Lab 4 note:

Central limit theorem:

$X_1, X_2, \ldots X_n$ have similar μ & σ

$$S_n = X_1 + X_2 + \ldots + X_n \qquad \mu_{S_n} = n\mu$$

$$\sigma_{S_n} = \sqrt{n}\,\sigma$$

$$Z = \frac{S_n - \mu_{S_n}}{\sigma_{S_n}} = \frac{S_n - n\mu}{\sqrt{n}\,\sigma}$$

→ can be approximated by normal distribution.

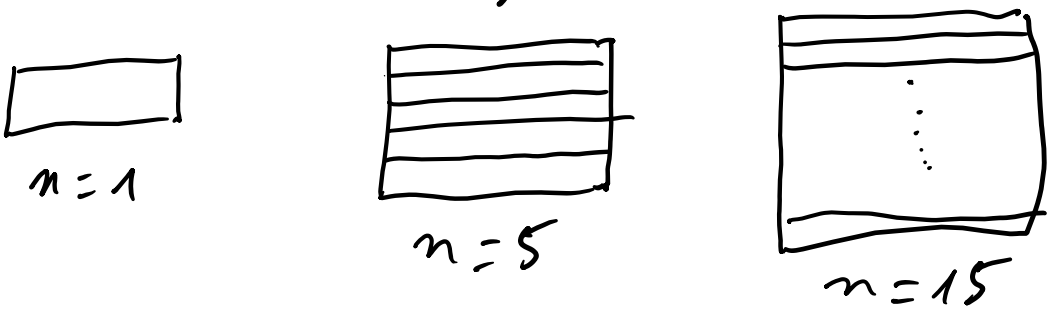
Imagine W (thickness of the book) is random variable X

R.V W is uniformly distributed in $[1, 3]$

$$\mu_W = \frac{1+3}{2} = 2 \qquad \sigma_W = \sqrt{\frac{1}{12}\cdot(3-1)^2} = 0.577$$

$$= \frac{a+b}{2}$$

S: the width of a stack of books



$n = 1$: $S = W_1$

$n = 5$: $S = W_1 + W_2 + \ldots + W_5$

$n = 15$: $S = W_1 + W_2 + \ldots + W_{15}$

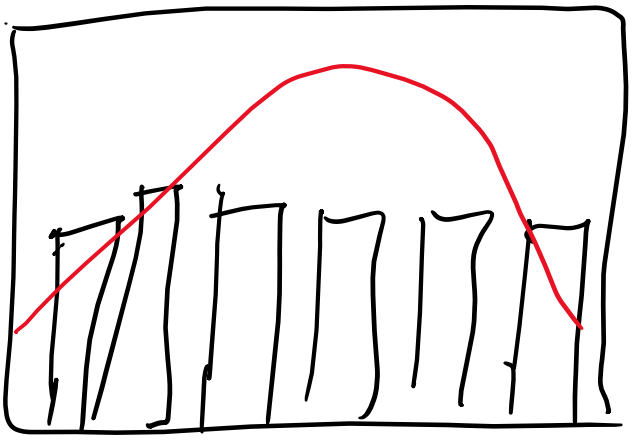
Simulation: $n = 1 \Rightarrow S = W_1$ 

- Generate $N = 10{,}000$ different values for the width of the book, which is btw $[1,3]$
  (use random.uniform(1, 3, n=1)
  These values will be assigned to S.
- Plot the histogram of S
- Plot the normal prob. function of S by using:

$$f(S_n) = \frac{1}{\sigma_{S_n}\sqrt{2\pi}} e^{\frac{-(S-\mu_S)^2}{2\sigma_S^2}}$$

- Plot them on the same figure to compare.

* Repeat with $n = 5$, $n = 15$



Question 3)

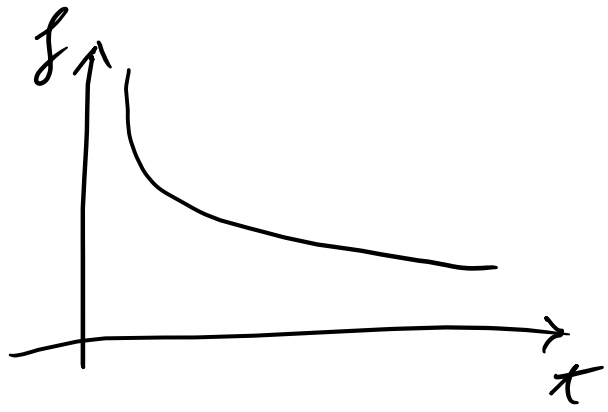
Exponential distribution:

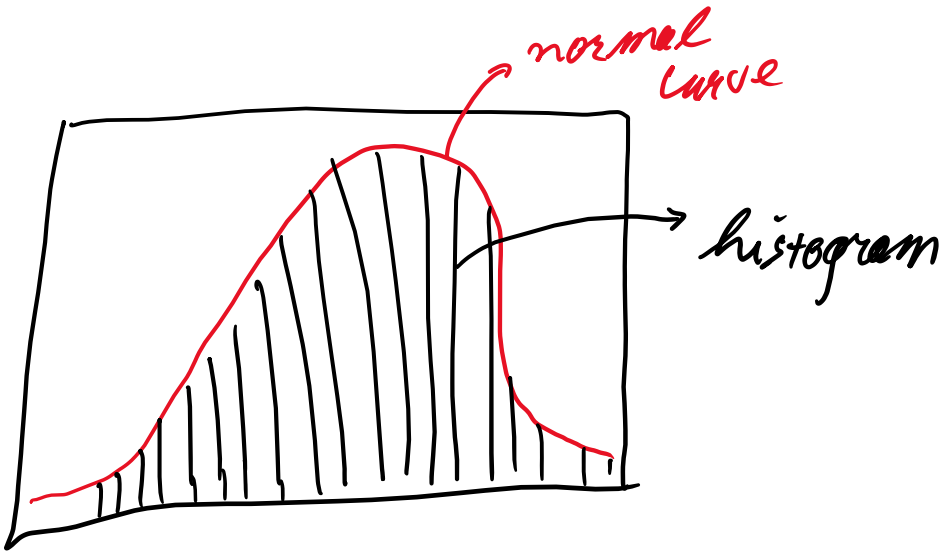
$$f(x) = \begin{cases} \alpha e^{-\alpha x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Mean $= \mu = \frac{1}{\alpha} = \beta = 45$

Variance: $\sigma^2 = \frac{1}{\alpha^2} = \beta^2$

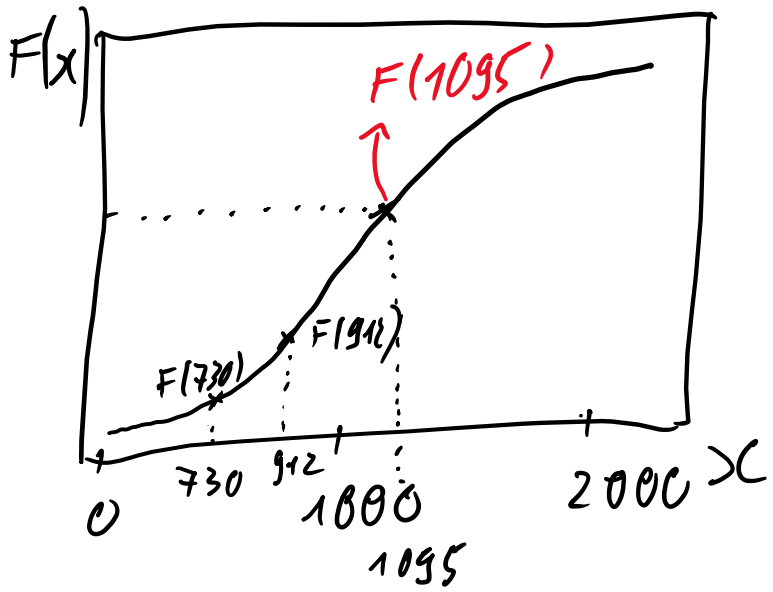
Standard deviation: $\sigma = \frac{1}{\alpha} = \beta$





e) Use "np.cumsum" to calculate the CDF of the carton lifetime

cdf = np.cumsum(h_1 * barwidth)



$P(S > 1095)$

$= 1 - F(1095)$

$= \;!$

b) $P(730 < S < 912)$

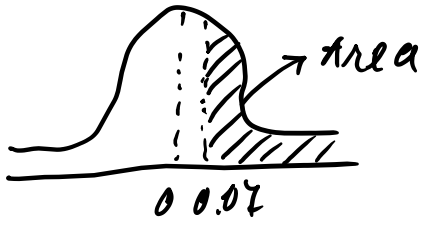
$= F(912) - F(730)$

Using normal distribution to check the results

$\mu = 24 \times 45 = 1080$

$\sigma = 45 \times \sqrt{24} = 220.454$

a) More than 1095:



$$\frac{1095.5 - 1080}{220.454} = 0.07$$

$P(X > 1095) = 0.5 - 0.0279$

$= 0.4721$