Given: $kJ := 1000J$ $USD := 1$

A cryogenic manufacturer handles liquid methane at 115 K and 5 MPa at a rate of 0.280 m^3/s. The process involves dropping the pressure to 1 MPa by means of a throttling value. An engineer proposes to replace the throttling value with a turbine so power can be produced from the pressure drop.

$$T_{in} := 115K \qquad P_{in} := 5MPa \qquad V'_{in} := 0.280\frac{m^3}{s} \qquad P_{out} := 1MPa$$

Properties of Liquid Methane					
Temp T, K	Pressure P, MPa	Density T, kg/m³	Enthalpy h, kJ/kg	Entropy s, kJ/kg K	Specific Heat c_p, kJ/kg K
110	0.5	425.3	208.3	4.878	3.476
	1	425.8	209.0	4.875	3.471
	2	426.6	210.5	4.867	3.460
	5	429.1	215.0	4.844	3.432
120	0.5	410.4	243.4	5.185	3.551
	1	411.0	244.1	5.180	3.543
	2	412.0	245.4	5.171	3.528
	5	415.2	249.6	5.145	3.486

Required:

What is the maximum amount of power that can be produced by the turbine? Given that the turbine operates 8760 hr/yr and the cost of electricity is $0.075/kWhr, what is the maximum savings for the company if they use the turbine?

Solution:

The operating time is defined as

$$\Delta t := 8760\frac{hr}{yr}$$

The cost of electricity is defined as

$$C_e := 0.075\cdot\frac{USD}{kW\cdot hr}$$

<u>1st Law</u> for a steady flow turbine that is adiabatic, and has no ΔKE and ΔPE shows

$$\frac{d}{dt}E_{sys} = \Sigma E'_{in} - \Sigma E'_{out}$$

$$0 = m'_{in}\cdot\left(h_{in} + \frac{V_{in}^{\ 2}}{2} + g\cdot z_{in}\right) - m'_{out}\cdot\left(h_{out} + \frac{V_{out}^{\ 2}}{2} + g\cdot z_{out}\right) - W'_{out}$$

$$0 = m'_{in}\cdot\left(h_{in} - h_{out}\right) - W'_{out}$$

Knowing that the device only has one inlet and outlet the work output of the device becomes

$$W'_{out} = m'\cdot\left(h_{in} - h_{out}\right)$$

Solution (contd.):

Going to the given table @ $T_{in} = 115K$ & $P_{in} = 5 \cdot MPa$ shows that interpolation is needed.

$$T_a := 110 \cdot K \qquad T_b := 120 \cdot K$$

$$\rho_a := 429.1 \frac{kg}{m^3} \qquad \rho_b := 415.2 \frac{kg}{m^3} \qquad h_a := 215 \frac{kJ}{kg} \qquad h_b := 249.6 \frac{kJ}{kg}$$

$$\rho_{in} := \frac{T_{in} - T_a}{T_b - T_a} \cdot (\rho_b - \rho_a) + \rho_a = 422.15 \frac{kg}{m^3} \qquad h_{in} := \frac{T_{in} - T_a}{T_b - T_a} \cdot (h_b - h_a) + h_a = 232.3 \cdot \frac{kJ}{kg}$$

$$s_a := 4.844 \frac{kJ}{kg \cdot K} \qquad s_b := 5.145 \frac{kJ}{kg \cdot K}$$

$$s_{in} := \frac{T_{in} - T_a}{T_b - T_a} \cdot (s_b - s_a) + s_a = 4.995 \cdot \frac{kJ}{kg \cdot K}$$

The mass flow rate can then be found by

$$m' := \rho_{in} \cdot V'_{in} = 118.202 \frac{kg}{s}$$

Desiring an upper limit to what work can be produced by a turbine, let's assume that the turbine is not only adiabatic but also reversible. It has been shown that a turbine that is both adiabatic and reversible is also isentropic so

$$s_{out} := s_{in} = 4.995 \cdot \frac{kJ}{kg \cdot K}$$

Going to the given table @ $P_{out} = 1 \cdot MPa$ & $s_{out} = 4.995 \cdot \frac{kJ}{kg \cdot K}$ shows that interpolation is needed.

$$s_a := 4.875 \frac{kJ}{kg \cdot K} \qquad s_b := 5.180 \frac{kJ}{kg \cdot K}$$

$$h_a := 209.0 \frac{kJ}{kg} \qquad h_b := 244.1 \frac{kJ}{kg}$$

$$h_{out} := \frac{s_{out} - s_a}{s_b - s_a} \cdot (h_b - h_a) + h_a = 222.752 \cdot \frac{kJ}{kg}$$

The maximum possible work that could be produced by a turbine can then be found by

$$\boxed{W'_{out} := m' \cdot (h_{in} - h_{out}) = 1128.6 \cdot kW}$$

The maximum amount of savings per year is then given by

$$\boxed{Savings := C_e \cdot W'_{out} \cdot \Delta t = 741462.49 \cdot \frac{USD}{yr}}$$