Given:

An insulated $8~{\rm m}^3$ rigid tank contains air at 600 kPa and 400 K. A valve connected to the tank is now opened, and air is allowed to escape until the pressure inside drops to 200 kPa. The air temperature during the process is maintained constant by an electric resistance heater placed in the tank.

$$V := 8 \text{ m}^{-3}$$

$$P_1 := 600 \text{ kPa}$$
 $T := 400 \text{ K}$

$$T := 400 \text{ I}$$

$$P_2 := 200 \text{ kPa}$$

Required:

Determine the electrical energy supplied to the air during this process.

Solution:

Mass Conservation

$$\frac{d}{dt} m_{sys} = \Sigma m'_{in} - \Sigma m'_{out}$$

$$\frac{d}{dt} m_{sys} = -m'_{out}$$



$$\frac{d}{dt} E_{sys} = \Sigma E'_{in} - \Sigma E'_{out}$$

$$\frac{d}{dt} U_{sys} = W'_{elec,in} - m'_{out} \cdot h_{out}$$

Substituting mass conservation into the 1st Law yields

$$\frac{d}{dt} U_{sys} = W'_{elec,in} + h_{out} \cdot \frac{d}{dt} m_{sys}$$

Integrating yields

$$\Delta U_{sys} = W_{elec,in} + h_{out} \cdot \Delta m_{sys}$$

$$\mathbf{m_2} \cdot \mathbf{u_2} - \mathbf{m_1} \cdot \mathbf{u_1} = \mathbf{W_{elect,in}} + \mathbf{h_{out}} \cdot \left(\mathbf{m_2} - \mathbf{m_1}\right)$$

Solving for the electrical work done shows

$$\mathbf{W}_{\texttt{elect,in}} = \left(\mathbf{m}_2 \cdot \mathbf{u}_2 - \mathbf{m}_1 \cdot \mathbf{u}_1\right) - \mathbf{h}_{\texttt{out}} \cdot \left(\mathbf{m}_2 - \mathbf{m}_1\right)$$

Going to Table A-17 @ T = 400 K shows

$$h_{out} := 400.98 \frac{\text{kJ}}{\text{kg}}$$
 $u_1 := 286.16 \frac{\text{kJ}}{\text{kg}}$ $u_2 := 286.16 \frac{\text{kJ}}{\text{kg}}$

Assuming the air behaves as an ideal gas, the initial and final masses may be found by

$$m = \frac{P \cdot V}{R \cdot T}$$

Going to Table A-1 @ air shows

$$R := 0.287 \frac{kJ}{kg K}$$

$$m_1 := \frac{P_1 \cdot V}{\dots} = 41.8118 \text{ kg}$$

$$m_1 := \frac{P_1 \cdot V}{R \cdot T} = 41.8118 \text{ kg}$$
 $m_2 := \frac{P_2 \cdot V}{R \cdot T} = 13.9373 \text{ kg}$

The electrical work done is then

$$\mathbf{W}_{\texttt{elec,in}} := \left(\mathbf{m}_2 \cdot \mathbf{u}_2 - \mathbf{m}_1 \cdot \mathbf{u}_1\right) - \mathbf{h}_{\texttt{out}} \cdot \left(\mathbf{m}_2 - \mathbf{m}_1\right) = 3201 \text{ kJ}$$

