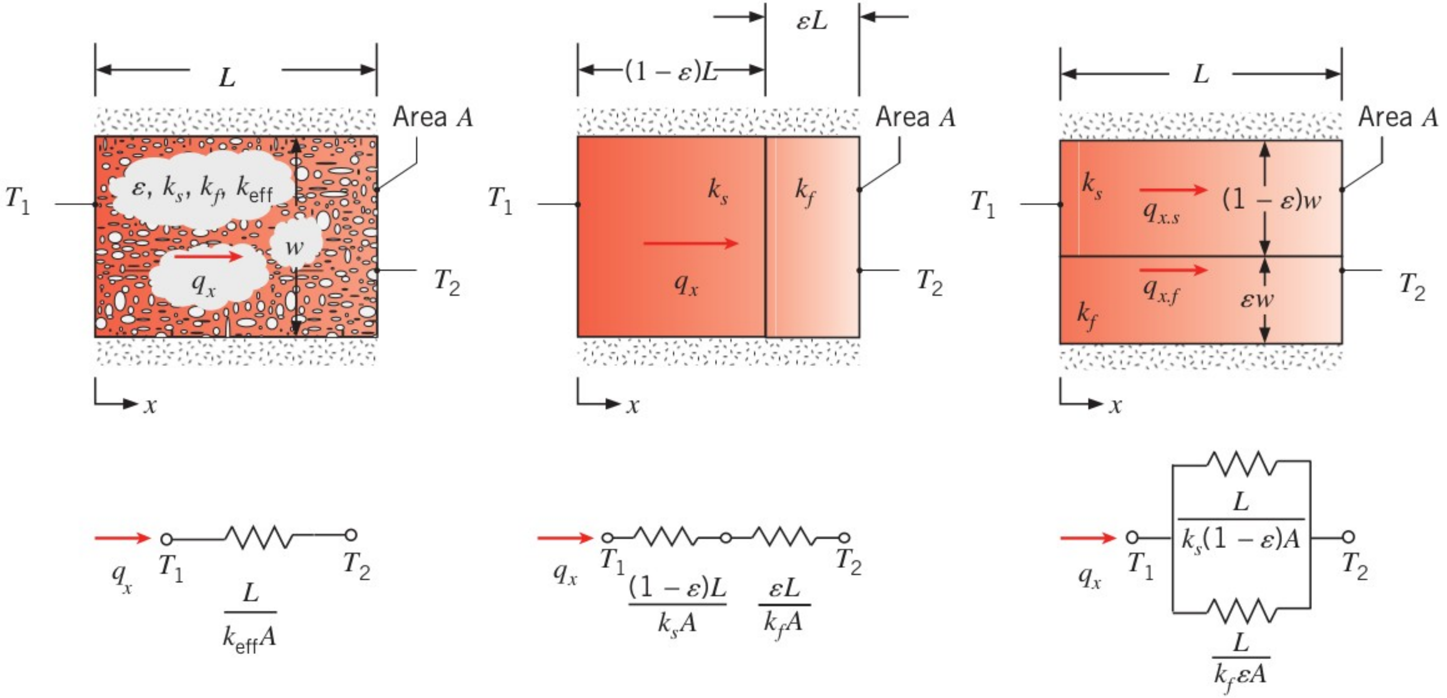
Thermal Resistance



Thermal Resistance

- Let's look at a special case of the Heat Diffusion Equation (HDE)

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

- For a plane 1D wall, steady state, and no heat generation, the HDE becomes

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=0$$

- The general solution of this ordinary differential equation is

$$T(x)=C_1x+C_2$$

where C_1 and C_2 are some constants determined by boundary conditions.

Thermal Resistance

- The constants C_1 and C_2 may be determined by applying a constant surface temperature boundary condition to each wall surface located at 0 and L.
- This yields

  $$C_1 = \frac{T_{s,2} - T_{s,1}}{L} \qquad C_2 = T_{s,1}$$

  where $T_{s,1}$, and $T_{s,2}$ is the surface temperature at the wall surface located at 0 and L, respectively.
- Substituting these into the temperature distribution expression shows

  $$T(x) = (T_{s,2} - T_{s,1}) \frac{x}{L} + T_{s,1}$$

Thermal Resistance

$$T(x) = (T_{s,2} - T_{s,1})\frac{x}{L} + T_{s,1}$$

- With this temperature distribution expression, we can see how conduction behaves

$$q = -kA\frac{\partial T}{\partial x} = -kA\frac{\partial}{\partial x}\left[(T_{s,2} - T_{s,1})\frac{x}{L} + T_{s,1}\right] = \frac{kA}{L}(T_{s,1} - T_{s,2})$$

- Rearranging shows

$$\frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{kA}$$

- This is what is known as thermal resistance for conduction.

$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{kA}$$

Thermal Resistance

$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{kA}$$

- Let's look at the units

$$\frac{K}{W} [=] \frac{K}{\frac{J}{s}} [=] \frac{m}{\left(\frac{W}{mK}\right)(m^2)} [=] \frac{m}{\frac{W\,m}{K}} [=] \frac{K}{W} [=] \frac{K}{\frac{J}{s}}$$

- Recall that electrical resistance is given by

$$R = \frac{\rho L}{A} = \frac{V}{I}$$

- A few analogies may be made to equate certain thermal elements to electrical elements

Type	Electrical	Thermal
Resistance	R	$R_{t,cond}$
Flow	I	q
Potential	V	T

Thermal Resistance

- This may be applied to other modes of heat transfer.
- Convection

$$R_{t,conv} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$

- Radiation between a surface and large surroundings

$$R_{t,rad} = \frac{T_s - T_{sur}}{q} = \frac{1}{h_r A}$$

where h_r is the radiation heat transfer coefficient and is given by

$$h_r = \varepsilon \sigma (T_s + T_{sur})(T_s^2 + T_{sur}^2)$$

Composite Wall (series)

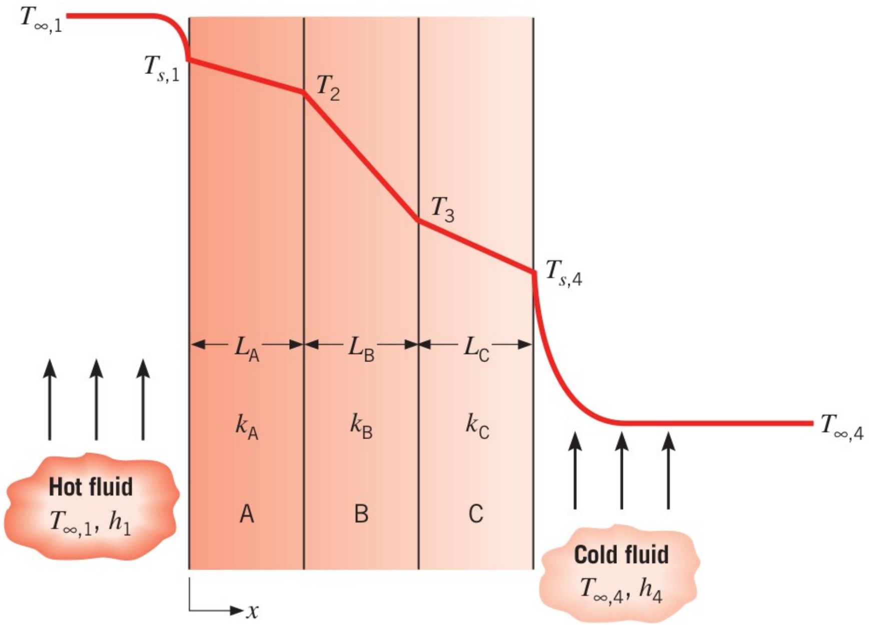
- Consider the composite wall shown to the right.
- The thermal resistance of wall A, B, and C is given by

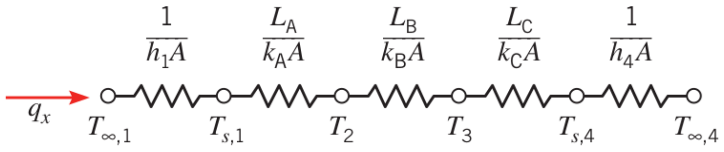
$$R_A = \frac{L_A}{k_A A} \qquad R_B = \frac{L_B}{k_B A} \qquad R_C = \frac{L_C}{k_C A}$$

- The thermal resistance of the two convection boundaries are

$$R_1 = \frac{1}{h_1 A} \qquad R_4 = \frac{1}{h_4 A}$$

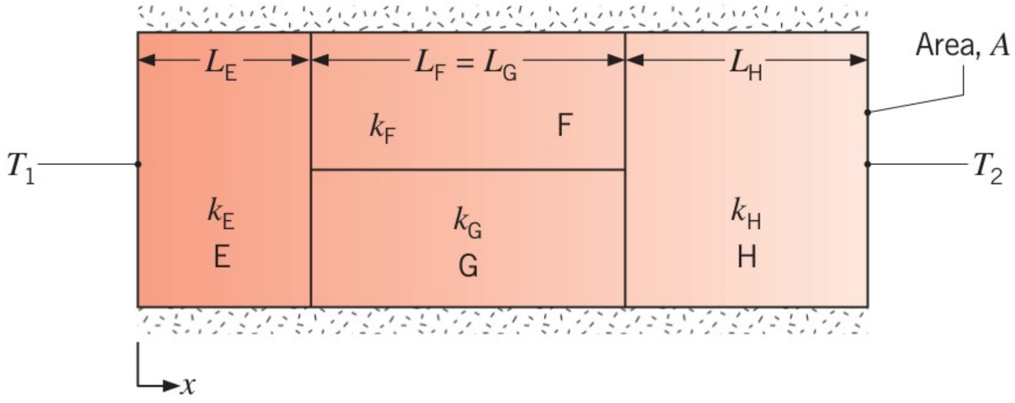
- The thermal circuit may then be drawn as
- The heat rate is then $q_x = \dfrac{T_{\infty,1} - T_{\infty,4}}{\sum R_t}$

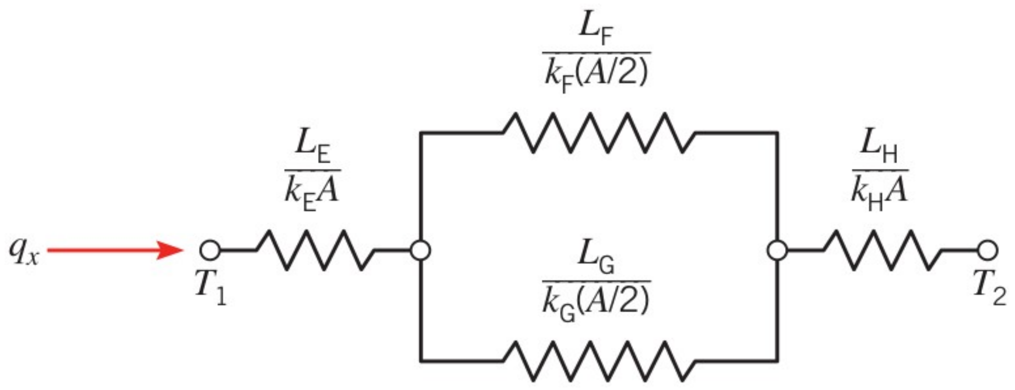




Composite Wall (parallel)

- A similar approach may be used for conduction in parallel.
- Consider the composite wall to the upper right.
- The equivalent thermal circuit would be as shown to the lower right.
- Remember from the initial analysis, 1D conduction was assumed so it is assumed that there is no heat transfer between wall F and G.





Composite Wall

- It is often convenient to work with an overall heat transfer coefficient.
- It is often denoted as U so

$$q = U\,A\,\Delta T$$

- U may then be defined as

$$U = \frac{1}{R_{tot}\,A}$$

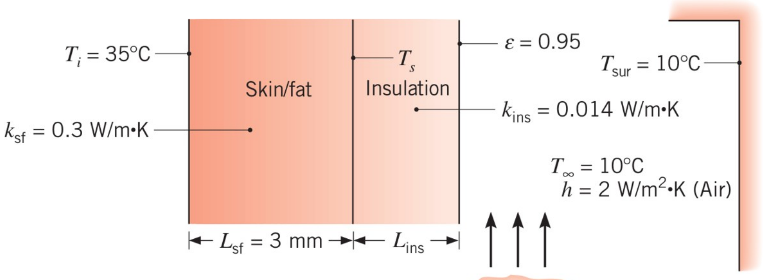
- Or more often represented as

$$R_{tot} = \Sigma R_t = \frac{1}{U\,A}$$

- R_{tot} would be equivalent to R_{eq} in an electrical circuit.

Example 1

Consider a person that has a layer of skin/fat covered with a special sporting gear suit, with the outer surface exposed to air at a temperature of $10\ °C$ with a convective heat transfer coefficient of $2\ W/m^2K$. The inner surface (under the layer of skin/fat) is maintained at a constant temperature of $35\ °C$ through a process called thermoregulation. The skin/fat layer has a thickness of $3\ mm$ and an effective thermal conductivity of $0.3\ W/mK$. The total surface area of the skin/fat is $1.8\ m^2$. The special sporting gear suit has a thermal conductivity of $0.014\ W/mK$. The emissivity of the outer surface of the special sporting gear suit is 0.95. What thickness of the sporting gear suit is needed to reduce the heat loss rate to $100\ W$? What is the resulting skin temperature?



Contact Resistance

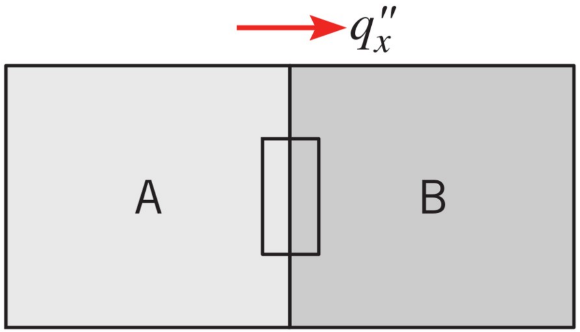
- Whenever composite materials are arranged, there exists a finite temperature drop across the touching surfaces of the materials.
- This finite temperature drop may be equated to a thermal contact resistance per unit area, $R''_{t,c}$.

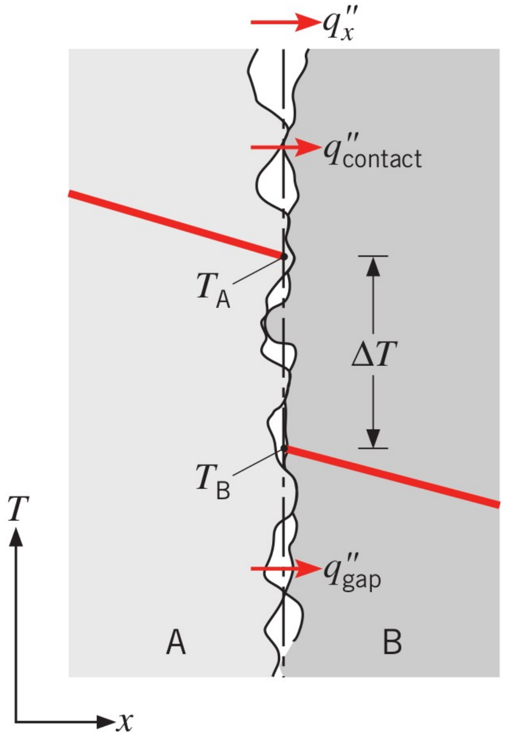
$$R''_{t,c} = \frac{T_A - T_B}{q''}$$

- The thermal contact resistance is a function of many different factors:
  - Material
  - Surface finish
  - Contact pressure
  - Interfacial fluid

Contact Resistance

- Consider the two materials shown to the upper right.
- If a section of the interface is closely examined, it may appear as the lower image shown to the right.
- As can be seen, there appears to points where there is material A to material B contact.
- However, there are also gaps.
- Heat will transfer differently at these locations.
- The effective (or "average") heat transfer is what causes thermal contact resistance.





Contact Resistance

- Empirical studies have been conducted to determine thermal contact resistance.
- For vacuum interfaces at different contact pressures

Table 5.1

Material	100 (kPa)	10,000 (kPa)
Stainless Steel	6-25	0.7-4.0
Copper	1-10	0.1-0.5
Magnesium	1.5-3.5	0.2-0.4
Aluminum	1.5-5.0	0.2-0.4

where values are $R''_{t,c} \times 10^4\ (m^2K/W)$

- For aluminum interface ($10\ \mu m$ surface roughness, $10\ kPa$ contact pressure)

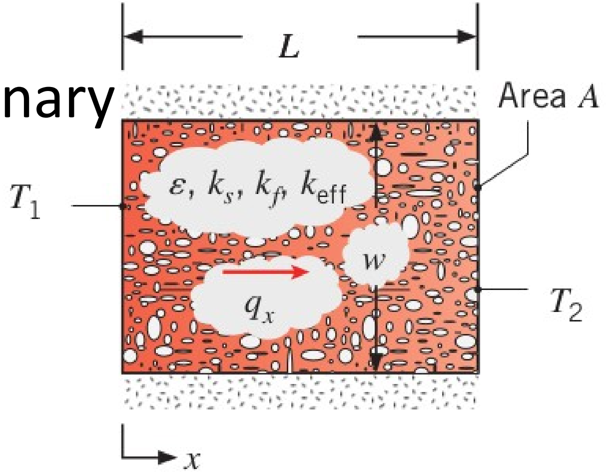
Table 5.2

Interfacial Fluid	$R''_{t,c} \times 10^4\ (m^2K/W)$
Air	2.75
Helium	1.05
Hydrogen	0.720
Silicone Oil	0.525
Glycerine	0.265

Example 2

A composite wall separates combustion gases at *2600 °C* from a liquid coolant at *100 °C*, with gas and liquid side convection coefficients of *50* and *1000* W/m^2K, respectively. The wall is composed of *10 mm* thick layer of beryllium oxide on the gas side and a *20 mm* thick slab of stainless steel (AISI 304) on the liquid side. The contact resistance between the oxide and the steel is *0.05* m^2K/W. What is the heat lost per unit surface area of the composite? Sketch the temperature distribution from the gas to the liquid.

Porous Materials



- Consider a material that consists of pockets of stationary solid and fluid states.
- The heat transfer may be expressed as

  $$q = \frac{k_{eff} A}{L}(T_1 - T_2)$$

  where k_{eff} is the effective thermal conductivity.
- Let k_s and k_f represent the thermal conductivity of the solid and fluid portions, respectively.
- Let ε represents the fluid volume fraction (fluid volume over total volume).
- The porous material may then be represented two different ways.

Porous Materials

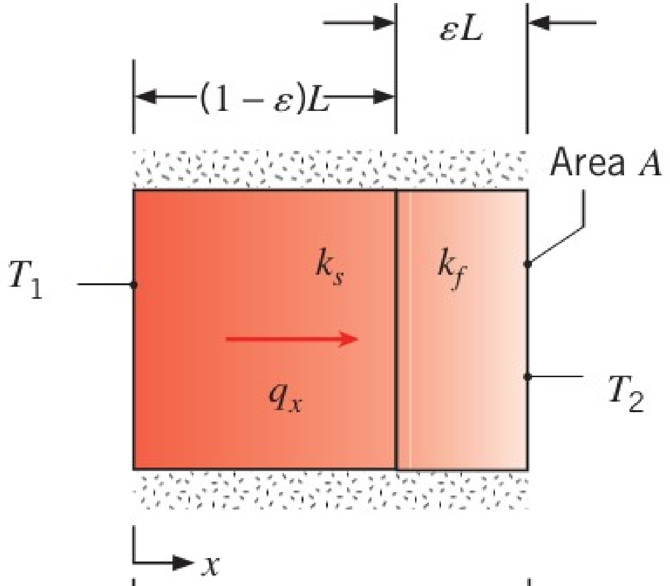
- The first representation is as if two materials are in series, the k_{eff} term is then

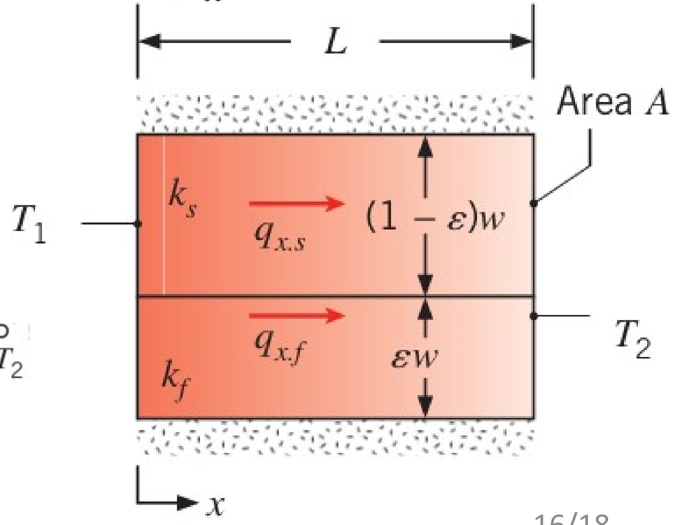
$$k_{eff,min} = \frac{1}{\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}}$$

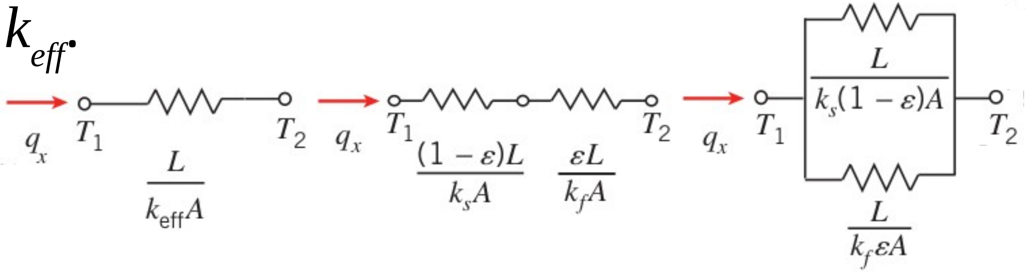
- The second representation is as if two materials are in parallel, the k_{eff} terms is then

$$k_{eff,max} = \varepsilon k_f + (1-\varepsilon) k_s$$

- This approach provides minimum and maximum values for k_{eff}.







Example 3

A batt of glass fiber insulation has a density of $28\ kg/m^3$. Determine the maximum and minimum possible values of the effective thermal conductivity of the insulation at a temperature of $300\ K$. How does this compare to the values reported in Table A.3?

Cylindrical Systems

- A similar approach as the 1D plane wall may be used to derived the thermal resistance in a cylindrical system.

$$R_{t,cond} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L}$$

$$R_{t,conv} = \frac{1}{h 2\pi r L}$$