$X = \left\{ \begin{matrix} 2, 4, 5, 1, 1, 3, 6, 6, 5 \\ 2 \end{matrix} \right\}$

$1, 2, 3, 4, 5, 6$

$$\frac{\sum_{n=1}^{10} X}{10}$$

$$E(X) = 2 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + \ldots \ldots 2 \times \frac{1}{6}$$

$$= \ldots \ldots$$

$$E(X) = \sum x f(x)$$

$Y = g(X)$ $\qquad$ $Y = X + 3$

$$E(Y) = \sum g(x) f(x)$$

$$E(Y) = 5 \times \frac{1}{6} + 7 \times \frac{1}{6} + 8 \times \frac{1}{6} + \ldots \ldots$$

Theorem 3-1: $Y = g(X) = cX$

$$E(cX) = \sum c x f(x)$$
$$= c \sum x f(x) = c\,E(X)$$

Theorem 3-2: $f(x,y)$: joint probability function

$$E(X+Y) = \sum_x \sum_y (x+y) f(x,y)$$
$$= \sum_x \sum_y x f(x,y) + \sum_x \sum_y y f(x,y)$$
$$= E(X) + E(Y)$$

Theorem 3-3:

$$f(x,y) = f_1(x) \cdot f_2(y)$$

$$E(XY) = \sum_x \sum_y x y f(x,y)$$
$$= \sum_x \sum_y x \cdot f_1(x) \cdot y \cdot f_2(y)$$
$$= \sum_x \left[x f_1(x) \cdot \sum_y y f_2(y) \right]$$
$$= \sum_x \left[x f_1(x) E(Y) \right]$$
$$= E(Y) \sum_x x f_1(x) = E(Y) \cdot E(X)$$

In-class exercise:

x	-2	3	1
$f(x)$	$1/3$	$1/2$	$1/6$

y	1	11	7
z	4	9	1

a) $E(X) = -2\left(\frac{1}{3}\right) + 3 \times \frac{1}{2} + 1 \times \frac{1}{6}$

$= \boxed{1}$

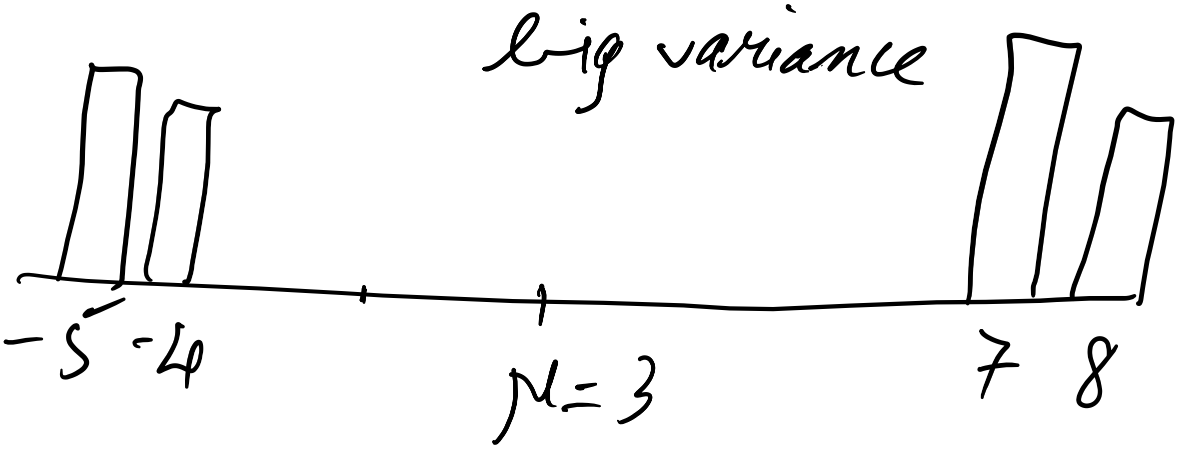
b) $E(2X+5) = 1 \times \frac{1}{3} + 11 \times \frac{1}{2} + 7\,\frac{1}{6}$

$= \boxed{7}$

c) $E(X^2) = 4 \times \frac{1}{3} + 9 \times \frac{1}{2} + 1 \times \frac{1}{6}$

$= \boxed{6}$

big variance



x	1	2	3	4	5
$(X-\mu)^2$	4	.	.	.	.

$\mu = 3$

$$Var(X) = E[(X-\mu)^2]$$

In-class exercise

b) $E(X) = 1 \times 0.1 + 2 \times 0.1 + \dots + 10 \times 0.2$

$= 6$

c) $E[(X-\mu)^2] = (1-6)^2 \times 0.1 + (2-6)^2 \times 0.1 + \dots + (10-6)^2 \times 0.2$

$= 11.2$

$$\sigma = \sqrt{Var(X)} = \sqrt{11.2} = 3.35$$

3.4)

$E[(X-\mu)^2] = E(X^2 - 2X\mu + \mu^2)$

$= E(X^2) - 2\mu E(X) + \mu^2$

$= E(X^2) - 2\mu^2 + \mu^2$

$= E(X^2) - \mu^2$

3.5) $Var(cX) = E[(c(X-\mu))^2]$

$$= E[c^2(X-\mu)^2]$$

$$= c^2 E[(X-\mu)^2]$$

$$= c^2 Var(cX)$$

3.7)

$$Var(X \mp Y) = E[\{(X \mp Y) - (\mu_X - \mu_Y)\}^2]$$

$$= E[\{(X-\mu_X) \mp (Y-\mu_Y)\}^2]$$

$$= E[(X-\mu)^2] \mp 2\underset{0}{\cancel{E\{(X-\mu_X)(Y-\mu_Y)\}}}$$

$$+ E[(Y-\mu_Y)^2]$$

$$= Var(X) + Var(Y)$$

$$X^* = \frac{X - \mu}{\sigma}$$

$$E(X^*) = E\left(\frac{X-\mu}{\sigma}\right)$$

$$= \frac{1}{\sigma} E(X - \mu)$$

$$= \frac{1}{\sigma}\left[E(X) - \cancel{E(\mu)}^{\mu}\right] = 0$$

$$E(X) = \mu$$

$$Var(X^*) = Var\left(\frac{X-\mu}{\sigma}\right)$$

$$= \frac{1}{\sigma^2} \underbrace{E\left[(X-\mu)^2\right]}_{Var = \sigma^2} = \frac{1}{\cancel{\sigma^2}} \times \cancel{\sigma^2} = 1$$

$$Var = \sigma^2$$

σ = std. deviation = $\sqrt{Var}$