

Given:

The electrical heating systems used in many houses consist of a simple duct with resistance heaters. Air is heated as it flows over the resistance wires. Consider a 15 kW electric heating system where air enters at 100 kPa and 17°C with a flow rate of $150 \frac{\text{m}^3}{\text{min}}$.

$$\dot{Q}'_{out} := 200 \text{ W} \quad T_1 := 17^\circ\text{C} \quad P_1 := 100 \text{ kPa}$$

$$\dot{W}'_{e,in} := 15 \text{ kW} \quad \dot{V}'_1 := 150 \frac{\text{m}^3}{\text{min}}$$

Required:

If the rate of heat loss from the air duct to the surroundings is 200 W, determine the final temperature of the air.

Solution:

1st Law (for rigid, steady flow device with no changes in kinetic and potential energy)

$$\frac{d}{dt} E_{sys} = \sum E'_{in} - \sum E'_{out}$$

$$E'_{in} = E'_{out}$$

$$\dot{W}'_{in} + \dot{m}' \cdot h_{in} = \dot{m}' \cdot h_{out} + \dot{Q}'_{out}$$

Rearranging yields

$$\dot{W}'_{in} - \dot{Q}'_{out} = \dot{m}' \cdot (h_{out} - h_{in})$$

Assuming the air behaves as an ideal gas in the process region and has a constant specific heat the first law becomes

$$\dot{W}'_{in} - \dot{Q}'_{out} = \dot{m}' \cdot c_p \cdot (T_{out} - T_{in})$$

Solving for the outlet temperature yields

$$T_{out} = \frac{\dot{W}'_{in} - \dot{Q}'_{out}}{\dot{m}' \cdot c_p} + T_{in}$$

Using the ideal gas law, the mass flow rate may be found by

$$\dot{m}' = \frac{P \cdot \dot{V}'}{R \cdot T}$$

Going to Table A-2(a) @ air shows

$$R := 0.287 \frac{\text{kJ}}{\text{kg K}} \quad c_p := 1.005 \frac{\text{kJ}}{\text{kg K}}$$

The mass flow rate is then

$$\dot{m}' := \frac{P_1 \cdot \dot{V}'_1}{R \cdot T_1} = 3.002 \frac{\text{kg}}{\text{s}}$$

The outlet temperature is then

$$T_2 := \frac{\dot{W}'_{e,in} - \dot{Q}'_{out}}{\dot{m}' \cdot c_p} + T_1 = 295.1 \text{ K}$$

$$T_2 = 21.91^\circ\text{C}$$

