

**ENGT 320 Applied Thermal Systems**  
**Quiz 2 Formula Sheet**

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$$m = \rho V$$

$$\dot{V} = \frac{V}{t}$$

$$q = mc_p \Delta T$$

$$q = -kA \frac{dT}{dt}$$

$$q = hA (T_s - T_{\infty})$$

$$q = \varepsilon \sigma A (T_s^4 - T_{sur}^4)$$

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

$$\Delta E_{sys} = \Sigma E_{in} - \Sigma E_{out}$$

$$\frac{dE_{sys}}{dt} = \Sigma \dot{E}_{in} - \Sigma \dot{E}_{out}$$

$$q'' = \frac{q}{A}$$

$$\hat{q}'' = -k_x \frac{\partial T}{\partial x} \hat{i} - k_y \frac{\partial T}{\partial y} \hat{j} - k_z \frac{\partial T}{\partial z} \hat{k}$$

$$k = \frac{9\gamma-5}{4} \frac{c_v}{\pi d^2} \sqrt{\frac{M_w k_B T}{N_A \pi}}$$

$$k_B = 1.381 \times 10^{-23} \frac{J}{K}$$

$$N_A = 6.022 \times 10^{23}$$

$$\alpha = \frac{k}{\rho c_p}$$

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = 0$$

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) +$$

$$\frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} +$$

$$\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( kr^2 \frac{\partial T}{\partial r} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$T(0, t) = T_s$$

$$-k \frac{\partial T}{\partial x} \Big|_{x=0} = q_s''$$

$$\frac{\partial T}{\partial x} \Big|_{x=0} = 0$$

$$-k \frac{\partial T}{\partial x} \Big|_{x=0} = h(T_{\infty} - T(0, t))$$

$$\Delta U = N(V^- - V^+)$$

$$q = \frac{\Delta U}{\Delta t} = \frac{N}{\Delta t} \Delta V$$

$$q = -IV = \frac{-V^2}{R} = -I^2 R$$

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$

$$S_{AB} = S_B - S_A = \frac{-\Delta V}{\Delta T}$$

$$C_1 = \frac{T_{s,2} - T_{s,1}}{L}$$

$$C_2 = T_{s,1}$$

$$T(x) = (T_{s,2} - T_{s,1}) \frac{x}{L} + T_{s,1}$$

$$q = \frac{kA}{L} (T_{s,1} - T_{s,2})$$

$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{kA}$$

$$R = \frac{V}{I}$$

$$R_{t,conv} = \frac{T_s - T_{\infty}}{q} = \frac{1}{hA}$$

$$R_{t,rad} = \frac{T_s - T_{sur}}{q} = \frac{1}{h_r A}$$

$$h_r = \varepsilon \sigma (T_s + T_{sur}) (T_s^2 + T_{sur}^2)$$

$$q = U A \Delta T$$

$$U = \frac{1}{R_{tot} A}$$

$$R_{tot} = \Sigma R_t = \frac{1}{U A}$$

$$R_{t,c}'' = \frac{T_A - T_B}{q''}$$

$$q = \frac{k_{eff} A}{L} (T_1 - T_2)$$

$$k_{eff,min} = \frac{1}{\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}}$$

$$k_{eff,max} = \varepsilon k_f + (1 + \varepsilon) k_s$$

$$R_{t,cond} = \frac{\ln \frac{r_2}{r_1}}{2\pi k L}$$

$$R_{t,conv} = \frac{1}{h 2\pi r L}$$

$$\frac{d^2 T}{dx^2} + \left( \frac{1}{A_c} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \left( \frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_{\infty}) = 0$$

$$A_s = Px$$

$$\frac{d^2 T}{dx^2} - \left( \frac{hP}{kA_c} \right) (T - T_{\infty})$$

$$\frac{d^2 \theta}{dx^2} - m^2 \theta = 0$$

$$\theta = T(x) - T_{\infty}$$

$$m^2 = \frac{hP}{kA_c}$$

$$P = 2w + 2t$$

$$A_c = wt$$

$$P = \pi D$$

$$A_c = \frac{\pi}{4} D^2$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta_b = T_b - T_{\infty}$$

$$M = \sqrt{h P K A_c} \theta_b$$

$$\varepsilon_f = \frac{q_f}{h A_{c,b} \theta_b}$$

$$\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$$

$$R_{t,f} = \frac{\theta_b}{q_f}$$

$$R_{t,b} = \frac{1}{h A_{c,b}}$$

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}$$

$$\eta_f = \frac{q_f}{q_{max}} = \frac{q_f}{h A_f \theta_b}$$

$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{h A_t \theta_b}$$

$$A_t = N A_f + A_b$$

$$q_t = N \eta_f h A_f \theta_b + h A_b \theta_b$$

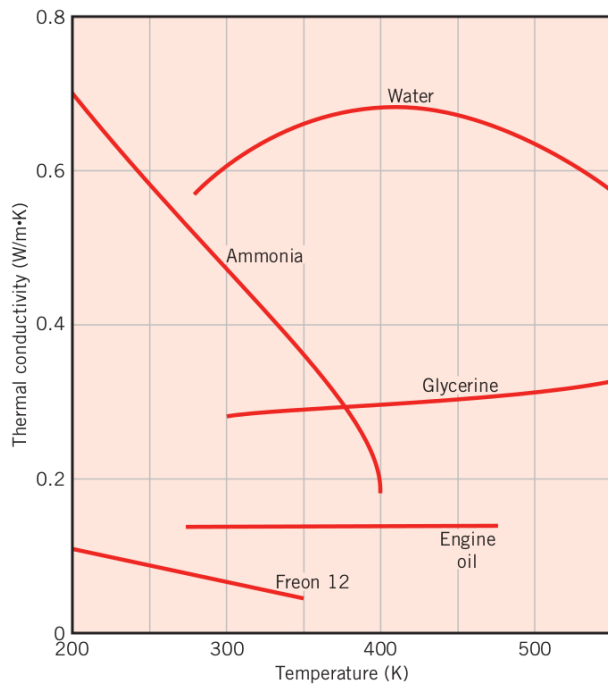
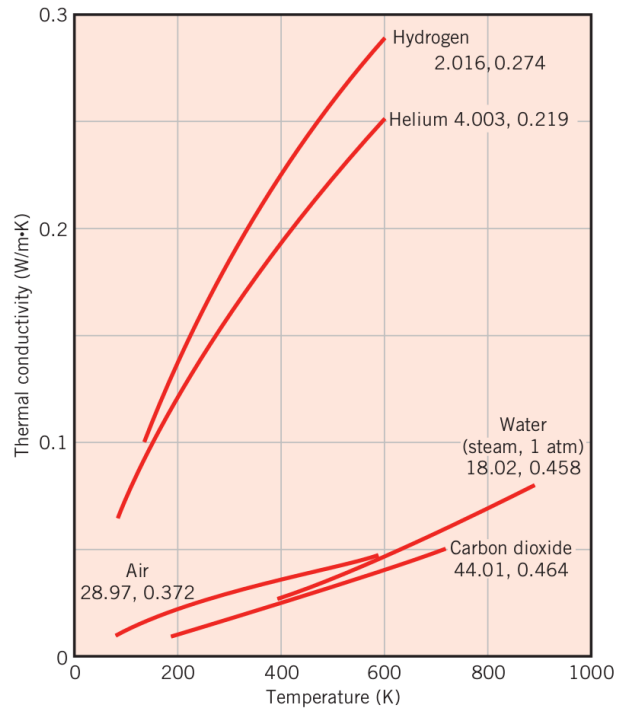
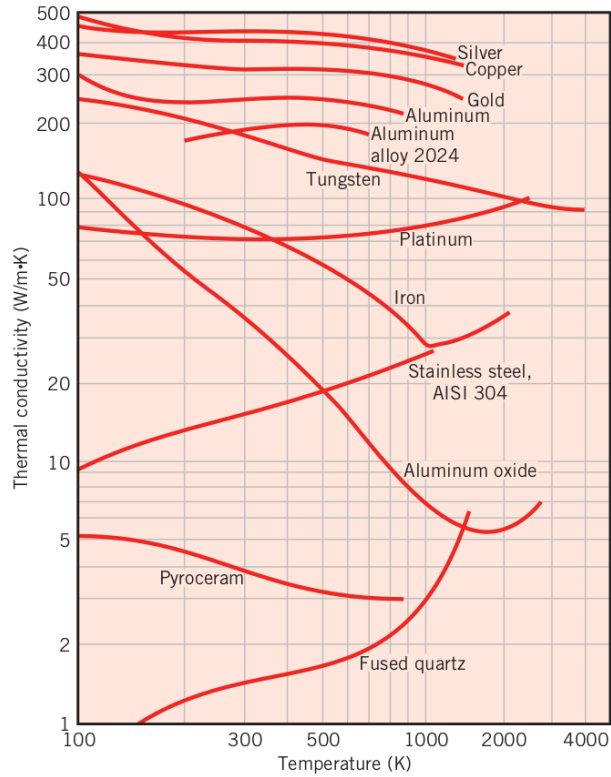
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$$\eta_o = 1 - \frac{NA_f}{A_t} (1 - \eta_f)$$

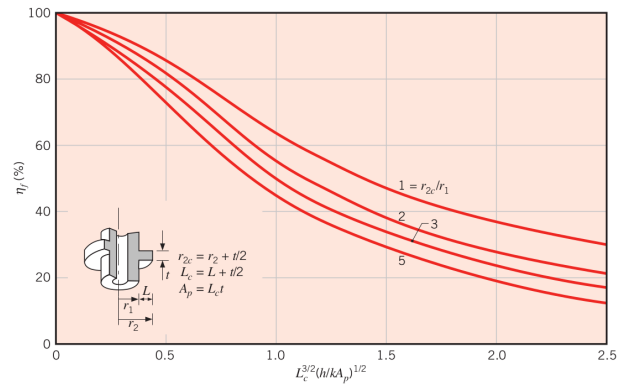
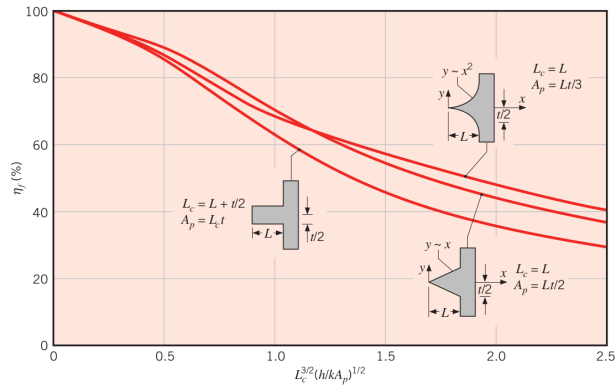
$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$$

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# ENGT 320 Applied Thermal Systems

## Quiz 2 Formula Sheet



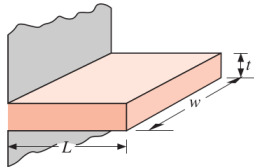
### Straight Fins

#### Rectangular

$$A_f = 2wL_c$$

$$L_c = L + (t/2)$$

$$A_p = tL$$

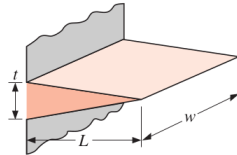


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

#### Triangular

$$A_f = 2w[L^2 + (t/2)^2]^{1/2}$$

$$A_p = (t/2)L$$



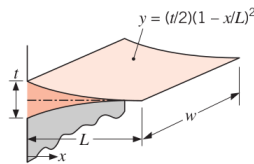
$$\eta_f = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$$

#### Parabolic

$$A_f = w[C_1 L + (L^2/t) \ln(t/L + C_1)]$$

$$C_1 = [1 + (t/L)^2]^{1/2}$$

$$A_p = (t/3)L$$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

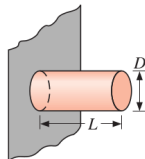
### Pin Fins

#### Rectangular

$$A_f = \pi D L_c$$

$$L_c = L + (D/4)$$

$$V = (\pi D^2/4)L$$

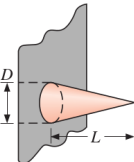


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

#### Triangular

$$A_f = \frac{\pi D}{2} [L^2 + (D/2)^2]^{1/2}$$

$$V = (\pi/12) D^2 L$$



$$\eta_f = \frac{2}{mL} \frac{I_2(2mL)}{I_1(2mL)}$$

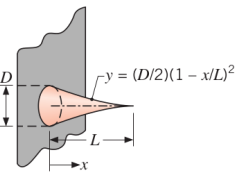
#### Parabolic

$$A_f = \frac{\pi L^3}{8D} \{C_3 C_4 - \frac{L}{2D} \ln [(2DC_4/L) + C_3]\}$$

$$C_3 = 1 + 2(D/L)^2$$

$$C_4 = [1 + (D/L)^2]^{1/2}$$

$$V = (\pi/20) D^2 L$$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$$

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Table 1: Contact Resistance for vacuum interfaces,  $R''_{t,c} \times 10^4 \left( \frac{m^2 K}{W} \right)$

Material	100 (kPa)	10,000 (kPa)
Stainless Steel	6-25	0.7-4.0
Copper	1-10	0.1-0.5
Magnesium	1.5-3.5	0.2-0.4
Aluminum	1.5-5.0	0.2-0.4

Table 2: Contact Resistance for aluminum ( $10\mu m$  surface roughness,  $10kPa$  contact pressure)

Interfacial Fluid	$R''_{t,c} \times 10^4 \left( \frac{m^2 K}{W} \right)$
Air	2.75
Helium	1.05
Hydrogen	0.720
Silicone Oil	0.525
Glycerine	0.265

Table 3: Fin Tip Conditions

Case	Tip Condition	Temperature Distribution $\frac{\theta}{\theta_b}$	Fin Heat Transfer Rate $q_f$
A	Convection	$\frac{\cosh m(L-x) + \left(\frac{h}{mk}\right) \sinh m(L-x)}{\cosh mL + \left(\frac{h}{mK}\right) \sinh mL}$	$M \frac{\sinh mL + \left(\frac{h}{mk}\right) \cosh mL}{\cosh mL + \left(\frac{h}{mK}\right) \sinh mL}$
B	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Constant Temperature	$\frac{\left(\frac{\theta_f}{\theta_b}\right) \sinh mx + \sinh m(L-x)}{\sinh mL}$	$M \frac{\cosh mL - \frac{\theta_f}{\theta_b}}{\sinh mL}$
D	Infinite Fin ( $L \rightarrow \infty$ )	$e^{-mx}$	$M$