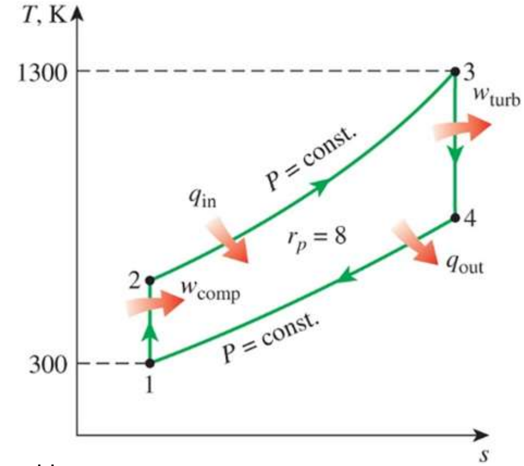
Given:
A gas turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet.

$r_p := 8 \quad T_1 := 300\ \text{K} \quad T_3 := 1300\ \text{K}$



Required:
Utilizing the air standard assumptions, determine
(a) the gas temperature at the exits of the compressor and the turbine,
(b) the back work ratio, and
(c) the thermal efficiency.

Solution:
Going to Table A-17 @ $T_1 = 300.0\ \text{K}$ shows

$h_1 := 300.19\ \frac{\text{kJ}}{\text{kg}} \qquad P_{r1} := 1.3860$

Using the relative pressure at state 1, the relative pressure at state 2 may be found by

$$P_{r2} := P_{r1} \cdot r_p = 11.088$$

Going to Table A-17 @ $P_{r2} = 11.09$ shows that interpolation is needed.

$P_{ra} := 10.37 \qquad P_{rb} := 11.10$

$h_a := 533.98\ \frac{\text{kJ}}{\text{kg}} \qquad h_b := 544.35\ \frac{\text{kJ}}{\text{kg}} \qquad T_a := 530\ \text{K} \qquad T_b := 540\ \text{K}$

$$h_2 := \frac{P_{r2} - P_{ra}}{P_{rb} - P_{ra}} \cdot \left(h_b - h_a\right) + h_a = 544.2\ \frac{\text{kJ}}{\text{kg}} \qquad \boxed{T_2 := \frac{P_{r2} - P_{ra}}{P_{rb} - P_{ra}} \cdot \left(T_b - T_a\right) + T_a = 539.8\ \text{K}} \quad \text{a)}$$

Going to Table A-17 @ $T_3 = 1300\ \text{K}$ shows

$h_3 := 1395.97\ \frac{\text{kJ}}{\text{kg}} \qquad P_{r3} := 330.9$

Using the relative pressure at state 3, the relative pressure at state 4 may be found by

$$P_{r4} := \frac{P_{r3}}{r_p} = 41.3625$$

Going to Table A-17 @ $P_{r4} = 41.36$ shows that interpolation is needed.

$P_{ra} := 39.27 \qquad P_{rb} := 43.35$

$h_a := 778.18\ \frac{\text{kJ}}{\text{kg}} \qquad h_b := 800.03\ \frac{\text{kJ}}{\text{kg}} \qquad T_a := 760\ \text{K} \quad T_b := 780\ \text{K}$

$$h_4 := \frac{P_{r4} - P_{ra}}{P_{rb} - P_{ra}} \cdot \left(h_b - h_a\right) + h_a = 789.4\ \frac{\text{kJ}}{\text{kg}} \qquad \boxed{T_4 := \frac{P_{r4} - P_{ra}}{P_{rb} - P_{ra}} \cdot \left(T_b - T_a\right) + T_a = 770.3\ \text{K}} \quad \text{a)}$$

To determine the back work ratio, the specific work of the compressor and turbine need to be determined. This is shown below.

$$w_c := h_2 - h_1 = 244.0\ \frac{\text{kJ}}{\text{kg}}$$

$$w_t := h_3 - h_4 = 606.6\ \frac{\text{kJ}}{\text{kg}}$$

Solution (cont.):

The back work ratio is then

$$\boxed{r_{bw} := \frac{w_c}{w_t} = 0.4022} \quad \text{b)}$$

The thermal efficiency may then be found by

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{out} - w_{in}}{h_3 - h_2} = \frac{w_t - w_c}{h_3 - h_2} \quad \text{or} \quad \boxed{\eta_{th} := \frac{w_t - w_c}{h_3 - h_2} = 42.57 \%} \quad \text{c)}$$

The thermal efficiency could have also been esitmated by the Brayton efficiency approximation with a specific heat ratio of 1.4 for air. This is shown below.

$k := 1.4$ (Table A-2(a) @ air)

$$\eta_{th,Brayton} := 1 - \frac{1}{r_p^{\frac{k-1}{k}}} = 44.80 \% \qquad \eta_{th} := 1 - \frac{T_4 - T_1}{T_3 - T_2} = 38.14 \%$$