ENGT 320 Applied Thermal Systems
Quiz 2 Formula Sheet

$$m = \rho \forall$$
$$\dot{\forall} = \frac{\forall}{t}$$
$$q = m c_p \Delta T$$
$$q = -kA\frac{dT}{dt}$$
$$q = hA\left(T_s - T_\infty\right)$$
$$q = \varepsilon \sigma A\left(T_s^4 - T_{sur}^4\right)$$
$$\sigma = 5.67 \times 10^{-8}\frac{W}{m^2K^4}$$
$$\Delta E_{sys} = \Sigma E_{in} - \Sigma E_{out}$$
$$\frac{dE_{sys}}{dt} = \Sigma \dot{E}_{in} - \Sigma \dot{E}_{out}$$
$$q^{''} = \frac{q}{A}$$
$$\hat{q}^{''} = -k_x\frac{\partial T}{\partial x}\hat{i} - k_y\frac{\partial T}{\partial y}\hat{j} - k_z\frac{\partial T}{\partial z}\hat{k}$$
$$k = \frac{9\gamma-5}{4}\frac{c_v}{\pi d^2}\sqrt{\frac{M_w k_B T}{N_A \pi}}$$
$$k_B = 1.381 \times 10^{-23}\frac{J}{K}$$
$$N_A = 6.022 \times 10^{23}$$
$$\alpha = \frac{k}{\rho c_p}$$
$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}dx$$
$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y}dy$$
$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z}dz$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = 0$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$T(0,t) = T_s$$
$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_s^{''}$$
$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$
$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h\left(T_\infty - T(0,t)\right)$$

$$\Delta U = N\left(V^- - V^+\right)$$
$$q = \frac{\Delta U}{\Delta t} = \frac{N}{\Delta t}\Delta V$$
$$q = -IV = \frac{-V^2}{R} = -I^2R$$
$$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$
$$S_{AB} = S_B - S_A = \frac{-\Delta V}{\Delta T}$$
$$C_1 = \frac{T_{s,2} - T_{s,1}}{L}$$
$$C_2 = T_{s,1}$$
$$T(x) = \left(T_{s,2} - T_{s,1}\right)\frac{x}{L} + T_{s,1}$$
$$q = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right)$$
$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{kA}$$
$$R = \frac{V}{I}$$
$$R_{t,conv} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$

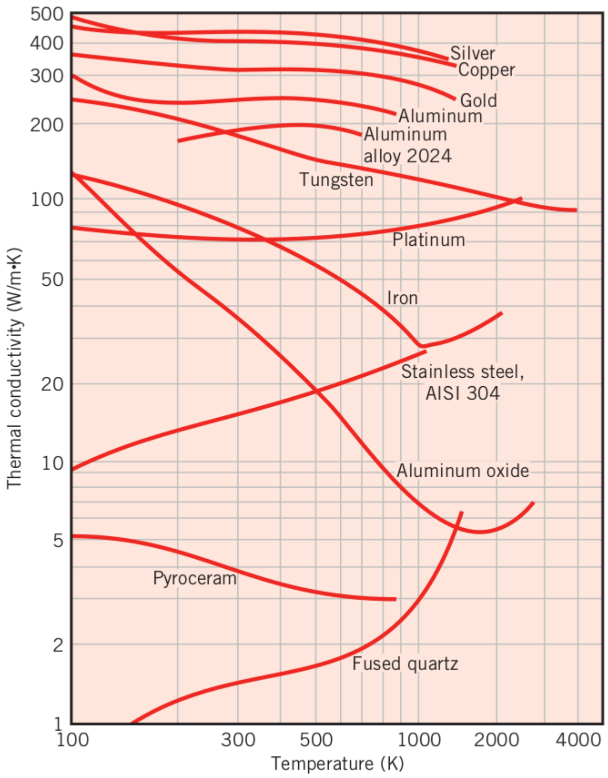
$$R_{t,rad} = \frac{T_s - T_{sur}}{q} = \frac{1}{h_r A}$$
$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right)$$
$$q = UA\Delta T$$
$$U = \frac{1}{R_{tot}A}$$
$$R_{tot} = \Sigma R_t = \frac{1}{UA}$$
$$R_{t,c}^{''} = \frac{T_A - T_B}{q^{''}}$$
$$q = \frac{k_{eff}A}{L}\left(T_1 - T_2\right)$$
$$k_{eff,min} = \frac{1}{\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}}$$
$$k_{eff,max} = \varepsilon k_f + (1-\varepsilon)k_s$$
$$R_{t,cond} = \frac{\ln\frac{r_2}{r_1}}{2\pi kL}$$
$$R_{t,conv} = \frac{1}{h2\pi rL}$$

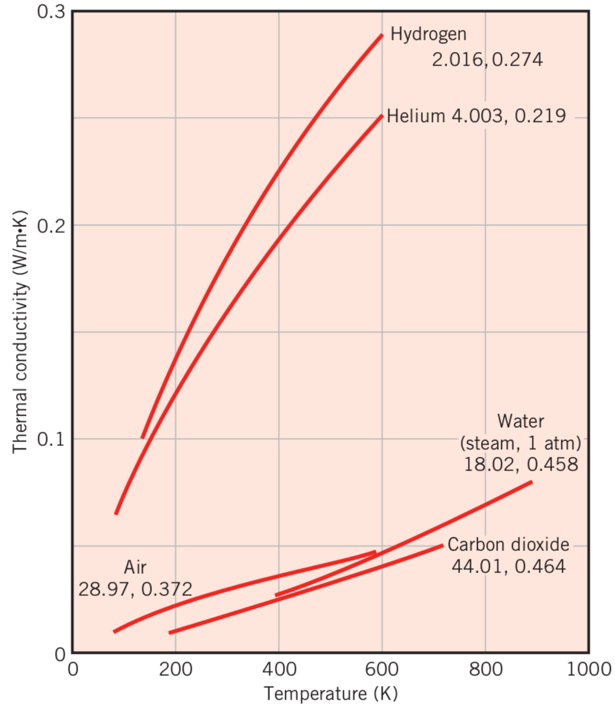
$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)\left(T - T_\infty\right) = 0$$

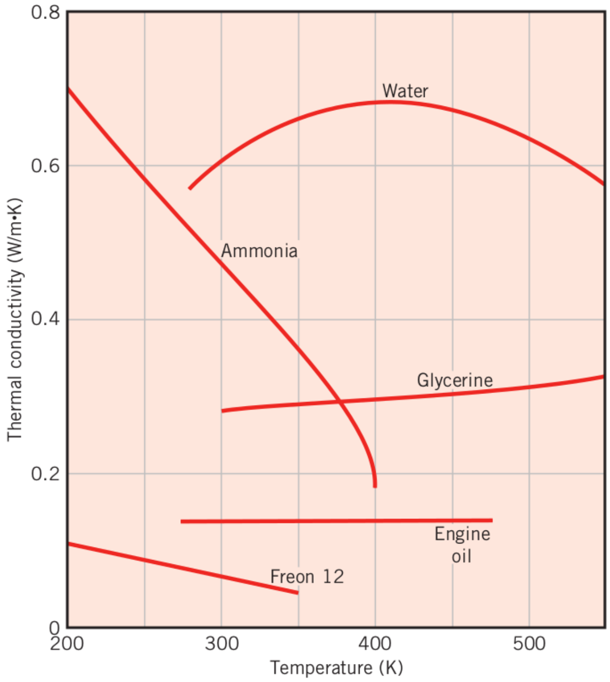
$$A_s = Px$$
$$\frac{d^2T}{dx^2} - \left(\frac{hP}{kA_c}\right)\left(T - T_\infty\right)$$
$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$
$$\theta = T(x) - T_\infty$$
$$m^2 = \frac{hP}{kA_c}$$
$$P = 2w + 2t$$
$$A_c = wt$$
$$P = \pi D$$
$$A_c = \frac{\pi}{4}D^2$$
$$\theta(x) = C_1e^{mx} + C_2e^{-mx}$$
$$\theta_b = T_b - T_\infty$$
$$M = \sqrt{hPKA_c}\theta_b$$
$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b}$$
$$\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$$
$$R_{t,f} = \frac{\theta_b}{q_f}$$
$$R_{t,b} = \frac{1}{hA_{c,b}}$$
$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}$$
$$\eta_f = \frac{q_f}{q_{max}} = \frac{q_f}{hA_f\theta_b}$$
$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{hA_t\theta_b}$$
$$A_t = NA_f + A_b$$
$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b$$

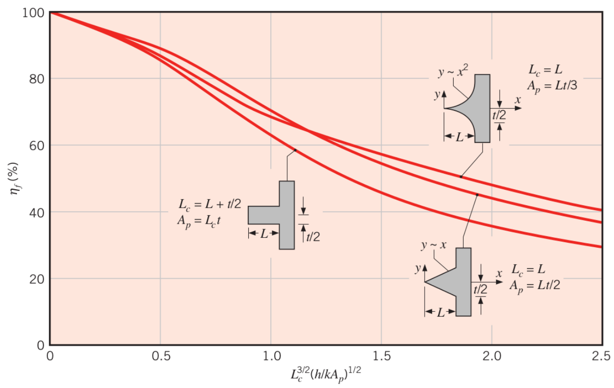
$$\eta_o = 1 - \frac{NA_f}{A_t}\left(1 - \eta_f\right)$$

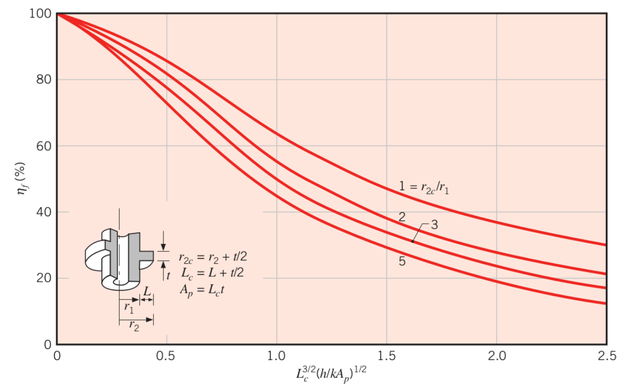
$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$$


500
400
300
200
100
50
20
10
5
2
1
Thermal conductivity (W/m•K)
Silver
Copper
Gold
Aluminum
Aluminum
alloy 2024
Tungsten
Platinum
Iron
Stainless steel,
AISI 304
Aluminum oxide
Pyroceram
Fused quartz
100
300
500
1000
2000
4000
Temperature (K)


0.3
0.2
0.1
0
Thermal conductivity (W/m•K)
Hydrogen
2.016, 0.274
Helium 4.003, 0.219
Water
(steam, 1 atm)
18.02, 0.458
Carbon dioxide
44.01, 0.464
Air
28.97, 0.372
0
200
400
600
800
1000
Temperature (K)


0.8
0.6
0.4
0.2
0
Thermal conductivity (W/m•K)
Water
Ammonia
Glycerine
Engine
oil
Freon 12
200
300
400
500
Temperature (K)





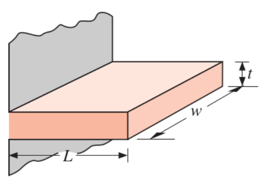
Straight Fins

Rectangular

$A_f = 2wL_c$

$L_c = L + (t/2)$

$A_p = tL$

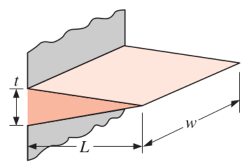


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$A_f = 2w[L^2 + (t/2)^2]^{1/2}$

$A_p = (t/2)L$



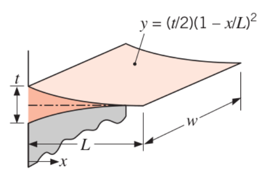
$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$

Parabolic

$A_f = w[C_1 L +$
$(L^2/t)\ln(t/L + C_1)]$

$C_1 = [1 + (t/L)^2]^{1/2}$

$A_p = (t/3)L$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

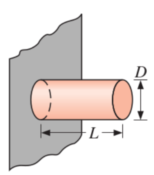
Pin Fins

Rectangular

$A_f = \pi D L_c$

$L_c = L + (D/4)$

$V = (\pi D^2/4)L$

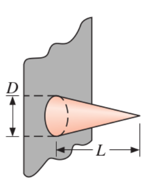


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$

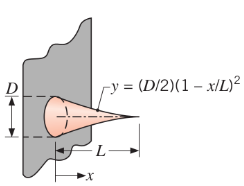
$V = (\pi/12)D^2 L$



$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$

Parabolic

$A_f = \frac{\pi L^3}{8D}\{C_3 C_4 -$
$\frac{L}{2D}\ln[(2DC_4/L) + C_3]\}$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$$

$C_3 = 1 + 2(D/L)^2$

$C_4 = [1 + (D/L)^2]^{1/2}$

$V = (\pi/20)D^2 L$

Table 1: Contact Resistance for vacuum interfaces, $R''_{t,c} \times 10^4 \left(\frac{m^2K}{W}\right)$

Material	$100\,(kPa)$	$10,000\,(kPa)$
Stainless Steel	6-25	0.7-4.0
Copper	1-10	0.1-0.5
Magnesium	1.5-3.5	0.2-0.4
Aluminum	1.5-5.0	0.2-0.4

Table 2: Contact Resistance for aluminum ($10\mu m$ surface roughness, $10kPa$ contact pressure)

Interfacial Fluid	$R''_{t,c} \times 10^4 \left(\frac{m^2K}{W}\right)$
Air	2.75
Helium	1.05
Hydrogen	0.720
Silicone Oil	0.525
Glycerine	0.265

Table 3: Fin Tip Conditions

Case	Tip Condition	Temperature Distribution $\frac{\theta}{\theta_b}$	Fin Heat Transfer Rate q_f
A	Convection	$\frac{\cosh m(L-x)+\left(\frac{h}{mk}\right)\sinh m(L-x)}{\cosh mL+\left(\frac{h}{mK}\right)\sinh mL}$	$M\frac{\sinh mL+\left(\frac{h}{mk}\right)\cosh mL}{\cosh mL+\left(\frac{h}{mk}\right)\sinh mL}$
B	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	$M\tanh mL$
C	Constant Temperature	$\frac{\left(\frac{\theta_L}{\theta_b}\right)\sinh mx+\sinh m(L-x)}{\sinh mL}$	$M\frac{\cosh mL-\frac{\theta_L}{\theta_b}}{\sinh mL}$
D	Infinite Fin ($L \to \infty$)	e^{-mx}	M