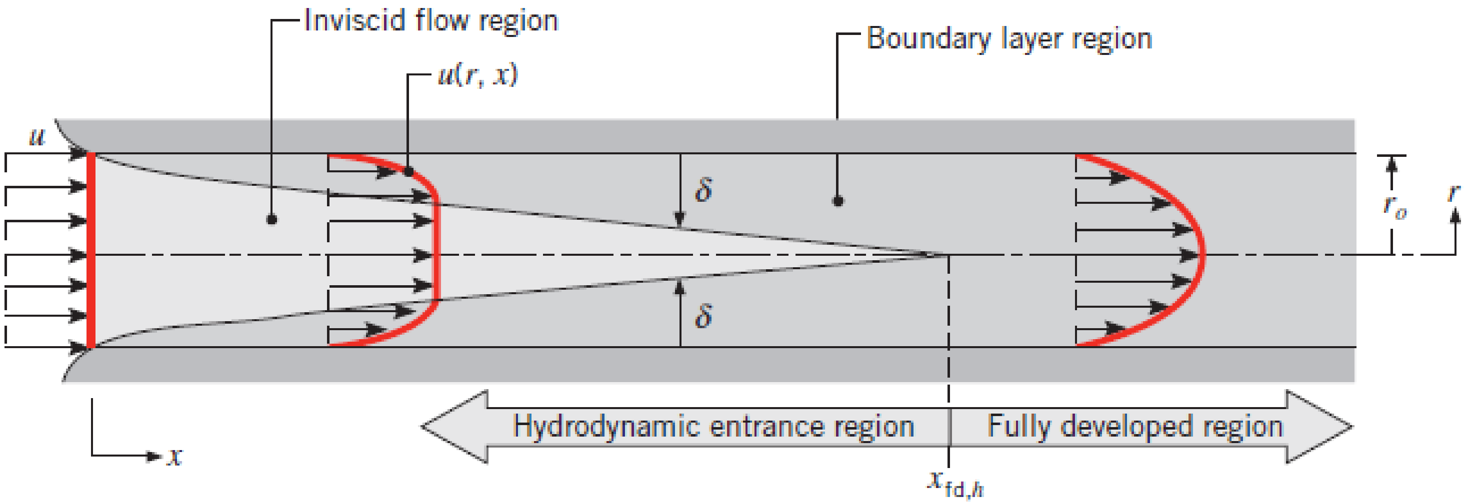
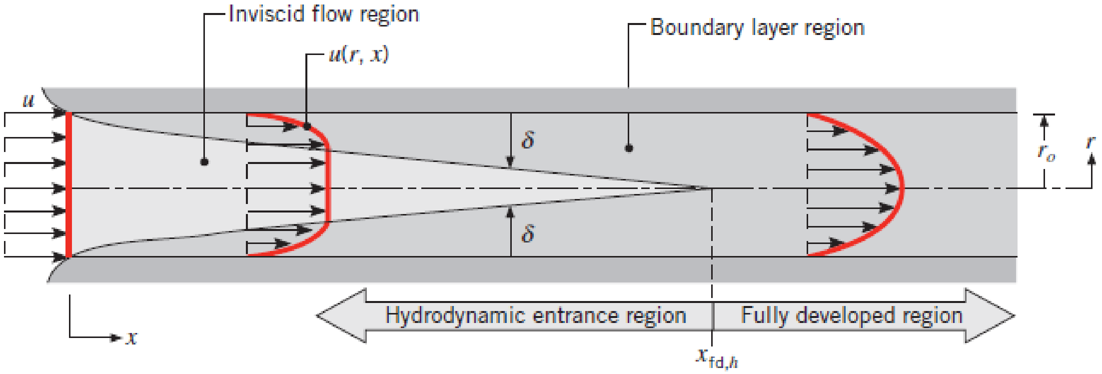
Internal Flow



Internal Flow

- Flow through a duct, internal flow, consists of two major regions
  - Entrance region
  - Fully developed region
- The entrance region consists of the velocity/thermal boundary layer developing from the walls of the duct.
- The fully developed region occurs when the velocity/thermal boundary layer meet each other.



Internal Flow

- Flow is described in terms of mean velocity u_m and is given by

  $$m = \rho u_m A_c$$

  where m is the mass flow rate, and A_c is the cross sectional area.

- The Reynolds number for a internal flow for a circular cross section is

  $$Re_D \equiv \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$$

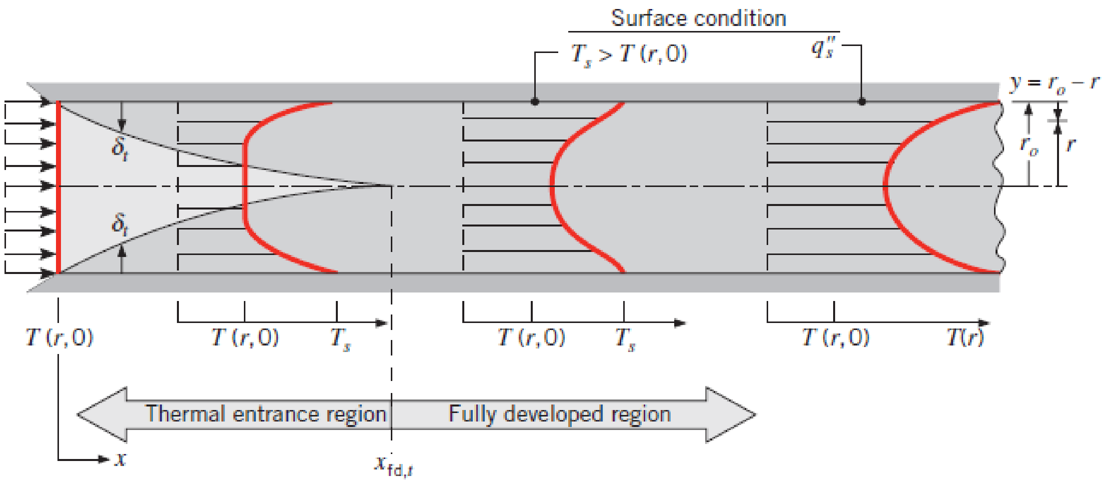
- The onset of turbulent flow for a circular cross section is

  $$Re_{D,c} \approx 2300$$

- Fully developed turbulent flow can be as high as $Re_D \approx 10000$
- For our purposes, fully developed turbulent will be considered $\frac{x}{D} > 10$

Internal Flow

- For the thermal boundary layer, the entrance length is given by
  $$\left(\frac{x_{fd,t}}{D}\right) \approx 0.05\, Re_D\, Pr$$
  where $x_{fd,t}$ is the position of the start of the fully developed region of the thermal boundary layer.



Pressure Drop

- Pressure drop is often of concern in duct flow.
- The friction factor is the quantity that dictates the pressure drop in a duct.
- This is not be confused with the friction coefficient C_f.
- The friction factor is defined as

$$f \equiv \frac{-\frac{dp}{dx} D}{\rho \frac{u_m^2}{2}}$$

  where dp/dx is the pressure drop in the x direction.
- For fully developed laminar flow, the friction factor is given by $f = \frac{64}{Re_D}$

Pressure Drop

- For fully developed turbulent flow, the following relationship is used

  $$\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{e/D}{3.7}+\frac{2.51}{Re_D\sqrt{f}}\right]$$

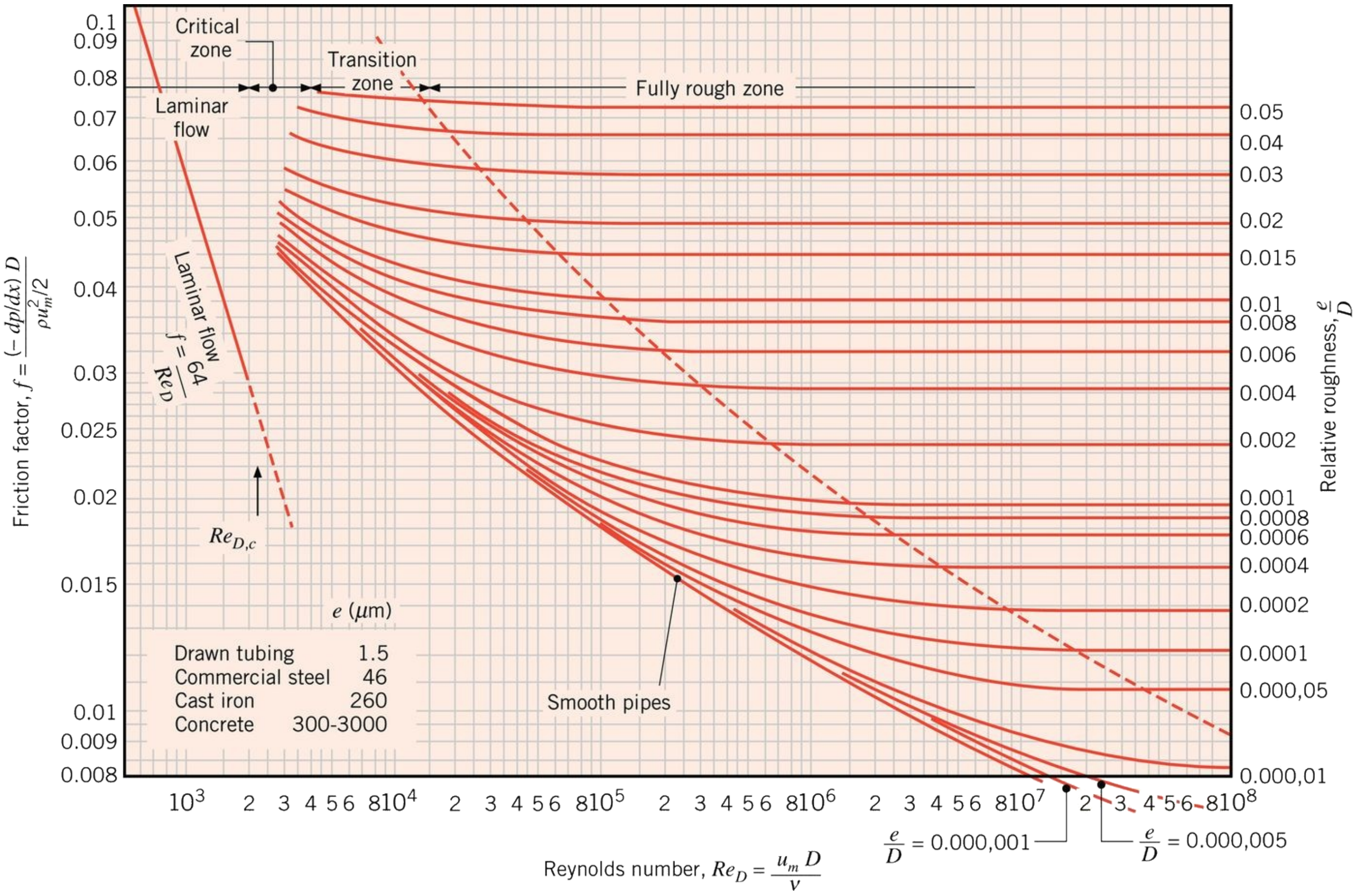
  where e is the surface roughness of the duct (a tabulated value).

- As can be seen this is not a closed form expression (i.e. f appears on both sides)

- For this reason, a plot called a *Moody* diagram is often used.

- The *Moody* diagram is named after Lewis Ferry Moody (1880-1953).

Moody Diagram



Energy Balance

- A differential energy balance of flow through a duct shows

$$dq_{conv} = m c_p \left[\left(T_m + dT_m \right) - T_m \right] = m c_p dT_m$$

where T_m is the mean temperature.

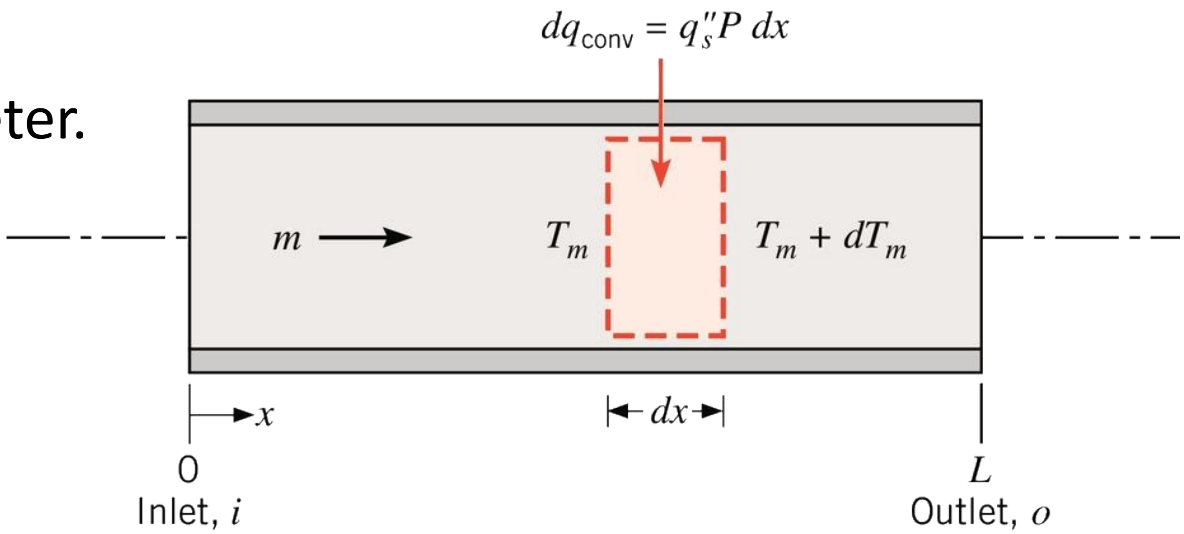
- The differential heat rate term may be written in terms of flux as

$$dq_{conv} = q_s'' P\, dx$$

where P is the surface perimeter.

- Rearranging shows

$$\frac{dT_m}{dx} = \frac{q_s'' P}{m c_p} = \frac{P}{m c_p} h \left(T_s - T_m \right)$$



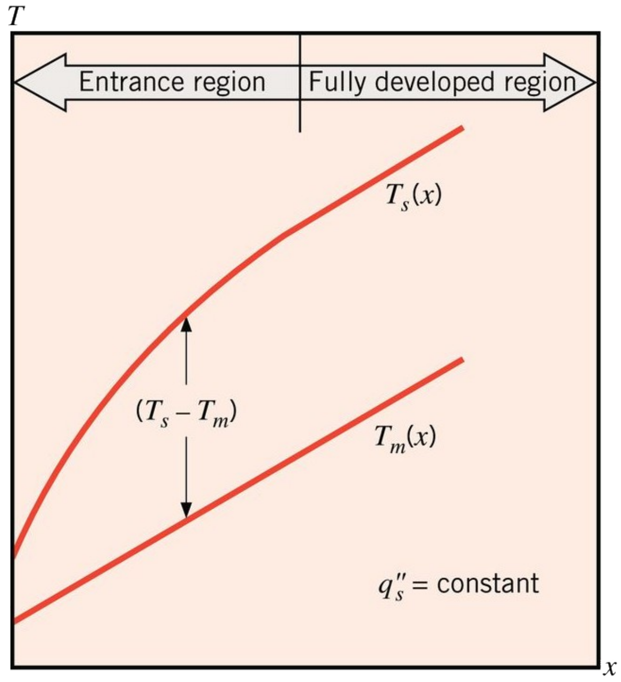
Energy Balance

- For a constant surface heat flux q''_s the energy balance becomes

  $$T_m(x) = T_{m,i} + \frac{q''_s P}{\dot{m} c_p} x$$

  where $T_{m,i}$ is the inlet mean temperature.

- The plot shows the axial temperature variations for heat transfer in a tube for a constant surface heat flux.



Example 1

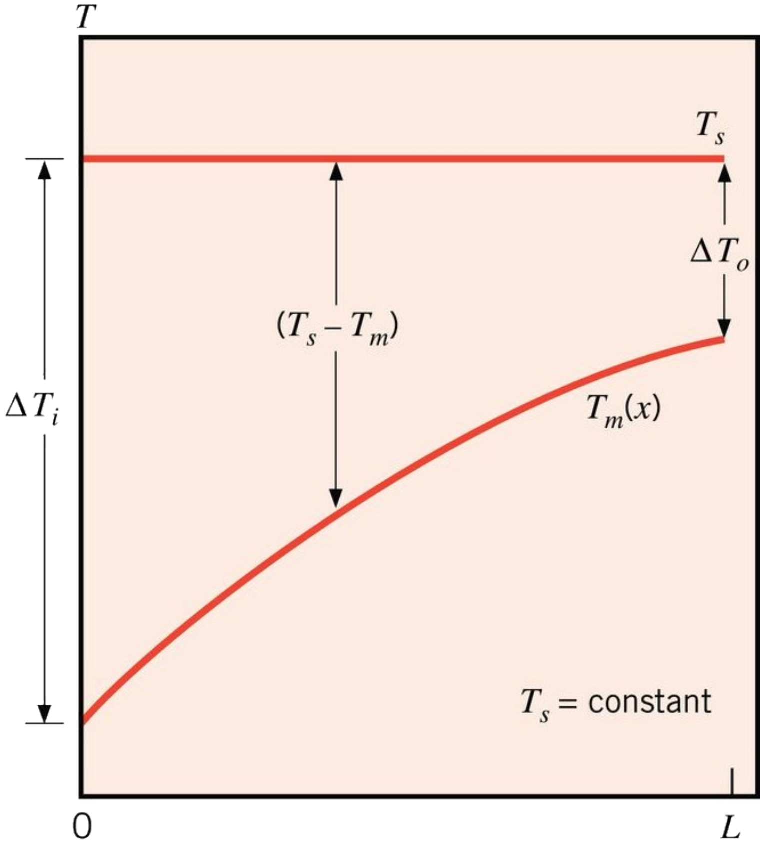
A system for heating water from an inlet temperature of $T_{m,i} = 20\ °C$ to an outlet temperature of $T_{m,o} = 60\ °C$ involves passing the water through a thick walled tube having inner and outer diameters of 20 and $40\ mm$. The outer surface of the tube is well insulated, and electrical heating within the wall provides for a uniform generation rate of $q = 10^6\ W/m^3$.

(a) For a water mass flow rate of $m = 0.1\ kg/s$, how long must the tube be to achieve the desired outlet temperature?

(b) If the inner surface temperature of the tube is $T_s = 70\ °C$ at the outlet, what is the local convection heat transfer coefficient at the outlet?

Energy Balance

- For a constant surface temperature, the term ΔT may be defined as
  $\Delta T = T_s - T_m$
- The energy balance then becomes
  $$\frac{dT_m}{dx} = \frac{-d(\Delta T)}{dx} = \frac{P}{\dot{m} c_p} h \Delta T$$
- Solving this shows
  $$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = e^{\frac{-Px}{\dot{m} c_p}\bar{h}}$$
- The plot shows the axial temperature variations for heat transfer in a tube for a constant surface heat flux



Energy Balance

- Sometimes the term called the log mean temperature is used.
- This is defined as

$$\Delta T_{lm} \equiv \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$$

where ΔT_o and ΔT_i are the temperature differences between the surface temperature and the outlet and inlet mean temperature, respectively.

- That is

$$\Delta T_o = T_s - T_{m,o} \qquad \Delta T_i = T_s - T_{m,i}$$

- The heat rate is then

$$q_{conv} = \bar{h} A_s \Delta T_{lm}$$

Example 2

Steam condensing on the outer surface of a thin walled circular tube of diameter $D = 50\ mm$ and length $L = 6\ m$ maintains a uniform outer surface temperature of $100\ °C$. Water flows through the tube at a rate of $m = 0.25\ kg/s$, and its inlet and outlet temperatures are $T_{m,i} = 15\ °C$ and $T_{m,o} = 57\ °C$. What is the average convection coefficient associated with the water flow?

Circular Tubes

- For fully developed laminar flow with constant surface heat flux

$$Nu_D \equiv \frac{hD}{k} = 4.36$$

- For fully developed laminar flow with constant surface temperature

$$Nu_D = 3.66$$

- For fully developed turbulent flow

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^n \qquad \left(0.6 \lesssim Pr \lesssim 160\right) \qquad \left(Re_D \gtrsim 10{,}000\right) \qquad \left(L/D \gtrsim 10\right)$$

where $n = 0.4$ when $T_s > T_m$ and $n = 0.3$ when $T_s < T_m$.

- This turbulent flow relation is good for small temperature differences and all properties are evaluated at T_m.

- This turbulent flow relation may be applied to both constant surface heat and temperature cases.

Circular Tubes

- For fully developed turbulent flow with large temperature difference

$$Nu_D = 0.027\, Re_D^{4/5}\, Pr^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad \left(0.7 \lesssim Pr \lesssim 16{,}700\right) \; \left(Re_D \gtrsim 10{,}000\right) \; \left(L/D \gtrsim 10\right)$$

- Again, all properties are evaluated at T_m except μ_s.
- Large temperature difference will be defined as $\Delta T \gtrsim 50\,K$

Example 3

Hot air flows with a mass rate of $m = 0.050\ kg/s$ through an uninsulated sheet metal duct of diameter $D = 0.15\ m$, which is in the crawlspace of a house. The hot air enters at $103\ °C$ and, after a distance of $L = 5\ m$, cools to $85\ °C$. The heat transfer coefficient between the duct outer surface and the ambient air at $T_\infty = 0\ °C$ is known to be $h_o = 6\ W/m^2K$.

(a) Calculate the heat loss (W) from the duct over the length L.

(b) Determine the heat flux and the duct surface temperature at $x = L$.

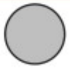
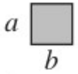
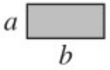
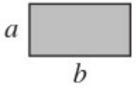
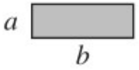
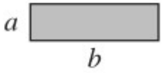
Noncircular Tubes

- For noncircular ducts, an effective diameter is used

  $$D_h \equiv \frac{4\,A_c}{P}$$

  where A_c is the flow cross sectional area and P is wetted perimeter.

Table 10.1

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q_s'')	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform T_s)	$f\,Re_{D_h}$
	—	4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	∞	8.23	7.54	96
	∞	5.39	4.86	96
	—	3.11	2.49	53

Circular Tube Annulus

- Applies to:
  - fully developed laminar flow
  - Annulus (outer section of concentric tubes)
  - Nu_i is for the inside surface
  - Nu_o is for the outside surface
  - Outside surface adiabatic
  - Inside surface at a constant temperature

Table 10.2

D_i/D_o	Nu_i	Nu_o
0	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
≈1.00	4.86	4.86