



DÀI HỌC ĐÀ NẴNG

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Chapter 5

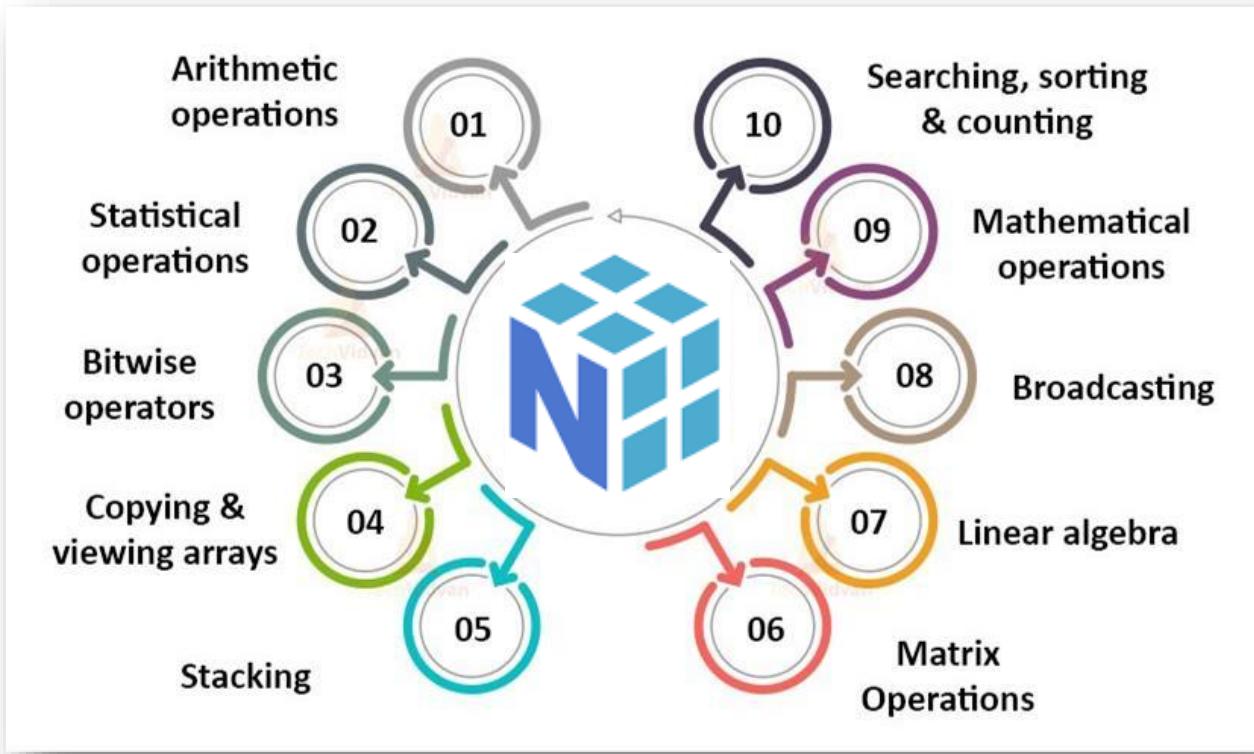
Numeric Computing with Numpy

- 1. Introduction to NumPy
- 2. Numpy Data Types
- 3. Numpy Getting Started
- 4. Numpy Array
- 5. Numpy Linear Algebra
- 6. Numpy Matrix Library - **Matlib**
- 7. Input/Output with Numpy

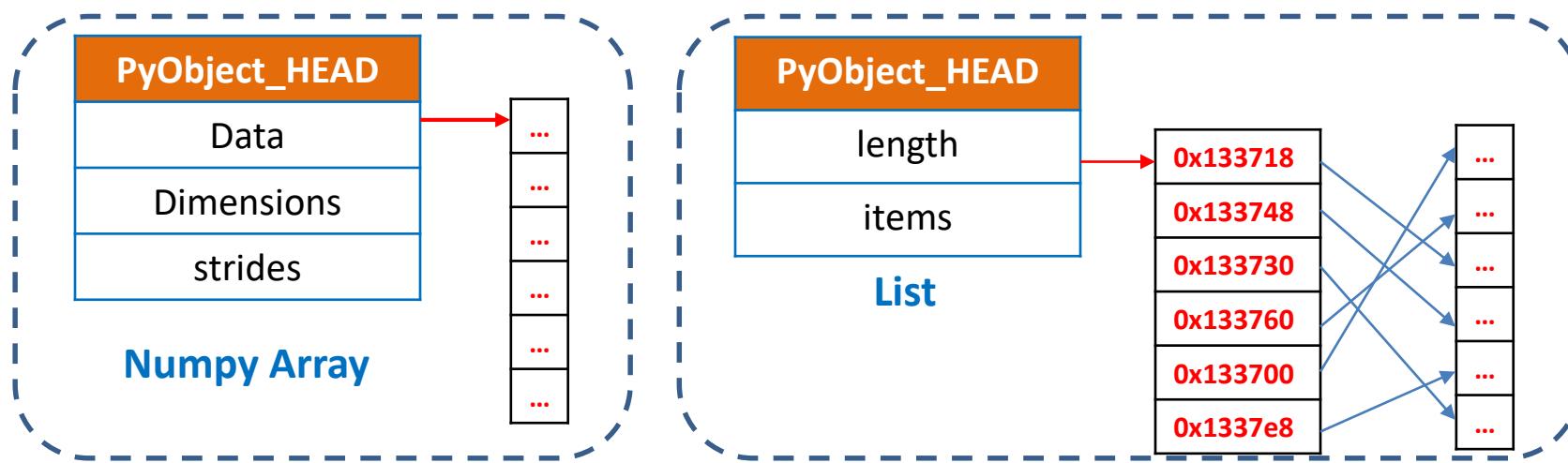
- In 2005, Travis Oliphant created NumPy
- Travis Oliphant is an American data scientist and businessman
- NumPy (**Numerical Python**) is an open source library in Python used to work with array data structures and related fast numerical routines
- Array oriented computing, Efficiently implemented multi-dimensional arrays



- NumPy is relied upon by scientists, engineers, and many other professionals around the world



- Convenient interface for working with multi-dimensional array data structures efficiently (**ndarray** object).
- Using fixed memory to store data (same data type) and less memory than Lists (different data type) .
- Allocate contiguous memory
- High computational efficiency



- Supports a much greater variety of numerical types than Python does

Data Types	Keywords/Sub Data Types
Boolean	<code>bool_</code>
Integer	<code>int_, intc, intp, int8, int16, int32, int64</code>
Unsigned Integer	<code>uint8, uint16, uint32, uint64</code>
Float	<code>float_, float16, float32, float64</code>
Complex	<code>complex_, complex64, complex128</code>

- Installing Numpy: `pip install numpy`
- Import Numpy: `import numpy`
- Alias of Numpy: `import numpy as np`
- Check Numpy version: `np.__version__`

- ndarray object (N-Dimensional array)

0-D Array

1

```
np.array(1)
```

2-D Array → Matrix

1	2	3
4	5	6
7	8	9

Axis 0

Axis 1

1-D Array → Vector

1	2	3
---	---	---

```
np.array([1, 2, 3])
```

3-D Array → Tensor

19	20	21
10	11	12
13	14	15

1	2	3
4	5	6
7	8	9

18		
26	27	28

Axis 0

Axis 1

Axis 2

```
np.array([[1, 2, 3],  
         [4, 5, 6],  
         [7, 8, 9]])
```

```
np.array([[[1, 2, 3],  
          [4, 5, 6],  
          [7, 8, 9]],  
        [[10, 11, 12],  
         [13, 14, 15],  
         [16, 17, 18]],  
        [[19, 20, 21],  
         [22, 23, 24],  
         [25, 26, 27]])
```

- Numpy Array Creation
- Numpy Array Indexing
- Numpy Array Slicing
- Numpy Arithmetic Operations
- Numpy Arithmetic Functions
- Numpy Array Manipulation Functions
- Numpy Broadcasting
- Numpy Statistical Operations

- Using the **array()** method

```
import numpy as np  
arr = np.array([1, 2, 3, 4, 5])
```

- Converting from lists, tuples → Create array from available data.
- Syntax:

```
np.array(Data)
```

Data: Input data (list, tuple, or other data types).

```
list1 = [1, 2, 3, 4, 5]  
arr = np.array(list1)
```

```
tuple1 = (1, 2, 3, 4, 5)  
arr = np.array(tuple1)
```

- Using special methods:
 - `randint(start, end)`: return a random integer in range [start, end] including the end points
 - `numpy.random.rand()`: return a array of defined shape, filled with random values.
 - `numpy.eye()`: returns a 2-D array with 1's as the diagonal and 0's elsewhere
 - `numpy.full(shape, fill_value, dtype=None, order = 'C')`: return a new array with the same shape and type as a given array filled with a fill_value
 - `numpy.random.random()`: return a array of random floats in the interval [0.0, 1.0).
 - `arange([start,] stop[,step,][,dtype])`: returns an array with evenly spaced elements as per the interval. The interval mentioned is half-opened i.e. [Start, Stop)
 - `numpy.zeros()`: returns a new array of given shape and type, with zeros.
 - `numpy.ones()`: returns a new array of given shape and type, with ones.
 - `numpy.empty(shape, dtype = float, order = 'C')`: return a new array of given shape and type, with random values.

- Example 1:

```
import numpy as np  
  
empty_array = np.empty(2, dtype=int) [1065353216 1065353216]  
  
zeros_array = np.zeros(5) [0., 0., 0., 0., 0.]  
  
[[1., 0., 0.],  
 [0., 1., 0.],  
 [0., 0., 1.]]  
  
arange_array = np.arange(0, 10) [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]  
  
eye_matrix = np.eye(3)  
  
[[5, 6, 2],  
 [9, 3, 3],  
 [9, 9, 1]]  
  
randint_array = np.random.randint(1, 10, (3, 3))  
  
rand_array = np.random.rand(2, 3) [[0.56549245, 0.06168123, 0.71656376],  
 [0.7808586, 0.53067687, 0.86072789]]
```

4. Numpy Array: Numpy Array Attributes

ndarray.shape

return the size of array
(Number of elements
of each dimension)

(3,3)

ndarray.ndim

return number of array
dimension

2

ndarray.dtype

return data type of
elements in the array

1	2	3
4	5	6
7	8	9

int64

ndarray.size

return number of
elements in the array

9

ndarray.itemsize

return the size (in bytes)
of elements in the array

8

- Example 2:

```
import numpy as np

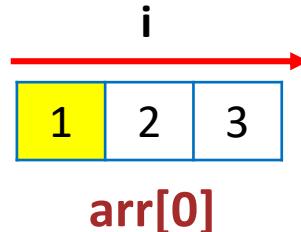
arr = np.array([1 , 2, 3, 4, 5])
print("arr :", arr)
print("Shape of arr:", arr.shape)
print("Size of arr:", arr.size)
print("Data type of arr :", arr.dtype)
print("Number of dimensions of arr:", arr.ndim)
```

arr : [1, 2, 3, 4, 5]
Shape of arr: (5 ,)
Size of arr: 5
Data type of arr: int64
Number of dimensions of arr: 1

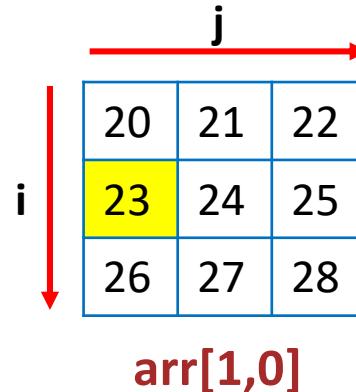
4. Numpy Array: Numpy Array Indexing

- NumPy allows to access each element of an array using its index

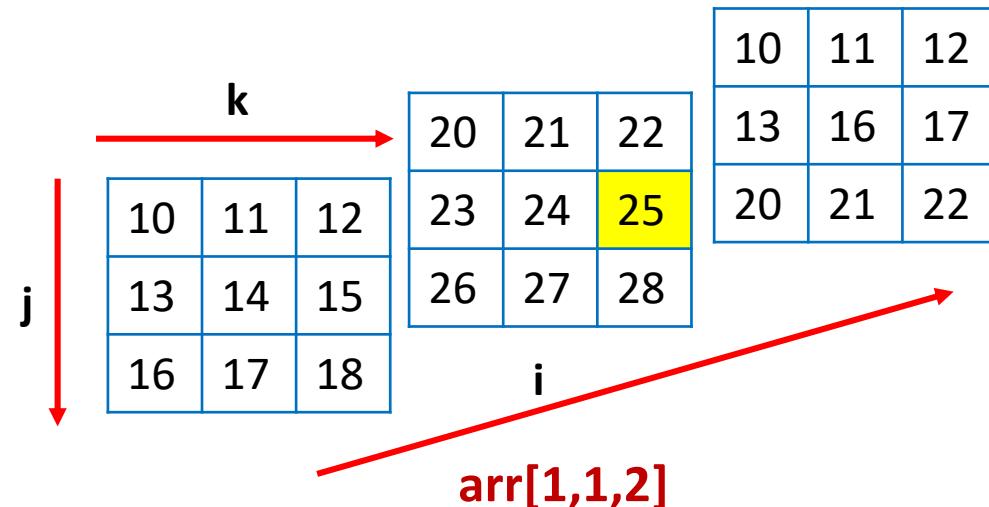
1-D Array: → `ArrayName[i]`



2-D Array: → `ArrayName[i,j]`



3-D Array: → `ArrayName[i,j,k]`



Note: The Index of first element is a 0
The Index of last element is a -1

- Example 3:

```
import numpy as np  
arr1d = np.array([1 , 2, 3, 4, 5])  
first_element = arr1d[0]  
print("The first element of arr1d: ", first_element )  
last_element = arr1d[-1]  
print("The last element of arr1d: ",last_element )
```

The first element of arr1d:1
The last element of arr1d:5

- Example 4:

```
import numpy as np  
  
arr2d = np.array([[1, 2, 3], [4, 5, 6]])  
print(arr2d)  
element_1_2 = arr2d[0, 1]  
print(element_1_2 )
```

[[1, 2, 3],
 [4, 5, 6]]

2

4. Numpy Array: Numpy Array Slicing

- Slicing in NumPy to extract a part of an array using its the index range

- 1-D Array

1	2	3
---	---	---

`arr[:1]`

- 3-D Array

k

10	11	12
13	14	15
16	17	18

j

`arr[2,:,1:,2]`

ArrayName_1D [start_Index : end_Index]

- 2-D Array

10	11	12
13	16	17
20	21	22

j

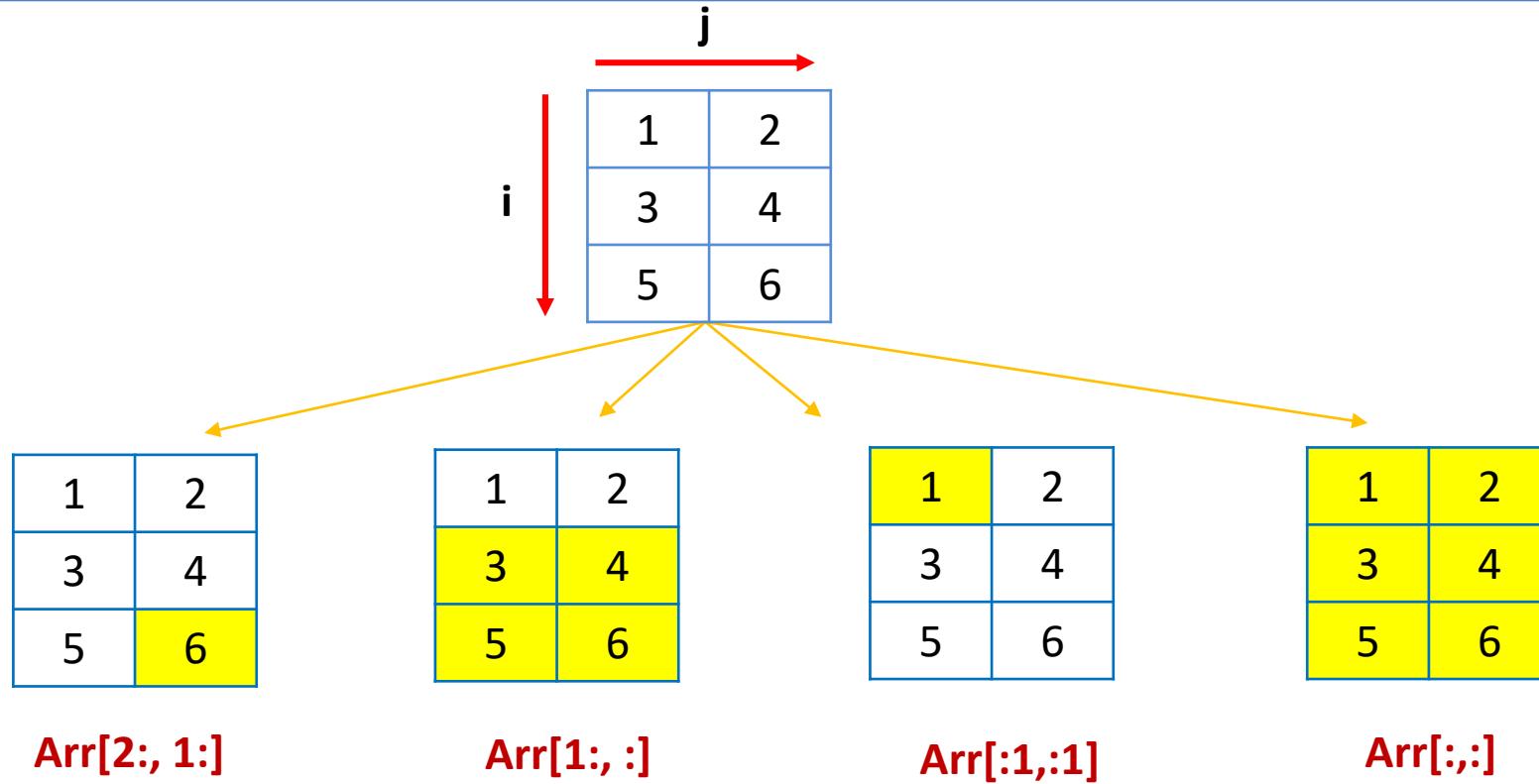
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24
25	26	27	28	29

i

`arr[1:,2:4]`

4. Numpy Array: Numpy Array Slicing

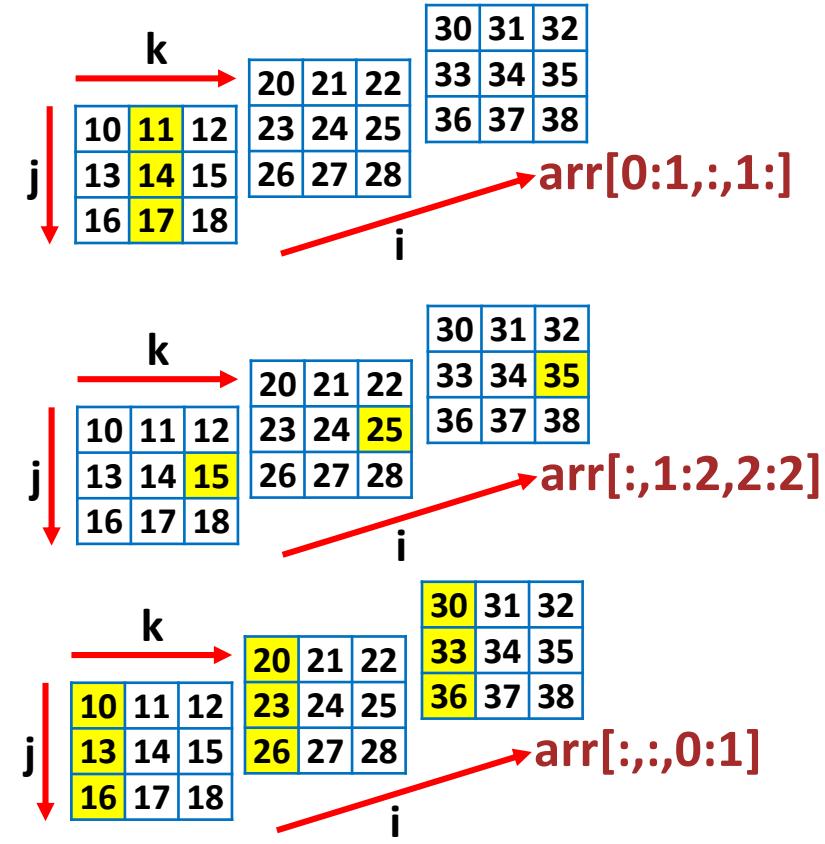
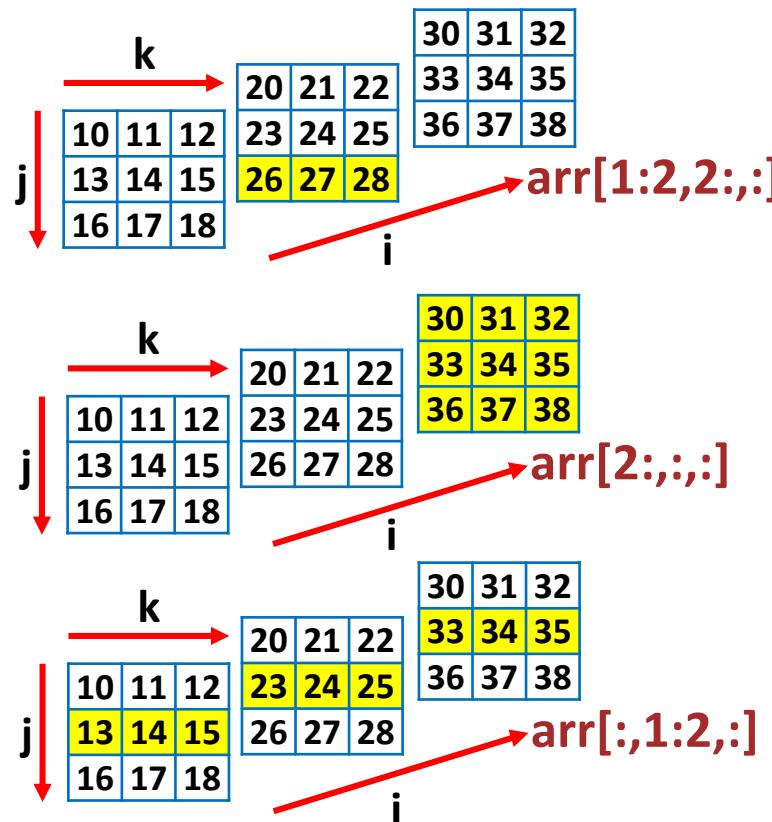
ArrayName_2D[start_Indexi:end_Indexi, start_Indexj:end_Indexj]



4. Numpy Array: Numpy Array Slicing

```
ArrayName_3D[start_Indexi:end_Indexi, start_Indexj:end_Indexj,  
              start_Indexk:end_Indexk ]
```

4. Numpy Array: Numpy Array Slicing



4. Numpy Array: Numpy Array Slicing

- Example 5:

```
import numpy as np
arr1d = np.array ([1 , 2, 3, 4, 5])
slice_arr1d = arr1d[1:4]
print("arr1d:", arr1d )
print("Slicing arr1d frome 1 to 3:", slice_arr1d)
```

arr1d: [1, 2, 3, 4, 5]
Slicing arr1d from 1 to 3: [2,
3, 4]

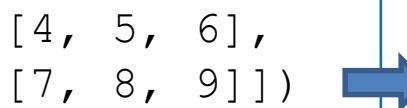
- Example 6:

```
import numpy as np
arr2d = np.array (
    [[1, 2, 3],
     [4, 5, 6],
     [7, 8, 9]])
slice_arr2d = arr2d[1:3 , 1:]
print("arr2d:\n", arr2d )
print("Slicing arr2d from row 2 to row  
3 and column 2 to column end:\n", slice_arr2d )
```

arr2d:
[[1, 2, 3],
 [4, 5, 6],
 [7, 8, 9]]
Slicing arr2d from row 2 to row
3 and column 2 to column end:
[[5, 6],
 [8, 9]]

- **transpose()** method:
 - Create a new array by transposing a 2-dimensional numpy array.
 - Syntax: `new_array = original_array.transpose()`
- Example 7:

```
import numpy as np
original_array = np.array([[1, 2, 3],
                           [4, 5, 6],
                           [7, 8, 9]])
new_array = original_array.transpose()
print(new_array)
```



```
[[1, 4, 7],
 [2, 5, 8],
 [3, 6, 9]]
```

- **reshape()** method:
 - Create a new array by changing the shape from another array
 - Syntax: `new_array = np.reshape(original_array, shape))`
- Example 8:

```
import numpy as np
original_array = np.array([[0, 1],
                           [2, 3],
                           [4, 5]])
new_array=np.reshape(original_array, (2,3))
print(new_array)
```



```
[[0, 2, 4],
 [1, 3, 5]]
```

- **reshape()** method
- Example 9:

```
import numpy as np  
  
arr_2D = np.array([[1 , 2, 3], [4, 5, 6]])  
# (2, 3) ->(3, 2)  
new_arr_2D = np.reshape(arr_2D , (3, 2))  
# 2D -> 1D  
  
arr_1D = np.reshape(arr_2D , newshape =(6 , ))  
print("array 2D:\n", arr_2D , arr_2D.shape )  
print("new array 2D :\n", new_arr_2D, new_arr_2D.shape)  
print("array 1D:\n", arr_1D, arr_1D.shape)
```

array 2D:
[[1,2,3],
 [4,5,6]] (2,3)
new array 2D :
[[1,2],
 [3,4],
 [5,6]] (3, 2)
array 1D:
[1,2,3,4,5,6] (6,)

- **resize()** method:
 - Change the size of an array
 - Syntax: `original_array.resize(row_number, column_number)`
- Example 10:

```
import numpy as np
original_array = np.array([[1, 2, 3],
                          [4, 5, 6],
                          [7, 8, 9]])
original_array.resize(3, 4)
print(original_array)
```



```
[[1, 2, 3, 4],
 [5, 6, 7, 8],
 [9, 0, 0, 0]]
```

- **insert()** method:

- Create a new array by inserting values from another array

- Syntax: `new_array = np.insert(original_array, index, value, axis)`

- Example 11:

```
import numpy as np
original_array = np.array([[1, 2, 3],
                           [4, 5, 6],
                           [7, 8, 9]])
axis = 0
axis = 1
new_array1 = np.insert(original_array, 0, [10, 10, 10], axis=0)
new_array2 = np.insert(original_array, 3, [10, 10, 10], axis=0)
new_array3 = np.insert(original_array, 0, [10, 10, 10], axis=1)
new_array4 = np.insert(original_array, 3, [10, 10, 10], axis=1)
```

[[10, 10, 10],
 [1, 2, 3],
 [4, 5, 6],
 [7, 8, 9]]

[[1, 2, 3],
 [4, 5, 6],
 [7, 8, 9],
 [10, 10, 10]]

[[10, 1, 2, 3],
 [10, 4, 5, 6],
 [10, 7, 8, 9]]

[[1, 2, 3, 10],
 [4, 5, 6, 10],
 [7, 8, 9, 10]]

- **append()** method:
 - Create a new array by appending values along the mentioned axis at the end of the array
 - Syntax: `new_array = np.append(original_array, value, axis)`
- Example 12:

```
import numpy as np
original_array1 = np.array([[1, 2, 3],
                           [4, 5, 6],
                           [7, 8, 9]])
axis = 0
axis = 1
[[ 1,2,3,10,20,30],
 [ 4,5,6,40,50,60],
 [ 7,8,9,70,80,90]]
original_array2 = np.array([[10, 20, 30],
                           [40, 50, 60],
                           [70, 80, 90]])
axis = 0
axis = 1
[[ 1,2,3],
 [ 4,5,6],
 [ 7,8,9],
 [10,20,30],
 [40,50,60],
 [70,80,90]]
new_array1 = np.append(original_array1,original_array2, axis=1)
new_array2 = np.append(original_array1,original_array2, axis=0)
```

- **delete()** method:
 - Returns a new array with the deletion of sub-arrays along with the mentioned axis
 - Syntax: `new_array = np.delete(original_array, index, axis)`
- Example 13:

```
import numpy as np
original_array = np.array([[1, 2, 3],
                           [4, 5, 6],
                           [7, 8, 9]])
axis = 0
new_array1 = np.delete(original_array, 0, axis=0)
new_array2 = np.delete(original_array, 0, axis=1)
new_array3 = np.delete(original_array, 2, axis=0)
new_array4 = np.delete(original_array, 2, axis=1)
```

axis = 0
↓
axis = 1

[[4, 5, 6],
 [7, 8, 9]]

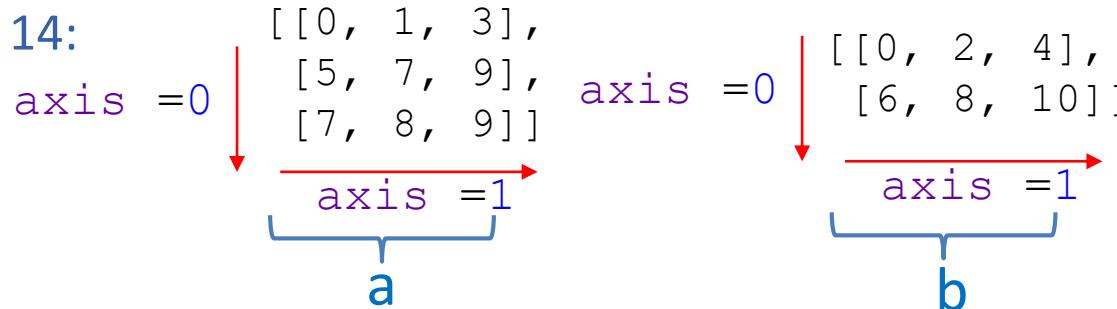
[[2, 3],
 [5, 6],
 [8, 9]]

[[1, 2, 3],
 [4, 5, 6]]

[[1, 2],
 [4, 5],
 [7, 8]]

4. Numpy Array: Manipulation Functions

- Example 14:



```
[[0, 1, 3, 0, 2, 4],  
 [5, 7, 9, 6, 8, 10]]
```

```
np.concatenate((a,b),axis=1)
```

```
[[0, 1, 3, 0, 2, 4],  
 [5, 7, 9, 6, 8, 10]]
```

```
np.hstack ((a , b))
```

```
[[0, 1, 3, ]]  
 [0, 2, 4],  
 [5, 7, 9],  
 [6, 8, 10] ]
```

```
np.vstack ((a , b))
```

- Example 15:

```
import numpy as np
arr_1 = np.array([1 , 2, 3])
arr_2 = np.array([4 , 5, 6])
arr_3 = np.hstack((arr_1 , arr_2))
print("arr 1:\n", arr_1)
print("arr 2:\n", arr_2)
print("Result_arr:\n", arr_3)
```



```
arr 1: [1, 2, 3]
arr 2: [4, 5, 6]
Result_arr:
[1, 2, 3, 4, 5, 6]
```

- Example 16:

```
import numpy as np
arr_1 = np.array([1 , 2, 3])
arr_2 = np.array([4 , 5, 6])
arr_3 = np.vstack((arr_1 , arr_2))
print("arr 1:", arr_1)
print("arr 2:", arr_2)
print("Result_arr:\n", arr_3)
```



```
arr 1:[1, 2, 3]
arr 2:[4, 5, 6]
Result_arr:
[[1, 2, 3],
 [4, 5, 6]]
```

- Example 17:

```
import numpy as np

arr_1 = np.array([[1 , 2, 3],
                  [4, 5, 6]])
arr_2 = np.array([[7 , 8, 9],
                  [10 , 11, 12]])

arr_3 = np.concatenate((arr_1, arr_2),axis =0)
arr_4 = np.concatenate((arr_1, arr_2),axis =1)

print("Array 1:\n", arr_1)
print("Array 2:\n", arr_2)
print("Result( axis =0) :\n",arr_3)
print("Result( axis =1) :\n",arr_4)
```



Array 1:
[[1,2,3],
 [4,5,6]]

Array 2:
[[7,8,9],
 [10,11,12]]

Result(axis =0):
[[1,2,3],
 [4,5,6],
 [7,8,9],
 [10,11,12]]

Result(axis =1):
[[1,2,3,7,8,9],
 [4,5,6,10,11,12]]

- Add a New Axis to a Numpy Array: There are various ways to add a new axis to a NumPy array
 - Using
 - `newaxis()` method
 - `expand_dims()` method
 - `reshape()` method

- **newaxis()** method
- Example 18:

```
import numpy as np

# array 1D
arr_1 = np.array([1 , 2, 3])

# 1D -> 2D
arr_2 = arr_1[np.newaxis, :]

# 2D -> 5D
arr_3 = arr_2[np.newaxis, :, np.newaxis ,:, np.newaxis]
print("array 1: ", arr_1 , arr_1.shape)
print("array 2: ", arr_2 , arr_2.shape)
print("array 3:\n ", arr_3 , arr_3.shape)
```



```
array 1:[1,2,3] (3,)
array 2:[[1,2,3],] (1, 3)
array 3:
[[[[[1]
   [2]
   [3]]]] (1, 1, 1, 3, 1)
```

- **expand_dims ()** method
- Example 19:

```
import numpy as np  
  
arr_1 = np.array([1 , 2, 3])  
  
# 1D -> 2D  
arr_2 = np.expand_dims(arr_1 , axis =0)  
  
# 2D -> 5D  
arr_3 = np.expand_dims(arr_2 ,axis =(0 , 2, 4))  
  
print("array 1:", arr_1 , arr_1.shape)  
print("array 2:", arr_2 , arr_2.shape)  
print("array 3:\n", arr_3 , arr_3.shape)
```

array 1: [1,2,3] (3,)
array 2: [[1,2,3],] (1, 3)
array 3:
[[[[[1]
[2]
[3]]]]] (1, 1, 1, 3, 1)

4. Numpy Array: Manipulation Functions

- Sort the elements of the array in ascending order .

- Syntax: `np.sort(arr , [axis])`

- `arr`: input array
- `axis`: sort direction

- Example 20:

```
import numpy as np
arr2d = np.array ([[1 , 7, 3],
                  [10, 8, 6],
                  [5, 15, 9]])
print("Original array\n",arr2d)
sort_in_row = np.sort(arr2d , axis =1)
sort_in_col = np.sort(arr2d , axis =0)
print("Result by row:\n", sort_in_row)
print("Result by col:\n", sort_in_col)
```

([[1 , 7, 3],
 [10, 8, 6],
 [5, 15, 9]])

axis =0
↓
axis =1

Original array:

[[1,7,3],
 [10,8,6],
 [5,15,9]]

Result by row:

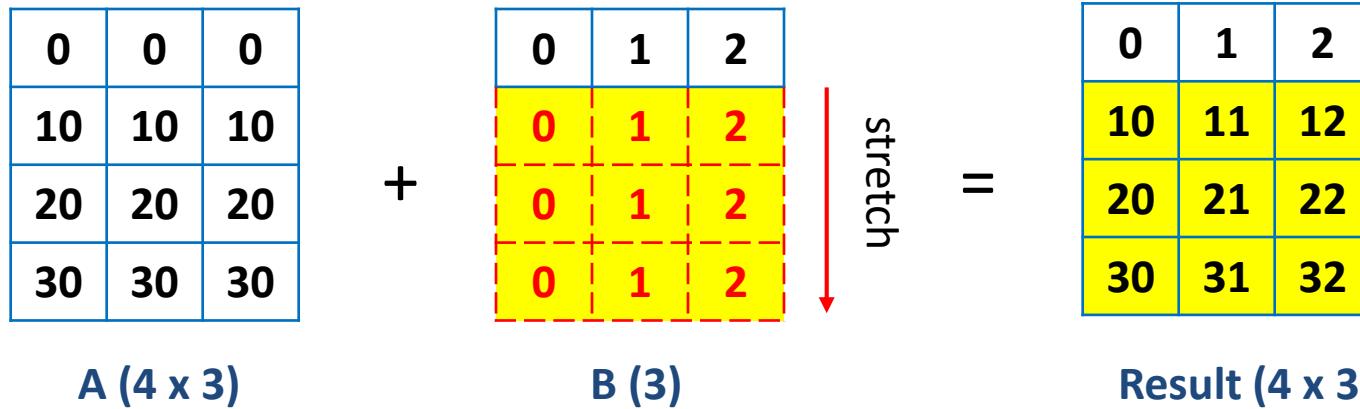
[[1,3,7],
 [6,8,10],
 [5,9,15]]

Result by col:

[[1,7,3],
 [5,8,6],
 [10,15,9]]



- Broadcasting:
 - Allows performing mathematical operations on arrays of different sizes
 - When done operations, NumPy automatically expands smaller arrays so that they are the same size as the larger array
 - → Thereby, saving memory and optimizing performance



- Broadcasting Rules (Simplified):
 - If array shapes differ in rank, prepend 1s to the smaller one until they match in length.
 - Two dimensions are compatible if they are equal or one of them is 1.
 - Arrays can be broadcast only if they are compatible in all dimensions.
 - After broadcasting, both arrays behave as if they had the shape of the element-wise maximum.
 - Any array with dimension 1 behaves as if it's copied to match the other array's size in that dimension.

4. Numpy Array: Numpy Arithmetic Operations

- Example 21:



```
np.add(a,b)
```

→ `[[10,8,12],
 13,12,16],
 [16,9,19]]`

```
np.subtract(a,b)
```

→ `[-10,-2,-10],
 [-7,2,-6],
 [-4,-1,-3]]`

```
np.multiply(a,b)
```

→ `[[0,15,11],
 [30,35,55],
 [60,20,88]]`

```
np.divide(a,b)
```



`[[0., 0.6, 0.09090909],
 [0.3, 1.4, 0.45454545],
 [0.6, 0.8, 0.72727273]]`

4. Numpy Array: Numpy Arithmetic Operations

- Example 22:

```
import numpy as np
a = np.array ([[0, 3, 1],
               [3, 7, 5],
               [6, 4, 8]])
b = [10, 5, 11]

print("add_array =\n",np.add(a,b))
print("subtract_array =\n",np.subtract(a,b))
print("multiply_array =\n",np.multiply(a,b))
print("divide_array =\n",np.divide(a,b))
```



```
add_array =
[[10, 8, 12],
 [13, 12, 16],
 [16, 9, 19]]
subtract_array =
[[-10, -2, -10],
 [-7, 2, -6],
 [-4, -1, -3]]
multiply_array =
[[ 0, 15, 11],
 [30, 35, 55],
 [60, 20, 88]]
divide_array =
[[0. , 0.6, 0.09090909],
 [0.3, 1.4, 0.45454545],
 [0.6, 0.8, 0.72727273]]
```

4. Numpy Array: Numpy Arithmetic Operations

- Example 23:

[7, 3, 4, 5, 1]
 └───┐
 a

[3, 4, 5, 6, 7]
 └───┐
 b

`np.remainder(a,b)`

→ [1, 3, 4, 5, 1]

`np.power(a,b)`

→ [343, 81, 1024, 15625, 1]

`np.mod(a,b)`

→ [1, 3, 4, 5, 1]

`np.reciprocal(a)`

→ [0, 0, 0, 0, 1]

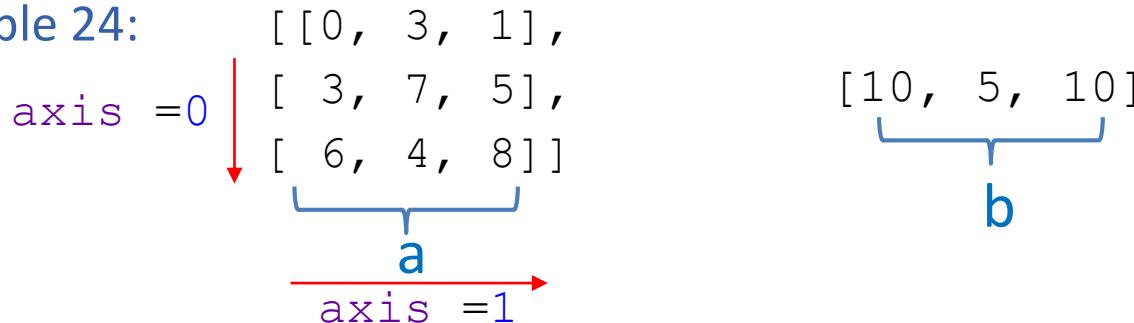
4. Numpy Array: Numpy Arithmetic Operations

- Return sum value in the array.

- Syntax:

```
np.sum(array, [axis])
```

- Example 24:



```
np.sum(a)
```

→ 37

```
np.sum(b)
```

→ 25

```
np.sum(a , axis =0)
```

→ [9, 14, 14]

```
np.sum(a , axis =1)
```

→ [4, 15, 18]

- array: input array
- axis: search direction

4. Numpy Array: Numpy Arithmetic Operations

- Example 25:

```
import numpy as np
a = np.array ([[0, 3, 1],
               [3, 7, 5],
               [6, 4, 8]])
b = [10, 5, 11]

print("Sum_a =", np.sum(a))
print("Sum_b =", np.sum(b))
print("Sum_a_axis_0 =", np.sum(a, axis=0))
print("Sum_a_axis_1 =", np.sum(a, axis = 1))
```



```
Sum_a = 37
Sum_b = 26
Sum_a_axis_0 = [ 9, 14, 14]
Sum_a_axis_1 = [ 4, 15, 18]
```

- Return the largest/smallest value in the array.

- Syntax:

```
np.max(array , [axis] )
```

```
np.min(array , [axis] )
```

- array: input array
- axis: search direction

- Example 26:

```
import numpy as np
arr = np. array ([1 , 2, 3, 4, 5])
max_value = np.max(arr)
print("largest value :", max_value)
min_value = np.min(arr)
print("Smallest value:", min_value)
```

largest value : 5
Smallest value: 1

4. Numpy Array: Numpy Statistical Operations

- Example 27:

axis = 0

[[1, 2, 3],
 [4, 5, 6]]

axis = 1

Maximum value in row: [3, 6]
Maximum value in col: [4, 5, 6]
Minimum value in row: [1, 4]
Minimum value in col: [1 2, 3]

```
import numpy as np  
  
arr2d = np.array ([[1 , 2, 3], [4, 5, 6]])  
max_value_row = np.max(arr2d , axis =1)  
max_value_col = np.max(arr2d , axis =0)  
print("Maximum value in row:", max_value_row)  
print("Maximum value in col:", max_value_col)
```

```
min_value_row = np.min(arr2d , axis =1)  
min_value_col = np.min(arr2d , axis =0)  
print("Minimum value in row:", min_value_row)  
print("Minimum value in col:", min_value_col)
```

4. Numpy Array: Numpy Statistical Operations

- Return the Index of the element that has the largest/smallest value.
- Syntax:
 - arr: input array
 - axis: search direction
- Example 28:

```

import numpy as np
arr2d = np.array ([[1 , 2, 3], [4, 5, 6]])
max_value_row = np.max(arr2d , axis =1)
max_index_row = np.argmax(arr2d , axis =1)
print("Maximum value in row:",max_value_row)
print("Index:", max_index_row)

max_value_col = np.max(arr2d , axis =0)
max_index_col = np.argmax(arr2d , axis =0)
print("Maximum value in col:", max_value_col)
print("Index:", max_index_col)
    
```

np. **argmax**(arr , [axis])
 np. **argmin**(arr , [axis])

axis =0

[[1 , 2, 3],
 [4, 5, 6]]
 axis =1



Maximum value in row:[3,6]
 Index: [2,2]
 Maximum value in col:[4,5,6]
 Index: [1,1,1]

4. Numpy Array: Numpy Statistical Operations

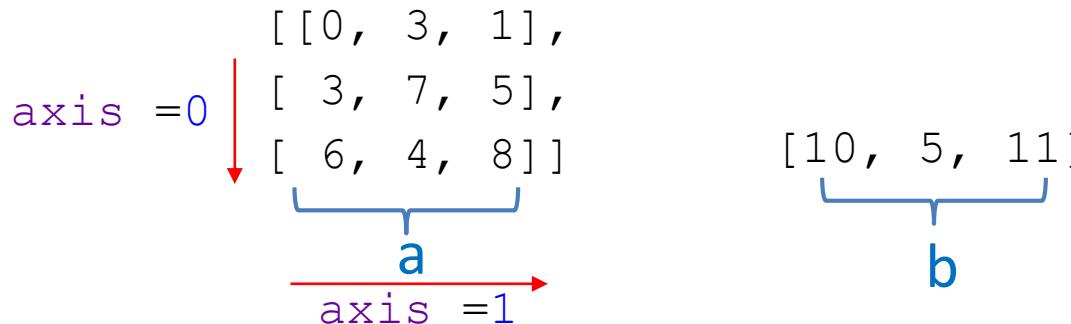
- Return median value in the array.

- Syntax:

```
np.median(array, [axis])
```

- array: input array
- axis: search direction

- Example 29:



np.median(a)	→	4.0
np.median(b)	→	10.0
np.median(a, axis =0)	→	[3.0, 4.0, 5.0]
np.median(a, axis =1)	→	[1.0, 5.0, 6.0]

- Example 30:

```
import numpy as np
a = np.array ([[0, 3, 1],
               [3, 7, 5],
               [6, 4, 8]])
b = [10, 5, 11]

print("Median value in row:", np.median(a , axis =1))
print("Median value in col:", np.median(a , axis =0))
print("Median array A:", np.median(a))
print("Median array B:", np.median(b))
```

Median value in row: [1., 5., 6.]
Median value in col: [3., 4., 5.]
Median array A: 4.0
Median array B: 10.0

4. Numpy Array: Numpy Statistical Operations

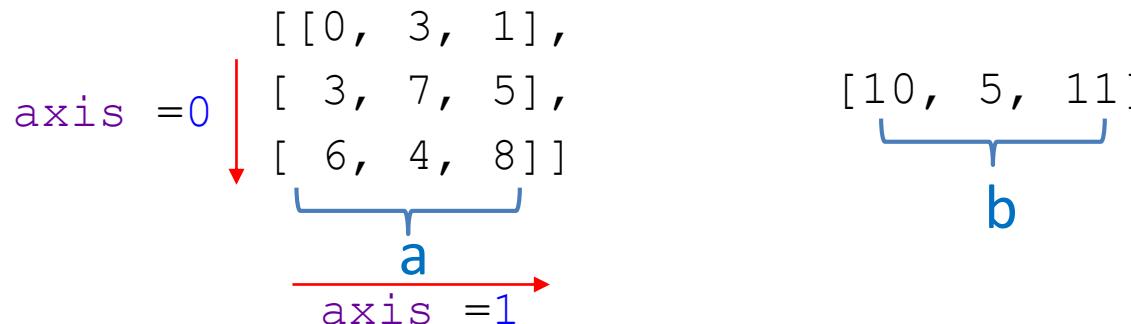
- Return mean value in the array.

- Syntax:

```
np.mean(array, [axis])
```

- array: input array
- axis: search direction

- Example 31:



np.mean(a)	→	4.11
np.mean(b)	→	8.66
np.mean(a , axis =0)	→	[3.0, 4.66, 4.66]
np.mean(a , axis =1)	→	[1.33, 5.0, 6.0]

- Example 32:

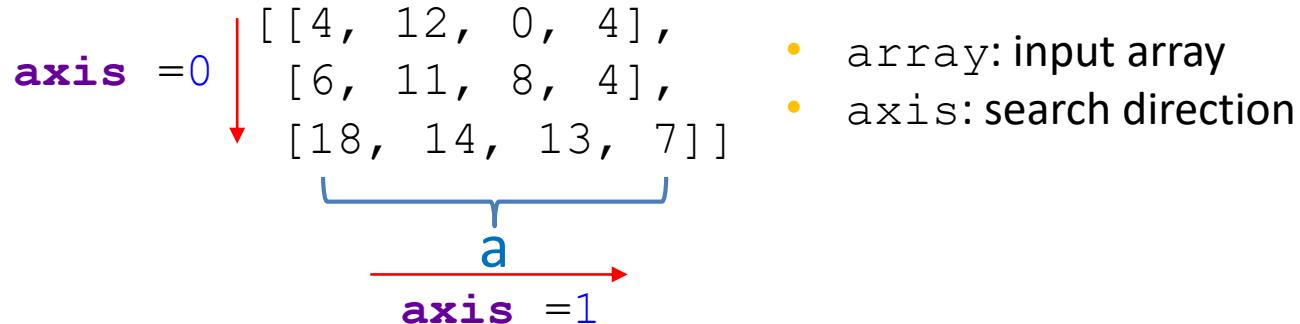
```
import numpy as np
a = np.array ([[0, 3, 1],
               [3, 7, 5],
               [6, 4, 8]])
b = [10, 5, 11]
print("Mean_a =", np.mean(a))
print("Mean_b =", np.mean(b))
print("Mean_a_axis_0 =", np.mean(a, axis = 0))
print("Mean_a_axis_1 =", np.mean(a, axis = 1))
```



```
Mean_a = 4.11
Mean_b = 8.66
Mean_a_axis_0 = [3., 4.66, 4.66]
Mean_a_axis_1 = [1.33, 5., 6. ]
```

4. Numpy Array: Numpy Statistical Operations

- Return standard deviation: `np.std(array, [axis])`
- Return variance : `np.var(array, [axis])`
- Example 33:



<code>np.std(a, axis=0)</code>	→ [6.18, 1.24, 5.35, 1.41]
<code>np.std(a, axis=1)</code>	→ [4.35, 2.58, 3.93]
<code>np.var(a, axis=0)</code>	→ [38.22, 1.55, 28.66, 2.]
<code>np.var(a, axis=1)</code>	→ [19., 6.68, 15.5]

- Example 34:

```
import numpy as np
a = np.array ([[4, 12, 0, 4],
               [6, 11, 8, 4],
               [18, 14, 13, 7]])

print("Standard Deviation_a_axis_0 =", np.std(a, axis=0))
print("Standard Deviation_a_axis_1 =", np.std(a, axis=1))
print("Variance_a_axis_0 =", np.var(a, axis=0))
print("Variance_a_axis_1 =", np.var(a, axis=1))
```

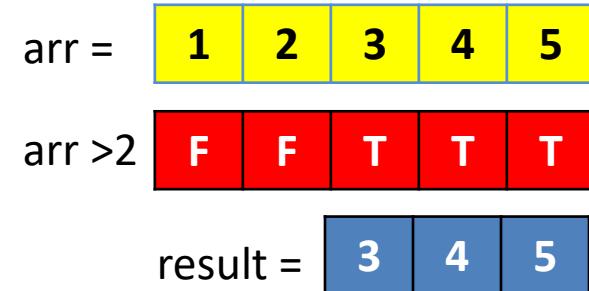


```
Standard Deviation_a_axis_0 = [6.18, 1.24, 5.35, 1.41]
Standard Deviation_a_axis_1 = [4.35, 2.58, 3.93]
Variance_a_axis_0 = [38.22, 1.55, 28.66, 2.      ]
Variance_a_axis_1 = [19.   , 6.68, 15.5    ]
```

4. Numpy Array: Numpy Statistical Operations

- Return the elements with conditions `where()`
- Syntax:
- Example 35:

```
np.where (condition)
```



```
import numpy as np

arr = np.array([1 , 2, 3, 4, 5])
result = np.where(arr > 2)
print("Position of elements greater than 2:",result)
print("The value of the elements at the found position:",arr[result])
```



Position of elements greater than 2: (array([2, 3, 4], dtype=int64),)
The value of the elements at the found position: [3, 4, 5]

- Linalg: the package in NumPy for Linear Algebra
 - `dot()` : Dot product of two arrays
 - `vdot()` : Return the dot product of two vectors.
 - `inner()` : Inner product of two arrays
 - `outer()` : Compute the outer product of two vectors.
 - `matmul()` : Matrix product of two arrays.
 - <https://numpy.org/doc/stable/reference/routines.linalg.html>

- Example 36:

```
import numpy as np
a = np.array([[1, 0],
              [0, 1]])
b = np.array([[4, 1],
              [2, 2]])

print("Dot_Result =\n", np.dot(a, b))
print("Vdot_Result =\n", np.vdot(a, b))
print("Inner_Result =\n", np.inner(a, b))
print("Outer_Result =\n", np.outer(a, b))
```



```
Dot_Result =
[[4, 1]
 [2, 2]]

Vdot_Result =
6

Inner_Result =
[[4, 2]
 [1, 2]]

Outer_Result =
[[4, 1, 2, 2]
 [0, 0, 0, 0]
 [0, 0, 0, 0]
 [4, 1, 2, 2]]
```

- Example 37:

```
import numpy as np

arr_1 = np.array([[1 , 2], [3, 4]])
arr_2 = np.array([[5 , 6], [7, 8]])
print("arr_1 :\n", arr_1 )
print("arr_2 :\n", arr_2 )
result_matmul = np.matmul(arr_1,arr_2)
print("Result:\n",result_matmul)
```



arr_1:
[[1, 2],
 [3, 4]]

arr_2:
[[5, 6],
 [7, 8]]

Result:
[[19, 22],
 [43, 50]]

• Example 38:

```
import math
def Euclidean_distance(point1, point2):
    return math.sqrt((point1[0]-point2[0])**2+ point1[1]-point2[1])**2)
point1 = (1, 2)
point2 = (4, 6)
distance = Euclidean_distance(point1, point2)
print("Euclidean distance:", distance)
# Result: Euclidean distance: 5.0
```

Euclidean Distance Using Python

```
import numpy as np
def Euclidean_distance_np(point1, point2):
    return np.linalg.norm(np.array(point1)-np.array(point2))
point1 = (1, 2)
point2 = (4, 6)
distance = Euclidean_distance_np(point1, point2)
print("Euclidean distance (NumPy) :", distance)
# Result: Euclidean distance (NumPy): 5.0
```

$$d(p, q) = \sqrt{\sum_{i=1}^N (q_i - p_i)^2}$$

Euclidean Distance Using Numpy

- Example 39:

```
def manhattan_distance(point1, point2):      Manhattan Distance Using Python
    return sum(abs(a - b) for a, b in zip(point1, point2))
point1 = (1, 2)
point2 = (4, 6)
distance = manhattan_distance(point1, point2)
print("Manhattan distance:", distance)
#Result: Manhattan distance: 7
```

$$d(p, q) = \sum_{i=1}^N |q_i - p_i|$$

```
import numpy as np
def manhattan_distance_np(point1, point2):
    return np.sum(np.abs(np.array(point1) - np.array(point2)))
point1 = (1, 2)
point2 = (4, 6)
distance = manhattan_distance_np(point1, point2)
print("Manhattan distance (NumPy):", distance)
# Result: Manhattan distance (NumPy): 7      Manhattan Distance Using NumPy
```

- Has functions that return matrices instead of `ndarray` objects
- Example 40:

```
import numpy as np
import numpy.matlib
#with the specified shape and type without initializing entries
mat_e = np.matlib.empty((3, 2), dtype=int)
#filled with 0
mat_zeros = np.matlib.zeros(5, 3)
#filled with 1
mat_ones = np.matlib.ones(4, 3)
#diagonal elements filled with 1, others with 0
mat_ones = np.matlib.eye(3, 5)
#create square matrix with 0, diagonal filled with 1, others with 0
mat_zeros = np.matlib.identity(5)
#filled with random data
mat_e = np.matlib.empty(3, 2)
```

- What are the I/O functions of NumPy?
- The I/O functions provided by NumPy are
 - `Load()` and `save()` methods handle numPy binary files (with npy extension)
 - `loadtxt()` and `savetxt()` methods handle normal text files

- Example 41: The input array is stored in a disk file with the **save()** method, file and is loaded by **load()** method

```
import numpy as np  
a = np.array([1,2,3,4,5])  
np.save("out.npy", a)  
b = np.load("out.npy")  
print(b)           → [1, 2, 3, 4, 5]
```

- Example 42: The input array is stored in a disk file with the **savetxt()** method, file and is loaded by **loadtxt()** method

```
import numpy as np  
a = np.array([1,2,3,4,5])  
np.savetxt("out.txt", a)  
b = np.loadtxt("out.txt")  
print(b)           → [1., 2., 3., 4., 5.]
```

- 1. **Practice:** Practice all the examples of Chapter 5
- 2. **Exercise:** in the next slide below

1. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

a = np.array([1, 2, 3])      # ...
print(type(a))              # ...
print(a.shape)               # ...
print(a[0], a[1], a[2])     # ...
a[0] = 5                     # ...
print(a)                     # ...

b = np.array([[1, 2, 3], [4, 5, 6]])    # ...
print(b.shape)                # ...
print(b[0, 0], b[0, 1], b[1, 0])  # ...
```

2. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

a = np.zeros((2,2))      # ...
b = np.ones((1,2))       # ...
c = np.full((2,2), 7)    # ...
d = np.eye(2)             # ...
e = np.random.random((2,2)) # ...
```

3. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

a = np.array([[1,2,3,4], [5,6,7,8], [9,10,11,12]]) #...

b = a[:2, 1:3]

print(a[0, 1])    # ...

b[0, 0] = 77      # ...

print(a[0, 1])    # ...
```

4. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

a = np.array([[1,2,3,4], [5,6,7,8], [9,10,11,12]]) #...

row_r1 = a[1, :]      # ...
row_r2 = a[1:2, :]    # ...
print(row_r1, row_r1.shape) # ...
print(row_r2, row_r2.shape) # ...

col_r1 = a[:, 1]
col_r2 = a[:, 1:2]
print(col_r1, col_r1.shape) # ...
print(col_r2, col_r2.shape) # ...
```

5. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

a = np.array([[1,2], [3, 4], [5, 6]]) # ...

print(a[[0, 1, 2], [0, 1, 0]]) # ...

print(np.array([a[0, 0], a[1, 1], a[2, 0]])) # ...

print(a[[0, 0], [1, 1]]) # ...

print(np.array([a[0, 1], a[0, 1]])) # ...
```

6. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

a = np.array([[1,2,3], [4,5,6], [7,8,9], [10, 11, 12]]) #...

print(a) #...

b = np.array([0, 2, 0, 1]) #...

print(a[np.arange(4), b]) # ...

a[np.arange(4), b] += 10 #...

print(a) #...
```

7. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

a = np.array([[1,2], [3, 4], [5, 6]])

bool_idx = (a > 2)      # ...

print(bool_idx)          # ...

print(a[bool_idx])       # ...

print(a[a > 2])         # ...
```

8. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

x = np.array([1, 2])      # ...
print(x.dtype)            # ...

x = np.array([1.0, 2.0])    # ...
print(x.dtype)            # ...

x = np.array([1, 2], dtype=np.int64)  # ...
print(x.dtype) # ...
```

9. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

x = np.array([[1,2],[3,4]], dtype=np.float64) #...
y = np.array([[5,6],[7,8]], dtype=np.float64) #...

print(x + y) #...
print(np.add(x, y)) #...
print(x - y) #...
print(np.subtract(x, y)) #...
print(x * y) #...
print(np.multiply(x, y)) #...
print(x / y) #...
print(np.divide(x, y)) #...
print(np.sqrt(x)) #...
```

9. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

x = np.array([[1,2],[3,4]]) #...
y = np.array([[5,6],[7,8]]) #...
v = np.array([9,10]) #...
w = np.array([11, 12]) #...

print(v.dot(w)) #...
print(np.dot(v, w)) #...
print(x.dot(v)) #...
print(np.dot(x, v)) #...
print(x.dot(y)) #...
print(np.dot(x, y)) #...
```

10. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

x = np.array([[1,2],[3,4]]) # ...

print(np.sum(x)) # ...
print(np.sum(x, axis=0)) # ...
print(np.sum(x, axis=1)) # ...
x = np.array([[1,2], [3,4]])
print(x) # ...
print(x.T) # ...
v = np.array([1,2,3])
print(v) # ...
print(v.T) # ...
```

11. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

x = np.array([[1,2,3], [4,5,6], [7,8,9], [10, 11, 12]])
v = np.array([1, 0, 1])
y = np.empty_like(x)    # ...

for i in range(4):
    y[i, :] = x[i, :] + v

print(y) #...
```

12. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np
x = np.array([[1,2,3], [4,5,6], [7,8,9], [10, 11, 12]])
v = np.array([1, 0, 1])
vv = np.tile(v, (4, 1))    # ...
print(vv)                  # ...
y = x + vv    # ...
print(y)    # ...
```

13. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np
x = np.array([[1,2,3], [4,5,6], [7,8,9], [10, 11, 12]])
v = np.array([1, 0, 1])
y = x + v    # ...
print(y)    # ...
```

14. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

# Compute outer product of vectors
v = np.array([1,2,3])    # v has shape (3,)
w = np.array([4,5])      # w has shape (2,)
print(np.reshape(v, (3, 1)) * w) #...

x = np.array([[1,2,3], [4,5,6]]) #...
print(x + v) #...

print((x.T + w).T) #...
print(x + np.reshape(w, (2, 1))) #...
```

15. Explain the meaning of each command line and show the result of the following code:

```
import numpy as np

v = np.array([1,2,3])    # ...
w = np.array([4,5])      # ...
print(np.reshape(v, (3, 1)) * w) # ...

x = np.array([[1,2,3], [4,5,6]]) # ...
print(x + v) # ...

print((x.T + w).T) # ...
print(x + np.reshape(w, (2, 1))) # ...

print(x * 2) # ...
```

The end of Chapter