

List of integrals of hyperbolic functions

The following is a list of integrals (anti-derivative functions) of hyperbolic functions. For a complete list of integral functions, see list of integrals.

In all formulas the constant a is assumed to be nonzero, and C denotes the constant of integration.

Integrals involving only hyperbolic sine functions

$$\begin{aligned}\int \sinh ax \, dx &= \frac{1}{a} \cosh ax + C \\ \int \sinh^2 ax \, dx &= \frac{1}{4a} \sinh 2ax - \frac{x}{2} + C \\ \int \sinh^n ax \, dx &= \begin{cases} \frac{1}{an} (\sinh^{n-1} ax)(\cosh ax) - \frac{n-1}{n} \int \sinh^{n-2} ax \, dx, & n > 0 \\ \frac{1}{a(n+1)} (\sinh^{n+1} ax)(\cosh ax) - \frac{n+2}{n+1} \int \sinh^{n+2} ax \, dx, & n < 0, n \neq -1 \end{cases} \\ \int \frac{dx}{\sinh ax} &= \frac{1}{a} \ln \left| \tanh \frac{ax}{2} \right| + C \\ &= \frac{1}{a} \ln \left| \frac{\cosh ax + 1}{\sinh ax} \right| + C \\ &= \frac{1}{a} \ln \left| \frac{\sinh ax}{\cosh ax + 1} \right| + C \\ &= \frac{1}{2a} \ln \left| \frac{\cosh ax - 1}{\cosh ax + 1} \right| + C \\ \int \frac{dx}{\sinh^n ax} &= -\frac{\cosh ax}{a(n-1) \sinh^{n-1} ax} - \frac{n-2}{n-1} \int \frac{dx}{\sinh^{n-2} ax} \quad (\text{for } n \neq 1) \\ \int x \sinh ax \, dx &= \frac{1}{a} x \cosh ax - \frac{1}{a^2} \sinh ax + C \\ \int (\sinh ax)(\sinh bx) \, dx &= \frac{1}{a^2 - b^2} (a(\sinh bx)(\cosh ax) - b(\cosh bx)(\sinh ax)) + C \quad (\text{for } a^2 \neq b^2)\end{aligned}$$

Integrals involving only hyperbolic cosine functions

$$\begin{aligned}\int \cosh ax \, dx &= \frac{1}{a} \sinh ax + C \\ \int \cosh^2 ax \, dx &= \frac{1}{4a} \sinh 2ax + \frac{x}{2} + C \\ \int \cosh^n ax \, dx &= \begin{cases} \frac{1}{an} (\sinh ax)(\cosh^{n-1} ax) + \frac{n-1}{n} \int \cosh^{n-2} ax \, dx, & n > 0 \\ -\frac{1}{a(n+1)} (\sinh ax)(\cosh^{n+1} ax) + \frac{n+2}{n+1} \int \cosh^{n+2} ax \, dx, & n < 0, n \neq -1 \end{cases} \\ \int \frac{dx}{\cosh ax} &= \frac{2}{a} \arctan e^{ax} + C \\ &= \frac{1}{a} \arctan(\sinh ax) + C \\ \int \frac{dx}{\cosh^n ax} &= \frac{\sinh ax}{a(n-1) \cosh^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\cosh^{n-2} ax} \quad (\text{for } n \neq 1) \\ \int x \cosh ax \, dx &= \frac{1}{a} x \sinh ax - \frac{1}{a^2} \cosh ax + C\end{aligned}$$

$$\int x^2 \cosh ax \, dx = -\frac{2x \cosh ax}{a^2} + \left(\frac{x^2}{a} + \frac{2}{a^3} \right) \sinh ax + C$$

$$\int (\cosh ax)(\cosh bx) \, dx = \frac{1}{a^2 - b^2} (a(\sinh ax)(\cosh bx) - b(\sinh bx)(\cosh ax)) + C \quad (\text{for } a^2 \neq b^2)$$

$$\int \frac{dx}{1 + \cosh(ax)} = \frac{2}{a} \frac{1}{1 + e^{-ax}} + C \quad \text{or } \frac{2}{a} \text{ times } \underline{\text{The Logistic Function}}$$

Other integrals

Integrals of hyperbolic tangent, cotangent, secant, cosecant functions

$$\int \tanh x \, dx = \ln \cosh x + C$$

$$\int \tanh^2 ax \, dx = x - \frac{\tanh ax}{a} + C$$

$$\int \tanh^n ax \, dx = -\frac{1}{a(n-1)} \tanh^{n-1} ax + \int \tanh^{n-2} ax \, dx \quad (\text{for } n \neq 1)$$

$$\int \coth x \, dx = \ln |\sinh x| + C, \text{ for } x \neq 0$$

$$\int \coth^n ax \, dx = -\frac{1}{a(n-1)} \coth^{n-1} ax + \int \coth^{n-2} ax \, dx \quad (\text{for } n \neq 1)$$

$$\int \operatorname{sech} x \, dx = \arctan(\sinh x) + C$$

$$\int \operatorname{csch} x \, dx = \ln \left| \tanh \frac{x}{2} \right| + C = \ln |\coth x - \operatorname{csch} x| + C, \text{ for } x \neq 0$$

Integrals involving hyperbolic sine and cosine functions

$$\int (\cosh ax)(\sinh bx) \, dx = \frac{1}{a^2 - b^2} (a(\sinh ax)(\sinh bx) - b(\cosh ax)(\cosh bx)) + C \quad (\text{for } a^2 \neq b^2)$$

$$\int \frac{\cosh^n ax}{\sinh^m ax} \, dx = \frac{\cosh^{n-1} ax}{a(n-m) \sinh^{m-1} ax} + \frac{n-1}{n-m} \int \frac{\cosh^{n-2} ax}{\sinh^m ax} \, dx \quad (\text{for } m \neq n)$$

$$= -\frac{\cosh^{n+1} ax}{a(m-1) \sinh^{m-1} ax} + \frac{n-m+2}{m-1} \int \frac{\cosh^n ax}{\sinh^{m-2} ax} \, dx \quad (\text{for } m \neq 1)$$

$$= -\frac{\cosh^{n-1} ax}{a(m-1) \sinh^{m-1} ax} + \frac{n-1}{m-1} \int \frac{\cosh^{n-2} ax}{\sinh^{m-2} ax} \, dx \quad (\text{for } m \neq 1)$$

$$\int \frac{\sinh^m ax}{\cosh^n ax} \, dx = \frac{\sinh^{m-1} ax}{a(m-n) \cosh^{n-1} ax} + \frac{m-1}{n-m} \int \frac{\sinh^{m-2} ax}{\cosh^n ax} \, dx \quad (\text{for } m \neq n)$$

$$= \frac{\sinh^{m+1} ax}{a(n-1) \cosh^{n-1} ax} + \frac{m-n+2}{n-1} \int \frac{\sinh^m ax}{\cosh^{n-2} ax} \, dx \quad (\text{for } n \neq 1)$$

$$= -\frac{\sinh^{m-1} ax}{a(n-1) \cosh^{n-1} ax} + \frac{m-1}{n-1} \int \frac{\sinh^{m-2} ax}{\cosh^{n-2} ax} \, dx \quad (\text{for } n \neq 1)$$

Integrals involving hyperbolic and trigonometric functions

$$\int \sinh(ax+b) \sin(cx+d) \, dx = \frac{a}{a^2 + c^2} \cosh(ax+b) \sin(cx+d) - \frac{c}{a^2 + c^2} \sinh(ax+b) \cos(cx+d) + C$$

$$\int \sinh(ax+b) \cos(cx+d) \, dx = \frac{a}{a^2 + c^2} \cosh(ax+b) \cos(cx+d) + \frac{c}{a^2 + c^2} \sinh(ax+b) \sin(cx+d) + C$$

$$\int \cosh(ax + b) \sin(cx + d) dx = \frac{a}{a^2 + c^2} \sinh(ax + b) \sin(cx + d) - \frac{c}{a^2 + c^2} \cosh(ax + b) \cos(cx + d) + C$$

$$\int \cosh(ax + b) \cos(cx + d) dx = \frac{a}{a^2 + c^2} \sinh(ax + b) \cos(cx + d) + \frac{c}{a^2 + c^2} \cosh(ax + b) \sin(cx + d) + C$$

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