

Given:

Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at a pressure of 1.2 MPa.

Required:

Determine the work output of the turbine per unit mass of steam if the process is reversible.

Solution:

The inlet pressure and temperature are defined as

$$P_{in} := 5 \text{ MPa} \quad T_{in} := 450 \text{ }^{\circ}\text{C} = 723.15 \text{ K}$$

The outlet pressure is defined as

$$P_{out} := 1.2 \text{ MPa}$$

1st Law (for steady state, adiabatic device with no ΔKE and ΔPE)

$$\frac{d}{dt} E_{sys} = \sum \dot{E}'_{in} - \sum \dot{E}'_{out}$$

$$0 = \dot{m}' \cdot h_{in} - \dot{m}' \cdot h_{out} - \dot{W}'_{out}$$

Solving for the specific work out shows

$$w_{out} = h_{in} - h_{out}$$

Going to Table A-5 @ $P_{in} = 5000 \text{ kPa}$ shows

$$T_{sat} := 263.94 \text{ }^{\circ}\text{C}$$

Since $T_{in} > T_{sat}$, the state is superheated. Going to Table A-6 @ $P_{in} = 5.000 \text{ MPa}$ & $T_{in} = 450.0 \text{ }^{\circ}\text{C}$ shows

$$h_{in} := 3317.2 \frac{\text{kJ}}{\text{kg}} \quad s_{in} := 6.8210 \frac{\text{kJ}}{\text{kg K}}$$

Since the process is adiabatic and reversible, it is isentropic. Thus $\Delta s = 0$ and

$$s_{out} := s_{in} = 6.821 \frac{\text{kJ}}{\text{kg K}}$$

Going to Table A-5 @ $P_{out} = 1200 \text{ kPa}$ shows

$$s_g := 6.5217 \frac{\text{kJ}}{\text{kg K}}$$

Since $s_{out} > s_g$, the state is superheated. Gong to Table A-6 @ $P_{out} = 1.200 \text{ MPa}$ & $s_{out} = 6.821 \frac{\text{kJ}}{\text{kg K}}$ shows that interpolation is needed. This is shown below.

$$s_a := 6.5909 \frac{\text{kJ}}{\text{kg K}} \quad s_b := 6.8313 \frac{\text{kJ}}{\text{kg K}}$$

$$h_a := 2816.1 \frac{\text{kJ}}{\text{kg}} \quad h_b := 2935.6 \frac{\text{kJ}}{\text{kg}}$$

$$h_{out} := \frac{s_{out} - s_a}{s_b - s_a} \cdot (h_b - h_a) + h_a = 2930 \frac{\text{kJ}}{\text{kg}}$$

The specific work out of the device is then

$$w_{out} := h_{in} - h_{out} = 386.7 \frac{\text{kJ}}{\text{kg}}$$