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| RESEARCH ARTICLE

Collatz Conjecture (3N+1) Solution

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| ABSTRACT

Collatz Conjecture (3x+1) or in some literature as 3N+1 is a problem because it works in the way that if you take any positive number, if it is an odd number you multiply it by three (3) then add one (1). On the other hand, if it is an even number, you divide it by two (2). Eventually, all positive numbers decrease to one (1). One (1) is odd, so multiply it by three (3) is three (3) and add one (1) is four (4). Four (4) is even, so divide it by two (2) is two (2). Two (2) is also even, so divide it by two (2) is one (1) again. All positive numbers end up in the loop (4-2-1). This loop is like a numerical lock. Therefore, the solution of this problem will have to be a numerical key results to all positive numbers.

| KEYWORDS

Collatz Conjecture, Positive numbers, Even number, Odd number, Numerical

| ARTICLE INFORMATION

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1. Introduction

The Collatz conjecture is one of the most famous unsolved problems in mathematics. The conjecture asks whether repeating two simple arithmetic operations will eventually transform every positive integer into1. The Collatz conjecture (or "Syracuse problem") considers recursively-defined sequences of positive integers where n is succeeded by n/2, if n is even, or (3n+1), if n is odd[Christian, 2023].

It concerns sequences of integers in which each term is obtained from the previous term as follows: if the previous term is even, the next term is one half of the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1.

The conjecture is that these sequences always reach 1, no matter which positive integer is chosen to start the sequence. It is named after the mathematician Lothar Collatz, who introduced the idea in 1937, two years after receiving his doctorate[Christian, 2023]. It is also known as the 3n

+ 1 problem (or conjecture), the 3x + 1 problem (or conjecture), the Ulam conjecture (after Stan is ław Ulam), Kakutani's problem (after Shizuo Kakutani), the Thwaites conjecture (after Sir Bryan Thwaites), Hasse's algorithm (after Helmut Hasse), or the Syracuse problem [Daniel, 2022; Mercedes, 2022; Barina, 2022].

The sequence of numbers involved is sometimes referred to as the hailstone sequence, hailstone numbers or hailstone numerals (because the values are usually subject to multiple descents and ascents like hailstones in a cloud),[Samtani, 2023] or as wondrous numbers[Pickover, 2001].

For the Collatz function in the form

$$f(x) = \begin{cases} x/2, \ if\ x = 0 \\ (3x+1)/1, \ if\ x = 1 \end{cases} \quad (1)$$

Hailstone sequences can be computed by the 2-tag system with production rules

a → bc, b → a, c → aaa (2)

In this system, the positive integer x is represented by a string of x copies of a, and iteration of the tag operation halts on any word of length less than 2. (Adapted from De Mol.)

The Collatz conjecture equivalently states that this tag system, with an arbitrary finite string of a as the initial word, eventually halts (see Tag system for a worked example).

As of 2020, the conjecture has been checked by computer for all starting values up to 268 ≈ 2.95×1020. All initial values tested so far eventually end in the repeating cycle (4; 2; 1) of period 3[Hofstadter, 1979].

This computer evidence is still not rigorous proof that the conjecture is true for all starting values, as counterexamples may be found when considering very large (or possibly immense) positive integers, as in the case of the disproven Pólya conjecture.
However, such verifications may have other implications. For example, one can derive additional constraints on the period and structural form of a non-trivial cycle. [Barina, 2020],[Michael, 2021],[Ma, 2019].

In this paper, we need to use about 56 multiples of three (3) to result in the positive numbers from one (1) to a hundred (100). This illustrates that only the powers of three (3) has the power to generate all positive numbers when applying the two basic rules of the problem as shown in the calculations section. The conjecture states that for all starting values n the sequence eventually reaches the trivial cycle 1, 2, 1, 2, ... eventually, the existence of nontrivial cycles is interested[John, 2022].

2. Literature Review

An account of several strategies to address the Collatz conjecture can be found in papers of Lagaris [Renza, 2019; Patrick, 2021] and citations therein. The Collatz conjecture is considered and the density of values is compared to Planck's black body radiation in physics, showing a remarkable agreement between the two [Nicola, 2023], Collatz process does not diverge to positive infinity and eventually reaches one digit in binary[Terence, 2022].

Since one digit obtained from Collatz process in binary is equal to 1 in decimal, number of times that the Collatz process reaches 1 is limited [Makoto, 2023]. Is an unsolved mathematical problem that states the following: for any natural number, can always be reduced to 1 by following a series of steps defined by operations math.

The steps are defined as follows: if the number is even, divide by 2; if it is odd, multiply by 3 and add 1.
The conjecture suggests that regardless of the starting number, we will arrive at eventually to number 1 after a finite number of steps, entering an infinite loop with the numbers 4→2→1→4 [17]. In [Eduardo, 2023], Trees evolution of the Collatz dynamics showing only the odd numbers as shown in Figure1.

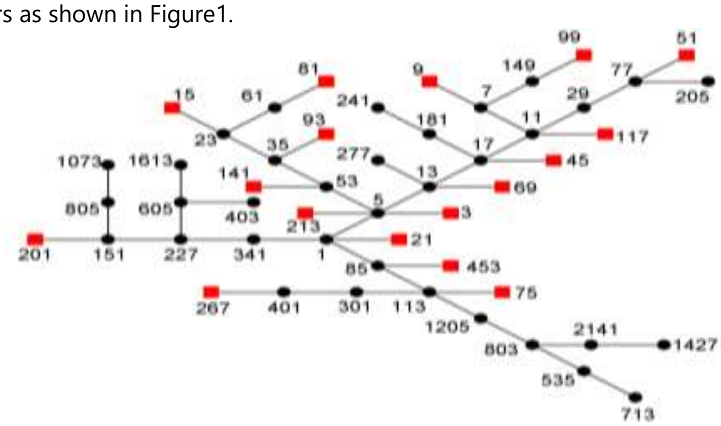

Figure.1 Trees evolution of the Collatz dynamics.

In Figure 1, Branches always start from a number multiple of 3, represented by red squares. Branches that start with black dots indicate that they are incomplète.

3. Methodology

First of all, the powers of two (2) is obvious in this problem. Many of the positive numbers after applying these two rules reach to either the number four (4). The number eight (8), the number sixteen (16), the number thirty-tow (32), or other numbers within the powers of two (2) tree. It is clear that these powers of tow take you all the way down to the number one (1) by dividing to two (2). Now, the collatz conjecture can be explained by the power of two (2).

However, the powers of three (3) is a more direct way to solve this problem. The solution of the Collatz Conjecture starts with a numerical key (the powers of three 3) which are all odd numbers.

Second step, we add one (1) to the powers of three (3) which results in even numbers. Thirdly, we divide the resulting numbers by two (2). Finally, if the result is an odd number we add one (1). If the result is an even number we divide by two (2). The result of this direct application of these two rules is all positive numbers.

The digit root of all powers of three (3) is always the number nine (9). This means that 9+1 are the two primary numbers (GOD's numbers) by Witch every other number can be produced. It is also possible that all negative numbers can be generated by this same relationship using the negative multiples of three (3) as such (-3x+1). The reason behind the second rule introduced in the problem (Dividing by 2) is that two numbers namely one (1) and nine (9) are needed to generate every other single number.
In this work, a novel layout of odd integers according to the equation (1) sequence can be shown in table 1.

4. Results and Discussion

A result between [1-100] demonstrates how certain numbers, establishing an associated instantiation of certain next odd integers to further Collatz subsequences as shown in table2[1-3-9-27-81]

Table 1: Calculations of the Collatz conjecture

3x	+1	÷2	Odd (+1) , Even (÷2)							
3	4	2	1							
9	10	5	6	3	4	2	1			
27	28	14	7	8	4	2	1			
81	82	41	42	21	22	11	12			
			6	3	4	2	1			
243	244	122	61	62	31	32	16			
			8	4	2	1				
729	730	365	366	183	184	92				
			46	24	12	6	3	4	2	1
2.187	2.188	1.094	547	548	274	137				
			138	69	70	35	36	18		
			9	10	5	6	3	4	2	1
6.561	6.562	3.281	3.282	1.641	1.642					
			821	822	411	412	206			
			103	104	52	26	13	14		
			7	8	4	2	1			
19.683	19.684	9.842	4.921	49.22	2.461	2.462				
			1.231	1.232	616	308	154			
			77	78	39	40	20	10		
			5	6	3	4	2	1		
59.049	59.050	29.525	29.526	14.763	14.764					
			7.382	3.691	3.692					
			1.846	923	924	462	231			
			232	116	58	29	30	15		
			16	8	4	2	1			
177.147	177.148	88.574	44.287	44.288	22.144					
			11.072	5.536	2.768					
			1.384	692	346	173				
			174	87	88	44	22	11		
			12	6	3	4	2	1		
531.441	531.442	265.721	265.722	132.861	132.862					
			66.431	66.432	33.216					
			16.608	8.304	4.152					
			2.076	1.038	519	520				
			260	130	65	66	33			
			34	17	18	9	10	5	6	
			3	4	2	1				
1.594.323	1.594.324	797.162	398.581	398.582						

			11.072	5.536	2.768					
			1.384	692	346	173				
			174	87	88	44	22	11		
			12	6	3	4	2	1		
531.441	531.442	265.721	265.722	132.861	132.862					
			66.431	66.432	33.216					
1.594.323	1.594.324	797.162	398.581	398.582						
			199.291	199.292						
			99.646	49.823						
			49.824	24.912	12.456					
			6.228	3.114	1.557					
			1.558	779	780	390				
			195	196	98	49	50			
			25	26	13	14	7	8		
			4	2	1					
4.782.969	4.782.970	2.391.485	2.391.486	1.195.743						
			1.195.744	597.872						
			298.936	149.468						
			74.734	37.367	37.368					
			18.684	9.342	4.671					
			4.672	2.336	1.168					
			584	292	146	73				
			74	37	38	19	20			
			10	5	6	3	4	2	1	

3x	+1	÷2	Odd (+1)		,	Even (÷2)				
14.348.907	110	55	56	28						
43.046.721	165	166	83	84	42					
129.140.163	124	62	31							
387.420.489	185	186	93	74	47	48				
1.162.261.467	139	140	70	35	36					
3.486.784.401	208	104	52							
10.460.353.203	156	78	39							
31.381.059.609	234	117	118	59	60					
94.143.178.827	176	88	44							
282.429.536.481	264	132	66							
847.288.609.443	198	99	100	50						
2.541.865.828.329	148	74								
7.625.597.484.987	222	111	112	56	23					
22.876.792.454.961	167	168	84							
68.630.377.364.883	250	125	126	63	64					
205.891.132.094.649	188	94	47							
617.673.396.283.947	141	142	71	72						
1.853.020.188.851.841	211	212	106	53	54					
5.559.060.566.555.523	158	79	80	40						
16.677.181.699.666.569	237	238	119	120	60					
50.031.545.098.999.707	178	89	90	45						
150.094.635.296.999.121	134	67	68							
450.283.905.890.997.363	200	100	50							
1.350.851.717.672.992.089	150	75	76							
4.052.555.153.018.976.267	113	114	57							
12.157.665.459.056.928.801	169	170	85	86	43					
36.472.996.377.170.786.403	127	128	64							
109.418.989.131.512.359.209	190	95	96	48						
328.256.967.394.537.077.627	143	144	72							
984.770.902.183.611.232.881	214	107	108	54						
2.954.312.706.550.833.698.643	161	162	81	82						
8.862.938.119.652.501.095.929	121	122	61	62						
26.588.814.358.957.503.287.787	181	182	91	92						
79.766.443.076.872.509.863.361	136	68								
239.299.329.230.617.529.590.083	102	51								
717.897.987.691.852.588.770.249	153	154	77							
2.153.693.963.075.557.766.310.747	115	116	58							
6.461.081.889.226.673.298.932.241	172	86								
19.383.245.667.680.019.896.796.723	129	130	65							
58.149.737.003.040.059.690.390.169	193	194	97							
174.449.211.009.120.179.071.170.507	145	146	83							
523.347.633.027.360.537.213.511.521	109	110	55							

]

Table 2 Odd numbers obtaining

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The simulation of the 3x values is shown in figure 2

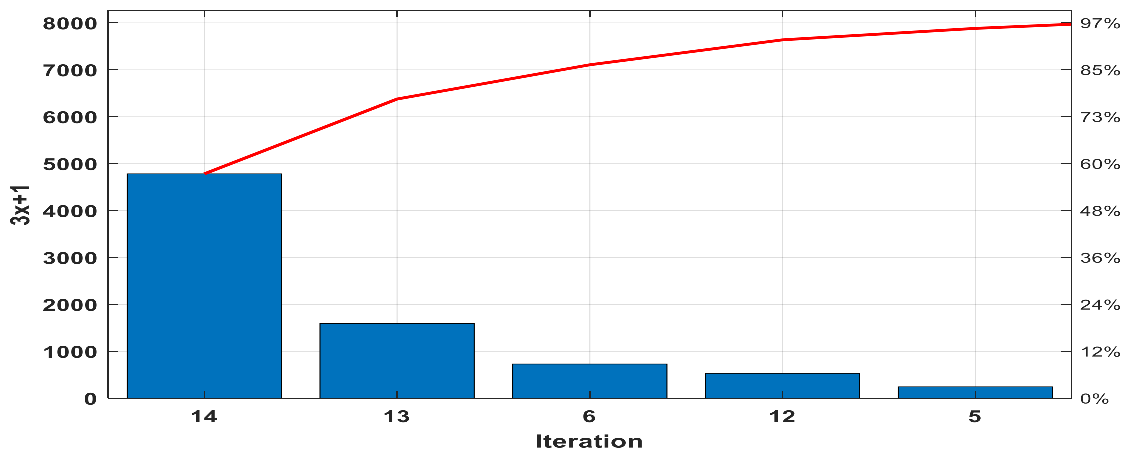

Figure2. A Pareto chart of the 3x+1 values

The validation of the (3x+1)/2 is shown in Figure3

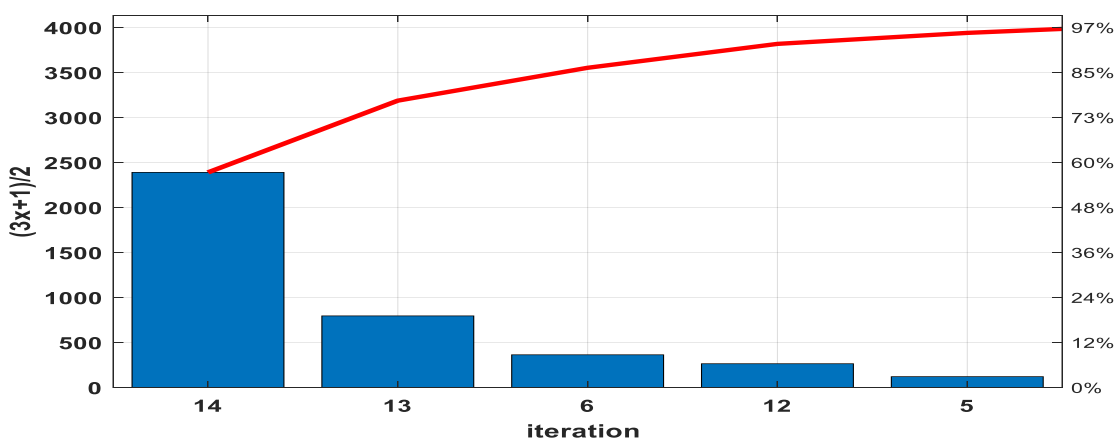

Figure3. A Pareto chart of the (3x+1)/2 values

A Pareto chart is a type of chart that contains both bars and a line graph, where individual values are represented in descending order by bars, and the cumulative total is represented by the line. The chart is named for the Pareto principle, which, in turn, derives its name from Vilfredo Pareto, a noted Italian economist.

The left vertical axis is the frequency of occurrence, but it can alternatively represent cost or another important unit of measure. The right vertical axis is the cumulative percentage of the total number of occurrences, total cost, or total of the particular unit of measure. Because the values are in decreasing order, the cumulative function is a concave function.

The purpose of the Pareto chart is to highlight the most important among a (typically large) set of factors. In quality control, Pareto charts are useful to find the defects to prioritize in order to observe the greatest overall improvement. It often represents the most common sources of defects, the highest occurring type of defect, or the most frequent reasons for customer complaints, and so on. Wilkinson (2006) devised an algorithm for producing statistically based acceptance limits (similar to confidence intervals) for each bar in the Pareto chart which is completely shows the effectiveness of the solution of the Collatz conjecture problem [19].

5. Conclusion

The colatz conjecture involves a numerical lock (4-2-1) loop. The solution starts with a numerical key (the powers of 3) and result in all positive numbers by direct application of the two basic rules introduced in the problem. In conclusion, the number one (1) and the number nine (9) are the origin of all numbers.

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