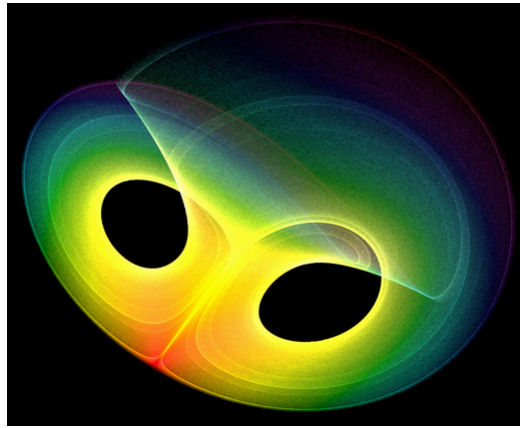


# Mathematical analysis

**Analysis** is the branch of [mathematics](#) dealing with [continuous functions](#), [limits](#), and related theories, such as [differentiation](#), [integration](#), measure, [infinite sequences](#), [series](#), and [analytic functions](#).<sup>[1][2]</sup>

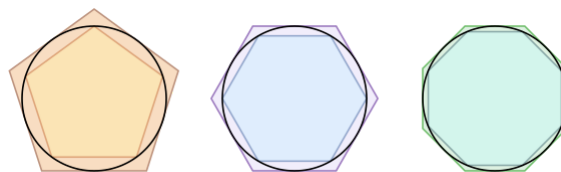


A [strange attractor](#) arising from a [differential equation](#). Differential equations are an important area of mathematical analysis with many applications in science and engineering.

These theories are usually studied in the context of [real](#) and [complex](#) numbers and [functions](#). Analysis evolved from [calculus](#), which involves the elementary concepts and techniques of analysis. Analysis may be distinguished from [geometry](#); however, it can be applied to any [space](#) of [mathematical objects](#) that has a definition of nearness (a [topological space](#)) or specific distances between objects (a [metric space](#)).

## History

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[Archimedes](#) used the [method of exhaustion](#) to compute the [area](#) inside a circle by finding the area of [regular polygons](#) with more and more sides. This was an early but informal example of a [limit](#), one of the most basic concepts in mathematical analysis.

## Ancient

Mathematical analysis formally developed in the 17th century during the [Scientific Revolution](#),<sup>[3]</sup> but many of its ideas can be traced back to earlier mathematicians. Early results in analysis were

implicitly present in the early days of [ancient Greek mathematics](#). For instance, an [infinite geometric sum](#) is implicit in [Zeno's paradox of the dichotomy](#).<sup>[4]</sup> (Strictly speaking, the point of the paradox is to deny that the infinite sum exists.) Later, [Greek mathematicians](#) such as [Eudoxus](#) and [Archimedes](#) made more explicit, but informal, use of the concepts of limits and convergence when they used the [method of exhaustion](#) to compute the area and volume of regions and solids.<sup>[5]</sup> The explicit use of [infinitesimals](#) appears in Archimedes' *The Method of Mechanical Theorems*, a work rediscovered in the 20th century.<sup>[6]</sup> In Asia, the [Chinese mathematician Liu Hui](#) used the method of exhaustion in the 3rd century CE to find the area of a circle.<sup>[7]</sup> From Jain literature, it appears that Hindus were in possession of the formulae for the sum of the [arithmetic](#) and [geometric](#) series as early as the 4th century BCE.<sup>[8]</sup> [Ācārya Bhadrabāhu](#) uses the sum of a geometric series in his *Kalpasūtra* in 433 BCE.<sup>[9]</sup>

## Medieval

[Zu Chongzhi](#) established a method that would later be called [Cavalieri's principle](#) to find the volume of a [sphere](#) in the 5th century.<sup>[10]</sup> In the 12th century, the [Indian mathematician Bhāskara II](#) used infinitesimal and used what is now known as [Rolle's theorem](#).<sup>[11]</sup>

In the 14th century, [Madhava of Sangamagrama](#) developed [infinite series](#) expansions, now called [Taylor series](#), of functions such as [sine](#), [cosine](#), [tangent](#) and [arctangent](#).<sup>[12]</sup> Alongside his development of Taylor series of [trigonometric functions](#), he also estimated the magnitude of the error terms resulting of truncating these series, and gave a rational approximation of some infinite series. His followers at the [Kerala School of Astronomy and Mathematics](#) further expanded his works, up to the 16th century.

## Modern

### Foundations

The modern foundations of mathematical analysis were established in 17th century Europe.<sup>[3]</sup> This began when [Fermat](#) and [Descartes](#) developed [analytic geometry](#), which is the precursor to modern calculus. Fermat's method of [adequality](#) allowed him to determine the maxima and minima of functions and the tangents of curves.<sup>[13]</sup> Descartes's publication of *La Géométrie* in 1637, which introduced the [Cartesian coordinate system](#), is considered to be the establishment of mathematical analysis. It would be a few decades later that [Newton](#) and [Leibniz](#) independently developed [infinitesimal calculus](#), which grew, with the stimulus of applied work that continued through the 18th century, into analysis topics such as the [calculus of variations](#), [ordinary](#) and [partial differential equations](#), [Fourier analysis](#), and [generating functions](#). During this period, calculus techniques were applied to approximate [discrete problems](#) by continuous ones.

## Modernization

In the 18th century, [Euler](#) introduced the notion of a [mathematical function](#).<sup>[14]</sup> Real analysis began to emerge as an independent subject when [Bernard Bolzano](#) introduced the modern definition of continuity in 1816,<sup>[15]</sup> but Bolzano's work did not become widely known until the 1870s. In 1821, [Cauchy](#) began to put calculus on a firm logical foundation by rejecting the principle of the [generality of algebra](#) widely used in earlier work, particularly by Euler. Instead, Cauchy formulated calculus in terms of geometric ideas and [infinitesimals](#). Thus, his definition of continuity required an infinitesimal change in  $x$  to correspond to an infinitesimal change in  $y$ . He also introduced the concept of the [Cauchy sequence](#), and started the formal theory of [complex analysis](#). [Poisson](#), [Liouville](#), [Fourier](#) and others studied partial differential equations and [harmonic analysis](#). The contributions of these mathematicians and others, such as [Weierstrass](#), developed the [\( \$\epsilon\$ ,  \$\delta\$ \)-definition of limit](#) approach, thus founding the modern field of mathematical analysis. Around the same time, [Riemann](#) introduced his theory of [integration](#), and made significant advances in complex analysis.

Towards the end of the 19th century, mathematicians started worrying that they were assuming the existence of a [continuum](#) of [real numbers](#) without proof. [Dedekind](#) then constructed the real numbers by [Dedekind cuts](#), in which irrational numbers are formally defined, which serve to fill the "gaps" between rational numbers, thereby creating a [complete](#) set: the continuum of real numbers, which had already been developed by [Simon Stevin](#) in terms of [decimal expansions](#). Around that time, the attempts to refine the [theorems](#) of [Riemann integration](#) led to the study of the "size" of the set of [discontinuities](#) of real functions.

Also, various [pathological objects](#), (such as [nowhere continuous functions](#), continuous but [nowhere differentiable functions](#), and [space-filling curves](#)), commonly known as "monsters", began to be investigated. In this context, [Jordan](#) developed his theory of [measure](#), [Cantor](#) developed what is now called [naive set theory](#), and [Baire](#) proved the [Baire category theorem](#). In the early 20th century, calculus was formalized using an axiomatic [set theory](#). [Lebesgue](#) greatly improved measure theory, and introduced his own theory of integration, now known as [Lebesgue integration](#), which proved to be a big improvement over Riemann's. [Hilbert](#) introduced [Hilbert spaces](#) to solve [integral equations](#). The idea of [normed vector space](#) was in the air, and in the 1920s [Banach](#) created [functional analysis](#).

## Important concepts

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### Metric spaces

In [mathematics](#), a metric space is a [set](#) where a notion of [distance](#) (called a [metric](#)) between elements of the set is defined.

Much of analysis happens in some metric space; the most commonly used are the [real line](#), the [complex plane](#), [Euclidean space](#), other [vector spaces](#), and the [integers](#). Examples of analysis without a metric include [measure theory](#) (which describes size rather than distance) and [functional analysis](#) (which studies [topological vector spaces](#) that need not have any sense of distance).

Formally, a metric space is an [ordered pair](#)  $(M, d)$  where  $M$  is a set and  $d$  is a [metric](#) on  $M$ , i.e., a [function](#)

$$d: M \times M \rightarrow \mathbb{R}$$

such that for any  $x, y, z \in M$ , the following holds:

1.  $d(x, y) \geq 0$ , with equality [if and only if](#)  $x = y$  ([identity of indiscernibles](#)),
2.  $d(x, y) = d(y, x)$  ([symmetry](#)), and
3.  $d(x, z) \leq d(x, y) + d(y, z)$  ([triangle inequality](#)).

By taking the third property and letting  $z = x$ , it can be shown that  $d(x, y) \geq 0$  ([non-negative](#)).

## Sequences and limits

A sequence is an ordered list. Like a [set](#), it contains [members](#) (also called *elements*, or *terms*). Unlike a set, order matters, and exactly the same elements can appear multiple times at different positions in the sequence. Most precisely, a sequence can be defined as a [function](#) whose domain is a [countable totally ordered](#) set, such as the [natural numbers](#).

One of the most important properties of a sequence is *convergence*. Informally, a sequence converges if it has a *limit*. Continuing informally, a ([singly-infinite](#)) sequence has a limit if it approaches some point  $x$ , called the limit, as  $n$  becomes very large. That is, for an abstract sequence  $(a_n)$  (with  $n$  running from 1 to infinity understood) the distance between  $a_n$  and  $x$  approaches 0 as  $n \rightarrow \infty$ , denoted

$$\lim_{n \rightarrow \infty} a_n = x.$$

## Main branches

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### Calculus

### Real analysis

Real analysis (traditionally, the "theory of functions of a real variable") is a branch of mathematical analysis dealing with the [real numbers](#) and real-valued functions of a real variable.<sup>[16][17]</sup> In

particular, it deals with the analytic properties of real [functions](#) and [sequences](#), including [convergence](#) and [limits](#) of [sequences](#) of real numbers, the [calculus](#) of the real numbers, and [continuity](#), [smoothness](#) and related properties of real-valued functions.

## Complex analysis

Complex analysis (traditionally known as the "theory of functions of a complex variable") is the branch of mathematical analysis that investigates [functions](#) of [complex numbers](#).<sup>[18]</sup> It is useful in many branches of mathematics, including [algebraic geometry](#), [number theory](#), [applied mathematics](#); as well as in [physics](#), including [hydrodynamics](#), [thermodynamics](#), [mechanical engineering](#), [electrical engineering](#), and particularly, [quantum field theory](#).

Complex analysis is particularly concerned with the [analytic functions](#) of complex variables (or, more generally, [meromorphic functions](#)). Because the separate [real](#) and [imaginary](#) parts of any analytic function must satisfy [Laplace's equation](#), complex analysis is widely applicable to two-dimensional problems in [physics](#).

## Functional analysis

Functional analysis is a branch of mathematical analysis, the core of which is formed by the study of [vector spaces](#) endowed with some kind of limit-related structure (e.g. [inner product](#), [norm](#), [topology](#), etc.) and the [linear operators](#) acting upon these spaces and respecting these structures in a suitable sense.<sup>[19][20]</sup> The historical roots of functional analysis lie in the study of [spaces of functions](#) and the formulation of properties of transformations of functions such as the [Fourier transform](#) as transformations defining [continuous](#), [unitary](#) etc. operators between function spaces. This point of view turned out to be particularly useful for the study of [differential](#) and [integral equations](#).

## Harmonic analysis

Harmonic analysis is a branch of mathematical analysis concerned with the representation of [functions](#) and [signals](#) as the superposition of basic [waves](#). This includes the study of the notions of [Fourier series](#) and [Fourier transforms](#) ([Fourier analysis](#)), and of their generalizations. Harmonic analysis has applications in areas as diverse as [music theory](#), [number theory](#), [representation theory](#), [signal processing](#), [quantum mechanics](#), [tidal analysis](#), and [neuroscience](#).

## Differential equations

A differential equation is a [mathematical equation](#) for an unknown [function](#) of one or several [variables](#) that relates the values of the function itself and its [derivatives](#) of various [orders](#).<sup>[21][22][23]</sup> Differential equations play a prominent role in [engineering](#), [physics](#), [economics](#), [biology](#), and other disciplines.

Differential equations arise in many areas of science and technology, specifically whenever a [deterministic](#) relation involving some continuously varying quantities (modeled by functions) and their rates of change in space or time (expressed as derivatives) is known or postulated. This is illustrated in [classical mechanics](#), where the motion of a body is described by its position and velocity as the time value varies. [Newton's laws](#) allow one (given the position, velocity, acceleration and various forces acting on the body) to express these variables dynamically as a differential equation for the unknown position of the body as a function of time. In some cases, this differential equation (called an [equation of motion](#)) may be solved explicitly.

## Measure theory

A measure on a [set](#) is a systematic way to assign a number to each suitable [subset](#) of that set, intuitively interpreted as its size.<sup>[24]</sup> In this sense, a measure is a generalization of the concepts of length, area, and volume. A particularly important example is the [Lebesgue measure](#) on a [Euclidean space](#), which assigns the conventional [length](#), [area](#), and [volume](#) of [Euclidean geometry](#) to suitable subsets of the  $n$ -dimensional Euclidean space  $\mathbb{R}^n$ . For instance, the Lebesgue measure of the [interval](#)  $[0, 1]$  in the [real numbers](#) is its length in the everyday sense of the word – specifically, 1.

Technically, a measure is a function that assigns a non-negative real number or  $+\infty$  to (certain) subsets of a set  $X$ . It must assign 0 to the [empty set](#) and be ([countably](#)) additive: the measure of a 'large' subset that can be decomposed into a finite (or countable) number of 'smaller' disjoint subsets, is the sum of the measures of the "smaller" subsets. In general, if one wants to associate a *consistent* size to *each* subset of a given set while satisfying the other axioms of a measure, one only finds trivial examples like the [counting measure](#). This problem was resolved by defining measure only on a sub-collection of all subsets; the so-called *measurable* subsets, which are required to form a  $\sigma$ -[algebra](#). This means that the empty set, countable [unions](#), countable [intersections](#) and [complements](#) of measurable subsets are measurable. [Non-measurable sets](#) in a Euclidean space, on which the Lebesgue measure cannot be defined consistently, are necessarily complicated in the sense of being badly mixed up with their complement. Indeed, their existence is a non-trivial consequence of the [axiom of choice](#).



## Numerical analysis

Numerical analysis is the study of [algorithms](#) that use numerical [approximation](#) (as opposed to general [symbolic manipulations](#)) for the problems of mathematical analysis (as distinguished from [discrete mathematics](#)).<sup>[25]</sup>

Modern numerical analysis does not seek exact answers, because exact answers are often impossible to obtain in practice. Instead, much of numerical analysis is concerned with obtaining approximate solutions while maintaining reasonable bounds on errors.

Numerical analysis naturally finds applications in all fields of engineering and the physical sciences, but in the 21st century, the life sciences and even the arts have adopted elements of scientific computations. [Ordinary differential equations](#) appear in [celestial mechanics](#) (planets, stars and galaxies); [numerical linear algebra](#) is important for data analysis; [stochastic differential equations](#) and [Markov chains](#) are essential in simulating living cells for medicine and biology.

## Vector analysis

*Vector analysis*, also called *vector calculus*, is a branch of mathematical analysis dealing with [vector-valued functions](#).<sup>[26]</sup>

## Scalar analysis

Scalar analysis is a branch of mathematical analysis dealing with values related to scale as opposed to direction. Values such as temperature are scalar because they describe the magnitude of a value without regard to direction, force, or displacement that value may or may not have.

## Tensor analysis

## Other topics

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- [Calculus of variations](#) deals with extremizing [functionals](#), as opposed to ordinary [calculus](#) which deals with [functions](#).
- [Harmonic analysis](#) deals with the representation of [functions](#) or signals as the [superposition](#) of basic [waves](#).
- [Geometric analysis](#) involves the use of geometrical methods in the study of [partial differential equations](#) and the application of the theory of partial differential equations to geometry.

- [Clifford analysis](#), the study of Clifford valued functions that are annihilated by Dirac or Dirac-like operators, termed in general as monogenic or Clifford analytic functions.
- [p-adic analysis](#), the study of analysis within the context of [p-adic numbers](#), which differs in some interesting and surprising ways from its real and complex counterparts.
- [Non-standard analysis](#), which investigates the [hyperreal numbers](#) and their functions and gives a [rigorous](#) treatment of [infinitesimals](#) and infinitely large numbers.
- [Computable analysis](#), the study of which parts of analysis can be carried out in a [computable](#) manner.
- [Stochastic calculus](#) – analytical notions developed for [stochastic processes](#).
- [Set-valued analysis](#) – applies ideas from analysis and topology to set-valued functions.
- [Convex analysis](#), the study of convex sets and functions.
- [Idempotent analysis](#) – analysis in the context of an [idempotent semiring](#), where the lack of an additive inverse is compensated somewhat by the idempotent rule  $A + A = A$ .
  - [Tropical analysis](#) – analysis of the idempotent semiring called the [tropical semiring](#) (or [max-plus algebra/min-plus algebra](#)).
- [Constructive analysis](#), which is built upon a foundation of [constructive](#), rather than classical, logic and set theory.
- [Intuitionistic analysis](#), which is developed from constructive logic like constructive analysis but also incorporates [choice sequences](#).
- [Paraconsistent analysis](#), which is built upon a foundation of [paraconsistent](#), rather than classical, logic and set theory.
- [Smooth infinitesimal analysis](#), which is developed in a smooth topos.

## Applications

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Techniques from analysis are also found in other areas such as:

### Physical sciences

The vast majority of [classical mechanics](#), [relativity](#), and [quantum mechanics](#) is based on applied analysis, and [differential equations](#) in particular. Examples of important differential equations include [Newton's second law](#), the [Schrödinger equation](#), and the [Einstein field equations](#).

[Functional analysis](#) is also a major factor in [quantum mechanics](#).



## Signal processing

When processing signals, such as [audio](#), [radio waves](#), light waves, [seismic waves](#), and even images, Fourier analysis can isolate individual components of a compound waveform, concentrating them for easier detection or removal. A large family of signal processing techniques consist of Fourier-transforming a signal, manipulating the Fourier-transformed data in a simple way, and reversing the transformation.<sup>[27]</sup>

## Other areas of mathematics

Techniques from analysis are used in many areas of mathematics, including:

- [Analytic number theory](#)
- [Analytic combinatorics](#)
- [Continuous probability](#)
- [Differential entropy](#) in information theory
- [Differential games](#)
- [Differential geometry](#), the application of calculus to specific mathematical spaces known as [manifolds](#) that possess a complicated internal structure but behave in a simple manner locally.
- [Differentiable manifolds](#)
- [Differential topology](#)
- [Partial differential equations](#)

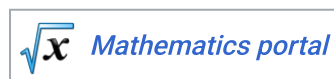
## Famous Textbooks

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- Foundation of Analysis: The Arithmetic of Whole Rational, Irrational and Complex Numbers, by Edmund Landau
- Introductory Real Analysis, by [Andrey Kolmogorov](#), [Sergei Fomin](#)<sup>[28]</sup>
- Differential and Integral Calculus (3 volumes), by [Grigorii Fichtenholz](#)<sup>[29][30][31]</sup>
- The Fundamentals of Mathematical Analysis (2 volumes), by [Grigorii Fichtenholz](#)<sup>[32][33]</sup>
- A Course Of Mathematical Analysis (2 volumes), by [Sergey Nikolsky](#)<sup>[34][35]</sup>
- Mathematical Analysis (2 volumes), by [Vladimir Zorich](#)<sup>[36][37]</sup>
- A Course of Higher Mathematics (5 volumes, 6 parts), by [Vladimir Smirnov](#)<sup>[38][39][40][41][42]</sup>
- Differential And Integral Calculus, by [Nikolai Piskunov](#)<sup>[43]</sup>

- A Course of Mathematical Analysis, by [Aleksandr Khinchin](#)<sup>[44]</sup>
- Mathematical Analysis: A Special Course, by [Georgiy Shilov](#)<sup>[45]</sup>
- Theory of Functions of a Real Variable (2 volumes), by [Isidor Natanson](#)<sup>[46][47]</sup>
- Problems in Mathematical Analysis, by [Boris Demidovich](#)<sup>[48]</sup>
- [Problems and Theorems in Analysis](#) (2 volumes), by [George Pólya](#), [Gábor Szegő](#)<sup>[49][50]</sup>
- Mathematical Analysis: A Modern Approach to Advanced Calculus, by [Tom Apostol](#)<sup>[51]</sup>
- Principles of Mathematical Analysis, by [Walter Rudin](#)<sup>[52]</sup>
- Real Analysis: Measure Theory, Integration, and Hilbert Spaces, by [Elias Stein](#)<sup>[53]</sup>
- Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable, by [Lars Ahlfors](#)<sup>[54]</sup>
- Complex Analysis, by [Elias Stein](#)<sup>[55]</sup>
- Functional Analysis: Introduction to Further Topics in Analysis, by [Elias Stein](#)<sup>[56]</sup>
- Analysis (2 volumes), by [Terence Tao](#)<sup>[57][58]</sup>
- Analysis (3 volumes), by [Herbert Amann](#), [Joachim Escher](#)<sup>[59][60][61]</sup>
- Real and Functional Analysis, by [Vladimir Bogachev](#), [Oleg Smolyanov](#)<sup>[62]</sup>
- Real and Functional Analysis, by [Serge Lang](#)<sup>[63]</sup>

## See also



- [Constructive analysis](#)
- [History of calculus](#)
- [Hypercomplex analysis](#)
- [Multiple rule-based problems](#)
- [Multivariable calculus](#)
- [Paraconsistent logic](#)
- [Smooth infinitesimal analysis](#)
- [Timeline of calculus and mathematical analysis](#)

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## External links

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- [Earliest Known Uses of Some of the Words of Mathematics: Calculus & Analysis \(http://www.economics.soton.ac.uk/staff/aldrich/Calculus%20and%20Analysis%20Earliest%20Uses.htm\)](http://www.economics.soton.ac.uk/staff/aldrich/Calculus%20and%20Analysis%20Earliest%20Uses.htm)
- [Basic Analysis: Introduction to Real Analysis \(http://www.jirka.org/ra/\)](http://www.jirka.org/ra/) by Jiri Lebl (Creative Commons BY-NC-SA)
- [Mathematical Analysis – Encyclopædia Britannica \(https://www.britannica.com/topic/analysis-mathematics\)](https://www.britannica.com/topic/analysis-mathematics)
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