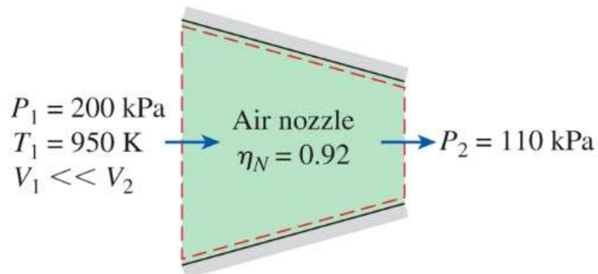
Given: $kJ := 1000J$

Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 110 kPa. Assume constant specific heat of air is 1.11 kJ/kgK and the specific heat ratio is 1.349.

$$P_1 := 200kPa \quad T_1 := 950K \quad P_2 := 110kPa \quad c_p := 1.11\frac{kJ}{kg\cdot K} \quad k := 1.349$$

Required:

If the isentropic efficiency of the nozzle is 92%, determine the maximum possible velocity, the exit temperature of the air, and the actual velocity of the air.



Solution:

The isentropic efficiency of the nozzle is defined as

$$\eta_N := 92\%$$

For an isentropic process the following is true

$$\left(\frac{T_2}{T_1}\right)_{s=const} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

Solving for the temperature at state 2 for an insentropic process is then

$$T_{2s} := T_1\cdot\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 813.9K$$

The <u>1st Law</u> for a nozzle is shown below when $V_2 >> V_1$ shows

$$\frac{d}{dt}E_{sys} = \Sigma E'_{in} - \Sigma E'_{out}$$

$$0 = m'\cdot\left(h_1 + \frac{V_1^{\,2}}{2}\right) - m'\cdot\left(h_2 + \frac{V_2^{\,2}}{2}\right)$$

$$h_1 = h_2 + \frac{V_2^{\,2}}{2}$$

Solving for the velocity at the outlet when the process is an isentropic process shows

$$V_{2s} = \sqrt{2\cdot(h_1 - h_{2s})}$$

Since the specific heat is constant, the maximum velocity at the exit can be expressed as

$$\boxed{V_{2s} := \sqrt{2\cdot c_p\cdot(T_1 - T_{2s})} = 549.7\frac{m}{s}}$$

Solution (contd.):

Starting with the definition of the isentropic efficiency of a nozzle

$$\eta_N = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Since the specific heat is constant, the issentropic efficiency of the nozzle can be expressed as

$$\eta_N = \frac{c_p \cdot (T_1 - T_{2a})}{c_p \cdot (T_1 - T_{2s})} = \frac{T_1 - T_{2a}}{T_1 - T_{2s}}$$

Solving for the actual temperature at the exit yields

$$\boxed{T_{2a} := T_1 - \eta_N \cdot (T_1 - T_{2s}) = 824.8\,\mathrm{K}}$$

The actual exit velocity may then be found by the alternate relation for the isentropic efficiency of a nozzle shown below.

$$\eta_N = \frac{V_{2a}^{\,2}}{V_{2s}^{\,2}}$$

Solving for the actual exit velocity yields

$$\boxed{V_{2a} := \sqrt{\eta_N \cdot V_{2s}^{\,2}} = 527.3\,\frac{\mathrm{m}}{\mathrm{s}}} \qquad V_{2a} := \sqrt{2 \cdot c_p \cdot (T_1 - T_{2a})} = 527.3\,\frac{\mathrm{m}}{\mathrm{s}}$$