

Given and Required:

Assuming a compressor efficiency of 80% and a turbine efficiency of 85%, determine the back work ratio, thermal efficiency, and the exit temperature of the Brayton gas-turbine cycle used in the previous problem.

$$\eta_c := 80 \% \quad \eta_t := 85 \%$$

Solution:

From the previous problem, the specific work of the compressor and turbine in the isentropic cases were found to be

$$w_{cs} := 243.99 \frac{\text{kJ}}{\text{kg}} \quad w_{ts} := 606.584 \frac{\text{kJ}}{\text{kg}}$$

From the previous problem, the specific enthalpy at state 1 and state 3 were found to be

$$h_1 := 300.19 \frac{\text{kJ}}{\text{kg}} \quad h_3 := 1395.97 \frac{\text{kJ}}{\text{kg}}$$

Using the efficiencies given, the actual work of the compressor and turbine may be found

$$w_{ca} := \frac{w_{cs}}{\eta_c} = 305.0 \frac{\text{kJ}}{\text{kg}} \quad w_{ta} := \eta_t \cdot w_{ts} = 515.6 \frac{\text{kJ}}{\text{kg}}$$

The back work ratio is then found by

$$r_{bw} := \frac{w_{ca}}{w_{ta}} = 0.5915$$

It should be mentioned that for the ideal Brayton cycle, the back work ratio was 0.402 (found in previous problem). The increase seen in the calculation above is a result of irreversibilities in the turbine and compressor.

The actual thermal efficiency is given by

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{out} - w_{in}}{h_3 - h_{2a}} = \frac{w_{ta} - w_{ca}}{h_3 - h_{2a}}$$

The actual specific enthalpy at state 2 may be found by

$$h_{2a} := h_1 + w_{ca} = 605.2 \frac{\text{kJ}}{\text{kg}} \quad (h_{2a} - h_1 = w_{ca})$$

Thus the actual thermal efficiency is found to be

$$\eta_{th} := \frac{w_{ta} - w_{ca}}{h_3 - h_{2a}} = 26.63 \%$$

Again it should be mentioned that for the ideal Brayton cycle, the thermal efficiency was 42.6%. The decrease seen in the calculation above is a result of the efficiencies of the turbine and compressor.

The actual specific enthalpy at state 4 may be found by

$$h_{4a} := h_3 - w_{ta} = 880.4 \frac{\text{kJ}}{\text{kg}} \quad (h_3 - h_{4a} = w_{ta})$$

Going to Table A-17 @ $h_{4a} = 880.4 \frac{\text{kJ}}{\text{kg}}$ shows that interpolation is needed.

$$h_a := 866.08 \frac{\text{kJ}}{\text{kg}} \quad h_b := 888.27 \frac{\text{kJ}}{\text{kg}}$$

$$T_a := 840 \text{ K} \quad T_b := 860 \text{ K}$$

$$T_{4a} := \frac{h_{4a} - h_a}{h_b - h_a} \cdot (T_b - T_a) + T_a = 852.9 \text{ K}$$