Given: $kJ := 1000J$

A piston cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is then compressed to 0.1 m^3 in such a way that the temperature of the air remains constant.

Required:

Determine the work done during the process.

Solution:

The initial volume, pressure, and temperature are defined below.

$$V_1 := 0.4m^3 \qquad P_1 := 100kPa \qquad T_0 := 80\,°C = 353.15\,K$$

The final volume is defined below.

$$V_2 := 0.1m^3$$

Beginning with the Ideal Gas Law (IGL) for a constant temperature process (i.e. isothermal process), the following is true.

$$PV = mRT \qquad \text{rearranging} \qquad P = \frac{mRT}{V} = \frac{¢}{V} \qquad \text{where } ¢ \text{ is a constant.}$$

Using this in the expression for boundary work shows

$$W_b = \int_1^2 P\,dV = \int_1^2 \frac{¢}{V}\,dV = ¢\cdot\int_1^2 \frac{1}{V}\,dV = ¢\cdot\left(\ln(V_2) - \ln(V_1)\right) = ¢\cdot\ln\left(\frac{V_2}{V_1}\right)$$

This constant ¢ may be found from

$$¢ = mRT = P_1\cdot V_1 = P_2\cdot V_2$$

Thus the boundary work may be expressed as

$$W_b = P_1\cdot V_1\cdot\ln\left(\frac{V_2}{V_1}\right) \qquad \text{or} \qquad W_b = P_2\cdot V_2\cdot\ln\left(\frac{V_2}{V_1}\right)$$

The ratio of the volumes may also be represented alternatively. This is shown below.

$$¢ = P_1\cdot V_1 = P_2\cdot V_2 \qquad \text{rearranging} \qquad \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

The boundary work expression could also be expressed as

$$W_b = P_1\cdot V_1\cdot\ln\left(\frac{P_1}{P_2}\right) \qquad \text{or} \qquad W_b = P_2\cdot V_2\cdot\ln\left(\frac{P_1}{P_2}\right)$$

All boundary work expressions are valid for the particular assumptions made for this system. To recap, those underlining assumptions are a closed isothermal system containing an ideal gas. Calculating the boundary work may now be done. This is shown below.

$$\boxed{W_b := P_1\cdot V_1\cdot\ln\left(\frac{V_2}{V_1}\right) = -55.45\cdot kJ}$$

Note: The boundary work is negative because work is being done to the system not by the system.