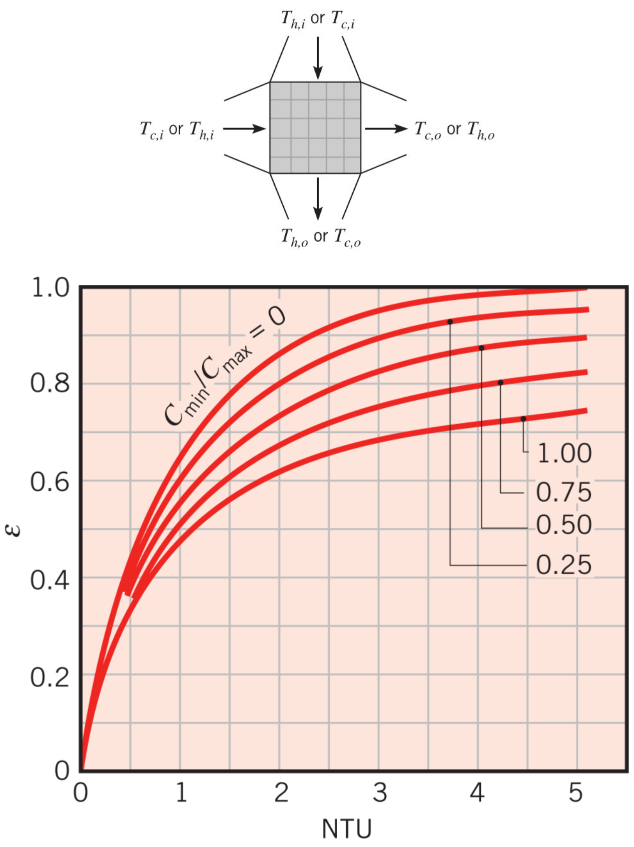
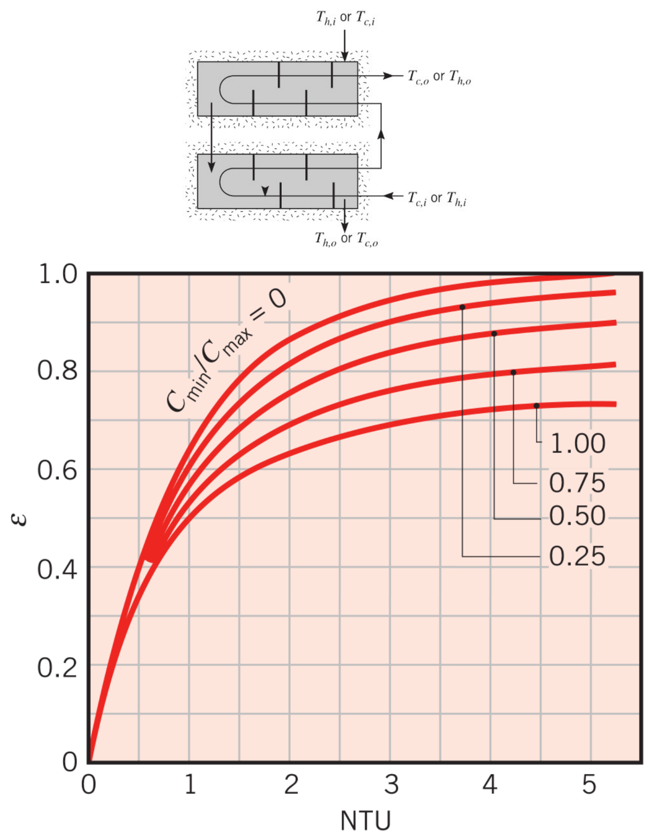
Effectiveness NTU Method





Effectiveness NTU Method

- ε-NTU method arises from the limitation of the log mean temperature difference method.
- The log mean temperature difference method works well when all inlet and outlet temperatures are known.
- If only the inlet temperatures are known, an iterative method must be used.
- This is not the case with the ε-NTU method.

Heat Exchanger Effectiveness

- To use the ε-NTU method, the effectiveness of a heat exchanger must first be defined.
- The effectiveness is given by

  $$\varepsilon \equiv \frac{q}{q_{max}}$$

  where q is the actual heat rate exchanged between the two fluids, and q_{max} is the maximum heat rate that could be exchanged between the two fluids.
- To determine the maximum heat rate that could be exchanged, a couple of different cases need to be considered.

Heat Exchanger Effectiveness

- First, let's consider an infinitely long counter flow heat exchanger.
- Second, consider the case where the heat capacity of the cold fluid is lower than the hot fluid (i.e. $C_c < C_h$).
- For a long enough length, the cold fluid outlet temperature $T_{c,o}$ will be equal to the hot fluid inlet temperature $T_{h,i}$.
- Thus the maximum heat rate would be given by
  $q_{max} = C_c (T_{h,i} - T_{c,i})$
- For the case where the heat capacity of the hot fluid is lower than the cold fluid (i.e. $C_h < C_c$), the maximum heat rate is given by
  $q_{max} = C_h (T_{h,i} - T_{c,i})$

Heat Exchanger Effectiveness

- Since the maximum heat rate is dependent on the minimum heat capacity, the maximum heat rate is typically defined as

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right)$$

where C_{min} is given by $C_{min} = min\left(C_c, C_h\right) = \begin{cases} C_c, C_c < C_h \\ C_h, C_h < C_c \end{cases}$

- Thus for a particular heat exchanger, the effectiveness is given by

$$\varepsilon = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} \qquad \varepsilon = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)}$$

- The actual heat can be written as

$$q = \varepsilon\, C_{min}\left(T_{h,i} - T_{c,i}\right)$$

Heat Exchanger Effectiveness

- For any heat exchanger it may be shown that

$$\varepsilon = f\left(NTU, \frac{C_{min}}{C_{max}}\right) = f\left(NTU, C_r\right)$$

  where NTU is the number of transfer units.

- NTU is defined as

$$NTU \equiv \frac{UA}{C_{min}}$$

- Expressions have been developed relating the two for different types of heat exchangers.

Heat Exchanger Effectiveness Relations

Table 12.1

Flow Arrangement	Relation	
Parallel ow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 + C_r)]}{1 + C_r}$	
Counterow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 - C_r)]}{1 - C_r \exp[-\mathrm{NTU}(1 - C_r)]}$	$(C_r < 1)$
	$\varepsilon = \dfrac{\mathrm{NTU}}{1 + \mathrm{NTU}}$	$(C_r = 1)$
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$	
n shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$	
Cross-ow (single pass)		
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(\mathrm{NTU})^{0.22}\{\exp[-C_r(\mathrm{NTU})^{0.78}] - 1\}\right]$	
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\mathrm{NTU})]\})$	
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\mathrm{NTU})]\})$	
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-\mathrm{NTU})$	

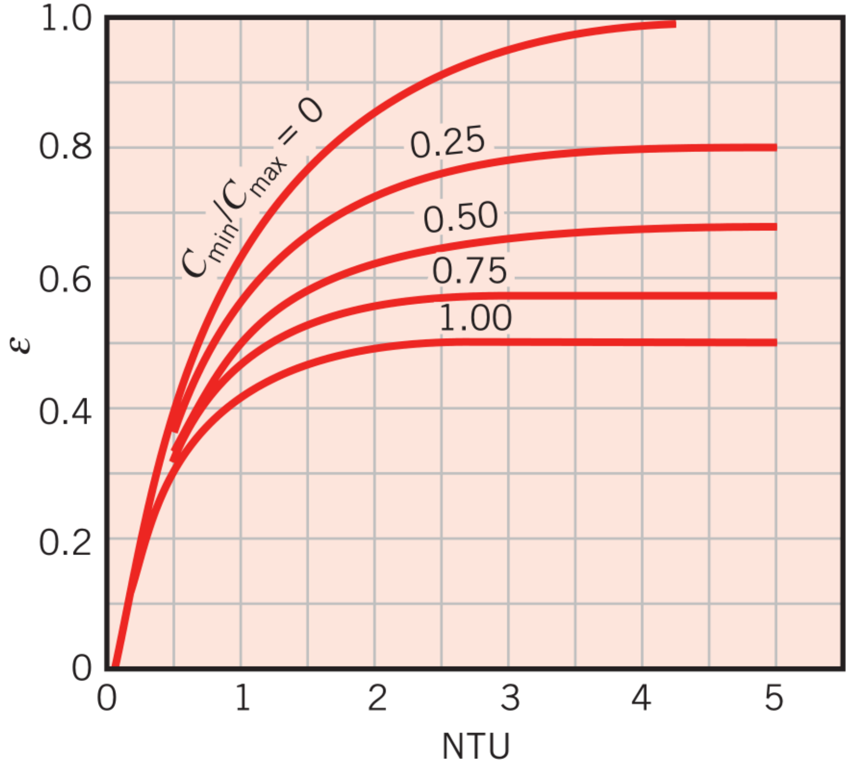
Heat Exchanger NTU Relationships

Table 12.2

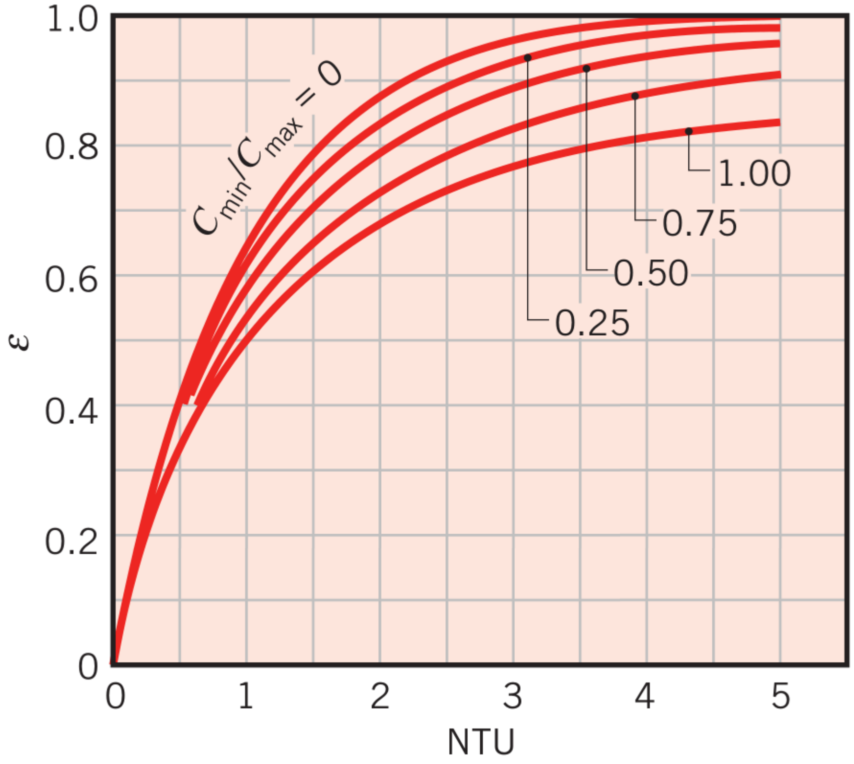
Flow Arrangement	Relation	
Parallel ow	$\mathrm{NTU} = -\dfrac{\ln[1-\varepsilon(1+C_r)]}{1+C_r}$	
Counterow	$\mathrm{NTU} = \dfrac{1}{C_r-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon C_r-1}\right)$	$(C_r < 1)$
	$\mathrm{NTU} = \dfrac{\varepsilon}{1-\varepsilon}$	$(C_r = 1)$
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$(\mathrm{NTU})_1 = -(1+C_r^2)^{-1/2}\ln\left(\dfrac{E-1}{E+1}\right)$ $E = \dfrac{2/\varepsilon_1-(1+C_r)}{(1+C_r^2)^{1/2}}$	
n shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F-1}{F-C_r} \quad F = \left(\dfrac{\varepsilon C_r-1}{\varepsilon-1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$	
Cross-ow (single pass)		
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\mathrm{NTU} = -\ln\left[1+\left(\dfrac{1}{C_r}\right)\ln(1-\varepsilon C_r)\right]$	
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r\ln(1-\varepsilon)+1]$	
All exchangers ($C_r = 0$)	$\mathrm{NTU} = -\ln(1-\varepsilon)$	

Effectiveness-NTU Relationships

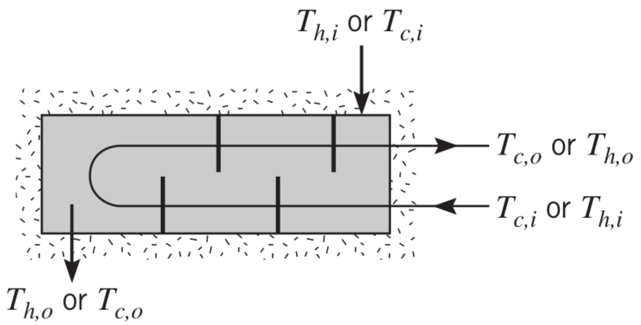
Parallel Flow

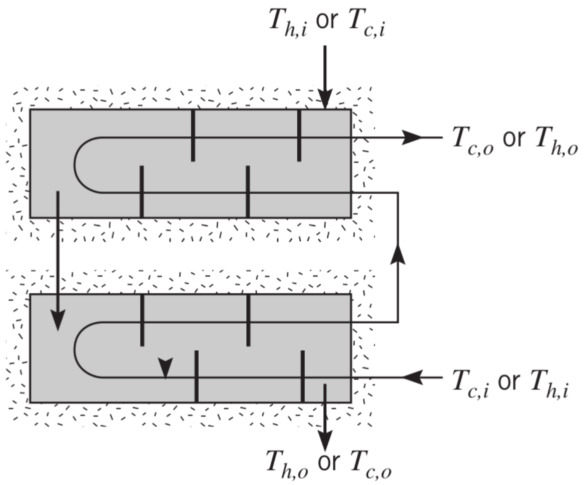


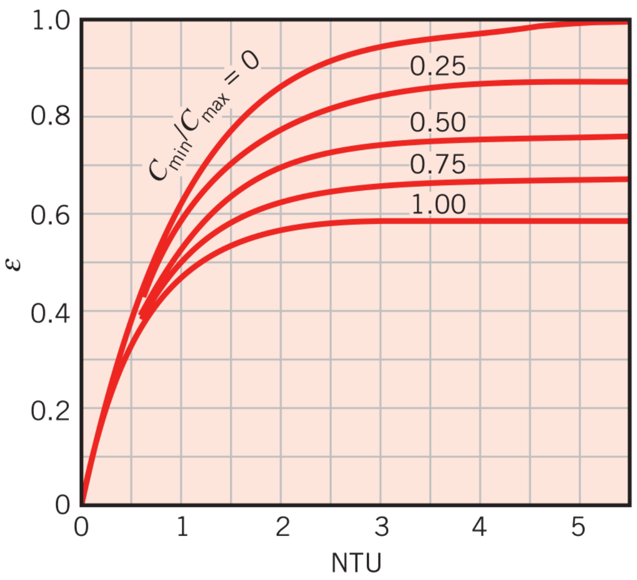
Counter Flow

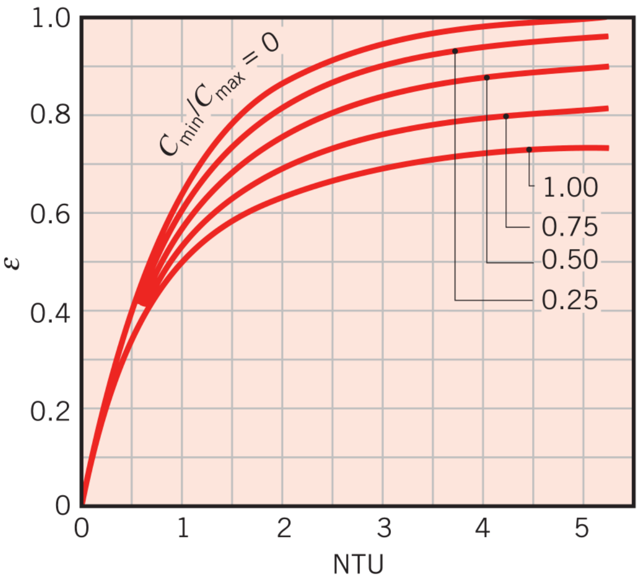


Effectiveness-NTU Relationships

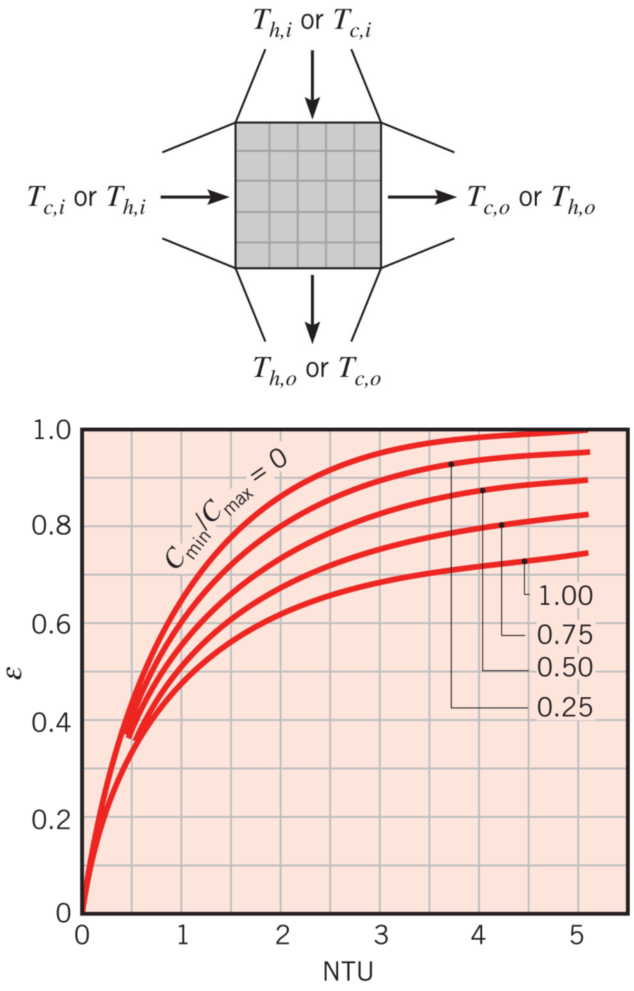


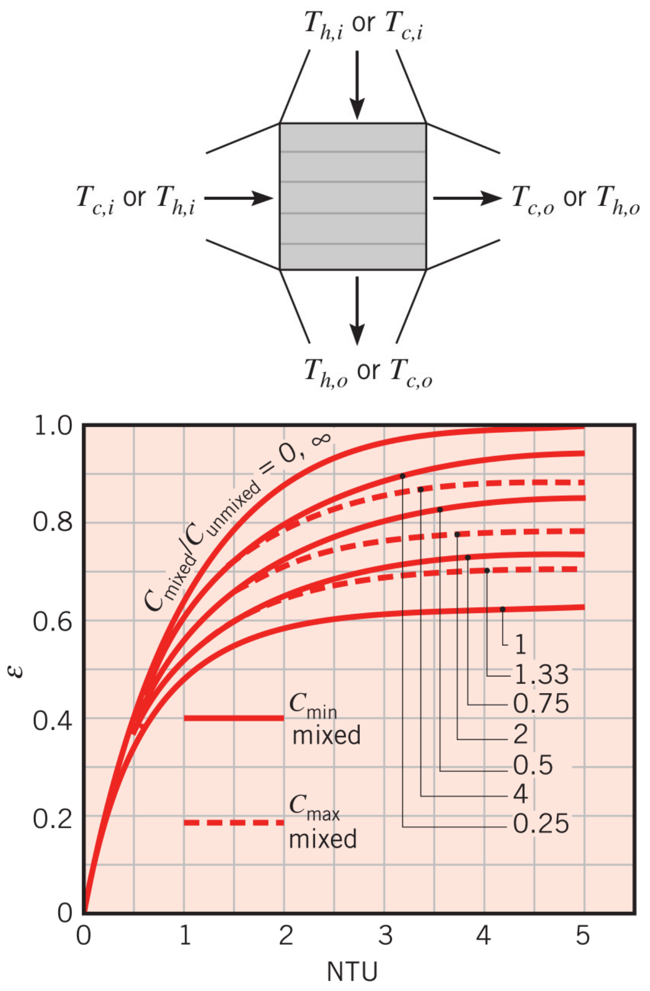






Effectiveness-NTU Relationships





Example 1

Hot exhaust gases, which enter a finned-tube, cross-flow heat exchanger at $300\ °C$ and leave at $100\ °C$, are used to heat pressurized water at a flow rate of $1\ kg/s$ from 35 to $125\ °C$. The overall heat transfer coefficient based on the gas-side surface area is $U_h = 100\ W/m^2K$. Determine the required gas-side surface area A_h using the NTU method.

Example 2

Consider the heat exchanger design from the previous example, that is, a finned-tube, cross-flow heat exchanger with a gas-side overall heat transfer coefficient and area of $100\ W/m^2K$ and $38.23\ m^2$, respectively. The water flow rate and inlet temperature remain at $1\ kg/s$ and $35\ °C$. However, a change in operating conditions for the hot gas generator causes the gases to now enter the heat exchanger with a flow rate of $1.5\ kg/s$ and a temperature of $250\ °C$. What is the rate of heat transfer by the exchanger, and what are the gas and water outlet temperatures?