

Given:

Air at 10°C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m². The air leaves the diffuser with a velocity that is very small compared with the inlet velocity.

$$T_1 := 10 \text{ }^{\circ}\text{C} \quad P_1 := 80 \text{ kPa} \quad V_1 := 200 \frac{\text{m}}{\text{s}} \quad A_1 := 0.4 \text{ m}^2$$

Required:

Determine the mass flow rate of air and the temperature of the air leaving the diffuser.

Solution:

The mass flow rate is given by

$$\dot{m} = \rho \cdot A \cdot V$$

Assuming air behaves as an ideal gas, the density of air may be found by

$$P \cdot V = m \cdot R \cdot T \quad \text{or} \quad v = \frac{V}{m} \quad \text{or} \quad \rho = \frac{1}{v}$$

$$v = \frac{R \cdot T}{P} \quad \rho = \frac{P}{R \cdot T}$$

Going to Table A-1 @ air shows

$$R := 0.287 \frac{\text{kJ}}{\text{kg K}}$$

The density is then given by

$$\rho_1 := \frac{P_1}{R \cdot T_1} = 0.9844 \frac{\text{kg}}{\text{m}^3}$$

The mass flow rate is then given by

$$\dot{m} := \rho_1 \cdot A_1 \cdot V_1 = 78.76 \frac{\text{kg}}{\text{s}}$$

1st Law for an adiabatic, rigid, steady flow device

$$\frac{d}{dt} E_{sys} = \sum \dot{E}'_{in} - \sum \dot{E}'_{out}$$

$$0 = \sum \dot{E}'_{in} - \sum \dot{E}'_{out}$$

$$0 = \dot{m}'_{in} \cdot \left(h_{in} + \frac{V_{in}^2}{2} \right) - \dot{m}'_{out} \cdot \left(h_{out} + \frac{V_{out}^2}{2} \right) \quad \text{assuming there is no change in potential energy}$$

Rearranging yields

$$h_{out} = h_{in} + \frac{V_{in}^2 - V_{out}^2}{2} \quad \text{knowing } \dot{m}'_{in} = \dot{m}'_{out}$$

Since it is known that $V_{out} < V_{in}$

$$h_{out} = h_{in} + \frac{V_{in}^2}{2}$$

Solution (cont.):

Going to Table A-17 @ $T := T_1 = 283.2 \text{ K}$ shows interpolation is needed.

$$T_a := 280 \text{ K} \qquad T_b := 285 \text{ K}$$

$$h_a := 280.13 \frac{\text{kJ}}{\text{kg}} \qquad h_b := 285.14 \frac{\text{kJ}}{\text{kg}}$$

$$h_1 := \frac{T_1 - T_a}{T_b - T_a} \cdot (h_b - h_a) + h_a = 283.3 \frac{\text{kJ}}{\text{kg}}$$

The enthalpy at the outlet is then given by

$$h_2 := h_1 + \frac{V_1^2}{2} = 303.3 \frac{\text{kJ}}{\text{kg}}$$

Going to Table A-17 @ $h := h_2 = 303.3 \frac{\text{kJ}}{\text{kg}}$ shows interpolation is needed.

$$h_a := 300.19 \frac{\text{kJ}}{\text{kg}} \qquad h_b := 305.22 \frac{\text{kJ}}{\text{kg}}$$

$$T_a := 300 \text{ K} \qquad T_b := 305 \text{ K}$$

$$T_2 := \frac{h_2 - h_a}{h_b - h_a} \cdot (T_b - T_a) + T_a = 303.1 \text{ K}$$

$$T_2 = 29.9 \text{ } ^\circ\text{C}$$