EE381 Homework #4 Solution

1) We have:

$$\sum_{n=0}^{\infty} f(x_n) = \sum_{n=0}^{\infty} (1-r)^n \times r$$

$$= \frac{r}{1-(1-r)} = 1$$

$\Rightarrow f(x)$ is the probability function of S.

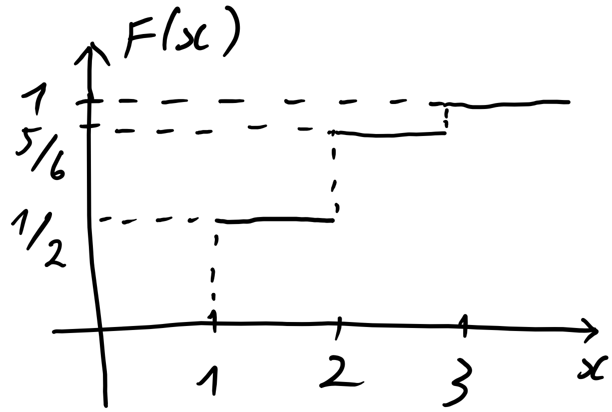
2) a & b)

$C(4,2) \times C(48,2)$ $\quad$ $C(4,3) \times 48$

x	0	1	2	3	4
$f(x)$	$\frac{194{,}580}{270{,}725}$	$\frac{69{,}184}{270{,}725}$	$\frac{6768}{270{,}725}$	$\frac{192}{270{,}725}$	$\frac{1}{270{,}725}$
$F(x)$	$\frac{194{,}580}{270{,}725}$	$\frac{263{,}764}{270{,}725}$	$\frac{270{,}532}{270{,}725}$	$\frac{270{,}724}{270{,}725}$	1

3)

x	1	2	3
$f(x)$	$1/2$	$1/3$	$1/6$
$F(x)$	$1/2$	$5/6$	1



4) a)

x	1	2	3	4
$F(x)$	1/8	3/8	3/4	1
$f(x)$	1/8	1/4	3/8	1/4

b) $P(1 \leq X \leq 3) = P(1) + P(2) + P(3)$
$= 1/8 + 1/4 + 3/8$
$= \boxed{3/4}$

c) $P(X \geq 2) = P(2) + P(3) + P(4)$
$= 1/4 + 3/8 + 1/4$
$= \boxed{7/8}$

d) $P(X < 3) = P(1) + P(2)$
$= 1/8 + 1/4$
$= \boxed{3/8}$

e) $P(X > 1.4) = 1 - P(X \leq 1.4)$
$= 1 - P(X \leq 1)$
$= 1 - F(1)$
$= 1 - 1/8 = \boxed{\frac{7}{8}}$

5) $f(x,y) = cxy$

$x \backslash y$	1	2	3
1	c	$2c$	$3c$
2	$2c$	$4c$	$6c$
3	$3c$	$6c$	$9c$

a) $36c = 1 \Rightarrow c = \boxed{1/36}$

b) $P(X=2, Y=3) = 6c = \frac{6}{36} = \boxed{\frac{1}{6}}$

c) $P(1 \leq X \leq 2, Y \leq 2)$

$= c + 2c + 2c + 4c$

$= 9c = 9/36 = \boxed{\frac{1}{4}}$

d) $P(X \geq 2) = 2c + 4c + 6c + 3c$
$+ 6c + 9c$

$= 30c = 30/36 = \boxed{\frac{5}{6}}$

e) $P(Y=3) = 3c + 6c + 9c = 18c = \frac{18}{36}$

$= \boxed{\frac{1}{2}}$

6) a) $\left.\begin{aligned} f_1(X=1) &= 6c \\ f_1(X=2) &= 12c \\ f_1(X=3) &= 18c \end{aligned}\right\} \Rightarrow \boxed{f_1(X) = 6xc}$

$\left.\begin{aligned} f_2(Y=1) &= 6c \\ f_2(Y=2) &= 12c \\ f_2(Y=3) &= 18c \end{aligned}\right\} \Rightarrow \boxed{f_2(Y) = 6yc}$

b) $P(X=2, Y=3) = \frac{1}{6}$

$P(X=2) = \frac{12}{36} = \frac{1}{3}$

$P(Y=3) = \frac{18}{36} = \frac{1}{2}$

$\Rightarrow \quad P(X=2, Y=3) \neq P(X=2) \cdot P(Y=3)$

$\Rightarrow$ X & Y are independent.

7) a) $f(X|Y) = \dfrac{f(x,y)}{f_2(Y)} = \dfrac{cxy}{6yc} = \boxed{\dfrac{x}{6}}$

b) $f(Y|X) = \dfrac{f(x,y)}{f_1(X)} = \dfrac{cxy}{6xc} = \boxed{\dfrac{y}{6}}$