Topic 6
CLOSED SYSTEMS

Objectives

- Examine the moving boundary work or $P \, dV$ work commonly encountered in reciprocating devices such as automotive engines and compressors.

- Identify the first law of thermodynamics as simply a statement of the conservation of energy principle for closed (fixed mass) systems.

- Develop the general energy balance applied to closed systems.

MOVING BOUNDARY WORK

Moving boundary work ($P \, dV$ work):
The expansion and compression work in a piston-cylinder device.
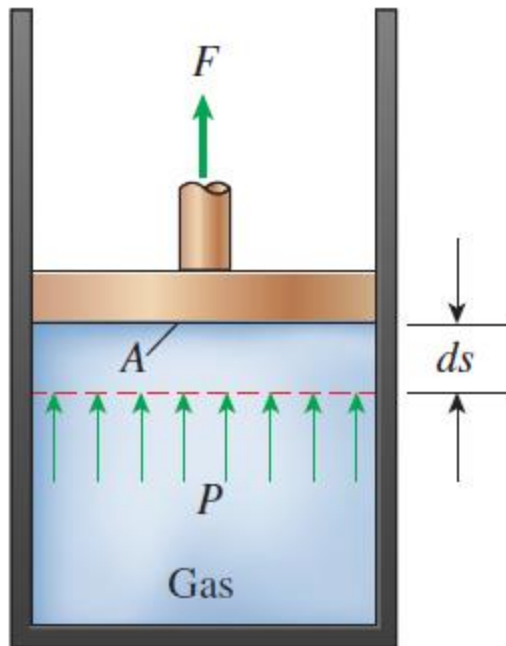
$$\delta W_b = F \, ds = PA \, ds = P \, dV$$

$$W_b = \int_1^2 P \, dV \qquad (\text{kJ})$$

Quasi-equilibrium process:
A process during which the system remains nearly in equilibrium at all times.

W_b **is positive $\rightarrow$ for expansion**
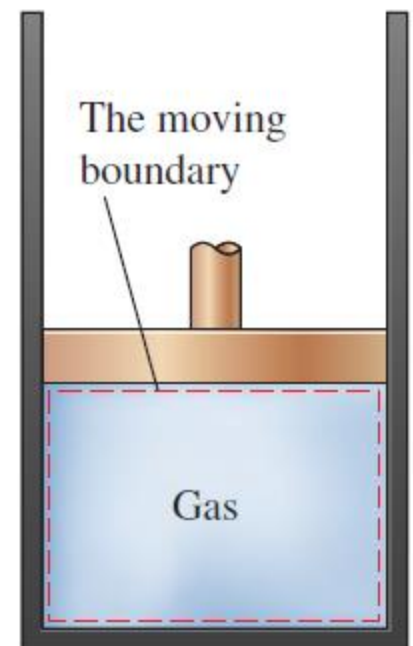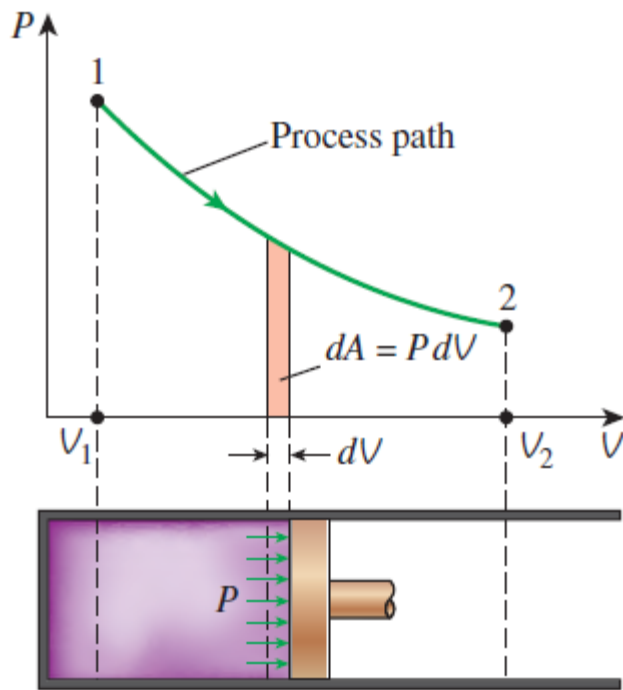W_b **is negative $\rightarrow$ for compression**

FIGURE 4–1
The work associated with a moving boundary is called *boundary work*.

FIGURE 4–2
A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount ds.

FIGURE 4–3

The area under the process curve on a P-V diagram represents the boundary work.

$$\text{Area} = A = \int_1^2 dA = \int_1^2 P \, dV$$

The area under the process curve on a P-V diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.

The boundary work done during a process depends on the path followed as well as the end states.

$W_A = 10 \text{ kJ}$
$W_B = 8 \text{ kJ}$
$W_C = 5 \text{ kJ}$
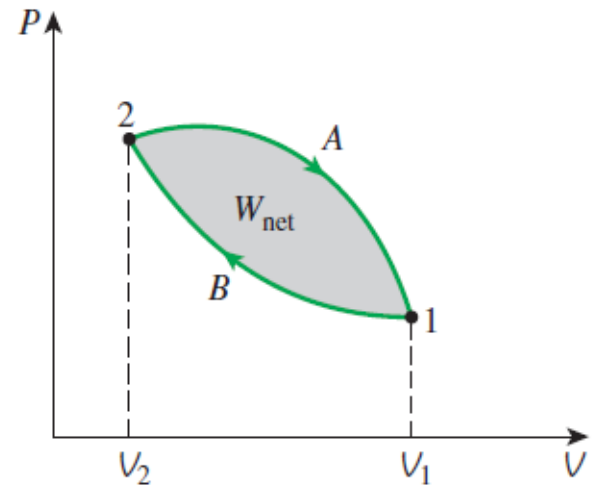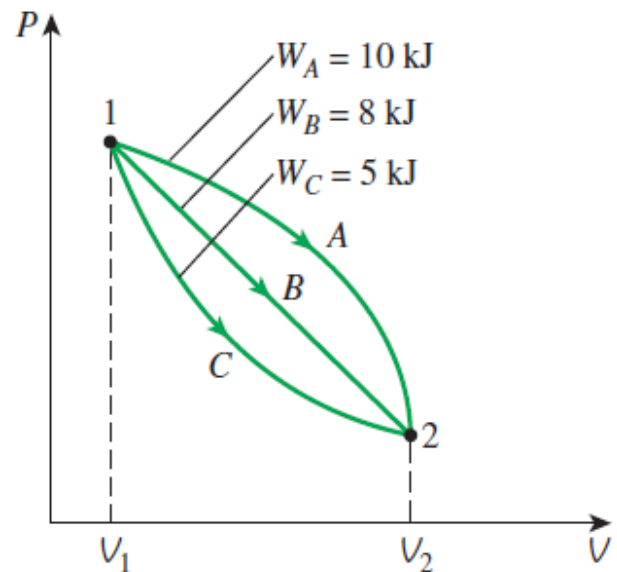
FIGURE 4–5

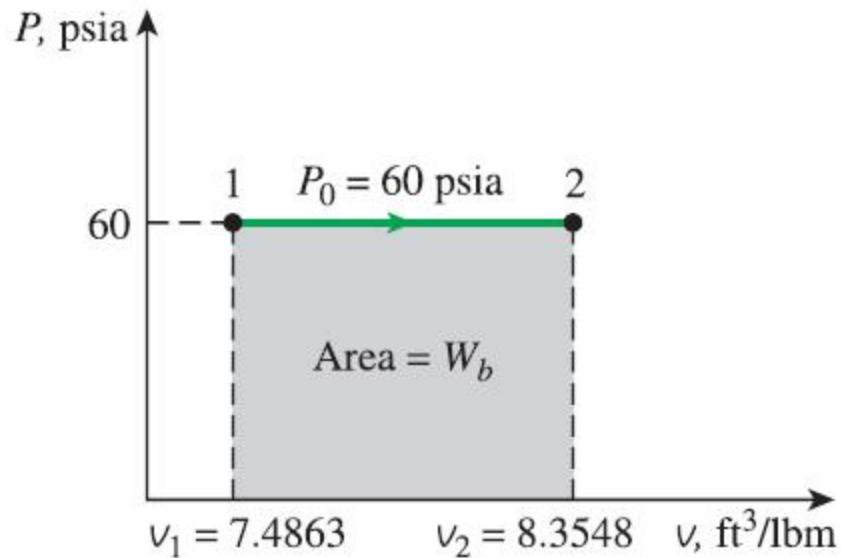The net work done during a cycle is the difference between the work done by the system and the work done on the system.

Boundary Work for a Constant-Pressure Process

H_2O

Heat
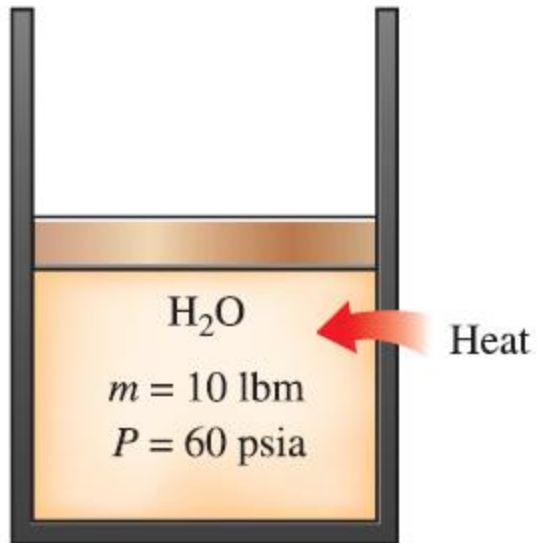
$m = 10$ lbm
$P = 60$ psia

$$W_b = \int_1^2 PdV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$$

$$W_b = mP_0(v_2 - v_1)$$

P, psia

1     $P_0 = 60$ psia   2

60

Area $= W_b$

$v_1 = 7.4863$ $v_2 = 8.3548$ v, ft^3/lbm

Boundary Work for a Constant-Pressure Process

A piston cylinder device contains 10 lbm of steam at 60 psia and 320ºF. Heat is now transferred to the steam until the temperature reaches 400ºF. Assuming the mass of the piston and the atmospheric pressure remain constant, determine the work done by the steam during the heating process.

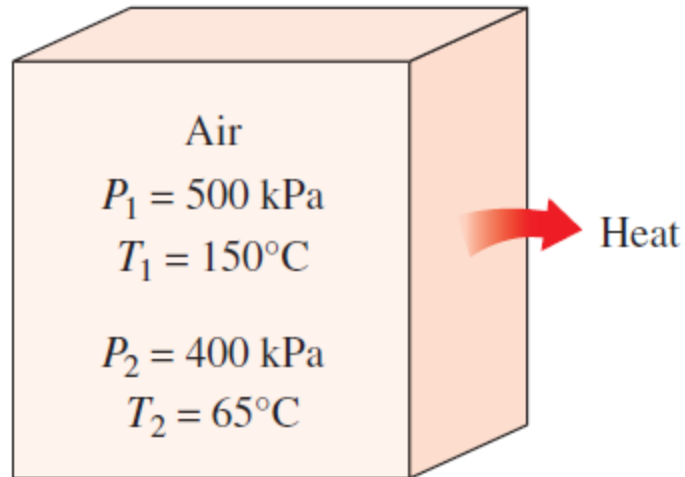**Example 1**

Isothermal compression of an Ideal Gas

A piston cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is then compressed to 0.1 m^3 in such a way that the temperature of the air remains constant. Determine the work done during the process.

Example 2
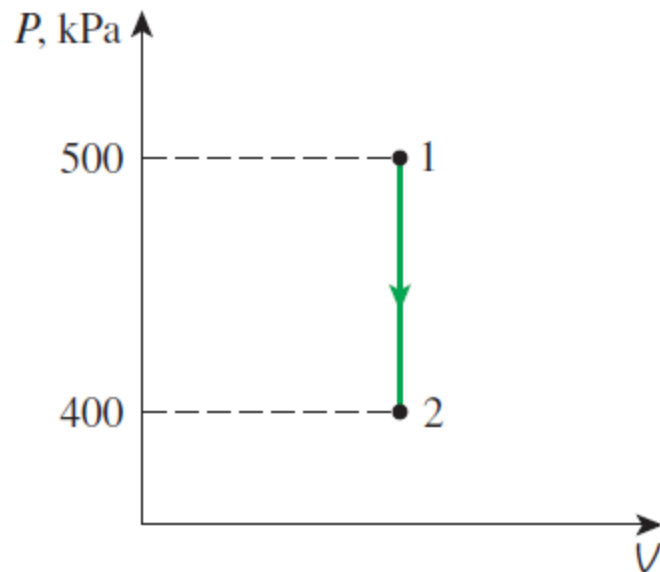
Boundary Work for a Constant-Volume Process

Air

$P_1 = 500 \text{ kPa}$
$T_1 = 150°C$

$P_2 = 400 \text{ kPa}$
$T_2 = 65°C$

Heat

P, kPa

500 — — — — — — • 1

400 — — — — — — • 2

V

What is the boundary work for a constant-volume process?

$$W_b = \int_1^2 P\, dV$$

$$dV = 0$$

$$\rightarrow\ W_b = 0$$

Boundary work done during a constant volume process is always <u>zero</u>.

Heating of a Gas at Constant Pressure

A piston cylinder device initially contains air at 150 kPa and 27°C. Initially, the piston is resting on stops (as shown below) and the enclosed volume is 400 L. The piston requires at least 350 kPa to move it. The air is then heated until the volume doubled. Determine the final temperature, the work done by the system, and the total heat transferred to the air.

Air

$V_1 = 400$ L
$P_1 = 150$ kPa
$T_1 = 27°C$

Q
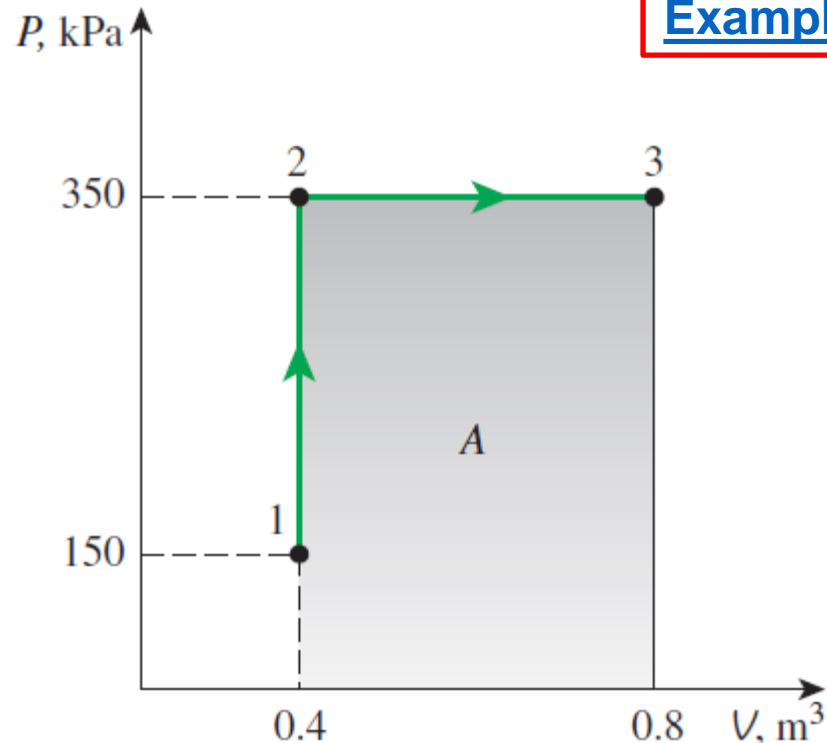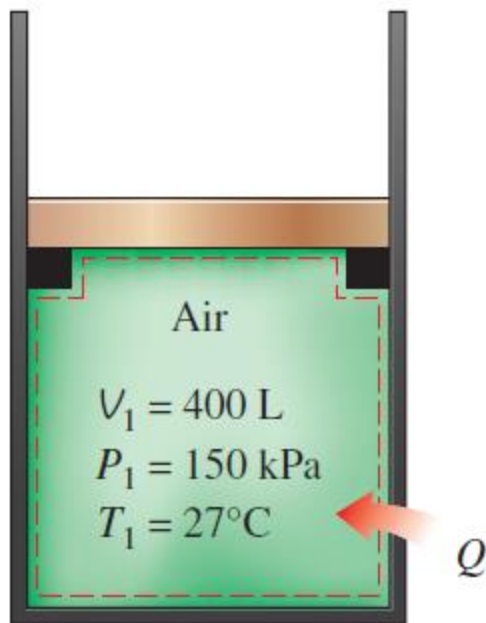
Boundary Work for an Isothermal Compression Process

Air
$V_1 = 0.4 \text{ m}^3$
$P_1 = 100 \text{ kPa}$
$T_0 = 80°C = \text{const.}$
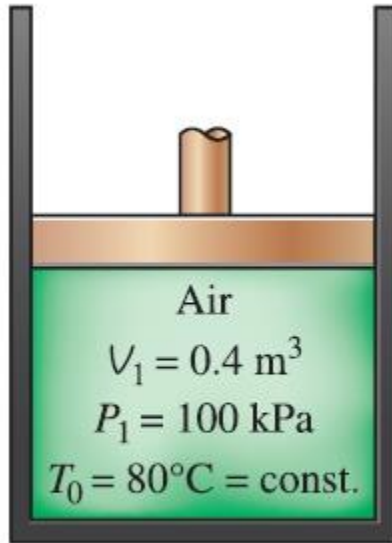
$$W_b = \int_1^2 P \, dV = \int_1^2 CV^{-1} \, dV = PV \ln\left(\frac{V_2}{V_1}\right)$$

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$
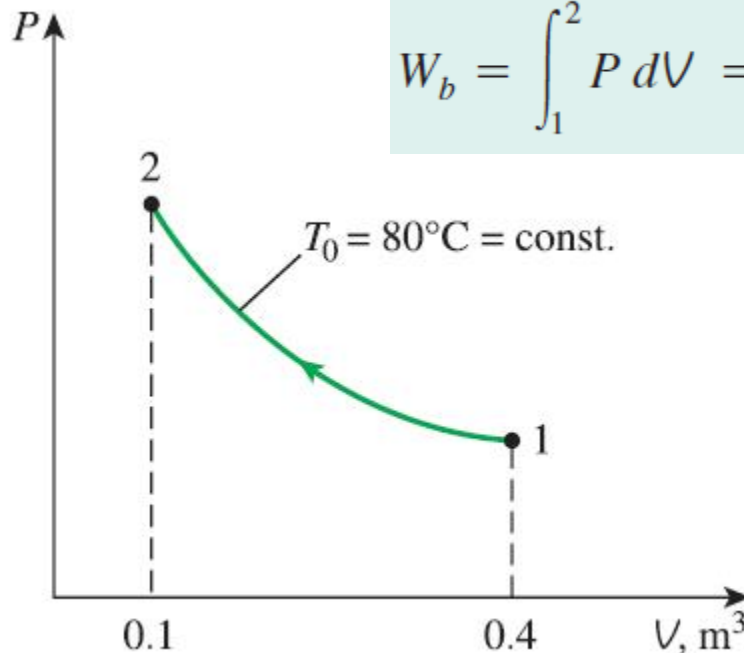
$$W_b = \int_1^2 P \, dV = \int_1^2 \frac{C}{V} \, dV = C \int_1^2 \frac{dV}{V} = C \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{V_2}{V_1}$$

P

2

$T_0 = 80°C = \text{const.}$

1

0.1

0.4

V, m^3

Boundary Work for a Polytropic Process

$$PV^n = C, \qquad P = CV^{-n}$$

$$W_b = \int_1^2 P \, dV = \int_1^2 CV^{-n} \, dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n + 1} = \frac{P_2 V_2 - P_1 V_1}{1 - n}$$

$$W_b = \frac{mR(T_2 - T_1)}{1 - n} \quad \text{For ideal gas}$$

Gas

$PV^n = C = \text{const.}$

$P_1 V_1^n = P_2 V_2^n$

$PV^n = \text{const.}$
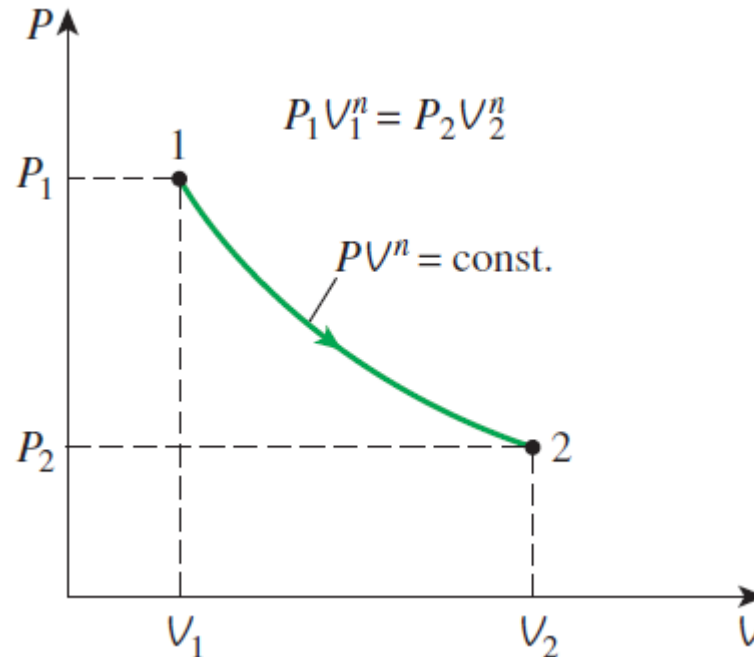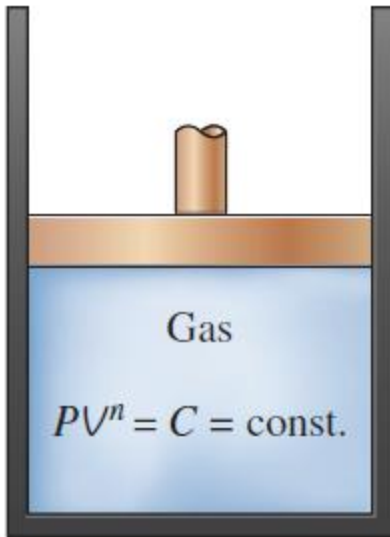
Expansion of gas against a spring

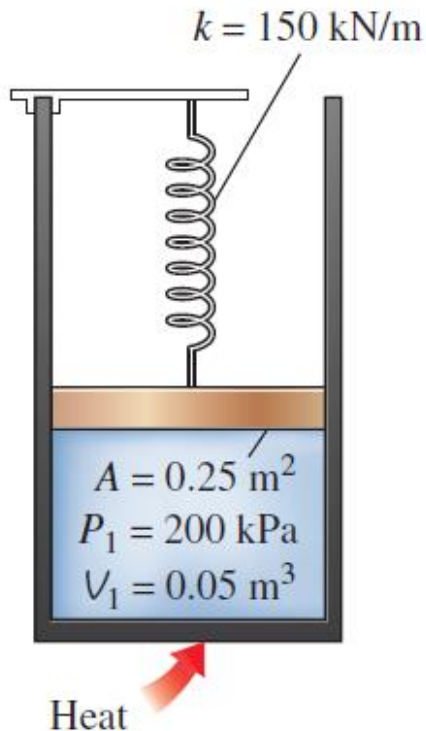A piston cylinder device contains 0.05 m³ of a gas initially at 200 kPa. At this initial state, the linear spring has a spring constant of 150 kN/m but is exerting no force on it. Heat is then transferred to the system causing the volume to double in size. As a result of the expansion, the piston rises and the spring is compressed. The cross sectional area of the piston is 0.25 m². Determine the final pressure of the gas inside the cylinder and the work done by the gas.

$k = 150 \text{ kN/m}$

$A = 0.25 \text{ m}^2$
$P_1 = 200 \text{ kPa}$
$V_1 = 0.05 \text{ m}^3$

Heat

Example 4

Expansion of a Gas against a Spring

$k = 150 \text{ kN/m}$

$A = 0.25 \text{ m}^2$
$P_1 = 200 \text{ kPa}$
$V_1 = 0.05 \text{ m}^3$

Heat

P, kPa

320

200

II

I

0.05

0.1 V, m^3
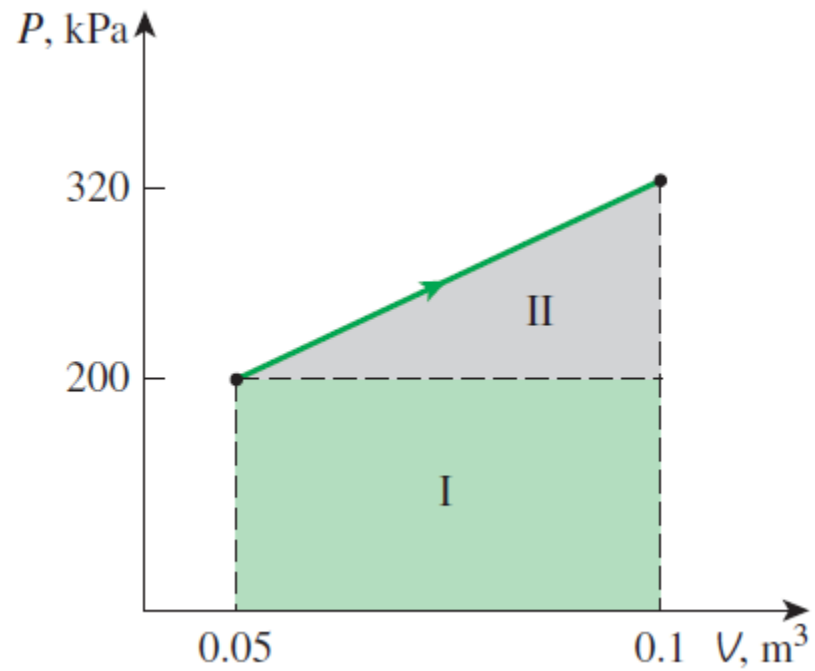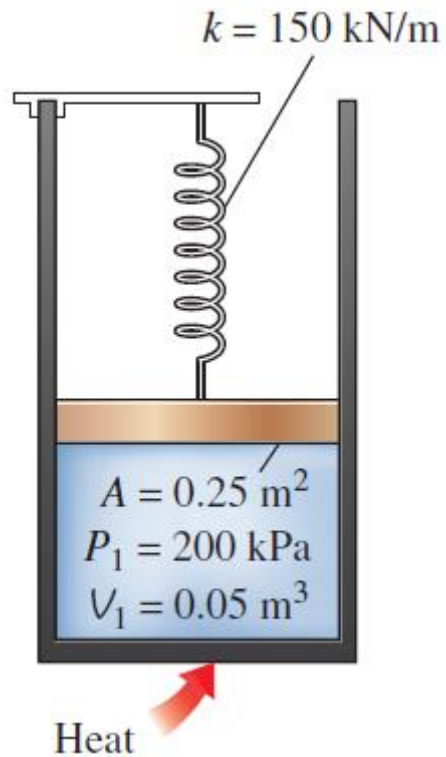
ENERGY BALANCE FOR CLOSED SYSTEMS

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}} \quad (\text{kJ})$$

Energy balance for any system undergoing any process

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal,} \\ \text{kinetic, potential, etc., energies}}} \quad (\text{kW})$$

Energy balance in the rate form

The total quantities are related to the quantities per unit time is

$$Q = \dot{Q}\,\Delta t, \quad W = \dot{W}\,\Delta t, \quad \text{and} \quad \Delta E = (dE/dt)\Delta t \quad (\text{kJ})$$

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}} \quad (\text{kJ/kg})$$

Energy balance per unit mass basis

$$\delta E_{\text{in}} - \delta E_{\text{out}} = dE_{\text{system}} \quad \text{or} \quad \delta e_{\text{in}} - \delta e_{\text{out}} = de_{\text{system}}$$

Energy balance in differential form

$$W_{\text{net,out}} = Q_{\text{net,in}} \quad \text{or} \quad \dot{W}_{\text{net,out}} = \dot{Q}_{\text{net,in}}$$
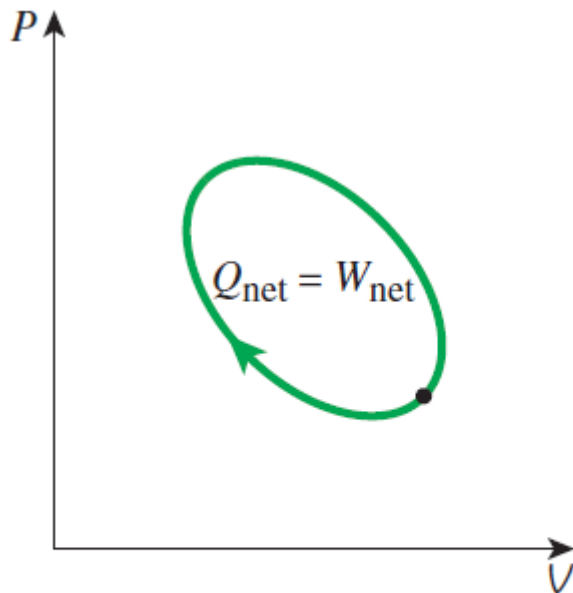
Energy balance for a cycle

$$Q_{net,in} - W_{net,out} = \Delta E_{system} \quad \text{or} \quad Q - W = \Delta E$$

$$Q = Q_{net,in} = Q_{in} - Q_{out}$$

$$W = W_{net,out} = W_{out} - W_{in}$$

Energy balance when sign convention is used: (i.e., heat input and work output are positive; heat output and work input are negative).

FIGURE 4–11
For a cycle $\Delta E = 0$, thus $Q = W$.

General $Q - W = \Delta E$

Stationary systems $Q - W = \Delta U$

Per unit mass $q - w = \Delta e$

Differential form $\delta q - \delta w = de$

Various forms of the first-law relation for closed systems when sign convention is used.

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof.

Energy Balance for Constant Pressure Process

$$E_{in} - E_{out} = \Delta E_{sys}$$

Closed system $\rightarrow \dot{m} = 0$

$$Q - W = \Delta U + \Delta PE + \Delta KE$$

System is stationary $\rightarrow \Delta KE = \Delta PE = 0$

$$Q - (W_b + W_{other}) = \Delta U$$

***W* is negative if work is done on system; *W* is positive if work is done by system.**

$$Q - W_{other} - P_0(V_2 - V_1) = U_2 - U_1$$

Energy Balance for Constant Pressure Process

$$Q - W_{other} - P_0(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{other} = U_2 + P_0V_2 - U_1 - P_0V_1$$

Recall $\quad H = U + PV$

$$Q - W_{other} = H_2 - H_1$$

For a constant pressure expansion/compression process:

$$\Delta U + W_b = \Delta H$$

Example: Heating of a gas at constant pressure

A piston cylinder device contains 25 g of saturated water vapor that is maintained at a constant pressure of 300 kPa. A resistance heater running off of 0.2 A and 120 V is used to heat the vapor for 5 minutes. At the same time, the system experiences a heat loss of 3.7 kJ. Determine the final temperature.

Example 5

H_2O

0.2 A

$m = 25$ g

$P_1 = P_2 = 300$ kPa

120 V

Sat. vapor

5 min

$Q_{out} = 3.7$ kJ
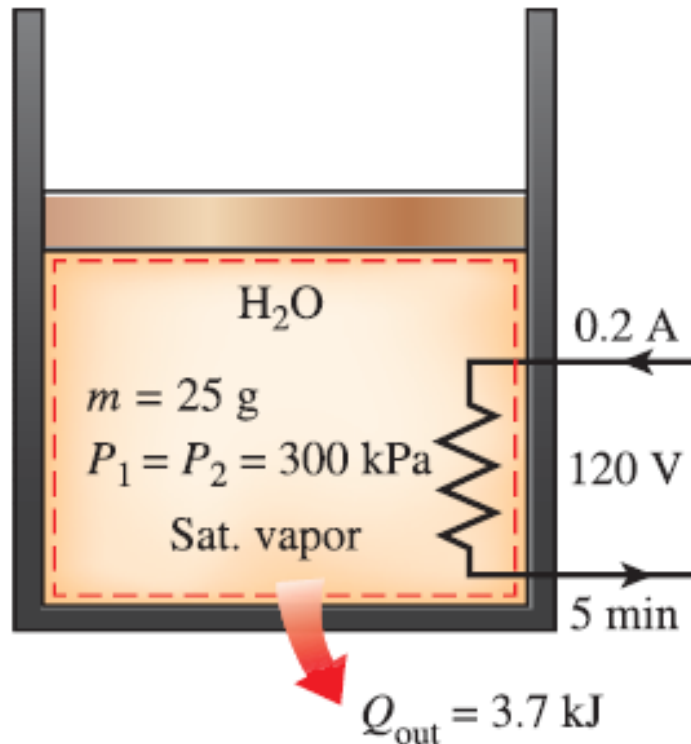
Energy balance for a constant-pressure expansion or compression process

General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process. *Q* is *to* the system and *W* is *from* the system.

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

Net energy transfer by heat, work, and mass

Change in internal, kinetic, potential, etc., energies

$$Q - W = \Delta U + \Delta KE^{0} + \Delta PE^{0}$$

$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

$$Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{\text{other}} = (U_2 + P_2 V_2) - (U_1 + P_1 V_1)$$

$$H = U + PV$$

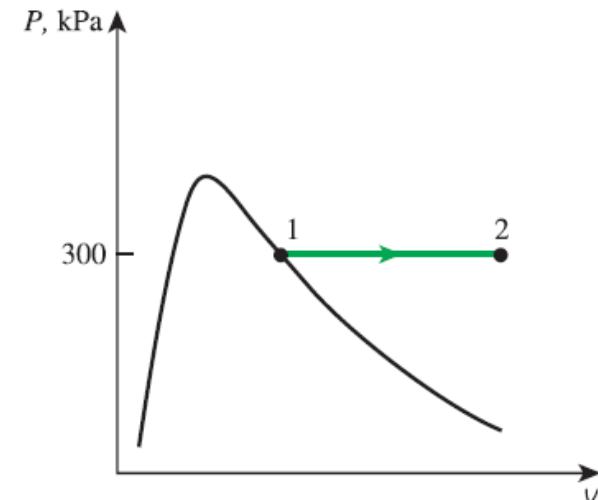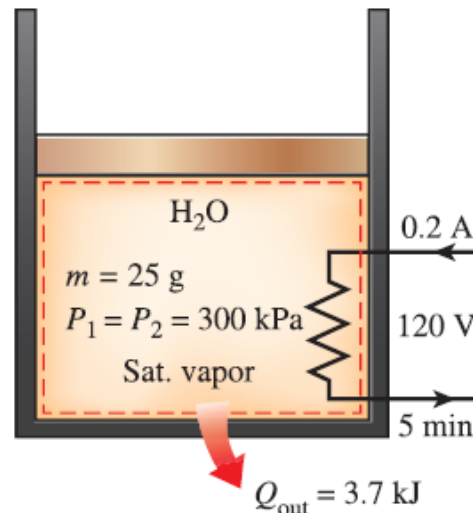$$Q - W_{\text{other}} = H_2 - H_1$$

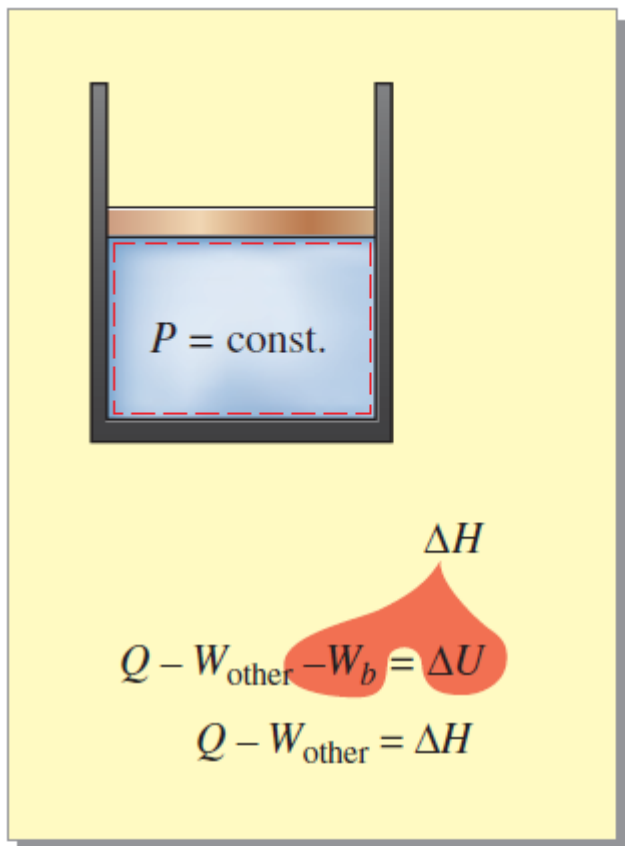**For a constant-pressure expansion or compression process:**

$$\Delta U + W_b = \Delta H$$

An example of constant-pressure process
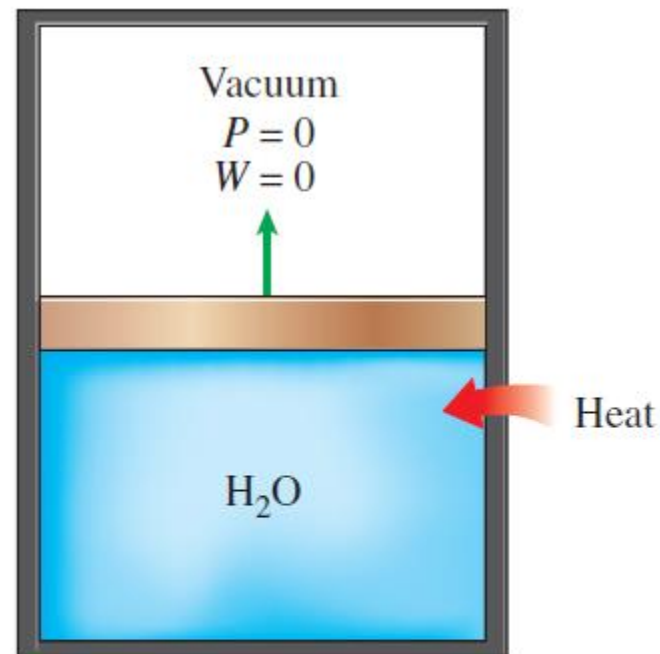
$$W_{e,\text{in}} - Q_{\text{out}} - W_b = \Delta U$$

$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1)$$

H_2O
$m = 25$ g
$P_1 = P_2 = 300$ kPa
Sat. vapor
0.2 A
120 V
5 min
$Q_{\text{out}} = 3.7$ kJ

P, kPa
300
1   2
V

FIGURE 4–14

For a closed system undergoing a quasi-equilibrium, $P = $ constant process, $\Delta U + W_b = \Delta H$. Note that this relation is NOT valid for closed systems processes during which pressure DOES NOT remain constant.
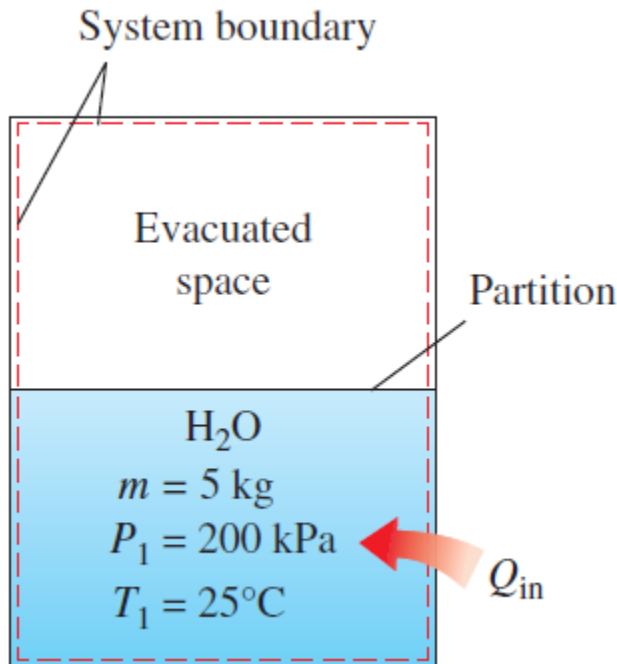
In figure: $P = $ const.

$$Q - W_{other} - W_b = \Delta U$$
$$Q - W_{other} = \Delta H$$
$$\Delta H$$

FIGURE 4–16

Expansion against a vacuum involves no work and thus no energy transfer.

In figure: Vacuum $P = 0$ $W = 0$; Heat; H_2O

Unrestrained Expansion of Water

A rigid tank is divided into two equal parts by a partition. Initially, one side contains 5 kg of water at 200 kPa and 25°C and the other side is evacuated (i.e. is a vacuum). Once the partition is removed, water expands into the evacuated space. During the expansion, the system is allowed to exchange heat with its surroundings to maintain its initial temperature of 25°C. Determine the final pressure and the heat transfer for this process.
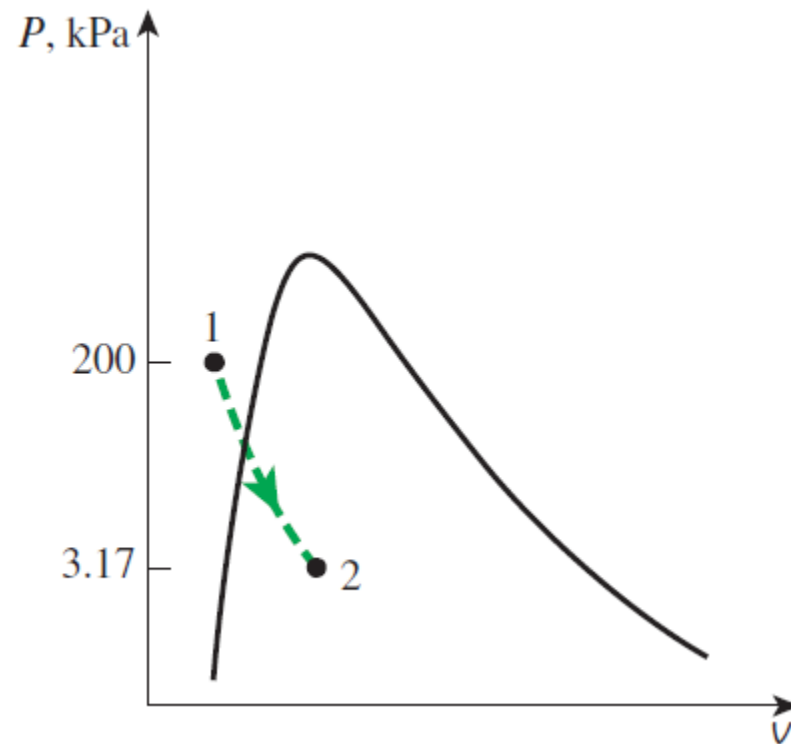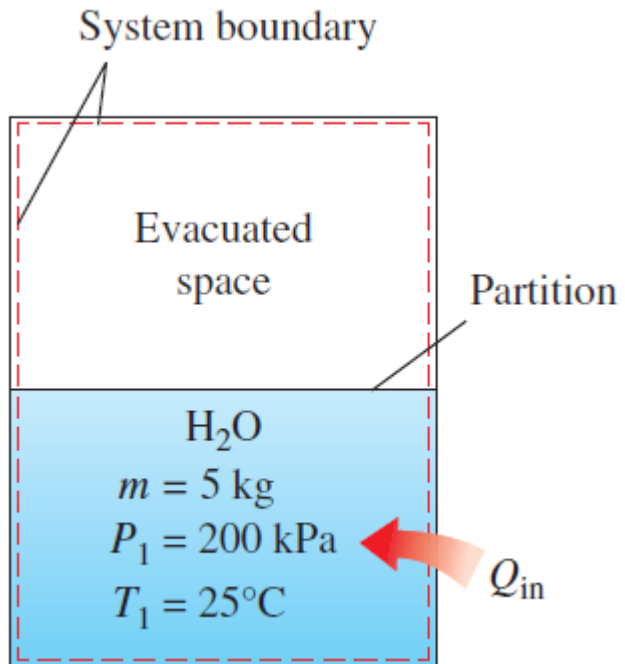
Example 6

System boundary

Evacuated space

Partition

H_2O

$m = 5$ kg

$P_1 = 200$ kPa

$T_1 = 25°C$

Q_{in}

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q_{in} = \Delta U = m(u_2 - u_1)$$

Unrestrained Expansion of Water

System boundary

Evacuated space

Partition

H_2O
$m = 5$ kg
$P_1 = 200$ kPa
$T_1 = 25°C$

Q_{in}

P, kPa

200

3.17

1

2

v

Summary

- Moving boundary work
  - W_b for an isothermal process
  - W_b for a constant-pressure process
  - W_b for a polytropic process
- Energy balance for closed systems
  - Energy balance for a constant-pressure expansion or compression process