

Given:

A piston cylinder device contains 0.05 m^3 of a gas initially at 200 kPa . At this initial state, the linear spring has a spring constant of $150 \frac{\text{kN}}{\text{m}}$ but is exerting no force on it. Heat is then transferred to the system causing the volume to double in size. As a result of the expansion, the piston rises and the spring is compressed. The cross sectional area of the piston is 0.25 m^2 .

Required:

Determine the final pressure of the gas inside the cylinder and the work by the gas.

Solution:

The initial volume pressure of the gas, spring constant, and cross-sectional area are defined as

$$V_1 := 0.05 \text{ m}^3 \quad P_1 := 200 \text{ kPa} \quad k := 150 \frac{\text{kN}}{\text{m}} \quad A := 0.25 \text{ m}^2$$

The final volume is found by

$$V_2 := 2 \cdot V_1 = 0.1 \text{ m}^3$$

The amount that the spring compresses may be found by

$$\Delta x = \frac{\Delta V}{A} \quad \text{or} \quad \Delta x := \frac{V_2 - V_1}{A} = 0.2 \text{ m}$$

Beginning with the expression for boundary work, the expression may be expressed in terms of the distance x that the piston travels. This is shown below.

$$W_b = \int_1^2 P \, dV \quad W_b = \int_{V_1}^{V_2} P \, dV \quad W_b = \int_{x_1}^{x_2} P \cdot A \, dx$$

The pressure that the spring and piston exert on the gas is given by

$$P = \frac{F_{\text{spring}}}{A} + P_1$$

Where F_{spring} is the force exerted by the spring and may be expressed by Hooke's Law. This is shown below.

$$F_{\text{spring}} = kx$$

The final pressure is then given by

$$P_2 := \frac{k \cdot \Delta x}{A} + P_1 = 320 \text{ kPa}$$

Substituting both of these expressions (the pressure expression and Hooke's Law) into the boundary work expression shows

$$W_b = \int_{x_1}^{x_2} P \cdot A \, dx \quad W_b = \int_{x_1}^{x_2} \left(\frac{F_{\text{spring}}}{A} + P_1 \right) \cdot A \, dx \quad W_b = \int_{x_1}^{x_2} (k \cdot x + P_1 \cdot A) \, dx$$

Integrating from 0 to Δx yields

$$W_b := \frac{k}{2} \cdot \Delta x^2 + P_1 \cdot A \cdot \Delta x = 13 \text{ kJ}$$

