

**ENGT 320 Applied Thermal Systems**  
**Quiz 5 Formula Sheet**

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$$m = \rho V$$

$$\dot{V} = \frac{V}{t}$$

$$q = mc_p \Delta T$$

$$q = -kA \frac{dT}{dt}$$

$$q = hA(T_s - T_\infty)$$

$$q = \varepsilon\sigma A(T_s^4 - T_{sur}^4)$$

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

$$\Delta E_{sys} = \Sigma E_{in} - \Sigma E_{out}$$

$$\frac{dE_{sys}}{dt} = \Sigma \dot{E}_{in} - \Sigma \dot{E}_{out}$$

$$q'' = \frac{q}{A}$$

$$\hat{q}'' = -k_x \frac{\partial T}{\partial x} \hat{i} - k_y \frac{\partial T}{\partial y} \hat{j} - k_z \frac{\partial T}{\partial z} \hat{k}$$

$$k = \frac{9\gamma-5}{4} \frac{c_v}{\pi d^2} \sqrt{\frac{M_w k_B T}{N_A \pi}}$$

$$k_B = 1.381 \times 10^{-23} \frac{J}{K}$$

$$N_A = 6.022 \times 10^{23}$$

$$\alpha = \frac{k}{\rho c_p}$$

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = 0$$

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\begin{aligned} & \frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \\ & \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = 0 \end{aligned}$$

$$R_{t,rad} = \frac{T_s - T_{sur}}{q} = \frac{1}{h_r A}$$

$$h_r = \varepsilon\sigma (T_s + T_{sur}) (T_s^2 + T_{sur}^2)$$

$$q = UA\Delta T$$

$$U = \frac{1}{R_{tot} A}$$

$$R_{tot} = \Sigma R_t = \frac{1}{UA}$$

$$R_{t,c}'' = \frac{T_A - T_B}{q''}$$

$$q = \frac{k_{eff} A}{L} (T_1 - T_2)$$

$$k_{eff,min} = \frac{1}{\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}}$$

$$k_{eff,max} = \varepsilon k_f + (1 - \varepsilon) k_s$$

$$R_{t,cond} = \frac{\ln \frac{r_2}{r_1}}{2\pi k L}$$

$$R_{t,conv} = \frac{1}{h 2\pi r L}$$

$$\begin{aligned} & \frac{d^2 T}{dx^2} + \left( \frac{1}{A_c} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \\ & \left( \frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_\infty) = 0 \end{aligned}$$

$$A_s = Px$$

$$\frac{d^2 T}{dx^2} - \left( \frac{hP}{kA_c} \right) (T - T_\infty)$$

$$\frac{d^2 \theta}{dx^2} - m^2 \theta = 0$$

$$\theta = T(x) - T_\infty$$

$$m^2 = \frac{hP}{kA_c}$$

$$P = 2w + 2t$$

$$A_c = wt$$

$$P = \pi D$$

$$A_c = \frac{\pi}{4} D^2$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta_b = T_b - T_\infty$$

$$M = \sqrt{hPKA_c} \theta_b$$

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b}$$

$$\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$$

$$R_{t,f} = \frac{\theta_b}{q_f}$$

$$R_{t,b} = \frac{1}{hA_{c,b}}$$

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}$$

$$\eta_f = \frac{q_f}{q_{max}} = \frac{q_f}{hA_f\theta_b}$$

$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{hA_t\theta_b}$$

$$A_t = NA_f + A_b$$

$$qt = N\eta_f hA_f\theta_b + hA_b\theta_b$$

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$$\eta_o = 1 - \frac{NA_f}{A_t} (1 - \eta_f)$$

$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{m,n} \approx \frac{\frac{\partial T}{\partial x} \Big|_{m+\frac{1}{2},n} - \frac{\partial T}{\partial x} \Big|_{m-\frac{1}{2},n}}{\Delta x}$$

$$\frac{\partial T}{\partial x} \Big|_{m+\frac{1}{2},n} \approx \frac{T \Big|_{m+1,n} - T \Big|_{m,n}}{\Delta x}$$

$$\frac{\partial T}{\partial x} \Big|_{m-\frac{1}{2},n} \approx \frac{T \Big|_{m,n} - T \Big|_{m-1,n}}{\Delta x}$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{m,n} \approx \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{(\Delta x)^2}$$

$$\frac{\partial^2 T}{\partial y^2} \Big|_{m,n} \approx \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{(\Delta y)^2}$$

$$\sum_{i=1}^4 q_{(i) \rightarrow (m,n)} + \dot{q} (\Delta x \cdot \Delta y \cdot 1) = 0$$

$$q_{(m-1,n) \rightarrow (m,n)} = \\ k (\Delta y \cdot 1) \frac{T_{m-1,n} - T_{m,n}}{\Delta x}$$

$$q_{(m+1,n) \rightarrow (m,n)} = \\ k (\Delta y \cdot 1) \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

$$q_{(m,n+1) \rightarrow (m,n)} = \\ k (\Delta x \cdot 1) \frac{T_{m,n+1} - T_{m,n}}{\Delta y}$$

$$q_{(m,n-1) \rightarrow (m,n)} = \\ k (\Delta x \cdot 1) \frac{T_{m,n-1} - T_{m,n}}{\Delta y}$$

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + \\ T_{m-1,n} + \frac{\dot{q} \cdot 1 \cdot (\Delta x)^2}{k} - 4T_{m,n} = 0$$

$$AT = C$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \dots & a_{NN} \end{bmatrix}$$

$$T = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix}$$

$$C = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{bmatrix}$$

$$T = A^{-1}C$$

$$\frac{u(y)}{u_\infty} = 0.99$$

$$C_f = \frac{\tau_s}{\rho u_\infty^2 / 2}$$

$$\tau_s = \mu \frac{\partial u}{\partial y} \Big|_{y=0}$$

$$\frac{T_s - T(y)}{T_s - T_\infty} = 0.99$$

$$q_s'' = -k \frac{\partial T}{\partial y} \Big|_{y=0}$$

$$q_s'' = h (T_s - T_\infty)$$

$$h = \frac{-k \frac{\partial T}{\partial y} \Big|_{y=0}}{T_s - T_\infty}$$

$$q = \int_{A_s} q'' dA_s = (T_s - T_\infty) \int_{A_s} h dA_s$$

$$\bar{q}'' = \bar{h} (T_s - T_\infty)$$

$$\bar{q} = \bar{q}'' A_s = \bar{h} A_s (T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h dA_s$$

$$\bar{h} = \frac{1}{L} \int_0^L h dx$$

$$Re_x = \frac{\rho u_\infty x}{\mu}$$

$$x^* = \frac{x}{L}$$

$$y^* = \frac{y}{L}$$

$$u^* = \frac{u}{V}$$

$$v^* = \frac{v}{V}$$

$$T^* = \frac{T - T_s}{T_\infty - T_s}$$

$$p^* = \frac{p_\infty}{\rho V^2}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = - \frac{dp^*}{dx^*} + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$Re_L = \frac{VL}{\nu}$$

$$\nu = \frac{\mu}{\rho}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$Pr = \frac{\nu}{\alpha}$$

$$Nu = \frac{hL}{k} = \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}$$

$$\overline{Nu} = \frac{\bar{h}L}{k}$$

$$T_f = \frac{T_s + T_\infty}{2}$$

$$\delta = \frac{5}{\sqrt{\frac{u_\infty}{\nu x}}} = \frac{5x}{\sqrt{Re_x}}$$

$$C_{f,x} = \frac{\tau_{s,x}}{\frac{\rho u_\infty^2}{2}} = 0.664 Re_x^{-1/2}$$

$$Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3}$$

( $Pr \gtrsim 0.6$ )

$$\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$$

$$\overline{Nu}_x = \frac{\bar{h}_x x}{k} = 0.664 Re_x^{1/2} Pr^{1/3}$$

( $Pr \gtrsim 0.6$ )

Turbulent:

$$\delta = 0.37 x Re_x^{-1/2}$$

$$C_{f,x} = 0.0592 Re_x^{-1/5}$$

$$(Re_{x,c} \lesssim Re_x \lesssim 10^8)$$

$$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$$

$$(0.6 \lesssim Pr \lesssim 60)$$

$$A = 0.037 Re_{x,c}^{4/5} - 0.664 Re_{x,c}^{1/2}$$

$$\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - \frac{2A}{Re_L}$$

$$(Re_{x,c} \lesssim Re_L \lesssim 10^8)$$

$$\overline{Nu}_L = \left( 0.037 Re_L^{4/5} - A \right) Pr^{1/3}$$

$$(0.6 \lesssim Pr \lesssim 60)$$

$$(Re_{x,c} \lesssim Re_L \lesssim 10^8)$$

$$Re_D = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

$$C_D = \frac{F_D}{A_f \left( \frac{\rho V^2}{2} \right)}$$

Cylinder

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C Re_D^m Pr^{1/3}$$

$$(Pr \gtrsim 0.7)$$

See Table 5

**Isothermal Flat Plate**

$$\frac{\delta}{\delta_t} \approx Pr^{1/3}$$

Laminar:

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$$\dots \dots \dots$$

**Various Geometries**

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C Re_D^m Pr^{1/3}$$

$$(Pr \gtrsim 0.7)$$

See Table 6

$$\dots \dots \dots$$

**Sphere**

$$A_s = \pi D^2$$

$$C_D = \frac{25}{Re_D}$$

$$(Re_D \lesssim 0.5)$$

$$\overline{Nu}_D = 2 + \left( 0.4 Re_D^{1/2} + 0.06 Re_D^{2/3} \right) Pr^{0.4} \left( \frac{\mu}{\mu_s} \right)^{1/4}$$

$$(0.71 \lesssim Pr \lesssim 380)$$

$$(3.5 \lesssim Re_D \lesssim 7.6 \times 10^4)$$

$$\left( 1.0 \lesssim \frac{\mu}{\mu_s} \lesssim 3.2 \right)$$

$$m = \rho u_m A_c$$

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$$

$$\left( \frac{x_{fd,t}}{D} \right) \approx 0.05 Re_D Pr$$

$$f = \frac{-\frac{dP}{dx} D}{\rho \frac{u_m^2}{2}}$$

$$f = \frac{64}{Re_D}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log \left[ \frac{e/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right]$$

$$dq_{conv} = mc_p [(T_m + dT_m) - T_m] =$$

$$mc_p dT_m$$

$$dq_{conv} = q''_s P dx$$

$$\frac{dT_m}{dx} = \frac{q''_s P}{mc_p} = \frac{P}{mc_p} h (T_s - T_m)$$

$$T_m(x) = T_{m,i} + \frac{q''_s P}{mc_p} x$$

$$\Delta T = T_s - T_m$$

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{mc_p} h \Delta T$$

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = e^{-\frac{P x}{mc_p} \bar{h}}$$

$$\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln \left( \frac{\Delta T_o}{\Delta T_i} \right)}$$

$$\Delta T_o = T_s - T_{m,o}$$

$$\Delta T_i = T_s - T_{m,i}$$

$$q_{conv} = \bar{h} A_s \Delta T_{lm}$$

$$q = m_h (i_{h,i} - i_{h,o})$$

$$q = m_c (i_{c,i} - i_{c,o})$$

$$q = m_h c_{p,h} (T_{h,i} - T_{h,o})$$

$$q = m_c c_{p,c} (T_{c,o} - T_{c,i})$$

$$q = UA \Delta T_m$$

$$q = UA \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$$

$$\Delta T_1 = T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i}$$

$$\Delta T_2 = T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o}$$

$$\Delta T_1 = T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o}$$

$$\Delta T_2 = T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i}$$

$$q_{max} = C_{min} (T_{h,i} - T_{c,i})$$

$$\varepsilon = \frac{C_h (T_{h,i} - T_{h,o})}{C_{min} (T_{h,i} - T_{c,i})}$$

$$\varepsilon = \frac{C_c (T_{c,o} - T_{c,i})}{C_{min} (T_{h,i} - T_{c,i})}$$

$$q = \varepsilon C_{min} (T_{h,i} - T_{c,i})$$

$$NTU = \frac{UA}{C_{min}}$$

$$\lambda = \frac{c}{\nu}$$

$$c_o = 2.998 \times 10^8 \frac{m}{s}$$

$$E = \varepsilon \sigma T^4$$

$$J = E + \rho G$$

$$q''_{rad} = J - G$$

$$q''_{rad} = \varepsilon \sigma T^4 - \alpha G$$

$$G = G_{ref} + G_{abs} + G_{tr}$$

$$\rho + \alpha + \tau = 1$$

$$\rho + \alpha = 1$$

$$d\omega = \frac{dA_n}{r^2}$$

$$dA_n = r^2 \sin \theta d\theta d\phi$$

$$d\omega = \sin \theta d\theta d\phi$$

$$\omega_{hemi} = \int_0^{2\pi} \int_0^{\pi/2} \sin \theta d\theta d\phi = 2\pi sr$$

$$I_{\lambda,e} (\lambda, \theta, \phi) = \frac{dq}{(dA_1 \cos \theta) d\omega d\lambda}$$

$$dq_\lambda = \frac{dq}{d\lambda} = I_{\lambda,e} (\lambda, \theta, \phi) dA_1 \cos \theta d\omega$$

$$dq''_\lambda = I_{\lambda,e} (\lambda, \theta, \phi) \cos \theta d\omega =$$

$$I_{\lambda,e} (\lambda, \theta, \phi) \cos \theta \sin \theta d\omega$$

**Noncircular Tubes**

$$D_h = \frac{4A_c}{P}$$

$$Nu_D = \frac{h D_h}{k}$$

See Table 7

$$\dots \dots \dots$$

$$\frac{1}{UA} = \frac{1}{U_c A_c} = \frac{1}{U_h A_h} =$$

$$\frac{1}{(hA)_c} + R_w + \frac{1}{(hA)_h}$$

$$\frac{1}{UA} = \frac{1}{(\eta_o h A)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w +$$

$$\frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o h A)_h}$$

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$$E_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi) \cos \theta \sin \theta d\theta d\phi \quad F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_j dA_i$$

$$F_{ji} = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_j dA_i$$

$$A_i F_{ij} = A_j F_{ji}$$

$$E = \int_0^\infty E_\lambda(\lambda) d\lambda \quad \sum_{j=1}^N F_{ij} = 1$$

$$E_\lambda(\lambda) = \pi I_{\lambda,e}(\lambda)$$

$$E = \pi I_e \quad q_{ij} = q_{i \rightarrow j} - q_{j \rightarrow i}$$

$$q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj}$$

$$q_{ij} = A_i F_{ij} \sigma(T_i^4 - T_j^4)$$

$$\int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,i}(\lambda, \theta, \phi) \cos \theta \sin \theta d\theta d\phi \quad q_i = \sum_{j=1}^N A_i F_{ij} \sigma(T_i^4 - T_j^4)$$

$$q_i = A_i (J_i - G_i)$$

$$G = \int_0^\infty G_\lambda(\lambda) d\lambda \quad q_i = A_i (E_i - \alpha_i G_i)$$

$$q_i = \frac{E_{bi} - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i}$$

$$J_\lambda(\lambda) =$$

$$\int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e+r}(\lambda, \theta, \phi) \cos \theta \sin \theta d\theta d\phi$$

$$J = \int_0^\infty J_\lambda(\lambda) d\lambda$$

$$q_i = \sum_{j=1}^N A_i F_{ij} (J_i - J_j) = \sum_{j=1}^N \frac{J_i - J_j}{(A_i F_{ij})^{-1}}$$

$$E_{\lambda,b}(\lambda, T) = \pi I_{\lambda,T}(\lambda, T) =$$

$$\frac{C_1}{\lambda^5 [e^{(C_2/\lambda T)} - 1]} \quad q_1 = -q_2 = q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

$$C_1 = 3.742 \times 10^8 W \mu m^4 / m^2$$

$$C_2 = 1.439 \times 10^4 \mu mK$$

$$\lambda_{max} T = C_3 = 2898 \mu mK$$

$$E_b = \pi I_b = \int_0^\infty E_{\lambda,b} d\lambda = \sigma T^4$$

$$q_1 = -q_2 = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12} + [(1/A_1 F_{1R}) + (1/A_2 F_{2R})]^{-1}} + \frac{1-\varepsilon_1}{\varepsilon_1 A_1}}$$

$$\frac{J_1 - J_R}{1/A_1 F_{1R}} = \frac{J_R - J_2}{1/A_2 F_{2R}} \\ T_R = \left( \frac{J_R}{\sigma} \right)^{1/4}$$

$$F_{(\lambda_1 - \lambda_2)} = F_{0 - \lambda_2} - F_{0 - \lambda_1} =$$

$$\frac{\int_0^{\lambda_2} E_{\lambda,b} d\lambda - \int_0^{\lambda_1} E_{\lambda,b} d\lambda}{\sigma T^4}$$

$$q_{i,ext} = q_{i,rad} + q_{i,conv} + q_{i,cond}$$

$$F_{(0 - \lambda)} = \frac{\int_0^{\lambda} E_{\lambda,b} d\lambda}{\sigma T^4}$$

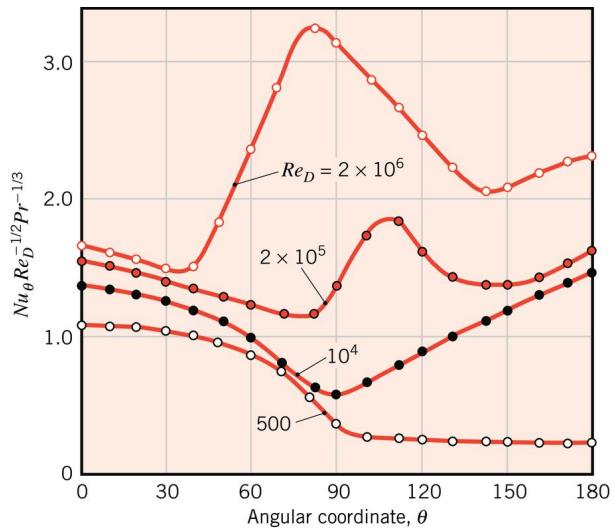
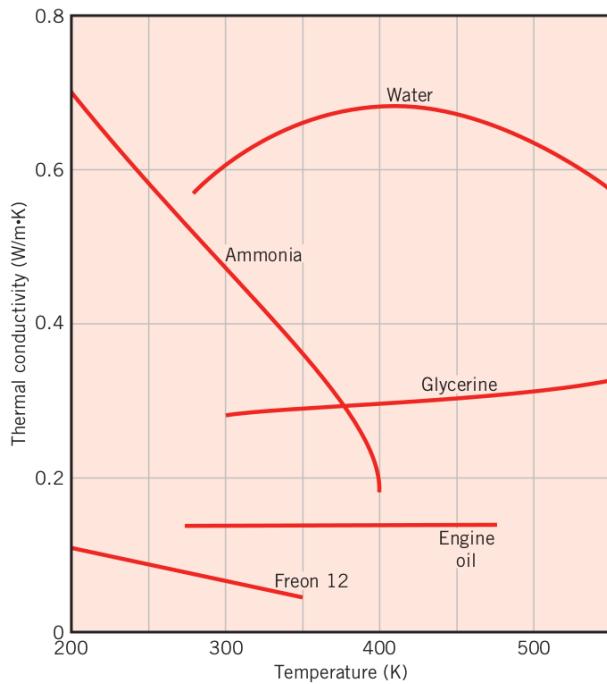
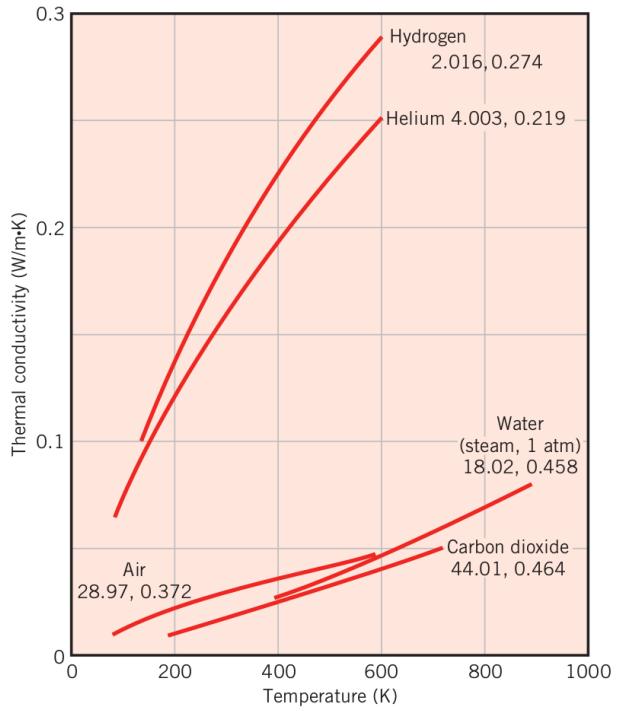
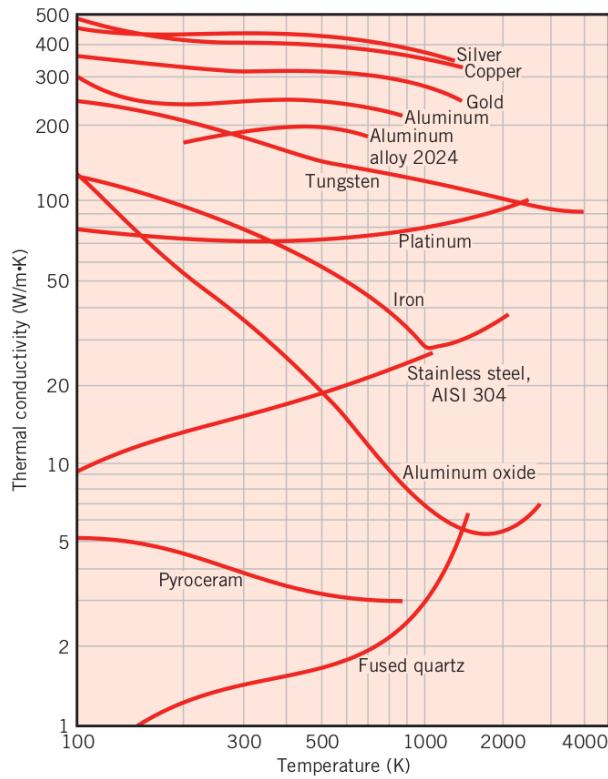
Lecture Note 14 formulas not needed

$$F_{ij} = \frac{q_{i \rightarrow j}}{A_i J_i}$$

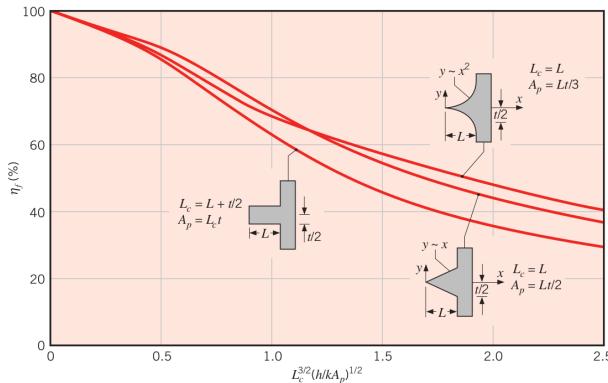
$$dq_{i \rightarrow j} = I_i \cos \theta_i dA_i d\omega_{j-i} =$$

$$J_i \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_j dA_i$$

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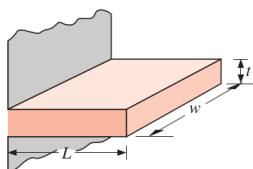
**Straight Fins**

*Rectangular*

$$A_f = 2wL_c$$

$$L_c = L + (t/2)$$

$$A_p = tL$$

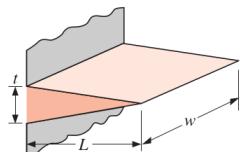


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

*Triangular*

$$A_f = 2w[L^2 + (t/2)^2]^{1/2}$$

$$A_p = (t/2)L$$



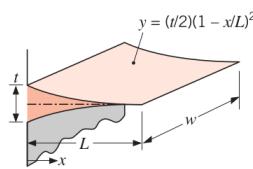
$$\eta_f = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$$

*Parabolic*

$$A_f = w[C_1L + (L^2/t)\ln(t/L + C_1)]$$

$$C_1 = [1 + (t/L)^2]^{1/2}$$

$$A_p = (t/3)L$$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

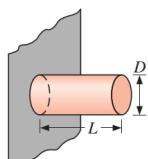
**Pin Fins**

*Rectangular*

$$A_f = \pi DL_c$$

$$L_c = L + (D/4)$$

$$V = (\pi D^2/4)L$$

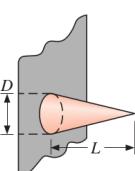


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

*Triangular*

$$A_f = \frac{\pi D}{2} [L^2 + (D/2)^2]^{1/2}$$

$$V = (\pi/12)D^2L$$



$$\eta_f = \frac{2}{mL} \frac{I_2(2mL)}{I_1(2mL)}$$

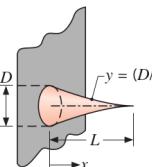
*Parabolic*

$$A_f = \frac{\pi L^3}{8D} \{C_3 C_4 - \frac{L}{2D} \ln[(2DC_4/L) + C_3]\}$$

$$C_3 = 1 + 2(D/L)^2$$

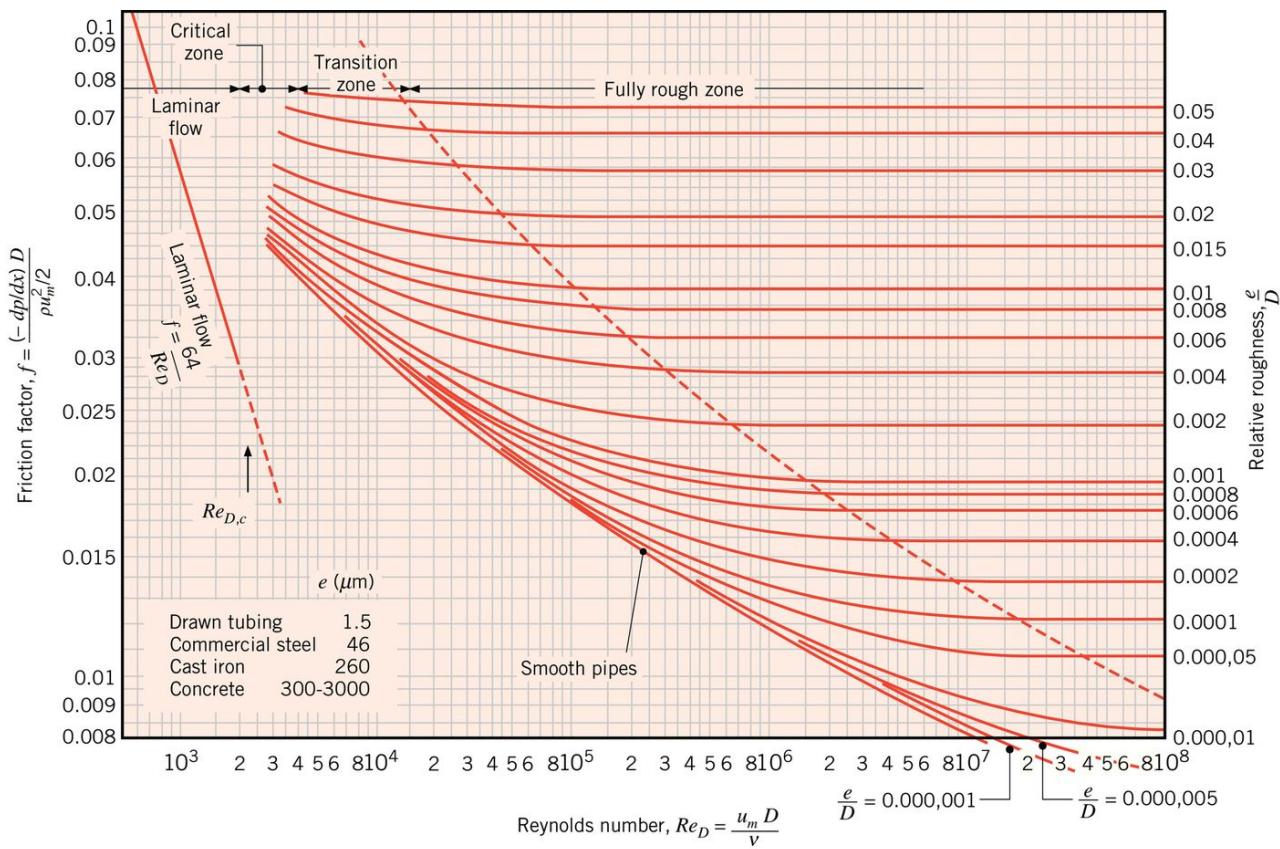
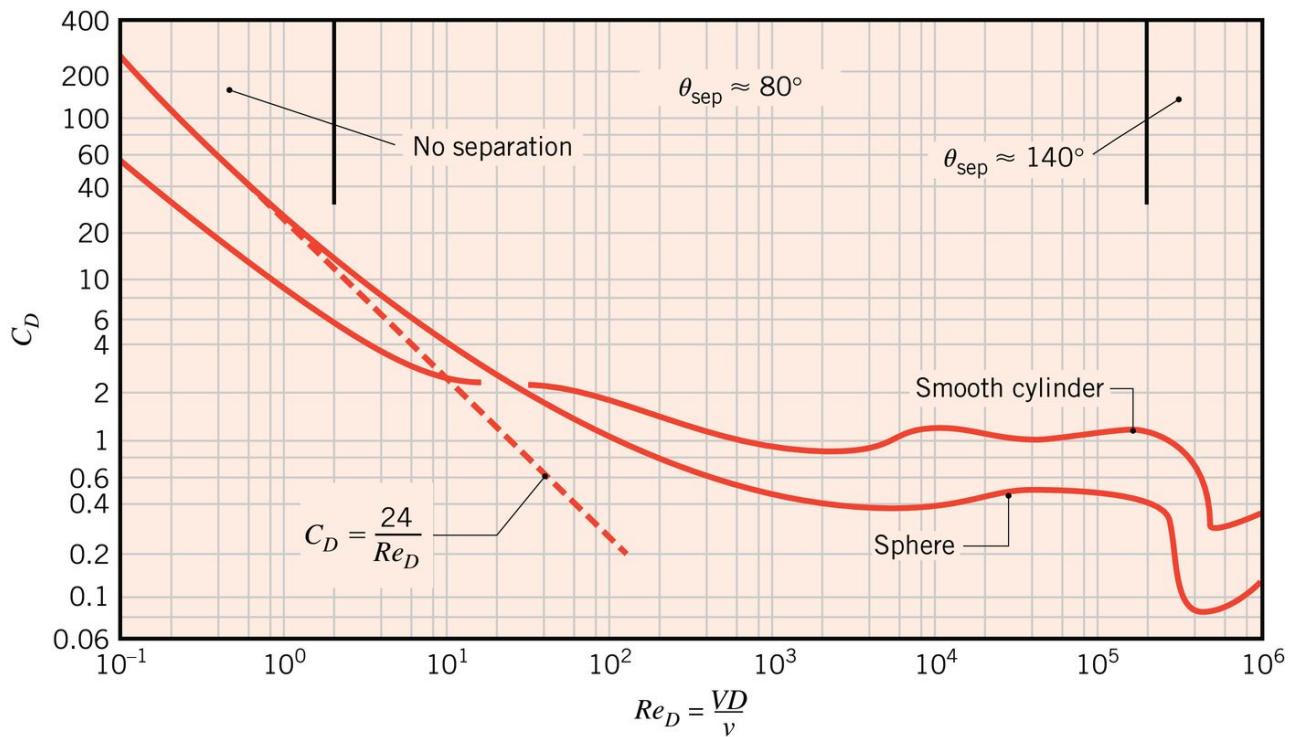
$$C_4 = [1 + (D/L)^2]^{1/2}$$

$$V = (\pi/20)D^2 L$$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$$

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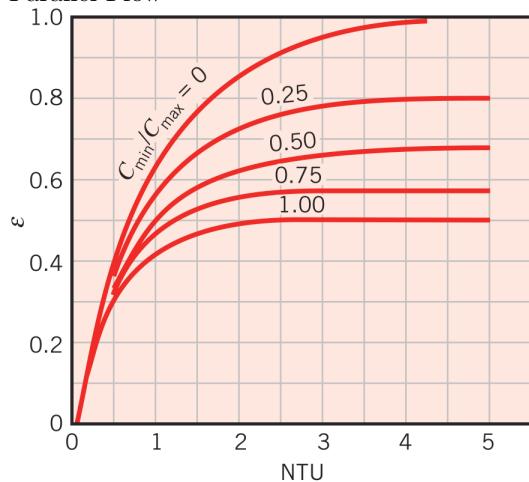
Flow Arrangement	Relation
<b>Parallel ow</b>	$\varepsilon = \frac{1 - \exp[-\text{NTU}(1 + C_r)]}{1 + C_r}$
<b>Counterow</b>	$\varepsilon = \frac{1 - \exp[-\text{NTU}(1 - C_r)]}{1 - C_r \exp[-\text{NTU}(1 - C_r)]} \quad (C_r < 1)$ $\varepsilon = \frac{\text{NTU}}{1 + \text{NTU}} \quad (C_r = 1)$
<b>Shell-and-tube</b>	
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2 \left\{ 1 + C_r + (1 + C_r^2)^{1/2} \times \frac{1 + \exp[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\text{NTU})_1(1 + C_r^2)^{1/2}]} \right\}^{-1}$
$n$ shell passes ( $2n, 4n, \dots$ tube passes)	$\varepsilon = \left[ \left( \frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - 1 \right] \left[ \left( \frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - C_r \right]^{-1}$
<b>Cross-ow (single pass)</b>	
Both fluids unmixed	$\varepsilon = 1 - \exp \left[ \left( \frac{1}{C_r} \right) (\text{NTU})^{0.22} \{ \exp[-C_r(\text{NTU})^{0.78}] - 1 \} \right]$
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\varepsilon = \left( \frac{1}{C_r} \right) (1 - \exp \{ -C_r [1 - \exp(-\text{NTU})] \})$
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1} \{ 1 - \exp[-C_r(\text{NTU})] \})$
All exchangers ( $C_r = 0$ )	$\varepsilon = 1 - \exp(-\text{NTU})$

Flow Arrangement	Relation
<b>Parallel ow</b>	$\text{NTU} = -\frac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$
<b>Counterow</b>	$\text{NTU} = \frac{1}{C_r - 1} \ln \left( \frac{\varepsilon - 1}{\varepsilon C_r - 1} \right) \quad (C_r < 1)$ $\text{NTU} = \frac{\varepsilon}{1 - \varepsilon} \quad (C_r = 1)$
<b>Shell-and-tube</b>	
One shell pass (2, 4, . . . tube passes)	$(\text{NTU})_1 = -(1 + C_r^2)^{-1/2} \ln \left( \frac{E - 1}{E + 1} \right)$ $E = \frac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$
$n$ shell passes ( $2n, 4n, \dots$ tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \frac{F - 1}{F - C_r} \quad F = \left( \frac{\varepsilon C_r - 1}{\varepsilon - 1} \right)^{1/n} \quad \text{NTU} = n(\text{NTU})_1$
<b>Cross-ow (single pass)</b>	
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\text{NTU} = -\ln \left[ 1 + \left( \frac{1}{C_r} \right) \ln(1 - \varepsilon C_r) \right]$
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\text{NTU} = -\left( \frac{1}{C_r} \right) \ln[C_r \ln(1 - \varepsilon) + 1]$
All exchangers ( $C_r = 0$ )	$\text{NTU} = -\ln(1 - \varepsilon)$

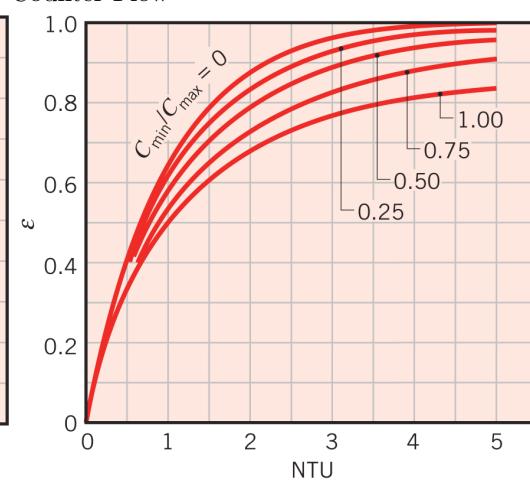
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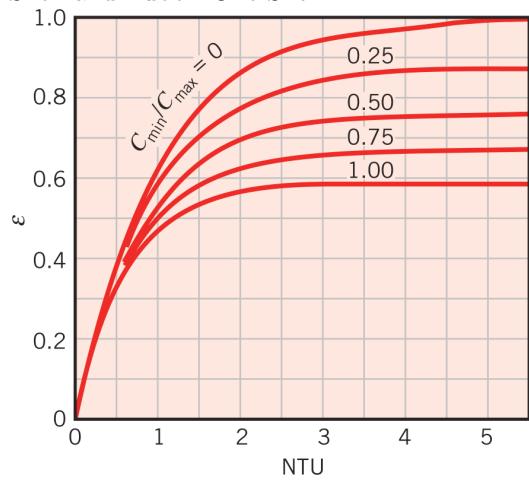
Parallel Flow



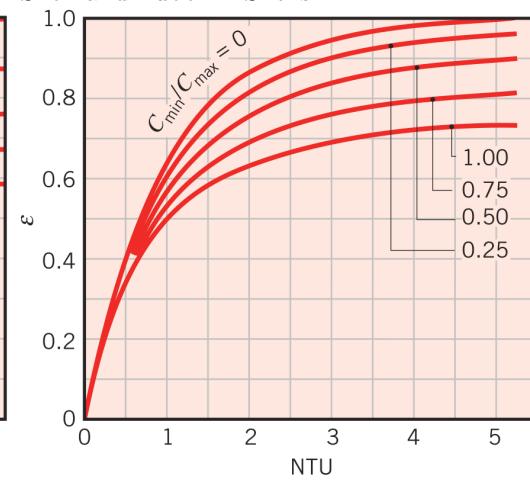
Counter Flow



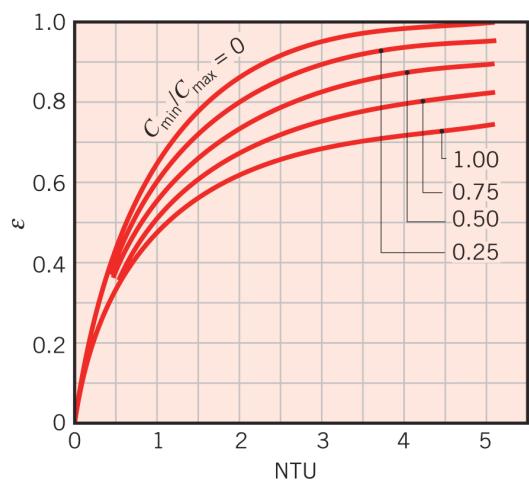
Shell and Tube - One Shell



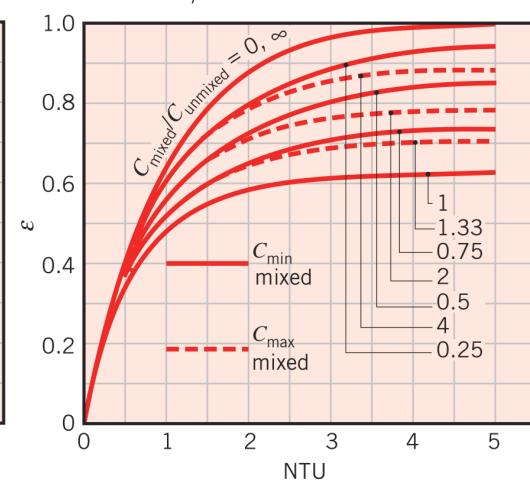
Shell and Tube - 2 Shells



Cross-flow Both Unmixed

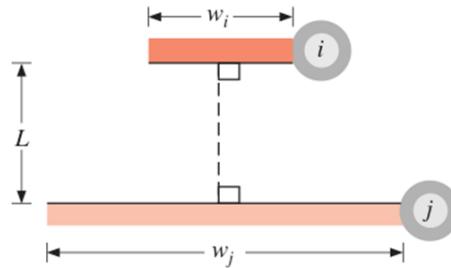


Cross-flow One Mixed/One Unmixed



### Geometry

#### Parallel Plates with Midlines Connected by Perpendicular

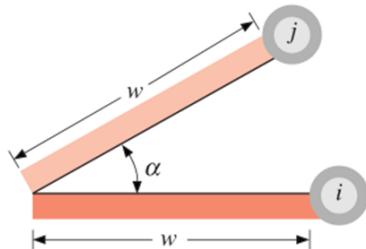


### Relation

$$F_{ij} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$$

$$W_i = w_i/L, W_j = w_j/L$$

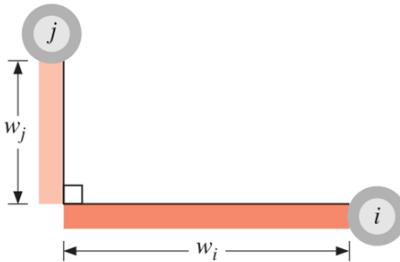
#### Inclined Parallel Plates of Equal Width and a Common Edge



$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$

### Geometry

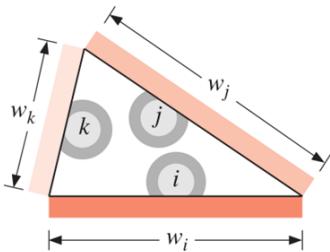
#### Perpendicular Plates with a Common Edge



### Relation

$$F_{ij} = \frac{1 + (w_j/w_i) - [1 + (w_j/w_i)^2]^{1/2}}{2}$$

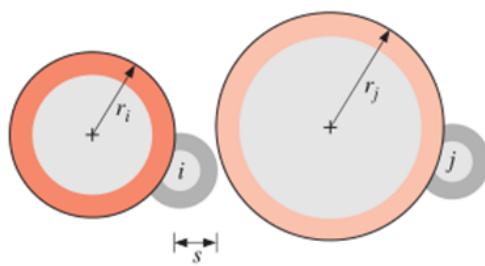
#### Three-Sided Enclosure



$$F_{ij} = \frac{w_i + w_j - w_k}{2w_i}$$

### Geometry

#### Parallel Cylinders of Different Radii



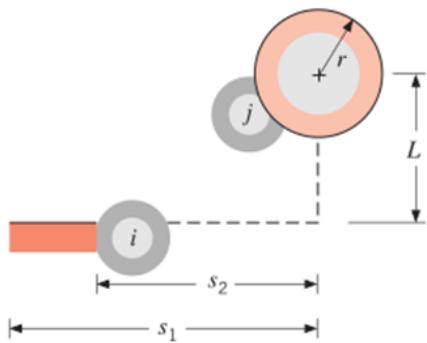
### Relation

$$F_{ij} = \frac{1}{2\pi} \left\{ \pi + [C^2 - (R + 1)^2]^{1/2} - [C^2 - (R - 1)^2]^{1/2} + (R - 1) \cos^{-1} \left[ \left( \frac{R}{C} \right) - \left( \frac{1}{C} \right) \right] - (R + 1) \cos^{-1} \left[ \left( \frac{R}{C} \right) + \left( \frac{1}{C} \right) \right] \right\}$$

$$R = r_j/r_i, S = s/r_i$$

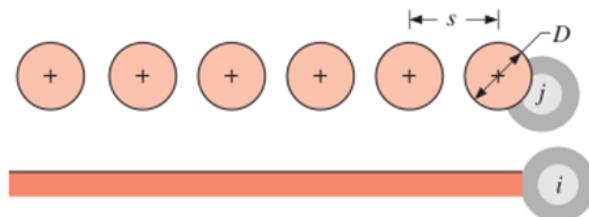
$$C = 1 + R + S$$

#### Cylinder and Parallel Rectangle



$$F_{ij} = \frac{r}{s_1 - s_2} \left[ \tan^{-1} \frac{s_1}{L} - \tan^{-1} \frac{s_2}{L} \right]$$

#### Infinite Plane and Row of Cylinders

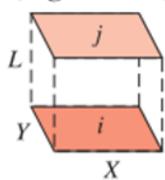


$$F_{ij} = 1 - \left[ 1 - \left( \frac{D}{s} \right)^2 \right]^{1/2} + \left( \frac{D}{s} \right) \tan^{-1} \left[ \left( \frac{s^2 - D^2}{D^2} \right)^{1/2} \right]$$

### Geometry

#### Aligned Parallel Rectangles

(Figure 13.4)

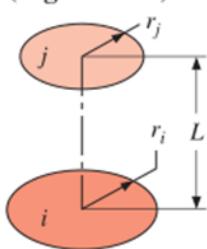


$$\bar{X} = X/L, \bar{Y} = Y/L$$

$$F_{ij} = \frac{2}{\pi \bar{X} \bar{Y}} \left\{ \ln \left[ \frac{(1 + \bar{X}^2)(1 + \bar{Y}^2)}{1 + \bar{X}^2 + \bar{Y}^2} \right]^{1/2} \right. \\ \left. + \bar{X}(1 + \bar{Y}^2)^{1/2} \tan^{-1} \frac{\bar{X}}{(1 + \bar{Y}^2)^{1/2}} \right. \\ \left. + \bar{Y}(1 + \bar{X}^2)^{1/2} \tan^{-1} \frac{\bar{Y}}{(1 + \bar{X}^2)^{1/2}} - \bar{X} \tan^{-1} \bar{X} - \bar{Y} \tan^{-1} \bar{Y} \right\}$$

#### Coaxial Parallel Disks

(Figure 13.5)



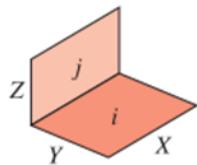
$$R_i = r_i/L, R_j = r_j/L$$

$$S = 1 + \frac{1 + R_j^2}{R_i^2}$$

$$F_{ij} = \frac{1}{2} \{ S - [S^2 - 4(r_j/r_i)^2]^{1/2} \}$$

#### Perpendicular Rectangles with a Common Edge

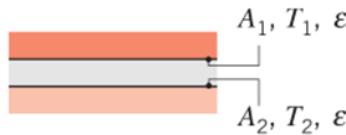
(Figure 13.6)



$$H = Z/X, W = Y/X$$

$$F_{ij} = \frac{1}{\pi W} \left( W \tan^{-1} \frac{1}{W} + H \tan^{-1} \frac{1}{H} \right. \\ \left. - (H^2 + W^2)^{1/2} \tan^{-1} \frac{1}{(H^2 + W^2)^{1/2}} \right. \\ \left. + \frac{1}{4} \ln \left\{ \frac{(1 + W^2)(1 + H^2)}{1 + W^2 + H^2} \left[ \frac{W^2(1 + W^2 + H^2)}{(1 + W^2)(W^2 + H^2)} \right]^{W^2} \right. \right. \\ \left. \left. \times \left[ \frac{H^2(1 + H^2 + W^2)}{(1 + H^2)(H^2 + W^2)} \right]^{H^2} \right\} \right)$$

### Large (Infinite) Parallel Planes

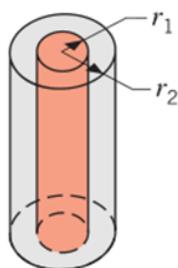


$$A_1 = A_2 = A$$

$$F_{12} = 1$$

$$q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

### Long (Infinite) Concentric Cylinders

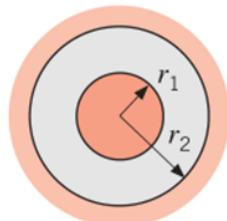


$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \left( \frac{r_1}{r_2} \right)}$$

### Concentric Spheres

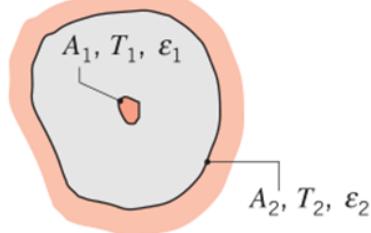


$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \left( \frac{r_1}{r_2} \right)^2}$$

### Small Convex Object in a Large Cavity



$$\frac{A_1}{A_2} \approx 0$$

$$F_{12} = 1$$

$$q_{12} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

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Table 1: Contact Resistance for vacuum interfaces,  $R''_{t,c} \times 10^4 \left( \frac{m^2 K}{W} \right)$

Material	100 (kPa)	10,000 (kPa)
Stainless Steel	6-25	0.7-4.0
Copper	1-10	0.1-0.5
Magnesium	1.5-3.5	0.2-0.4
Aluminum	1.5-5.0	0.2-0.4

Table 2: Contact Resistance for aluminum ( $10\mu m$  surface roughness,  $10kPa$  contact pressure)

Interfacial Fluid	$R''_{t,c} \times 10^4 \left( \frac{m^2 K}{W} \right)$
Air	2.75
Helium	1.05
Hydrogen	0.720
Silicone Oil	0.525
Glycerine	0.265

Table 3: Fin Tip Conditions

Case	Tip Condition	Temperature Distribution $\frac{\theta}{\theta_b}$	Fin Heat Transfer Rate $q_f$
A	Convection	$\frac{\cosh m(L-x) + \left(\frac{h}{mk}\right) \sinh m(L-x)}{\cosh mL + \left(\frac{h}{mk}\right) \sinh mL}$	$M \frac{\sinh mL + \left(\frac{h}{mk}\right) \cosh mL}{\cosh mL + \left(\frac{h}{mk}\right) \sinh mL}$
B	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Constant Temperature	$\frac{\left(\frac{\theta_L}{\theta_b}\right) \sinh mx + \sinh m(L-x)}{\sinh mL}$	$M \frac{\cosh mL - \frac{\theta_L}{\theta_b}}{\sinh mL}$
D	Infinite Fin ( $L \rightarrow \infty$ )	$e^{-mx}$	$M$

Table 4: Energy Balance Method Case Summary

Case	Diagram	Equation
1		$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0$
2		$2(T_{m-1,n} - T_{m,n+1}) + (T_{m+1,n} + T_{m,n-1}) + 2 \frac{h\Delta x}{k} T_{\infty} - 2(3 + \frac{h\Delta x}{k}) T_{m,n} = 0$
3		$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + 2 \frac{h\Delta x}{k} T_{\infty} - 2(2 + \frac{h\Delta x}{k}) T_{m,n} = 0$
4		$(T_{m,n-1} + T_{m-1,n}) + 2 \frac{h\Delta x}{k} T_{\infty} - 2(1 + \frac{h\Delta x}{k}) T_{m,n} = 0$

Table 5: Cylinder In Cross Flow

$Re_D$	$C$	$m$
0.4 – 4	0.989	0.330
4 – 40	0.911	0.385
40 – 4000	0.683	0.466
4000 – 40,000	0.193	0.618
40,000 – 400,000	0.027	0.805

Table 6: Various Geometries In Cross Flow

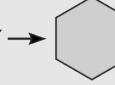
Geometry		$Re_D$	$C$	$m$
		6000 – 60,000	0.304	0.59
		5000 – 60,000	0.158	0.66
		5200 – 20,400 20,400 – 105,000	0.164 0.039	0.638 0.78
		4500 – 90,700	0.150	0.638
	Front Back	10,000 – 50,000 7,000 – 80,000	0.667 0.191	0.500 0.667

Table 7: Noncircular Tubes

Cross Section	$\frac{b}{a}$	Uniform $q_s''$	Uniform $T_s$	$fRe_{D_h}$
		4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	$\infty$	8.23	7.54	96
	$\infty$	5.39	4.86	96
		3.11	2.49	53

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Table 8: Fouling Factors

<i>Fluid</i>	$R_f'' \text{ (m}^2\text{K/W)}$
Seawater and treated boiler feedwater (below 50°C)	0.001
Seawater and treated boiler feedwater (above 50°C)	0.002
River water (below 50°C)	0.0002 - 0.001
Fuel Oil	0.0009
Refrigerating Liquids	0.0002
Steam (nonoil bearing)	0.0001

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Table 9: Blackbody Radiation Functions

$\lambda T (\mu mK)$	$F_{(0 \rightarrow \lambda)}$	$I_{\lambda,b}(\lambda, T) / \sigma T^5 (\mu mK sr)^{-1}$	$\frac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	$0.375034E - 27$	0.000000
400	0.000000	$0.490335E - 13$	0.000000
600	0.000000	$0.104046E - 8$	0.000014
800	0.000016	$0.991126E - 7$	0.001372
1000	0.000321	$0.118505E - 5$	0.016406
1200	0.002134	$0.523927E - 5$	0.072534
1400	0.007790	$0.134411E - 4$	0.186082
1600	0.019718	$0.249130E - 4$	0.344904
1800	0.039341	$0.375568E - 4$	0.519949
2000	0.066728	$0.493432E - 4$	0.683123
2200	0.100888	$0.589649E - 4$	0.816329
2400	0.140256	$0.658866E - 4$	0.912155
2600	0.183120	$0.701292E - 4$	0.970891
2898	0.250108	$0.722318E - 4$	1.000000
3000	0.273232	$0.720254E - 4$	0.997143
3400	0.361735	$0.681544E - 4$	0.943551
3800	0.443382	$0.615225E - 4$	0.851737
4200	0.516014	$0.540394E - 4$	0.748139
4600	0.579280	$0.467343E - 4$	0.647004
5000	0.633747	$0.400813E - 4$	0.554898
5400	0.680360	$0.342445E - 4$	0.474092
5800	0.720158	$0.292301E - 4$	0.404671
6200	0.754140	$0.249723E - 4$	0.345724
6600	0.783199	$0.213786E - 4$	0.295973
7000	0.808109	$0.183534E - 4$	0.254090
7400	0.829527	$0.158073E - 4$	0.218842
7800	0.848005	$0.136621E - 4$	0.189143
8000	0.856288	$0.127185E - 4$	0.176079
9000	0.890029	$0.901463E - 5$	0.124801
10000	0.914199	$0.653279E - 5$	0.090442
20000	0.985602	$0.623310E - 6$	0.008629
30000	0.995340	$0.140469E - 6$	0.001945
40000	0.997967	$0.473891E - 7$	0.000656
50000	0.998953	$0.201605E - 7$	0.000279
75000	0.999713	$0.418597E - 8$	0.000058
100000	0.999905	$0.135752E - 8$	0.000019