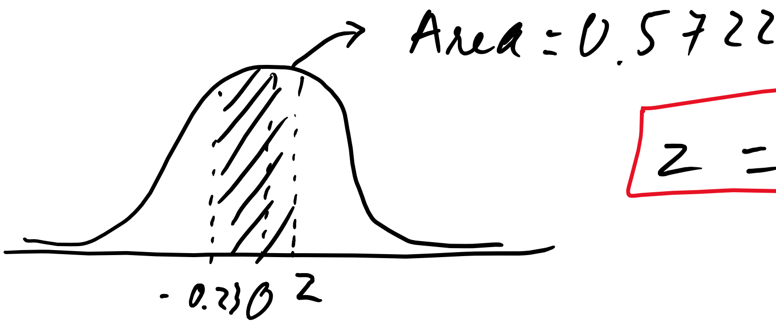
$\boxed{z = 0.75}$

b)



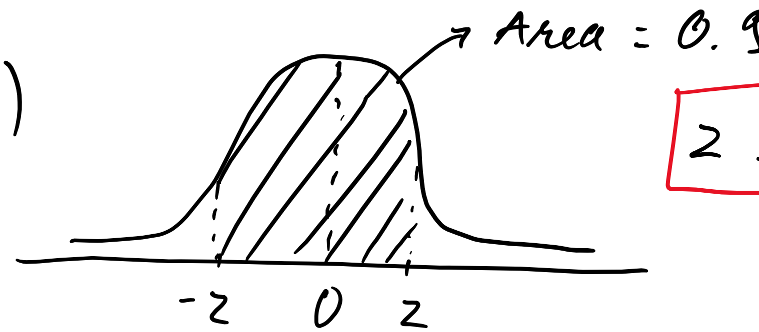
$\boxed{z = -1.86}$

c)



$\boxed{z = 2.08}$

e)



$\boxed{z = \pm 1.65}$

4) $\mu = 0.614$ $\sigma = 0.0025$

a) $0.61 < X < 0.618$

0.61 in standard unit : $\dfrac{0.6095 - 0.614}{0.0025} = -1.8$

0.618 " : $\dfrac{0.6185 - 0.614}{0.0025} = 1.8$

$P(-1.8 < z < 1.8) = 0.4641 + 0.4641 = \boxed{0.9282}$

b) 0.617 in standard unit: $\dfrac{0.6175 - 0.614}{0.0025} = 1.4$

$P(z > 1.4) = 0.5 - 0.4192 = \boxed{0.0808}$

c) $0.608 \rightarrow \dfrac{0.6075 - 0.614}{0.0025} = -2.6$

$P(z < -2.6) = 0.5 - 0.4953 = \boxed{0.0047}$

d) Discrete: Continuous

$X = 0.615$ $\quad 0.6145 < X < 0.6155$

0.6145 in standard unit: $\dfrac{0.6145 - 0.614}{0.0025} = 0.2$

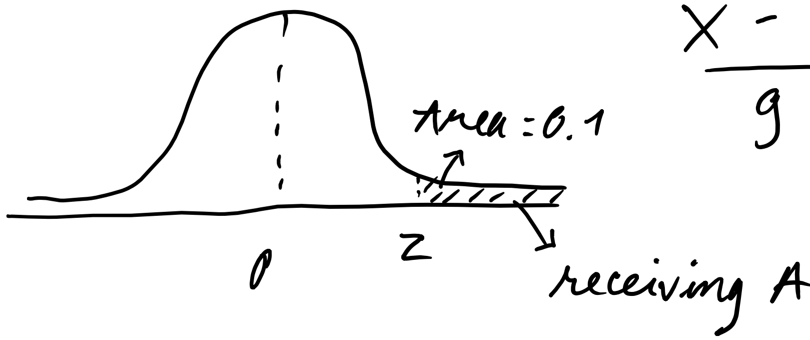
0.6155 " : $\dfrac{0.6155 - 0.614}{0.0025} = 0.6$

$P(0.2 < z < 0.6) = 0.2258 - 0.0793$

$= \boxed{0.1465}$

* Note: Results without applying correction factor are also acceptable in this case.

5) $\mu = 72 \quad \sigma = 9$



$$\frac{X - 72}{9} = z$$

$z = 1.28$

$\Rightarrow \quad X = 9 \times 1.28 + 72 = 83.52$

6) 3% of 100 bulbs : 3 bulbs (defective)

$\lambda = \mu = 3 \qquad \sigma = \sqrt{npq} = \sqrt{100 \times 0.03 \times 0.97}$

$= 1.71$

a) $P(X=0) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{3^0 \times 2.71828^{-3}}{1}$

$= \boxed{0.0498}$

b) $P(X=5) = \dfrac{3^5 \times 2.71828^{-3}}{5!} = \dfrac{243 \times 0.0498}{120}$

$= \boxed{0.1}$

c) $P(X > 5) = 1 - P(X \leq 5)$

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)]$$

$$= \boxed{0.0838}$$

d) $P(1 \leq X \leq 3) = P(X=1) + P(X=2) + P(X=3)$

$$= \boxed{0.5976}$$

7) a)

$$\mu = E(X) = \int_a^b \frac{x}{b-a}\,dx = \frac{x^2}{2(b-a)}\Big|_a^b = \frac{a+b}{2}$$

b) $E(X^2) = \int_a^b \frac{x^2}{b-a}\,dx = \frac{b^2+ab+a^2}{3}$

$$\sigma^2 = E(X^2) - \mu^2 = \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{1}{12}(b-a)^2$$

8) X is uniformly distributed in $-2 < x < 2$

$$f(x) = \begin{cases} \frac{1}{4} & -2 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

a) $$P(X<1) = \int_{-2}^{1} \frac{1}{4}\,dx = \frac{1}{4}x \Big|_{-2}^{1} = \boxed{\frac{3}{4}}$$

b) $$P\left(|X-1| \geqslant \frac{1}{2}\right)$$

$$|X-1| \geqslant \frac{1}{2} \Rightarrow \left[\begin{array}{l} X-1 \leqslant -\frac{1}{2} \\ X-1 \geqslant \frac{1}{2} \end{array}\right. \Rightarrow \left[\begin{array}{l} X \leqslant \frac{1}{2} \\ X \geqslant \frac{3}{2} \end{array}\right.$$

$$P\left(|X-1| \geqslant \frac{1}{2}\right) = P\left(X \leqslant \frac{1}{2}\right) + P\left(X \geqslant \frac{3}{2}\right)$$

$$= \int_{-2}^{1/2} \frac{1}{4}\,dx + \int_{3/2}^{2} \frac{1}{4}\,dx$$

$$= \frac{5}{8} + \frac{1}{8} = \boxed{\frac{3}{4}}$$