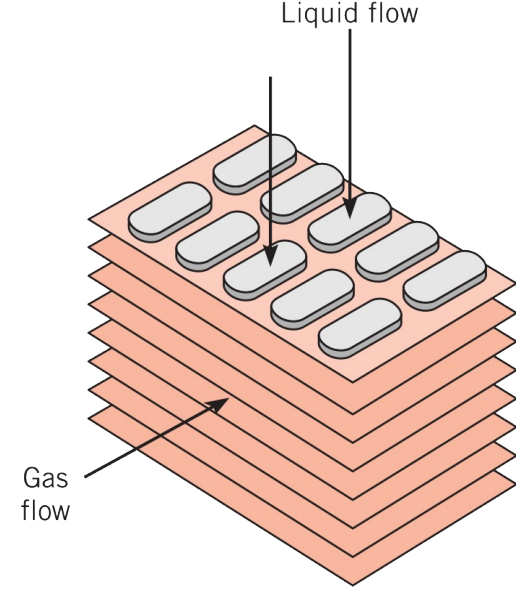
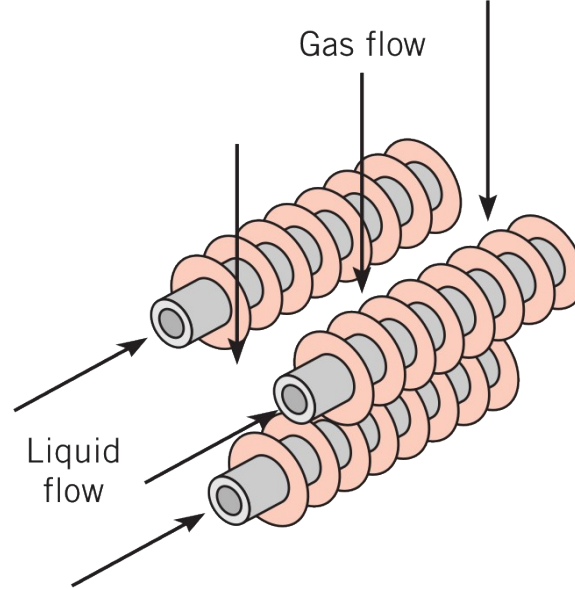
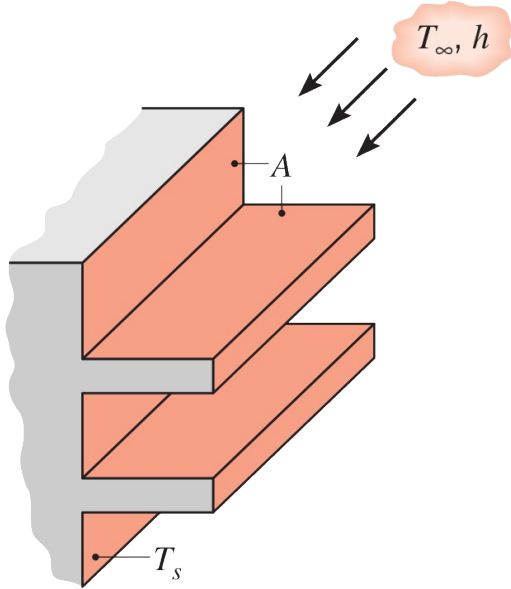
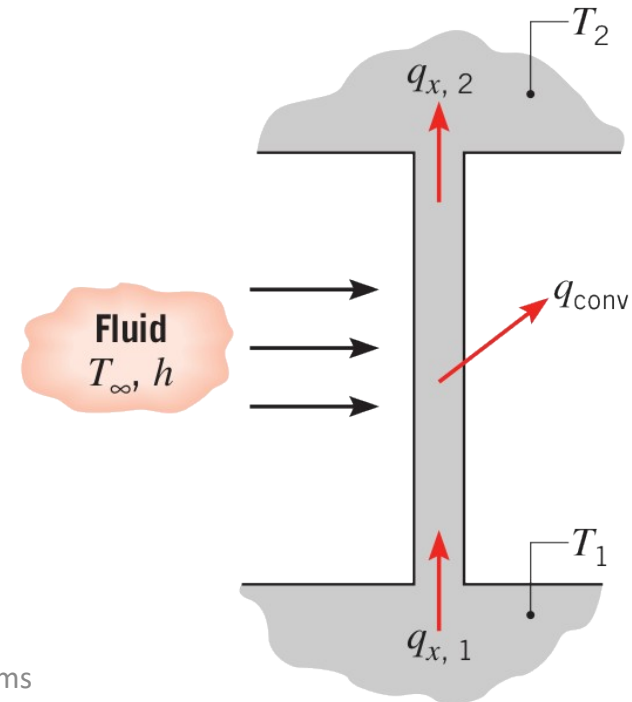
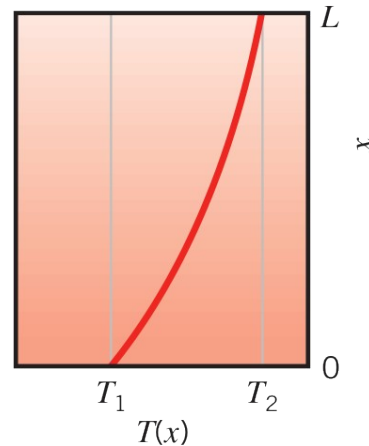


Extended Surfaces



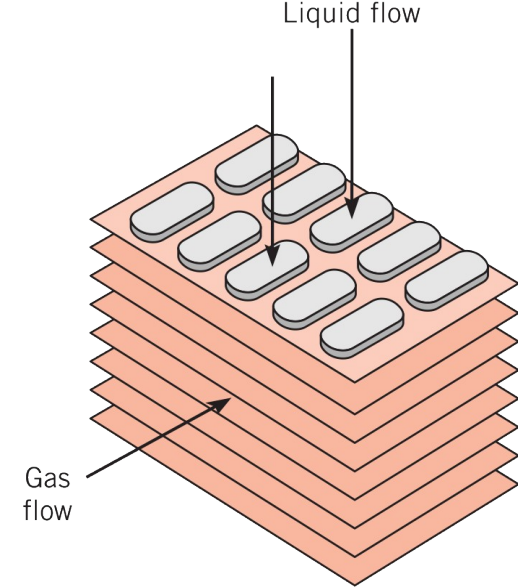
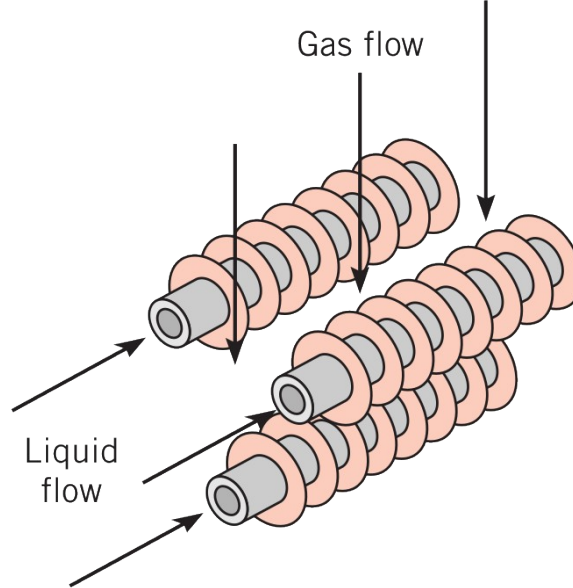
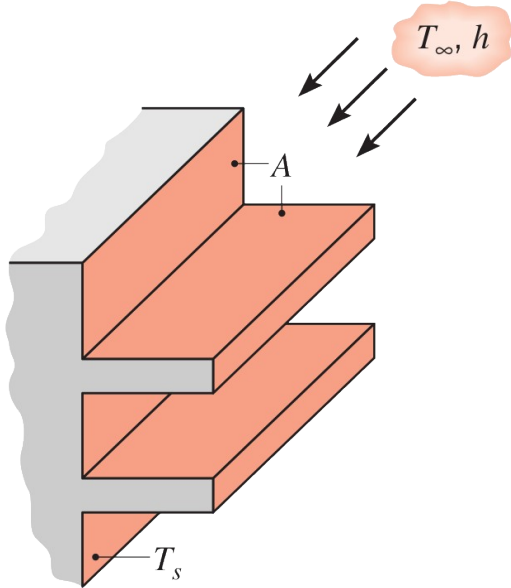
Extended Surfaces

- Until now, the heat transfer from solid boundaries have been in the same direction as the heat transfer in the solid (by conduction).
- For an extended surface, the direction from the solid boundaries are perpendicular to the direction of the heat transfer in the solid.
- Consider a strut:
 - Connects two walls at different temperature.
 - Fluid flows across strut.
 - $T_1 > T_2 > T_\infty$



Extended Surfaces

- Main purpose is to enhance heat transfer between solid and fluid.
- An extended surface is known as a n .



Extended Surfaces

- A 1st Law look at an extended surface shows

$$\frac{d^2 T}{dx^2} + \left(\frac{1}{A_c} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \left(\frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_\infty) = 0$$

where A_c is the cross sectional area,
 A_s is the surface area.

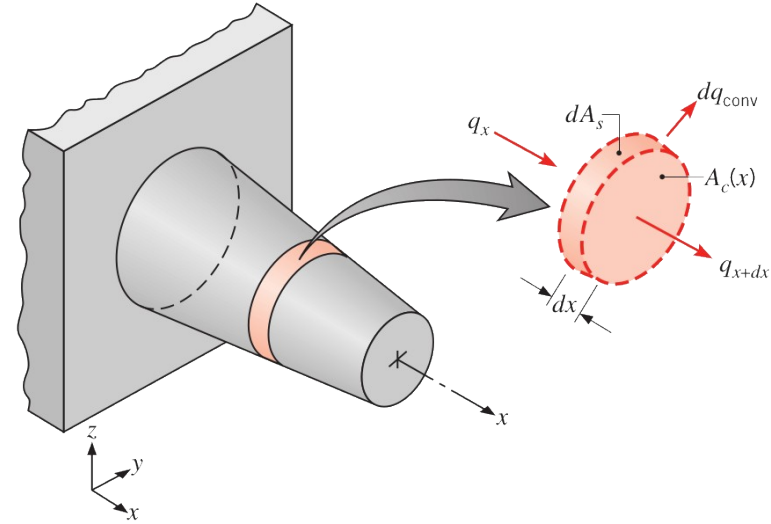
- If the fin has a uniform cross section

- A_c is constant

- $A_s = Px$

- P is the fin perimeter

$$\frac{d^2 T}{dx^2} - \left(\frac{hP}{k A_c} \right) (T - T_\infty) = 0$$



- Often times this may be written as

$$\frac{d^2 \theta}{dx^2} - m^2 \theta = 0$$

where $\theta \equiv T(x) - T_\infty$ and $m^2 \equiv hP/kA_c$

Extended Surfaces

- Consider the two fins shown to the right.

- The rectangular fin has

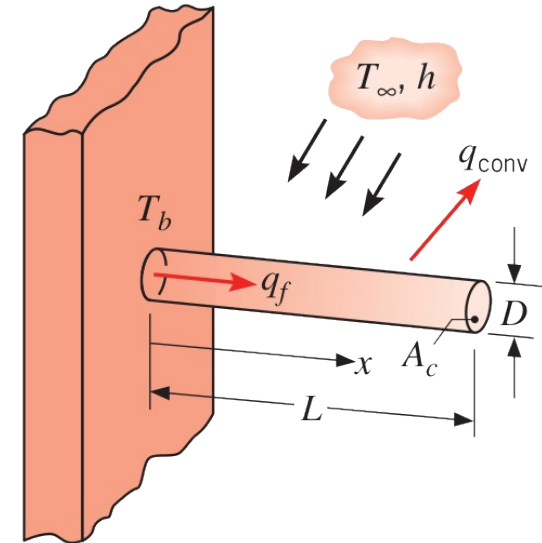
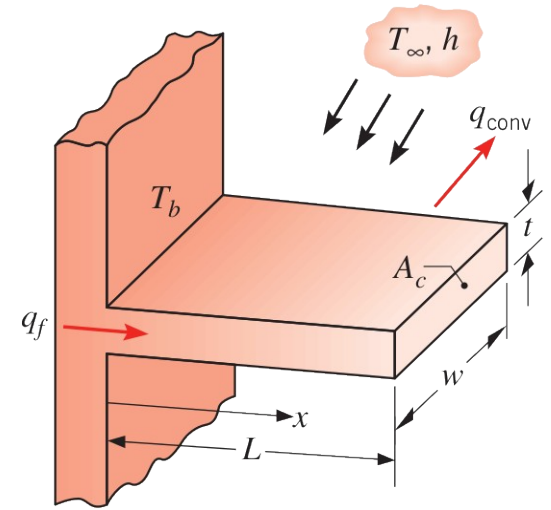
$$P = 2w + 2t$$

$$A_c = wt$$

- The cylindrical fin has

$$P = \pi D$$

$$A_c = \frac{\pi}{4} D^2$$



Extended Surfaces

$$\frac{d^2 \theta}{dx^2} - m^2 \theta = 0$$

- The solution to governing differential equation is

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

where C_1 and C_2 are constants determined by boundary conditions.

- 1st Boundary Condition: Constant temperature at base, T_b

$$\theta(0) = T_b - T_\infty \equiv \theta_b$$

- 2nd Boundary Condition: At tip
 - Convection
 - Adiabatic
 - Constant Temperature
 - Infinite Long Fin

Tip Conditions

Case	Tip Condition	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection	$\frac{\cosh m(L-x) + (h/mk) \sinh m(L-x)}{\cosh mL + (h/mk) \sinh mL}$	$M \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$
B	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Constant Temperature	$\frac{(\theta_L/\theta_b) \sinh mx + \sinh m(L-x)}{\sinh mL}$	$M \frac{\cosh mL - \theta_L/\theta_b}{\sinh mL}$
D	Infinite Fin ($L \rightarrow \infty$)	e^{-mx}	M

$$\theta(x) = T(x) - T_\infty \quad \theta_b = \theta(0) = T_b - T_\infty \quad m^2 = \frac{hP}{kA_c} \quad M = \sqrt{hPkA_c} \theta_b$$

Example 1

A very long rod 5 mm in diameter has one end maintained at $100\text{ }^{\circ}\text{C}$. The surface of the rod is exposed to ambient air at $25\text{ }^{\circ}\text{C}$ with a convection heat transfer coefficient of $100\text{ W/m}^2\text{K}$.

- (a) Determine the temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?
- (b) Estimate how long the rods must be for the assumption of infinite length to yield an accurate estimate of the heat loss.

Fin Performance

- A fin does not guarantee an increase of heat transfer.
- A fin adds a conduction resistance and needs to be less than the gains of the increased convection effects.
- Fin performance may be defined in terms of effectiveness.
- The effectiveness ε_f of a single n fin may be defined as

$$\varepsilon_f = \frac{q_f}{h A_{c,b} \theta_b}$$

where $A_{c,b}$ is the fin cross-sectional area at the base.

- Effectiveness is the ratio of the actual heat transfer over heat transfer without fin.
- Generally, fins are rarely justified unless $\varepsilon_f > 2$.

Fin Effectiveness

- For case D (infinite long fin), the effectiveness is

$$\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$$

- This illustrates that fin performance is due to:
 - Material properties
 - Geometry
- For geometry, the ratio of perimeter to cross sectional area is important.
- For this reason, thin but closely spaced fins is typically preferred.
- However, too closely spaced fins may start to impact convection.

Fin Effectiveness

- The effectiveness of a fin may also be expressed in terms of thermal resistances.

- The thermal resistance of a fin is

$$R_{t,f} = \frac{\theta_b}{q_f}$$

- The thermal resistance of the convection if the fin was not present is

$$R_{t,b} = \frac{1}{h A_{c,b}}$$

- The effectiveness in terms of thermal resistance is then

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}$$

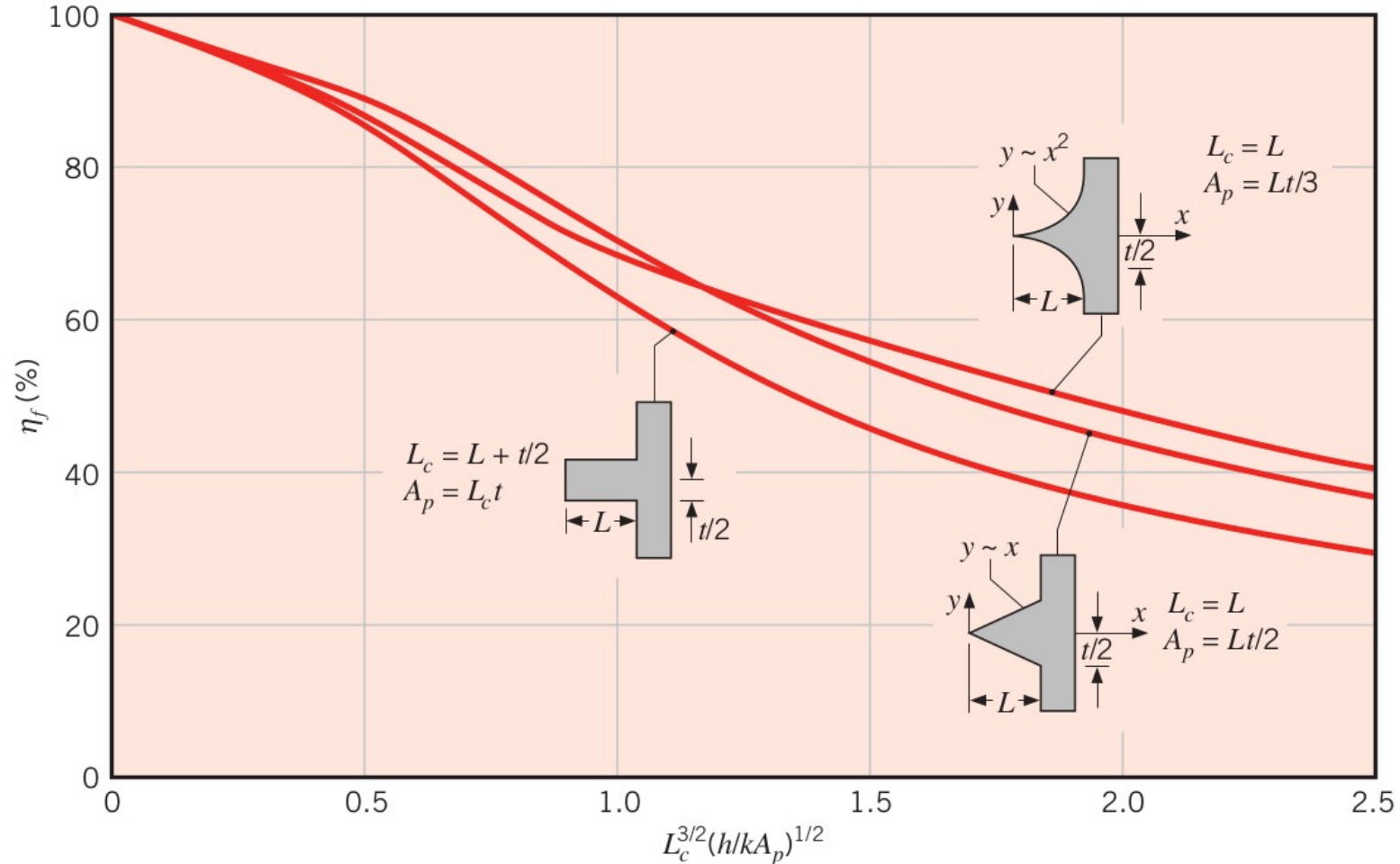
Fin Efficiency

- Another measure of fin performance is fin efficiency.
- Fin efficiency is defined in terms of the maximum heat transfer possible.
- The maximum heat transfer possible would occur if the entire fin, not just the base, was at the base temperature.
- This is written as

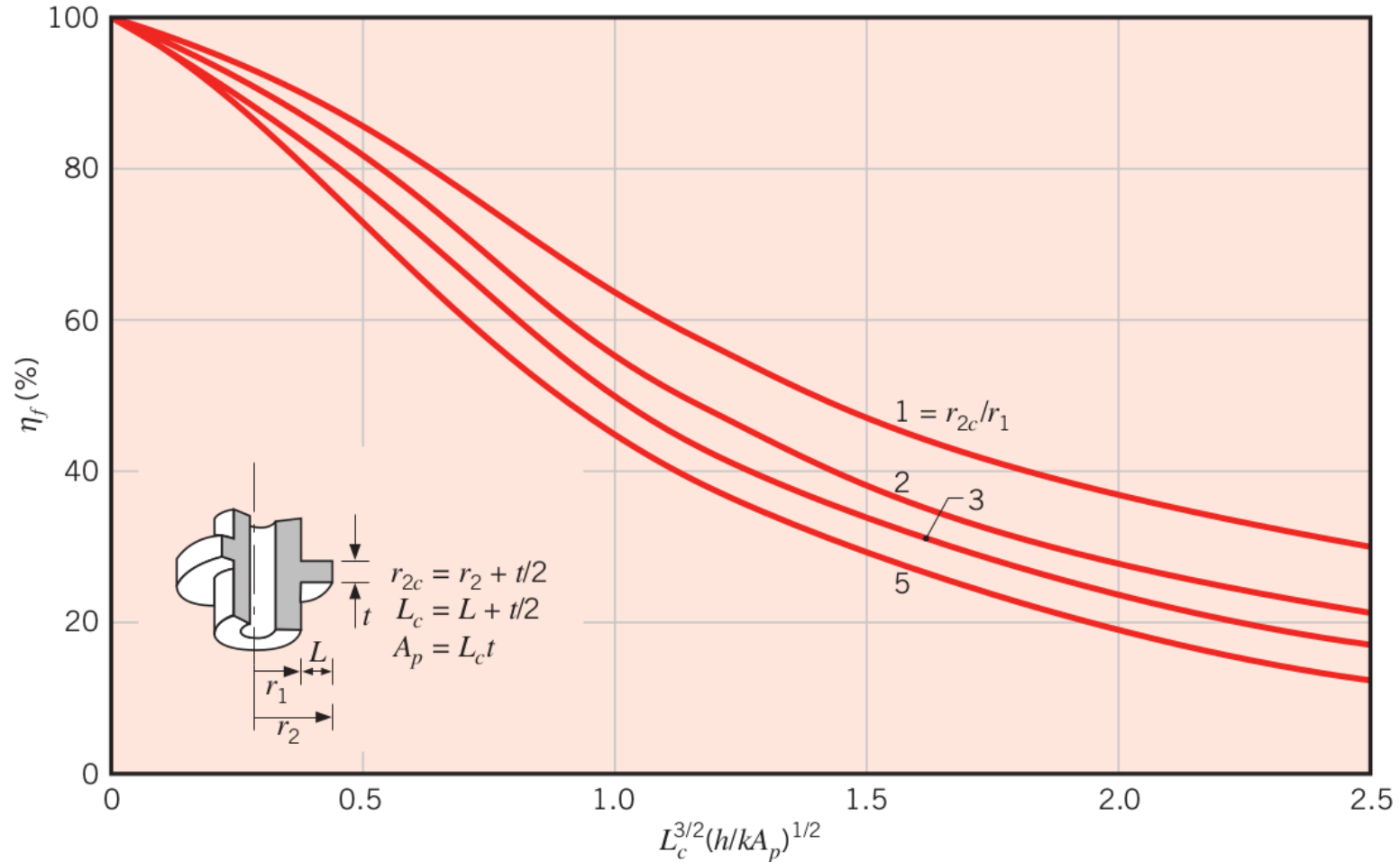
$$\eta_f \equiv \frac{q_f}{q_{max}} = \frac{q_f}{h A_f \theta_b}$$

where A_f is the surface area of the fin.

Fin Efficiency



Fin Efficiency



Fin Efficiency

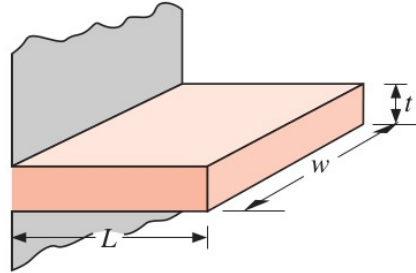
Straight Fins

Rectangular

$$A_f = 2wL_c$$

$$L_c = L + (t/2)$$

$$A_p = tL$$

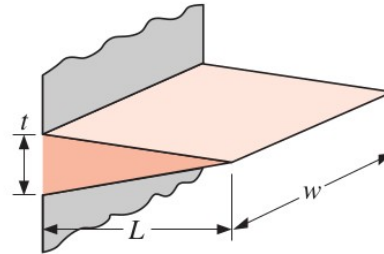


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$$A_f = 2w[L^2 + (t/2)^2]^{1/2}$$

$$A_p = (t/2)L$$



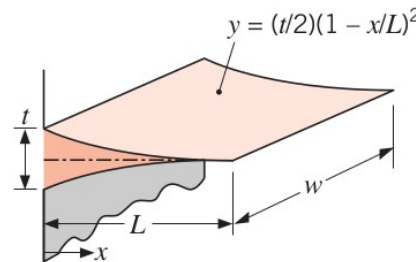
$$\eta_f = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$$

Parabolic

$$A_f = w[C_1L + (L^2/t)\ln(t/L + C_1)]$$

$$C_1 = [1 + (t/L)^2]^{1/2}$$

$$A_p = (t/3)L$$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

Fin Efficiency

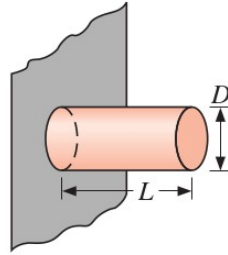
Pin Fins

Rectangular

$$A_f = \pi D L_c$$

$$L_c = L + (D/4)$$

$$V = (\pi D^2/4)L$$

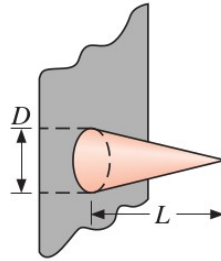


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$$A_f = \frac{\pi D}{2} [L^2 + (D/2)^2]^{1/2}$$

$$V = (\pi/12)D^2L$$



$$\eta_f = \frac{2}{mL} \frac{I_2(2mL)}{I_1(2mL)}$$

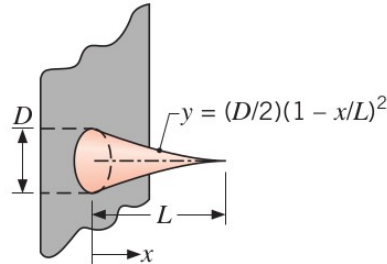
Parabolic

$$A_f = \frac{\pi L^3}{8D} \{ C_3 C_4 - \frac{L}{2D} \ln [(2DC_4/L) + C_3] \}$$

$$C_3 = 1 + 2(D/L)^2$$

$$C_4 = [1 + (D/L)^2]^{1/2}$$

$$V = (\pi/20)D^2 L$$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$$

Overall Surface Efficiency

- η_f denotes the efficiency of a single fin.
- An arrangement of fins may be described by the overall surface efficiency

$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{h A_t \theta_b}$$

where q_t is the total heat rate from the surface A_t associated with both the fin and the exposed portion at the base.

- The exposed portion is known as the prime surface, A_b .
- If there are N fins in an array, then the total area is
$$A_t = N A_f + A_b$$
- The total rate of heat transfer is
$$q_t = N \eta_f h A_f \theta_b + h A_b \theta_b$$

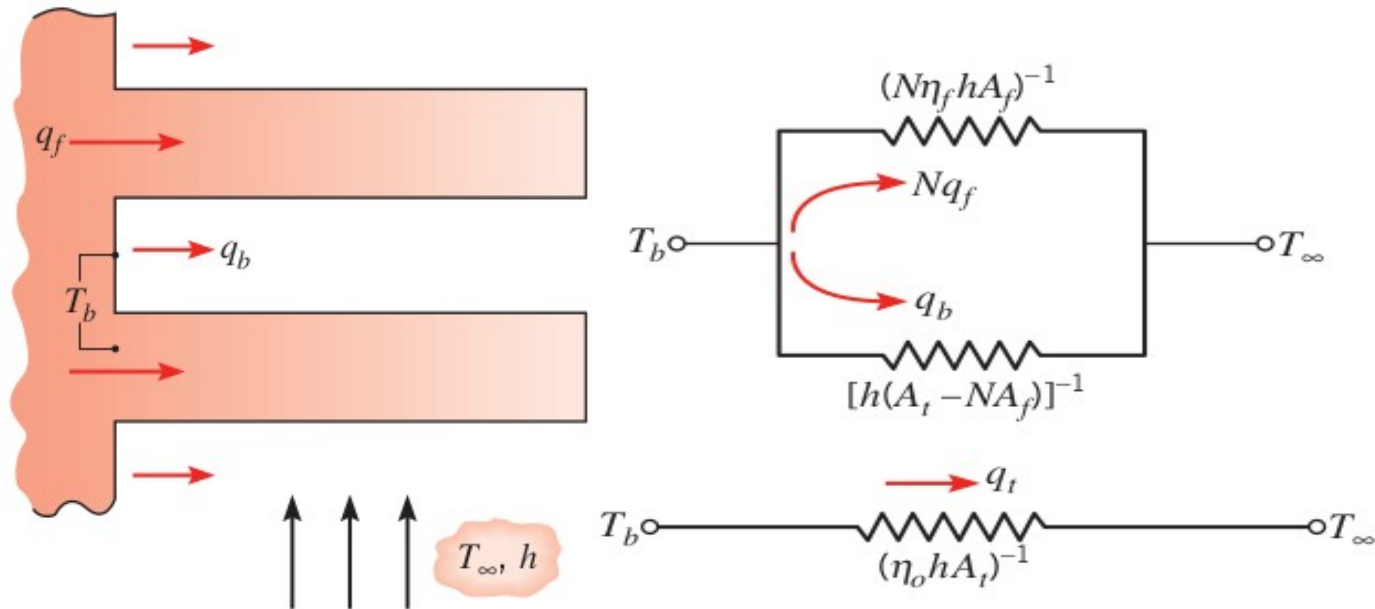
Overall Surface Efficiency

- After some manipulation, the overall surface efficiency becomes

$$\eta_o = 1 - \frac{N A_f}{A_t} (1 - \eta_f)$$

- It may also be related to thermal resistance of the fin array.

$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$$



Example 2

The engine cylinder of a motorcycle is constructed of 2024-T6 aluminum alloy is of height $H=0.15\text{ m}$ and outside diameter $D=50\text{ mm}$. Under typical operating conditions the outer surface of the cylinder is at a temperature of 500 K and is exposed to ambient air at 300 K , with a convection coefficient of $50\text{ W/m}^2\text{K}$. Annular fins are integrally cast with the cylinder to increase heat transfer to the surroundings. Consider five such fins, which are of thickness $t=6\text{ mm}$, length $L=20\text{ mm}$, and equally spaced. What is the increase in heat transfer due to use of the fins?

Example 3, part 1

To generate a power of $P = 9 \text{ W}$, the temperature of a fuel cell must be maintained at $T_c = 56.4 \text{ }^\circ\text{C}$, which requires removal of 11.25 W from the fuel cell and a cooling air velocity of $V = 9.4 \text{ m/s}$ for $T_\infty = 25 \text{ }^\circ\text{C}$. To provide these convective conditions, the fuel cell is centered in a $50 \text{ mm} \times 26 \text{ mm}$ rectangular duct, with 10 mm gaps between the exterior of the $50 \text{ mm} \times 50 \text{ mm} \times 6 \text{ mm}$ fuel cell and the top and bottom of the well insulated duct wall. A small fan, powered by the fuel cell, is used to circulate the cooling air. Inspection of a particular fan vendor's data sheets suggests that the ratio of the fan power consumption to the fan's volumetric flow rate is $P_f/V_f = C = 1000 \text{ W}/(\text{m}^3/\text{s})$ for the range of $10^{-4} \leq V_f \leq 10^{-2} \text{ m}^3/\text{s}$. Determine the net electric power produced by the fuel cell fan system, $P_{net} = P - P_f$.

Example 3, part 2

Consider the effect of attaching an aluminum ($k = 200 \text{ W/mK}$) finned heat sink, of identical top and bottom sections, onto the fuel cell body. The base of the heat sink is of thickness $t_b = 2 \text{ mm}$. Each of the N rectangular fins is of length $L_f = 8 \text{ mm}$ and thickness $t_f = 1 \text{ mm}$, and spans the entire length of the fuel cell, $L_c = 50 \text{ mm}$. With the heat sink in place, radiation losses are negligible and the convective heat transfer coefficient may be related to the size and geometry of a typical air channel by an expression of the form $h = 1.78 k_{air} (L_f + a)/(L_f a)$, where a is the distance between fins. Draw an equivalent thermal circuit for part 2 and determine the total number fins needed to reduce the fan power consumption to half of the value found in part 1.