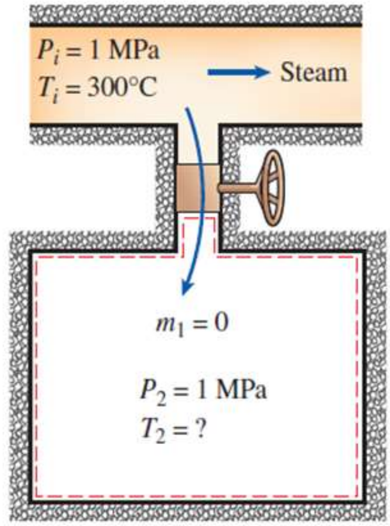
Given:

A rigid, insulated tank that is initially evacuated is connected through a valve to a supply line that carries steam at 1 MPa and 300°C. Now the valve is opened, and steam is allowed to flow slowly into the tank until the pressure reaches 1 MPa, at which point the valve is closed.

$P_i := 1\ \mathrm{MPa}$ $\quad T_i := 300\ °\mathrm{C}$ $\quad P_2 := 1\ \mathrm{MPa}$



(*a*) Flow of steam into an evacuated tank

Required:
Determine the temperature of the steam in the tank.

Solution:
Mass Conservation

$$\frac{d}{d\,t} m_{sys} = \Sigma m'_{in} - \Sigma m'_{out}$$

Since there is no mass leaving, the mass conservation becomes

$$\frac{d}{d\,t} m_{sys} = m'_i$$

1st Law (for adiabatic, rigid with no changes in KE and PE)

$$\frac{d}{d\,t} E_{sys} = \Sigma E'_{in} - \Sigma E'_{out}$$

$$\frac{d}{d\,t} U_{sys} = m'_i \cdot h_i$$

Subsitituing the mass conservation into the 1st Law yields

$$\frac{d}{d\,t} U_{sys} = h_i \cdot \frac{d}{d\,t} m_{cv}$$

Integrating yields

$$\Delta U_{sys} = h_i \cdot \Delta m_{sys}$$

$$m_2 \cdot u_2 - m_1 \cdot u_2 = h_i \cdot \left(m_2 - m_1\right)$$

Knowing that the initial mass is zero

$$m_2 \cdot u_2 = h_i \cdot m_2$$

Simplifying yields

$$u_2 = h_i$$

Going to Table A-4 @ $T := T_i = 300\ °\mathrm{C}$ and $P := P_i = 1000\ \mathrm{kPa}$ shows that the state is superheated.

Going to Table A-6 @ $T := T_i = 300\ °\mathrm{C}$ and $P := P_i = 1\ \mathrm{MPa}$ shows

$h_i := 3051.6\ \frac{\mathrm{kJ}}{\mathrm{kg}}$ and $u_2 := h_i = 3051.6\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

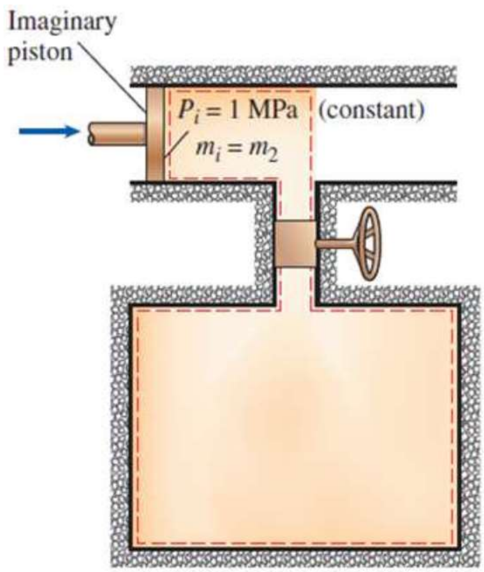
Solution (contd.):
Going to Table A-6 @ $P := P_2 = 1\ \text{MPa}$ and $u := u_2 = 3051.6\ \frac{\text{kJ}}{\text{kg}}$ shows that interpolation is needed.

$u_a := 2957.9\ \frac{\text{kJ}}{\text{kg}}$ $\qquad u_b := 3125.0\ \frac{\text{kJ}}{\text{kg}}$

$T_a := 400\ °\text{C}$ $\qquad T_b := 500\ °\text{C}$

$$\boxed{T_2 := \frac{u_2 - u_a}{u_b - u_a} \cdot \left(T_b - T_a\right) + T_a = 456.1\ °\text{C}}$$

Discussion:
There is another approach that could have been used to reach the same solution. Consider the boundary work needed to push the steam into the tank. This may be visualized as an imaginary piston push the steam into the tank. This is shown in the figure below.



The boundary work being done on the system is

$$W_{b,in} = -\int_{State1}^{State2} P_i \, \mathrm{d}\,V$$

$$W_{b,in} = -P_i \cdot \left(V_2 - V_1\right)$$

$$W_{b,in} = -P_i \cdot \left(V_{tank} - \left(V_{tank} + V_i\right)\right)$$

$$W_{b,in} = P_i \cdot V_i$$

<u>1st Law</u> (for steady state, adaibatic with no changes in KE and PE)

$$\Delta E_{sys} = \Sigma E_{in} - \Sigma E_{out}$$

$$\Delta U = W_{bin}$$

$$m_2 \cdot u_2 - m_i \cdot u_i = m_i \cdot P_i \cdot v_i$$

$$m_2 \cdot u_2 = m_i \cdot \left(u_i + P_i \cdot v_i\right) = m_i \cdot h_i$$

Knowing that $m_2 = m_i$ shows

$$u_2 = h_i$$