

Given: $\text{kJ} := 1000\text{J}$

The electrical heating systems used in many houses consist of a simple duct with resistance heaters. Air is heated as it flows over the resistance wires. Consider a 15 kW electric heating system where air enters at 100 kPa and 17°C with a flow rate of 150 m³/min.

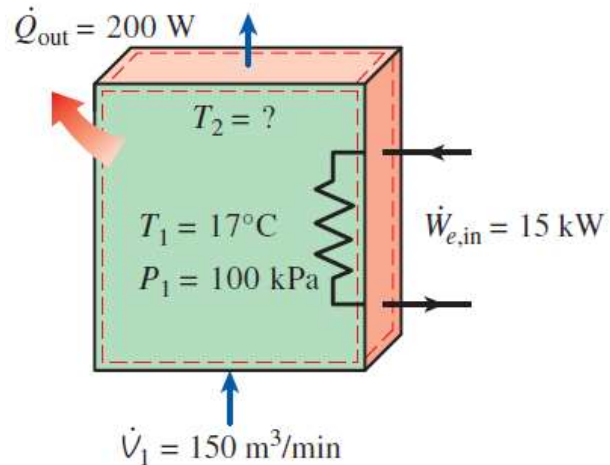
$$\dot{Q}'_{\text{out}} := 200\text{W}$$

$$T_1 := 17^\circ\text{C}$$

$$P_1 := 100\text{kPa}$$

$$\dot{W}'_{e,\text{in}} := 15\text{kW}$$

$$\dot{V}_1 := 150 \frac{\text{m}^3}{\text{min}}$$



Required:

If the rate of heat loss from the air duct to the surroundings is 200 W, determine the final temperature of the air.

Solution:

1st Law (for rigid, steady flow device with no changes in kinetic and potential energy)

$$E'_{\text{in}} = E'_{\text{out}}$$

$$\dot{W}'_{\text{in}} + \dot{m}' \cdot h_{\text{in}} = \dot{m}' \cdot h_{\text{out}} + \dot{Q}'_{\text{out}}$$

Rearranging yields

$$\dot{W}'_{\text{in}} - \dot{Q}'_{\text{out}} = \dot{m}' (h_{\text{out}} - h_{\text{in}})$$

Assuming the air behaves as an ideal gas in the process region and has a constant specific heat the first law becomes

$$\dot{W}'_{\text{in}} - \dot{Q}'_{\text{out}} = \dot{m}' \cdot c_p \cdot (T_{\text{out}} - T_{\text{in}})$$

Solving for the outlet temperature yields

$$T_{\text{out}} = \frac{\dot{W}'_{\text{in}} - \dot{Q}'_{\text{out}}}{\dot{m}' \cdot c_p} + T_{\text{in}}$$

Using the ideal gas law, the mass flow rate may be found by

$$\dot{m}' = \frac{P \cdot \dot{V}'}{R \cdot T}$$

Going to Table A-2(a) @ air shows

$$R := 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad c_p := 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Solution (cont.):

The mass flow rate is then

$$\dot{m} := \frac{P_1 \cdot V_1}{R \cdot T_1} = 3.002 \frac{\text{kg}}{\text{s}}$$

The outlet temperature is then

$$T_2 := \frac{\dot{W}_{e,\text{in}} - \dot{Q}_{\text{out}}}{\dot{m} \cdot c_p} + T_1 = 21.9 \cdot ^\circ\text{C}$$