Given:

The temperature distribution across a wall 1 m thick at a certain instant of time is given as

$$T(x) = a + b \cdot x + c \cdot x^2$$

where T is in degrees Celsius and x is in meters, while a, b, and c are listed below. The wall has an area of 10 m² and a thermal conductivity of 40 $\frac{W}{m \text{ K}}$.

$$a := 900 \, \Delta^{\circ} C$$
 $b := -300 \, \frac{\Delta^{\circ} C}{m}$ $c := -50 \, \frac{\Delta^{\circ} C}{m^2}$

Required:

Determine the rate of heat transfer entering the wall and leaving the wall. Is the wall gaining or losing energy?

Solution:

The wall thickness, area, and thermal conductivity is defined as

$$L := 1 \text{ m} \qquad \qquad A_{_{\hspace{-.1em}W}} := 10 \text{ m}^{\hspace{-.1em} 2} \qquad \qquad k_{_{\hspace{-.1em}t}} := 40 \text{ } \frac{\text{W}}{\text{m K}}$$

The temperature distribution is defined as

$$T(x) := a + b \cdot x + c \cdot x^2$$

The rate of heat transfer as a function of position within the wall is then

$$Q'(x) := -\left(k_t \cdot A_w \cdot \frac{d}{dx} T(x)\right) \qquad \frac{d}{dx} T(x) = -\frac{100 K \cdot (x+3 m)}{m^2} \qquad \frac{d}{dx} T(x) := b + 2 \cdot c \cdot x$$

The rate of heat transfer entering the wall is then

$$Q'_{in} := Q' (0 \text{ m}) = 120 \text{ kW}$$

$$Q'_{in} := -k_t \cdot A_w \cdot \left(-\frac{100 \text{ K} \cdot (0 \text{ m} + 3 \text{ m})}{\text{m}^2} \right) = 120 \text{ kW}$$

The rate of heat transfer leaving the wall is then

$$Q'_{in} := -k_{t} \cdot A_{w} \cdot \left(-\frac{100 \text{ K} \cdot (L+3 \text{ m})}{m^{2}}\right) = 160 \text{ kW}$$

Since $Q'_{out} > Q'_{in}$, the wall is losing energy.