

Given:

In an ordinary shower, hot water at 140°F is mixed with cold water at 50°F. The desired temperature of the water stream leaving the shower head is 110°F.

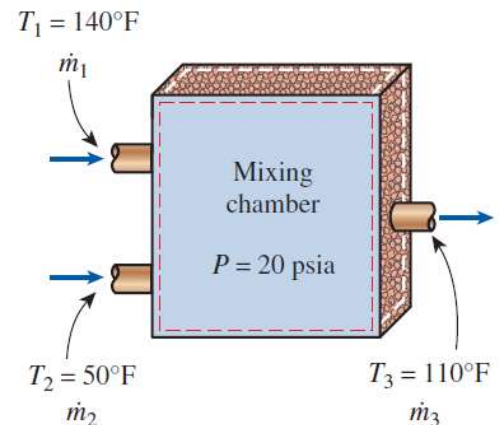
$$T_1 := 140 \text{ } ^\circ\text{F}$$

$$T_2 := 50 \text{ } ^\circ\text{F} \quad P := 20 \text{ psi}$$

$$T_3 := 110 \text{ } ^\circ\text{F}$$

Required:

Assuming there is no heat loss and the mixing place is at a pressure of 20 psia, determine the mass flow ratio of hot to cold water.

**Solution:**

Mass Conservation (for a steady flow device)

$$\frac{d}{dt} m_{sys} = \sum \dot{m}'_{in} - \sum \dot{m}'_{out}$$

$$0 = \sum \dot{m}'_{in} - \sum \dot{m}'_{out}$$

$$0 = \dot{m}'_1 + \dot{m}'_2 - \dot{m}'_3$$

1st Law (for adiabatic, rigid, steady flow device with no changes in kinetic and potential energy)

$$\frac{d}{dt} E_{sys} = \sum \dot{E}'_{in} - \sum \dot{E}'_{out}$$

$$0 = \sum \dot{E}'_{in} - \sum \dot{E}'_{out}$$

$$0 = \dot{m}'_1 \cdot h_1 + \dot{m}'_2 \cdot h_2 - \dot{m}'_3 \cdot h_3$$

Dividing both the mass conservation and first law by \dot{m}'_2 yields

$$0 = \frac{\dot{m}'_1}{\dot{m}'_2} + \frac{\dot{m}'_2}{\dot{m}'_2} - \frac{\dot{m}'_3}{\dot{m}'_2}$$

$$0 = \frac{\dot{m}'_1}{\dot{m}'_2} + 1 - \frac{\dot{m}'_3}{\dot{m}'_2}$$

$$0 = \frac{\dot{m}'_1}{\dot{m}'_2} \cdot h_1 + \frac{\dot{m}'_2}{\dot{m}'_2} \cdot h_2 - \frac{\dot{m}'_3}{\dot{m}'_2} \cdot h_3$$

$$0 = \frac{\dot{m}'_1}{\dot{m}'_2} \cdot h_1 + h_2 - \frac{\dot{m}'_3}{\dot{m}'_2} \cdot h_3$$

Solving for the ratio of the mass flow rate of the hot to cold water (i.e., $\frac{\dot{m}'_1}{\dot{m}'_2}$)

$$\frac{\dot{m}'_1}{\dot{m}'_2} = \frac{h_3 - h_2}{h_1 - h_3}$$

Going to Table A-4E @ $T := T_1 = 140 \text{ } ^\circ\text{F}$ shows that the state is a compressed liquid. However, the compressed liquid table (A-7E) does not have the necessary pressure values so the state will be approximated by the saturated liquid value of Table A-4E. This is shown below.

$$h_1 := 107.99 \frac{\text{BTU}}{\text{lbm}}$$

Solution (cont.)

Going to Table A-4E @ $T := T_2 = 50^\circ\text{F}$ shows that the state is a compressed liquid. However, the compress liquid table (A-7E) does not have the necessary pressure values so the state will be approximated by the saturated liquid value of Table A-4E. This is shown below.

$$h_2 := 18.07 \frac{\text{BTU}}{\text{lbm}}$$

Going to Table A-4E @ $T := T_3 = 110^\circ\text{F}$ shows that the state is a compressed liquid. However, the compress liquid table (A-7E) does not have the necessary pressure values so the state will be approximated by the saturated liquid value of Table A-4E. This is shown below.

$$h_3 := 78.02 \frac{\text{BTU}}{\text{lbm}}$$

The ratio of the mass flow rate of the hot to cold water is then

$$r_{hc} := \frac{h_3 - h_2}{h_1 - h_3} = 2.000$$