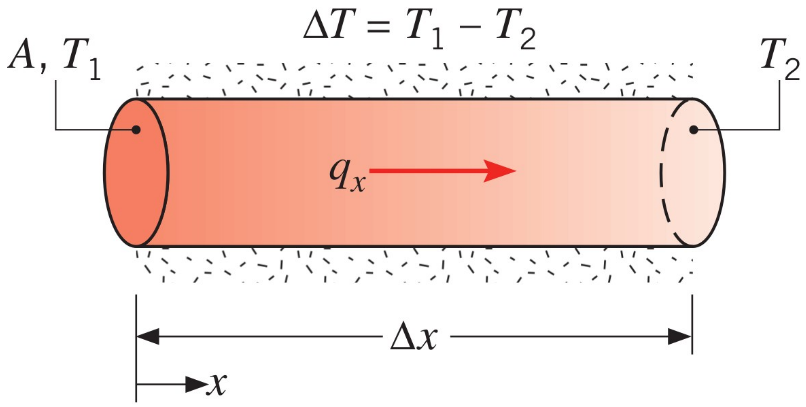
Conduction
Introduction

Fourier's Law

- Describes how heat transfer due to conduction behaves
- Name after Jean-Baptiste Joseph Fourier
- Is *phenomenological*; Developed from observed phenomena rather than derived
- Fourier found that $\dot{Q} \propto A \dfrac{\Delta T}{\Delta x}$
- Varied area, temperature, and distance
- Found that the material made a difference



Fourier's Law

- Since material affected the performance, Fourier created a property called thermal conductivity and said

$$\dot{Q} = k\,A\,\frac{\Delta T}{\Delta x}$$

- If the change in length is allowed to approach zero, we get

$$\dot{Q} = -k\,A\,\frac{dT}{dx}$$

- The negative sign is there because heat is transferred in the direction of decreasing temperature
- Thermal conductivity has units of $k = \frac{\dot{Q}}{A\,\frac{dT}{dx}} [=] \frac{W}{m^2\,\frac{K}{m}} [=] \frac{W}{m\,K}$

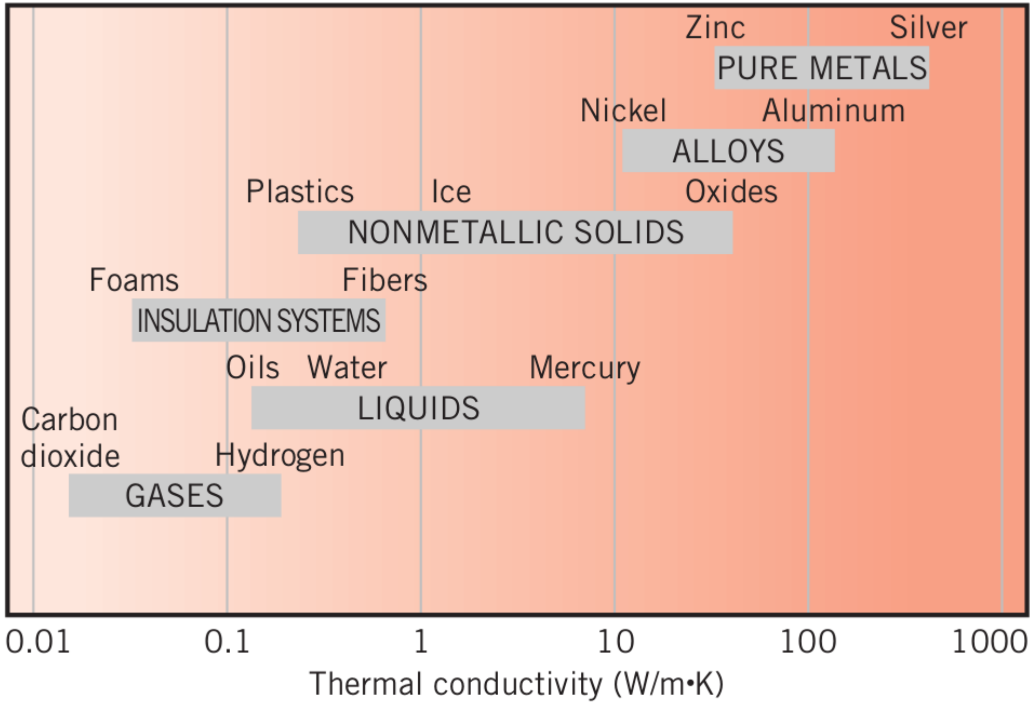
Fourier's Law

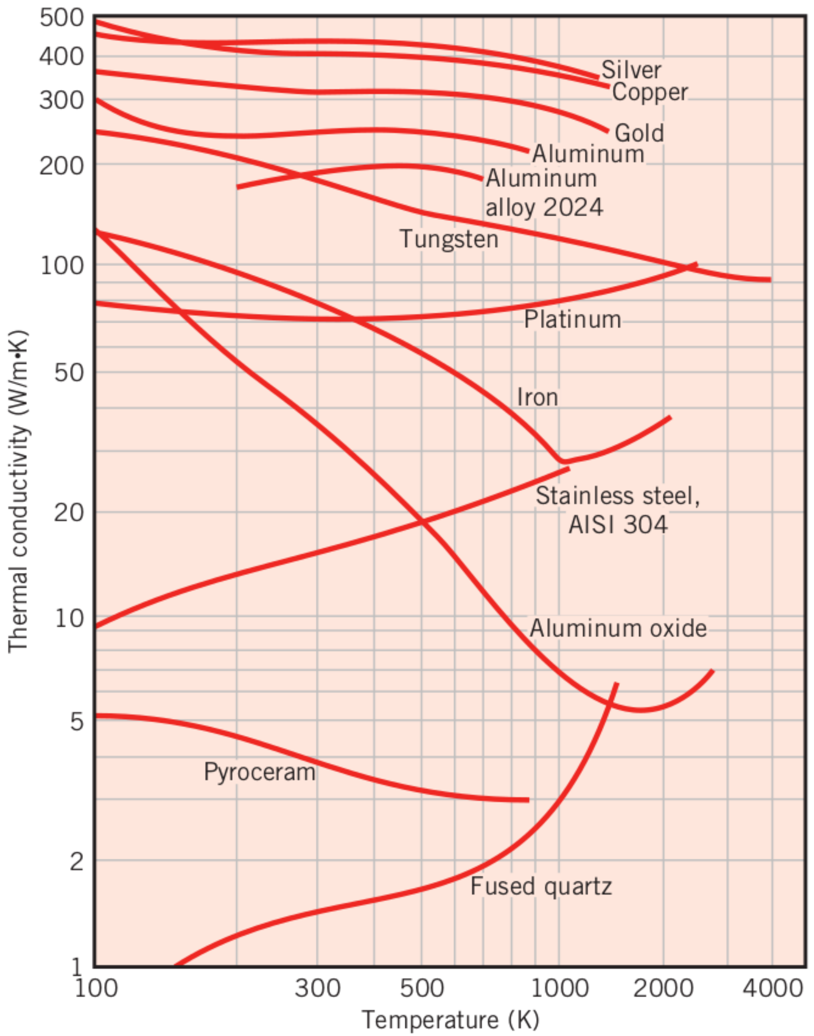
- The expression may be expanded to 3D

$$\hat{q}''=-k_x\frac{\partial T}{\partial x}\hat{i}-k_y\frac{\partial T}{\partial y}\hat{j}-k_z\frac{\partial T}{\partial z}\hat{k}$$

- Notice the thermal conductivity is different in the 3 directions
- If $k_x = k_y = k_z$, known as an isotropic material
- The thermal conductivity is dependent on many things such as material, material state, temperature, etc.

Thermal Conductivity

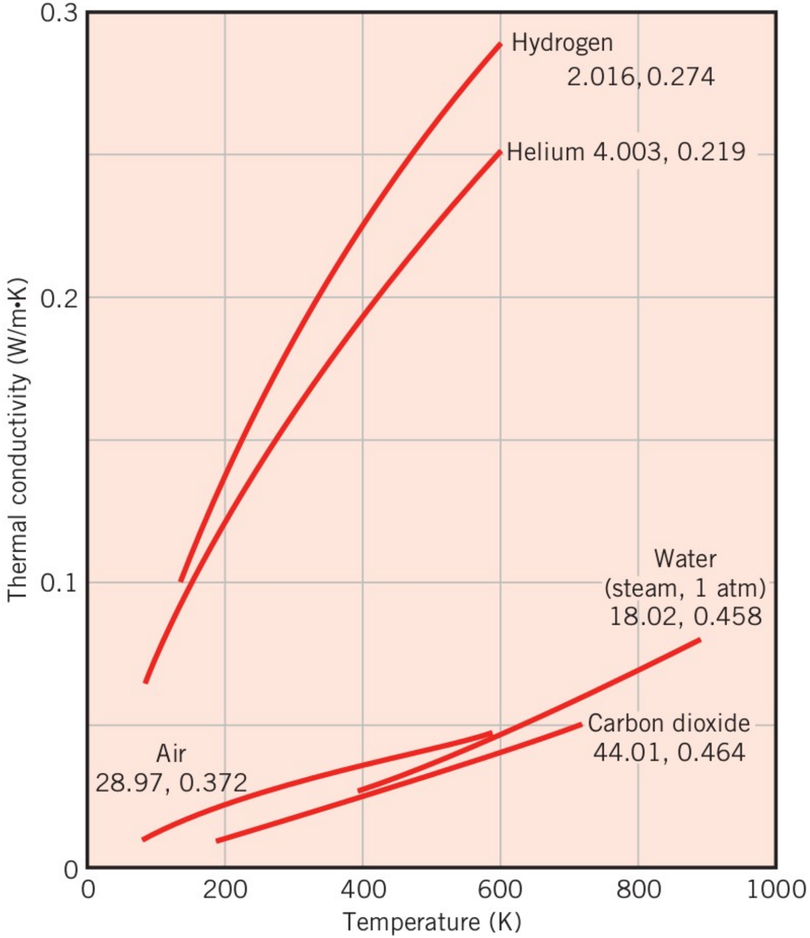




Thermal Conductivity

$$k_B = 1.381 \times 10^{-23} \frac{J}{K} \qquad N_A = 6.022 \times 10^{23}$$

- For gases, $$k = \frac{9\gamma - 5}{4} \frac{c_v}{\pi d^2} \sqrt{\frac{M_w k_B T}{N_A \pi}}$$
  - γ is specific heat ratio
  - c_v is specific heat at constant volume
  - d is diameter of gas molecule
  - M_w is molecular weight
  - k_B is Boltzmann's constant
  - T is temperature
  - N_A is Avogadro's number



Note: 1st number is M_w, and 2nd number is d in nm.

Example 1

Use the thermal conductivity expression for gases to determine the specific heat ratio for the following gases at the given temperatures.

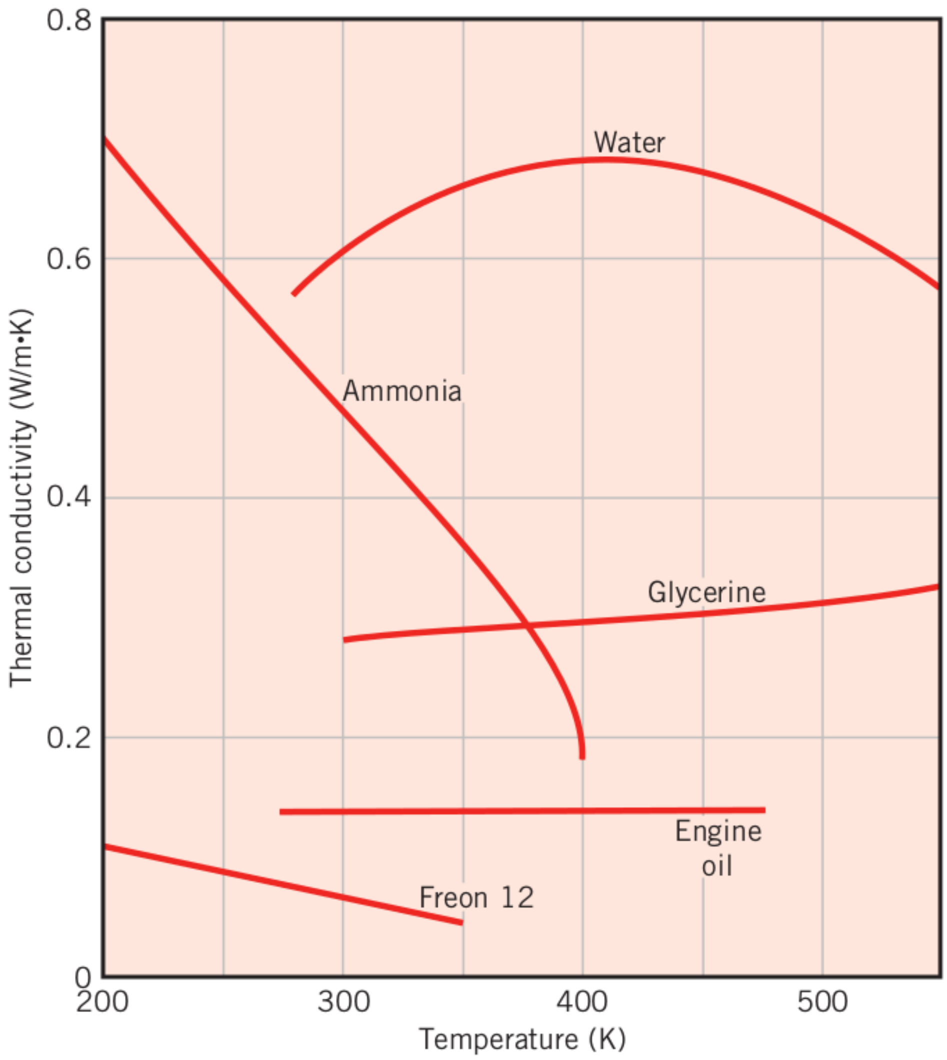
(a) Air, $500\ K$

(b) Carbon dioxide, $800\ K$

(c) Hydrogen, $375\ K$

Thermal Conductivity

- Shows temperature dependence of selected nonmetal liquids under saturated conditions



Thermal Diffusivity

- Ratio of the thermal conductivity to heat capacity

$$\alpha = \frac{k}{\rho c_p}$$

- Describes a materials ability to conduct energy relative to its ability to store thermal energy

- Looking at the units

$$\alpha = \frac{k}{\rho c_p} [=] \frac{\frac{W}{m\,K}}{\frac{kg}{m^3}\frac{J}{kg\,K}} [=] \frac{\frac{W}{m\,K}}{\frac{J}{m^3\,K}} [=] \frac{W\,m^3\,K}{m\,K\,J} [=] \frac{m^2}{s}$$

Example 2

Using appropriate values of k, ρ, and c_p from Appendix A, calculate α for the following materials at the prescribed temperatures:

(a) pure aluminum, 300 and $700\ K$

(b) silicon carbide, $1000\ \mathrm{K}$

(c) paraffin, $300\ K$

Example 3

The thermal conductivity of a sheet of rigid, extruded insulation is reported to be $k = 0.029\ W/mK$. The measured temperature difference across a $20\ mm$ thick sheet of material is $T_1 - T_2 = 10°C$.

(a) What is the heat flux through $2m$ x $2m$ sheet of insulation?

(b) What is the rate of heat transfer through the sheet of insulation?