Lecture 10 note (11/3/20)

X	x_1	x_2	....	x_∞
	5'7''	5'6''	......	
	3000	5000	.....	
$f(x)$	$\frac{3000}{12000} \to f(x_1)$	$f(x_2)$	...	$f(x_\infty)$

$f(x)$ — μ, σ^2, σ, $\mu(x)$ $\Rightarrow$ population parameters

1^{st} sample data (50 stocks) $\to$ average return $\frac{6+5+\ldots-3}{50}$
(Tesla, amazon, ...)

X_1 $\quad X_2 \quad X_{50}$

$x_1 = 6\%$ $\quad 5\% \quad -3\%$

$\to \mu_1 = 7.5\%$

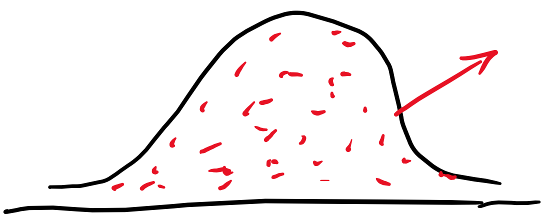
$\sigma^2 \quad \sigma$

2^{nd} sample data (50 stocks) $\to \mu_2 = 8\%$

⋮

100^{th} sample data (50 stocks) $\rightarrow \mu_{100} = 10\%$

S	μ_1	μ_2	....	μ_{100}
$f(x)$	$f(\mu_1)$	$f(\mu_2)$	....	$f(\mu_{100})$



sampling distribution of means
variance
standard deviation

$$E(S^2) = \frac{n-1}{n}\sigma^2$$

variance of the population

$$= \mu_{S^2}$$

mean of S^2

$$\underline{\hat{S}^2} = \frac{n}{n-1} S^2 = \frac{n}{n-1} \cdot \frac{n-1}{n} \cdot \sigma^2$$

$$= \underline{\sigma^2}$$

Theorem 5-1:

$$\mu_{\bar{X}} = \mu$$

mean of population

mean of sampling distribution of means

Theorem 5-2: sampling with replacement

$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n}$$

variance of population

no. of member of sample

variance of sampling distribution of means

Theorem 5-3: sampling without replacement

$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$$

$$\mu_{\bar{X}} = \mu$$

$$\mu_{\bar{X}} = \frac{2 + 2.5 + 4 + \dots + 11}{25} = \frac{150}{25} = 6$$

$$\sigma^2_{\bar{X}} = \frac{(2-6)^2 + (2.5-6)^2 + \dots + (11-6)^2}{25} = 5.4$$

$$\sigma_{\bar{X}} = \sqrt{5.4} = 2.32 \qquad \frac{10.8}{2}$$

$$S_n = X_1 + X_2 + X_3 + \dots + X_n$$

$$Z = \frac{S_n - n\mu}{\sigma\sqrt{n}} = \frac{\frac{S_n - n\mu}{n}}{\frac{\sigma\sqrt{n}}{n}} = \frac{\frac{S_n}{n} - \mu_{\bar{X}}}{\sigma/\sqrt{n}}$$

$\frac{S_n}{n} \to \bar{X}$

$\mu_{\bar{X}} \to$ population mean

$\sigma/\sqrt{n} \to \sigma_{\bar{X}}$

$\sigma \to$ variance of population

$n \to$ sample size

$$\mu_{\bar{X}} = \mu$$

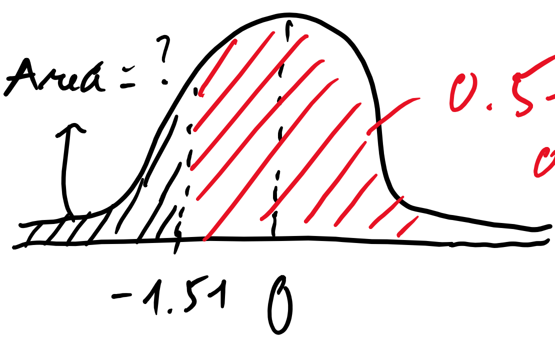
$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} \to \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$\mu = 29{,}321 \qquad \sigma = 2120$

$n = 100$

29,000 in standard unit:

$$\frac{29{,}000 - 29{,}321}{2120/\sqrt{100}} = -1.51$$



$0.5 + 0.4345$

0.9345

$$\begin{array}{r} 0.4345 \\ +\ 0.5 \end{array}$$

$0.5 - 0.4345 = 0.0655$

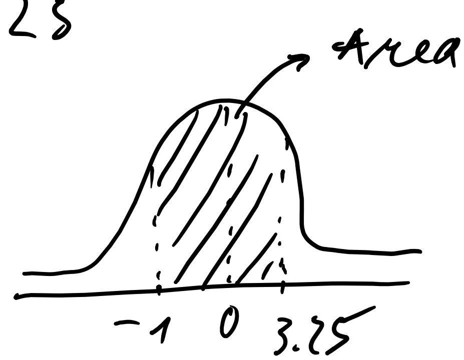
In-class exercise:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad \mu = 12 \quad \sigma = 8$$

$$n = 4$$

$$25 \rightarrow \frac{25 - 12}{8/\sqrt{4}} = 3.25$$

$$8 \rightarrow \frac{8 - 12}{8/\sqrt{4}} = -1$$



$$P(-1 < z < 3.25) = 0.4994 + 0.3413 = \boxed{0.8407}$$

3% of 400 : 12 defective tools

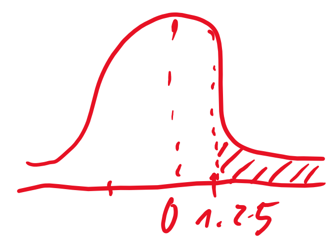
On a continuous basis, 12 or more also means 11,5 or more

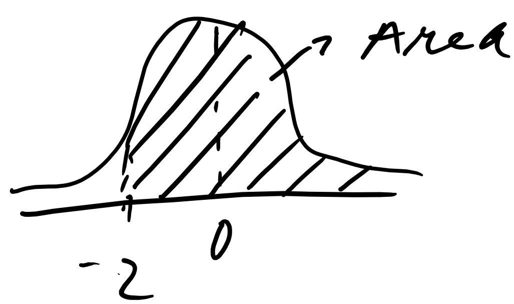
$$\mu = (2\% \text{ of } 400) = 8$$

$$\sigma = \sqrt{npq} = \sqrt{400 \times 0.02 \times 0.98}$$
$$= 2.8$$

$$11{,}5 \rightarrow \frac{11.5 - 8}{2.8} = 1.25$$

$$P(z > 1.25) = 0.1056$$





Population 1

$S_1 \rightarrow$ sampling distribution of means

$\downarrow$

μ_{s_1}, σ_{s_1}

Population 2

$S_2 \rightarrow$ sampling : means

$\downarrow$

μ_{s_2}, σ_{s_2}

Class interval: $15.3 - 17.7$

$0.05 \quad 0.05$

Class boundaries: ?

$15.25 \longrightarrow 17.75$

$\cancel{62 - 60} \rightarrow 2$

$62.5 - 59.5 \rightarrow 3$ → width of class interval "C"

$17.75 - 15.25 \rightarrow 2.5$

X	x_1	x_2	x_3
	$f(x_1)$	$f(x_2)$	$\ldots$

$$\mu = f(x_1) \cdot x_1 + f(x_2) \cdot x_2 \ldots$$

$$= \frac{f_1}{n} x_1 + \frac{f_2}{n} \cdot x_2 \ldots$$

$x = a + c\,u$ → width of class interval "$c = 3$"

$x = 67$ for $u = 0$

$x = 64$

$64 = 67 - 3 \cdot 1$

c

$u = -1$

$x = 67 + 3u$

$x = 64 \to 64 = 67 + 3u$

$\to u = -1$

$x = 70 \to 70 = 67 + 3u$

$\to u = 1$

$c = 3/4$

0.75

$x = 61$ for $u = 0$

$x = 61 + 3u$

when $x = 64 \to 64 = 61 + 3u$

$\to u = 1$

$x = 67 \to 67 = 61 + 3u$

$\to u = 2$