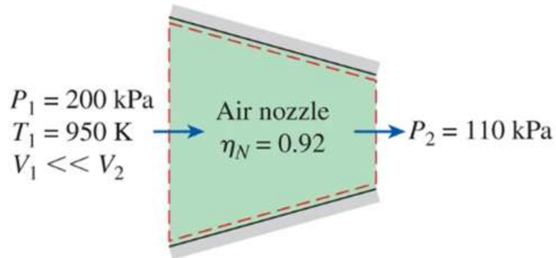
Given:
Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 110 kPa. Assume constant specific heat of air is 1.11 kJ/kgK and the specific heat ratio is 1.349.

$P_1 := 200\ \text{kPa}$ $\quad T_1 := 950\ \text{K}$ $\quad P_2 := 110\ \text{kPa}$ $\quad c_p := 1.11\ \frac{\text{kJ}}{\text{kg K}}$ $\quad k := 1.349$

Required:

If the isentropic efficiency of the nozzle is 92%, determine the maximum possible velocity, the exit temperature of the air, and the actual velocity of the air.



Solution:
The isentropic efficiency of the nozzle is defined as

$$\eta_N := 92\ \%$$

For an isentropic process the following is true

$$\left(\frac{T_2}{T_1}\right)_{s = const} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

Solving for the temperature at state 2 for an insentropic process is then

$$T_{2s} := T_1 \cdot \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 813.9\ \text{K}$$

The 1st Law for a nozzle is shown below when $V_2 >> V_1$ shows

$$\frac{d}{d\ t} E_{sys} = \Sigma E'_{in} - \Sigma E'_{out}$$

$$0 = m' \cdot \left(h_1 + \frac{V_1^{\ 2}}{2}\right) - m' \cdot \left(h_2 + \frac{V_2^{\ 2}}{2}\right)$$

$$h_1 = h_2 + \frac{V_2^{\ 2}}{2}$$

Solving for the velocity at the outlet when the process is an isentropic process shows

$$V_{2s} = \sqrt{2 \cdot \left(h_1 - h_{2s}\right)}$$

Since the specific heat is constant, the maximum velocity at the exit can be expressed as

$$\boxed{V_{2s} := \sqrt{2 \cdot c_p \cdot \left(T_1 - T_{2s}\right)} = 549.7\ \frac{\text{m}}{\text{s}}}$$

Starting with the definition of the isentropic efficiency of a nozzle

$$\eta_N = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Since the specific heat is constant, the issentropic efficiency of the nozzle can be expressed as

$$\eta_N = \frac{c_p \cdot \left(T_1 - T_{2a}\right)}{c_p \cdot \left(T_1 - T_{2s}\right)} = \frac{T_1 - T_{2a}}{T_1 - T_{2s}}$$

Solution (contd.):
Solving for the actual temperature at the exit yields

$$\boxed{T_{2a} := T_1 - \eta_N \cdot \left(T_1 - T_{2s}\right) = 824.8\ K}$$

The actual exit velocity may then be found by the alternate relation for the isentropic efficiency of a nozzle shown below.

$$\eta_N = \frac{V_{2a}^{\ 2}}{V_{2s}^{\ 2}}$$

Solving for the actual exit velocity yields

$$\boxed{V_{2a} := \sqrt{\eta_N \cdot V_{2s}^{\ 2}} = 527.3\ \frac{m}{s}} \qquad V_{2a} := \sqrt{2 \cdot c_p \cdot \left(T_1 - T_{2a}\right)} = 527.3\ \frac{m}{s}$$