Given: $USD := 1$

A cryogenic manufacturer handles liquid methane at 115 K and 5 MPa at a rate of 0.280 m3/s. The process involves dropping the pressure to 1 MPa by means of a throttling value. An engineer proposes to replace the throttling value with a turbine so power can be produced from the pressure drop.

$$T_{in} := 115\ \text{K}$$

$$P_{in} := 5\ \text{MPa}$$

$$V'_{in} := 0.280\ \frac{\text{m}^3}{\text{s}}$$

$$P_{out} := 1\ \text{MPa}$$

Properties of Liquid Methane

Temp T, K	Pressure P, MPa	Density ρ, kg/m³	Enthalpy h, kJ/kg	Entropy s, kJ/kg K	Specific Heat c_p, kJ/kg K
110	0.5	425.3	208.3	4.878	3.476
	1	425.8	209.0	4.875	3.471
	2	426.6	210.5	4.867	3.460
	5	429.1	215.0	4.844	3.432
120	0.5	410.4	243.4	5.185	3.551
	1	411.0	244.1	5.180	3.543
	2	412.0	245.4	5.171	3.528
	5	415.2	249.6	5.145	3.486

Required:

What is the maximum amount of power that can be produced by the turbine? Given that the turbine operates 8760 hr/yr and the cost of electricity is $0.075/kWhr, what is the maximum savings for the company if they use the turbine?

Solution:

The operating time is defined as

$$\Delta t := 8760\ \frac{\text{hr}}{\text{yr}}$$

The cost of electricity is defined as

$$C_e := 0.075 \cdot \frac{USD}{\text{kW hr}}$$

1st Law for a steady flow turbine that is adiabatic, and has no ΔKE and ΔPE shows

$$\frac{d}{d\ t} E_{sys} = \Sigma E'_{in} - \Sigma E'_{out}$$

$$0 = m'_{in} \cdot \left(h_{in} + \frac{V_{in}^{\ 2}}{2} + g_e \cdot z_{in} \right) - m'_{out} \cdot \left(h_{out} + \frac{V_{out}^{\ 2}}{2} + g_e \cdot z_{out} \right) - W'_{out}$$

$$0 = m'_{in} \cdot \left(h_{in} - h_{out} \right) - W'_{out}$$

Knowing that the device only has one inlet and outlet the work output of the device becomes

$$W'_{out} = m' \cdot \left(h_{in} - h_{out} \right)$$

Solution (contd.):
Going to the given table @ $T_{in} = 115.0\ \mathrm{K}$ and $P_{in} = 5.000\ \mathrm{MPa}$ shows that interpolation is needed.

$$T_a := 110\ \mathrm{K} \qquad T_b := 120\ \mathrm{K}$$

$$\rho_a := 429.1\ \frac{\mathrm{kg}}{\mathrm{m}^3} \qquad \rho_b := 415.2\ \frac{\mathrm{kg}}{\mathrm{m}^3} \qquad h_a := 215\ \frac{\mathrm{kJ}}{\mathrm{kg}} \qquad h_b := 249.6\ \frac{\mathrm{kJ}}{\mathrm{kg}}$$

$$\rho_{in} := \frac{T_{in} - T_a}{T_b - T_a} \cdot \left(\rho_b - \rho_a\right) + \rho_a = 422.15\ \frac{\mathrm{kg}}{\mathrm{m}^3} \qquad h_{in} := \frac{T_{in} - T_a}{T_b - T_a} \cdot \left(h_b - h_a\right) + h_a = 232.3\ \frac{\mathrm{kJ}}{\mathrm{kg}}$$

$$s_a := 4.844\ \frac{\mathrm{kJ}}{\mathrm{kg\ K}} \qquad s_b := 5.145\ \frac{\mathrm{kJ}}{\mathrm{kg\ K}}$$

$$s_{in} := \frac{T_{in} - T_a}{T_b - T_a} \cdot \left(s_b - s_a\right) + s_a = 4.994\ \frac{\mathrm{kJ}}{\mathrm{kg\ K}}$$

The mass flow rate can then be found by

$$m' := \rho_{in} \cdot V'_{in} = 118.202\ \frac{\mathrm{kg}}{\mathrm{s}}$$

Desiring an upper limit to what work can be produced by a turbine, let's assume that the turbine is not only adiabatic but also reversible. It has been shown that a turbine that is both adiabatic and reversible is also isentropic so

$$s_{out} := s_{in} = 4.994\ \frac{\mathrm{kJ}}{\mathrm{kg\ K}}$$

Going to the given table @ $P_{out} = 1.000\ \mathrm{MPa}$ and $s_{out} = 4.994\ \frac{\mathrm{kJ}}{\mathrm{kg\ K}}$ shows that interpolation is needed.

$$s_a := 4.875\ \frac{\mathrm{kJ}}{\mathrm{kg\ K}} \qquad s_b := 5.180\ \frac{\mathrm{kJ}}{\mathrm{kg\ K}}$$

$$h_a := 209.0\ \frac{\mathrm{kJ}}{\mathrm{kg}} \qquad h_b := 244.1\ \frac{\mathrm{kJ}}{\mathrm{kg}}$$

$$h_{out} := \frac{s_{out} - s_a}{s_b - s_a} \cdot \left(h_b - h_a\right) + h_a = 222.8\ \frac{\mathrm{kJ}}{\mathrm{kg}}$$

The maximum possible work that could be produced by a turbine can then be found by

$$\boxed{W'_{out} := m' \cdot \left(h_{in} - h_{out}\right) = 1129\ \mathrm{kW}}$$

The maximum amount of savings per year is then given by

$$\boxed{Savings := C_e \cdot W'_{out} \cdot \Delta t = 7.415 \cdot 10^5\ \frac{USD}{\mathrm{yr}}}$$