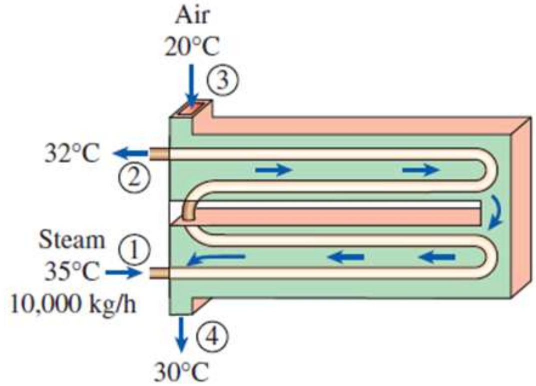
Given:

Air in a large building is kept warm by heating it with steam in a heat exchanger. Saturated water vapor enters the unit at 35°C at a rate of 10,000 kg/hr and leaves as saturated liquid at 32°C. Air at 1 atm enters the unit at 20°C and leaves at 30°C at about the same pressure.

$T_1 := 35\ °C \qquad T_2 := 32\ °C \qquad m'_s := 10000\ \frac{kg}{hr}$

$T_3 := 20\ °C \qquad T_4 := 30\ °C \qquad P_{air} := 1\ atm$



Required:

Determine the rate of entropy generated during this process.

Solution:

Starting with an entropy balance for a steady flow device shows

$$\frac{d}{dt} S_{sys} = \Sigma S'_{in} - \Sigma S'_{out} + S'_{gen}$$

$$0 = \Sigma S'_{in} - \Sigma S'_{out} + S'_{gen}$$

$$S'_{gen} = \Sigma S'_{out} - \Sigma S'_{in} = m'_s \cdot s_2 + m'_{air} \cdot s_4 - m'_s \cdot s_1 - m'_{air} \cdot s_3 = m'_s \cdot (s_2 - s_1) + m'_{air} \cdot (s_4 - s_3)$$

If the air is assumed to behave as an ideal gas with a constant specific heat (this is known as the approximate method), the rate of entropy generation becomes

$$S'_{gen} = m'_s \cdot (s_2 - s_1) + m'_{air} \cdot \left(c_{pavg} \cdot \ln\left(\frac{T_4}{T_3}\right) - R \cdot \ln\left(\frac{P_4}{P_3}\right)\right)$$

Knowing the pressure of the air remains constant throughout the process, the rate of entropy generation becomes

$$S'_{gen} = m'_s \cdot (s_2 - s_1) + m'_{air} \cdot \left(c_{pavg} \cdot \ln\left(\frac{T_4}{T_3}\right) - R \cdot \ln\left(\frac{P_{air}}{P_{air}}\right)\right) = m'_s \cdot (s_2 - s_1) + m'_{air} \cdot c_{pavg} \cdot \ln\left(\frac{T_4}{T_3}\right)$$

Going to Table A-4 @ $T_1 = 35.00\ °C$ and $x_1 = 1$ shows

$s_1 := 8.3517\ \frac{kJ}{kg\ K} \qquad h_1 := 2564.6\ \frac{kJ}{kg}$

Going to Table A-4 @ $T_2 = 32.00\ °C$ and $x_2 = 0$ shows that interpolation is needed.

$T_a := 30\ °C \qquad T_b := 35\ °C$

$s_a := 0.4368\ \frac{kJ}{kg\ K} \qquad s_b := 0.5051\ \frac{kJ}{kg\ K} \qquad h_a := 125.74\ \frac{kJ}{kg} \qquad h_b := 146.64\ \frac{kJ}{kg}$

$s_2 := \frac{T_2 - T_a}{T_b - T_a} \cdot (s_b - s_a) + s_a = 0.4641\ \frac{kJ}{kg\ K} \qquad h_2 := \frac{T_2 - T_a}{T_b - T_a} \cdot (h_b - h_a) + h_a = 134.1\ \frac{kJ}{kg}$

The mass flow rate of the air may be found by performing an energy balance on the system. This is shown below for a steady flow device with negligible changes in KE and PE.

$$\frac{d}{dt} E_{sys} = \Sigma E'_{in} - \Sigma E'_{out}$$

$$0 = m'_{sin} \cdot h_1 + m'_{airin} \cdot h_3 - m'_{sout} \cdot h_2 - m'_{airout} \cdot h_4$$

Realizing the steam and air mass streams remain constant, the energy balance becomes

$$0 = m'_s \cdot (h_1 - h_2) + m'_{air} \cdot (h_3 - h_4)$$

Solution (contd.):
Solving for the mass flow rate of the air shows

$$m'_{air} = \frac{m'_s \cdot (h_2 - h_1)}{h_3 - h_4}$$

Assuming air has a constant specific heat over the range of the process, the mass flow rate of air becomes

$$m'_{air} = \frac{m'_s \cdot (h_2 - h_1)}{c_{pavg} \cdot (T_3 - T_4)}$$

Going to Table A-2(a) @ air shows

$$c_{p,avg} := 1.005 \ \frac{kJ}{kg\ K}$$

The mass flow rate of air may then be found by

$$m'_{air} := \frac{m'_s \cdot (h_2 - h_1)}{c_{p,avg} \cdot (T_3 - T_4)} = 671.8 \ \frac{kg}{s}$$

The rate of entropy generation is then found to be

$$\boxed{S'_{gen} := m'_s \cdot (s_2 - s_1) + m'_{air} \cdot c_{p,avg} \cdot \ln\left(\frac{T_4}{T_3}\right) = 0.7364 \ \frac{kW}{K}}$$

Discussion:
A more accurate method would be to account for variable specific heat. Beginning with the derived entropy balance found above shows

$$S'_{gen} = m'_s \cdot (s_2 - s_1) + m'_{air} \cdot (s_4 - s_3)$$

Instead of using a constant specific heat approach as previously shown, a variable specific heat will be used.

$$S'_{gen} = m'_s \cdot (s_2 - s_1) + m'_{air} \cdot \left(s°_4 - s°_3 - R \cdot \ln\left(\frac{P_4}{P_3}\right)\right)$$

Knowing the pressure of the air remains constant throughout the process, the rate of entropy generation becomes

$$S'_{gen} = m'_s \cdot (s_2 - s_1) + m'_{air} \cdot (s°_4 - s°_3)$$

Going to Table A-17 @ $T_3 = 293.2\ K$ shows that interpolation is needed.

$$T_a := 290\ K \qquad T_b := 295\ K$$

$$s°_a := 1.66802 \ \frac{kJ}{kg\ K} \qquad s°_b := 1.68515 \ \frac{kJ}{kg\ K} \qquad h_a := 290.16 \ \frac{kJ}{kg} \qquad h_b := 295.17 \ \frac{kJ}{kg}$$

$$s°_3 := \frac{T_3 - T_a}{T_b - T_a} \cdot (s°_b - s°_a) + s°_a = 1.679 \ \frac{kJ}{kg\ K} \qquad h_3 := \frac{T_3 - T_a}{T_b - T_a} \cdot (h_b - h_a) + h_a = 293.3 \ \frac{kJ}{kg}$$

Going to Table A-17 @ $T_4 = 303.2\ K$ shows that interpolation is needed.

$$T_a := 300\ K \qquad T_b := 305\ K$$

$$s°_a := 1.70203 \ \frac{kJ}{kg\ K} \qquad s°_b := 1.71865 \ \frac{kJ}{kg\ K} \qquad h_a := 300.19 \ \frac{kJ}{kg} \qquad h_b := 305.22 \ \frac{kJ}{kg}$$

$$s°_4 := \frac{T_4 - T_a}{T_b - T_a} \cdot (s°_b - s°_a) + s°_a = 1.713 \ \frac{kJ}{kg\ K} \qquad h_4 := \frac{T_4 - T_a}{T_b - T_a} \cdot (h_b - h_a) + h_a = 303.4 \ \frac{kJ}{kg}$$

Discussion (cont.):
Solving for the mass flow rate of air while accounting for variable specific heat shows

$$m'_{air} := \frac{m'_s \cdot \left(h_2 - h_1\right)}{h_3 - h_4} = 672.3\ \frac{\text{kg}}{\text{s}}$$

The rate of entropy generated is then given by

$$\boxed{S'_{gen,var\text{-}cp} := m'_s \cdot \left(s_2 - s_1\right) + m'_{air} \cdot \left(s°_4 - s°_3\right) = 0.7381\ \frac{\text{kW}}{\text{K}}}$$

Comparing the two methods shows a percent difference of

$$\%diff := \frac{\left|S'_{gen} - S'_{gen,var\text{-}cp}\right|}{S'_{gen,var\text{-}cp}} = 0.2274\ \%$$