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RANDOM POLYNOMIALS

A.T. BHARUCHA-REID / M. SAMBANDHAM

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To
Kurush and Rustam;
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Preface

The algebraic properties and utility of deterministic algebraic polynomials are well known. Similarly, the properties of orthogonal and trigonometric polynomials are also well understood, and many books treating these deterministic theories are available. But this book is the first of its kind in presenting a fairly rigorous and comprehensive treatment of random algebraic, orthogonal, and trigonometric polynomials.

Random polynomials have applications in several fields of physics, engineering, and economics. Therefore, this book is addressed to probabilists, statisticians, physicists, engineers, and economists alike. The book describes several basic probabilistic properties of random algebraic polynomials (such as the measurability of zeros) and provides an in-depth treatment of expectation, variance, maxima, and distribution of the number of real zeros of random polynomials. Several theoretical results have been verified through numerical work, and independent numerical studies have led the authors to conjecture and prove certain theoretical results, which are presented herein.

As early as 1782 probabilistic methods were in use, and a few simple probabilistic results were obtained in connection with the study of complex zeros of deterministic algebraic polynomials. However, it was not until 1932 that a systematic study of random algebraic polynomials was undertaken. Since then, the theory of random polynomials has developed considerably from the study of the expected number of real zeros into complex limit theorems. Despite active research in this field in the United States, Great Britain, and India, no comprehensive treatment of this subject is as yet available in book form. The senior author, who kept up with these developments, has properly arranged and presented these results in several special lectures and seminar courses. Thus, this monograph began to take shape over a decade ago.