Given:
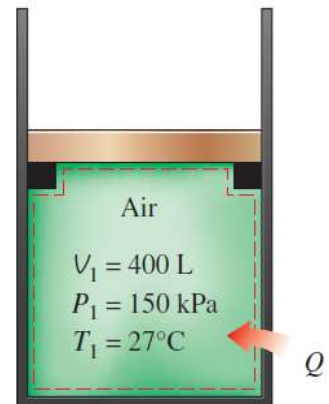
A piston cylinder device initially contains air at `150 kPa` and `27 °C`. Initially, the piston is resting on stops (as shown below) and the enclosed volume is `400 L`. The piston requires at least `350 kPa` to move it. The air is then heated until the volume doubled.

$$P_1 := 150 \text{ kPa} \qquad\qquad P_2 := 350 \text{ kPa} \qquad P_3 := P_2 = 350 \text{ kPa}$$

$$T_1 := 27 \text{ °C} = 300.15 \text{ K}$$

$$V_1 := 400 \text{ L} = 0.4 \text{ m}^3 \qquad\qquad V_3 := 2 \cdot V_1 = 0.8 \text{ m}^3$$

Air

$V_1 = 400$ L
$P_1 = 150$ kPa
$T_1 = 27°C$

Q

Required:

Determine the final temperature, the work done by the system and the total heat.

Solution:

Assuming the air behaves as an ideal gas throughout the process

$$P \cdot V = m \cdot R \cdot T$$

The pressure, volume, and temperature all are changing while the mass and ideal gas constant remains constant so

$$\frac{P \cdot V}{T} = m \cdot R \qquad\qquad \frac{P_1 \cdot V_1}{T_1} = \frac{P_3 \cdot V_3}{T_3}$$

Solving for the final temperature shows

$$\boxed{T_3 := \frac{P_3 \cdot V_3 \cdot T_1}{P_1 \cdot V_1} = 1128 \text{ °C}}$$

The work done by the system is given by

$$\boxed{W_b := P_2 \cdot \left(V_3 - V_1 \right) = 140 \text{ kJ}}$$

Starting with the first law

$$\Delta E_{sys} = \Sigma E_{in} - \Sigma E_{out}$$

Assuming there is no change in kinectic and potential energy and the only work being done is by raising the piston, the first law becomes

$$\Delta U = Q_{add} - W_b$$

$$m \cdot \left(u_3 - u_1 \right) = Q_{add} - W_b$$

Solving for the heat added yields

$$Q_{add} = m \cdot \left(u_3 - u_1 \right) + W_b$$

The mass in the cylinder may be found by using the ideal gas law.

$$P \cdot V = m \cdot R \cdot T \quad \text{ or } \quad m = \frac{P_1 \cdot V_1}{R \cdot T_1}$$

Going to Table A-3(a) @ air shows

$$R := 0.287 \, \frac{\text{kJ}}{\text{kg K}}$$

The mass is then found to be

$$m := \frac{P_1 \cdot V_1}{R \cdot T_1} = 0.6965 \text{ kg}$$

Solution (cont.):

Going to Table A-17 @ $T := T_1 = 27 \ °\text{C}$ and $T := T_3 = 1128 \ °\text{C}$ shows

$$T_1 = 300.15 \ \text{K} \qquad u_1 := 214.07 \ \frac{\text{kJ}}{\text{kg}}$$

$$T_3 = 1400.7 \ \text{K} \qquad u_3 := 1113.52 \ \frac{\text{kJ}}{\text{kg}}$$

Note: The table data doesn't match up exactly with the temperature values but interpolating between table rows will produce very similar results as the ones shown here. This could be a time saver on a quiz or exam.

The heat added is then

$$\boxed{Q_{add} := m \cdot \left(u_3 - u_1 \right) + W_b = 766.5 \ \text{kJ}}$$