EE 351 HW #4

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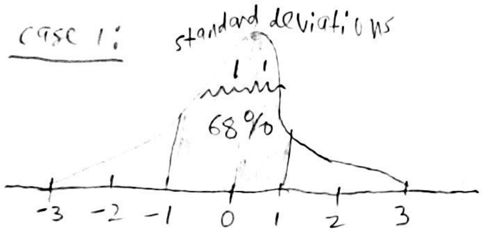
11/1/20

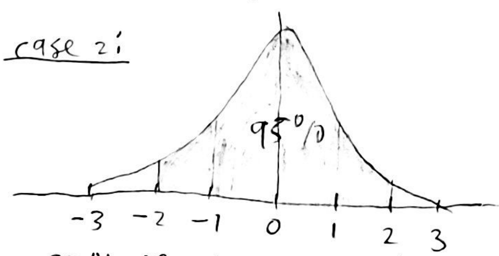
1. Find mean, and standard deviation on an examination in which grades of 70 and 88 correspond to the standard scores of -0.6 and 1.4 respectively

case 1:



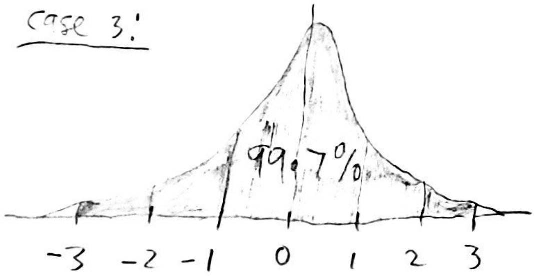
68% of values → 1 std deviation of the mean

case 2:



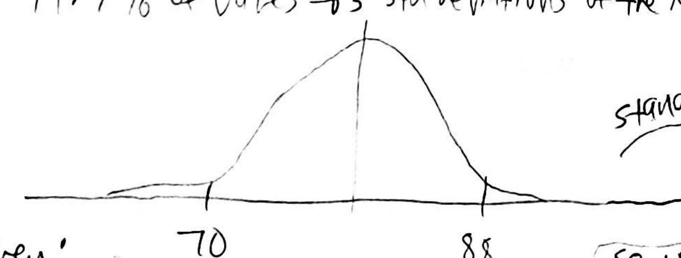
95% of values → 2 std deviations of the mean

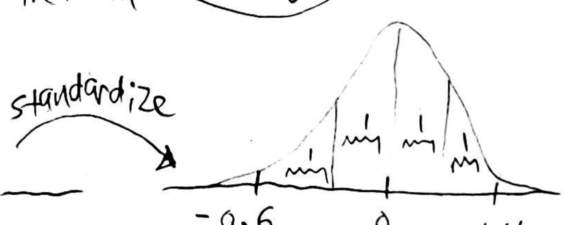
case 3:



99.7% of vales → 3 std deviations of the mean

$$Z = \frac{X - \mu}{\sigma}$$





Given:

Z values are -0.6 and 1.4
X values are 70 and 88

EQ 1: $Z = \frac{X-\mu}{\sigma} = -0.6 = \frac{70-\mu}{\sigma}$

EQ 2: $Z = \frac{X-\mu}{\sigma} = 1.4 = \frac{88-\mu}{\sigma}$

EQ 1:

$\sigma \cdot -0.6 = \frac{70-\mu}{\cancel{\sigma}} \cdot \cancel{\sigma} \rightarrow$

$-0.6\sigma = \cancel{70} - \mu$
$-70 \quad -\cancel{70}$

$\rightarrow -1 \cdot (-0.6\sigma - 70) = (-\mu) \cdot (-1)$

$= \boxed{\mu = 0.6\sigma + 70}$

Plug into EQ 2: $1.4 = \frac{88 - (0.6\sigma + 70)}{\sigma}$

1. (continued!)

plug into EQ 2:

$\sigma \cdot 1.4 = \frac{88 - 0.6\sigma - 70}{\cancel{\sigma}} \cdot \cancel{\sigma}$

$1.4\sigma = 88 - 0.6\sigma - 70$

$= 1.4\sigma = 18 - 0.6\sigma$
$+0.6\sigma \quad +0.6\sigma$

$\frac{2\sigma}{2} = \frac{18}{2}$; $\boxed{\sigma = 9 = \text{std deviation}}$

$\boxed{\mu = 75.4 = \text{mean} \quad \sigma = 9 = \text{std dev}}$

plug in
EQ 1: $-0.6 = \frac{70 - \mu}{\sigma}$

// plug in $\sigma = 9$

$-0.6 = \frac{70 - \mu}{(9)}$

$9 \cdot -0.6 = \frac{70 - \mu}{\cancel{9}} \cdot \cancel{9}$

$-5.4 = 70 - \mu$
$-70 \quad -70$

$\frac{-75.4}{-1} = \frac{-\mu}{-1}$; $\boxed{\mu = 75.4 = \text{mean}}$

check: $z = \frac{X - \mu}{\sigma}$

EQ 1: $-0.6 = \frac{70 - 75.4}{9}$ ✓

$-0.6 = \frac{-5.4}{9}$

$-0.6 = -0.6$ ✓

EQ 2: $1.4 = \frac{88 - 75.4}{9}$ ✓

$1.4 = \frac{12.6}{9}$

$1.4 = 1.4$ ✓

plug in
EQ 2: $1.4 = \frac{88 - \mu}{\sigma}$

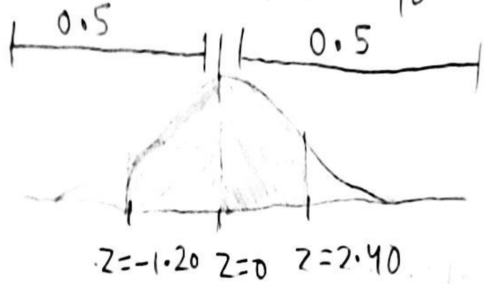
$9 \cdot 1.4 = \frac{88 - \mu}{\cancel{9}} \cdot \cancel{9}$

$12.6 = 88 - \mu$
$-88 \quad -88$

$\frac{-75.4}{-1} = \frac{-\mu}{-1}$; $\boxed{\mu = 75.4 = \text{mean}}$

Find the area under the normal curve between

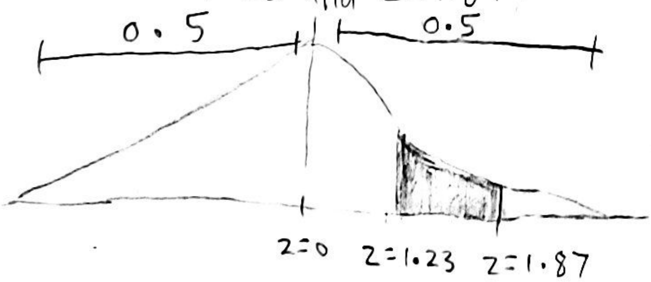
a) $z=-1.20$ and $z=2.40$



area btw $z=-1.20$ and $z=0$ = $z=1.20=0.3849$
area btw $z=0$ and $z=2.40=0.4918$

area = (area btw $z=-1.20$ and $z=0$) + (area btw $z=0$ and $z=2.40$)
$= 0.3849+0.4918 = \boxed{0.8767}$

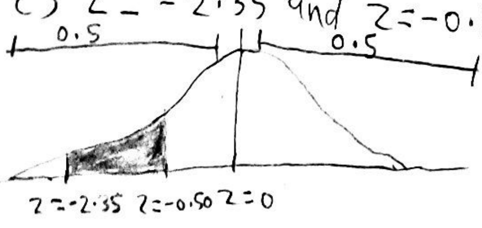
b) $z=1.23$ and $z=1.87$



area btw $z=0$ and $z=1.23=0.3907$
area btw $z=0$ and $z=1.87=0.4693$

area = (area btw $z=0$ and $z=1.87$ - area btw $z=0$ and $z=1.23$
$= 0.4693-0.3907 = \boxed{0.0786}$

c) $z=-2.35$ and $z=-0.50$

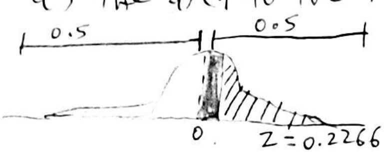


area btw $z=-0.50$ and $z=0 = z=0.50=0.191$
area btw $z=-2.35$ and $z=0= z=2.35=0.49$

area = (area btw $z=-2.35$ and $z=0$ -
area btw $z=-0.50$ and $z=0$) $= 0.4906-0.1915$
$\boxed{=0.2991}$

3. Find the values of z such that

a) The area to the right of z is 0.2266



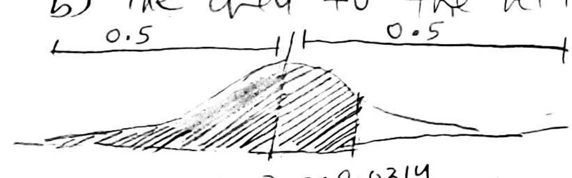
Is area = 0.2266 > 0.5, X NO

Area btw 0 and z:

0.5 - 0.2266 = 0.2734

$\boxed{x = z = 0.2734 = 0.75}$

b) The area to the left of 0.0314



Area btw 0 and z:

Is area = 0.0314 > 0.5, X NO

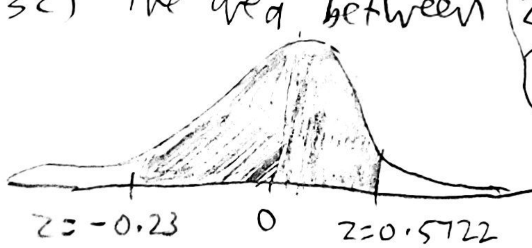
0.5 - 0.0314 = 0.4686

$\boxed{x = z = 0.4686 = -1.86}$

3c) the area between z = -0.23 and z is 0.5722 = P(-0.23 < z < x)

area btw z = -0.23 and z = 0 = z = 0.23

= 0.0910 = P(-0.23 < z)



P(-0.23 < z < x) = 0.5722

EQ: P(z < x) + P(-0.23 < z) = 0.5722

P(z < x) + ~~0.0910~~ = 0.5722 $\boxed{z = 2.08}$

~~-0.0910~~ -0.0910

$\boxed{0.4812 = P(z < x)}$

The area between -z and z is 0.9

$P(-z<Z<z) = 0.9$

$= \underbrace{P(-z<-Z) + P(Z<z)}_{p} = 0.9$

$= \frac{\cancel{2}\cdot p}{\cancel{2}} = \frac{0.9}{2}$; $p = 0.45$

$z = 1.64$; $\boxed{z = -1.64, 1.64}$

4. If the diameters of the ball bearings are normally distributed, with mean 0.6140 inches and standard deviation 0.0025 inches, determine the percentage of ball bearings

Given: $\mu = 0.6140$ inches, $\sigma = 0.0025$ inches

Z-Score Formula: $Z = \dfrac{X - \mu}{\sigma}$

a) Between 0.610 and 0.618 inches

X values: 0.610 and 0.618 // Finding Z values

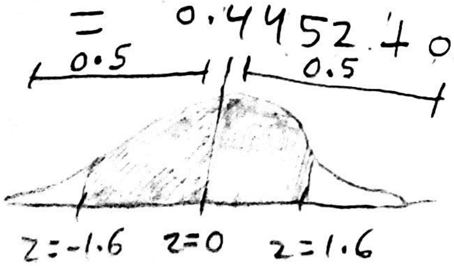
1st Z-value: $Z = \dfrac{0.610 - 0.6140}{0.0025} = -1.6 = Z$

2nd Z-value: $Z = \dfrac{0.618 - 0.6140}{0.0025} = 1.6 = Z$

$= P(0.610 \leq X \leq 0.618) = P(-1.6 \leq Z \leq 1.6)$

$=$ (area between $Z = -1.6$ and $Z = 0$) + (area between $Z = 0$ and $Z = 1.6$)

$= 0.4452 + 0.4452 = \boxed{0.8904 = 89.04\%}$



Given: $\mu = 0.6140$ inches, $\sigma = 0.0025$ inches

(continued!)

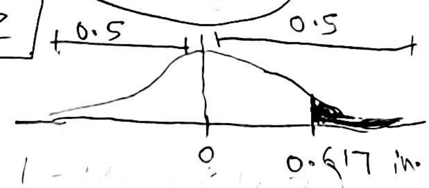
b) greater than 0.617 inches

$X = 0.617$ inches

$$z = \frac{0.617 - 0.6140}{0.0025} = \boxed{1.2 = z}$$

$$z = \frac{x - \mu}{\sigma}$$

= 0.617 in standard



$P(0 \leq z \leq \infty) - P(0 \leq z \leq 1.2)$

$= 0.5 -$ (area btw $z=0$ and $z=1.2$)

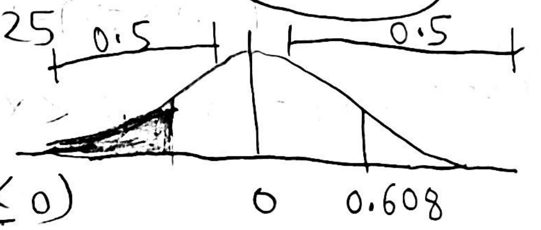
$= 0.5 - 0.3849 = 0.1151 = \boxed{11.51\%}$

$z = 1.2 = 0.3849$

c) less than 0.608 inches

$X = 0.608$

$$0.608 \text{ in standard} = z = \frac{0.608 - 0.6140}{0.0025} = -2.4 = z$$



$P(X < 0.68) = P(z < -2.4)$

$= P(-\infty \leq z \leq 0) - P(-2.4 \leq z \leq 0)$

$= 0.5 -$ (area btw $z=0$ and $z=-2.4$)

$= 0.5 - 0.4918 = 0.0082$

$= \boxed{0.82\%}$

$z = -2.4 =$
$z = 2.4 = 0.4918$

$\mu = 0.6140$ inches, $\sigma = 0.0025$

4.d) $P(X=0.615)$ $\quad z = \frac{x-\mu}{\sigma}$

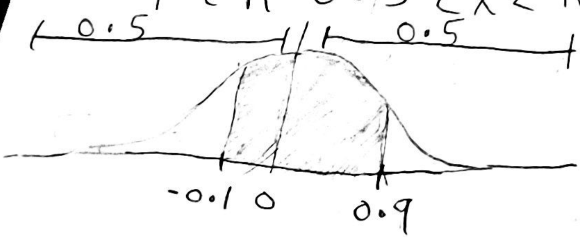
x value $= 0.615$

$$z = \frac{0.615 - 0.6140}{0.0025} = \boxed{0.4 = z}$$

$= 0.615$ in standard

$P(X=0.615) = P(z=0.4) =$

$P(n-0.5 < x < n+0.5) = P(-0.1 < x < 0.9)$



$z = -0.1 = z = 0.1 = 0.0398$
$z = 0.9 = 0.3159$

$P(-0.1 < X < 0.9) =$ (area btw $z=-0.1$ and $z=0$) $+$
(area btw $z=0$ and $z=0.9$) $= 0.0398 + 0.3159$
$= 0.3557 = \boxed{35.57\%}$

The mean grade on a final eximination was 72 and standard deviation was 9, The top 10% of the students are to recieve A's. what is the minimum grade a student must get to recieve an A?

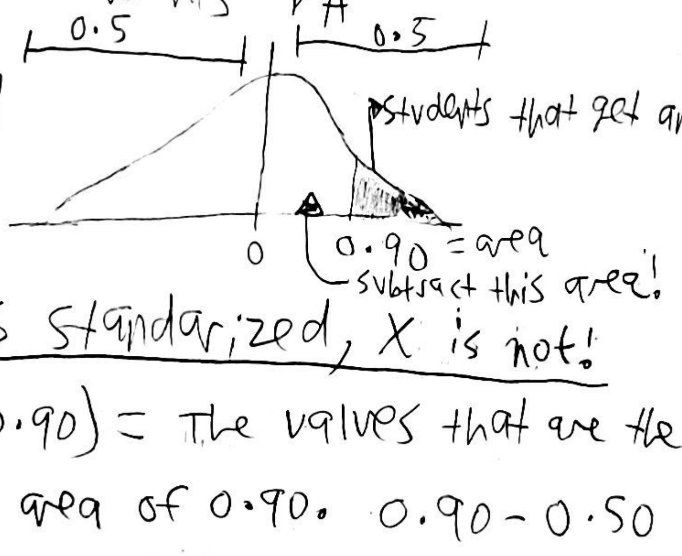
Given:

$\mu = 72$

$\sigma = 9$

TOP 10% of students → A

$$\boxed{Z = \frac{X - \mu}{\sigma}}$$



NOTE: z is standarized, X is not!

$P(0 < X < 0.90)$ = The values that are the closest to make the area of 0.90. 0.90 − 0.50 = 0.40.

which area is closest to 0.40.

Possible values: [.3944, .3962, .3980, .3997, ~~.4015~~]

0.3997 = X → Z = 1.28

$$9 \cdot 1.28 = \frac{X - 72}{\cancel{9}} \cdot \cancel{9}$$

$$\begin{array}{r} 11.52 \\ +72.00 \\ \hline 83.52 \end{array} \quad \begin{array}{l} = X - \cancel{72} \\ \quad\ + \cancel{72} \\ \hline = X \end{array}$$

; X = 83.52 % least in order to get an A.

6. If 3% of the electric bulbs manufactured by company are defective, find the probability that in a sample of 100 bulbs, (a) 0, (b) 5, (c) more than 5, (d) between 1 and 3 are defective.

// Poisson Distribution:

X: a random variable takes in the values 0, 1, 2, ..., etc.

Probability function: $f(x) = P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$

λ: given mean μ of X

e is 2.71828 : base of natural log system

Mean	$\mu = \lambda$
Variance	$\sigma^2 = \lambda$
Std dev.	$\sigma = \sqrt{\lambda}$
MTG	$M(t) = e^{\lambda(e^t - 1)}$

a) 0 bulbs defective

given: $\lambda = 3$ % of elec. bulbs defective

$x = 0$; $e = 2.71828$

$$f(0) = P(X = 0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^0 e^{-3}}{0!} \approx \boxed{0.04979}$$

(continued!)

b) 5 bulbs defective

Given: $\lambda = 3\%$ of elec. bulbs defective

$x = 5$; $e = 2.71828$

$$f(5) = P(X=5) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^5 e^{-3}}{5!} = \boxed{0.1008}$$

c) More than 5 bolts are defective

Given: we have a total of 100 bolts. we are not gonna add up all 100 of them. we will use the poisson distribution

$$\cancel{0}\ \cancel{1}\ \cancel{2}\ \cancel{3}\ \cancel{4}\ \cancel{5}\ \underbrace{6\ 7\ 8\ \dots\ 100}_{>5}$$

Range: $(5, 100]$

5 is exclusive

Given: $\lambda = 3\%$ of elec. bulbs def., $e = 2.71828$

$$f(0) = P(X=0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^0 e^{-3}}{0!} = 0.0498$$

$$f(1) = P(X=1) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^1 e^{-3}}{1!} = 0.1494$$

$$f(2) = P(X=2) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^2 e^{-3}}{2!} = 0.2240$$

$$f(3) = P(X=3) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^3 e^{-3}}{3!} = 0.2240$$

6. (continued!)

c) (continued!)

$f(4) = P(X=4) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^4 e^{-3}}{4!} = 0.1680$

$f(5) = P(X=5) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^5 e^{-3}}{5!} = 0.1008$

$= f(0) + f(1) + f(2) + f(3) + f(4) + f(5)$

$= 0.04979 + 0.1494 + 0.2240 + 0.2240 + 0.1680 + 0.1008 = 0.91599 = P(0 \leq X \leq 5)$

$1 - P(0 \leq X \leq 5) = 1 - 0.916 = \boxed{0.084 = P(X > 5)}$

d) Between 1 and 3: [1, 3]; Range; Inclusive

$f(1) = P(X=1) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^1 e^{-3}}{1!} = 0.1497$

$f(2) = P(X=2) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^2 e^{-3}}{2!} = 0.2240$

$f(3) = P(X=3) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^3 e^{-3}}{3!} = 0.2240$

$P(1 \leq X \leq 3) = 0.1497 + 0.2240 + 0.2240 = \boxed{0.5977}$

Show that the mean and variance of the <u>uniform distribution</u> are given respectively by:

<u>Uniform distribution:</u> A random variable X is said to be uniformly distributed in $a \le x \le b$ if its density function is

lower lim. upper lim.

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

continuous

a) $\mu = \frac{1}{2}(a+b)$

$\mu = E(X)$

$$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\sigma^2 = (E-\mu)^2$$

$$E(X) = \mu = \int_a^b x \cdot \left(\frac{1}{b-a}\right) dx$$

constant

$$\mu = \int_a^b x \cdot \boxed{\left(\frac{1}{b-a}\right)} dx$$

$$\mu = \left(\frac{1}{b-a}\right) \int_a^b x^1\, dx = \frac{x^{1+1}}{1+1} = \frac{x^2}{2}\Big|_a^b =$$

$$\mu = \left(\frac{1}{b-a}\right) \cdot \frac{x^2}{2}\Big|_a^b = \left[\frac{(b)^2}{2} - \frac{(a)^2}{2}\right] \cdot \left(\frac{1}{b-a}\right)$$

$\boxed{\text{Difference of sqs: } (b^2-a^2) = (b-a)(b+a)}$

$$\mu = \frac{b^2}{2} - \frac{a^2}{2} \cdot \left(\frac{1}{b-a}\right) = \mu = \frac{b^2-a^2}{2}\left(\frac{1}{b-a}\right) = \mu = \frac{1}{2}(b^2-a^2)\left(\frac{1}{b-a}\right)$$

$$= \mu = \frac{1}{2}(b+a)\cancel{(b-a)}\left(\frac{1}{\cancel{b-a}}\right) = \boxed{\mu = \frac{1}{2}(b+a)} \checkmark$$

7. (continued!)

$$\sigma^2 = \frac{1}{12}(b-a)^2$$

$$\sigma^2 = \text{variance} = E[(X-\mu)^2] = E(X^2) - E(X)^2$$

$$\mu = \frac{1}{2}(b+a) = E(X) \text{ from part A}$$

$$\sigma^2 = E(x^2) - \left(\frac{1}{2}(b+a)\right)^2$$

Density Function: $f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_a^b x^2 \cdot \underbrace{\left(\frac{1}{b-a}\right)}_{\text{constant}} dx$$

$$= \frac{1}{b-a}\int_a^b x^2\,dx = \frac{1}{b-a}\cdot\frac{x^{2+1}}{2+1}\Big|_a^b = \frac{1}{b-a}\cdot\frac{x^3}{3}\Big|_a^b$$

$$= \frac{1}{b-a}\left[\frac{b^3}{3} - \frac{a^3}{3}\right] = E(x^2)$$

$$= \frac{1}{b-a}\left[\frac{b^3-a^3}{3}\right] - \left(\frac{1}{2}(b+a)\right)^2 \qquad = \boxed{\frac{1}{12}(b-a)^2}$$

$$= \frac{1}{b-a}\left[\frac{1}{3}(b^3-a^3)\right] - \frac{1}{4}(b+a)^2$$

cubes: $b^3 - a^3 = (b-a)(b^2+a^2+ab)$

$$= \frac{1}{\cancel{b-a}}\left[\frac{1}{3}\cancel{(b-a)}(a^2+ab+b^2)\right] - \frac{1}{4}(b^2+2ab+a^2)$$

$$= \frac{1}{3}(a^2+ab+b^2) - \frac{1}{4}(b^2+2ab+a^2)$$

$$= \frac{4}{4}\cdot\left[\frac{1}{3}a^2 + \frac{1}{3}ab + \frac{1}{3}b^2\right] - \frac{3}{3}\cdot\left[\frac{1}{4}b^2 + \frac{2}{4}ab + \frac{1}{4}a^2\right]$$

$$= \frac{4}{12}a^2 + \frac{4}{12}ab + \frac{4}{12}b^2 - \frac{3}{12}b^2 - \frac{6}{12}ab - \frac{3}{12}a^2 = \frac{1}{12}b^2 - \frac{2}{12}ab - \frac{1}{12}a^2$$

$$= \frac{1}{12}(b^2 - ab + a^2)$$

5. Let X be uniformly distributed in $-2 \le x \le 2$.

a b

Find

a) $P(X<1)$

b) $P(|X-1| \ge \frac{1}{2})$

Distribution Function of uniform Distribution:

$$F(x) = P(X \le x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x < b \\ 1 & x \ge b \end{cases}$$

a) $P(X<1)$

Given: $-2 \le x \le 2$

$x = 1, a = -2, b = 2$

$$P(X<1) = F(X \le 1) = \frac{x-a}{b-a} = \frac{1-(-2)}{2-(-2)} = \frac{1+2}{2+2} = \boxed{\frac{3}{4} = P(X<1)}$$

8. (continued!) Range: $-2 \le x \le 2$

b) $P(|X-1| \ge \frac{1}{2})$

$$|X-1| \ge \frac{1}{2}$$

$$X-1 \ge \frac{1}{2} \qquad X-1 \le \frac{1}{2}$$

$$= P(|X-1| \ge \frac{1}{2}) = P(X-1 \ge \frac{1}{2}) + P(X-1 \le \frac{1}{2})$$

$$P(X-\cancel{1} \ge \frac{1}{2}) + P(X-1 \le \frac{1}{2})$$
$$+1 \quad +1 \qquad\qquad +1 \quad +1$$

$$\rightarrow \underbrace{P(X \ge \frac{3}{2})} + P(X \le 3/2)$$

// like problem 6, to find the bolts greater than 6 via poisson distribution,
we won't add $P(X=6) + P(X=7) \ldots + P(X=100)$;
instead, we took $P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$ and subtracted it from 1.

$$= 1 - P(X < \frac{3}{2}) + P(X \le \frac{3}{2})$$
$$= \boxed{1}$$