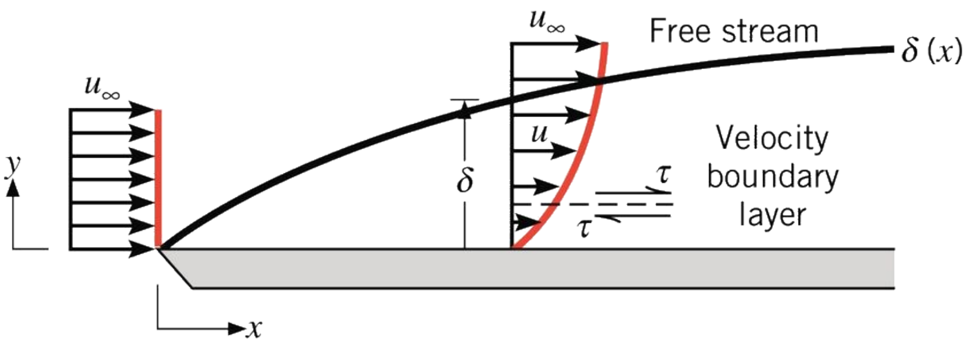
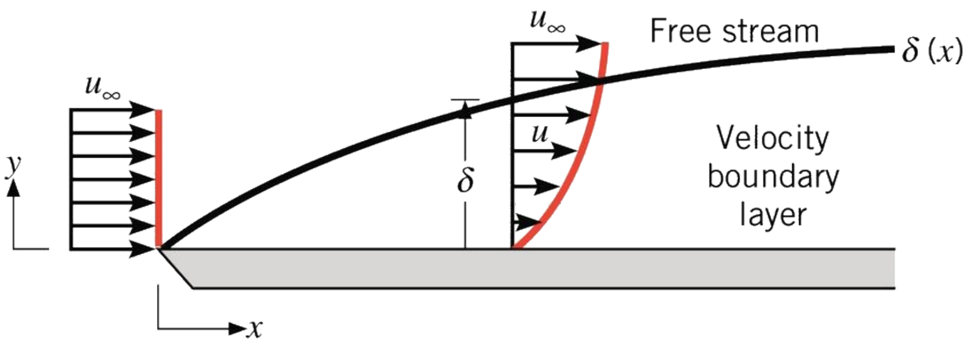
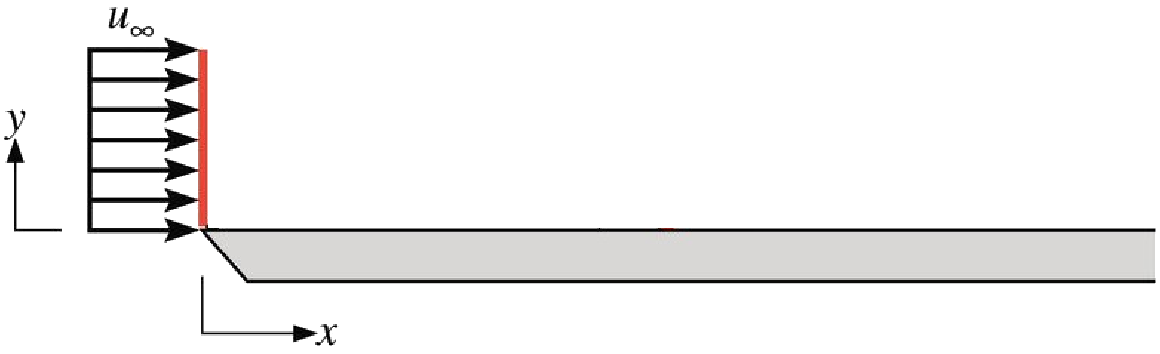
Convection Introduction





Convection

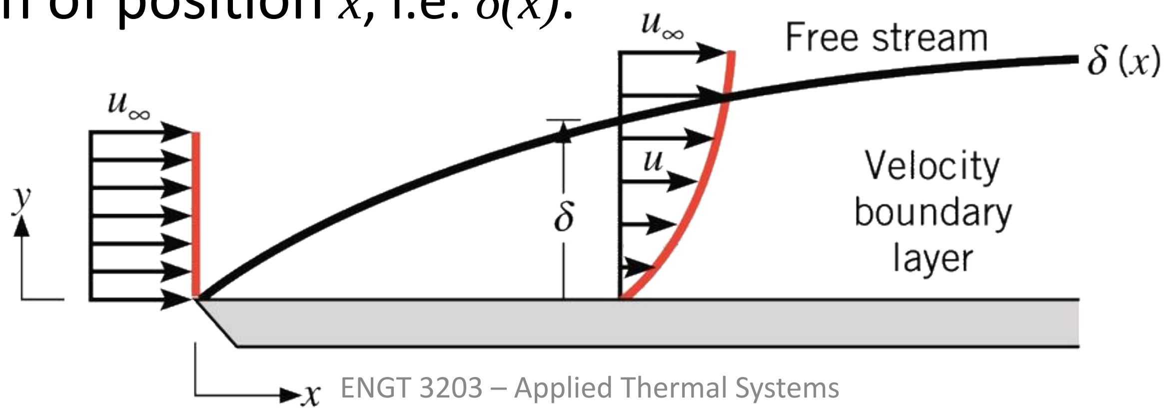
- Convection analysis involves understanding how flow develops over different objects.
- The simplest flow arrangement is flow over a heated flat plate.



- The uniform flow u_∞ becomes distorted.
- A velocity boundary layer is said to develop.

Velocity Boundary Layer

- The velocity boundary layer forms as a result of viscous effects in the fluid.
- At the surface of the plate, the velocity is zero. This is known as having a no slip condition.
- The further away from the plate, the closer the velocity is to the far field velocity, u_∞.
- The boundary layer δ is defined as the y location when $\dfrac{u(y)}{u_\infty} = 0.99$
- δ is a function of position x, i.e. $\delta(x)$.



Velocity Boundary Layer

- The friction coefficient of an object may be defined in terms of the far field velocity of the fluid as

$$C_f \equiv \frac{\tau_s}{\rho u_\infty^2 / 2}$$

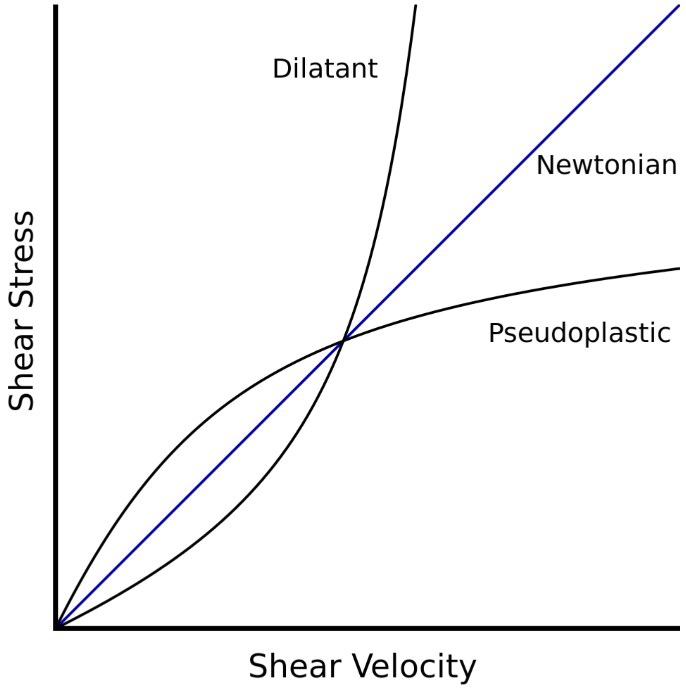
where τ_s is the surface shear stress.

- For a Newtonian Fluid, τ_s is given by

$$\tau_s = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}$$

where μ is the dynamic viscosity of the fluid.

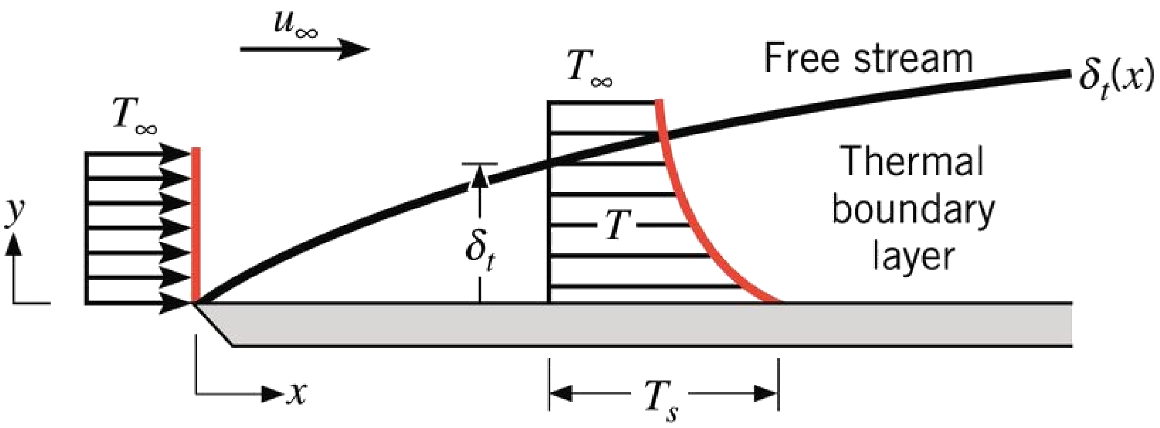
- μ is a tabulated property of the fluid.



Thermal Boundary Layer

- The temperature of the fluid flow across the flat plate also forms a boundary layer.

- The thermal boundary layer is denoted as δ_t.



- It is defined as the y location where $\dfrac{T_s - T(y)}{T_s - T_\infty} = 0.99$ and is a function of x.

Thermal Boundary Layer

- Using Fourier's law at the surface of the flat plate shows

$$q_s'' = -k \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

where the s denotes surface. This will be dropped in later expressions.

- Knowing Newton's law of cooling also as

$$q_s'' = h\left(T_s - T_\infty\right)$$

- The convective heat transfer coefficient h becomes

$$h = \frac{-k \left.\dfrac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}$$

Local and Average Terms

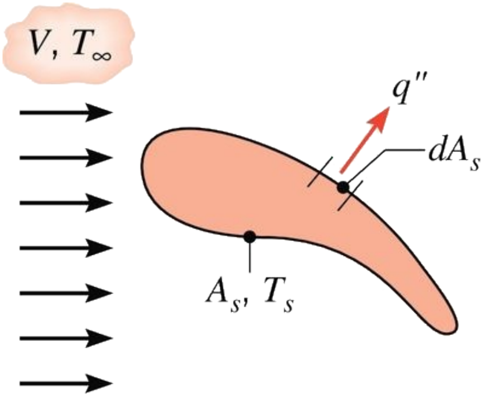
- When using heat rates and convective coefficients, special attentions must be given to whether local or average values are being used.
- Consider the objects shown to the right.
- The local heat flux is

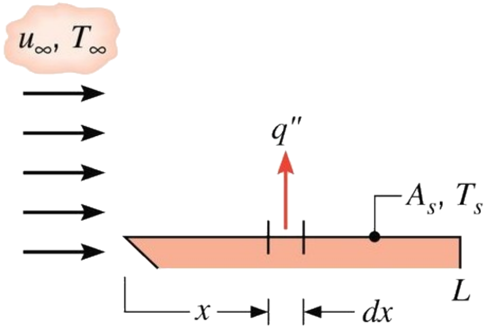
$$q'' = h(T_s - T_\infty)$$

- The local heat rate is

$$q = \int_{A_s} q''\, dA_s = (T_s - T_\infty) \int_{A_s} h\, dA_s$$

where h is the local convective coefficient.





Local and Average Terms

- Average values are denoted by an over bar.
- The average heat flux is

$$\bar{q}'' = \bar{h}(T_s - T_\infty)$$

- The average heat rate is

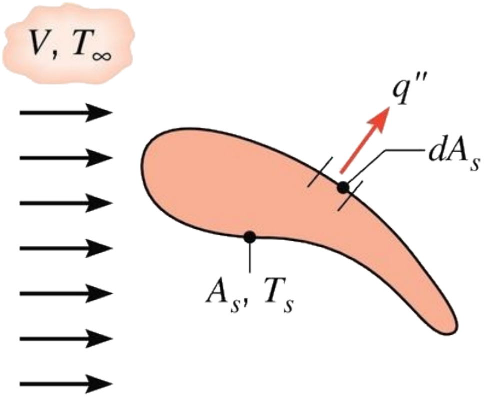
$$\bar{q} = \bar{q}'' A_s = \bar{h} A_s (T_s - T_\infty)$$

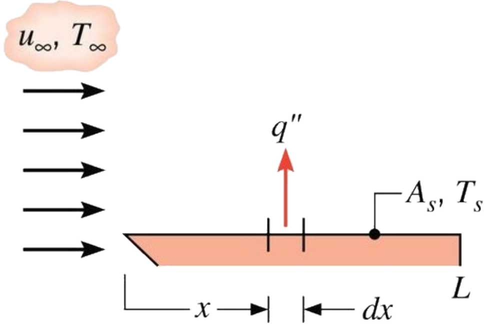
where the average convective coefficient is

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h \, dA_s$$

- For a flat plate in parallel flow this becomes

$$\bar{h} = \frac{1}{L} \int_0^L h \, dx$$



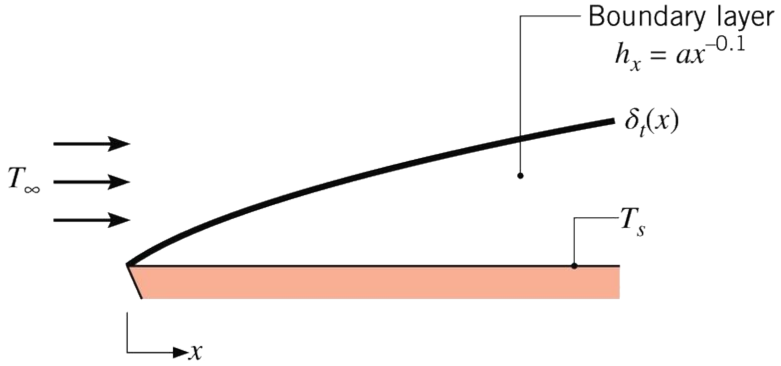


Example 1

Experimental results for the local heat transfer coefficient h_x for flow over a flat plate with an extremely rough surface were found to fit the relation

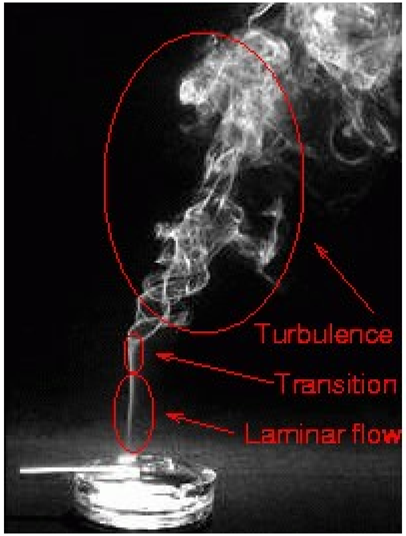
$$h_x(x) = a\,x^{-0.1}$$

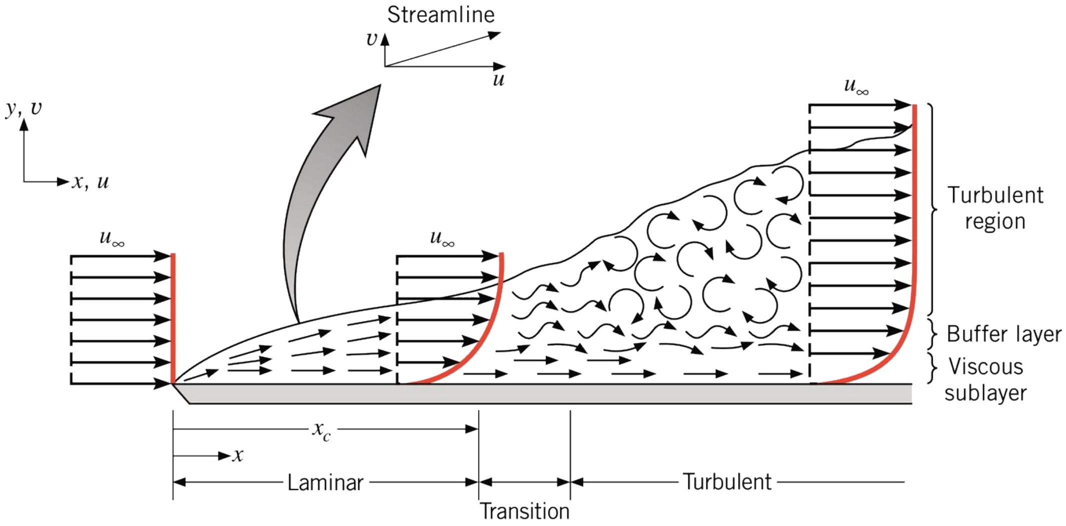
where a is a coefficient ($W/m^{1.9}K$) and x (m) is the distance from the leading edge of the plate. Develop an expression for the ratio of the average heat transfer coefficient $\bar{h}_x$ for a plate of length x to the local heat transfer coefficient h_x at x. Plot the variation of h_x and $\bar{h}_x$ as a function of x.



Flow Conditions

- There are 3 main types of flow that will be discussed:
  - Laminar
  - Transition
  - Turbulent
- In each of these regions, the heat transfer characteristics will be different.



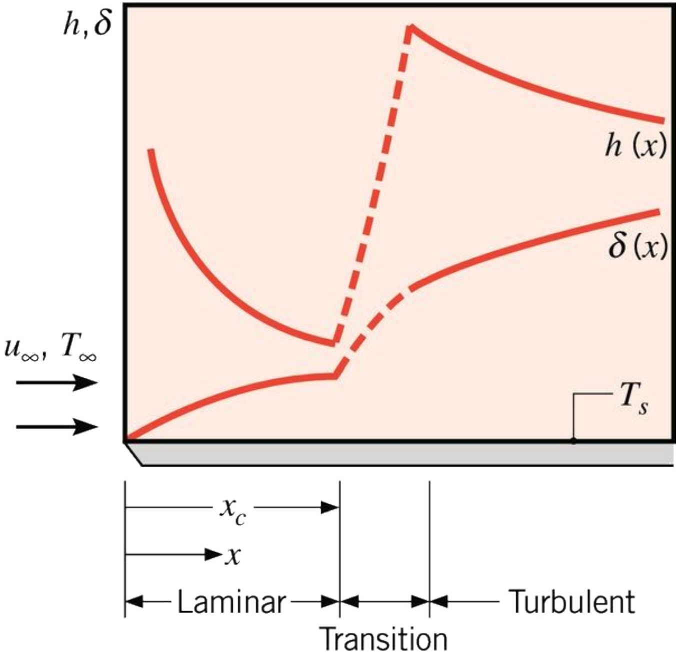


Flow Conditions

- The Reynolds number is a dimensionless number and is used to characterize the flow region.
- The Reynolds number at a particular location x is defined as

$$Re_x \equiv \frac{\rho u_\infty x}{\mu} [=] \frac{\frac{kg}{m^3}\frac{m}{s}m}{\frac{Ns}{m^2}} [=] \frac{\frac{kg}{ms}}{\frac{Ns}{m^2}} [=] \frac{kg\,m}{N\,s^2} [=] \frac{\frac{kg\,m}{s^2}}{\frac{kg\,m}{s^2}} [=] 1$$

- The start of the transition regions is described by the critical location x_c.
- This is found by the critical Reynolds number.
- For flow over a flat plate this is $Re_{x,c} \equiv 5\times10^5 = \frac{\rho u_\infty x_c}{\mu}$



The Reynolds number is named after Osborne Reynolds.

Example 2

Water flows at a velocity $u_\infty = 1\ m/s$ over a flat plate of length $L = 0.6\ m$. Consider two cases, one for which the water temperature is approximately $300\ K$ and the other for an approximate water temperature of $350\ K$. In the laminar and turbulent regions, experimental measurements show that the local convection coefficients are well described by

$$h_{lam}(x) = C_{lam}\, x^{-0.5} \qquad h_{turb}(x) = C_{turb}\, x^{-0.2}$$

where x has units of m. At $300\ K$,

$$C_{lam,300} = 395 \frac{W}{m^{1.5}K} \qquad C_{turb,300} = 2330 \frac{W}{m^{1.8}K}$$

while at $350\ K$,

$$C_{lam,350} = 477 \frac{W}{m^{1.5}K} \qquad C_{turb,350} = 3600 \frac{W}{m^{1.8}K}$$

Example 2 (contd.)

As is evident, the constant C depends on the nature of the flow as well as the water temperature because of the thermal dependence of various properties of the fluid. Determine the average convection coefficient, $\bar{h}$, over the entire plate for the two water temperatures. Plot the convective coefficients as a function of x for the entire length of the flat plate. On the plot, show the average convective coefficient values.

Normalizing Boundary Layer Equations

- For different flow arrangements, the governing boundary layer equations will differ.
- The governing boundary layer equations can be normalized such that one set of governing equations may be solved for practically any flow arrangement.
- This is accomplished by use of similarity parameters.
- The similarity parameters are dimensionless independent variables.
- The similarity parameters for positions are

  $$x^* \equiv \frac{x}{L} \qquad\qquad y^* \equiv \frac{y}{L}$$

  where L is the characteristic length for the surface of interest (e.g., the length of a flat plate).

Normalizing Boundary Layer Equations

- The similarity parameters for velocities are

$$u^* \equiv \frac{u}{V} \qquad \mathrm{v}^* \equiv \frac{\mathrm{v}}{V}$$

where V is the upstream flow velocity, and u and v are the velocities in the x and y directions, respectively.

- The similarity parameter for temperature is

$$T^* = \frac{T - T_s}{T_\infty - T_s}$$

- The similarity parameter for pressure is

$$p^* = \frac{p_\infty}{\rho V^2} [=] \frac{Pa}{\frac{kg}{m^3}\left(\frac{m}{s}\right)^2} [=] \frac{\frac{N}{m^2}}{\frac{kg}{m s^2}} [=] \frac{N s^2}{kg\, m} [=] \frac{\frac{kg\, m}{s^2} s^2}{kg\, m} [=] 1$$

Normalizing Boundary Layer Equations

- Using the similarity parameters, the governing boundary layer equations become

Velocity: $$u^* \frac{\partial u^*}{\partial x^*} + \mathrm{v}^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$ $$Re_L = \frac{V\,L}{\nu} \qquad \nu = \frac{\mu}{\rho}$$

Thermal: $$u^* \frac{\partial T^*}{\partial x^*} + \mathrm{v}^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L\,Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$ $$Pr = \frac{\nu}{\alpha}$$

where v is the kinematic viscosity (a material property), and Pr is the Prandtl number.

- The Prandtl number is named after Ludwig Prandtl.

Other Useful Dimensionless Number

- The Nusselt number is another dimensionless number often used.
- The Nusselt number is named after Wilhelm Nusselt.
- The Nusselt number is defined as

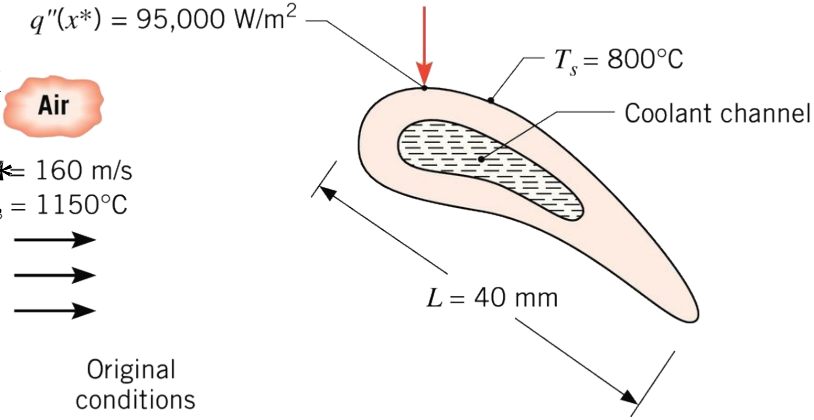
$$Nu \equiv \frac{hL}{k} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \qquad Nu = f\left(x^*, Re_L, Pr\right)$$ For a given geometry.

- The average Nusselt number may also be defined as

$$\bar{N}u \equiv \frac{\bar{h}L}{k} = f\left(Re_L, Pr\right)$$

Example 3

Experimental tests using air as the working fluid are conducted on a portion of the turbine blade shown in the sketch. The heat flux to the blade at a particular point (x^*) on the surface is measured to be $q'' = 95{,}000\ W/m^2$. To maintain a steady-state surface temperature of $800\ °C$, heat transferred to the blade is removed by circulating a coolant inside the blade. Determine the heat flux to the blade at x^* if its temperature is reduced to $T_{s,1} = 700\ °C$ by increasing the coolant flow. Determine the heat flux at the same dimensionless location x^* for a similar turbine blade having a chord length of $L = 80\ mm$, when the blade operates in an airflow at $T_\infty = 1150\ °C$ and $V = 80\ m/s$, with $T_s = 800\ °C$.



Example 4

Consider convective cooling of a two-dimensional streamlined strut of characteristic length $L_{H2} = 40\ mm$. The strut is exposed to hydrogen flowing at $p_{H2} = 2\ atm$, $V_{H2} = 8.1\ m/s$ and $T_{\infty,H2} = -30\ °C$. Of interest is the value of the average heat transfer coefficient $\bar{h}_{H2}$, when the surface temperature is $T_{s,H2} = -15\ °C$. Rather than conducting expensive experiments involving pressurized hydrogen, an engineer proposes to take advantage of similarity by performing wind tunnel experiments using air at atmospheric pressure with $T_{\infty,Air} = 23\ °C$. A geometrically similar strut of characteristic length $L_{Air} = 60\ mm$ and perimeter $P = 150\ mm$ is placed in the wind tunnel. Measurements reveal a surface temperature of $T_{s,Air} = 30\ °C$ when the heat loss per unit object length (into the page) is $q'_{Air} = 50\ W/m$. Determine the required air velocity in the wind tunnel experiment V_{Air} and the average convective heat transfer coefficient in the hydrogen $\bar{h}_{H2}$.