Given:

A piston cylinder device initially contains $0.4\ m^3$ of air at $100\ kPa$ and $80\ °C$. The air is then compressed to $0.1\ m^3$ in such a way that the temperature of the air remains constant.

Required:

Determine the work done during the process.

Solution:

The initial volume, pressure, and temperature are defined below.

$$V_1 := 0.4\ m^3 \qquad P_1 := 100\ kPa \qquad T_0 := 80\ °C = 353.15\ K$$

The final volume is defined below.

$$V_2 := 0.1\ m^3$$

Beginning with the Ideal Gas Law (IGL) for a constant temperature process (i.e. isothermal process), the following is true.

$$PV = mRT \qquad \text{rearranging} \qquad \left(P = \frac{mRT}{V}\right) = \frac{¢}{V} \qquad \text{where } ¢ \text{ is a constant.}$$

Using this in the expression for boundary work shows

$$W_b = \int_1^2 P\ \mathrm{d}V \qquad W_b = \int_1^2 \frac{¢}{V}\ \mathrm{d}V \qquad W_b = ¢ \cdot \int_1^2 \frac{1}{V}\ \mathrm{d}V$$

$$W_b = ¢ \cdot \left(\ln\left(V_2\right) - \ln\left(V_1\right)\right) \qquad W_b = ¢ \cdot \ln\left(\frac{V_2}{V_1}\right)$$

This constant $¢$ may be found from

$$¢ = mRT = P_1 \cdot V_1 = P_2 \cdot V_2$$

Thus the boundary work may be expressed as

$$W_b = P_1 \cdot V_1 \cdot \ln\left(\frac{V_2}{V_1}\right) \qquad \text{or} \qquad W_b = P_2 \cdot V_2 \cdot \ln\left(\frac{V_2}{V_1}\right)$$

The ratio of the volumes may also be represented alternatively. This is shown below.

$$¢ = P_1 \cdot V_1 = P_2 \cdot V_2 \qquad \text{rearranging} \qquad \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

The boundary work expression could also be expressed as

$$W_b = P_1 \cdot V_1 \cdot \ln\left(\frac{P_1}{P_2}\right) \qquad \text{or} \qquad W_b = P_2 \cdot V_2 \cdot \ln\left(\frac{P_1}{P_2}\right)$$

All boundary work expressions are valid for the particular assumptions made for this system. To recap, those underlining assumptions are a closed isothermal system containing an ideal gas. Calculating the boundary work may now be done. This is shown below.

$$\boxed{W_b := P_1 \cdot V_1 \cdot \ln\left(\frac{V_2}{V_1}\right) = -55.45\ kJ}$$

Note: The boundary work is negative because work is being done to the system not by the system.