

# Thermodynamics: An Engineering Approach

8th Edition

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## **Topic 15**

## **Reversible Work**

# Objectives

- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Apply the second law of thermodynamics to processes.

# REVERSIBLE STEADY-FLOW WORK

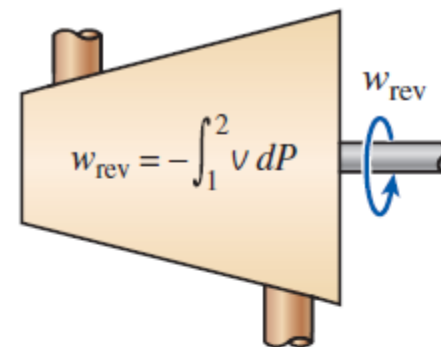
Recall the formula for boundary work:

$$W_b = \int_1^2 P dV$$

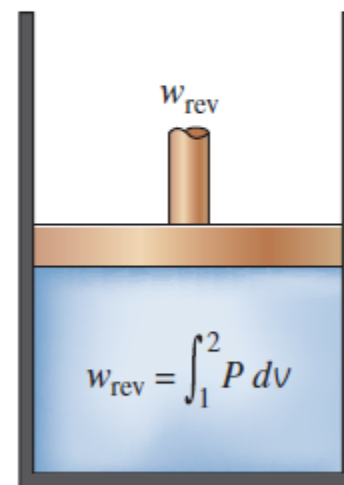
This was useful for closed systems like piston cylinders. But what about open, steady flow systems like turbines?

Total Energy Balance (differential form):

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe$$



(a) Steady-flow system



(b) Closed system

**FIGURE 7–40**

Reversible work relations for steady-flow and closed systems.

# Reversible Steady-flow Work

$$\delta q_{rev} = Tds = dh - \nu dP$$

$$dh - \nu dP - \delta w_{rev} = dh + dke + dpe$$

$$-\delta w_{rev} = \nu dP + dke + dpe$$

If kinetic and potential energies are ignored, then:

$$w_{rev,out} = - \int_1^2 \nu dP$$

$$w_{rev,in} = \int_1^2 \nu dP$$

# Reversible Steady-flow Work

$$-\delta w_{rev} = \nu dP + dke + dpe$$

$$\dot{W}_{in} = \dot{m} [\nu \Delta P + \Delta ke + \Delta pe]$$

$$\dot{W}_{in} = \dot{m} \left[ \nu (P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g (z_2 - z_1) \right]$$

$$\dot{W}_{in} = \dot{m} \left[ \frac{(P_2 - P_1)}{\rho} + \frac{V_2^2 - V_1^2}{2} + g (z_2 - z_1) \right]$$

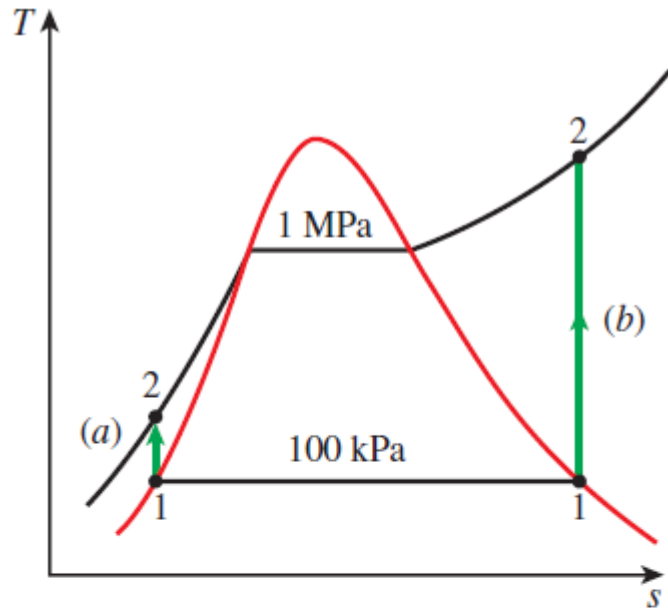
Look familiar?

What if there is no work?

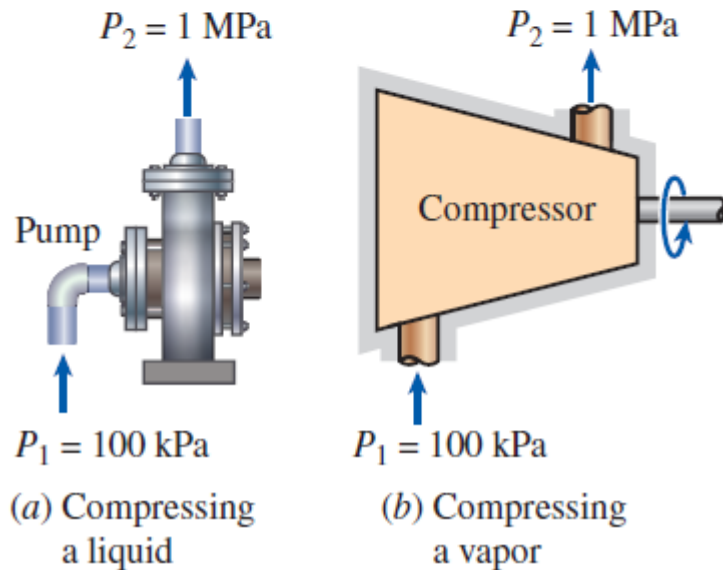
$$0 = \left[ \frac{(P_2 - P_1)}{\rho} + \frac{V_2^2 - V_1^2}{2} + g (z_2 - z_1) \right]$$

**Bernoulli Equation**

# Compressing a Liquid vs. Gas



Determine the work input required to compress steam isentropically from 100 kPa to 1 MPa, assuming that the steam exists as (a) saturated liquid and (b) as saturated vapor.



## Example 1

# Proof that Reversible Process is more Efficient

$$\delta q_{act} - \delta w_{act} = dh + dke + dpe$$

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe$$

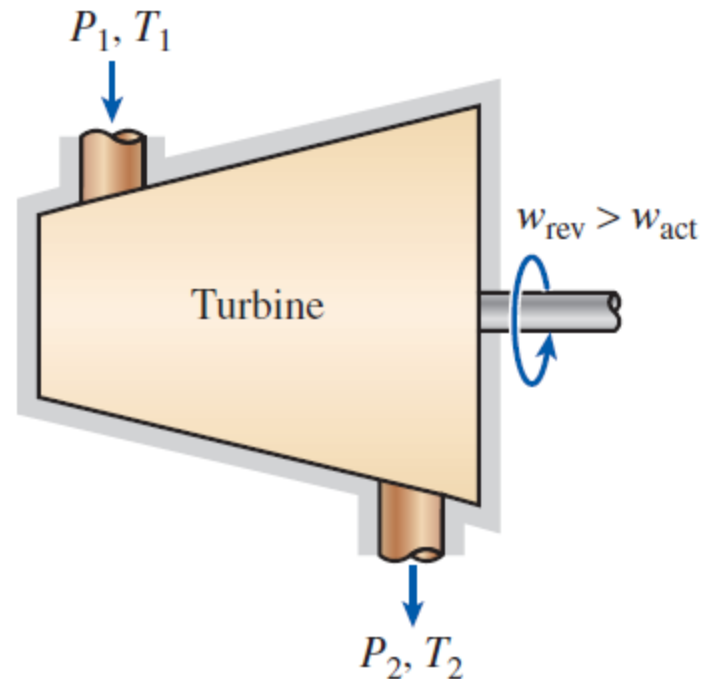
$$\delta q_{act} - \delta w_{act} = \delta q_{rev} - \delta w_{rev}$$

$$\delta w_{rev} - \delta w_{act} = \delta q_{rev} - \delta q_{act}$$

$$\delta q_{rev} = T ds$$

$$\delta w_{rev} - \delta w_{act} = T ds - \delta q_{act}$$

$$ds \geq \frac{\delta q_{act}}{T}$$



**FIGURE 7–43**

A reversible turbine delivers more work than an irreversible one if both operate between the same end states.

# MINIMIZING THE COMPRESSOR WORK

$$w_{\text{rev,in}} = \int_1^2 v \, dP \quad \begin{array}{l} \text{When kinetic and} \\ \text{potential energies} \\ \text{are negligible} \end{array}$$

Isentropic ( $Pv^k = \text{constant}$ ):

$$w_{\text{comp,in}} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]$$

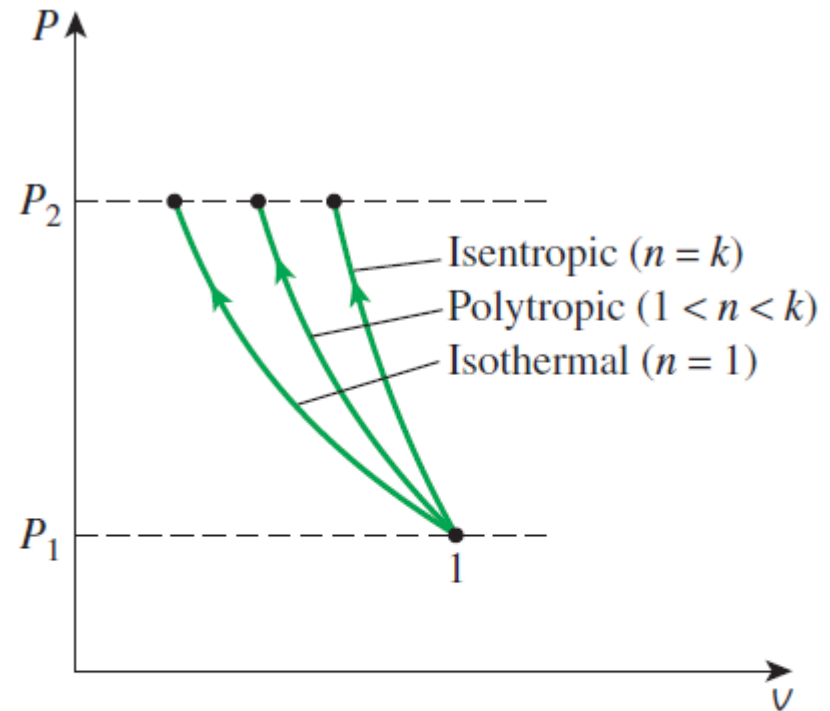
Polytropic ( $Pv^n = \text{constant}$ ):

$$w_{\text{comp,in}} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]$$

Isothermal ( $Pv = \text{constant}$ ):

$$w_{\text{comp,in}} = RT \ln \frac{P_2}{P_1}$$

The adiabatic compression ( $Pv^k = \text{constant}$ ) requires the maximum work and the isothermal compression ( $T = \text{constant}$ ) requires the minimum. **Why?**



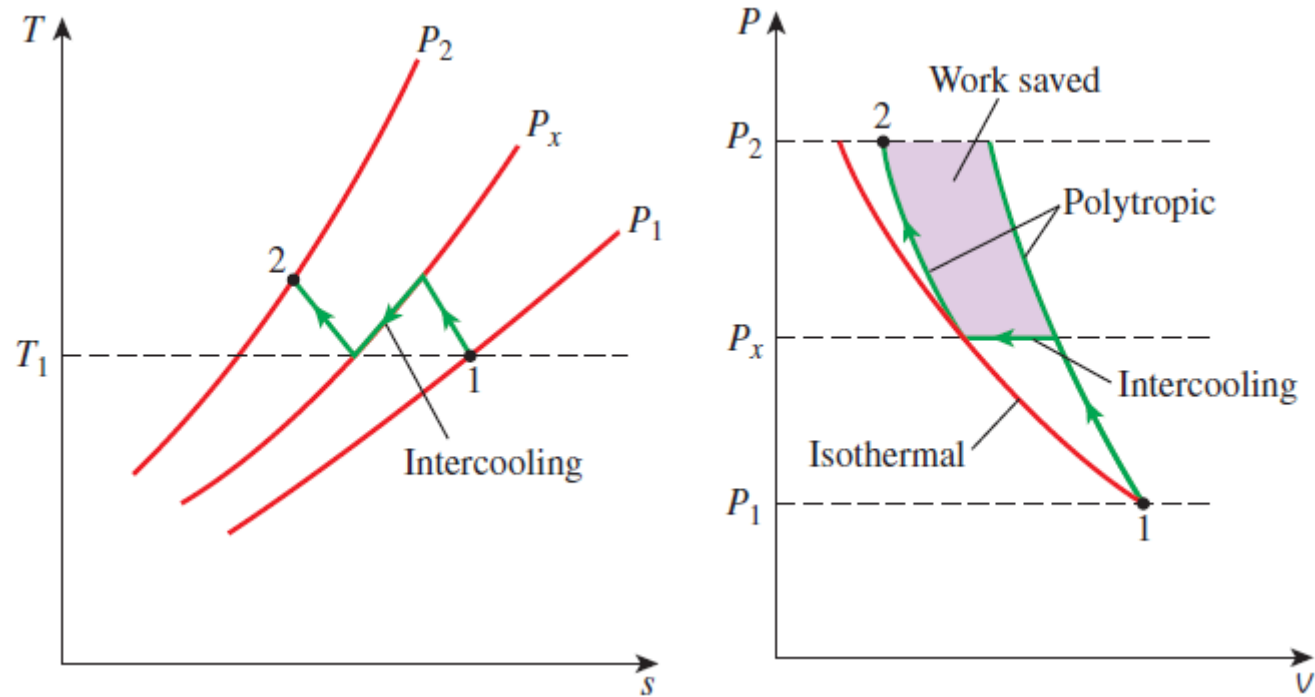
**FIGURE 7-44**

$P$ - $v$  diagrams of isentropic, polytropic, and isothermal compression processes between the same pressure limits.



# Multistage Compression with Intercooling

The gas is compressed in stages and cooled between each stage by passing it through a heat exchanger called an intercooler.



**FIGURE 7-45**

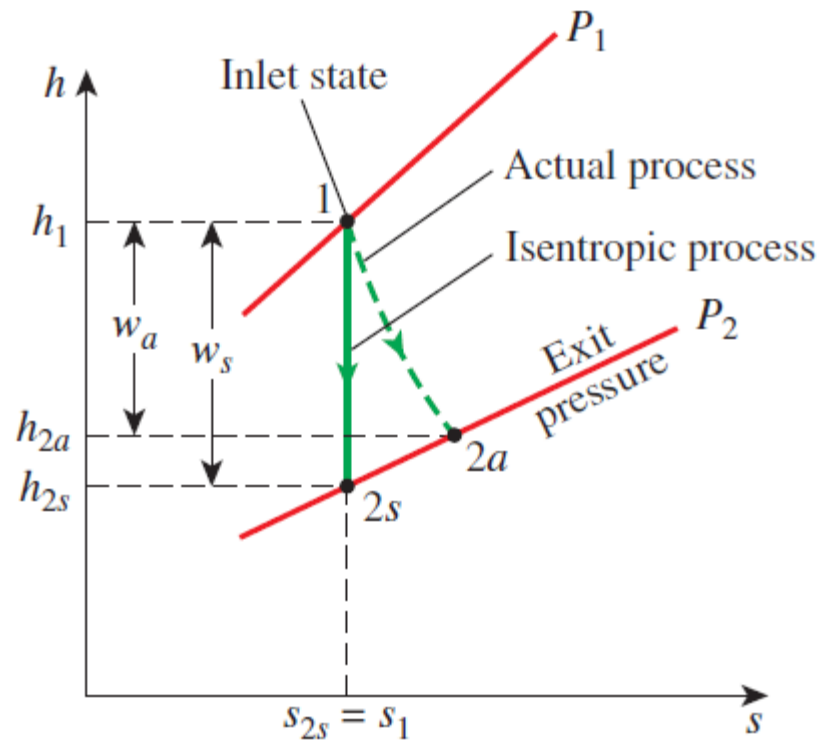
*P-v and T-s diagrams for a two-stage steady-flow compression process.*

$$W_{\text{comp, in}} = W_{\text{comp I, in}} + W_{\text{comp II, in}}$$

$$= \frac{nRT_1}{n-1} \left[ \left( \frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] + \frac{nRT_1}{n-1} \left[ \left( \frac{P_2}{P_x} \right)^{(n-1)/n} - 1 \right]$$

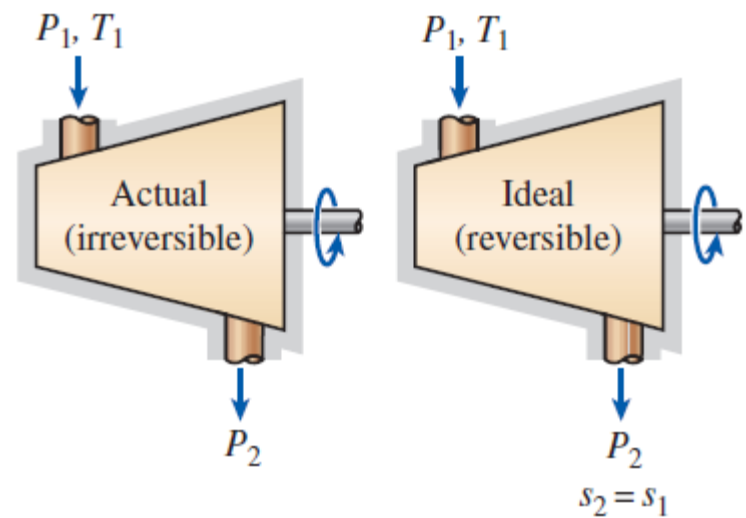
$P_x = (P_1 P_2)^{1/2}$  or  $\frac{P_x}{P_1} = \frac{P_2}{P_x}$  *To minimize compression work during two-stage compression, the pressure ratio across each stage of the compressor must be the same.*

# ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES



**FIGURE 7–48**

The  $h$ - $s$  diagram for the actual and isentropic processes of an adiabatic turbine.



**FIGURE 7–47**

The isentropic process involves no irreversibilities and serves as the ideal process for adiabatic devices.

## Isentropic Efficiency of Turbines

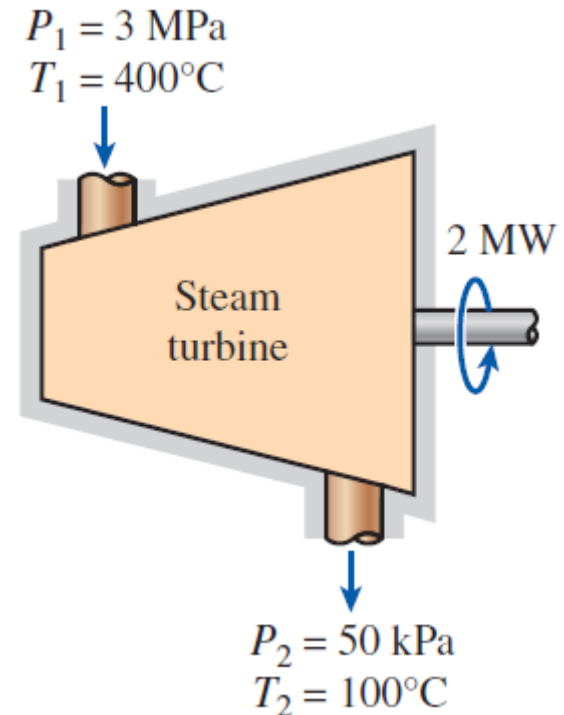
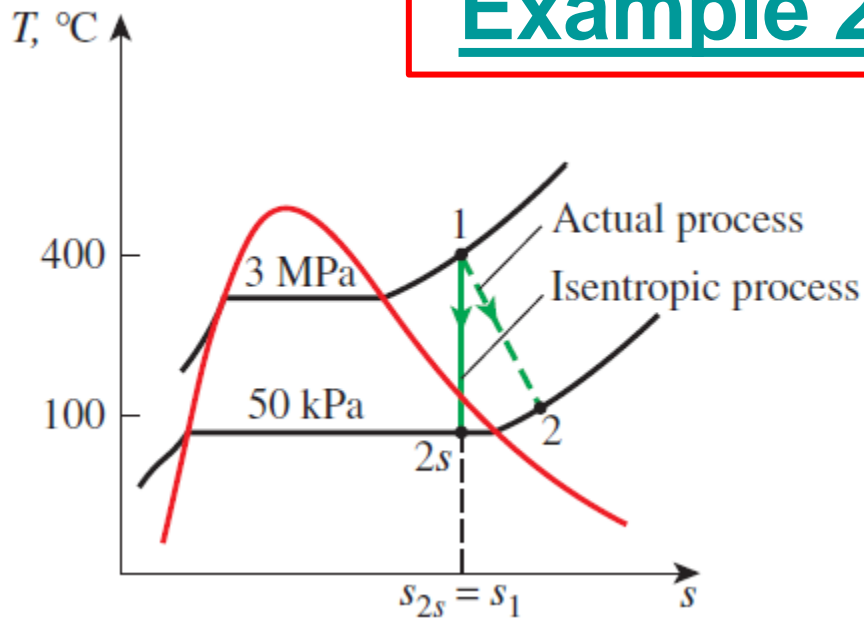
$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

# Isentropic Efficiency of a Turbine

Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2 MW, determine the isentropic efficiency and the mass flow rate of the steam.

## Example 2



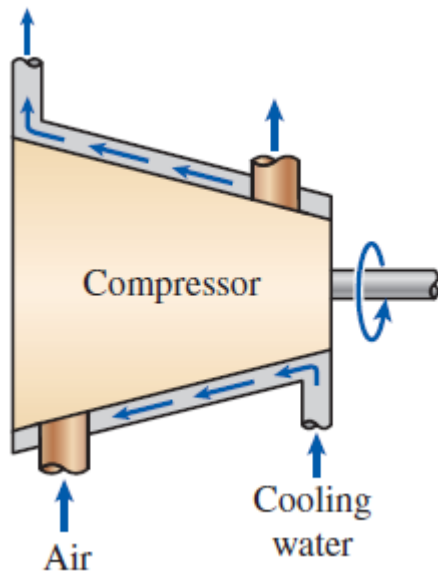
# Isentropic Efficiencies of Compressors and Pumps

$$\eta_c = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

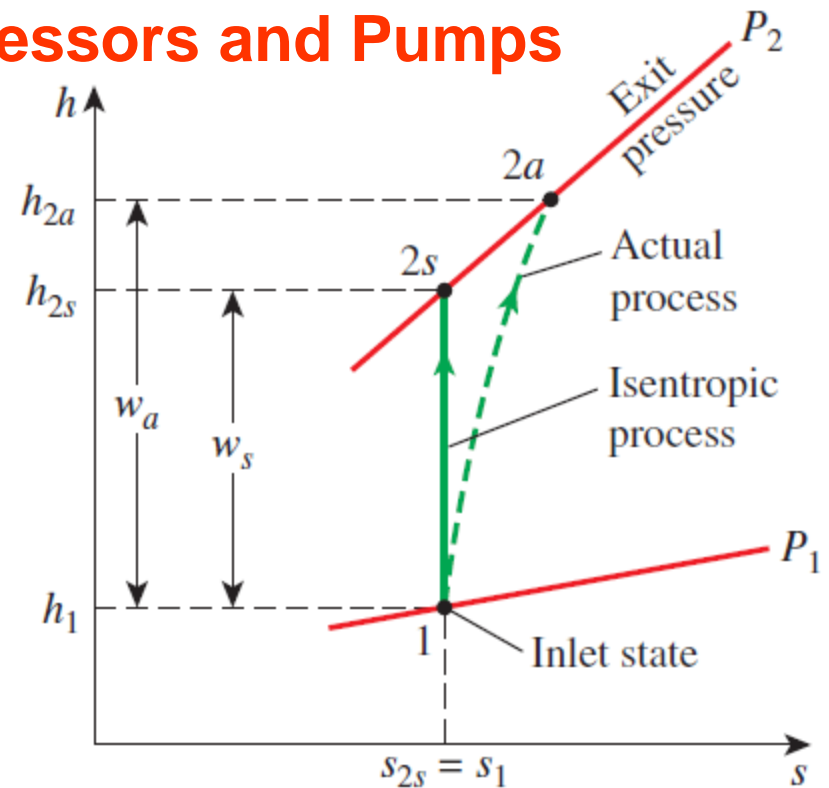
$$\eta_c \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad \begin{array}{l} \text{When kinetic and} \\ \text{potential energies} \\ \text{are negligible} \end{array}$$

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1} \quad \begin{array}{l} \text{For a} \\ \text{pump} \end{array}$$

$$\eta_c = \frac{w_t}{w_a} \quad \begin{array}{l} \text{Isothermal} \\ \text{efficiency} \end{array}$$



Compressors are sometimes intentionally cooled to minimize the work input.



**FIGURE 7-50**

The  $h$ - $s$  diagram of the actual and isentropic processes of an adiabatic compressor.

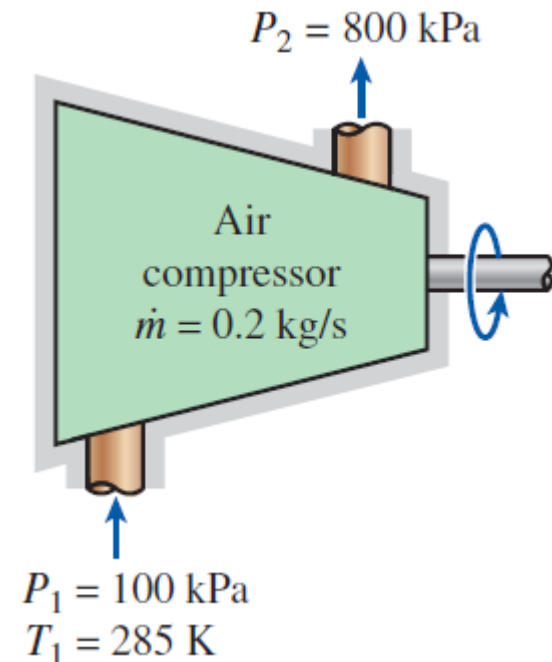
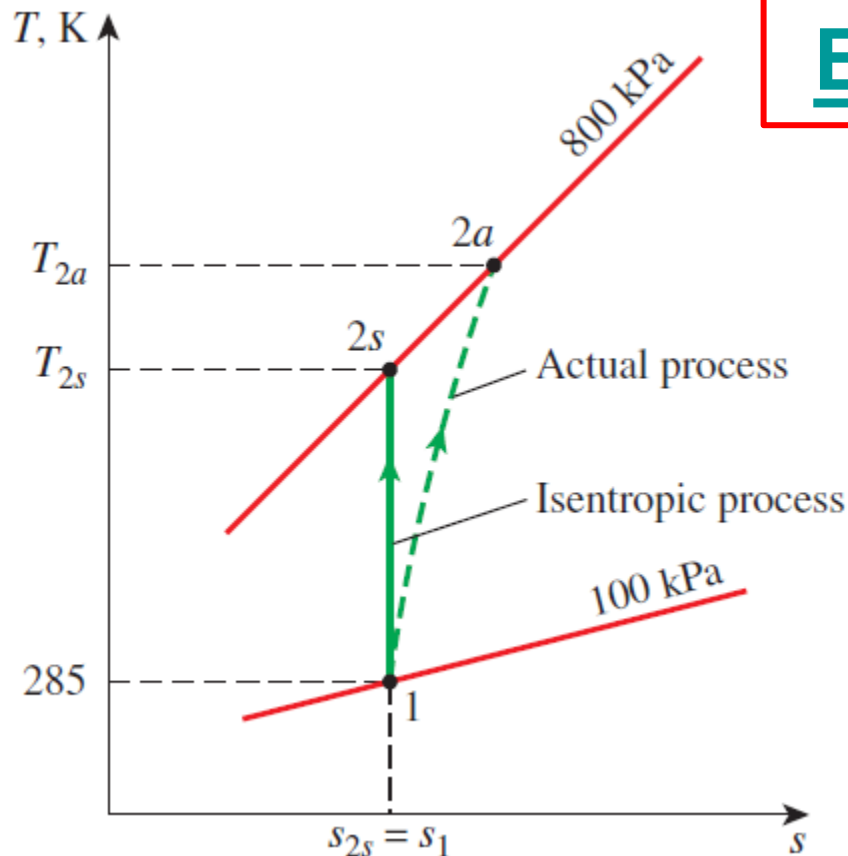
Can you use isentropic efficiency for a non-adiabatic compressor?

Can you use isothermal efficiency for an adiabatic compressor?

## Effect of Efficiency on Compressor Power Input

Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency is 80%, determine the air temperature at the exit and the require power input for the compressor.

### Example 3



# Isentropic Efficiency of Nozzles

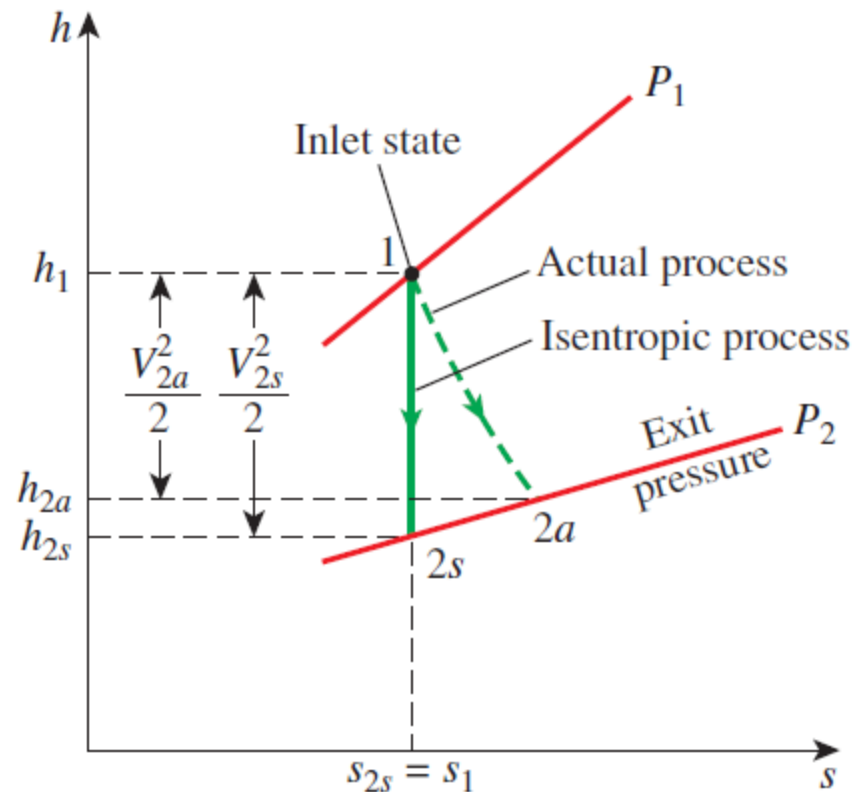
$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2}$$

If the inlet velocity of the fluid is small relative to the exit velocity, the energy balance is

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

Then,

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

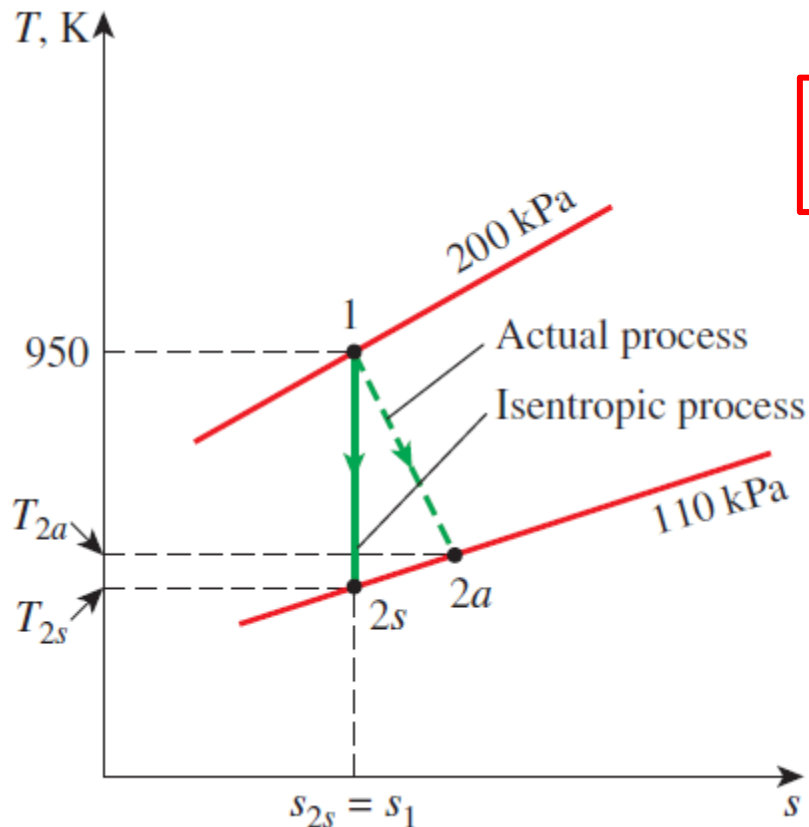


**FIGURE 7–53**

The  $h$ - $s$  diagram of the actual and isentropic processes of an adiabatic nozzle.

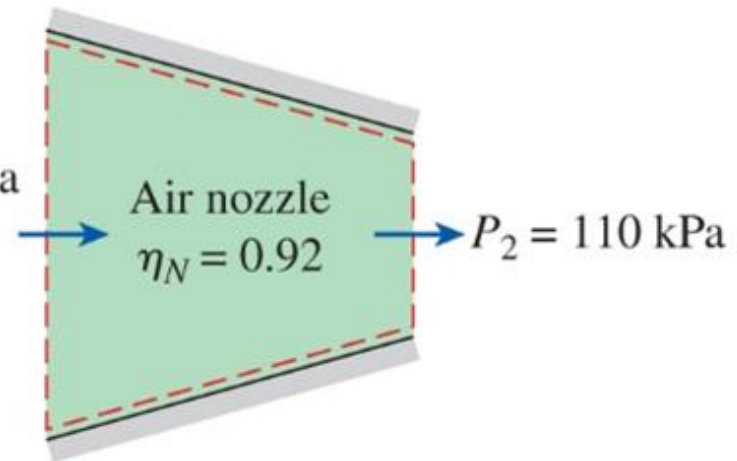
# Effect of Efficiency on Nozzle Exit Velocity

Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 110 kPa. If the isentropic efficiency of the nozzle is 92%, determine the maximum possible velocity, the exit temperature of the air, and the actual velocity of the air. Assume constant specific heat of air is 1.11 kJ/ kg K and the specific heat ratio is 1.349.



## Example 4

$$\begin{aligned} P_1 &= 200 \text{ kPa} \\ T_1 &= 950 \text{ K} \\ V_1 &\ll V_2 \end{aligned}$$



# Summary

- Reversible steady-flow work
- Minimizing the compressor work
- Isentropic efficiencies of steady-flow devices
- The increase of entropy principle