Given: $kJ := 1000J$

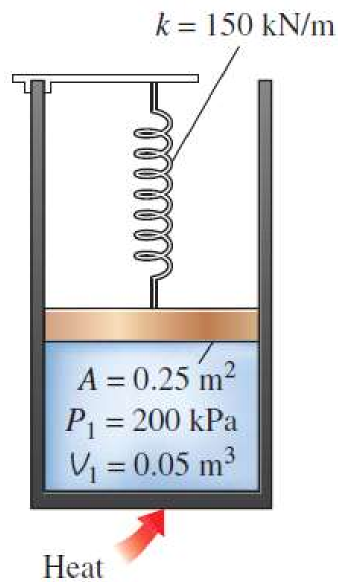
A piston cylinder device contains 0.05 m^3 of a gas initially at 200 kPa. At this initial state, the linear spring has a spring constant of 150 kN/m but is exerting no force on it. Heat is then transferred to the system causing the volume to double in size. As a result of the expansion, the piston rises and the spring is compressed. The cross sectional area of the piston is 0.25 m^2.

Required:

Determine the final pressure of the gas inside the cylinder and the work by the gas.

Solution:

The initial volume and pressure of the gas are defined as



$$V_1 := 0.05m^3 \qquad P_1 := 200kPa$$

The spring constant is defined as

$$k := 150\frac{kN}{m}$$

The cross sectional area of the piston is

$$A := 0.25m^2$$

The final volume is found by

$$V_2 := 2 \cdot V_1 = 0.1 \cdot m^3$$

The amount that the spring compresses may be found by

$$\Delta x = \frac{\Delta V}{A} \qquad \text{or} \qquad \Delta x := \frac{V_2 - V_1}{A} = 0.2\,m$$

Beginning with the expression for boundary work, the expression may be expressed in terms of the distance x that the piston travels. This is shown below.

$$W_b = \int_1^2 P\,dV = \int_{V_1}^{V_2} P\,dV = \int_{x_1}^{x_2} P \cdot A\,dx$$

The pressure that the spring and piston exert on the gas is given by

$$P = \frac{F_{spring}}{A} + P_1$$

Where F_{spring} is the force exerted by the spring and may be expressed by Hooke's Law. This is shown below.

$$F_{spring} = kx$$

The final pressure is then given by

$$\boxed{P_2 := \frac{k \cdot \Delta x}{A} + P_1 = 320 \cdot kPa}$$

Solution (contd.):

Substituting both of these expressions (the pressure expression and Hooke's Law) into the boundary work expression shows

$$W_b = \int_{x_1}^{x_2} P \cdot A \, dx = \int_{x_1}^{x_2} \left(\frac{F_{spring}}{A} + P_1 \right) \cdot A \, dx = \int_{x_1}^{x_2} \left(k \cdot x + P_1 \cdot A \right) dx$$

Integrating from 0 to Δx yields

$$\boxed{W_b := \frac{k}{2} \cdot \Delta x^2 + P_1 \cdot A \cdot \Delta x = 13 \cdot kJ}$$