Lecture 7 (10/13/20)

A : event that head will occur when tossing the coin.

p : probability that head occurs
(probability of success)

q : " tail occurs
(" failure)

$q = 1 - p$

$$f(x) = \binom{n}{x} \cdot p^{x} q^{n-x}$$

x → no. of time the event occurs

n → no. of trials.

$$\begin{cases} f(x) \geq 0 \\ \sum f(x) = 1 \end{cases}$$

✓ ✓

$p + q = 1$

$$\sum f(x) = \sum_{x=0}^{n} \binom{n}{x} \cdot p^{x} q^{n-x}$$

$$= q^n + \binom{n}{1} q^{n-1} p + \binom{n}{2} q^{n-2} p^2 + \dots + p^n$$

$$= (q+p)^n = 1^n = 1$$

In-class exercise:

$p = 10^{-3}$: probability of getting error when 1 bit (prob. of success)

$q = 1 - 10^{-3}$: prob. of failure

a) $P(X=1) = \binom{\overset{n}{32}}{\underset{x}{1}} \cdot p^{x} \cdot q^{n-x}$

$= \binom{32}{1} \cdot (10^{-3})^{1} \cdot (1-10^{-3})^{31}$

$= 32 \cdot 10^{-3} \cdot 0.9695$

$= 0.031$

b) $P(X=0) = \binom{32}{0} \cdot (10^{-3})^{0} \cdot (1-10^{-3})^{32}$

$= 0.9685$

c) $P(X \geq 1) = 1 - P(X=0)$

$= 1 - 0.9685$

$= 0.0315$

$\{3, 4, 1, 2, 3, 6, 5,$

$1, 4, 5\}$

$\frac{2}{10} = \frac{1}{5}$

A_1 : event that 3's occurs

A_2 : " 4 "

33444 $\quad X_1 = 2$

34443 $\quad X_2 = 3$

34344

⋮

In-class exercise:

$n = 20$

$p_0 = 0.4$

$p_1 = 0.3$

$\vdots$

$X_0 = 4 \qquad X_2 = 6$

$X_1 = 8 \qquad X_3 = 2$

$$P(X_0 = \underline{4}, X_1 = \underline{8}, X_2 = \underline{6}, X_3 = \underline{2})$$

$$= \frac{20!}{4!\,8!\,6!\,2!} \times 0.4^4 \times 0.3^8 \times 0.2^6 \times 0.1^2$$

$$= 0.00187$$

prob. of choosing blue ball

$$P(X = x) = \binom{n}{x} \left(\frac{b}{b+r}\right)^x \left(\frac{r}{b+r}\right)^{n-x}$$

prob. of choosing red ball

$$= \binom{n}{x} \frac{b^x r^{n-x}}{(b+r)^n}$$

$b = 3$: three blues

$r = 4$: four reds

$x = 2$: choose exact 2 blues
in 6 selection

n

$$\frac{C(3,2) \times C(4,4)}{C(7,6)}$$

$x = \max(0, 2) = 2$ ✓

✓

$\min(6, 3) = 3$

$x = 1$? → 1 blue
5 reds ✗

$x = 4$ → ✗

$x = 5$? → ✗

$x = 0$? → ✗

Theorem 4-1 :

Chebyshev's inequality :

$$P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$$

If X is binomially distributed :

$$\mu = np \qquad \sigma = \sqrt{npq}$$

$$P(|X-np| \geq k\sqrt{npq}) \leq \frac{1}{k^2}$$

$$\Rightarrow P\left(\left|\frac{X}{n}-p\right| \geq k\sqrt{\frac{pq}{n}}\right) \leq \frac{1}{k^2}$$

If we let $\varepsilon = k\sqrt{\frac{pq}{n}}$, then : $\rightarrow k^2$

$$P\left(\left|\frac{X}{n}-p\right| \geq \varepsilon\right) \leq \frac{pq}{n\varepsilon^2}$$

$n \to \infty$

$$\lim_{n\to\infty} P\left(\left|\frac{X}{n}-p\right| \geq \varepsilon\right) \approx 0$$