Tuần 6

Ứng dụng xấp xỉ đạo hàm

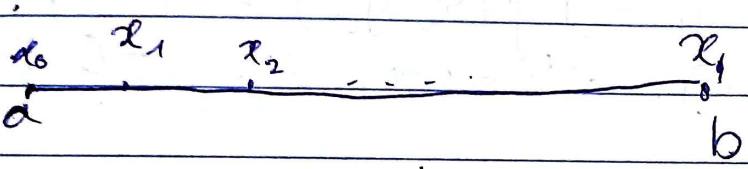
1) Tính xấp xỉ nghiệm của phương trình phân cấp 1

$$\begin{cases} y'(x) = f(x, y(x)) \\ y(a) = y_0 \end{cases}$$

VD: $\begin{cases} y'(x) = 2x - y(x) \\ y(0) = -1 \end{cases}$ Ta cần tính ra nghiệm xấp xỉ nghiệm chính xác sau:

$$y(x) = e^{-x} + 2x - 2 \quad \forall x \in [a,b]$$

~~Chia~~ $[a,b]$.



$x_i = a + ih$ với $h = \frac{b-a}{N}$ Tìm $\{y(x_i)\}$ $y(x_0) = y_0$

Từ bài toán vô hạn điểm, ta chuyển về bài toán trên các điểm x_i. Tìm $y(x_i)$.

Ta có $y'(x_i) = f(x, y(x_i))$ áp dụng công thức sai phân tiến $y'(x_0) = \frac{y(x_1) - y(x_0)}{h} + O(h)$

do đó $f(x_0, y(x_0)) = \frac{y(x_1) - y(x_0)}{h} + O(h)$

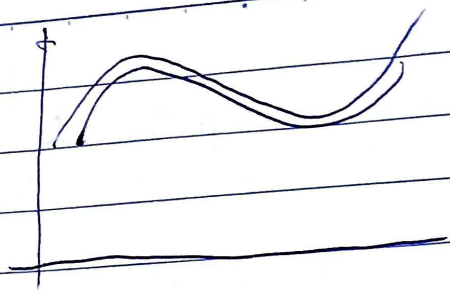
$$y(x_1) - y_0 + O(h^2) = hf(x_0, y_0)$$

Đặt y_1 giá trị xấp xỉ của $y(x_1)$, thoả

$$y_1 = \underbrace{y_0 + hf(x_0, y_0)}_{\text{tính được}}$$

Ta kí hiệu $y_h = (y_i)_i$ là giá trị xấp xỉ

$y_e = (y(x_i))_i$ là giá trị chính xác



$$\|Y_h - Y_e\|_2 = \sqrt{\sum_{i=1}^{n} |y_i - y(x_i)|^2}$$

$$\|y_h - y\| = \left(\int_a^b |y_h(x) - y(x)|^2 dx\right)^{\frac{1}{2}}$$

VD: $\begin{cases} y'(x) = 2x - y(x) \\ y(0) = -1. \end{cases}$

$y_1 = y_0 + h f(x_0, y_0) = -1 + h(2.0 + 1) = -0{,}9$

$y(x_1) = e^{-x_1} + 2x_1 - 2 = -0{,}89516258.$

$y_2 = y_1 + h f(x_1, y_1) = -0{,}9 + 0{,}1(2.0{,}1 + 0{,}9)$

$= -0{,}79$

$y(x_2) = e^{-x_2} + 2x_2 - 2 = -0{,}78126925$

2) Tính ng0 xấp xỉ PTVP cấp 2

$\begin{cases} y''(x) + \left(\frac{2}{x}\right) y'(x) - \left(\frac{2}{x^2}\right) y(x) = \frac{\sin(\ln x)}{x^2} \quad x \in (1,2) \\ y(1) = 1, \ y(2) = 2. \end{cases}$

$x \in (1,2)$

$y''(x_1) + \frac{2}{x_1} y'(x_1) - \frac{2}{(x_1^2)} y(x_1) = \frac{\sin(\ln(x_1))}{x_1^2}$

$\Rightarrow y''(x_1) + 1{,}6 y'(x_1) - 1{,}28 y(x_1) = \frac{\sin(\ln(1{,}25))}{x^2}$

$\Rightarrow \left[\frac{y(x_2) - 2y(x_1) + y(x_0)}{h^2} + O(h^2)\right]$

$+ 1{,}6 \cdot \left[\frac{y(x_2) - y(x_0)}{2h} + O(h^2)\right] - 1{,}28 y(x_1) = f_1$

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$$\Rightarrow y(x_2)\left[\frac{1}{h^2}+\frac{1{,}6}{2h}\right]-y(x_1)\left[\frac{-2}{h^2}-1{,}28\right]$$

$$+\dots+y(x_0)\left[\frac{1}{h^2}-\frac{1{,}6}{2h}\right]+O(h^2)=f_1$$

$$\Rightarrow 19{,}2y_2-33{,}28y_1=-12{,}65837035+O(h^2)$$

$* \; x=x_2=1{,}5.$

$$y''(x_2)+\left(\frac{2}{x^2}\right)y'(x_2)-\left(\frac{2}{x^2}\right)y(x_2)=\frac{\sin(\ln(x_2))}{x_2^2}.$$

$$\Rightarrow y''(x_2)+\frac{4}{3}y'(x_2)-\frac{8}{9}y(x_2)=\overbrace{\frac{\sin(\ln(1{,}5))}{1{,}5}}^{f_2}$$

$$\Rightarrow \frac{y(x_3)-2y(x_2)+y(x_1)}{h^2}+O(h^2)+\dots$$

$$+\frac{4}{3}\left[\frac{y(x_3)-y(x_1)}{2h}+O(h^2)\right]-\frac{8}{9}y(x_2)=f_2$$

$$\Rightarrow y(x_3)\left[\frac{1}{h^2}+\frac{4}{6h}\right]+y(x_2)\left[\frac{-2}{h^2}-\frac{8}{9}\right]+\dots$$

$$+\dots+y(x_1)\left[\frac{1}{h^2}-\frac{4}{6h}\right]+O(h^2)=f_2$$

$$\Rightarrow \frac{56}{3}y(x_3)-\frac{296}{9}y(x_2)+\frac{40}{3}y(x_1)+O(h^2)$$

$$=0{,}17530942$$

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$x = x_3 = 1,75$

$$y''(x_3) + \left(\frac{2}{x_3}\right) y'(x_3) - \frac{2}{x_3^2} y(x_3) = \frac{\sin(\ln x_3)}{x_3^2}$$

$$\Rightarrow \left(\frac{y(x_4) - 2y(x_3) + y(x_2)}{h^2} + O(h^2) \right)$$

$$+ \frac{8}{7} \left(\frac{y(x_4) - y(x_2)}{2h} + O(h^2) \right)$$

$$- \frac{32}{49} y(x_3) = 0,17334225$$

$$\Rightarrow y(x_4)\left(\frac{1}{h^2} + \frac{4}{7h}\right) + y(x_3)\left(\frac{2}{h^2} - \frac{32}{49}\right) + O(h^2)$$

$$+ y(x_2)\left(\frac{1}{h^2} - \frac{4}{7h}\right) = 0,17334225$$