Given:

The temperature distribution across a wall $1\ \mathrm{m}$ thick at a certain instant of time is given as

$$T(x) = a + b \cdot x + c \cdot x^2$$

where T is in degrees Celsius and x is in meters, while a, b, and c are listed below. The wall has an area of $10\ \mathrm{m}^2$ and a thermal conductivity of $40\ \frac{\mathrm{W}}{\mathrm{m\ K}}$.

$$a := 900\ \Delta°\mathrm{C} \qquad b := -300\ \frac{\Delta°\mathrm{C}}{\mathrm{m}} \qquad c := -50\ \frac{\Delta°\mathrm{C}}{\mathrm{m}^2}$$

Required:

Determine the rate of heat transfer entering the wall and leaving the wall. Is the wall gaining or losing energy?

Solution:

The wall thickness, area, and thermal conductivity is defined as

$$L := 1\ \mathrm{m} \qquad A_w := 10\ \mathrm{m}^2 \qquad k_t := 40\ \frac{\mathrm{W}}{\mathrm{m\ K}}$$

The temperature distribution is defined as

$$T(x) := a + b \cdot x + c \cdot x^2$$

The rate of heat transfer as a function of position within the wall is then

$$Q'(x) := -\left(k_t \cdot A_w \cdot \frac{d}{d\ x} T(x)\right) \qquad \frac{d}{d\ x} T(x) = -\frac{100\ \mathrm{K} \cdot (x + 3\ \mathrm{m})}{\mathrm{m}^2} \qquad \frac{d}{d\ x} T(x) := b + 2 \cdot c \cdot x$$

The rate of heat transfer entering the wall is then

$$\boxed{Q'_{in} := Q'(0\ \mathrm{m}) = 120\ \mathrm{kW}} \qquad Q'_{in} := -k_t \cdot A_w \cdot \left(-\frac{100\ \mathrm{K} \cdot (0\ \mathrm{m} + 3\ \mathrm{m})}{\mathrm{m}^2}\right) = 120\ \mathrm{kW}$$

The rate of heat transfer leaving the wall is then

$$\boxed{Q'_{out} := Q'(L) = 160\ \mathrm{kW}} \qquad Q'_{in} := -k_t \cdot A_w \cdot \left(-\frac{100\ \mathrm{K} \cdot (L + 3\ \mathrm{m})}{\mathrm{m}^2}\right) = 160\ \mathrm{kW}$$

Since $Q'_{out} > Q'_{in}$, the wall is losing energy.