

Given: $\text{kJ} := 1000\text{J}$

Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C.

$$P_1 := 3\text{ MPa} \quad T_1 := 400^\circ\text{C} \quad P_2 := 50\text{ kPa} \quad T_2 := 100^\circ\text{C}$$

Required:

If the power output of the turbine is 2 MW, determine the isentropic efficiency and the mass flow rate of the steam.

Solution:

The power output of the turbine is defined as

$$W'_{\text{out}} := 2\text{ MW}$$

The isentropic efficiency is given by

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Going to Table A-5 @ $P_1 = 3000\text{ kPa}$ shows

$$T_{\text{sat}} := 233.85^\circ\text{C}$$

Since $T_1 > T_{\text{sat}}$ the state is superheated. Going to Table A-6 @ $P_1 = 3\text{ MPa}$ & $T_1 = 400^\circ\text{C}$ shows

$$h_1 := 3231.7 \frac{\text{kJ}}{\text{kg}} \quad s_1 := 6.9235 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Going to Table A-4 @ $T_2 = 100^\circ\text{C}$ shows

$$P_{\text{sat}} := 101.42\text{ kPa}$$

Since $P_2 < P_{\text{sat}}$ the state is superheated. Going to Table A-6 @ $T_2 = 100^\circ\text{C}$ & $P_2 = 0.05\text{ MPa}$ shows

$$h_{2a} := 2682.4 \frac{\text{kJ}}{\text{kg}}$$

The enthalpy h_{2s} is the final state of an isentropic process. Thus

$$s_{2s} := s_1 = 6.923 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

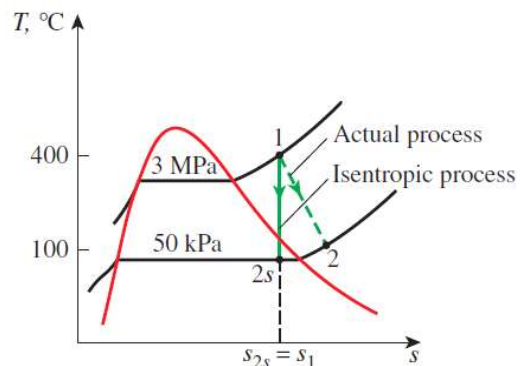
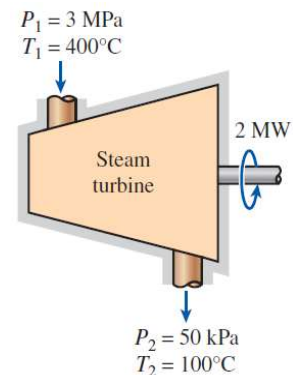
Going to Table A-5 @ $P_2 = 50\text{ kPa}$ & $s_{2s} = 6.923 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$ shows

$$s_f := 1.0912 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad s_g := 7.5931 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$h_f := 340.54 \frac{\text{kJ}}{\text{kg}} \quad h_g := 2645.2 \frac{\text{kJ}}{\text{kg}}$$

$$x_{2s} := \frac{s_{2s} - s_f}{s_g - s_f} = 0.897$$

$$h_{2s} := h_f + x_{2s} \cdot (h_g - h_f) = 2407.9 \frac{\text{kJ}}{\text{kg}}$$



Solution (contd.):

Thus the isentropic efficiency is

$$\eta_T := \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = 66.68\%$$

1st Law for a steady state adiabatic turbine with negligible changes in KE and PE shows

$$\begin{aligned} \frac{d}{dt} E_{\text{sys}} &= \Sigma E'_{\text{in}} - \Sigma E'_{\text{out}} \\ 0 &= m'_{\text{in}} \cdot \left(h_{\text{in}} + \frac{V_{\text{in}}^2}{2} + g \cdot z_{\text{in}} \right) - m'_{\text{out}} \cdot \left(h_{\text{out}} + \frac{V_{\text{out}}^2}{2} + g \cdot z_{\text{out}} \right) - W'_{\text{out}} \\ 0 &= m'_{\text{in}} \cdot h_{\text{in}} - m'_{\text{out}} \cdot h_{\text{out}} - W'_{\text{out}} \end{aligned}$$

Since the turbine has a single inlet and outlet mass stream, the mass flow rates are the same. Thus

$$0 = m' \cdot (h_{\text{in}} - h_{\text{out}}) - W'_{\text{out}}$$

$$m' := \frac{W'_{\text{out}}}{h_1 - h_{2a}} = 3.641 \frac{\text{kg}}{\text{s}}$$