Tuần 7.(8/4)

9.1)

Vì đa thức P_2 nội suy f tại x_i , $i = 0,1,2,n$

Với $x_i = a + ih$ nên ta được :

$$\begin{cases} P_2(x) = a_0 + a_1 x + a_2 x \\ P_2(x_0) = f(x_0) \\ P_2(x_1) = f(x_1) \\ P_2(x_2) = f(x_2) \end{cases}$$

$$\Rightarrow \begin{cases} a_0 + a_1 x_0 + a_2 x_0^2 = f(x_0) \\ a_0 + a_1 x_1 + a_2 x_1^2 = f(x_1) \\ a_0 + a_1 x_2 + a_2 x_2^2 = f(x_2) \end{cases}$$

$$\Rightarrow \begin{pmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} - \begin{pmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{pmatrix} = 0$$

$P_2'(x) = a_1 + 2a_2 x$

$P_2''(x) = 2a_2$

$f(x_1 + h) = f(a + h + h) = f(x_2) = P_2(x_2)$

$f(x_1 - h) = f(a + h - h) = f(x_0) = P_2(x_0)$

$$\frac{f(x_1 + h) - f(x_1 + h)}{h^2} = \frac{P_2(x_2) - P_2(x_0)}{h^2}$$

$$P_2(x) = f(x_0)\frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + f(x_1)\frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} + \dots$$

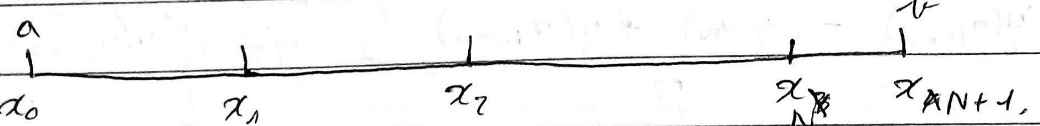
$$\dots + f(x_2)\frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

$$a_2 = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)} + \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)} + \frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)}$$

$$= \frac{f(x_0)}{2h^2} - \frac{f(x_1)}{h^2} + \frac{f(x_2)}{2h^2} = \frac{f(x_0) - 2f(x_1) + f(x_2)}{2h^2}.$$

Phương trình vi phân cấp 2 tổng quát

$$\begin{cases} y''(x) + p(x)y'(x) + q(x)y(x) = r(x), & x \in (a, b) \\ y(a) = \alpha_1, \ y(b) = \alpha_2 \end{cases}$$



$$h = \frac{b - a}{N + 1}$$

$y(x_0) = \alpha_1$, $y(x_{N+1}) = \alpha_2$.

Bài toán là $y(x_i) = ?$ $i = \cancel{\overline{0, N+1}}\ \overline{1, N}$

Đầu tiên, xét $i = 1$.

$$y''(x_1) + p(x_1)\,y'(x_1) + q(x_1)\,y(x_1) = r(x_1)$$

$$\Rightarrow \frac{y(x_2) - 2y(x_1) + y(x_0)}{h^2} + p(x_1)\left[\frac{y(x_2) - y(x_0)}{2h}\right] + \dots$$

$$\dots + q(x_1)\,y(x_1) + O(h^2) = r(x_1)$$

$$\Rightarrow \left(\frac{-2}{h^2} + q(x_1)\right) y(x_1) + \left(\frac{1}{h^2} + \frac{p(x_1)}{2h}\right) y(x_2) + O(h^2) + \dots$$

$$\dots = r(x_1) - \left[\frac{1}{h^2} - \frac{p(x_1)}{2h}\right] y(x_0)$$

Giờ ta xét $i \in [2, N-1]$:

$$y''(x_i) + p(x_i)\,y'(x_i) + q(x_i)\,y(x_i) = r(x_i)$$

$$\Rightarrow \frac{y(x_{i+1}) - 2y(x_i) + y(x_{i-1})}{h^2} + p(x_i)\left[\frac{y(x_{i+1}) - y(x_{i-1})}{2h}\right] + \dots$$

$$\dots + q(x_i)\,y(x_i) + O(h^2) = r(x_i)$$

$$\Rightarrow y(x_{i-1})\left[\frac{1}{h^2} - \frac{p(x_i)}{2h}\right] + y(x_i)\left[\frac{-2}{h^2} + q(x_i)\right] + \dots$$

$$\dots + y(x_{i+1})\left[\frac{1}{h^2} + \frac{p(x_i)}{2h}\right] + O(h^2) = r(x_i)$$

Cuối cùng khi $i = N$.

$$y''(x_N) + p(x_N)\,y'(x_N) + q(x_N)\,y(x_N) = r(x_N)$$

$$\Rightarrow \frac{y(x_{N+1}) - 2y(x_N) + y(x_{N-1})}{h^2} + p(x_N)\left[\frac{y(x_{N+1}) - y(x_{N-1})}{2h}\right] + \dots$$

$$\dots + q(x_N)\,y(x_N) + O(h^2) = r(x_N).$$

$$\Rightarrow y(x_{N-1})\left[\frac{1}{h^2} - \frac{p(x_N)}{2h}\right] + y(x_N)\left[\frac{-2}{h^2} + q(x_N)\right]$$

$$+ O(h^2) = r(x_N) - \left[\frac{1}{h^2} + \frac{p(x_N)}{2h}\right] y(x_{N+1})$$

Ta đặt $y(x_i) \approx y_i$, $i = \overline{1,N}$ với $\{y_i\}_{i=\overline{1,N}}$ thoả:

$$\underbrace{\left[-\frac{2}{h^2} + q(x_i)\right]}_{A_{11}} y_1 + \underbrace{\left[\frac{1}{h^2} + \frac{p(x_i)}{2h}\right]}_{A_{12}} y_2 = \dots$$

$$\dots = r(x_1) - \underbrace{\left[\frac{1}{h^2} - \frac{p(x_1)}{2h}\right]}_{B_1} \alpha_1$$

Đồng thời với $2 \leq i \leq N-1$:

$$\underbrace{\left[\frac{1}{h^2} - \frac{p(x_i)}{2h}\right]}_{A_{i,i-1}} y_{i-1} + \underbrace{\left[-\frac{2}{h^2} + q(x_i)\right]}_{A_{i,i}} y_i + \dots$$

$$\dots + \underbrace{\left[\frac{1}{h^2} + \frac{p(x_i)}{2h}\right]}_{A_{i,i+1}} y_{i+1} = r(x_i)$$

và

$$\left[\frac{1}{h^2} - \frac{p(x_N)}{2h}\right] y_{N-1} + \left[-\frac{2}{h^2} + q(x_N)\right] y_N = \dots$$

$$\dots = r(x_N) - \left[\frac{1}{h^2} + \frac{p(x_N)}{2h}\right] \alpha_2.$$

Từ đây, ta xây dựng các ma trận dựa trên các phương trình trên:

$$A \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix} = B \Rightarrow \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix} = A^{-1} B$$

$$A_{11} = -\frac{2}{h^2} + q(x_1), \quad A_{12} = \frac{1}{h^2} + \frac{p(x_i)}{2h}$$

với $A_{i,i-1} = \dfrac{1}{h^2} - \dfrac{p(x_i)}{2h}$, $A_{i,i} = \dfrac{-2}{h^2} + q(x_i)$,

$$A_{i,i+1} = \frac{1}{h^2} + \frac{p(x_2)}{2h}$$

$$B_1 = r(x_1) - \left[\frac{1}{h^2} - \frac{p(x_1)}{2h}\right] \alpha, \quad B_i = r(x_i) \quad i = \overline{2, N-1}$$

$$B_N = r(x_N) - \left[\frac{1}{h^2} + \frac{p(x_N)}{2h}\right] \alpha_2.$$

MATLAB code

input: a, b, α_1, α_2, N

function: r, q, p

$$h = \frac{b-a}{N+1}, \quad x_0 = a,$$

$$x_i = x_0 + i.h, \quad x_{N+1} = b, \quad A = \text{space}(N,N)$$

$$A(1,1) = \frac{-2}{h^2} + q(x(1))$$

$$A(1,2) = \frac{1}{h^2} + p(x(1))$$

$$A(i,i-1) = \frac{1}{h^2} - \frac{p(x(i))}{2h}$$

$$A(i,i) = -\frac{2}{h^2} + q(x(i))$$

$$A(i,i+1) = \frac{1}{h^2} + \frac{p(x(i))}{2h}.$$

Bài tập về nhà, viết chương trình thực hiện thuật toán trên bằng MATLAB. hạn 5h chiều mai (9/4) Ex 13.1 (bắt buộc)

$$A(N,N-1) = \frac{1}{h^2} - \frac{p(x(N))}{2h}$$

$$A(N,N) = \frac{-2}{h^2} + q(x(N))$$

$$B(1) = r(x(1)) - \left(\frac{1}{h^2} - \frac{p(x(1))}{2h}\right)\alpha_1$$

$$B(N) = r(x(N)) - \left[\frac{1}{h^2} + \frac{p(x(N))}{2h}\right]\alpha_2$$

$$B(i) = r(x(i))$$

$U = A \backslash B$

$Y = [\alpha_1 ; U_j \cancel{\alpha} \alpha_2]$ trên $\mathbb{R}$.

$\mathbb{R}^{N+2}$

11h (9/4) ~~Hết~~ ~~Btvn~~. Hết bài toán (không bắt buộc)

$$\begin{cases} y'' + p(x)\, y'(x) + q(x)\, y(x) = r(x) & x \in (a, b) \\ y(a) = \alpha_1 \\ y'(b) = \alpha_2 \end{cases}$$

bậc hội tụ = 2.