Given: $kJ := 1000J$

Air is compressed from an initial state of 100 kPa and 17°C to a final state of 600 kPa and 57°C.

$$P_1 := 100kPa \qquad T_1 := 17\,°C = 290.15\,K \qquad P_2 := 600kPa \qquad T_2 := 57\,°C = 330.15\,K$$

Required:

Determine the entropy change of air during this compression process

(a) by using the property tables and

(b) by using an average specific heat.

Solution:

Going to Table A-2(a) @ air shows

$$R_{air} := 0.287\frac{kJ}{kg \cdot K}$$

Going to Table A-17 @ $T_1 = 290.15K$ shows that interpolation is needed. This is shown below.

$$T_a := 290K \qquad T_b := 295K$$

$$s°_a := 1.66802\frac{kJ}{kg \cdot K} \qquad s°_b := 1.68515\frac{kJ}{kg \cdot K}$$

$$s°_1 := \frac{T_1 - T_a}{T_b - T_a} \cdot \left(s°_b - s°_a\right) + s°_a = 1.669 \cdot \frac{kJ}{kg \cdot K}$$

Going to Table A-17 @ $T_2 = 330.15K$ shows that interpolation is needed. This is shown below.

$$T_a := 330K \qquad T_b := 340K$$

$$s°_a := 1.79783\frac{kJ}{kg \cdot K} \qquad s°_b := 1.82790\frac{kJ}{kg \cdot K}$$

$$s°_2 := \frac{T_2 - T_a}{T_b - T_a} \cdot \left(s°_b - s°_a\right) + s°_a = 1.798 \cdot \frac{kJ}{kg \cdot K}$$

The change in entropy is then found by

$$\boxed{\Delta s_{table} := s°_2 - s°_1 - R_{air} \cdot \ln\left(\frac{P_2}{P_1}\right) = -0.38449 \cdot \frac{kJ}{kg \cdot K}}$$

The average temperature is found by

$$T_{avg} := \frac{T_1 + T_2}{2} = 310.15\,K$$

Going to Table A-2(b) @ $T_{avg} = 310.15K$ and air shows that interpolation is needed. This is shown below.

$$T_a := 300K \qquad T_b := 350K$$

$$c_a := 1.005\frac{kJ}{kg \cdot K} \qquad c_b := 1.008\frac{kJ}{kg \cdot K}$$

$$c_{p,avg} := \frac{T_{avg} - T_a}{T_b - T_a} \cdot \left(c_b - c_a\right) + c_a = 1.006 \cdot \frac{kJ}{kg \cdot K}$$

Solution (contd.):

The change in entropy when using an average specific heat value is then found by

$$\boxed{\Delta s_{avgcp} := c_{p,avg} \cdot \ln\left(\frac{T_2}{T_1}\right) - R_{air} \cdot \ln\left(\frac{P_2}{P_1}\right) = -0.38436 \cdot \frac{kJ}{kg \cdot K}}$$

The percent difference is found by

$$\%diff := \left|\frac{\Delta s_{table} - \Delta s_{avgcp}}{\Delta s_{table}}\right| = 0.033 \cdot \%$$

Alternatively, the average specific heat value could have been found by looking up the specific heat value at the temperature at state 1 and 2 and then averaging those values. This is shown below.

Going to Table A-2(b) @ $T_1 = 290.15\,K$ shows that interpolation is needed. This is shown below.

$$T_a := 250K \qquad T_b := 300K$$

$$c_{p,a} := 1.003 \frac{kJ}{kg \cdot K} \qquad c_{p,b} := 1.005 \frac{kJ}{kg \cdot K}$$

$$c_{p,1} := \frac{T_1 - T_a}{T_b - T_a} \cdot \left(c_{p,b} - c_{p,a}\right) + c_{p,a} = 1.005 \cdot \frac{kJ}{kg \cdot K}$$

Going to Table A-2(b) @ $T_2 = 330.15\,K$ shows that interpolation is needed. This is shown below.

$$T_a := 300K \qquad T_b := 350K$$

$$c_{p,a} := 1.005 \frac{kJ}{kg \cdot K} \qquad c_{p,b} := 1.008 \frac{kJ}{kg \cdot K}$$

$$c_{p,2} := \frac{T_2 - T_a}{T_b - T_a} \cdot \left(c_{p,b} - c_{p,a}\right) + c_{p,a} = 1.007 \cdot \frac{kJ}{kg \cdot K}$$

The average specific heat value using the two table lookups is then

$$c_{p,avg2} := \frac{c_{p,1} + c_{p,2}}{2} = 1.006 \cdot \frac{kJ}{kg \cdot K}$$

The change in entropy when using an average specific heat value from the two table lookups is then found by

$$\Delta s_{avgcp2} := c_{p,avg2} \cdot \ln\left(\frac{T_2}{T_1}\right) - R_{air} \cdot \ln\left(\frac{P_2}{P_1}\right) = -0.38435 \cdot \frac{kJ}{kg \cdot K}$$

The percent difference between the two methods is

$$\%diff := \left|\frac{\Delta s_{avgcp2} - \Delta s_{avgcp}}{\left(\frac{\Delta s_{avgcp2} + \Delta s_{avgcp}}{2}\right)}\right| = 3.31 \times 10^{-3} \cdot \%$$

The percent difference shows that the two methods do produce difference. However, the difference is very small. The first method only requires one table lookup while the second method required two table lookups. For this reason, the first method is preferred.