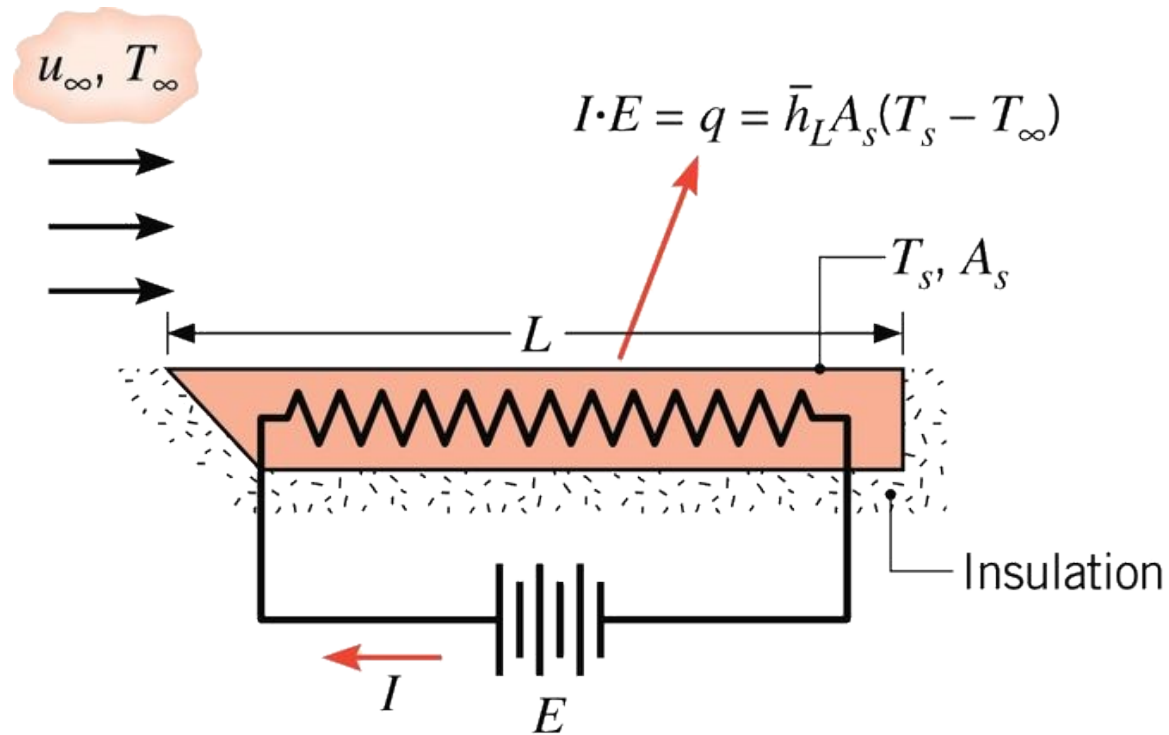
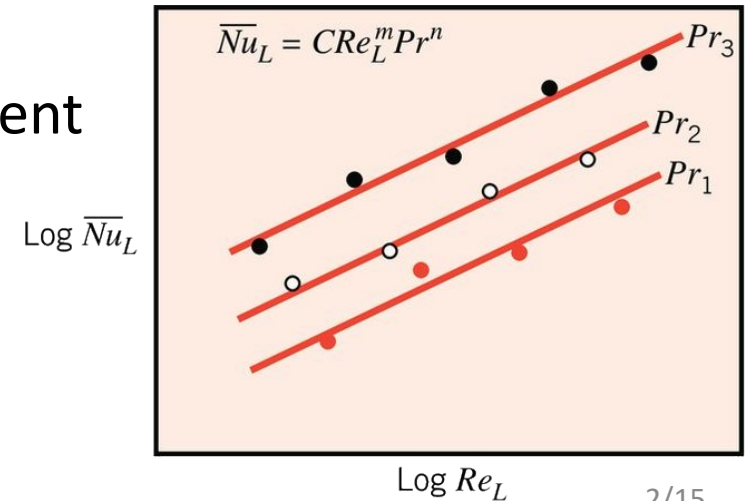
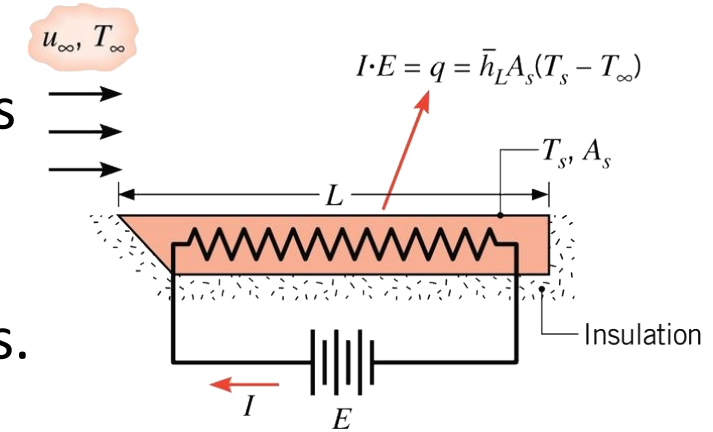


External Flow



Flow Experiment

- Suppose flow over a heated plate is arranged as shown to the right.
- The Reynolds, average Nusselt, and Prandtl numbers are noted for different flow conditions.
- The average Nusselt numbers may then be plotted against the Reynolds number on a log-log scale.
- The different color dots here represent different fluids.
- The C , m , and n are empirical coefficients.



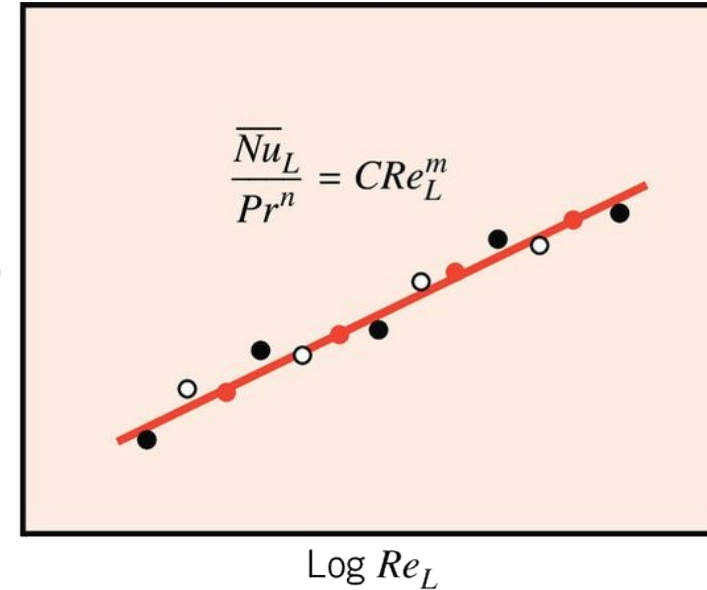
Flow Experiment

- The plot relation may be rewritten to be applicable for any type of fluid.
- This indicates that there is a functional relationship between the various dimensionless numbers.

$$\frac{\bar{Nu}_L}{Pr^n} = C Re_L^m$$

- There is an underlying constant fluid property assumption with this relation.
- This is be handled in one of two ways.
 - Film Temperature method $T_f \equiv \frac{T_s + T_\infty}{2}$
 - Property variation method

$$\text{Log} \left(\frac{\bar{Nu}_L}{Pr^n} \right)$$



Property Variation Method

- The property variation method involves evaluated all material properties at the far field temperature T_{∞} .
- The right hand side of the functional relation between the Reynolds, average Nusselt, and Prandtl numbers are then multiplied by various ratios of material properties.
- For example, the ratios shown below may be used.

$$\left(\frac{Pr_{\infty}}{Pr_s}\right)^r \quad \left(\frac{\mu_{\infty}}{\mu_s}\right)^r$$

Flat Plate: Isothermal Surface

- From empirical studies of parallel flow with an isothermal plate (i.e. T_s is constant), the following relations have been discovered.

Laminar

$$\delta = \frac{5}{\sqrt{\frac{u_\infty}{\nu x}}} = \frac{5x}{\sqrt{Re_x}}$$

$$\frac{\delta}{\delta_t} \approx Pr^{1/3}$$

$$C_{f,x} \equiv \frac{\tau_{s,x}}{\frac{\rho u_\infty^2}{2}} = 0.664 Re_x^{-1/2}$$

$$Nu_x \equiv \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3} \quad (Pr \gtrsim 0.6)$$

Turbulent

$$\delta = 0.37 x Re_x^{-1/2}$$

$$C_{f,x} = 0.0592 Re_x^{-1/5} \quad (Re_{x,c} \lesssim Re_x \lesssim 10^8)$$

$$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3} \quad (0.6 \lesssim Pr \lesssim 60)$$

Flat Plate: Isothermal Surface

- For the average friction coefficient and Nusselt number, an intermediate parameter A is used for the case of turbulent flow.

$$A = 0.037 Re_{x,c}^{4/5} - 0.664 Re_{x,c}^{1/2}$$

Laminar

$$\bar{C}_{f,x} = 1.328 Re_x^{-1/2}$$

$$\bar{Nu}_x \equiv \frac{\bar{h}_x x}{k} = 0.664 Re_x^{1/2} Pr^{1/3} \quad (Pr \gtrsim 0.6)$$

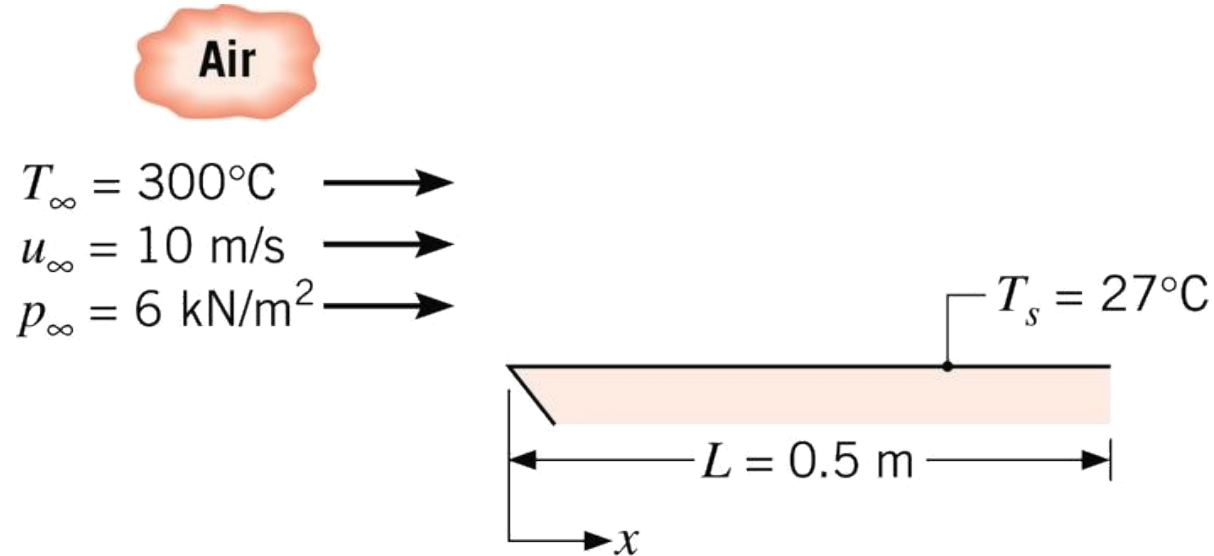
Turbulent

$$\bar{C}_{f,L} = 0.074 Re_L^{-1/5} - \frac{2A}{Re_L} \quad (Re_{x,c} \lesssim Re_L \lesssim 10^8)$$

$$\bar{Nu}_L = (0.037 Re_L^{4/5} - A) Pr^{1/3} \quad (0.6 \lesssim Pr \lesssim 60) \\ (Re_{x,c} \lesssim Re_L \lesssim 10^8)$$

Example 1

Air at a pressure of 6 kPa and a temperature of 300°C flows with a velocity of 10 m/s over a flat plate 0.5 m long. Estimate the cooling rate per unit width of the plate needed to maintain it at a surface temperature of 27°C .



Cylinder in Cross Flow

- Consider a cylinder experiencing cross flow.
- The Reynolds number Re_D for this type of flow may be defined as

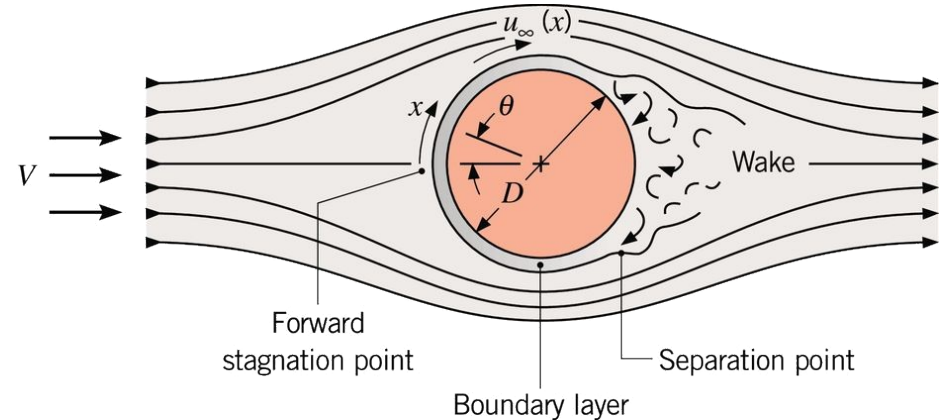
$$Re_D \equiv \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

- The drag coefficient C_D is defined as

$$C_D \equiv \frac{F_D}{A_f \left(\frac{\rho V^2}{2} \right)}$$

where A_f is the cylinder frontal area.

- The frontal area is the area projected perpendicular to the flow.
- The separation point is a function of Reynolds number.



Separation Point

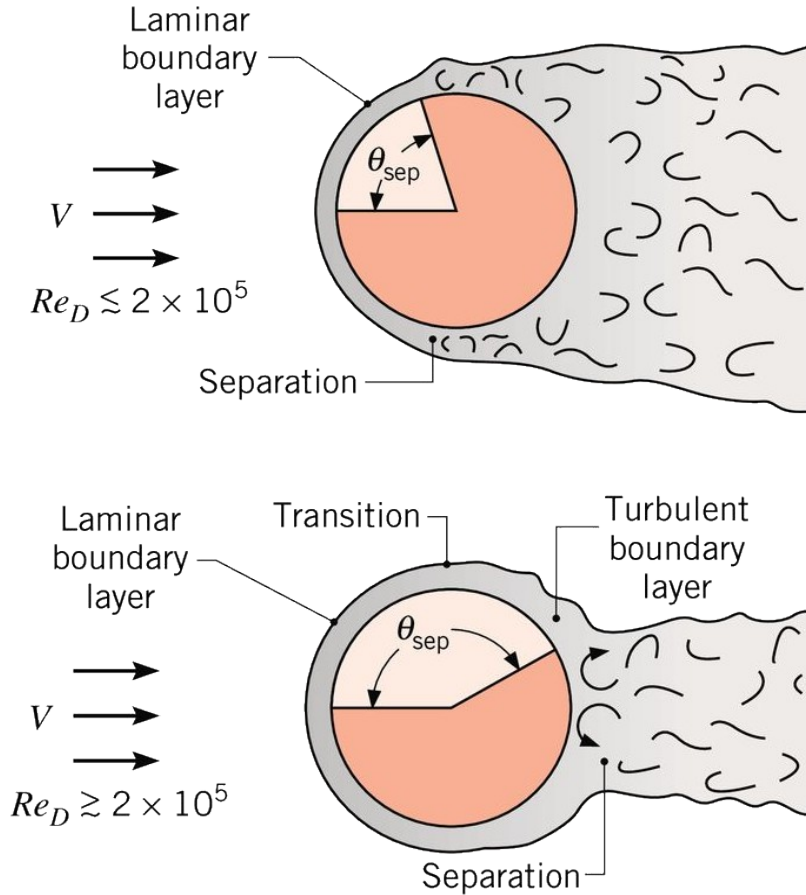
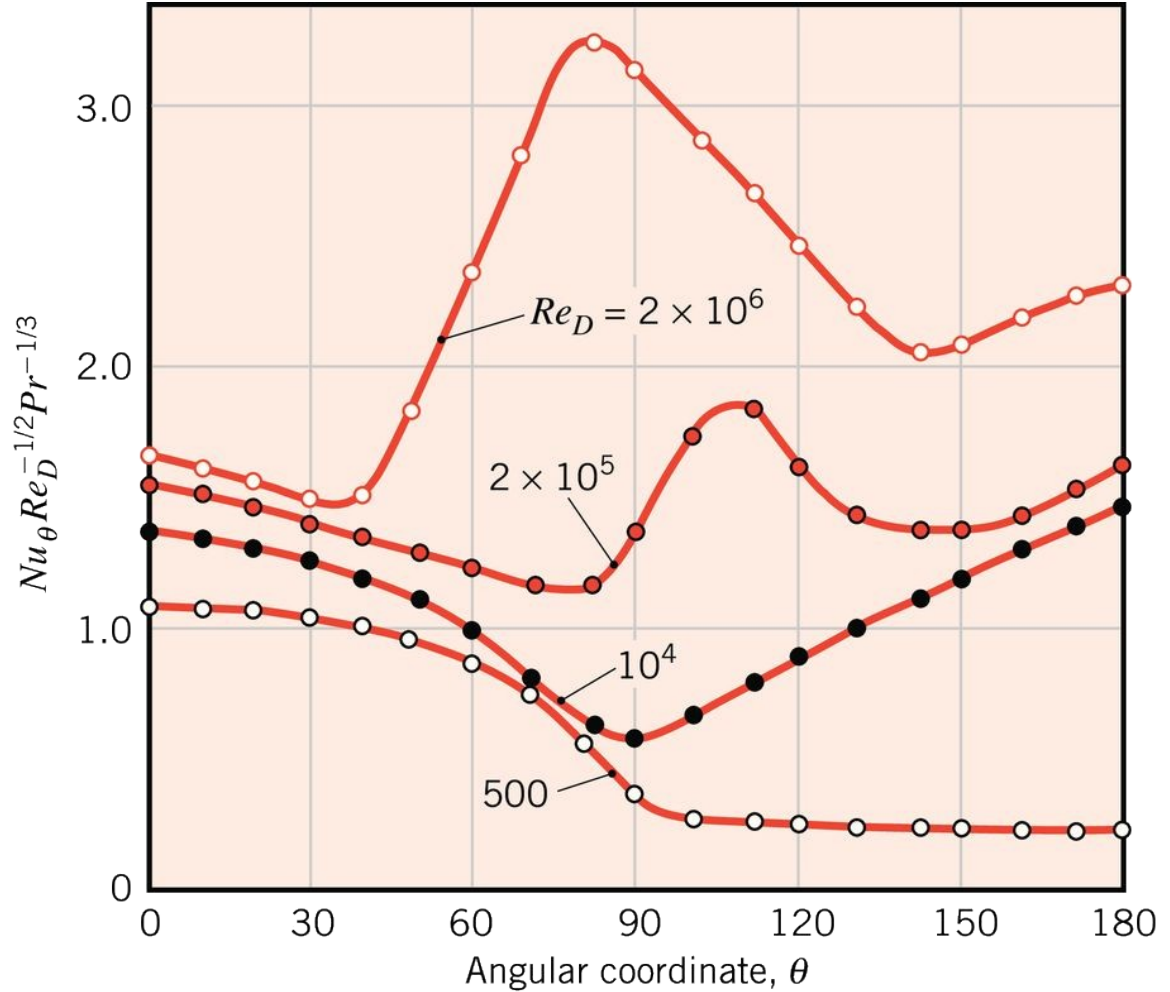
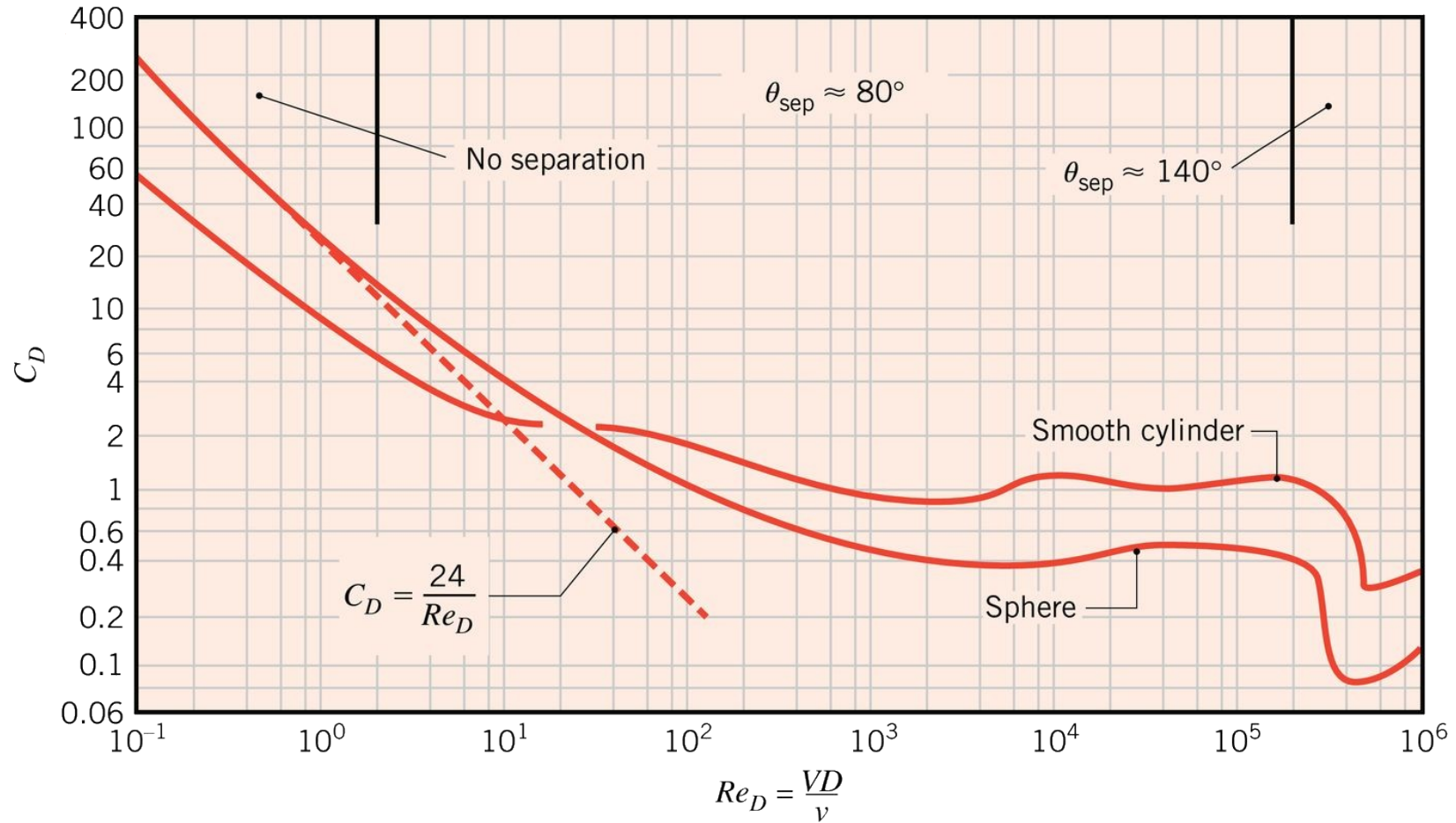


Figure 9.1



Drag Coefficient

Figure 9.2



Cylinder in cross flow

- Rarely is it of concern the local Nusselt number for a cylinder in cross flow.
- However, the average Nusselt number is often used and is given by

$$\bar{Nu}_D \equiv \frac{\bar{h} D}{k} = C Re_D^m Pr^{1/3} \quad (Pr \gtrsim 0.7)$$

Table 9.1

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805







Various geometries in cross flow

- The same relation may be used for various geometries in cross flow.

$$\bar{Nu}_D \equiv \frac{\bar{h} D}{k} = C Re_D^m Pr^{1/3}$$

$(Pr \gtrsim 0.7)$

Table 9.2

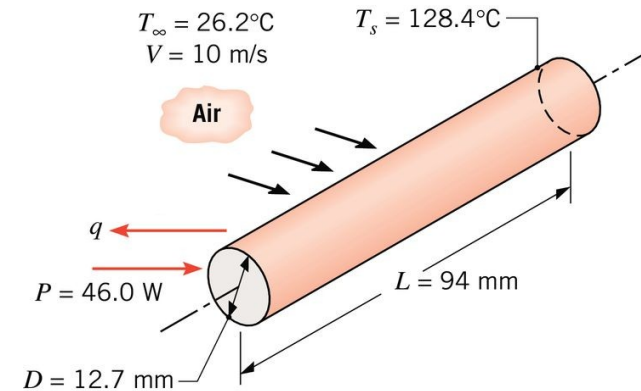
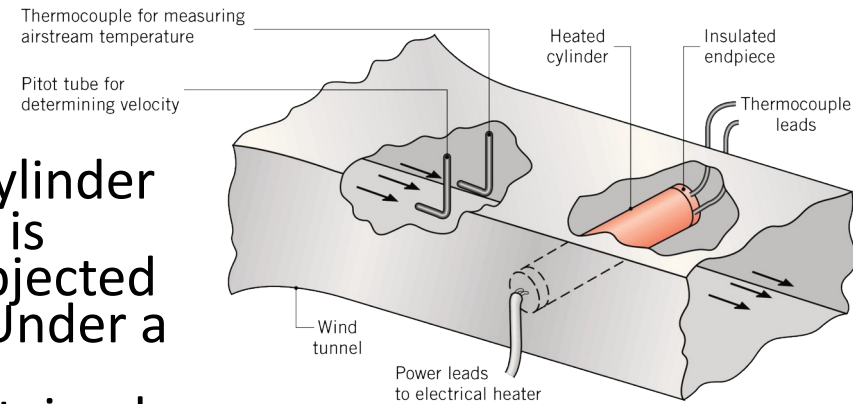
Geometry	Re_D	C	m
Square			
$V \rightarrow$  \overline{D}	6000–60,000	0.304	0.59
$V \rightarrow$  \overline{D}	5000–60,000	0.158	0.66
Hexagon			
$V \rightarrow$  \overline{D}	5200–20,400	0.164	0.638
$V \rightarrow$  \overline{D}	20,400–105,000	0.039	0.78
$V \rightarrow$  \overline{D}	4500–90,700	0.150	0.638
Thin plate perpendicular to flow			
$V \rightarrow$  \overline{D}	Front 10,000–50,000	0.667	0.500
	Back 7000–80,000	0.191	0.667

Example 2

Pitot Tube?

Experiments have been conducted on a metallic cylinder 12.7 mm in diameter and 94 mm long. The cylinder is heated internally by an electrical heater and is subjected to a cross flow of air in a low-speed wind tunnel. Under a specific set of operating conditions for which the upstream air velocity and temperature were maintained at $V = 10\text{ m/s}$ and $26.2\text{ }^{\circ}\text{C}$, respectively, the heater power dissipation was measured to be $P = 46\text{ W}$, while the average cylinder surface temperature was determined to be $T_s = 128.4\text{ }^{\circ}\text{C}$. It is estimated that 15% of the power dissipation is lost through the cumulative effect of surface radiation and conduction through the end pieces.

- Determine the convection heat transfer coefficient from the experimental observations.
- Compare the experimental result with the convection coefficient computed from an appropriate correlation.



The Sphere

- Boundary layers develop very similarly to that of the circular cylinder.
- For very small Reynolds numbers, the drag coefficient is

$$C_D = \frac{25}{Re_D} \quad (Re_D \lesssim 0.5)$$

- The average Nusselt number is given by

$$\bar{Nu}_D = 2 + \left(0.4 Re_D^{1/2} + 0.06 Re_D^{2/3} \right) Pr^{0.4} \left(\frac{\mu}{\mu_s} \right)^{1/4} \quad \begin{aligned} & (0.71 \lesssim Pr \lesssim 380) \\ & (3.5 \lesssim Re_D \lesssim 7.6 \times 10^4) \\ & (1.0 \lesssim \frac{\mu}{\mu_s} \lesssim 3.2) \end{aligned}$$

where all properties except for μ_s are evaluated at T_∞ .

Example 3

Air at $25\text{ }^{\circ}\text{C}$ flows over a 10 mm diameter sphere with a velocity of 15 m/s , while the surface of the sphere is maintained at $75\text{ }^{\circ}\text{C}$.

- (a) What is the drag force on the sphere?
- (b) What is the rate of heat transfer from the sphere?
- (c) Generate a plot of the heat transfer from the sphere as a function of the air velocity for the range 1 to 25 m/s .