

# List of integrals of logarithmic functions

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The following is a list of integrals (antiderivative functions) of logarithmic functions. For a complete list of integral functions, see list of integrals.

*Note:*  $x > 0$  is assumed throughout this article, and the constant of integration is omitted for simplicity.

## Integrals involving only logarithmic functions

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$$\int \log_a x \, dx = x \log_a x - \frac{x}{\ln a} = \frac{x}{\ln a} (\ln x - 1)$$

$$\int \ln(ax) \, dx = x \ln(ax) - x = x(\ln(ax) - 1)$$

$$\int \ln(ax + b) \, dx = \frac{ax + b}{a} (\ln(ax + b) - 1)$$

$$\int (\ln x)^2 \, dx = x(\ln x)^2 - 2x \ln x + 2x$$

$$\int (\ln x)^n \, dx = (-1)^n n! x \sum_{k=0}^n \frac{(-\ln x)^k}{k!}$$

$$\int \frac{dx}{\ln x} = \ln |\ln x| + \ln x + \sum_{k=2}^{\infty} \frac{(\ln x)^k}{k \cdot k!}$$

$$\int \frac{dx}{\ln x} = \operatorname{li}(x), \text{ the } \text{logarithmic integral}.$$

$$\int \frac{dx}{(\ln x)^n} = -\frac{x}{(n-1)(\ln x)^{n-1}} + \frac{1}{n-1} \int \frac{dx}{(\ln x)^{n-1}} \quad (\text{for } n \neq 1)$$

$$\int \ln f(x) \, dx = x \ln f(x) - \int x \frac{f'(x)}{f(x)} \, dx \quad (\text{for differentiable } f(x) > 0)$$

## Integrals involving logarithmic and power functions

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$$\int x^m \ln x \, dx = x^{m+1} \left( \frac{\ln x}{m+1} - \frac{1}{(m+1)^2} \right) \quad (\text{for } m \neq -1)$$

$$\int x^m (\ln x)^n \, dx = \frac{x^{m+1} (\ln x)^n}{m+1} - \frac{n}{m+1} \int x^m (\ln x)^{n-1} \, dx \quad (\text{for } m \neq -1)$$

$$\int \frac{(\ln x)^n dx}{x} = \frac{(\ln x)^{n+1}}{n+1} \quad (\text{for } n \neq -1)$$

$$\int \frac{\ln x dx}{x^m} = -\frac{\ln x}{(m-1)x^{m-1}} - \frac{1}{(m-1)^2 x^{m-1}} \quad (\text{for } m \neq 1)$$

$$\int \frac{(\ln x)^n dx}{x^m} = -\frac{(\ln x)^n}{(m-1)x^{m-1}} + \frac{n}{m-1} \int \frac{(\ln x)^{n-1} dx}{x^m} \quad (\text{for } m \neq 1)$$

$$\int \frac{x^m dx}{(\ln x)^n} = -\frac{x^{m+1}}{(n-1)(\ln x)^{n-1}} + \frac{m+1}{n-1} \int \frac{x^m dx}{(\ln x)^{n-1}} \quad (\text{for } n \neq 1)$$

$$\int \frac{dx}{x \ln x} = \ln|\ln x|$$

$$\int \frac{dx}{x \ln x \ln \ln x} = \ln|\ln|\ln x||, \text{ etc.}$$

$$\int \frac{dx}{x \ln \ln x} = \text{li}(\ln x)$$

$$\int \frac{dx}{x^n \ln x} = \ln|\ln x| + \sum_{k=1}^{\infty} (-1)^k \frac{(n-1)^k (\ln x)^k}{k \cdot k!}$$

$$\int \frac{dx}{x(\ln x)^n} = -\frac{1}{(n-1)(\ln x)^{n-1}} \quad (\text{for } n \neq 1)$$

$$\int \ln(x^2 + a^2) dx = x \ln(x^2 + a^2) - 2x + 2a \tan^{-1} \frac{x}{a}$$

$$\int \frac{x}{x^2 + a^2} \ln(x^2 + a^2) dx = \frac{1}{4} \ln^2(x^2 + a^2)$$

## **Integrals involving logarithmic and trigonometric functions**

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$$\int \sin(\ln x) dx = \frac{x}{2} (\sin(\ln x) - \cos(\ln x))$$

$$\int \cos(\ln x) dx = \frac{x}{2} (\sin(\ln x) + \cos(\ln x))$$

## **Integrals involving logarithmic and exponential functions**

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$$\int e^x \left( x \ln x - x - \frac{1}{x} \right) dx = e^x (x \ln x - x - \ln x)$$

$$\int \frac{1}{e^x} \left( \frac{1}{x} - \ln x \right) dx = \frac{\ln x}{e^x}$$

$$\int e^x \left( \frac{1}{\ln x} - \frac{1}{x(\ln x)^2} \right) dx = \frac{e^x}{\ln x}$$

## ***n* consecutive integrations**

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For *n* consecutive integrations, the formula

$$\int \ln x \, dx = x(\ln x - 1) + C_0$$

generalizes to

$$\int \cdots \int \ln x \, dx \cdots dx = \frac{x^n}{n!} \left( \ln x - \sum_{k=1}^n \frac{1}{k} \right) + \sum_{k=0}^{n-1} C_k \frac{x^k}{k!}$$

## **See also**

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- [List of mathematical identities](#)
- [Lists of mathematics topics](#)

## **References**

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- [Milton Abramowitz and Irene A. Stegun, \*Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables\*, 1964. A few integrals are listed on page 69 \(\[http://www.math.sfu.ca/~cbm/aands/page\\\_69.htm\]\(http://www.math.sfu.ca/~cbm/aands/page\_69.htm\)\).](#)
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