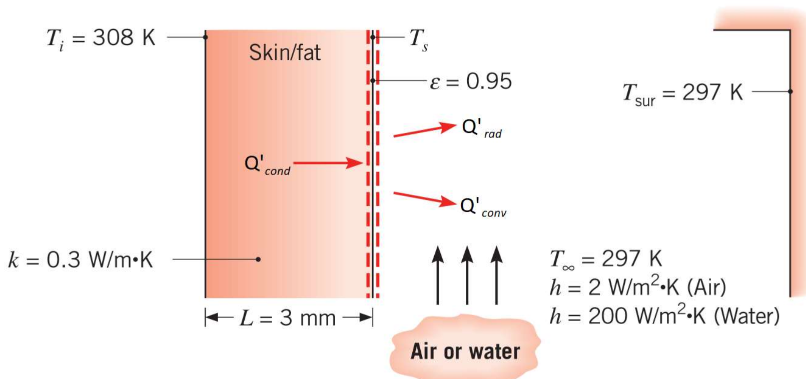
Given:

Humans can control their heat production rate and heat loss rate to maintain a nearly constant core temperature under a wide range of environmental conditions. The process is called thermoregulation. From the perspective of calculating heat transfer between a human body and its surroundings, we focus on a layer of skin and fat, with its outer surface exposed to the environment and its inner surface at a temperature slightly less than the core temperature of 308 K. Consider a person with a skin/fat layer thickness of 3 mm and effective thermal conductivity of 0.3 W/mK. The person has surface area of 1.8 m^2 and is dressed in a bathing suit. The emissivity of the skin is 0.95.

Required:

When the person is in still air at 297 K, what is the skin surface temperature and rate of heat loss to the environment? Assume the convection coefficient of the air is 2 W/m^2K. How are the surface temperature and rate of heat loss effected when the person is in water with a convection coefficient of 200 W/m^2K?



Solution:

The core body temperature is defined as

$$T_i := 308K$$

The skin/fat layer thickness is defined as

$$L := 3mm$$

The effective thermal conductivity

$$k := 0.3 \frac{W}{m \cdot K}$$

The surface area of the person is defined as

$$A_s := 1.8m^2$$

The skin emissivity is defined as

$$\varepsilon := 0.95$$

The still air temperature and surrounding temperatures are defined as

$$T_\infty := 297K \qquad T_{sur} := 297K$$

The convection coefficients of the air and water are defined as

$$h_{air} := 2 \frac{W}{m^2 \cdot K} \qquad h_{water} := 200 \frac{W}{m^2 \cdot K}$$

Solution (contd.):

1st Law for a steady state system as shown in the diagram (the space enclosed in the dotted lines) shows

$$\frac{d}{dt}E_{sys} = \Sigma E'_{in} - \Sigma E'_{out}$$

$$0 = Q'_{cond} - Q'_{conv} - Q'_{rad}$$

$$0 = k \cdot A_s \cdot \frac{\Delta T}{\Delta x} - h \cdot A_s \cdot (T_s - T_\infty) - \varepsilon \cdot \sigma \cdot A_s \cdot \left(T_s^{\,4} - T_{sur}^{\,4}\right)$$

$$0 = k \cdot A \cdot \frac{T_i - T_s}{L} - h \cdot A \cdot (T_s - T_\infty) - \varepsilon \cdot \sigma \cdot A \cdot \left(T_s^{\,4} - T_{sur}^{\,4}\right)$$

where σ is the Stefan-Boltzman's constant. $\sigma := 5.67 \cdot 10^{-8} \cdot \frac{W}{m^2 \cdot K^4}$

Inspecting the expression, it may be seen that every variable is known except for the surface temperature T_s, so this expression becomes just a simple matter of one equation and one unknown. However, it is no trivial task to solve for the surface temperature T_s due to it being raised to the fourth power. There are various techniques that may be used to solve for the surface temperature T_s but for this example, Mathcad will be used to solve for the T_s. This is shown below using a Mathcad Solve Block.

$T_s := 1K$ <--- Initial guesss for the surface temperature T_s

Given

$$0 = k \cdot A_s \cdot \frac{T_i - T_s}{L} - h_{air} \cdot A_s \cdot (T_s - T_\infty) - \varepsilon \cdot \sigma \cdot A_s \cdot \left(T_s^{\,4} - T_{sur}^{\,4}\right)$$

$$\boxed{T_{s,air} := \text{Find}(T_s) = 307.2\,K}$$

The rate of heat due to conduction, convection, and radiation may then be found by

$$Q'_{cond,air} := k \cdot A_s \cdot \frac{T_i - T_{s,air}}{L} = 145.679\,W$$

$$Q'_{conv,air} := h_{air} \cdot A_s \cdot (T_{s,air} - T_\infty) = 36.686\,W$$

$$Q'_{rad,air} := \varepsilon \cdot \sigma \cdot A_s \cdot \left(T_{s,air}^{\,4} - T_{sur}^{\,4}\right) = 108.993\,W$$

The rate of heat loss is then

$$\boxed{Q'_{loss,air} := Q'_{conv,air} + Q'_{rad,air} = 145.7\,W}$$

This may be seen as equal to the rate of heat conduction term. This should be the case seeing as the rate of heat conduction is the only energy supplied to the system so it must equal what is lost by the system if the system is to be steady state.

Solution (contd.):

This process may be repeat for the second case of when the person is in water. This is shown below.

$T_s := 1K$ <--- Initial guesss for the surface temperature T_s

Given

$$0 = k \cdot A_s \cdot \frac{T_i - T_s}{L} - h_{water} \cdot A_s \cdot (T_s - T_\infty) - \varepsilon \cdot \sigma \cdot A_s \cdot \left(T_s^4 - T_{sur}^4\right)$$

$$\boxed{T_{s,water} := \text{Find}(T_s) = 300.6\,K}$$

The rate of heat due to conduction, convection, and radiation may then be found by

$$Q'_{cond,water} := k \cdot A_s \cdot \frac{T_i - T_{s,water}}{L} = 1.332 \cdot kW$$

$$Q'_{conv,water} := h_{water} \cdot A_s \cdot (T_{s,water} - T_\infty) = 1.295 \cdot kW$$

$$Q'_{rad,water} := \varepsilon \cdot \sigma \cdot A_s \cdot \left(T_{s,water}^4 - T_{sur}^4\right) = 0.037 \cdot kW$$

The rate of heat loss is then

$$\boxed{Q'_{loss,water} := Q'_{conv,water} + Q'_{rad,water} = 1332.4\,W}$$

Comparing the two solutions shows a decrease in temperature and an increase in rate of heat loss.

$$\Delta T := T_{s,water} - T_{s,air} = -6.593\,K$$

$$\Delta Q'_{loss} := Q'_{loss,water} - Q'_{loss,air} = 1.187 \cdot kW$$

Discussion:

If the problem were to be worked by hand, an iterative approach must be used to find the temperatures. The first step in an iterative approach is to provide an initial guess for the temperature value. This is show below.

$$T_{s,air} := 320K$$

The second step is to solve the fourth order equation for a single temperature variable. This is shown below.

$$0 = k \cdot A_s \cdot \frac{T_i - T_s}{L} - h_{air} \cdot A_s \cdot (T_s - T_\infty) - \varepsilon \cdot \sigma \cdot A_s \cdot \left(T_s^4 - T_{sur}^4\right)$$

$$T_{s,air} = T_i - \frac{L}{k \cdot A_s} \cdot \left[h_{air} \cdot A_s \cdot (T_{s,air} - T_\infty) + \varepsilon \cdot \sigma \cdot A_s \cdot \left(T_{s,air}^4 - T_{sur}^4\right)\right]$$

Now that a single temperature variable has been solved for, iterations may be performed to reach a solution. This is shown below.

Iteration 1: $$T_{s,air} := T_i - \frac{L}{k \cdot A_s} \cdot \left[h_{air} \cdot A_s \cdot (T_{s,air} - T_\infty) + \varepsilon \cdot \sigma \cdot A_s \cdot \left(T_{s,air}^4 - T_{sur}^4\right)\right] = 306.1\,K$$

Since the calculated temperature value doesn't match the initial guess value, a solution has not been reached.

Discussion:

Another iteration must be performed. This time the calculated value will be used as the initial guess. This is shown below.

Iteration 2: $$T_{s,air} := T_i - \frac{L}{k \cdot A_s} \cdot \left[h_{air} \cdot A_s \cdot \left(T_{s,air} - T_\infty \right) + \varepsilon \cdot \sigma \cdot A_s \cdot \left({T_{s,air}}^4 - {T_{sur}}^4 \right) \right] = 307.3\,K$$

Since the calculated temperature value doesn't match the second guess value (iteration 1 value), a solution has not been reached. Another iteration must be performed. This is shown below.

Iteration 3: $$T_{s,air} := T_i - \frac{L}{k \cdot A_s} \cdot \left[h_{air} \cdot A_s \cdot \left(T_{s,air} - T_\infty \right) + \varepsilon \cdot \sigma \cdot A_s \cdot \left({T_{s,air}}^4 - {T_{sur}}^4 \right) \right] = 307.2\,K$$

This time the calculated temperature value is very close to the guessed value (iteration 2 value). A solution has almosted been reached. Another iteration must be performed. This is shown below.

Iteration 4: $$T_{s,air} := T_i - \frac{L}{k \cdot A_s} \cdot \left[h_{air} \cdot A_s \cdot \left(T_{s,air} - T_\infty \right) + \varepsilon \cdot \sigma \cdot A_s \cdot \left({T_{s,air}}^4 - {T_{sur}}^4 \right) \right] = 307.2\,K$$

Since the calculated temperature value (iteration 4) is the same as the guessed value (iteration 3), a solution has been reached. This may be seen as the same answer reached useing the Mathcad Solve Block method. A similar process could be used to find the temperature when in the person is in water.