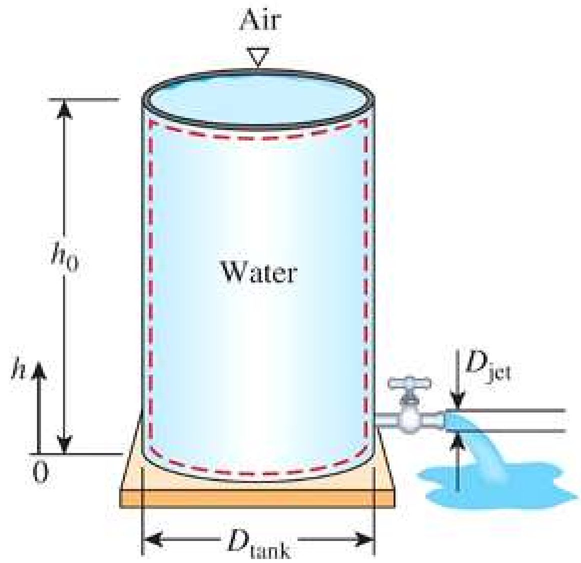
Given:

A 4 ft high, 3 ft diameter cylindrical water tank whose top is open to the atmosphere is being drained. The diameter of the water jet that streams out the bottom is 0.5 in.

$h_0 := 4ft \qquad D_{tank} := 3ft \qquad D_{jet} := 0.5in$



Required:

Determine the velocity of the water leaving the tank and the time it takes to drain half of the tank.

Solution:

If a particular particle that flows from the top surface (state 1) through the jet (state 2) is analyzed, the first law is

$$\Delta E_{sys} = \Sigma E_{in} - \Sigma E_{out}$$

Assuming there is no heat or work being done on that particular particle, and there is no change in internal energy, the first law becomes

$$\Delta KE + \Delta PE = 0 = m\cdot\left(\frac{V_2^{\,2} - V_1^{\,2}}{2}\right) + m\cdot g\cdot\left(z_2 - z_1\right)$$

The first law can be solved for V_2 as shown below.

$V_2 = \sqrt{2\cdot g\cdot z_1}$ This assumes that V_1 is negligible and z_2 is zero.

The velocity at the jet when the tank is full is then

$$\boxed{V_2 := \sqrt{2\cdot g\cdot h_0} = 4.89\,\frac{m}{s}}$$

The mass conservation equation may now be used on the entire body of water (the dashed line in the diagram)

$$\frac{d}{dt}m_{cv} = \Sigma m'_{in} - \Sigma m'_{out}$$

There is no mass entering the system, and the mass leaving the system is given by

$m'_{out} = \rho\cdot V'_{out} = \rho\cdot A\cdot V_{out} = \rho\cdot\frac{\pi}{4}\cdot D_{jet}^{\,2}\cdot\sqrt{2\cdot g\cdot z_1}$ where $\rho := 1000\frac{kg}{m^3}$ is the density of water.

The mass in the control volume is given by

$$m_{cv} = \rho\cdot V = \rho\cdot A\cdot H = \rho\cdot\frac{\pi}{4}\cdot D_{tank}^{\,2}\cdot z_1$$

The mass conservation equation is then

$$\frac{d}{dt}m_{cv} = -m'_{out} \qquad \text{or} \qquad \frac{d}{dt}\left(\rho\cdot\frac{\pi}{4}\cdot D_{tank}^{\,2}\cdot z_1\right) = -\rho\cdot\frac{\pi}{4}\cdot D_{jet}^{\,2}\cdot\sqrt{2\cdot g\cdot z_1}$$

Solution (cont.):

The density of water, and tank diameter remain constant so z_1 (the height of the water in the tank) is the only thing that is dependent on time. This is shown below.

$$\rho \cdot \frac{\pi}{4} \cdot D_{tank}^{2} \frac{d}{dt} z_1 = -\rho \cdot \frac{\pi}{4} \cdot D_{jet}^{2} \cdot \sqrt{2 \cdot g \cdot z_1}$$

Rearranging shows

$$dt = \frac{-D_{tank}^{2}}{D_{jet}^{2}} \cdot \frac{dz_1}{\sqrt{2g \cdot z_1}}$$

Integrating from 0 to t and 4 ft to 2 ft yield

$$\int_0^t 1 \, dt = \Delta t = \frac{-D_{tank}^{2}}{D_{jet}^{2} \cdot \sqrt{2g}} \cdot \int_{h_0}^{\frac{h_0}{2}} \frac{1}{\sqrt{z_1}} \, dz_1$$

$$\boxed{\Delta t := \frac{-D_{tank}^{2}}{D_{jet}^{2} \cdot \sqrt{2g}} \cdot \int_{h_0}^{\frac{h_0}{2}} \frac{1}{\sqrt{z_1}} \, dz_1 = 757.1 \, s}$$

$$\boxed{\Delta t = 12.6 \cdot min}$$