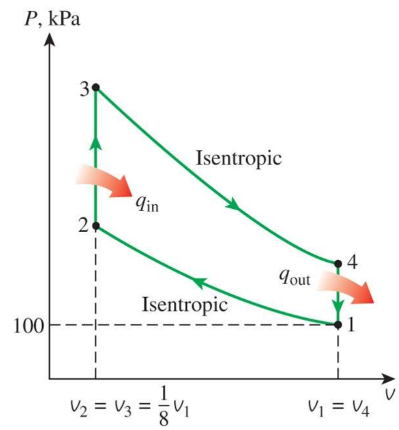
Given: $kJ := 1000J$ $cycle := 1$

An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 100 kPa and 17°C, and 800 kJ/kg of heat is transferred to the air during the constant volume heat addition process.

$r := 8$ $P_1 := 100kPa$ $T_1 := 17\,°C$ $q_{in} := 800\dfrac{kJ}{kg}$



Required:

Accounting for the variation of specific heats of air with temperature, determine (a) the maximum temperature and pressure that occur during the cycle, (b) the net work output, (c) the thermal efficiency, and (d) the mean effective pressure for the cycle. Also, (e) determine the power output from the cycle, in kW, for an engine speed of 4000 rpm (rev/min). Assume the cycle is operated on an engine that has four cylinders with a total displacement of 1.6 L.

Solution:

The angular speed of the engine and the number of revolutions per cycle is defined as

$\omega := 4000rpm$ $n_{rev} := 2\dfrac{rev}{cycle}$

The total displacement volume of the four cylinder engine is

$V_d := 1.6L$

Going to Table A-17 @ $T_1 = 290.15K$ shows that interpolation is needed.

$T_a := 290K$ $T_b := 295K$

$u_a := 206.91\dfrac{kJ}{kg}$ $u_b := 210.49\dfrac{kJ}{kg}$ $\nu_{ra} := 676.1$ $\nu_{rb} := 647.9$

$$u_1 := \frac{T_1 - T_a}{T_b - T_a}\left(u_b - u_a\right) + u_a = 207.017\cdot\frac{kJ}{kg} \qquad \nu_{r1} := \frac{T_1 - T_a}{T_b - T_a}\cdot\left(\nu_{rb} - \nu_{ra}\right) + \nu_{ra} = 675.3$$

The relative volume at state 2 is found by

$\nu_{r2} := \dfrac{1}{r}\cdot\nu_{r1} = 84.41$ since the inverse of r is v_2/v_1.

Going to Table A-17 @ $\nu_{r2} = 84.41$ shows that interpolation is needed.

$\nu_{ra} := 85.34$ $\nu_{rb} := 81.89$

$T_a := 650K$ $T_b := 660K$ $u_a := 481.01\dfrac{kJ}{kg}$ $u_b := 481.01\dfrac{kJ}{kg}$

$$T_2 := \frac{\nu_{r2} - \nu_{ra}}{\nu_{rb} - \nu_{ra}}\cdot\left(T_b - T_a\right) + T_a = 652.7K \qquad u_2 := \frac{\nu_{r2} - \nu_{ra}}{\nu_{rb} - \nu_{ra}}\cdot\left(u_b - u_a\right) + u_a = 481.01\cdot\frac{kJ}{kg}$$

The pressure at state 2 may be found by using the Ideal Gas Law.

$P_1\cdot\nu_1 = R\cdot T_1$ and $P_2\cdot\nu_2 = R\cdot T_2$

Solution (contd.):

Since the Ideal Gas constant remains the same, the following is true.

$$\frac{P_1 \cdot \nu_1}{T_1} = \frac{P_2 \cdot \nu_2}{T_2}$$

Rearranging yields

$$P_2 = P_1 \cdot \left(\frac{T_2}{T_1}\right) \cdot \left(\frac{\nu_1}{\nu_2}\right) = P_1 \cdot \left(\frac{T_2}{T_1}\right) \cdot r \qquad \text{or} \qquad P_2 := P_1 \cdot \left(\frac{T_2}{T_1}\right) \cdot r = 1800 \cdot \text{kPa}$$

The process from state 2 to 3 is a constant volume, heat addition process so the following is true.

$$q_{in} = u_3 - u_2 \qquad \text{or} \qquad u_3 := u_2 + q_{in} = 1281.0 \cdot \frac{\text{kJ}}{\text{kg}}$$

Going to Table A-17 @ $u_3 = 1281.0 \cdot \frac{\text{kJ}}{\text{kg}}$ shows that interpolation is needed.

$$u_a := 1279.65 \frac{\text{kJ}}{\text{kg}} \qquad u_b := 1298.30 \frac{\text{kJ}}{\text{kg}}$$

$$T_a := 1580\text{K} \qquad T_b := 1600\text{K} \qquad \nu_{ra} := 6.046 \qquad \nu_{rb} := 5.804$$

(a) $$\boxed{T_3 := \frac{u_3 - u_a}{u_b - u_a} \cdot (T_b - T_a) + T_a = 1581\,\text{K}} \qquad \nu_{r3} := \frac{u_3 - u_a}{u_b - u_a} \cdot (\nu_{rb} - \nu_{ra}) + \nu_{ra} = 6.028$$

Using the Ideal Gas Law again, the pressure at state 3 may be found. This is shown below.

(a) $$\boxed{P_3 := P_2 \cdot \left(\frac{T_3}{T_2}\right) \cdot (1) = 4.360 \cdot \text{MPa}}$$ since from 2 to 3 is a constant volume process.

The relative volume at state 4 is found by

$\nu_{r4} := \nu_{r3} \cdot r = 48.227$ since r is also v_4/v_3.

Going to Table A-17 @ $\nu_{r4} = 48.227$ shows that interpolation is needed.

$$\nu_{ra} := 48.08 \qquad \nu_{rb} := 44.84$$

$$T_a := 800\text{K} \qquad T_b := 820\text{K} \qquad u_a := 592.30 \frac{\text{kJ}}{\text{kg}} \qquad u_b := 608.59 \frac{\text{kJ}}{\text{kg}}$$

$$T_4 := \frac{\nu_{r4} - \nu_{ra}}{\nu_{rb} - \nu_{ra}} \cdot (T_b - T_a) + T_a = 799.1\,\text{K} \qquad u_4 := \frac{\nu_{r4} - \nu_{ra}}{\nu_{rb} - \nu_{ra}} \cdot (u_b - u_a) + u_a = 591.6 \cdot \frac{\text{kJ}}{\text{kg}}$$

The heat rejected by the cycle is given by

$$q_{out} := u_4 - u_1 = 384.5 \cdot \frac{\text{kJ}}{\text{kg}}$$

The specific net work output provided by the cycle is then given by

(b) $$\boxed{w_{net} := q_{in} - q_{out} = 415.5 \cdot \frac{\text{kJ}}{\text{kg}}}$$

Solution (contd.):

The thermal efficiency of the cycle is then given by

(c) $$\boxed{\eta_{th} := \frac{w_{net}}{q_{in}} = 51.9\cdot\%}$$

Going to Table A-2(a) @ air shows

$$R := 0.287\frac{kJ}{kg\cdot K} \qquad k := 1.4$$

If the cold air assumptions where utilizied, the thermal efficiency of the Otto would be given by

$$\eta_{th,Otto} := 1 - \frac{1}{r^{k-1}} = 56.5\cdot\%$$

The specific volume at state 1 may be found by using the Ideal Gas Law. This is shown below.

$$\nu_1 := \frac{R\cdot T_1}{P_1} = 0.833\frac{m^3}{kg}$$

Rearranging the mean effective pressure (MEP) equation yields

$$MEP = \frac{w_{net}}{\nu_1 - \nu_2} = \frac{w_{net}}{\nu_1 - \frac{\nu_1}{r}} = \frac{w_{net}}{\nu_1\cdot\left(1 - \frac{1}{r}\right)}$$

or

$$\boxed{MEP := \frac{w_{net}}{\nu_1\cdot\left(1 - \frac{1}{r}\right)} = 570.2\cdot kPa}$$ (d)

The total mass in all four cylinders is given by

$$m := \frac{V_d}{\nu_1} = 0.001921\,kg$$

The net work output per cycle is then

$$W_{net} := \frac{m}{cycle}\cdot w_{net} = 0.798\cdot\frac{kJ}{cycle}$$

The net rate of work output of the engine is then

(e) $$\boxed{W'_{net} := \frac{W_{net}\cdot\omega}{n_{rev}} = 26.61\cdot kW}$$