

**Given:**

The temperature distribution across a wall 1 m thick at a certain instant of time is given as

$$T(x) = a + b \cdot x + c \cdot x^2$$

where T is in degrees Celsius and x is in meters, while a, b, and c are listed below. The wall has an area of 10 m<sup>2</sup> and a thermal conductivity of 40  $\frac{\text{W}}{\text{m K}}$ .

$$a := 900 \text{ } ^\circ\text{C} \quad b := -300 \frac{^\circ\text{C}}{\text{m}} \quad c := -50 \frac{^\circ\text{C}}{\text{m}^2}$$

**Required:**

Determine the rate of heat transfer entering the wall and leaving the wall. Is the wall gaining or losing energy?

**Solution:**

The wall thickness, area, and thermal conductivity is defined as

$$L := 1 \text{ m} \quad A_w := 10 \text{ m}^2 \quad k_t := 40 \frac{\text{W}}{\text{m K}}$$

The temperature distribution is defined as

$$T(x) := a + b \cdot x + c \cdot x^2$$

The rate of heat transfer as a function of position within the wall is then

$$Q'(x) := - \left( k_t \cdot A_w \cdot \frac{d}{dx} T(x) \right) \quad \frac{d}{dx} T(x) = - \frac{100 \text{ K} \cdot (x + 3 \text{ m})}{\text{m}^2} \quad \frac{d}{dx} T(x) := b + 2 \cdot c \cdot x$$

The rate of heat transfer entering the wall is then

$$Q'_{in} := Q'(0 \text{ m}) = 120 \text{ kW} \quad Q'_{in} := -k_t \cdot A_w \cdot \left( - \frac{100 \text{ K} \cdot (0 \text{ m} + 3 \text{ m})}{\text{m}^2} \right) = 120 \text{ kW}$$

The rate of heat transfer leaving the wall is then

$$Q'_{out} := Q'(L) = 160 \text{ kW} \quad Q'_{out} := -k_t \cdot A_w \cdot \left( - \frac{100 \text{ K} \cdot (L + 3 \text{ m})}{\text{m}^2} \right) = 160 \text{ kW}$$

Since  $Q'_{out} > Q'_{in}$ , the wall is losing energy.