List of integrals of inverse hyperbolic functions

The following is a list of indefinite integrals (antiderivatives) of expressions involving the inverse hyperbolic functions. For a complete list of integral formulas, see lists of integrals.

- In all formulas the constant a is assumed to be nonzero, and C denotes the constant of integration.
- For each inverse hyperbolic integration formula below there is a corresponding formula in the list of integrals of inverse trigonometric functions.
- The ISO 80000-2 standard uses the prefix "ar-" rather than "arc-" for the inverse hyperbolic functions; we do that here.

Inverse hyperbolic sine integration formulas

$$\int \operatorname{arsinh}(ax)\,dx = x\operatorname{arsinh}(ax) - \frac{\sqrt{a^2x^2+1}}{a} + C$$

$$\int x\operatorname{arsinh}(ax)\,dx = \frac{x^2\operatorname{arsinh}(ax)}{2} + \frac{\operatorname{arsinh}(ax)}{4a^2} - \frac{x\sqrt{a^2x^2+1}}{4a} + C$$

$$\int x^2\operatorname{arsinh}(ax)\,dx = \frac{x^3\operatorname{arsinh}(ax)}{3} - \frac{\left(a^2x^2-2\right)\sqrt{a^2x^2+1}}{9a^3} + C$$

$$\int x^m\operatorname{arsinh}(ax)\,dx = \frac{x^{m+1}\operatorname{arsinh}(ax)}{m+1} - \frac{a}{m+1}\int \frac{x^{m+1}}{\sqrt{a^2x^2+1}}\,dx \quad (m \neq -1)$$

$$\int \operatorname{arsinh}(ax)^2\,dx = 2x + x\operatorname{arsinh}(ax)^2 - \frac{2\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}{a} + C$$

$$\int \operatorname{arsinh}(ax)^n\,dx = x\operatorname{arsinh}(ax)^n - \frac{n\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^{n-1}}{a} + n(n-1)\int \operatorname{arsinh}(ax)$$

$$\int \operatorname{arsinh}(ax)^n\,dx = -\frac{x\operatorname{arsinh}(ax)^{n+2}}{(n+1)(n+2)} + \frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^{n+1}}{a(n+1)} + \frac{1}{(n+1)(n+2)}\int$$

Inverse hyperbolic cosine integration formulas

$$\int \operatorname{arcosh}(ax)\,dx = x\operatorname{arcosh}(ax) - \frac{\sqrt{ax+1}\sqrt{ax-1}}{a} + C$$

$$\int x\operatorname{arcosh}(ax)\,dx = \frac{x^2\operatorname{arcosh}(ax)}{2} - \frac{\operatorname{arcosh}(ax)}{4a^2} - \frac{x\sqrt{ax+1}\sqrt{ax-1}}{4a} + C$$

$$\int x^2\operatorname{arcosh}(ax)\,dx = \frac{x^3\operatorname{arcosh}(ax)}{3} - \frac{\left(a^2x^2+2\right)\sqrt{ax+1}\sqrt{ax-1}}{9a^3} + C$$

$$\int x^m\operatorname{arcosh}(ax)\,dx = \frac{x^{m+1}\operatorname{arcosh}(ax)}{m+1} - \frac{a}{m+1}\int\frac{x^{m+1}}{\sqrt{ax+1}\sqrt{ax-1}}\,dx \quad (m\neq -1)$$

$$\int \operatorname{arcosh}(ax)^2\,dx = 2x + x\operatorname{arcosh}(ax)^2 - \frac{2\sqrt{ax+1}\sqrt{ax-1}\operatorname{arcosh}(ax)}{a} + C$$

$$\int \operatorname{arcosh}(ax)^n\,dx = x\operatorname{arcosh}(ax)^n - \frac{n\sqrt{ax+1}\sqrt{ax-1}\operatorname{arcosh}(ax)^{n-1}}{a} + n(n-1)\int \operatorname{arc}$$

$$\int \operatorname{arcosh}(ax)^n\,dx = -\frac{x\operatorname{arcosh}(ax)^{n+2}}{(n+1)(n+2)} + \frac{\sqrt{ax+1}\sqrt{ax-1}\operatorname{arcosh}(ax)^{n+1}}{a(n+1)} + \frac{1}{(n+1)(n+}$$

Inverse hyperbolic tangent integration formulas

$$\int \operatorname{artanh}(ax)\,dx = x\operatorname{artanh}(ax) + \frac{\ln\left(1-a^2x^2\right)}{2a} + C$$

$$\int x \operatorname{artanh}(ax)\, dx = \frac{x^2 \operatorname{artanh}(ax)}{2} - \frac{\operatorname{artanh}(ax)}{2a^2} + \frac{x}{2a} + C$$

$$\int x^2 \operatorname{artanh}(ax)\, dx = \frac{x^3 \operatorname{artanh}(ax)}{3} + \frac{\ln\left(1 - a^2 x^2\right)}{6a^3} + \frac{x^2}{6a} + C$$

$$\int x^m \operatorname{artanh}(ax)\, dx = \frac{x^{m+1} \operatorname{artanh}(ax)}{m+1} - \frac{a}{m+1} \int \frac{x^{m+1}}{1 - a^2 x^2}\, dx \quad (m \neq -1)$$

Inverse hyperbolic cotangent integration formulas

$$\int \operatorname{arcoth}(ax)\, dx = x \operatorname{arcoth}(ax) + \frac{\ln\left(a^2 x^2 - 1\right)}{2a} + C$$

$$\int x \operatorname{arcoth}(ax)\, dx = \frac{x^2 \operatorname{arcoth}(ax)}{2} - \frac{\operatorname{arcoth}(ax)}{2a^2} + \frac{x}{2a} + C$$

$$\int x^2 \operatorname{arcoth}(ax)\, dx = \frac{x^3 \operatorname{arcoth}(ax)}{3} + \frac{\ln\left(a^2 x^2 - 1\right)}{6a^3} + \frac{x^2}{6a} + C$$

$$\int x^m \operatorname{arcoth}(ax)\, dx = \frac{x^{m+1} \operatorname{arcoth}(ax)}{m+1} + \frac{a}{m+1} \int \frac{x^{m+1}}{a^2 x^2 - 1}\, dx \quad (m \neq -1)$$

Inverse hyperbolic secant integration formulas

$$\int \operatorname{arsech}(ax)\, dx = x \operatorname{arsech}(ax) - \frac{2}{a} \arctan \sqrt{\frac{1 - ax}{1 + ax}} + C$$

$$\int x \operatorname{arsech}(ax)\, dx = \frac{x^2 \operatorname{arsech}(ax)}{2} - \frac{(1 + ax)}{2a^2} \sqrt{\frac{1 - ax}{1 + ax}} + C$$

$$\int x^2 \operatorname{arsech}(ax)\,dx = \frac{x^3 \operatorname{arsech}(ax)}{3} - \frac{1}{3a^3} \arctan\sqrt{\frac{1-ax}{1+ax}} - \frac{x(1+ax)}{6a^2}\sqrt{\frac{1-ax}{1+ax}} + C$$

$$\int x^m \operatorname{arsech}(ax)\,dx = \frac{x^{m+1} \operatorname{arsech}(ax)}{m+1} + \frac{1}{m+1}\int \frac{x^m}{(1+ax)\sqrt{\frac{1-ax}{1+ax}}}\,dx \quad (m \neq -1)$$

Inverse hyperbolic cosecant integration formulas

$$\int \operatorname{arcsch}(ax)\,dx = x \operatorname{arcsch}(ax) + \frac{1}{a} \operatorname{arcoth}\sqrt{\frac{1}{a^2x^2} + 1} + C$$

$$\int x \operatorname{arcsch}(ax)\,dx = \frac{x^2 \operatorname{arcsch}(ax)}{2} + \frac{x}{2a}\sqrt{\frac{1}{a^2x^2} + 1} + C$$

$$\int x^2 \operatorname{arcsch}(ax)\,dx = \frac{x^3 \operatorname{arcsch}(ax)}{3} - \frac{1}{6a^3} \operatorname{arcoth}\sqrt{\frac{1}{a^2x^2} + 1} + \frac{x^2}{6a}\sqrt{\frac{1}{a^2x^2} + 1} + C$$

$$\int x^m \operatorname{arcsch}(ax)\,dx = \frac{x^{m+1} \operatorname{arcsch}(ax)}{m+1} + \frac{1}{a(m+1)}\int \frac{x^{m-1}}{\sqrt{\frac{1}{a^2x^2} + 1}}\,dx \quad (m \neq -1)$$

-