

Given: $\text{kJ} := 1000\text{J}$

Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at a pressure of 1.2 MPa.

Required:

Determine the work output of the turbine per unit mass of steam if the process is reversible.

Solution:

The inlet pressure and temperature are defined as

$$P_{\text{in}} := 5\text{MPa} \quad T_{\text{in}} := 450^\circ\text{C} = 723.15\text{K}$$

The outlet pressure is defined as

$$P_{\text{out}} := 1.2\text{MPa}$$

1st Law (for steady state, adiabatic device with no ΔKE and ΔPE)

$$\frac{d}{dt}E_{\text{sys}} = \sum \dot{E}'_{\text{in}} - \sum \dot{E}'_{\text{out}}$$

$$0 = \dot{m} \cdot h_{\text{in}} - \dot{m} \cdot h_{\text{out}} - \dot{W}'_{\text{out}}$$

Solving for the specific work out shows

$$w_{\text{out}} = h_{\text{in}} - h_{\text{out}}$$

Going to Table A-5 @ $P_{\text{in}} = 5000\text{kPa}$ shows

$$T_{\text{sat}} := 263.94^\circ\text{C}$$

Since $T_{\text{in}} > T_{\text{sat}}$ the state is superheated. Going to Table A-6 @ $P_{\text{in}} = 5\text{MPa}$ & $T_{\text{in}} = 450^\circ\text{C}$ shows

$$h_{\text{in}} := 3317.2 \frac{\text{kJ}}{\text{kg}} \quad s_{\text{in}} := 6.8210 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Since the process is adiabatic and reversible, it is isentropic. Thus $\Delta s = 0$ and

$$s_{\text{out}} := s_{\text{in}} = 6.821 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Going to Table A-5 @ $P_{\text{out}} = 1200\text{kPa}$ shows

$$s_{\text{g}} := 6.5217 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Solution (contd.):

Since $s_{\text{out}} > s_g$ the state is superheated. Gong to Table A-6 @ $P_{\text{out}} = 1.2 \text{ MPa}$ & $s_{\text{out}} = 6.821 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ shows that interpolation is needed. This is shown below.

$$s_a := 6.5909 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad s_b := 6.8313 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$h_a := 2816.1 \frac{\text{kJ}}{\text{kg}} \quad h_b := 2935.6 \frac{\text{kJ}}{\text{kg}}$$

$$h_{\text{out}} := \frac{s_{\text{out}} - s_a}{s_b - s_a} \cdot (h_b - h_a) + h_a = 2930.5 \frac{\text{kJ}}{\text{kg}}$$

The specific work out of the device is then

$$w_{\text{out}} := h_{\text{in}} - h_{\text{out}} = 386.72 \frac{\text{kJ}}{\text{kg}}$$