List of integrals of rational functions

The following is a list of integrals (antiderivative functions) of rational functions. Any rational function can be integrated by partial fraction decomposition of the function into a sum of functions of the form:

$$\frac{a}{(x-b)^n}, \text{ and } \frac{ax+b}{\left((x-c)^2+d^2\right)^n}.$$

which can then be integrated term by term.

For other types of functions, see lists of integrals.

Miscellaneous integrands

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C = \begin{cases} -\dfrac{1}{a}\,\operatorname{artanh}\dfrac{x}{a} + C = \dfrac{1}{2a}\ln\dfrac{a-x}{a+x} + C & (\text{for } |x|<|a|) \\[2ex] -\dfrac{1}{a}\,\operatorname{arcoth}\dfrac{x}{a} + C = \dfrac{1}{2a}\ln\dfrac{x-a}{x+a} + C & (\text{for } |x|>|a|) \end{cases}$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C = \begin{cases} \dfrac{1}{a}\,\operatorname{artanh}\dfrac{x}{a} + C = \dfrac{1}{2a}\ln\dfrac{a+x}{a-x} + C & (\text{for } |x|<|a|) \\[2ex] \dfrac{1}{a}\,\operatorname{arcoth}\dfrac{x}{a} + C = \dfrac{1}{2a}\ln\dfrac{x+a}{x-a} + C & (\text{for } |x|>|a|) \end{cases}$$

$$\int \frac{dx}{x^{2^n}+1} = \frac{1}{2^{n-1}}\sum_{k=1}^{2^{n-1}} \sin\left(\frac{2k-1}{2^n}\pi\right)\arctan\left[\left(x-\cos\left(\frac{2k-1}{2^n}\pi\right)\right)\csc\left(\frac{2k-1}{2^n}\pi\right)\right] - \frac{1}{2}\cos\left(\frac{2k-1}{2^n}\pi\right)\ln\left|x^2-2x\cos\left(\frac{2k-1}{2^n}\pi\right)+1\right| -$$

Integrands of the form $x^m(a\,x+b)^n$

Many of the following antiderivatives have a term of the form ln |*ax* + *b*|. Because this is undefined when *x* = −*b* / *a*, the most general form of the antiderivative replaces the constant of integration with a locally constant function.[1] However, it is conventional to omit this from the notation. For example,

$$\int \frac{1}{ax+b}\,dx = \begin{cases} \dfrac{1}{a}\ln(-(ax+b)) + C^- & ax+b<0 \\ \dfrac{1}{a}\ln(ax+b) + C^+ & ax+b>0 \end{cases}$$

is usually abbreviated as

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C,$$

where *C* is to be understood as notation for a locally constant function of *x*. This convention will be adhered to in the following.

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C \qquad (\text{for } n \neq -1)$$ (Cavalieri's quadrature formula)

$$\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C$$

$$\int \frac{mx+n}{ax+b}\,dx = \frac{m}{a}x + \frac{an-bm}{a^2}\ln|ax+b| + C$$

$$\int \frac{x}{(ax+b)^2}\,dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln|ax+b| + C$$

$$\int \frac{x}{(ax+b)^n}\,dx = \frac{a(1-n)x-b}{a^2(n-1)(n-2)(ax+b)^{n-1}} + C \qquad (\text{for } n \notin \{1,2\})$$

$$\int x(ax+b)^n\,dx = \frac{a(n+1)x-b}{a^2(n+1)(n+2)}(ax+b)^{n+1} + C \qquad (\text{for } n \notin \{-1,-2\})$$

$$\int \frac{x^2}{ax+b}\,dx = \frac{b^2\ln(|ax+b|)}{a^3} + \frac{ax^2-2bx}{2a^2} + C$$

$$\int \frac{x^2}{(ax+b)^2}\,dx = \frac{1}{a^3}\left(ax - 2b\ln|ax+b| - \frac{b^2}{ax+b}\right) + C$$

$$\int \frac{x^2}{(ax+b)^3}\,dx = \frac{1}{a^3}\left(\ln|ax+b| + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2}\right) + C$$

$$\int \frac{x^2}{(ax+b)^n}\,dx = \frac{1}{a^3}\left(-\frac{(ax+b)^{3-n}}{(n-3)} + \frac{2b(ax+b)^{2-n}}{(n-2)} - \frac{b^2(ax+b)^{1-n}}{(n-1)}\right) + C \qquad (\text{for } n \notin \{1,2,3\})$$

$$\int \frac{1}{x(ax+b)}\,dx = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right| + C$$

$$\int \frac{1}{x^2(ax+b)}\,dx = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right| + C$$

$$\int \frac{1}{x^2(ax+b)^2}\,dx = -a\left(\frac{1}{b^2(ax+b)} + \frac{1}{ab^2x} - \frac{2}{b^3}\ln\left|\frac{ax+b}{x}\right|\right) + C$$

Integrands of the form x^m / $(a\,x^2 + b\,x + c)^n$

For $a \neq 0$:

$$\int \frac{1}{ax^2+bx+c}dx = \begin{cases} \dfrac{2}{\sqrt{4ac-b^2}}\arctan\dfrac{2ax+b}{\sqrt{4ac-b^2}} + C \\[2ex] \dfrac{1}{\sqrt{b^2-4ac}}\ln\left|\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| + C = \begin{cases} -\dfrac{2}{\sqrt{b^2-4ac}}\operatorname{artanh}\dfrac{2ax+b}{\sqrt{b^2-4ac}} + C & (\text{for } |2ax+b| < \sqrt{b^2-4ac} \\ -\dfrac{2}{\sqrt{b^2-4ac}}\operatorname{arcoth}\dfrac{2ax+b}{\sqrt{b^2-4ac}} + C & (\text{else}) \end{cases} \\[2ex] -\dfrac{2}{2ax+b} + C \end{cases}$$

$$\int \frac{x}{ax^2+bx+c}\,dx = \frac{1}{2a}\ln\left|ax^2+bx+c\right| - \frac{b}{2a}\int\frac{dx}{ax^2+bx+c} + C$$

$$\int \frac{mx+n}{ax^2+bx+c}\,dx = \begin{cases} \dfrac{m}{2a}\ln\left|ax^2+bx+c\right| + \dfrac{2an-bm}{a\sqrt{4ac-b^2}}\arctan\dfrac{2ax+b}{\sqrt{4ac-b^2}} + C \\[2ex] \dfrac{m}{2a}\ln\left|ax^2+bx+c\right| + \dfrac{2an-bm}{2a\sqrt{b^2-4ac}}\ln\left|\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| + C = \begin{cases} \dfrac{m}{2a}\ln\left|ax^2+bx+c\right| - \dfrac{2an-bm}{a\sqrt{b^2-4ac}}\operatorname{artanh} \\ \dfrac{m}{2a}\ln\left|ax^2+bx+c\right| - \dfrac{2an-bm}{a\sqrt{b^2-4ac}}\operatorname{arcoth} \end{cases} \\[2ex] \dfrac{m}{2a}\ln\left|ax^2+bx+c\right| - \dfrac{2an-bm}{a(2ax+b)} + C = \dfrac{m}{a}\ln\left|x+\dfrac{b}{2a}\right| - \dfrac{2an-bm}{a(2ax+b)} + C \end{cases}$$

$$\int \frac{1}{(ax^2+bx+c)^n}\,dx = \frac{2ax+b}{(n-1)(4ac-b^2)(ax^2+bx+c)^{n-1}} + \frac{(2n-3)2a}{(n-1)(4ac-b^2)}\int\frac{1}{(ax^2+bx+c)^{n-1}}\,dx + C$$

$$\int \frac{x}{(ax^2+bx+c)^n}\,dx = -\frac{bx+2c}{(n-1)(4ac-b^2)(ax^2+bx+c)^{n-1}} - \frac{b(2n-3)}{(n-1)(4ac-b^2)}\int\frac{1}{(ax^2+bx+c)^{n-1}}\,dx + C$$

$$\int \frac{1}{x(ax^2+bx+c)}\,dx = \frac{1}{2c}\ln\left|\frac{x^2}{ax^2+bx+c}\right| - \frac{b}{2c}\int\frac{1}{ax^2+bx+c}\,dx + C$$

Integrands of the form $x^m\,(a + b\,x^n)^p$

- The resulting integrands are of the same form as the original integrand, so these reduction formulas can be repeatedly applied to drive the exponents *m* and *p* toward 0.
- These reduction formulas can be used for integrands having integer and/or fractional exponents.

$$\int x^m(a+b\,x^n)^p dx = \frac{x^{m+1}(a+b\,x^n)^p}{m+n\,p+1} + \frac{a\,n\,p}{m+n\,p+1}\int x^m(a+b\,x^n)^{p-1}dx$$

$$\int x^m(a+b\,x^n)^p dx = -\frac{x^{m+1}(a+b\,x^n)^{p+1}}{a\,n(p+1)} + \frac{m+n(p+1)+1}{a\,n(p+1)}\int x^m(a+b\,x^n)^{p+1}dx$$

$$\int x^m(a+b\,x^n)^p dx = \frac{x^{m+1}(a+b\,x^n)^p}{m+1} - \frac{b\,n\,p}{m+1}\int x^{m+n}(a+b\,x^n)^{p-1}dx$$

$$\int x^m(a+b\,x^n)^p dx = \frac{x^{m-n+1}(a+b\,x^n)^{p+1}}{b\,n(p+1)} - \frac{m-n+1}{b\,n(p+1)}\int x^{m-n}(a+b\,x^n)^{p+1}dx$$

$$\int x^m(a+b\,x^n)^p dx = \frac{x^{m-n+1}(a+b\,x^n)^{p+1}}{b(m+n\,p+1)} - \frac{a(m-n+1)}{b(m+n\,p+1)}\int x^{m-n}(a+b\,x^n)^p dx$$

$$\int x^m (a+b x^n)^p dx = \frac{x^{m+1}(a+b x^n)^{p+1}}{a(m+1)} - \frac{b(m+n(p+1)+1)}{a(m+1)} \int x^{m+n}(a+b x^n)^p dx$$

Integrands of the form $(A + B x)\ (a + b x)^m\ (c + d x)^n\ (e + f x)^p$

- The resulting integrands are of the same form as the original integrand, so these reduction formulas can be repeatedly applied to drive the exponents m, n and p toward 0.
- These reduction formulas can be used for integrands having integer and/or fractional exponents.
- Special cases of these reductions formulas can be used for integrands of the form $(a+bx)^m(c+dx)^n(e+fx)^p$ by setting B to 0.

$$\int (A+Bx)(a+bx)^m(c+dx)^n(e+fx)^p dx = -\frac{(Ab-aB)(a+bx)^{m+1}(c+dx)^n(e+fx)^{p+1}}{b(m+1)(af-be)} + \frac{1}{b(m+1)(af-be)}\cdot$$

$$\int (bc(m+1)(Af-Be)+(Ab-aB)(nde+cf(p+1))+d(b(m+1)(Af-Be)+f(n+p+1)(Ab-aB))x)(a+bx)^{m+1}(c+dx$$

$$\int (A+Bx)(a+bx)^m(c+dx)^n(e+fx)^p dx = \frac{B(a+bx)^m(c+dx)^{n+1}(e+fx)^{p+1}}{df(m+n+p+2)} + \frac{1}{df(m+n+p+2)}\cdot$$

$$\int (Aadf(m+n+p+2)-B(bcem+a(de(n+1)+cf(p+1)))+(Abdf(m+n+p+2)+B(adfm-b(de(m+n+1)+cf(n$$

$$\int (A+Bx)(a+bx)^m(c+dx)^n(e+fx)^p dx = \frac{(Ab-aB)(a+bx)^{m+1}(c+dx)^{n+1}(e+fx)^{p+1}}{(m+1)(ad-bc)(af-be)} + \frac{1}{(m+1)(ad-bc)(af-be)}\cdot$$

$$\int ((m+1)(A(adf-b(cf+de))+Bbce)-(Ab-aB)(de(n+1)+cf(p+1))-df(m+n+p+3)(Ab-aB)x)(a+bx)^{m+1}(c-$$

Integrands of the form $x^m\ (A + B x^n)\ (a + b x^n)^p\ (c + d x^n)^q$

- The resulting integrands are of the same form as the original integrand, so these reduction formulas can be repeatedly applied to drive the exponents m, p and q toward 0.
- These reduction formulas can be used for integrands having integer and/or fractional exponents.
- Special cases of these reductions formulas can be used for integrands of the form $(a+bx^n)^p(c+dx^n)^q$ and $x^m(a+bx^n)^p(c+dx^n)^q$ by setting m and/or B to 0.

$$\int x^m\,(A+Bx^n)\,(a+bx^n)^p(c+dx^n)^q dx = -\frac{(Ab-aB)x^{m+1}(a+bx^n)^{p+1}(c+dx^n)^q}{abn(p+1)} + \frac{1}{abn(p+1)}\cdot$$

$$\int x^m\,(c(Abn(p+1)+(Ab-aB)(m+1))+d(Abn(p+1)+(Ab-aB)(m+nq+1))x^n)\,(a+bx^n)^{p+1}(c+dx^n)^{q-1}dx$$

$$\int x^m\,(A+Bx^n)\,(a+bx^n)^p(c+dx^n)^q dx = \frac{Bx^{m+1}(a+bx^n)^{p+1}(c+dx^n)^q}{b(m+n(p+q+1)+1)} + \frac{1}{b(m+n(p+q+1)+1)}\cdot$$

$$\int x^m\,(c((Ab-aB)(1+m)+Abn(1+p+q))+(d(Ab-aB)(1+m)+Bnq(bc-ad)+Abdn(1+p+q))\,x^n)\,(a+bx^n)^p(c+d$$

$$\int x^m\,(A+Bx^n)\,(a+bx^n)^p(c+dx^n)^q dx = -\frac{(Ab-aB)x^{m+1}(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{an(bc-ad)(p+1)} + \frac{1}{an(bc-ad)(p+1)}\cdot$$

$$\int x^m\,(c(Ab-aB)(m+1)+An(bc-ad)(p+1)+d(Ab-aB)(m+n(p+q+2)+1)x^n)\,(a+bx^n)^{p+1}(c+dx^n)^q dx$$

$$\int x^m\,(A+Bx^n)\,(a+bx^n)^p(c+dx^n)^q dx = \frac{Bx^{m-n+1}(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{bd(m+n(p+q+1)+1)} - \frac{1}{bd(m+n(p+q+1)+1)}\cdot$$

$$\int x^{m-n}\,(aBc(m-n+1)+(aBd(m+nq+1)-b(-Bc(m+np+1)+Ad(m+n(p+q+1)+1)))x^n)\,(a+bx^n)^p(c+dx^n)^q dx$$

$$\int x^m\,(A+Bx^n)\,(a+bx^n)^p(c+dx^n)^q dx = \frac{Ax^{m+1}(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{ac(m+1)} + \frac{1}{ac(m+1)}\cdot$$

$$\int x^{m+n}\,(aBc(m+1)-A(bc+ad)(m+n+1)-An(bcp+adq)-Abd(m+n(p+q+2)+1)x^n)\,(a+bx^n)^p(c+dx^n)^q dx$$

$$\int x^m\,(A+Bx^n)\,(a+bx^n)^p(c+dx^n)^q dx = \frac{Ax^{m+1}(a+bx^n)^{p+1}(c+dx^n)^q}{a(m+1)} - \frac{1}{a(m+1)}\cdot$$

$$\int x^{m+n}\,(c(Ab-aB)(m+1)+An(bc(p+1)+adq)+d((Ab-aB)(m+1)+Abn(p+q+1))x^n)\,(a+bx^n)^p(c+dx^n)^{q-1}dx$$

$$\int x^m\,(A+Bx^n)\,(a+bx^n)^p(c+dx^n)^q dx = \frac{(Ab-aB)x^{m-n+1}(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{bn(bc-ad)(p+1)} - \frac{1}{bn(bc-ad)(p+1)}\cdot$$

$$\int x^{m-n}\,(c(Ab-aB)(m-n+1)+(d(Ab-aB)(m+nq+1)-bn(Bc-Ad)(p+1))x^n)\,(a+bx^n)^{p+1}(c+dx^n)^q dx$$

Integrands of the form $(d + e x)^m\ (a + b x + c x^2)^p$ when $b^2 - 4\,a\,c = 0$

- The resulting integrands are of the same form as the original integrand, so these reduction formulas can be repeatedly applied to drive the exponents m and p toward 0.
- These reduction formulas can be used for integrands having integer and/or fractional exponents.

- Special cases of these reductions formulas can be used for integrands of the form $\left(a+b x+c x^2\right)^p$ when $b^2-4 a c=0$ by setting *m* to 0.

$$\int (d+e x)^m \left(a+b x+c x^2\right)^p dx = \frac{(d+e x)^{m+1}\left(a+b x+c x^2\right)^p}{e(m+1)} - \frac{p(d+e x)^{m+2}(b+2cx)\left(a+b x+c x^2\right)^{p-1}}{e^2(m+1)(m+2p+1)} + \frac{p(2p-1)(2cd-be)}{e^2(m+1)(m+2p+1)}$$

$$\int (d+e x)^m \left(a+b x+c x^2\right)^p dx = \frac{(d+e x)^{m+1}\left(a+b x+c x^2\right)^p}{e(m+1)} - \frac{p(d+e x)^{m+2}(b+2cx)\left(a+b x+c x^2\right)^{p-1}}{e^2(m+1)(m+2)} + \frac{2cp(2p-1)}{e^2(m+1)(m+2)} \int (a$$

$$\int (d+e x)^m \left(a+b x+c x^2\right)^p dx = -\frac{e(m+2p+2)(d+e x)^{m}\left(a+b x+c x^2\right)^{p+1}}{(p+1)(2p+1)(2cd-be)} + \frac{(d+e x)^{m+1}(b+2cx)\left(a+b x+c x^2\right)^{p}}{(2p+1)(2cd-be)} + \frac{e^2m(}{(p+1)(2p}$$

$$\int (d+e x)^m \left(a+b x+c x^2\right)^p dx = -\frac{em(d+e x)^{m-1}\left(a+b x+c x^2\right)^{p+1}}{2c(p+1)(2p+1)} + \frac{(d+e x)^{m}(b+2cx)\left(a+b x+c x^2\right)^{p}}{2c(2p+1)} + \frac{e^2m(m-1)}{2c(p+1)(2p+1)} \int ($$

$$\int (d+e x)^m \left(a+b x+c x^2\right)^p dx = \frac{(d+e x)^{m+1}\left(a+b x+c x^2\right)^p}{e(m+2p+1)} - \frac{p(2cd-be)(d+e x)^{m+1}(b+2cx)\left(a+b x+c x^2\right)^{p-1}}{2ce^2(m+2p)(m+2p+1)} + \frac{p(2p-1)}{2ce^2(m+2}$$

$$\int (d+e x)^m \left(a+b x+c x^2\right)^p dx = -\frac{2ce(m+2p+2)(d+e x)^{m+1}\left(a+b x+c x^2\right)^{p+1}}{(p+1)(2p+1)(2cd-be)^2} + \frac{(d+e x)^{m+1}(b+2cx)\left(a+b x+c x^2\right)^{p}}{(2p+1)(2cd-be)} + \frac{2ce^2(}{(p+}$$

$$\int (d+e x)^m \left(a+b x+c x^2\right)^p dx = \frac{(d+e x)^{m}(b+2cx)\left(a+b x+c x^2\right)^p}{2c(m+2p+1)} + \frac{m(2cd-be)}{2c(m+2p+1)} \int (d+e x)^{m-1}\left(a+b x+c x^2\right)^p dx$$

$$\int (d+e x)^m \left(a+b x+c x^2\right)^p dx = -\frac{(d+e x)^{m+1}(b+2cx)\left(a+b x+c x^2\right)^p}{(m+1)(2cd-be)} + \frac{2c(m+2p+2)}{(m+1)(2cd-be)} \int (d+e x)^{m+1}\left(a+b x+c x^2\right)^p dx$$

Integrands of the form $(d + e\, x)^m\, (A + B\, x)\, (a + b\, x + c\, x^2)^p$

- The resulting integrands are of the same form as the original integrand, so these reduction formulas can be repeatedly applied to drive the exponents *m* and *p* toward 0.
- These reduction formulas can be used for integrands having integer and/or fractional exponents.
- Special cases of these reductions formulas can be used for integrands of the form $\left(a+b x+c x^2\right)^p$ and $(d+e x)^m\left(a+b x+c x^2\right)^p$ by setting *m* and/or *B* to 0.

$$\int (d+e x)^m (A+Bx)\left(a+b x+c x^2\right)^p dx = \frac{(d+e x)^{m+1}(Ae(m+2p+2)-Bd(2p+1)+eB(m+1)x)\left(a+b x+c x^2\right)^p}{e^2(m+1)(m+2p+2)} + \frac{}{e^2(m+1)(}$$
$$\int (d+e x)^{m+1}(B(bd+2ae+2aem+2bdp)-Abe(m+2p+2)+(B(2cd+be+bem+4cdp)-2Ace(m+2p+2))x)\left(a+bx+\right.$$

$$\int (d+e x)^m (A+Bx)\left(a+b x+c x^2\right)^p dx = \frac{(d+e x)^{m}(Ab-2aB-(bB-2Ac)x)\left(a+b x+c x^2\right)^{p+1}}{(p+1)\left(b^2-4ac\right)} + \frac{1}{(p+1)\left(b^2-4ac\right)}\cdot$$
$$\int (d+e x)^{m-1}(B(2aem+bd(2p+3))-A(bem+2cd(2p+3))+e(bB-2Ac)(m+2p+3)x)\left(a+b x+c x^2\right)^{p+1}dx$$

$$\int (d+e x)^m (A+Bx)\left(a+b x+c x^2\right)^p dx = \frac{(d+e x)^{m+1}(Ace(m+2p+2)-B(cd+2cdp-bep)+Bce(m+2p+1)x)\left(a+bx+cx^2\right.}{ce^2(m+2p+1)(m+2p+2)}$$
$$\int (d+e x)^{m}(Ace(bd-2ae)(m+2p+2)+B(ae(be-2cdm+bem)+bd(bep-cd-2cdp))+$$
$$\left(Ace(2cd-be)(m+2p+2)-B\left(-b^2e^2(m+p+1)+2c^2d^2(1+2p)+ce(bd(m-2p)+2ae(m+2p+1))\right)\right)x)\left(a+bx+cx^2\right)$$

$$\int (d+e x)^m (A+Bx)\left(a+b x+c x^2\right)^p dx = \frac{(d+e x)^{m+1}\left(A\left(bcd-b^2e+2ace\right)-aB(2cd-be)+c(A(2cd-be)-B(bd-2ae))x\right)(a}{(p+1)\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$
$$\frac{1}{(p+1)\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}\cdot$$
$$\int (d+e x)^{m}(A\left(bcde(2p-m+2)+b^2e^2(m+p+2)-2c^2d^2(3+2p)-2ace^2(m+2p+3)\right)-$$
$$B(ae(be-2cdm+bem)+bd(-3cd+be-2cdp+bep))+ce(B(bd-2ae)-A(2cd-be))(m+2p+4)x)\left(a+bx+ca\right.$$

$$\int (d+e x)^m (A+Bx)\left(a+b x+c x^2\right)^p dx = \frac{B(d+e x)^{m}\left(a+b x+c x^2\right)^{p+1}}{c(m+2p+2)} + \frac{1}{c(m+2p+2)}\cdot$$
$$\int (d+e x)^{m-1}(m(Acd-aBe)-d(bB-2Ac)(p+1)+((Bcd-bBe+Ace)m-e(bB-2Ac)(p+1))x)\left(a+b x+c x^2\right)^{p}dx$$

$$\int (d+e x)^m (A+Bx)\left(a+b x+c x^2\right)^p dx = -\frac{(Bd-Ae)(d+e x)^{m+1}\left(a+b x+c x^2\right)^{p+1}}{(m+1)\left(cd^2-bde+ae^2\right)} + \frac{1}{(m+1)\left(cd^2-bde+ae^2\right)}\cdot$$
$$\int (d+e x)^{m+1}((Acd-Abe+aBe)(m+1)+b(Bd-Ae)(p+1)+c(Bd-Ae)(m+2p+3)x)\left(a+b x+c x^2\right)^{p}dx$$

Integrands of the form $x^m\, (a + b\, x^n + c\, x^{2n})^p$ when $b^2 - 4\, a\, c = 0$

- The resulting integrands are of the same form as the original integrand, so these reduction formulas can be repeatedly applied to drive the exponents *m* and *p* toward 0.
- These reduction formulas can be used for integrands having integer and/or fractional exponents.

- Special cases of these reductions formulas can be used for integrands of the form $\left(a+b\,x^n+c\,x^{2n}\right)^p$ when $b^2-4\,a\,c=0$ by setting m to 0.

$$\int x^m\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{x^{m+1}\left(a+b\,x^n+c\,x^{2n}\right)^p}{m+2n\,p+1}+\frac{n\,p\,x^{m+1}\left(2a+b\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^{p-1}}{(m+1)(m+2n\,p+1)}-\frac{b\,n^2p(2p-1)}{(m+1)(m+2n\,p+1)}\int x^{m+n}(a$$

$$\int x^m\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{(m+n(2p-1)+1)x^{m+1}\left(a+b\,x^n+c\,x^{2n}\right)^p}{(m+1)(m+n+1)}+\frac{n\,p\,x^{m+1}\left(2a+b\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^{p-1}}{(m+1)(m+n+1)}+\frac{2c\,p\,n^2(2p}{(m+1)(m+}$$

$$\int x^m\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{(m+n(2p+1)+1)x^{m-n+1}\left(a+b\,x^n+c\,x^{2n}\right)^{p+1}}{b\,n^2(p+1)(2p+1)}-\frac{x^{m+1}\left(b+2c\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p}{b\,n(2p+1)}-\frac{(m-n+1)(m}{b\,n^2(p+}$$

$$\int x^m\left(a+b\,x^n+c\,x^{2n}\right)^p dx=-\frac{(m-3n-2n\,p+1)x^{m-2n+1}\left(a+b\,x^n+c\,x^{2n}\right)^{p+1}}{2c\,n^2(p+1)(2p+1)}-\frac{x^{m-2n+1}\left(2a+b\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p}{2c\,n(2p+1)}+\frac{(m-n+}{2c\,n^2(}$$

$$\int x^m\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{x^{m+1}\left(a+b\,x^n+c\,x^{2n}\right)^p}{m+2n\,p+1}+\frac{n\,p\,x^{m+1}\left(2a+b\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^{p-1}}{(m+2n\,p+1)(m+n(2p-1)+1)}+\frac{2a\,n^2p(2p-1)}{(m+2n\,p+1)(m+n(2p-1)+1}$$

$$\int x^m\left(a+b\,x^n+c\,x^{2n}\right)^p dx=-\frac{(m+n+2n\,p+1)x^{m+1}\left(a+b\,x^n+c\,x^{2n}\right)^{p+1}}{2a\,n^2(p+1)(2p+1)}-\frac{x^{m+1}\left(2a+b\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p}{2a\,n(2p+1)}+\frac{(m+n(2p+1)}{2a\,n^2(}$$

$$\int x^m\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{x^{m-n+1}\left(b+2c\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p}{2c(m+2n\,p+1)}-\frac{b(m-n+1)}{2c(m+2n\,p+1)}\int x^{m-n}\left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

$$\int x^m\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{x^{m+1}\left(b+2c\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p}{b(m+1)}-\frac{2c(m+n(2p+1)+1)}{b(m+1)}\int x^{m+n}\left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

Integrands of the form $x^m\,(A+B\,x^n)\,(a+b\,x^n+c\,x^{2n})^p$

- The resulting integrands are of the same form as the original integrand, so these reduction formulas can be repeatedly applied to drive the exponents m and p toward 0.
- These reduction formulas can be used for integrands having integer and/or fractional exponents.
- Special cases of these reductions formulas can be used for integrands of the form $\left(a+b\,x^n+c\,x^{2n}\right)^p$ and $x^m\left(a+b\,x^n+c\,x^{2n}\right)^p$ by setting m and/or B to 0.

$$\int x^m\left(A+B\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{x^{m+1}\left(A(m+n(2p+1)+1)+B(m+1)x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p}{(m+1)(m+n(2p+1)+1)}+\frac{n\,p}{(m+1)(m+n(2p+1)+1}$$
$$\int x^{m+n}\left(2a\,B(m+1)-A\,b(m+n(2p+1)+1)+(b\,B(m+1)-2\,A\,c(m+n(2p+1)+1))x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^{p-1}dx$$

$$\int x^m\left(A+B\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{x^{m-n+1}\left(A\,b-2a\,B-(b\,B-2A\,c)x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^{p+1}}{n(p+1)\left(b^2-4a\,c\right)}+\frac{1}{n(p+1)\left(b^2-4a\,c\right)}\cdot$$
$$\int x^{m-n}\left((m-n+1)(2a\,B-A\,b)+(m+2n(p+1)+1)(b\,B-2A\,c)x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^{p+1}dx$$

$$\int x^m\left(A+B\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{x^{m+1}\left(b\,B\,n\,p+A\,c(m+n(2p+1)+1)+B\,c(m+2n\,p+1)x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p}{c(m+2n\,p+1)(m+n(2p+1)+1)}+\frac{}{c(m+2}$$
$$\int x^m\left(2a\,A\,c(m+n(2p+1)+1)-a\,b\,B(m+1)+\left(2a\,B\,c(m+2n\,p+1)+A\,b\,c(m+n(2p+1)+1)-b^2B(m+n\,p+1)\right)x^n\right)(a+b$$

$$\int x^m\left(A+B\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p dx=-\frac{x^{m+1}\left(A\,b^2-a\,b\,B-2a\,A\,c+(A\,b-2a\,B)c\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^{p+1}}{a\,n(p+1)\left(b^2-4a\,c\right)}+\frac{1}{a\,n(p+1)\left(b^2-4a\,c\right.}$$
$$\int x^m\left((m+n(p+1)+1)A\,b^2-a\,b\,B(m+1)-2(m+2n(p+1)+1)a\,A\,c+(m+n(2p+3)+1)(A\,b-2a\,B)c\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right.$$

$$\int x^m\left(A+B\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{B\,x^{m-n+1}\left(a+b\,x^n+c\,x^{2n}\right)^{p+1}}{c(m+n(2p+1)+1)}-\frac{1}{c(m+n(2p+1)+1)}\cdot$$
$$\int x^{m-n}\left(a\,B(m-n+1)+(b\,B(m+n\,p+1)-A\,c(m+n(2p+1)+1))x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

$$\int x^m\left(A+B\,x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p dx=\frac{A\,x^{m+1}\left(a+b\,x^n+c\,x^{2n}\right)^{p+1}}{a(m+1)}+\frac{1}{a(m+1)}\cdot$$
$$\int x^{m+n}\left(a\,B(m+1)-A\,b(m+n(p+1)+1)-A\,c(m+2n(p+1)+1)x^n\right)\left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

References

1. "Reader Survey: log|*x*| + *C* (http://golem.ph.utexas.edu/category/2012/03/reader_survey_logx_c.html)", Tom Leinster, *The* n*-category Café*, March 19, 2012

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