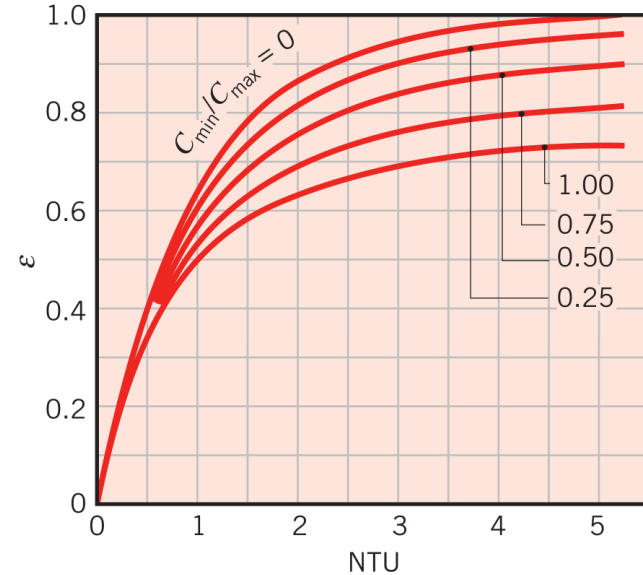
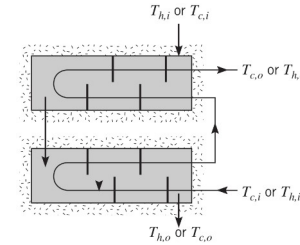
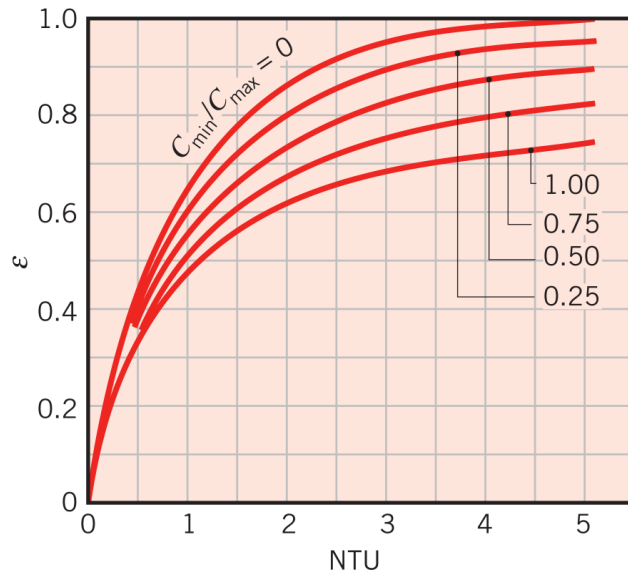
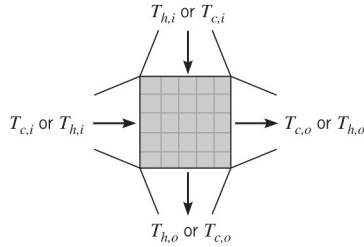


# Effectiveness NTU Method



# Effectiveness NTU Method

- $\varepsilon$ - $NTU$  method arises from the limitation of the log mean temperature difference method.
- The log mean temperature difference method works well when all inlet and outlet temperatures are known.
- If only the inlet temperatures are known, an iterative method must be used.
- This is not the case with the  $\varepsilon$ - $NTU$  method.

# Heat Exchanger Effectiveness

- To use the  $\varepsilon$ - $NTU$  method, the effectiveness of a heat exchanger must first be defined.
- The effectiveness is given by

$$\varepsilon \equiv \frac{q}{q_{max}}$$

where  $q$  is the actual heat rate exchanged between the two fluids, and  $q_{max}$  is the maximum heat rate that could be exchanged between the two fluids.

- To determine the maximum heat rate that could be exchanged, a couple of different cases need to be considered.

# Heat Exchanger Effectiveness

- First, let's consider an infinitely long counter flow heat exchanger.
- Second, consider the case where the heat capacity of the cold fluid is lower than the hot fluid (i.e.  $C_c < C_h$ ).
- For a long enough length, the cold fluid outlet temperature  $T_{c,o}$  will be equal to the hot fluid inlet temperature  $T_{h,i}$ .
- Thus the maximum heat rate would be given by
$$q_{max} = C_c (T_{h,i} - T_{c,i})$$
- For the case where the heat capacity of the hot fluid is lower than the cold fluid (i.e.  $C_h < C_c$ ), the maximum heat rate is given by
$$q_{max} = C_h (T_{h,i} - T_{c,i})$$

# Heat Exchanger Effectiveness

- Since the maximum heat rate is dependent on the minimum heat capacity, the maximum heat rate is typically defined as

$$q_{max} = C_{min}(T_{h,i} - T_{c,i})$$

where  $C_{min}$  is given by  $C_{min} = \min(C_c, C_h) = \begin{cases} C_c, & C_c < C_h \\ C_h, & C_h < C_c \end{cases}$

- Thus for a particular heat exchanger, the effectiveness is given by

$$\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})} \qquad \varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})}$$

- The actual heat can be written as

$$q = \varepsilon C_{min}(T_{h,i} - T_{c,i})$$

# Heat Exchanger Effectiveness

- For any heat exchanger it may be shown that

$$\varepsilon = f\left(NTU, \frac{C_{min}}{C_{max}}\right) = f(NTU, C_r)$$

where  $NTU$  is the number of transfer units.

- $NTU$  is defined as

$$NTU \equiv \frac{UA}{C_{min}}$$

- Expressions have been developed relating the two for different types of heat exchangers.

# Heat Exchanger Effectiveness Relations

Table 12.1

Flow Arrangement	Relation
<b>Parallel ow</b>	$\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}$
<b>Counterow</b>	$\varepsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]} \quad (C_r < 1)$ $\varepsilon = \frac{NTU}{1 + NTU} \quad (C_r = 1)$
<b>Shell-and-tube</b>	
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2 \left\{ 1 + C_r + (1 + C_r^2)^{1/2} \times \frac{1 + \exp[-(NTU)_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(NTU)_1(1 + C_r^2)^{1/2}]} \right\}^{-1}$
$n$ shell passes ( $2n, 4n, . . .$ tube passes)	$\varepsilon = \left[ \left( \frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - 1 \right] \left[ \left( \frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - C_r \right]^{-1}$
<b>Cross-ow (single pass)</b>	
Both fluids unmixed	$\varepsilon = 1 - \exp \left[ \left( \frac{1}{C_r} \right) (NTU)^{0.22} \{ \exp[-C_r(NTU)^{0.78}] - 1 \} \right]$
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\varepsilon = \left( \frac{1}{C_r} \right) (1 - \exp \{ -C_r [1 - \exp(-NTU)] \})$
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1} \{ 1 - \exp[-C_r(NTU)] \})$
<b>All exchangers (<math>C_r = 0</math>)</b>	$\varepsilon = 1 - \exp(-NTU)$

# Heat Exchanger NTU Relationships

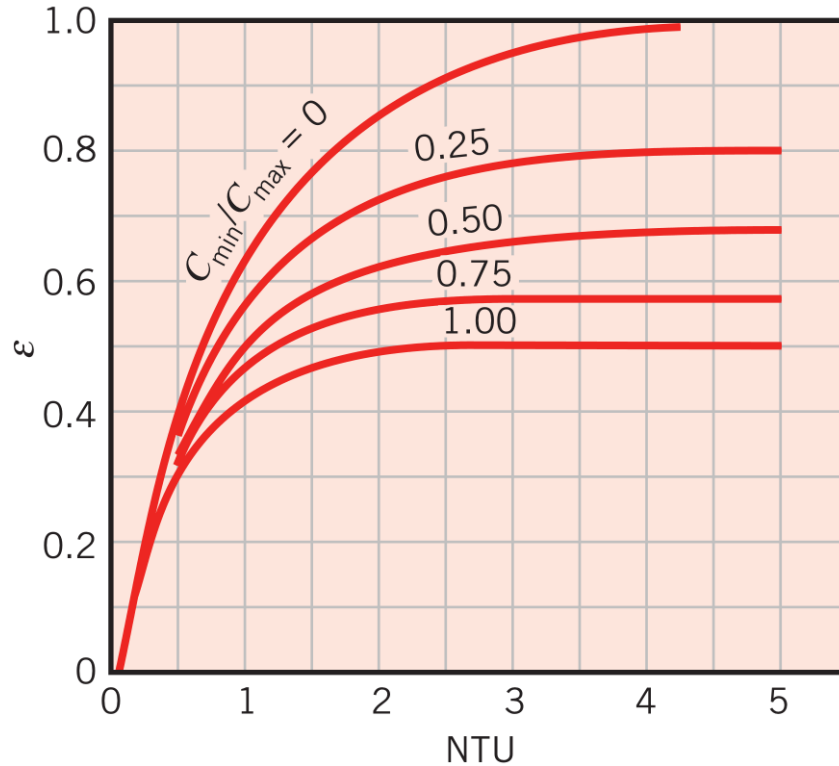
Table 12.2

Flow Arrangement	Relation
<b>Parallel ow</b>	$NTU = -\frac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$
<b>Counterow</b>	$NTU = \frac{1}{C_r - 1} \ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \quad (C_r < 1)$ $NTU = \frac{\varepsilon}{1 - \varepsilon} \quad (C_r = 1)$
<b>Shell-and-tube</b>	
One shell pass (2, 4, . . . tube passes)	$(NTU)_1 = -(1 + C_r^2)^{-1/2} \ln\left(\frac{E - 1}{E + 1}\right)$ $E = \frac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$
$n$ shell passes (2n, 4n, . . . tube passes)	<p>Use Equations 11.30b and 11.30c with</p> $\varepsilon_1 = \frac{F - 1}{F - C_r} \quad F = \left(\frac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad NTU = n(NTU)_1$
<b>Cross-ow (single pass)</b>	
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$NTU = -\ln\left[1 + \left(\frac{1}{C_r}\right) \ln(1 - \varepsilon C_r)\right]$
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$NTU = -\left(\frac{1}{C_r}\right) \ln[C_r \ln(1 - \varepsilon) + 1]$
<b>All exchangers (<math>C_r = 0</math>)</b>	$NTU = -\ln(1 - \varepsilon)$

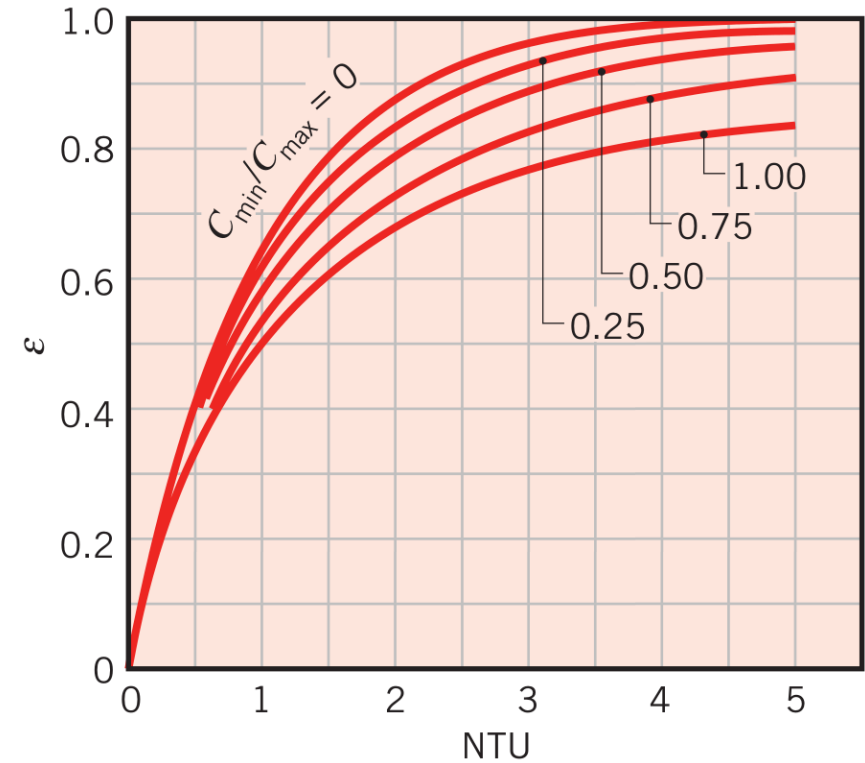


# Effectiveness-NTU Relationships

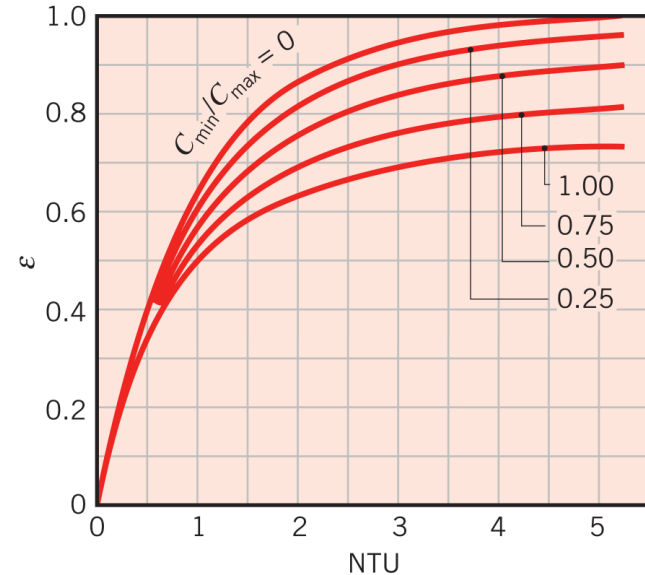
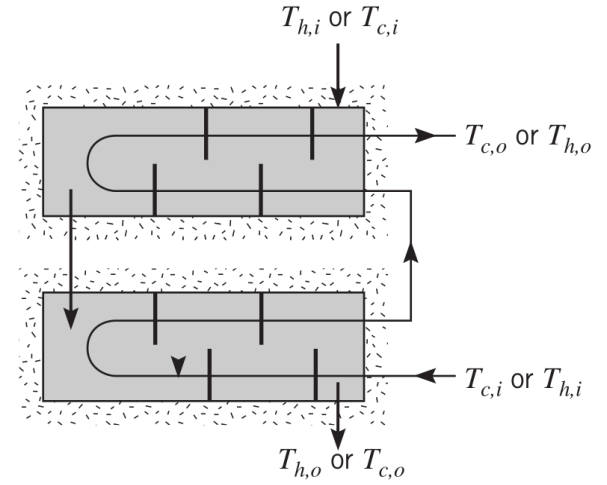
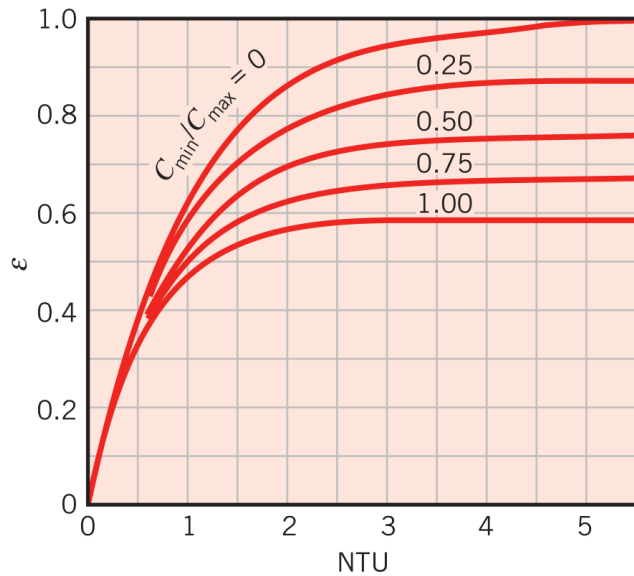
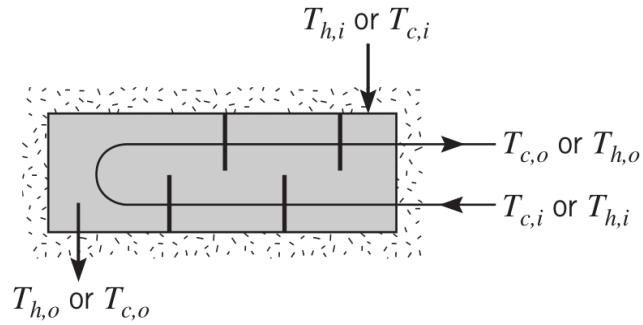
## Parallel Flow



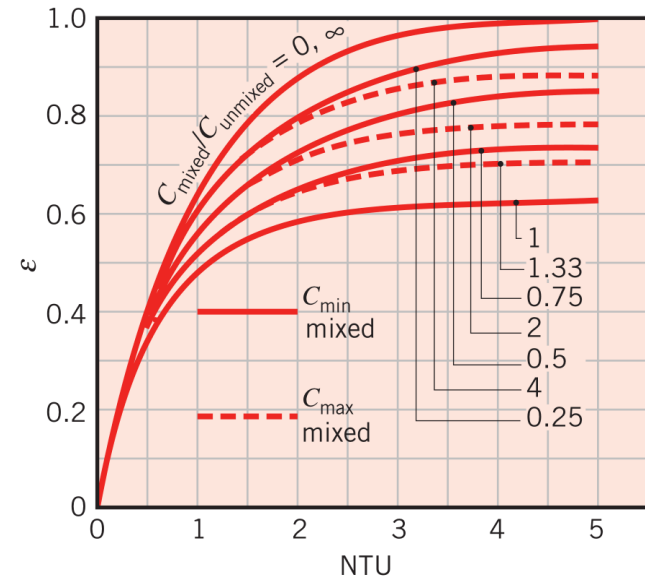
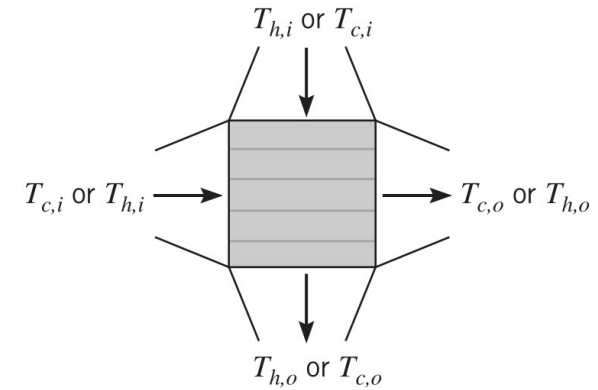
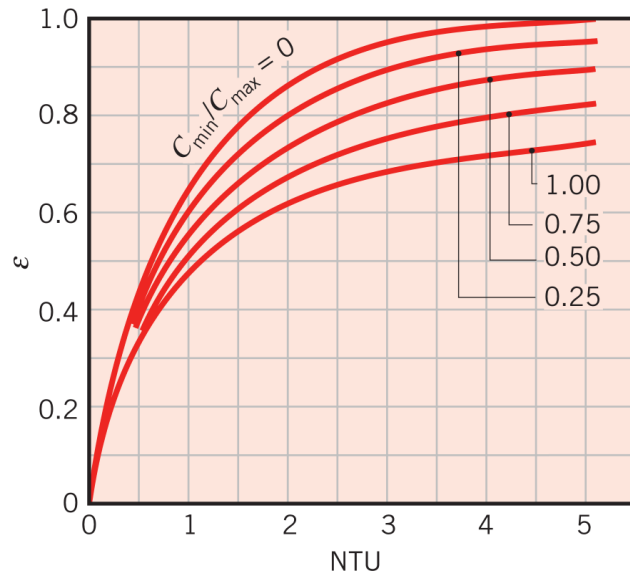
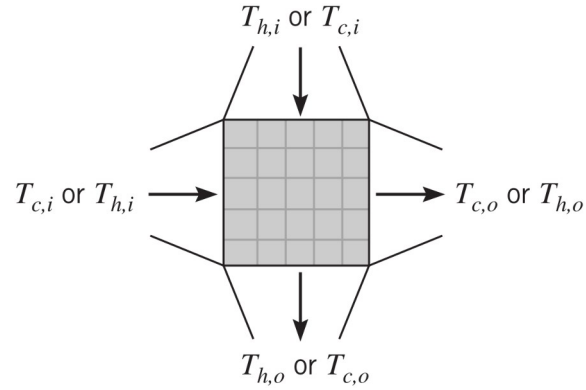
## Counter Flow



# Effectiveness-NTU Relationships



# Effectiveness-NTU Relationships



# Example 1

Hot exhaust gases, which enter a finned-tube, cross-flow heat exchanger at  $300\text{ }^{\circ}\text{C}$  and leave at  $100\text{ }^{\circ}\text{C}$ , are used to heat pressurized water at a flow rate of  $1\text{ kg/s}$  from  $35$  to  $125\text{ }^{\circ}\text{C}$ . The overall heat transfer coefficient based on the gas-side surface area is  $U_h = 100\text{ W/m}^2\text{K}$ . Determine the required gas-side surface area  $A_h$  using the  $NTU$  method.

## Example 2

Consider the heat exchanger design from the previous example, that is, a finned-tube, cross-flow heat exchanger with a gas-side overall heat transfer coefficient and area of  $100 \text{ W/m}^2\text{K}$  and  $38.23 \text{ m}^2$ , respectively. The water flow rate and inlet temperature remain at  $1 \text{ kg/s}$  and  $35 \text{ }^\circ\text{C}$ . However, a change in operating conditions for the hot gas generator causes the gases to now enter the heat exchanger with a flow rate of  $1.5 \text{ kg/s}$  and a temperature of  $250 \text{ }^\circ\text{C}$ . What is the rate of heat transfer by the exchanger, and what are the gas and water outlet temperatures?