Given:

A piston cylinder device contains 0.05 m of a gas initially at 200 kPa. At this initial state, the linear spring has a spring constant of 150 kN but is exerting no force on it. Heat is then transferred to the system causing the volume to double in size. As a result of the expansion, the piston rises and the spring is compressed. The cross sectional area of the piston is $0.25 \,\mathrm{m}^2$.

Required:

Determine the final pressure of the gas inside the cylinder and the work by the gas.

Solution:

The initial volume pressure of the gas, spring constant, and cross-sectional area are defined as

$$V_1 := 0.05 \text{ m}^3$$

$$P_1 := 200 \text{ kPa}$$

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 $P_1 := 200 \text{ kPa}$ $k := 150 \frac{\text{kN}}{\text{m}}$ $A := 0.25 \text{ m}^2$

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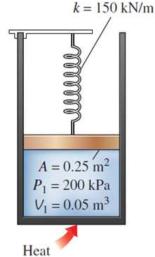
The final volume is found by

$$V_2 := 2 \cdot V_1 = 0.1 \text{ m}^3$$

The amount that the spring compresses may be found by

$$\Delta x = \frac{\Delta V}{A}$$

$$\Delta x = \frac{\Delta V}{\Delta}$$
 or $\Delta x := \frac{V_2 - V_1}{A} = 0.2 \text{ m}$



Beginning with the expression for boundary work, the expression may be expressed in terms of the distance x that the piston travels. This is shown below.

$$W_b = \int_{1}^{2} P \, \mathrm{d} V$$

$$W_b = \int_{V_{-}}^{V_2} P \, \mathrm{d} \, V$$

$$W_b = \int_{1}^{2} P dV \qquad W_b = \int_{V_a}^{V_2} P dV \qquad W_b = \int_{X_a}^{X_2} P \cdot A dX$$

The pressure that the spring and piston exert on the gas is given by

$$P = \frac{F_{spring}}{A} + P_1$$

Where F_{spring} is the force exerted by the spring and may be expressed by Hooke's Law. This is shown below.

$$F_{spring} = kx$$

The final pressure is then given by

$$P_2 := \frac{k \cdot \Delta x}{A} + P_1 = 320 \text{ kPa}$$

Substituting both of these expressions (the pressure expression and Hooke's Law) into the boundary work expression shows

$$W_{b} = \int_{X_{1}}^{X_{2}} P \cdot A \, dx \qquad W_{b} = \int_{X_{1}}^{X_{2}} \left(\frac{F_{spring}}{A} + P_{1} \right) \cdot A \, dx \qquad W_{b} = \int_{X_{1}}^{X_{2}} \left(k \cdot x + P_{1} \cdot A \right) dx$$

Integrating from 0 to Δx yields

$$W_b := \frac{k}{2} \cdot \Delta x^2 + P_1 \cdot A \cdot \Delta x = 13 \text{ kJ}$$