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List of integrals of trigonometric functions

The following is a list of integrals (antiderivative functions) of trigonometric functions. For antiderivatives involving both exponential and trigonometric functions, see List of integrals of exponential functions. For a complete list of antiderivative functions, see Lists of integrals. For the special antiderivatives involving trigonometric functions, see Trigonometric integral.

Generally, if the function $\sin x$ is any trigonometric function, and $\cos x$ is its derivative,

$$\int a\cos nx\,dx = \frac{a}{n}\sin nx + C$$

In all formulas the constant a is assumed to be nonzero, and C denotes the constant of integration.

Integrands involving only sine

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax + C = \frac{x}{2} - \frac{1}{2a}\sin ax\cos ax + C$$

$$\int \sin^3 ax\,dx = \frac{\cos 3ax}{12a} - \frac{3\cos ax}{4a} + C$$

$$\int x\sin^2 ax\,dx = \frac{x^2}{4} - \frac{x}{4a}\sin 2ax - \frac{1}{8a^2}\cos 2ax + C$$

$$\int x^2\sin^2 ax\,dx = \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3}\right)\sin 2ax - \frac{x}{4a^2}\cos 2ax + C$$

$$\int x\sin ax\,dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a} + C$$

$$\int (\sin b_1x)(\sin b_2x)\,dx = \frac{\sin((b_2-b_1)x)}{2(b_2-b_1)} - \frac{\sin((b_1+b_2)x)}{2(b_1+b_2)} + C \qquad (\text{for } |b_1| \neq |b_2|)$$

$$\int \sin^n ax\,dx = -\frac{\sin^{n-1} ax\cos ax}{na} + \frac{n-1}{n}\int \sin^{n-2} ax\,dx \qquad (\text{for } n>0)$$

$$\int \frac{dx}{\sin ax} = -\frac{1}{a}\ln|\csc ax + \cot ax| + C$$

$$\int \frac{dx}{\sin^n ax} = \frac{\cos ax}{a(1-n)\sin^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} ax} \qquad (\text{for } n>1)$$

$$\begin{aligned}\int x^n\sin ax\,dx &= -\frac{x^n}{a}\cos ax + \frac{n}{a}\int x^{n-1}\cos ax\,dx \\ &= \sum_{k=0}^{2k\le n}(-1)^{k+1}\frac{x^{n-2k}}{a^{1+2k}}\frac{n!}{(n-2k)!}\cos ax + \sum_{k=0}^{2k+1\le n}(-1)^k\frac{x^{n-1-2k}}{a^{2+2k}}\frac{n!}{(n-2k-1)!}\sin ax \\ &= -\sum_{k=0}^{n}\frac{x^{n-k}}{a^{1+k}}\frac{n!}{(n-k)!}\cos\left(ax + k\frac{\pi}{2}\right) \qquad (\text{for } n>0)\end{aligned}$$

$$\int \frac{\sin ax}{x}\,dx = \sum_{n=0}^{\infty}(-1)^n\frac{(ax)^{2n+1}}{(2n+1)\cdot(2n+1)!} + C$$

$$\int \frac{\sin ax}{x^n}\,dx = -\frac{\sin ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\cos ax}{x^{n-1}}\,dx$$

$$\int \sin(ax^2+bx+c)dx = \begin{cases} \sqrt{a}\sqrt{\frac{\pi}{2}}\cos\left(\frac{b^2-4ac}{4a}\right)S\left(\frac{2ax+b}{\sqrt{2a\pi}}\right) + \sqrt{a}\sqrt{\frac{\pi}{2}}\sin\left(\frac{b^2-4ac}{4a}\right)C\left(\frac{2ax+b}{\sqrt{2a\pi}}\right) & to\ b^2-4ac>0 \\ \sqrt{a}\sqrt{\frac{\pi}{2}}\cos\left(\frac{b^2-4ac}{4a}\right)S\left(\frac{2ax+b}{\sqrt{2a\pi}}\right) - \sqrt{a}\sqrt{\frac{\pi}{2}}\sin\left(\frac{b^2-4ac}{4a}\right)C\left(\frac{2ax+b}{\sqrt{2a\pi}}\right) & to\ b^2-4ac<0 \end{cases} \quad for\ a\neq0,\ a>0$$

$$\int \frac{dx}{1\pm\sin ax} = \frac{1}{a}\tan\left(\frac{ax}{2}\mp\frac{\pi}{4}\right) + C$$

$$\int \frac{x\,dx}{1+\sin ax} = \frac{x}{a}\tan\left(\frac{ax}{2}-\frac{\pi}{4}\right) + \frac{2}{a^2}\ln\left|\cos\left(\frac{ax}{2}-\frac{\pi}{4}\right)\right| + C$$

$$\int \frac{x\,dx}{1-\sin ax} = \frac{x}{a}\cot\left(\frac{\pi}{4}-\frac{ax}{2}\right) + \frac{2}{a^2}\ln\left|\sin\left(\frac{\pi}{4}-\frac{ax}{2}\right)\right| + C$$

$$\int \frac{\sin ax\,dx}{1 \pm \sin ax} = \pm x + \frac{1}{a}\tan\left(\frac{\pi}{4} \mp \frac{ax}{2}\right) + C$$

Integrands involving only cosine

$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax + C = \frac{x}{2} + \frac{1}{2a}\sin ax\cos ax + C$$

$$\int \cos^n ax\,dx = \frac{\cos^{n-1} ax\sin ax}{na} + \frac{n-1}{n}\int \cos^{n-2} ax\,dx \qquad (\text{for } n > 0)$$

$$\int x\cos ax\,dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a} + C$$

$$\int x^2\cos^2 ax\,dx = \frac{x^3}{6} + \left(\frac{x^2}{4a} - \frac{1}{8a^3}\right)\sin 2ax + \frac{x}{4a^2}\cos 2ax + C$$

$$\begin{aligned}\int x^n\cos ax\,dx &= \frac{x^n\sin ax}{a} - \frac{n}{a}\int x^{n-1}\sin ax\,dx \\ &= \sum_{k=0}^{2k+1\le n}(-1)^k\frac{x^{n-2k-1}}{a^{2+2k}}\frac{n!}{(n-2k-1)!}\cos ax + \sum_{k=0}^{2k\le n}(-1)^k\frac{x^{n-2k}}{a^{1+2k}}\frac{n!}{(n-2k)!}\sin ax \\ &= \sum_{k=0}^{n}(-1)^{\lfloor k/2\rfloor}\frac{x^{n-k}}{a^{1+k}}\frac{n!}{(n-k)!}\cos\left(ax - \frac{(-1)^k+1}{2}\frac{\pi}{2}\right) \\ &= \sum_{k=0}^{n}\frac{x^{n-k}}{a^{1+k}}\frac{n!}{(n-k)!}\sin\left(ax + k\frac{\pi}{2}\right) \qquad (\text{for } n > 0)\end{aligned}$$

$$\int \frac{\cos ax}{x}\,dx = \ln|ax| + \sum_{k=1}^{\infty}(-1)^k\frac{(ax)^{2k}}{2k\cdot(2k)!} + C$$

$$\int \frac{\cos ax}{x^n}\,dx = -\frac{\cos ax}{(n-1)x^{n-1}} - \frac{a}{n-1}\int \frac{\sin ax}{x^{n-1}}\,dx \qquad (\text{for } n \neq 1)$$

$$\int \frac{dx}{\cos ax} = \frac{1}{a}\ln\left|\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)\right| + C$$

$$\int \frac{dx}{\cos^n ax} = \frac{\sin ax}{a(n-1)\cos^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} ax} \qquad (\text{for } n > 1)$$

$$\int \frac{dx}{1+\cos ax} = \frac{1}{a}\tan\frac{ax}{2} + C$$

$$\int \frac{dx}{1-\cos ax} = -\frac{1}{a}\cot\frac{ax}{2} + C$$

$$\int \frac{x\,dx}{1+\cos ax} = \frac{x}{a}\tan\frac{ax}{2} + \frac{2}{a^2}\ln\left|\cos\frac{ax}{2}\right| + C$$

$$\int \frac{x\,dx}{1-\cos ax} = -\frac{x}{a}\cot\frac{ax}{2} + \frac{2}{a^2}\ln\left|\sin\frac{ax}{2}\right| + C$$

$$\int \frac{\cos ax\,dx}{1+\cos ax} = x - \frac{1}{a}\tan\frac{ax}{2} + C$$

$$\int \frac{\cos ax\,dx}{1-\cos ax} = -x - \frac{1}{a}\cot\frac{ax}{2} + C$$

$$\int(\cos a_1x)(\cos a_2x)\,dx = \frac{\sin((a_2-a_1)x)}{2(a_2-a_1)} + \frac{\sin((a_2+a_1)x)}{2(a_2+a_1)} + C \qquad (\text{for } |a_1| \neq |a_2|)$$

Integrands involving only tangent

$$\int \tan ax\,dx = -\frac{1}{a}\ln|\cos ax| + C = \frac{1}{a}\ln|\sec ax| + C$$

$$\int \tan^2 x\,dx = \tan x - x + C$$

$$\int \tan^n ax\,dx = \frac{1}{a(n-1)}\tan^{n-1} ax - \int \tan^{n-2} ax\,dx \qquad (\text{for } n \neq 1)$$

$$\int \frac{dx}{q\tan ax + p} = \frac{1}{p^2+q^2}\left(px + \frac{q}{a}\ln|q\sin ax + p\cos ax|\right) + C \qquad (\text{for } p^2+q^2 \neq 0)$$

$$\int \frac{dx}{\tan ax \pm 1} = \pm\frac{x}{2} + \frac{1}{2a}\ln|\sin ax \pm \cos ax| + C$$

$$\int \frac{\tan ax\,dx}{\tan ax \pm 1} = \frac{x}{2} \mp \frac{1}{2a} \ln|\sin ax \pm \cos ax| + C$$

Integrands involving only secant

$$\int \sec ax\,dx = \frac{1}{a} \ln|\sec ax + \tan ax| + C = \frac{1}{a} \ln\left|\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)\right| + C = \frac{1}{a} \operatorname{artanh}(\sin ax) + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec^3 x\,dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C.$$

$$\int \sec^n ax\,dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} \;+\; \frac{n-2}{n-1} \int \sec^{n-2} ax\,dx \qquad (\text{for } n \neq 1)$$

$$\int \frac{dx}{\sec x + 1} = x - \tan\frac{x}{2} + C$$

$$\int \frac{dx}{\sec x - 1} = -x - \cot\frac{x}{2} + C$$

$$\int \frac{\sin x}{\cos x} = \int \tan x$$

Integrands involving only cosecant

$$\int \csc ax\,dx = -\frac{1}{a} \ln|\csc ax + \cot ax| + C = \frac{1}{a} \ln|\csc ax - \cot ax| + C = \frac{1}{a} \ln\left|\tan\left(\frac{ax}{2}\right)\right| + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \csc^3 x\,dx = -\frac{1}{2} \csc x \cot x - \frac{1}{2} \ln|\csc x + \cot x| + C = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln|\csc x - \cot x| + C$$

$$\int \csc^n ax\,dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} \;+\; \frac{n-2}{n-1} \int \csc^{n-2} ax\,dx \qquad (\text{for } n \neq 1)$$

$$\int \frac{dx}{\csc x + 1} = x - \frac{2}{\cot\frac{x}{2} + 1} + C$$

$$\int \frac{dx}{\csc x - 1} = -x + \frac{2}{\cot\frac{x}{2} - 1} + C$$

Integrands involving only cotangent

$$\int \cot ax\,dx = \frac{1}{a} \ln|\sin ax| + C$$

$$\int \cot^2 x\,dx = -\cot x - x + C$$

$$\int \cot^n ax\,dx = -\frac{1}{a(n-1)} \cot^{n-1} ax - \int \cot^{n-2} ax\,dx \qquad (\text{for } n \neq 1)$$

$$\int \frac{dx}{1 + \cot ax} = \int \frac{\tan ax\,dx}{\tan ax + 1} = \frac{x}{2} - \frac{1}{2a} \ln|\sin ax + \cos ax| + C$$

$$\int \frac{dx}{1 - \cot ax} = \int \frac{\tan ax\,dx}{\tan ax - 1} = \frac{x}{2} + \frac{1}{2a} \ln|\sin ax - \cos ax| + C$$

Integrands involving both sine and cosine

An integral that is a rational function of the sine and cosine can be evaluated using Bioche's rules.

$$\int \frac{dx}{\cos ax \pm \sin ax} = \frac{1}{a\sqrt{2}} \ln\left|\tan\left(\frac{ax}{2} \pm \frac{\pi}{8}\right)\right| + C$$

$$\int \frac{dx}{(\cos ax \pm \sin ax)^2} = \frac{1}{2a} \tan\left(ax \mp \frac{\pi}{4}\right) + C$$

$$\int \frac{dx}{(\cos x + \sin x)^n} = \frac{1}{2(n-1)} \left(\frac{\sin x - \cos x}{(\cos x + \sin x)^{n-1}} + (n-2) \int \frac{dx}{(\cos x + \sin x)^{n-2}}\right)$$

$$\int \frac{\cos ax\,dx}{\cos ax + \sin ax} = \frac{x}{2} + \frac{1}{2a} \ln|\sin ax + \cos ax| + C$$

$$\int\frac{\cos ax\,dx}{\cos ax-\sin ax}=\frac{x}{2}-\frac{1}{2a}\ln\left|\sin ax-\cos ax\right|+C$$

$$\int\frac{\sin ax\,dx}{\cos ax+\sin ax}=\frac{x}{2}-\frac{1}{2a}\ln\left|\sin ax+\cos ax\right|+C$$

$$\int\frac{\sin ax\,dx}{\cos ax-\sin ax}=-\frac{x}{2}-\frac{1}{2a}\ln\left|\sin ax-\cos ax\right|+C$$

$$\int\frac{\cos ax\,dx}{(\sin ax)(1+\cos ax)}=-\frac{1}{4a}\tan^2\frac{ax}{2}+\frac{1}{2a}\ln\left|\tan\frac{ax}{2}\right|+C$$

$$\int\frac{\cos ax\,dx}{(\sin ax)(1-\cos ax)}=-\frac{1}{4a}\cot^2\frac{ax}{2}-\frac{1}{2a}\ln\left|\tan\frac{ax}{2}\right|+C$$

$$\int\frac{\sin ax\,dx}{(\cos ax)(1+\sin ax)}=\frac{1}{4a}\cot^2\left(\frac{ax}{2}+\frac{\pi}{4}\right)+\frac{1}{2a}\ln\left|\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right|+C$$

$$\int\frac{\sin ax\,dx}{(\cos ax)(1-\sin ax)}=\frac{1}{4a}\tan^2\left(\frac{ax}{2}+\frac{\pi}{4}\right)-\frac{1}{2a}\ln\left|\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right|+C$$

$$\int(\sin ax)(\cos ax)\,dx=\frac{1}{2a}\sin^2 ax+C$$

$$\int(\sin a_1x)(\cos a_2x)\,dx=-\frac{\cos((a_1-a_2)x)}{2(a_1-a_2)}-\frac{\cos((a_1+a_2)x)}{2(a_1+a_2)}+C\qquad(\text{for }|a_1|\neq|a_2|)$$

$$\int(\sin^n ax)(\cos ax)\,dx=\frac{1}{a(n+1)}\sin^{n+1}ax+C\qquad(\text{for }n\neq-1)$$

$$\int(\sin ax)(\cos^n ax)\,dx=-\frac{1}{a(n+1)}\cos^{n+1}ax+C\qquad(\text{for }n\neq-1)$$

$$\begin{aligned}\int(\sin^n ax)(\cos^m ax)\,dx&=-\frac{(\sin^{n-1}ax)(\cos^{m+1}ax)}{a(n+m)}+\frac{n-1}{n+m}\int(\sin^{n-2}ax)(\cos^m ax)\,dx\qquad(\text{for }m,n>0)\\&=\frac{(\sin^{n+1}ax)(\cos^{m-1}ax)}{a(n+m)}+\frac{m-1}{n+m}\int(\sin^n ax)(\cos^{m-2}ax)\,dx\qquad(\text{for }m,n>0)\end{aligned}$$

$$\int\frac{dx}{(\sin ax)(\cos ax)}=\frac{1}{a}\ln\left|\tan ax\right|+C$$

$$\int\frac{dx}{(\sin ax)(\cos^n ax)}=\frac{1}{a(n-1)\cos^{n-1}ax}+\int\frac{dx}{(\sin ax)(\cos^{n-2}ax)}\qquad(\text{for }n\neq1)$$

$$\int\frac{dx}{(\sin^n ax)(\cos ax)}=-\frac{1}{a(n-1)\sin^{n-1}ax}+\int\frac{dx}{(\sin^{n-2}ax)(\cos ax)}\qquad(\text{for }n\neq1)$$

$$\int\frac{\sin ax\,dx}{\cos^n ax}=\frac{1}{a(n-1)\cos^{n-1}ax}+C\qquad(\text{for }n\neq1)$$

$$\int\frac{\sin^2 ax\,dx}{\cos ax}=-\frac{1}{a}\sin ax+\frac{1}{a}\ln\left|\tan\left(\frac{\pi}{4}+\frac{ax}{2}\right)\right|+C$$

$$\int\frac{\sin^2 ax\,dx}{\cos^n ax}=\frac{\sin ax}{a(n-1)\cos^{n-1}ax}-\frac{1}{n-1}\int\frac{dx}{\cos^{n-2}ax}\qquad(\text{for }n\neq1)$$

$$\begin{aligned}\int\frac{\sin^2 x}{1+\cos^2 x}\,dx&=\sqrt{2}\,\text{arctangant}\left(\frac{\tan x}{\sqrt{2}}\right)-x\qquad(\text{for x in}]-\frac{\pi}{2};+\frac{\pi}{2}[)\\&=\sqrt{2}\,\text{arctangant}\left(\frac{\tan x}{\sqrt{2}}\right)-\text{arctangant}(\tan x)\qquad(\text{this time x being any real number })\end{aligned}$$

$$\int\frac{\sin^n ax\,dx}{\cos ax}=-\frac{\sin^{n-1}ax}{a(n-1)}+\int\frac{\sin^{n-2}ax\,dx}{\cos ax}\qquad(\text{for }n\neq1)$$

$$\int\frac{\sin^n ax\,dx}{\cos^m ax}=\begin{cases}\frac{\sin^{n+1}ax}{a(m-1)\cos^{m-1}ax}-\frac{n-m+2}{m-1}\int\frac{\sin^n ax\,dx}{\cos^{m-2}ax}&(\text{for }m\neq1)\\\frac{\sin^{n-1}ax}{a(m-1)\cos^{m-1}ax}-\frac{n-1}{m-1}\int\frac{\sin^{n-2}ax\,dx}{\cos^{m-2}ax}&(\text{for }m\neq1)\\-\frac{\sin^{n-1}ax}{a(n-m)\cos^{m-1}ax}+\frac{n-1}{n-m}\int\frac{\sin^{n-2}ax\,dx}{\cos^m ax}&(\text{for }m\neq n)\end{cases}$$

$$\int\frac{\cos ax\,dx}{\sin^n ax}=-\frac{1}{a(n-1)\sin^{n-1}ax}+C\qquad(\text{for }n\neq1)$$

$$\int\frac{\cos^2 ax\,dx}{\sin ax}=\frac{1}{a}\left(\cos ax+\ln\left|\tan\frac{ax}{2}\right|\right)+C$$

$$\int\frac{\cos^2 ax\,dx}{\sin^n ax}=-\frac{1}{n-1}\left(\frac{\cos ax}{a\sin^{n-1}ax}+\int\frac{dx}{\sin^{n-2}ax}\right)\qquad(\text{for }n\neq1)$$

$$\int\frac{\cos^n ax\,dx}{\sin^m ax}=\begin{cases}-\frac{\cos^{n+1}ax}{a(m-1)\sin^{m-1}ax}-\frac{n-m+2}{m-1}\int\frac{\cos^n ax\,dx}{\sin^{m-2}ax} & (\text{for } n\neq 1)\\ -\frac{\cos^{n-1}ax}{a(m-1)\sin^{m-1}ax}-\frac{n-1}{m-1}\int\frac{\cos^{n-2}ax\,dx}{\sin^{m-2}ax} & (\text{for } m\neq 1)\\ \frac{\cos^{n-1}ax}{a(n-m)\sin^{m-1}ax}+\frac{n-1}{n-m}\int\frac{\cos^{n-2}ax\,dx}{\sin^{m}ax} & (\text{for } m\neq n)\end{cases}$$

Integrands involving both sine and tangent

$$\int(\sin ax)(\tan ax)\,dx=\frac{1}{a}(\ln|\sec ax+\tan ax|-\sin ax)+C$$

$$\int\frac{\tan^n ax\,dx}{\sin^2 ax}=\frac{1}{a(n-1)}\tan^{n-1}(ax)+C\qquad(\text{for } n\neq 1)$$

Integrand involving both cosine and tangent

$$\int\frac{\tan^n ax\,dx}{\cos^2 ax}=\frac{1}{a(n+1)}\tan^{n+1}ax+C\qquad(\text{for } n\neq -1)$$

Integrand involving both sine and cotangent

$$\int\frac{\cot^n ax\,dx}{\sin^2 ax}=-\frac{1}{a(n+1)}\cot^{n+1}ax+C\qquad(\text{for } n\neq -1)$$

Integrand involving both cosine and cotangent

$$\int\frac{\cot^n ax\,dx}{\cos^2 ax}=\frac{1}{a(1-n)}\tan^{1-n}ax+C\qquad(\text{for } n\neq 1)$$

Integrand involving both secant and tangent

$$\int(\sec x)(\tan x)\,dx=\sec x+C$$

Integrand involving both cosecant and cotangent

$$\int(\csc x)(\cot x)\,dx=-\csc x+C$$

Integrals in a quarter period

Using the beta function $B(a,b)$ one can write

$$\int_0^{\frac{\pi}{2}}\sin^n x\,dx=\int_0^{\frac{\pi}{2}}\cos^n x\,dx=\frac{1}{2}B\left(\frac{n+1}{2},\frac{1}{2}\right)=\begin{cases}\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}, & \text{if } n \text{ is even}\\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{4}{5}\cdot\frac{2}{3}, & \text{if } n \text{ is odd and more than } 1\\ 1, & \text{if } n=1\end{cases}$$

Integrals with symmetric limits

$$\int_{-c}^{c}\sin x\,dx=0$$

$$\int_{-c}^{c}\cos x\,dx=2\int_0^c\cos x\,dx=2\int_{-c}^{0}\cos x\,dx=2\sin c$$

$$\int_{-c}^{c}\tan x\,dx=0$$

$$\int_{-\frac{a}{2}}^{\frac{a}{2}}x^2\cos^2\frac{n\pi x}{a}\,dx=\frac{a^3(n^2\pi^2-6)}{24n^2\pi^2}\qquad(\text{for } n=1,3,5...)$$

$$\int_{\frac{-a}{2}}^{\frac{a}{2}}x^2\sin^2\frac{n\pi x}{a}\,dx=\frac{a^3(n^2\pi^2-6(-1)^n)}{24n^2\pi^2}=\frac{a^3}{24}(1-6\frac{(-1)^n}{n^2\pi^2})\qquad(\text{for } n=1,2,3,\ldots)$$

Integral over a full circle

$$\int_0^{2\pi}\sin^{2m+1}x\cos^n x\,dx=0\qquad n,m\in\mathbb{Z}$$

$$\int_0^{2\pi} \sin^m x \cos^{2n+1} x \, dx = 0 \qquad n, m \in \mathbb{Z}$$

See also

- Trigonometric integral

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