EE381 Homework #8

1) A random variable X has density function:

$$f(x) = \begin{cases} ce^{-3x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

Find

a) The constant *c*
b) P(1< X <2)
c) $P(X \geq 3)$
d) P(X < 1)

2) A random variable X has density function

$$f(x) = \begin{cases} cx^2 & 1 \le x \le 2 \\ cx & 2 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find

a) The constant *c*
b) P(X > 2)
c) P(1/2 < X < 3/2)

3) Can the function:

$$F(x) = \begin{cases} c(1 - x^2) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

be a distribution function? Explain.

4) Let X and Y be continuous random variables having joint density function

$$f(x, y) = \begin{cases} c(x^2 + y^2) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine

a) The constant c
b) $P(X < \frac{1}{2}, Y > \frac{1}{2})$
c) $P(\frac{1}{4} < X < \frac{3}{4})$
d) $P(Y < \frac{1}{2})$
e) Whether X and Y are independent

5) Let X have density function

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

Find the density function of $Y = X^2$

6) Let X and Y have joint density function:

$$f(x,y) = \begin{cases} e^{-(x+y)} & x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

If $U = \frac{X}{Y}$ and $V = X + Y$, find the joint density function of U and V.

7) Let

$$f(x,y) = \begin{cases} x + y & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the conditional density function of

a) X given Y
b) Y given X

8) Let X be a random variable having density function

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find

a) $E(X)$, $E(X^2)$ and $E[(X-1)^2]$
b) $Var(X)$ and σ_X

9) Let X be a random variable having density function

$$f(x) = \begin{cases} x/2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the moment generating function of random variable X
b) Find the first four moments about the origin.

10) Let X and Y be random variables having joint density function

$$f(x,y) = \begin{cases} x + y & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find

a) $Var(X)$ and $Var(Y)$
b) σ_X and σ_Y
c) σ_{XY}
d) ρ

Note: Your answers should show your step-by-step work. Answers which have only final results are not accepted.