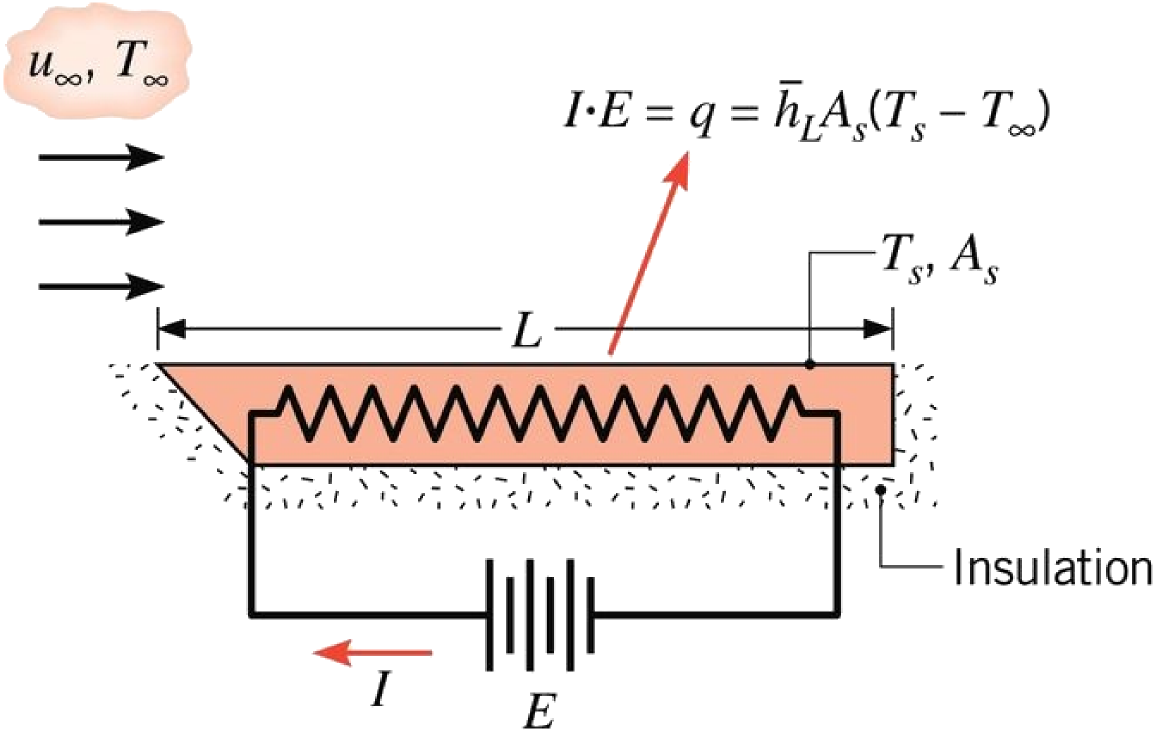
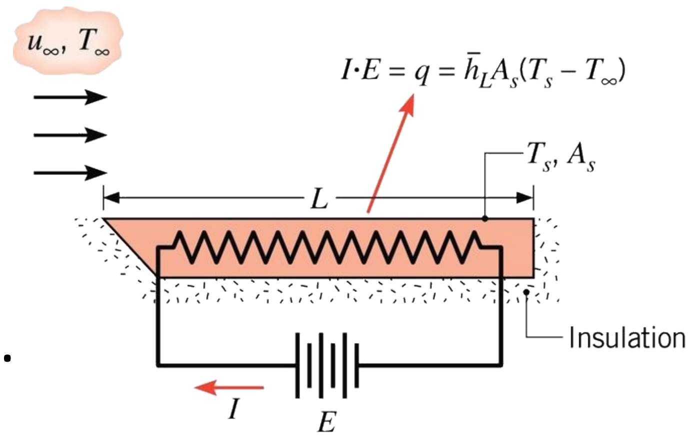
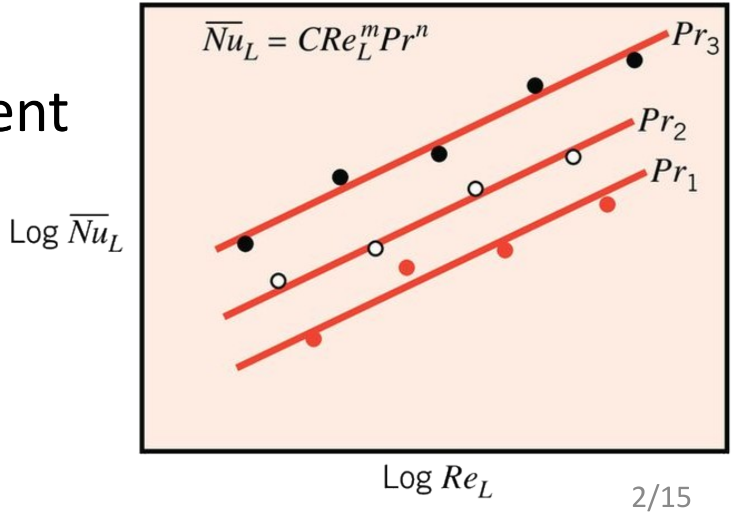
External Flow



Flow Experiment

- Suppose flow over a heated plate is arranged as shown to the right.
- The Reynolds, average Nusselt, and Prandtl numbers are noted for different flow conditions.
- The average Nusselt numbers may then be plotted against the Reynolds number on a log-log scale.
- The different color dots here represent different fluids.
- The C, m, and n are empirical coefficients.

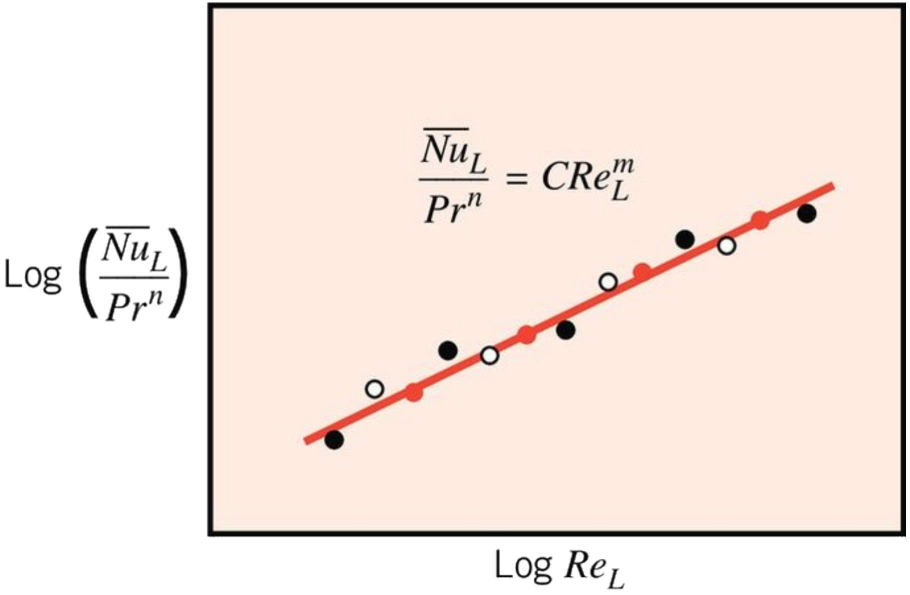




Flow Experiment

- The plot relation may be rewritten to be applicable for any type of fluid.
- This indicates that there is a functional relationship between the various dimensionless numbers.

$$\frac{\bar{N}u_L}{Pr^n} = C\,Re_L^m$$



- There is an underlying constant fluid property assumption with this relation.
- This is be handled in one of two ways.
  - Film Temperature method $T_f \equiv \frac{T_s + T_\infty}{2}$
  - Property variation method

Property Variation Method

- The property variation method involves evaluated all material properties at the far field temperature T_∞.
- The right hand side of the functional relation between the Reynolds, average Nusselt, and Prandtl numbers are then multiplied by various ratios of material properties.
- For example, the ratios shown below may be used.

$$\left(\frac{Pr_\infty}{Pr_s}\right)^r \qquad \left(\frac{\mu_\infty}{\mu_s}\right)^r$$

Flat Plate: Isothermal Surface

- From empirical studies of parallel flow with an isothermal plate (i.e. T_s is constant), the following relations have been discovered.

Laminar

$$\delta = \frac{5}{\sqrt{\frac{u_\infty}{\nu x}}} = \frac{5x}{\sqrt{Re_x}}$$

$$\frac{\delta}{\delta_t} \approx Pr^{1/3}$$

$$C_{f,x} \equiv \frac{\tau_{s,x}}{\frac{\rho u_\infty^2}{2}} = 0.664\, Re_x^{-1/2}$$

$$Nu_x \equiv \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3} \quad \left(Pr \gtrsim 0.6\right)$$

Turbulent

$$\delta = 0.37\, x\, Re_x^{-1/2}$$

$$C_{f,x} = 0.0592\, Re_x^{-1/5} \quad \left(Re_{x,c} \lesssim Re_x \lesssim 10^8\right)$$

$$Nu_x = 0.0296\, Re_x^{4/5}\, Pr^{1/3} \quad \left(0.6 \lesssim Pr \lesssim 60\right)$$

Flat Plate: Isothermal Surface

- For the average friction coefficient and Nusselt number, an intermediate parameter A is used for the case of turbulent flow.

$$A = 0.037\, Re_{x,c}^{4/5} - 0.664\, Re_{x,c}^{1/2}$$

Laminar

$$\bar{C}_{f,x} = 1.328\, Re_x^{-1/2}$$

$$\bar{Nu}_x \equiv \frac{\bar{h}_x x}{k} = 0.664\, Re_x^{1/2}\, Pr^{1/3} \quad \left(Pr \gtrsim 0.6\right)$$

Turbulent

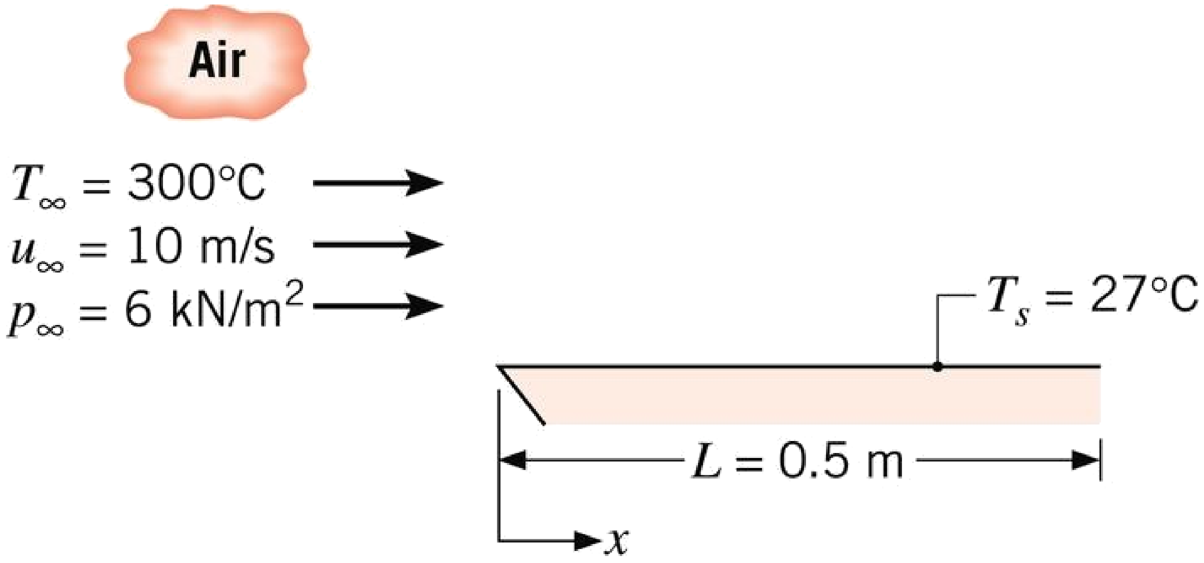
$$\bar{C}_{f,L} = 0.074\, Re_L^{-1/5} - \frac{2A}{Re_L} \quad \left(Re_{x,c} \lesssim Re_L \lesssim 10^8\right)$$

$$\bar{Nu}_L = \left(0.037\, Re_L^{4/5} - A\right) Pr^{1/3} \quad \left(0.6 \lesssim Pr \lesssim 60\right)$$

$$\left(Re_{x,c} \lesssim Re_L \lesssim 10^8\right)$$

Example 1

Air at a pressure of $6\ kPa$ and a temperature of $300\ °C$ flows with a velocity of $10\ m/s$ over a flat plate $0.5\ m$ long. Estimate the cooling rate per unit width of the plate needed to maintain it at a surface temperature of $27\ °C$.



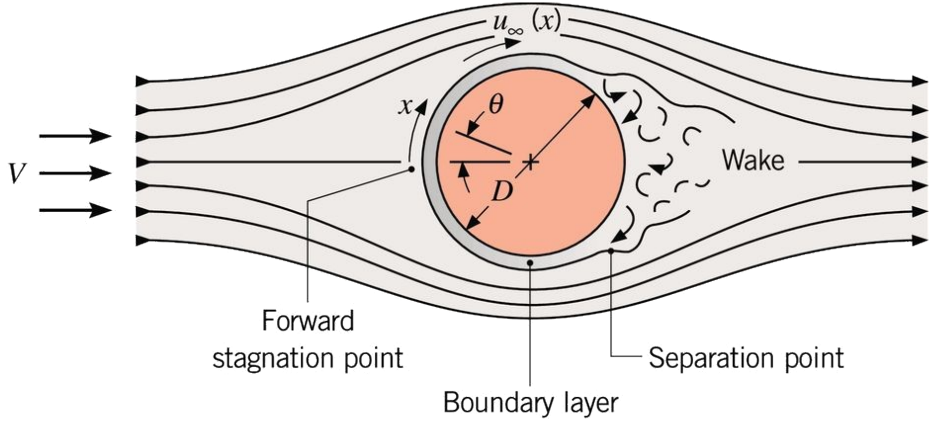
Cylinder in Cross Flow

- Consider a cylinder experiencing cross flow.
- The Reynolds number Re_D for this type of flow may be defined as

$$Re_D \equiv \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

- The drag coefficient C_D is defined as

$$C_D \equiv \frac{F_D}{A_f\left(\frac{\rho V^2}{2}\right)}$$



where A_f is the cylinder frontal area.

- The frontal area is the area projected perpendicular to the flow.
- The separation point is a function of Reynolds number.

Separation Point

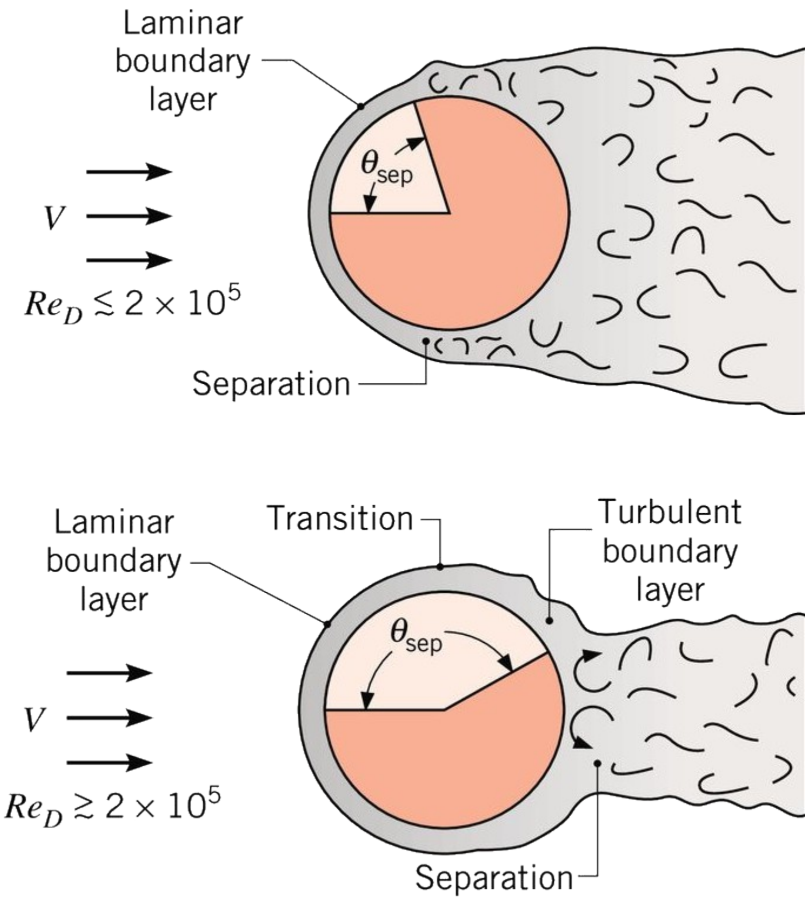
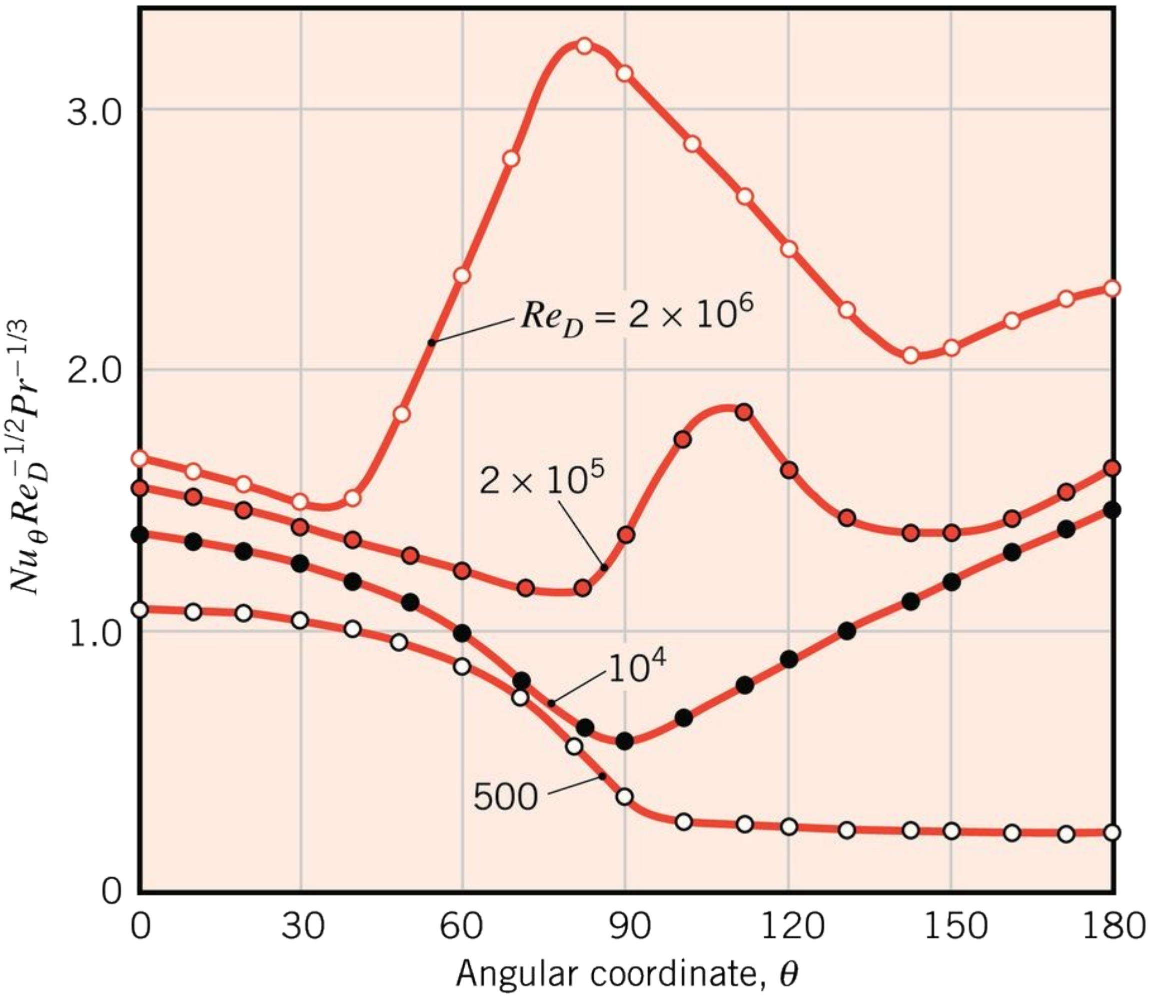
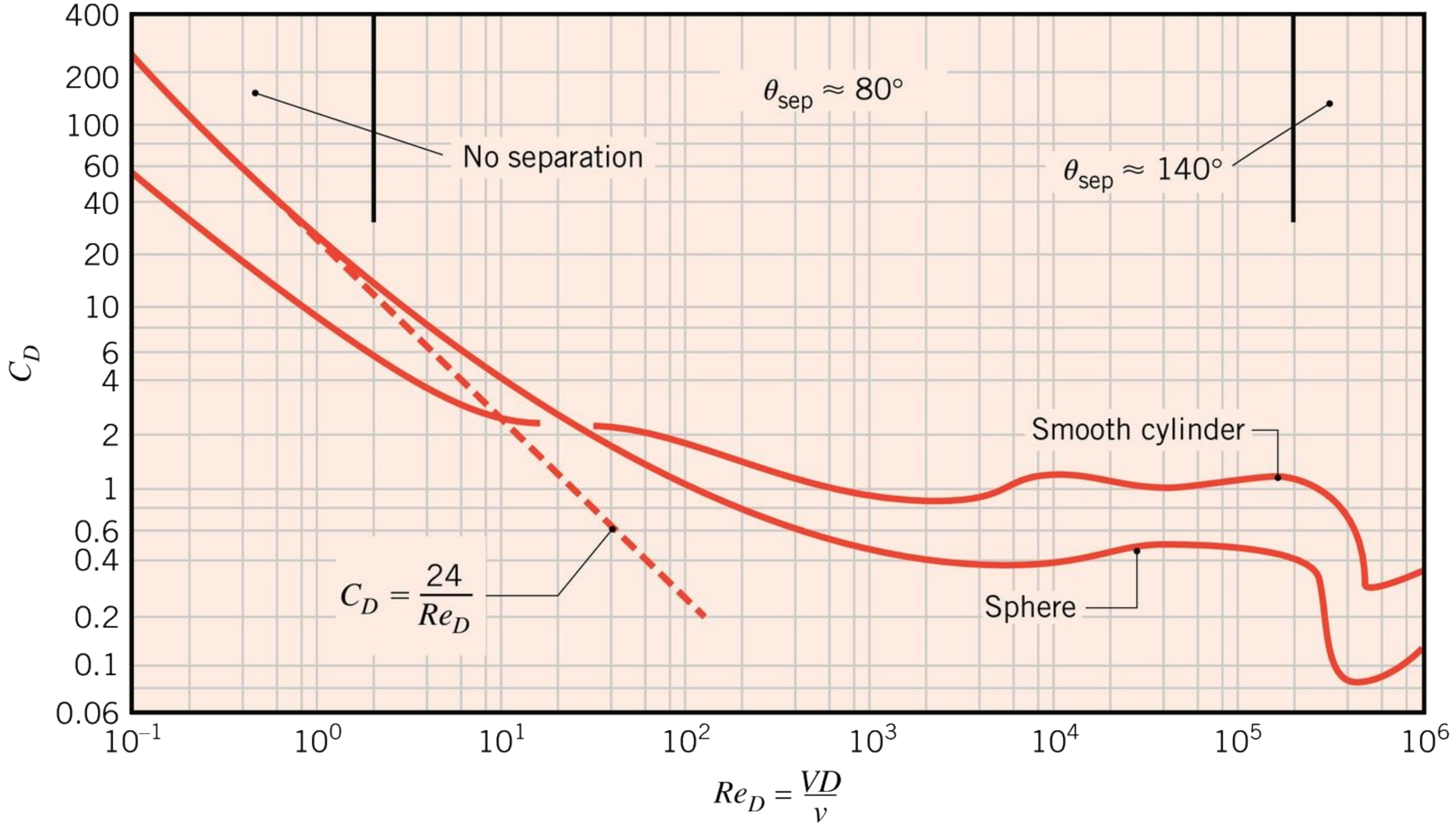


Figure 9.1



Drag Coefficient

Figure 9.2



Cylinder in cross flow

- Rarely is it of concern the local Nusselt number for a cylinder in cross flow.
- However, the average Nusselt number is often used and is given by

$$\overline{Nu}_D \equiv \frac{\bar{h} D}{k} = C\, Re_D^m\, Pr^{1/3} \qquad \left(Pr \gtrsim 0.7\right)$$

Table 9.1

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

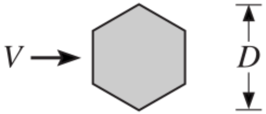
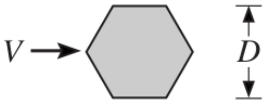
Various geometries in cross flow

- The same relation may be used for various geometries in cross flow.

$$\overline{Nu}_D \equiv \frac{\bar{h} D}{k} = C\, Re_D^m\, Pr^{1/3}$$

$$\left(Pr \gtrsim 0.7\right)$$

Table 9.2

Geometry		Re_D	C	m
Square				
		6000–60,000	0.304	0.59
		5000–60,000	0.158	0.66
Hexagon				
		5200–20,400	0.164	0.638
		20,400–105,000	0.039	0.78
		4500–90,700	0.150	0.638
Thin plate perpendicular to flow				
	Front	10,000–50,000	0.667	0.500
	Back	7000–80,000	0.191	0.667

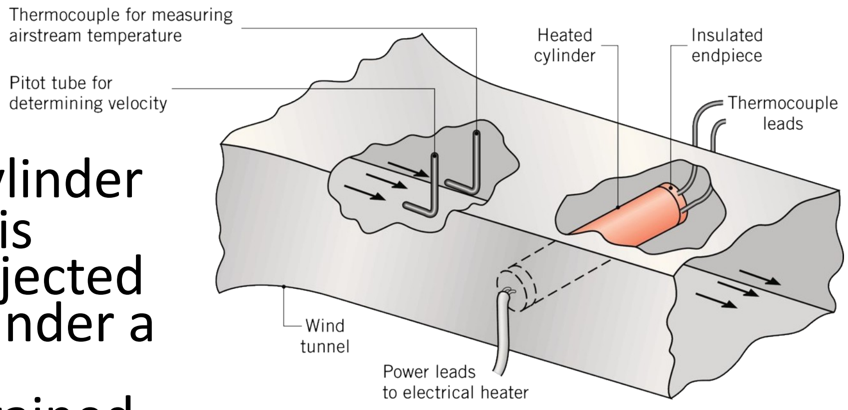
Example 2

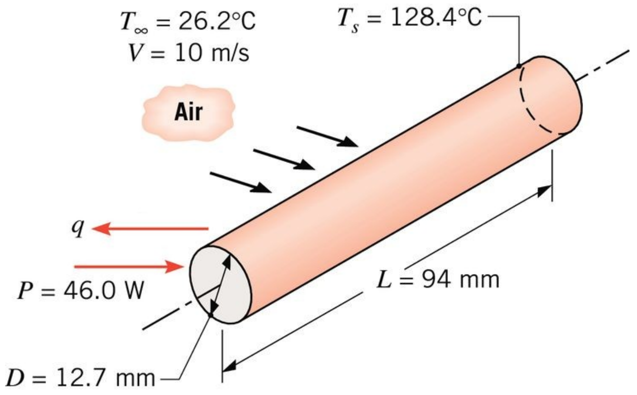
Pitot Tube?

Experiments have been conducted on a metallic cylinder $12.7\ mm$ in diameter and $94\ mm$ long. The cylinder is heated internally by an electrical heater and is subjected to a cross flow of air in a low-speed wind tunnel. Under a specific set of operating conditions for which the upstream air velocity and temperature were maintained at $V = 10\ m/s$ and $26.2\ °C$, respectively, the heater power dissipation was measured to be $P = 46\ W$, while the average cylinder surface temperature was determined to be $T_s = 128.4\ °C$. It is estimated that 15% of the power dissipation is lost through the cumulative effect of surface radiation and conduction through the end pieces.

(a) Determine the convection heat transfer coefficient from the experimental observations.

(b) Compare the experimental result with the convection coefficient computed from an appropriate correlation.





The Sphere

- Boundary layers develop very similarly to that of the circular cylinder.
- For very small Reynolds numbers, the drag coefficient is

$$C_D = \frac{25}{Re_D} \qquad \left(Re_D \lesssim 0.5\right)$$

- The average Nusselt number is given by

$$\bar{N}u_D = 2 + \left(0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right) Pr^{0.4} \left(\frac{\mu}{\mu_s}\right)^{1/4} \qquad \begin{array}{l} \left(0.71 \lesssim Pr \lesssim 380\right) \\ \left(3.5 \lesssim Re_D \lesssim 7.6\times 10^4\right) \\ \left(1.0 \lesssim \dfrac{\mu}{\mu_s} \lesssim 3.2\right) \end{array}$$

where all properties except for μ_s are evaluated at T_∞.

Example 3

Air at $25\ °C$ flows over a $10\ mm$ diameter sphere with a velocity of $15\ m/s$, while the surface of the sphere is maintained at $75\ °C$.

(a) What is the drag force on the sphere?

(b) What is the rate of heat transfer from the sphere?

(c) Generate a plot of the heat transfer from the sphere as a function of the air velocity for the range 1 to $25\ m/s$.