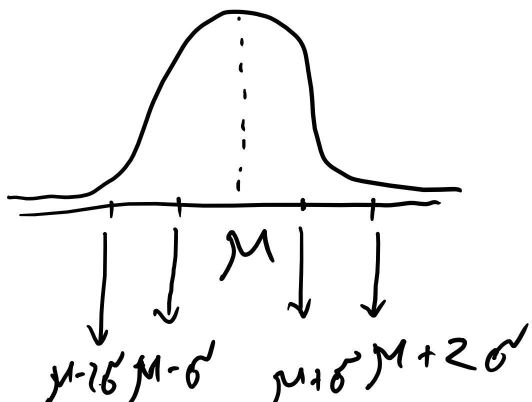
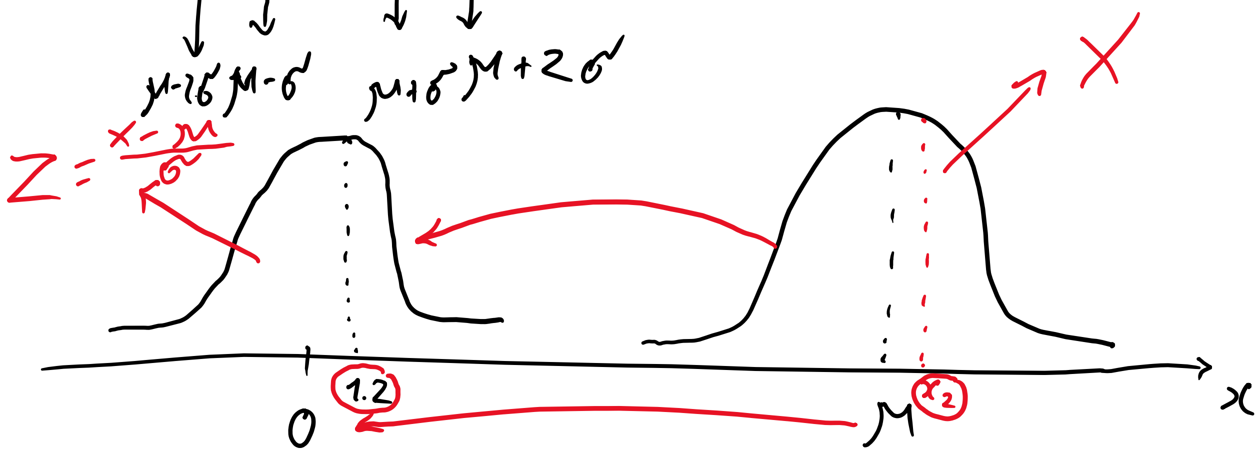
Lecture 9 note (10/27/20)

$X = \{x_1, x_2 \ldots\ldots x_n\}$ $\quad \mu = 1 \quad \sigma = 1.5$

$P(\mu \leq X \leq x_2)$





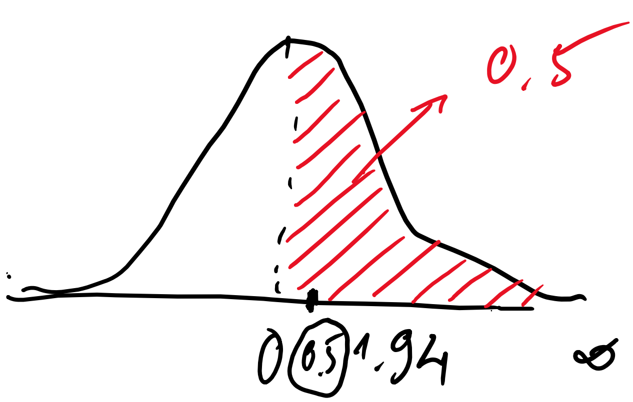
$\mu_Z = 0$

$\sigma_Z^2 = 1$

$f(x) = e^x$

$$E(X) = \int x f(x)\, dx$$

$$\int x e^x \, dx$$

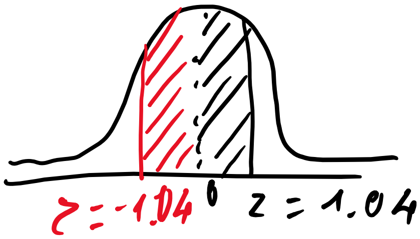


$P(1.94 \leq X)$

$0.5 - P(0 \leq X \leq 1.94)$

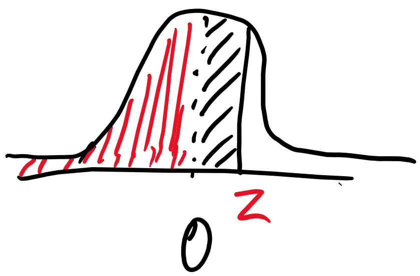
In-class exercise:

a) Area = 0.35 < 0.5



$\boxed{z = \pm 1.04}$

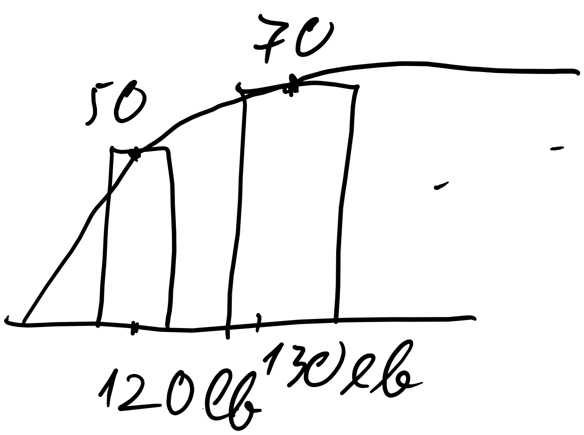
b) Area = 0.8665 > 0.5

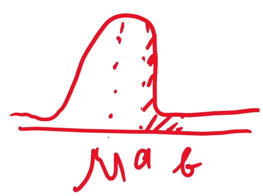


Area btw 0 & z:

0.8665 − 0.5 = 0.3665

→ $\boxed{z = 1.11}$





$P(117.5 < X < 118.5)$

z-score: $z_1 = \frac{117.5 - \mu}{\sigma} = a$

z-score: $z_2 = \frac{118.5 - \mu}{\sigma} = b$

In-class exercise:

$P(X < 2) = P(X = 0) + P(X = 1)$

$= 0.406$

Binomial distribution:

n is large ($n \geq 50$)

$p \approx 0 \qquad q \approx 1$

$\mu = np \qquad \sigma^2 = np\underbrace{q}_{\approx 1} \qquad \sigma = \sqrt{npq}$

$\mu = np \qquad \sigma^2 = np = \mu \qquad \sigma = \sqrt{np} = \sqrt{\mu}$

Poisson distribution

$$Z = \frac{X - \mu}{\sigma}$$

X is Poisson R.V $\Rightarrow \mu_X = \lambda, \; \sigma = \sqrt{\lambda}$

Central limit theorem:

$X_1, X_2, \ldots X_n.$

$\downarrow \quad \downarrow \quad \downarrow$

$\mu \quad \mu \quad \mu$

$\sigma^2 \quad \sigma^2 \quad \sigma^2$

$$S_n = X_1 + X_2 + \dots X_n$$

$$E(S_n) = \mu_s = E(X_1) + E(X_2) + \dots + E(X_n)$$
$$= \mu + \mu + \dots + \mu$$
$$= n\mu$$

$$Var(S_n) = Var(X_1) + Var(X_2) + \dots + Var(X_n)$$
$$= \sigma^2 + \sigma^2 + \dots + \sigma^2$$
$$= n\sigma^2$$

$$Z = \frac{\overset{S_n}{X} - \overset{\mu_s}{\mu}}{\underset{\sigma_s}{\sigma}} = \frac{S_n - n\mu}{\sigma\sqrt{n}}$$

$$= \frac{\frac{S_n - n\mu}{n}}{\frac{\sigma\sqrt{n}}{n}} = \frac{\overset{s'}{\frac{S_n}{n}} - \mu}{\sigma/\sqrt{n}}$$

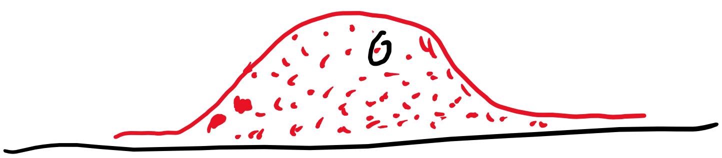
1st sample data (50 stocks) → produce an average return of 7.5%.
(Tesla, amazon, walmart ....)

2nd sample data (50 stocks) → produce an average return of 7.8%.
(......)

3^{rd}

$\vdots$

1000^{th} . . . .



$x_1 [0, 10]$

$x [5, 20]$

Def $\text{Normal\_pdf}(x, \mu, \sigma^2)$

$f_1 = \text{Normal\_pdf}(x_1, 0, 1)$

$\vdots$

$f_6 = \text{Normal\_pdf}(x_1, -3, 10^{-2})$

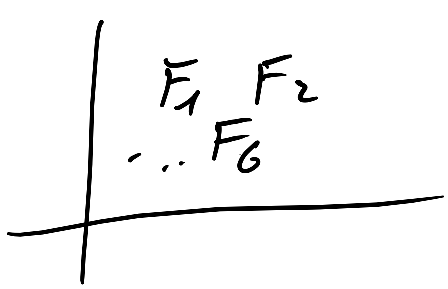
Def $\text{Normal\_cdf}(x, \mu, \sigma^2)$

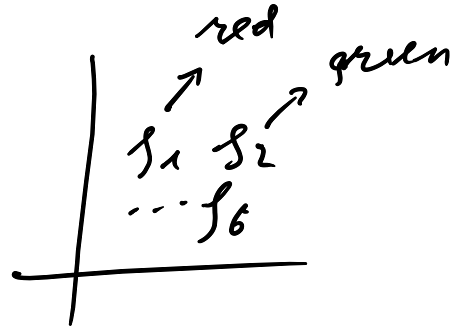
$F_1 = \text{Normal\_cdf}(X_1, 0, 1)$

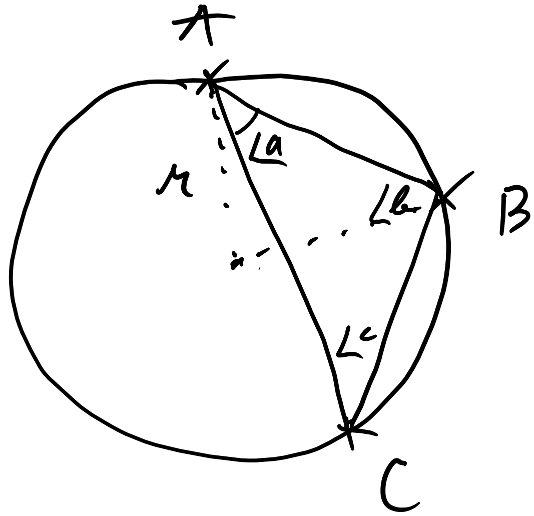
$F_2 = \text{Normal\_cdf}(X_1, 0, 10^{-1})$

$\vdots$

$F_6 = \text{Normal\_cdf}(X_1, -3, 10^{-2})$







If $\angle a \geq 90^\circ$

or $\angle b \geq 90^\circ$

or $\angle c \geq 90^\circ$

$\rightarrow$ A, B, C lie on the same semicircle!

AB? BC?

AC?