## Introduction to Stochastic Processes

## Contents

Preface	ix
Chapter 1 Probability Spaces and Random Variables	1
<ol> <li>Probability Spaces 1</li> <li>Random Variables and Stochastic Processes 6</li> <li>Conditional Probability 14</li> <li>Exercises 17</li> </ol>	
Chapter 2 Expectations and Independence	21
<ol> <li>Expected Value 22</li> <li>Conditional Expectations 33</li> <li>Exercises 40</li> </ol>	
Chapter 3 Bernoulli Processes and Sums of Independent Random Variables	43
<ol> <li>Bernoulli Process 44</li> <li>Numbers of Successes 45</li> </ol>	
<ul> <li>3. Times of Successes 53</li> <li>4. Sums of Independent Random Variables 60</li> <li>5. Exercises 66</li> </ul>	

Chapter 4 Poisson Processes	70
<ol> <li>Arrival Counting Process 71</li> <li>Times of Arrivals 79</li> <li>Forward Recurrence Times 85</li> <li>Superposition of Poisson Processes 87</li> <li>Decomposition of Poisson Processes 88</li> <li>Compound Poisson Processes 90</li> <li>Non-stationary Poisson Processes 94</li> <li>Exercises 101</li> </ol>	
Chapter 5 Markov Chains	106
<ol> <li>Introduction 106</li> <li>Visits to a Fixed State 119</li> <li>Classification of States 125</li> <li>Exercises 138</li> </ol>	
Chapter 6 Limiting Behavior and Applications of Markov Chains	<b>14</b> 4
<ol> <li>Computation of R and F 144</li> <li>Recurrent States and the Limiting Probabilities 152</li> <li>Periodic States 160</li> <li>Transient States 166 -</li> <li>Applications to Queueing Theory: M/G/1 Queue 169</li> <li>Queueing System G/M/1 178</li> <li>Branching Processes 183</li> <li>Exercises 188</li> </ol>	
Chapter 7 Potentials, Excessive Functions, and Optimal Stopping of Markov Chains	
<ol> <li>Potentials 195</li> <li>Excessive Functions 204</li> <li>Optimal Stopping 208</li> <li>Games with Discounting and Fees 220</li> <li>Exercises 226</li> </ol>	
Chapter 8 Markov Processes	232
<ol> <li>Markov Processes 233</li> <li>Sample Path Behavior 239</li> <li>Structure of a Markov Process 246</li> <li>Potentials and Generators 253</li> <li>Limit Theorems 261</li> <li>Birth and Death Processes 271</li> <li>Exercises 277</li> </ol>	

<b>Chapter 9 Renewal Theory</b>			283
<ol> <li>Renewal Processes 283</li> <li>Regenerative Processes and Rer</li> <li>Delayed and Stationary Process</li> <li>Exercises 307</li> </ol>	•	293	
Chapter 10 Markov Renewal Th	neory		313
<ol> <li>Markov Renewal Processes</li> <li>Markov Renewal Functions and Classification of States</li> <li>Markov Renewal Equations</li> <li>Limit Theorems 328</li> <li>Semi-Markov Processes 337</li> <li>Semi-Regenerative Processes</li> <li>Applications to Queueing Theorems</li> <li>Exercises 357</li> </ol>	313 318 323 343 ry350		
Afterword			363
Appendix. Non-Negative Matrice	es		364
<ol> <li>Eigenvalues and Eigenvectors</li> <li>Spectral Representations</li> <li>Positive Matrices 371</li> <li>Non-Negative Matrices 373</li> <li>Limits and Rates of Convergence</li> </ol>	364 367 ce 378		
References			383
Answers to Selected Exercises			387
Index of Notations			393
Subject Index			395