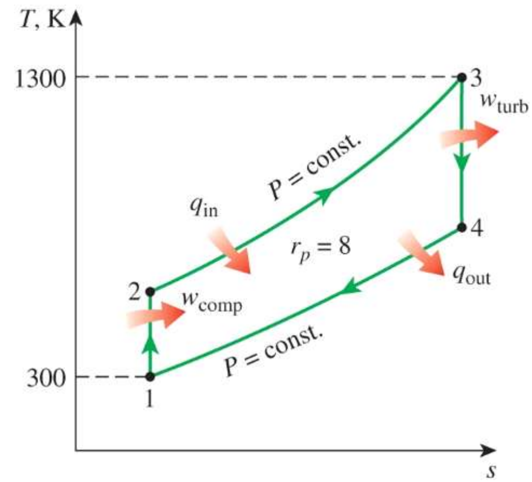
Given: $kJ := 1000J$

A gas turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet.

$r_p := 8 \qquad T_1 := 300K \qquad T_3 := 1300K$



Required:

Utilizing the air standard assumptions, determine

(a) the gas temperature at the exits of the compressor and the turbine,

(b) the back work ratio, and

(c) the thermal efficiency.

Solution:

Going to Table A-17 @ $T_1 = 300K$ shows

$$h_1 := 300.19\frac{kJ}{kg} \qquad P_{r1} := 1.3860$$

Using the relative pressure at state 1, the relative pressure at state 2 may be found by

$$P_{r2} := P_{r1} \cdot r_p = 11.088$$

Going to Table A-17 @ $P_{r2} = 11.088$ shows that interpolation is needed.

$$P_{ra} := 10.37 \qquad P_{rb} := 11.10$$

$$h_a := 533.98\frac{kJ}{kg} \qquad h_b := 544.35\frac{kJ}{kg} \qquad T_a := 530K \qquad T_b := 540K$$

$$h_2 := \frac{P_{r2} - P_{ra}}{P_{rb} - P_{ra}} \cdot (h_b - h_a) + h_a = 544.18 \cdot \frac{kJ}{kg} \qquad \boxed{T_2 := \frac{P_{r2} - P_{ra}}{P_{rb} - P_{ra}} \cdot (T_b - T_a) + T_a = 539.8\,K} \quad \text{a)}$$

Going to Table A-17 @ $T_3 = 1300\,K$ shows

$$h_3 := 1395.97\frac{kJ}{kg} \qquad P_{r3} := 330.9$$

Using the relative pressure at state 3, the relative pressure at state 4 may be found by

$$P_{r4} := \frac{P_{r3}}{r_p} = 41.362$$

Going to Table A-17 @ $P_{r4} = 41.362$ shows that interpolation is needed.

$$P_{ra} := 39.27 \qquad P_{rb} := 43.35$$

$$h_a := 778.18\frac{kJ}{kg} \qquad h_b := 800.03\frac{kJ}{kg} \qquad T_a := 760K \qquad T_b := 780K$$

$$h_4 := \frac{P_{r4} - P_{ra}}{P_{rb} - P_{ra}} \cdot (h_b - h_a) + h_a = 789.386 \cdot \frac{kJ}{kg} \qquad \boxed{T_4 := \frac{P_{r4} - P_{ra}}{P_{rb} - P_{ra}} \cdot (T_b - T_a) + T_a = 770.3\,K} \quad \text{a)}$$

Solution (contd.):

To determine the back work ratio, the specific work of the compressor and turbine need to be determined. This is shown below.

$$w_c := h_2 - h_1 = 243.99 \cdot \frac{kJ}{kg}$$

$$w_t := h_3 - h_4 = 606.584 \cdot \frac{kJ}{kg}$$

The back work ratio is then

$$\boxed{r_{bw} := \frac{w_c}{w_t} = 0.4022} \qquad \text{b)}$$

The thermal efficiency may then be found by

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{out} - w_{in}}{h_3 - h_2} = \frac{w_t - w_c}{h_3 - h_2} \qquad \text{or} \qquad \boxed{\eta_{th} := \frac{w_t - w_c}{h_3 - h_2} = 42.6 \cdot \%} \qquad \text{c)}$$

The thermal efficiency could have also been esitmated by the Brayton efficiency approximation with a specific heat ratio of 1.4 for air. This is shown below.

$k := 1.4$ (Table A-2(a) @ air)

$$\eta_{th,Brayton} := 1 - \frac{1}{r_p^{\frac{k-1}{k}}} = 44.8 \cdot \% \qquad \eta_{th} := 1 - \frac{T_4 - T_1}{T_3 - T_2} = 38.1 \cdot \%$$