

Given: $\text{kJ} := 1000\text{J}$

A piston cylinder device initially contains air at 150 kPa and 27°C. Initially, the piston is resting on stops (as shown below) and the enclosed volume is 400 L. The piston requires at least 350 kPa to move it. The air is then heated until the volume doubled.

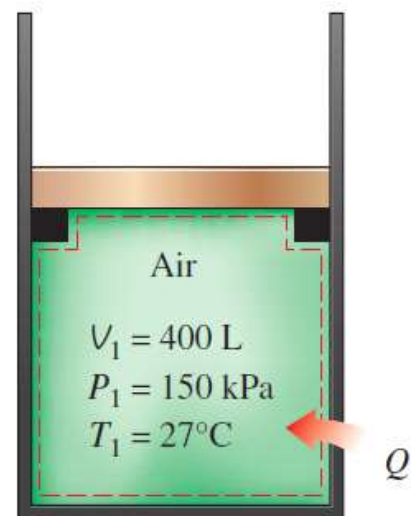
$$P_1 := 150\text{kPa} \quad P_2 := 350\text{kPa} \quad P_3 := P_2 = 350\text{kPa}$$

$$T_1 := 27^\circ\text{C} = 300.15\text{ K}$$

$$V_1 := 400\text{L} = 0.4\cdot\text{m}^3$$

$$V_3 := 2\cdot V_1 = 800\text{ L}$$

$$V_3 = 0.8\cdot\text{m}^3$$



Required:

Determine the final temperature, the work done by the system and the total heat.

Solution:

Assuming the air behaves as an ideal gas throughout the process

$$P\cdot V = m\cdot R\cdot T$$

The pressure, volume, and temperature all are changing while the mass and ideal gas constant remains constant so

$$\frac{P\cdot V}{T} = m\cdot R\cdot T = \frac{P_1\cdot V_1}{T_1} = \frac{P_3\cdot V_3}{T_3}$$

Solving for the final temperature shows

$$T_3 := \frac{P_3\cdot V_3\cdot T_1}{P_1\cdot V_1} = 1127.6^\circ\text{C}$$

The work done by the system is given by

$$W_b := P_2\cdot(V_3 - V_1) = 140\cdot\text{kJ}$$

Starting with the first law

$$\Delta E_{\text{sys}} = \Sigma E_{\text{in}} - \Sigma E_{\text{out}}$$

Assuming there is no change in kinetic and potential energy and the only work being done is by raising the piston, the first law becomes

$$\Delta U = Q_{\text{add}} - W_b$$

$$m\cdot(u_3 - u_1) = Q_{\text{add}} - W_b$$

Solution (cont.):

Solving for the heat added yields

$$Q_{\text{add}} = m \cdot (u_3 - u_1) + W_b$$

The mass in the cylinder may be found by using the ideal gas law.

$$P \cdot V = m \cdot R \cdot T \quad \text{or} \quad m = \frac{P_1 \cdot V_1}{R \cdot T_1}$$

Going to Table A-3(a) @ air shows

$$R := 0.287 \cdot \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

The mass is then found to be

$$m := \frac{P_1 \cdot V_1}{R \cdot T_1} = 0.697 \text{ kg}$$

Going to Table A-17 @ 27°C and 1127.6°C shows

$$T_1 = 300.15 \text{ K} \quad u_1 := 214.07 \frac{\text{kJ}}{\text{kg}}$$

$$T_3 = 1400.7 \text{ K} \quad u_3 := 1113.52 \frac{\text{kJ}}{\text{kg}}$$

Note: The table data doesn't match up exactly with the temperature values but actually interpolating between table rows will produce very similar results as the ones shown here. This could be a time saver on an exam.

The heat added is then

$$Q_{\text{add}} := m \cdot (u_3 - u_1) + W_b = 766.5 \text{ kJ}$$