EE 381 HW #5

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Expectation: AKA mean; notation: E

$$E(x) = \sum_{j=0}^{n} x_j f(x_j) = x_1 f(x_1) + x_2 f(x_2) + \dots + x_n f(x_n)$$

OR

$$E(x) = \frac{x_1 + x_2 + \dots + x_n}{n}$$ (if all probabilities are equal)

variance: notation: var(X)

$\mu = E(X)$

$var(X) = E[(X-\mu)^2]$

If X is a discrete random variable taking the values $x_1, x_2, \dots, x_n$, and having probability function (x), then the variance is

$$\sigma^2 f(x) = E[(X-\mu)^2] = \sum_{j=1}^{n} (x_j - \mu)^2 f(x_j)$$

Std deviation: sqrt of the variance.

$$\sigma_x = \sqrt{var(X)} = \sqrt{E[(X-\mu)^2]}$$

notation: σ

thms about variance:

Thm 3.4: $\sigma^2 = E[(X-\mu)^2] = E(X^2) - \mu^2 = E(X^2) - [E(X)]^2$
where $\mu = E(X)$

thm 3.5: If c is any constant, $var(cX) = c^2 var(X)$

Thm 3.6: Quantity $E[(X-a)^2]$ is a minimum when
$a = \mu = E(X)$

Thms. about variance

Thm 3.7: If X and Y are independent Random variables:

1. $var(X+Y) = var(X) + var(Y)$ OR

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$$

2. $var(X-Y) = var(X) + var(Y)$ OR

$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$$

Some special sums:

1. $$\sum_{k=1}^{m} k = 1+2+3+\dots+m = \frac{m(m+1)}{2}$$

2. $$\sum_{k=1}^{m} k^2 = 1^2+2^2+3^2+\dots+m^2 = \frac{(2m+1)(m+1)m}{6}$$

3. $$\sum_{k=0}^{\infty} r^k = 1+r+r^2+\dots = \frac{1}{1-r} \text{ for } |r|<1$$

Standarized Random variables:

Std dev σ is greater than 0 and X is random variable w/ mean μ defined by

$$X^* = \frac{X-\mu}{\sigma}$$ w/ properties of $E(X^*) = 0$, $var(X^*) = 1$

Chebyshev's Inequality:

X is a random variable having mean μ and variance σ^2, which is finite.

Then if ϵ is any positive number.

1. $P(|X-\mu| \geq \epsilon) \leq \frac{\sigma^2}{\epsilon^2}$

OR w/ $\epsilon = k\sigma$

2. $P(|X-\mu| \geq \epsilon) \leq \frac{1}{k^2}$

1. Find the expectation of a discrete random variable X whose probability function is given by:

$$f(x) = \left(\tfrac{1}{2}\right)^x \quad (x=1,2,3,\ldots)$$

First, we do not have the probability when $x=1$, $x=2$, $x=3$, and so on, so we need to determine that first.

prob. funct table

X	1	2	3	4	5	...	n
f(x)	1/2	1/4	1/8	1/16	1/32	...	1/2ⁿ

$f(x_1) = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$ $\quad f(x_4) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$f(x_2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ $\quad f(x_5) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

$f(x_3) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

2. Thms of Expectation

Thm 3.1: If c is any constant, then

$$E(cX) = cE(X)$$

$$Y = g(X) = cX$$

$$E(cX) = \sum c\,x\,f(x) = c\sum x\,f(x) = cE(X)$$

Thm 3.2: If X and Y are any random variables, then:

$$E(X+Y) = E(X)+E(Y)$$

$f(x,y)$: joint probability function

$$E(X+Y) = \sum_X \sum_Y (x+y)\,f(x,y)$$

$$= \sum_X \sum_Y x\,f(x,y) + \sum_X \sum_Y y\,f(x,y) = E(X)+E(Y)$$

Thm 3.3: If X and Y are independent random variables, then $E(XY) = E(X)E(Y)$

$$f(x,y) = f_1(x)\cdot f_2(y)$$

$$E(X,Y) = \sum_X \sum_Y xy\,f(x,y) = \sum_X \sum_Y x\cdot f_1(x)\cdot y\cdot f_2(y)$$

$$= \sum_X \left[x f_1(x)\cdot \sum_y y f_2(y)\right]$$

$$= \sum_X \left[x f_1(x) E(y)\right] = E(y)\cdot \sum_X x f_1(x) = E(Y)\cdot E(X)$$

1. (continued!)

$$E(X) = \sum_{i=0}^{n} x_i P(x_i)$$

$$= x_1 f(x_1) + x_2 f(x_2) + x_3 f(x_3) + x_4 f(x_4) + x_5 f(x_5) + \dots + x_n f(x_n)$$

$$= 1\left(\tfrac{1}{2}\right) + 2\left(\tfrac{1}{4}\right) + 3\left(\tfrac{1}{8}\right) + 4\left(\tfrac{1}{16}\right) + 5\left(\tfrac{1}{32}\right) + \dots + n\left(\tfrac{1}{2^n}\right)$$

$$= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \dots + n\left(\frac{1}{2^n}\right)$$

// general summation

$$= \sum_{k=1}^{m} \frac{k}{2^k} = \frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \frac{5}{2^5} + \dots + \frac{n}{2^n}$$

$$= \frac{1}{2^1} + \overset{1+2}{\frac{2}{2^2}} + \overset{1+2}{\frac{3}{2^3}} + \overset{1+3}{\frac{4}{2^4}} + \overset{1+4}{\frac{5}{2^5}} + \dots + \frac{n}{2^n} = S$$

$\boxed{1/2 = 1/2^1}$

$$= \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{2}{2^3} + \frac{1}{2^4} + \frac{3}{2^4} + \dots = S$$

// we factor out a 1/2 bc we see that for every iteration, it increases by 1/2 (denom.)

$$= \left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \dots\right) + \frac{1}{2}\underbrace{\left(\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \dots\right)}_{S} = S$$

$$= \frac{\frac{1}{2}}{1 - \frac{1}{2}} + \frac{1}{2} S = S$$

1. (continued!)

$$= \frac{\frac{1}{2}}{\frac{1}{2}} + \frac{1}{2}S = S$$

$$= 1 + \frac{1}{2}S = S$$

$$- \frac{1}{2}S \qquad - \frac{1}{2}S$$

$$1 = \frac{2}{2}S - \frac{1}{2}S$$

$$1 = \frac{1}{2}S$$

$$(2)\cdot 1 = \frac{1}{\cancel{2}}S\cdot(\cancel{2})$$

$$\boxed{S = 2}$$

$$\frac{d}{dx}\left(\frac{1}{1-x} = \sum_{n=0}^{\infty} x^{n}\right)$$

$$= \frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1} \quad * \text{if } |x|<1$$

let $X = 1/2$

$$\sum_{n=1}^{\infty} n\left(\tfrac{1}{2}\right)^{n-1} = \frac{1}{(1-(\frac{1}{2}))^2}$$

$$= \sum_{n=1}^{\infty} n\left(\tfrac{1}{2}\right)^{n-1} = \frac{1}{1/4} =$$

$$= \sum_{n=1}^{\infty} \frac{n}{2^{n-1}} = 4$$

$$= \sum_{n=1}^{\infty} \frac{n}{2^{n}2^{-1}} = 4$$

$$= \left(\frac{1}{\cancel{2}}\right)\cdot \cancel{2}\sum_{n=1}^{\infty} \frac{n}{2^n} = 4\cdot\left(\frac{1}{2}\right)$$

$$= \sum_{n=1}^{\infty} \frac{n}{2^n} = \boxed{2}$$

2. Let X and Y be independent random variables such that:

$$X = \begin{cases} 1 & \text{prob. } 1/3 \\ 0 & \text{prob. } 2/3 \end{cases} \qquad Y = \begin{cases} 2 & \text{prob. } 3/4 \\ -3 & \text{prob. } 1/4 \end{cases}$$

original

X	1	0
f(x)	1/3	2/3

original

Y	2	-3
f(y)	3/4	1/4

a) $E(3X+2Y)$

using

Thm 3.2: $E(X+Y) = E(X)+E(Y)$

in this case, $E(3X+2Y) = E(3X)+E(2Y)$

need to find $E(3X)$ and $E(2Y)$ seperately

1. $E(3X) \rightarrow$: $1: 3(1)=3$

$0: 3(0)=0$

3X	3	0
f(x)	1/3	2/3

(x_1, x_2)

$$E(3X) = \sum_{j=0}^{n} x_j f(x_j) = x_1 f(x_1) + x_2 f(x_2)$$

$$= \cancel{3}^{1}(1/\cancel{3}) + 0(2/3) = 1+0 = 1$$

$$= E(3X) = 1$$

$$E(3X)+E(Y) = 1+\frac{3}{2} = \frac{2}{2}+\frac{3}{2} = \boxed{\frac{5}{2}}$$

2. $E(2Y) \rightarrow$: $2: 2(2)=4$

$-3: 2(-3)=-6$

2Y	4	-6
f(y)	3/4	1/4

(y_1, y_2)

$$E(2Y) = \sum_{j=0}^{n} y_j f(y_j)$$

$$= y_1 f(y_1) + y_2 f(y_2) =$$

$$\cancel{4}\left(\frac{3}{\cancel{4}}\right) + -\cancel{6}^{3}\left(\frac{1}{\cancel{4}_2}\right) =$$

$$3 + -\frac{3}{2} = \frac{6}{2} - \frac{3}{2} = \frac{3}{2}$$

$E(Y) = 3/2$

Page: 44 2b) $E(2X^2 - Y^2)$

$E(2X^2) - E(Y^2)$

using Thm 3.2: $E(X+Y) = E(X) + E(Y)$

By applying thm 3.2: $E(2X^2 - Y^2) = E(2X^2) - E(Y^2)$

1. $E(2X^2)$

original

X	1	0
$f(x)$	1/3	2/3

$E(2X^2) \rightarrow$ * $1: (2(1)^2) = (2(1)) = 2$

$0: (2(0)^2) = (2(0)) = 0$

$2X^2$	2 (x_1)	0 (x_2)
$f(x)$	1/3	2/3

$E(2X^2) = \sum_{j=0}^{n} x_j f(x_j)$

$= x_1 f(x_1) + x_2 f(x_2)$

$= 2(1/3) + 0(2/3) = \frac{2}{3}$

$E(2X^2) = \frac{2}{3}$

2. $E(Y^2)$

original

Y	2	-3
$f(Y)$	3/4	1/4

$Y^2 \rightarrow 2: (2)^2 = 4$

$-3: (-3)^2 = 9$

Y^2	4 (y_1)	9 (y_2)
$f(y)$	3/4	1/4

$E(Y^2) = \sum_{j=0}^{n} y_j f(y_j)$

$= y_1 f(y_1) + y_2 f(y_2)$

$= \cancel{4}(3/\cancel{4}) + 9(1/4) = 3 + \frac{9}{4}$

$= \frac{12}{4} + \frac{9}{4} = \frac{21}{4}$

$E(Y^2) = \frac{21}{4}$

$E(2X^2 - Y^2) = \frac{2}{3} - \frac{21}{4} = {}^{*}\frac{8}{12} - \frac{63}{12} = \boxed{\frac{-55}{12} = E(2X^2 - Y^2)}$

2c) $E(XY)$

using thm 3.3: when x and Y are independent variables, then $E(XY)=E(X)E(Y)$

$f(x,y)=f_1(x)\cdot f_2(y)$

$E(X,Y)=\sum_X\sum_Y xy f(x,y)=\sum_X\sum_Y x\cdot f_1(x)\cdot y\cdot f_2(y)$

$=\sum_x[x\, f_1(x)\cdot\sum_y y\, f_2(y)]$

$=\sum_x[x\, f_1(x)E(y)]=E(y)\sum_x x\, f_1(x)=E(Y)\cdot E(X)$

$E(XY)=E(X)\cdot E(Y)$

original $E(X)$

	x_1	x_2
X	1	0
$f(x)$	1/3	2/3

$E(X)=\sum_{j=0}^{n} x_j f(x_j)$

$=x_1 f(x_1)+x_2 f(x_2)$

$=1(1/3)+0(2/3)=\frac{1}{3}+0=\frac{1}{3}$

$\frac{1}{3}=E(X)$

$E(Y)$

original

	y_1	y_2
Y	2	-3
$f(y)$	3/4	1/4

$E(Y)=\sum_{j=0}^{n} y_j f(y_j)$

$=y_1 f(y_1)+y_2 f(y_2)$

$=\cancel{2}(3/\cancel{4}_2)+-3(1/4)=$

$\frac{3}{2}+\frac{-3}{4}=\frac{6}{4}-\frac{3}{4}=\frac{3}{4}$

$\frac{3}{4}=E(Y)$

$E(XY)=E(X)\cdot E(Y)$

$=\frac{1}{\cancel{3}}\cdot\frac{\cancel{3}^1}{4}=\boxed{\frac{1}{4}=E(XY)}$

2d) $E(X^2Y)$

Using thm 3.3: If X and Y are independent, then $E(XY) = E(X)E(Y)$

$$E(X^2Y) = E(X^2)E(Y)$$

1. $E(X^2)$

original

X	1	0
$f(X)$	1/3	2/3

$E(X^2) \rightarrow$: $1: (1)^2 = 1$, $0: (0)^2 = 0$

	x_1	x_2
X^2	1	0
$f(X)$	1/3	2/3

$$E(X^2) = \sum_{j=0}^{n} x_j f(x_j)$$

$= x_1 f(x_1) + x_2 f(x_2)$

$= 1(1/3) + 0(2/3)$

$= \frac{1}{3} = E(X^2)$

2. $E(Y)$

original

	y_1	y_2
Y	2	-3
$f(Y)$	3/4	1/4

$$E(Y) = \sum_{j=0}^{n} y_j f(y_j)$$

$= y_1 f(y_1) + y_2 f(y_2)$

$= \cancel{2}(3/\cancel{4}_2) + -3(1/4)$

$= 3/2 - 3/4 = 6/4 - 3/4 = 3/4$

$= 3/4 = E(Y)$

$$E(X^2Y) = E(X^2)E(Y) = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{4} = \boxed{\frac{1}{4}}$$

$$\boxed{\frac{1}{4} = E(X^2Y)}$$

3) Let $x1, x2, \dots, x_n$ be n random variables which are identically distributed such that

$$X_k = \begin{cases} 1 & \text{Prob. } 1/2 \\ 2 & \text{Prob. } 1/3 \\ -1 & \text{Prob. } 1/6 \end{cases}$$

original

	x_1	x_2	x_3
X	1	2	-1
$f(x)$	1/2	1/3	1/6

Find

a) $E(x_1+x_2+\dots+x_n)$

$$E(X) = \sum_{j=1}^{n} X_j f(x_j) = x_1 f(x_1) + x_2 f(x_2) + \dots + x_n f(x_n)$$

$$E(X) = \sum_{j=1}^{n} x_1 f(x1) + x_2 f(x2) + x_3 f(x_3)$$

$$= \sum_{j=1}^{n} 1\left(\tfrac{1}{2}\right) + 2\left(\tfrac{1}{3}\right) + -1\left(\tfrac{1}{6}\right) = \sum_{j=1}^{n} \tfrac{1}{2} + \tfrac{2}{3} - \tfrac{1}{6} =$$

$$= \sum_{j=1}^{n} \tfrac{3}{6} + \tfrac{4}{6} - \tfrac{1}{6} = \boxed{\sum_{j=1}^{n} \tfrac{6}{6} = 1 = n = E(x_1+x_2+\dots+x_n)}$$

3. b) Find

$E(x_1^2 + x_2^2 + \dots + x_n^2)$

original

X	1 (x_1)	2 (x_2)	-1 (x_3)
$f(x)$	1/2	1/3	1/6

$\dots \; x_n$

$x^2 \to$ $1: (1)^2 = 1$

$2: (2)^2 = 4$

$-1: (-1)^2 = 1$

X^2	1 (x_1^2)	4 (x_2^2)	1 (x_3^2)
$f(x)$	1/2	1/3	1/6

$E(x^2) = \sum_{j=1}^{n} x_j^2 f(x_j) = x_1^2 f(x_1) + x_2^2 f(x_2) +$

$x_3^2 f(x_3) = \sum_{j=1}^{n} 1(\frac{1}{2}) + 4(\frac{1}{3}) + 1(\frac{1}{6}) =$

$\sum_{j=1}^{n} \frac{1}{2} + \frac{4}{3} + \frac{1}{6} = \sum_{j=1}^{n} \frac{3}{6} + \frac{8}{6} + \frac{1}{6} = \sum_{j=1}^{n} \frac{12}{6} = 2 = 2n$

$= E(x_1^2 + x_2^2 + \dots + x_n^2)$

4. A random variable X is defined by:

$$X = \begin{cases} -2 & \text{Prob. } 1/3 \\ 3 & \text{prob. } 1/2 \\ 1 & \text{prob. } 1/6 \end{cases}$$

original Table

X	-2	3	1
f(X)	1/3	1/2	1/6
	x_1	x_2	x_3

Determine the variance and standard deviation for X.

// we must find the expectation first.

$$E(X) = \sum_{j=1}^{n} x_j f(x_j) = x_1 f(x_1) + x_2 f(x_2) + x_3 f(x_3)$$

$$= -2\left(\frac{1}{3}\right) + 3\left(\frac{1}{2}\right) + 1\left(\frac{1}{6}\right) = \frac{-2}{3} + \frac{3}{2} + \frac{1}{6}$$

$$= \frac{-4}{6} + \frac{9}{6} + \frac{1}{6} = \frac{6}{6} = \boxed{1 = E(X) = \mu}$$

variance: $\mu = E(X) = 1$

$$Var(X) = E[(X-\mu)^2]$$

$$\sigma^2 f(x) = Var(X) = E[(X-\mu)^2] = \sum_{j=1}^{n} (x_j - \mu)^2 f(x_j)$$

4. (continued!) $\mu = 1$

$var(X) = E[(X-\mu)^2] = \sum_{j=1}^{n} (x_j - \mu)^2 f(x_j)$

$= (x_1 - 1)^2 f(x_1) + (x_2 - 1)^2 f(x_2) +$
$(x_3 - 1)^2 f(x_3)$

$= (-2-1)^2 (\frac{1}{3}) + (3-1)^2 (\frac{1}{2}) + (1-1)^2 (\frac{1}{6})$

$= (-3)^2 (\frac{1}{3}) + (2)^2 (\frac{1}{2}) - \cancel{(0)^2 (\frac{1}{6})}$

$= (9)(\frac{1}{3}) + (4)(\frac{1}{2}) - \cancel{0}$

$= \frac{9}{3} + \frac{4}{2} = 3 + 2 = \boxed{5 = Var(X)}$

<u>std deviation:</u> $\sqrt{var(X)} = \sqrt{E(X-\mu)^2}$

std deviation $= \sqrt{var(X)} = \boxed{\sqrt{5} \approx 2.24}$

5. If a random variable X is such that $E[(X-1)^2] = 10$ and $E[(X-2)^2] = 6$.

a) * $E[(X-1)^2] = 10$ *** property:** $E(aX^2+bX+c) = aE(X^2)+bE(X)+c$

FOIL

$= E[(X-1)(X-1)] = 10$ $E(X) = \mu_X$

$= E[(X^2-X-X+1)] = 10$

$= E[(X^2-2X+1)] = 10$

$= E(X^2) - 2E(X) + 1 = 10$

$E(X^2) - 2E(X) + \cancel{1} = 10$

$\cancel{-1} \quad -1$

eq 1: $E(X^2) - 2E(X) = 9 \rightarrow E(X^2) - 2\mu_X = 9$

* $E[(X-2)^2] = 6$

FOIL

$E[(X-2)(X-2)] = 6$

$E[(X^2-2X-2X+4)] = 6$

$E[(X^2-4X+4)] = 6$

$E(X^2) - 4E(X) + 4 = 6$

$E(X^2) - 4E(X) + \cancel{4} = 6$

$\cancel{-4} \quad -4$

eq 2: $E(X^2) - 4E(X) = 2 \rightarrow E(X^2) - 4\mu_X = 2$

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5. a) $E(x)$: expectation/mean
b) $Var(x)$: variance
c) σ_x : standard deviation

a) eq 1: $E(x^2) - 2\mu_x = 9$
eq 2: $E(x^2) - 4\mu_x = 2$

system of equations:

$-1\,[E(x^2) - 2\mu_x = 9]$
$E(x^2) - 4\mu x = 2$

$-\cancel{E(x^2)} + 2\mu x = -9$
$\cancel{E(x^2)} - 4\mu x = 2$

$\frac{-\cancel{2}\mu x}{-\cancel{2}} = \frac{+7}{+2}$

PLUG BACK μx in to either

$\boxed{\mu x = \frac{7}{2} = \text{mean} = E(x)}$

eq 1: $E(x^2) - 2\left(\frac{7}{2}\right) = 9$

$E(x^2) - \cancel{7} = 9$
$\cancel{+7} \quad +7$

$\boxed{E(x^2) = 16}$

OR

eq 2: $E(x^2) - \cancel{4}^{2}\left(\frac{7}{\cancel{2}}\right) = 2$

$E(x^2) - \cancel{14} = 2$
$+\cancel{14} \quad +14$

$\boxed{E(x^2) = 16}$

5. b) var - variance

$E(X^2) = 16$

$\mu = \frac{7}{2}$ from part A

Formula: Thm 3.4 $\sigma_x^2 = E[(X-\mu)^2]$

$\sigma_x^2 = Var(X) = E[(X-\mu)^2]$

$= E[(X-\mu)(X-\mu)] = E(X^2 - 2\mu X + \mu^2)$

$= E(X^2) - 2\mu E(X) + \mu^2$

$= E(X^2) - 2\mu^2 + \mu^2$

$= E(X^2) - \mu^2$

$\sigma_x^2 = Var(X) = E(X^2) - \mu^2$

$= 16 - \left(\frac{7}{2}\right)^2 = 16 - \frac{49}{4} = \frac{64}{4} - \frac{49}{4}$

$\boxed{= \frac{15}{4} = Var(X) = \sigma_2 X}$

5. c) σ_X : standard deviation

$$\sigma_X = \sqrt{var(X)} = \sqrt{\sigma_X^2} = \sqrt{E[(X-\mu)^2]}$$

$$\sigma_X = \sqrt{\frac{15}{4}} = \sqrt{\frac{15}{4}} = \boxed{\frac{\sqrt{15}}{2}}$$

6. Three dice, assumed fair, tossed successfully. Find

a) the mean/expectation

b) the variance of the sum

for 1 die, each die has a 1/6 chance of revealing each die.

6 faces in 1 die.

$$E(X) = \sum_{j=1}^{n} x_j f(x_j) = x_1 f(x_1) + x_2 f(x_1)$$

$$+ \dots + x_6 f(x_6) = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = E(x) = 3.5 = \mu$$

for 1 die

a) Because we have 3 dices that are assumed fair,

$$E(X) = E(Y) = E(Z)$$

Thm 3.2

$$E(X+Y) = E(X) + E(Y)$$

$$E(X+Y+Z) = E(X) + E(Y) + E(Z)$$

6. Three dice, assumed fair, tossed successfully. Find
a) the mean/expectation
b) The variance of the sum

for 1 die, each die has a 1/6 chance of revealing each die.
6 faces in 1 die.

$$E(X) = \sum_{j=1}^{n} x_j f(x_j) = x_1 f(x_1) + x_2 f(x_1)$$

$$+ \dots + x_6 f(x_6) = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = E(X)$$

$= 3.5 = \mu$

for 1 die

a) Because we have 3 dices that are assumed fair,

$E(X) = E(Y) = E(Z)$

Thm 3.2

$E(X+Y) = E(X) + E(Y)$

$E(X+Y+Z) = E(X) + E(Y) + E(Z)$

6. a) (continued!)

$E(X) + E(Y) + E(Z) = \frac{21}{6} + \frac{21}{6} + \frac{21}{6}$

$= \frac{63}{6} = \boxed{\frac{21}{2} = E(X+Y+Z) = \mu}$

b) variance of the sum

$\mathrm{var}(X+Y+Z) = \mathrm{var}(X) + \mathrm{var}(Y) + \mathrm{var}(Z)$

$\mathrm{var}(X) = \mathrm{var}(Y) = \mathrm{var}(Z) = E[(X-\mu)^2]$

$\mu = 3.5$ from finding expectation of 1 fair die. $f(x) = 1/6$

$= E[(X-3.5)^2] = \sum (x-\mu)^2 f(x)$

$= (1-3.5)^2(\frac{1}{6}) + (2-3.5)^2(\frac{1}{6}) + (3-3.5)^2(\frac{1}{6}) + (4-3.5)^2(\frac{1}{6}) + (5-3.5)^2(\frac{1}{6}) + (6-3.5)^2(\frac{1}{6})$

$= \frac{1}{6}[(1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2]$

$= \frac{35}{12} = 1.71^2 = \mathrm{var}(X)$

$\mathrm{var}(X+Y+Z) = \mathrm{var}(X) + \mathrm{var}(Y) + \mathrm{var}(Z)$

$= \frac{35}{12} + \frac{35}{12} + \frac{35}{12} = \frac{105}{12} = \boxed{\frac{35}{4} = \mathrm{var}(X+Y+Z)}$

7. A random variable X has mean 3 and variance 2. Use Chebyshev's Inequality to obtain an upper bound for:

$P(|X-\mu| \geq \epsilon) \leq \frac{\sigma^2}{\epsilon^2}$

OR w/ $\epsilon = k\sigma$

$P(|X-\mu| \geq \epsilon) \leq \frac{1}{k^2}$

$\mu = E(X) = \text{mean} = 3$

$\sigma_2{}^x = \text{var}(x) = 2 = \sigma^2$

$\sqrt{\sigma_2{}^x} = \text{std dev} = \sigma_x = \sqrt{2}$

$\epsilon = k\sigma$

a) $P(|X-3| \geq 2)$; $\epsilon = 1 \times \sigma^2$ $\boxed{P(|X-3| \geq 2) \leq 1}$

$\epsilon = 2$

$P(|X-3| \geq 2 \leq \frac{\sigma^2}{\epsilon^2} = \frac{(2)^2}{(2)^2} = \frac{4}{4} = \boxed{1}$

b) $P(|X-3| \geq 1)$ $\boxed{P(|X-3| \geq 1) \leq 2}$

$\epsilon = 1$ $P(|X-3| \geq 1 \leq \frac{\sigma^2}{\epsilon^2} = \frac{2^2}{1^2} = \boxed{2}$