

ENGT 320 Applied Thermal Systems
Quiz 4 Formula Sheet

$$m = \rho V$$

$$\dot{V} = \frac{V}{t}$$

$$q = mc_p \Delta T$$

$$q = -kA \frac{dT}{dt}$$

$$q = hA (T_s - T_{\infty})$$

$$q = \varepsilon \sigma A (T_s^4 - T_{sur}^4)$$

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

$$\Delta E_{sys} = \Sigma E_{in} - \Sigma E_{out}$$

$$\frac{dE_{sys}}{dt} = \Sigma \dot{E}_{in} - \Sigma \dot{E}_{out}$$

$$q'' = \frac{q}{A}$$

$$\hat{q}'' = -k_x \frac{\partial T}{\partial x} \hat{i} - k_y \frac{\partial T}{\partial y} \hat{j} - k_z \frac{\partial T}{\partial z} \hat{k}$$

$$k = \frac{9\gamma-5}{4} \frac{c_v}{\pi d^2} \sqrt{\frac{M_w k_B T}{N_A \pi}}$$

$$k_B = 1.381 \times 10^{-23} \frac{J}{K}$$

$$N_A = 6.022 \times 10^{23}$$

$$\alpha = \frac{k}{\rho c_p}$$

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = 0$$

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) +$$

$$\frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(kr^2 \frac{\partial T}{\partial r} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$T(0, t) = T_s$$

$$-k \frac{\partial T}{\partial x} \Big|_{x=0} = q_s''$$

$$\frac{\partial T}{\partial x} \Big|_{x=0} = 0$$

$$-k \frac{\partial T}{\partial x} \Big|_{x=0} = h(T_{\infty} - T(0, t))$$

$$\Delta U = N(V^- - V^+)$$

$$q = \frac{\Delta U}{\Delta t} = \frac{N}{\Delta t} \Delta V$$

$$q = -IV = \frac{-V^2}{R} = -I^2 R$$

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$

$$S_{AB} = S_B - S_A = \frac{-\Delta V}{\Delta T}$$

$$C_1 = \frac{T_{s,2} - T_{s,1}}{L}$$

$$C_2 = T_{s,1}$$

$$T(x) = (T_{s,2} - T_{s,1}) \frac{x}{L} + T_{s,1}$$

$$q = \frac{kA}{L} (T_{s,1} - T_{s,2})$$

$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{kA}$$

$$R = \frac{V}{I}$$

$$R_{t,conv} = \frac{T_s - T_{\infty}}{q} = \frac{1}{hA}$$

$$R_{t,rad} = \frac{T_s - T_{sur}}{q} = \frac{1}{h_r A}$$

$$h_r = \varepsilon \sigma (T_s + T_{sur}) (T_s^2 + T_{sur}^2)$$

$$q = U A \Delta T$$

$$U = \frac{1}{R_{tot} A}$$

$$R_{tot} = \Sigma R_t = \frac{1}{U A}$$

$$R_{t,c}'' = \frac{T_A - T_B}{q''}$$

$$q = \frac{k_{eff} A}{L} (T_1 - T_2)$$

$$k_{eff,min} = \frac{1}{\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}}$$

$$k_{eff,max} = \varepsilon k_f + (1 - \varepsilon) k_s$$

$$R_{t,cond} = \frac{\ln \frac{r_2}{r_1}}{2\pi k L}$$

$$R_{t,conv} = \frac{1}{h 2\pi r L}$$

$$\frac{d^2 T}{dx^2} + \left(\frac{1}{A_c} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \left(\frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_{\infty}) = 0$$

$$A_s = Px$$

$$\frac{d^2 T}{dx^2} - \left(\frac{hP}{kA_c} \right) (T - T_{\infty})$$

$$\frac{d^2 \theta}{dx^2} - m^2 \theta = 0$$

$$\theta = T(x) - T_{\infty}$$

$$m^2 = \frac{hP}{kA_c}$$

$$P = 2w + 2t$$

$$A_c = wt$$

$$P = \pi D$$

$$A_c = \frac{\pi}{4} D^2$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta_b = T_b - T_{\infty}$$

$$M = \sqrt{h P K A_c} \theta_b$$

$$\varepsilon_f = \frac{q_f}{h A_{c,b} \theta_b}$$

$$\varepsilon_f = \sqrt{\frac{kP}{h A_c}}$$

$$R_{t,f} = \frac{\theta_b}{q_f}$$

$$R_{t,b} = \frac{1}{h A_{c,b}}$$

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}$$

$$\eta_f = \frac{q_f}{q_{max}} = \frac{q_f}{h A_f \theta_b}$$

$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{h A_t \theta_b}$$

$$A_t = N A_f + A_b$$

$$q_t = N \eta_f h A_f \theta_b + h A_b \theta_b$$

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$$\eta_o = 1 - \frac{NA_f}{A_t} (1 - \eta_f)$$

$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{m,n} \approx \frac{\frac{\partial T}{\partial x} \Big|_{m+\frac{1}{2},n} - \frac{\partial T}{\partial x} \Big|_{m-\frac{1}{2},n}}{\Delta x}$$

$$\frac{\partial T}{\partial x} \Big|_{m+\frac{1}{2},n} \approx \frac{T \Big|_{m+1,n} - T \Big|_{m,n}}{\Delta x}$$

$$\frac{\partial T}{\partial x} \Big|_{m-\frac{1}{2},n} \approx \frac{T \Big|_{m,n} - T \Big|_{m-1,n}}{\Delta x}$$

$$\frac{\partial^2 T}{\partial x^2} \Big|_{m,n} \approx \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{(\Delta x)^2}$$

$$\frac{\partial^2 T}{\partial y^2} \Big|_{m,n} \approx \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{(\Delta y)^2}$$

$$\sum_{i=1}^4 q_{(i) \rightarrow (m,n)} + \dot{q} (\Delta x \cdot \Delta y \cdot 1) = 0$$

$$q_{(m-1,n) \rightarrow (m,n)} = k (\Delta y \cdot 1) \frac{T_{m-1,n} - T_{m,n}}{\Delta x}$$

$$q_{(m+1,n) \rightarrow (m,n)} = k (\Delta y \cdot 1) \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

$$q_{(m,n+1) \rightarrow (m,n)} = k (\Delta x \cdot 1) \frac{T_{m,n+1} - T_{m,n}}{\Delta y}$$

$$q_{(m,n-1) \rightarrow (m,n)} = k (\Delta x \cdot 1) \frac{T_{m,n-1} - T_{m,n}}{\Delta y}$$

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} + \frac{\dot{q} \cdot 1 \cdot (\Delta x)^2}{k} - 4T_{m,n} = 0$$

$$A = \begin{bmatrix} AT = C \\ a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix}$$

$$T = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix}$$

$$C = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{bmatrix}$$

$$T = A^{-1}C$$

$$\frac{u(y)}{u_\infty} = 0.99$$

$$C_f = \frac{\tau_s}{\rho u_\infty^2/2}$$

$$\tau_s = \mu \frac{\partial u}{\partial y} \Big|_{y=0}$$

$$\frac{T_s - T(y)}{T_s - T_\infty} = 0.99$$

$$q_s'' = -k \frac{\partial T}{\partial y} \Big|_{y=0}$$

$$q_s'' = h (T_s - T_\infty)$$

$$h = \frac{-k \frac{\partial T}{\partial y} \Big|_{y=0}}{T_s - T_\infty}$$

$$q = \int_{A_s} q'' dA_s = (T_s - T_\infty) \int_{A_s} h dA_s$$

$$\bar{q}'' = \bar{h} (T_s - T_\infty)$$

$$\bar{q} = \bar{q}'' A_s = \bar{h} A_s (T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h dA_s$$

$$\bar{h} = \frac{1}{L} \int_0^L h dx$$

$$Re_x = \frac{\rho u_\infty x}{\mu}$$

$$x^* = \frac{x}{L}$$

$$y^* = \frac{y}{L}$$

$$u^* = \frac{u}{V}$$

$$v^* = \frac{v}{V}$$

$$T^* = \frac{T - T_s}{T_\infty - T_s}$$

$$p^* = \frac{p - p_\infty}{\rho V^2/2}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$Re_L = \frac{VL}{\nu}$$

$$\nu = \frac{\mu}{\rho}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$Pr = \frac{\nu}{\alpha}$$

$$Nu = \frac{hL}{k} = \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}$$

$$\overline{Nu} = \frac{\bar{h}L}{k}$$

$$T_f = \frac{T_s + T_\infty}{2}$$

Isothermal Flat Plate

$$\frac{\delta}{\delta_t} \approx Pr^{1/3}$$

Laminar:

$$\delta = \frac{5}{\sqrt{\frac{u_\infty}{\nu x}}} = \frac{5x}{\sqrt{Re_x}}$$

$$C_{f,x} = \frac{\tau_{s,x}}{\frac{\rho u_\infty^2}{2}} = 0.664 Re_x^{-1/2}$$

$$Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3}$$

$$(Pr \gtrsim 0.6)$$

$$\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$$

$$\overline{Nu}_x = \frac{\bar{h}_x x}{k} = 0.664 Re_x^{1/2} Pr^{1/3}$$

$$(Pr \gtrsim 0.6)$$

Turbulent:

$$\delta = 0.37 x Re_x^{-1/2}$$

$$C_{f,x} = 0.0592 Re_x^{-1/5}$$

$$(Re_{x,c} \lesssim Re_x \lesssim 10^8)$$

$$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$$

$$(0.6 \lesssim Pr \lesssim 60)$$

$$A = 0.037 Re_{x,c}^{4/5} - 0.664 Re_{x,c}^{1/2}$$

$$\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - \frac{2A}{Re_L}$$

$$(Re_{x,c} \lesssim Re_L \lesssim 10^8)$$

$$\overline{Nu}_L = (0.037 Re_L^{4/5} - A) Pr^{1/3}$$

$$(0.6 \lesssim Pr \lesssim 60)$$

$$(Re_{x,c} \lesssim Re_L \lesssim 10^8)$$

$$Re_D = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

$$C_D = \frac{F_D}{A_f \left(\frac{\rho V^2}{2} \right)}$$

Cylinder

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = C Re_D^m Pr^{1/3}$$

$$(Pr \gtrsim 0.7)$$

See Table 5

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Various Geometries

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = CRe_D^m Pr^{1/3}$$

$$(Pr \gtrsim 0.7)$$

See Table 6

Sphere

$$A_s = \pi D^2$$

$$C_D = \frac{25}{Re_D}$$

$$(Re_D \lesssim 0.5)$$

$$\overline{Nu}_D = 2 +$$

$$\left(0.4Re_D^{1/2} + 0.06Re_D^{2/3}\right) Pr^{0.4} \left(\frac{\mu}{\mu_s}\right)^{1/4}$$

$$(0.71 \lesssim Pr \lesssim 380)$$

$$(3.5 \lesssim Re_D \lesssim 7.6 \times 10^4)$$

$$\left(1.0 \lesssim \frac{\mu}{\mu_s} \lesssim 3.2\right)$$

$$\Delta T_i = T_s - T_{m,i}$$

$$q_{conv} = \bar{h}A_s\Delta T_{lm}$$

Circular Tubes

Fully dev. lam. w/ $q'' = \mathbb{C}$:

$$Nu_D = 4.36$$

Fully dev. lam. w/ $T_s = \mathbb{C}$:

$$Nu_D = 3.66$$

Fully dev. turb. w/ small ΔT :

$$Nu_D = 0.023Re_D^{4/5} Pr^n$$

$$n = 0.4, \quad T_s > T_m$$

$$n = 0.3, \quad T_s < T_m$$

$$(0.6 \lesssim Pr \lesssim 160)$$

$$(Re_D \gtrsim 10,000)$$

$$(L/D \gtrsim 10)$$

$$q = m_h (i_{h,i} - i_{h,o})$$

$$q = m_c (i_{c,i} - i_{c,o})$$

$$q = m_h c_{p,h} (T_{h,i} - T_{h,o})$$

$$q = m_c c_{p,c} (T_{c,o} - T_{c,i})$$

$$q = UA\Delta T_m$$

$$q = UA\Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$$

$$\Delta T_1 = T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i}$$

$$\Delta T_2 = T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o}$$

$$\Delta T_1 = T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o}$$

$$\Delta T_2 = T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i}$$

$$q_{max} = C_{min} (T_{h,i} - T_{c,i})$$

$$\varepsilon = \frac{C_h (T_{h,i} - T_{h,o})}{C_{min} (T_{h,i} - T_{c,i})}$$

$$\varepsilon = \frac{C_c (T_{c,o} - T_{c,i})}{C_{min} (T_{h,i} - T_{c,i})}$$

$$q = \varepsilon C_{min} (T_{h,i} - T_{c,i})$$

$$NTU = \frac{UA}{C_{min}}$$

$$m = \rho u_m A_c$$

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$$

$$\left(\frac{x_{fd,t}}{D}\right) \approx 0.05 Re_D Pr$$

$$f = \frac{-\frac{dP}{dx} D}{\rho \frac{u_m^2}{2}}$$

$$f = \frac{64}{Re_D}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log \left[\frac{e/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right]$$

Fully dev. turb. w/ large ΔT :

$$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$(0.7 \lesssim Pr \lesssim 16,700)$$

$$(Re_D \gtrsim 10,000)$$

$$(L/D \gtrsim 10)$$

$$dq_{conv} = mc_p [(T_m + dT_m) - T_m] =$$

$$mc_p dT_m$$

$$dq_{conv} = q_s'' P dx$$

$$\frac{dT_m}{dx} = \frac{q_s'' P}{mc_p} = \frac{P}{mc_p} h (T_s - T_m)$$

$$T_m(x) = T_{m,i} + \frac{q_s'' P}{mc_p} x$$

$$\Delta T = T_s - T_m$$

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{mc_p} h \Delta T$$

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = e^{-\frac{Px}{mc_p} \bar{h}}$$

$$\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln \left(\frac{\Delta T_o}{\Delta T_i} \right)}$$

$$\Delta T_o = T_s - T_{m,o}$$

Noncircular Tubes

$$D_h = \frac{4A_c}{P}$$

$$Nu_D = \frac{hD_h}{k}$$

See Table 7

$$\frac{1}{UA} = \frac{1}{U_c A_c} = \frac{1}{U_h A_h} =$$

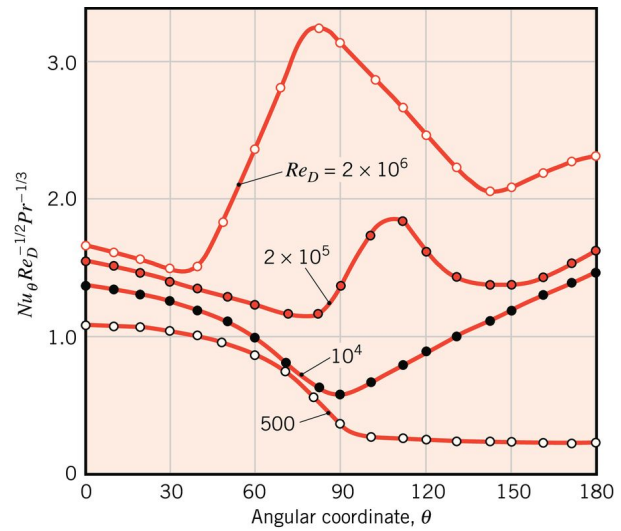
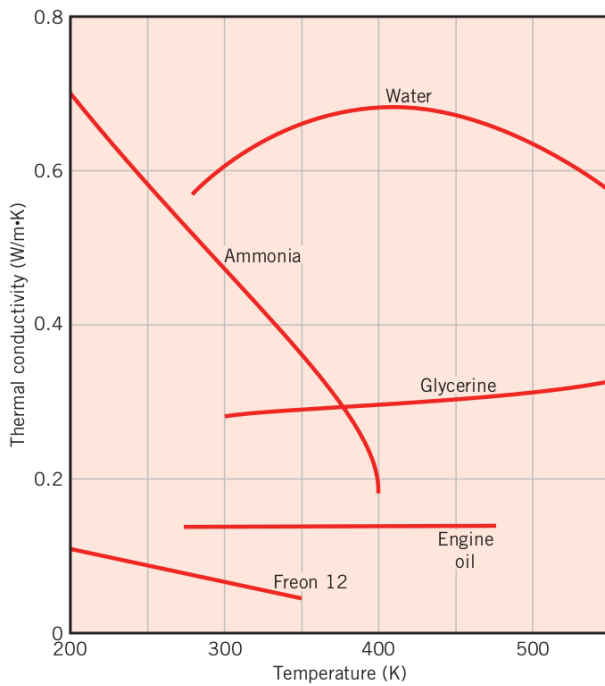
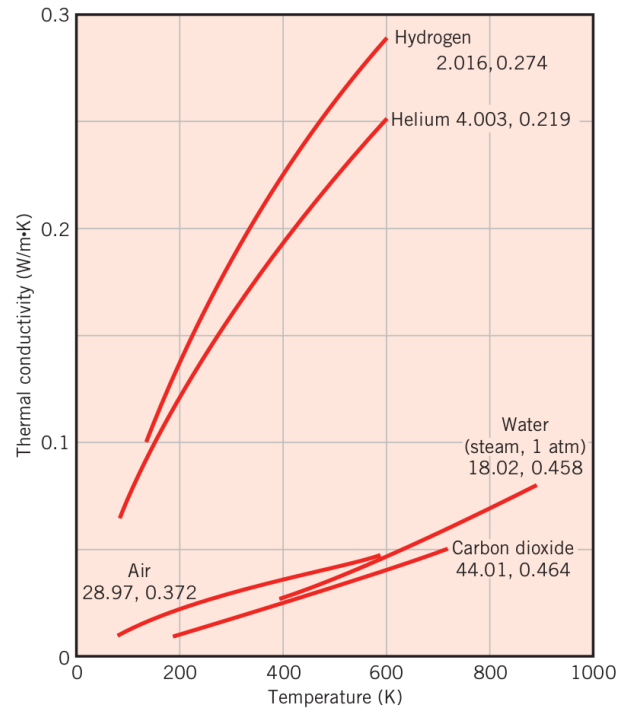
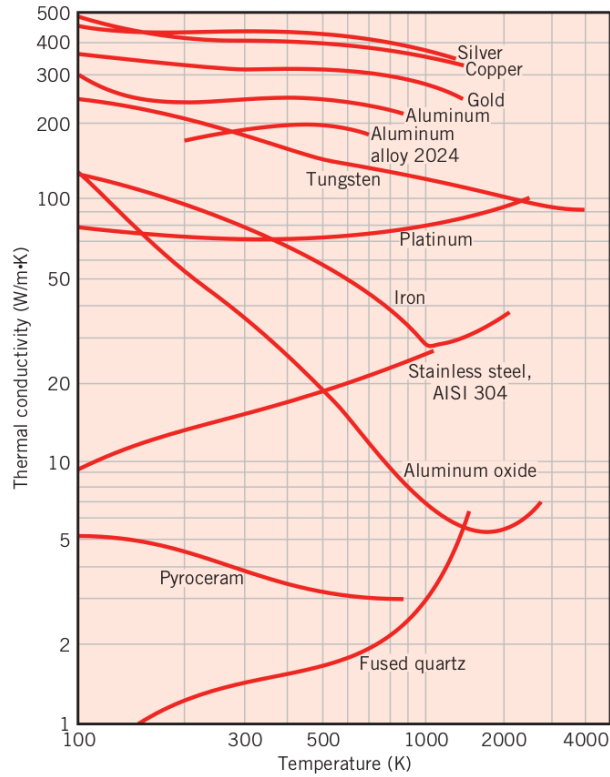
$$\frac{1}{(hA)_c} + R_w + \frac{1}{(hA)_h}$$

$$\frac{1}{UA} = \frac{1}{(\eta_o hA)_c} + \frac{R_{f,c}''}{(\eta_o A)_c} + R_w +$$

$$\frac{R_{f,h}''}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h}$$

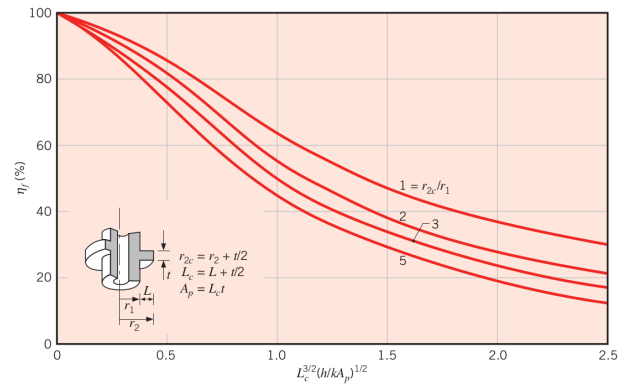
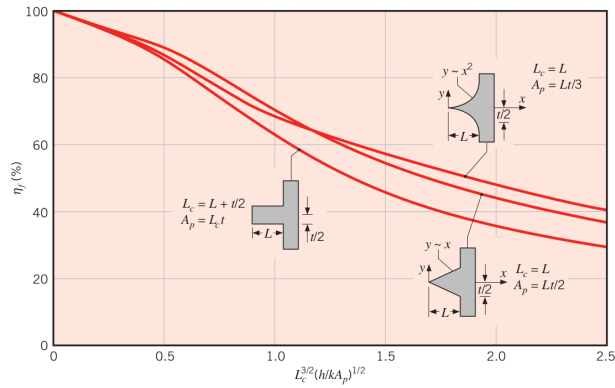
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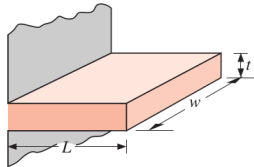
Straight Fins

Rectangular

$$A_f = 2wL_c$$

$$L_c = L + (t/2)$$

$$A_p = tL$$

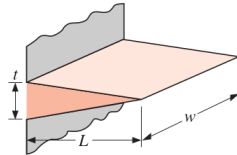


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$$A_f = 2w[L^2 + (t/2)^2]^{1/2}$$

$$A_p = (t/2)L$$



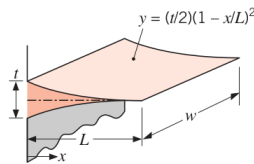
$$\eta_f = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$$

Parabolic

$$A_f = w[C_1 L + (L^2/t) \ln(t/L + C_1)]$$

$$C_1 = [1 + (t/L)^2]^{1/2}$$

$$A_p = (t/3)L$$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

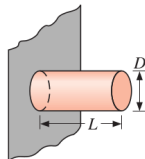
Pin Fins

Rectangular

$$A_f = \pi D L_c$$

$$L_c = L + (D/4)$$

$$V = (\pi D^2/4)L$$

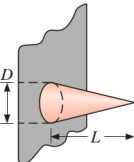


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$$A_f = \frac{\pi D}{2} [L^2 + (D/2)^2]^{1/2}$$

$$V = (\pi/12) D^2 L$$



$$\eta_f = \frac{2}{mL} \frac{I_2(2mL)}{I_1(2mL)}$$

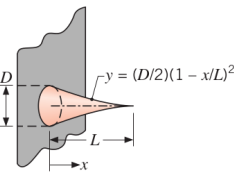
Parabolic

$$A_f = \frac{\pi L^3}{8D} \{C_3 C_4 - \frac{L}{2D} \ln [(2DC_4/L) + C_3]\}$$

$$C_3 = 1 + 2(D/L)^2$$

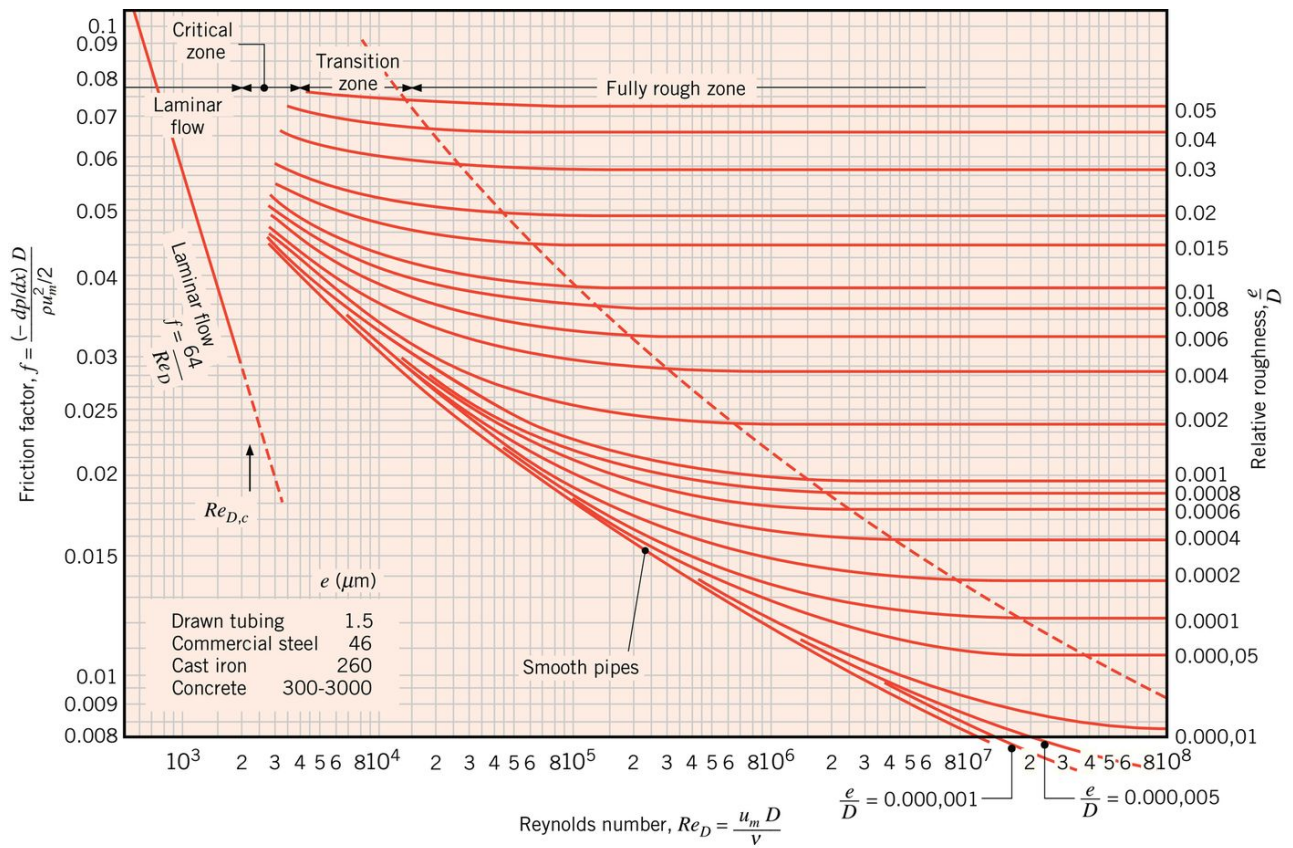
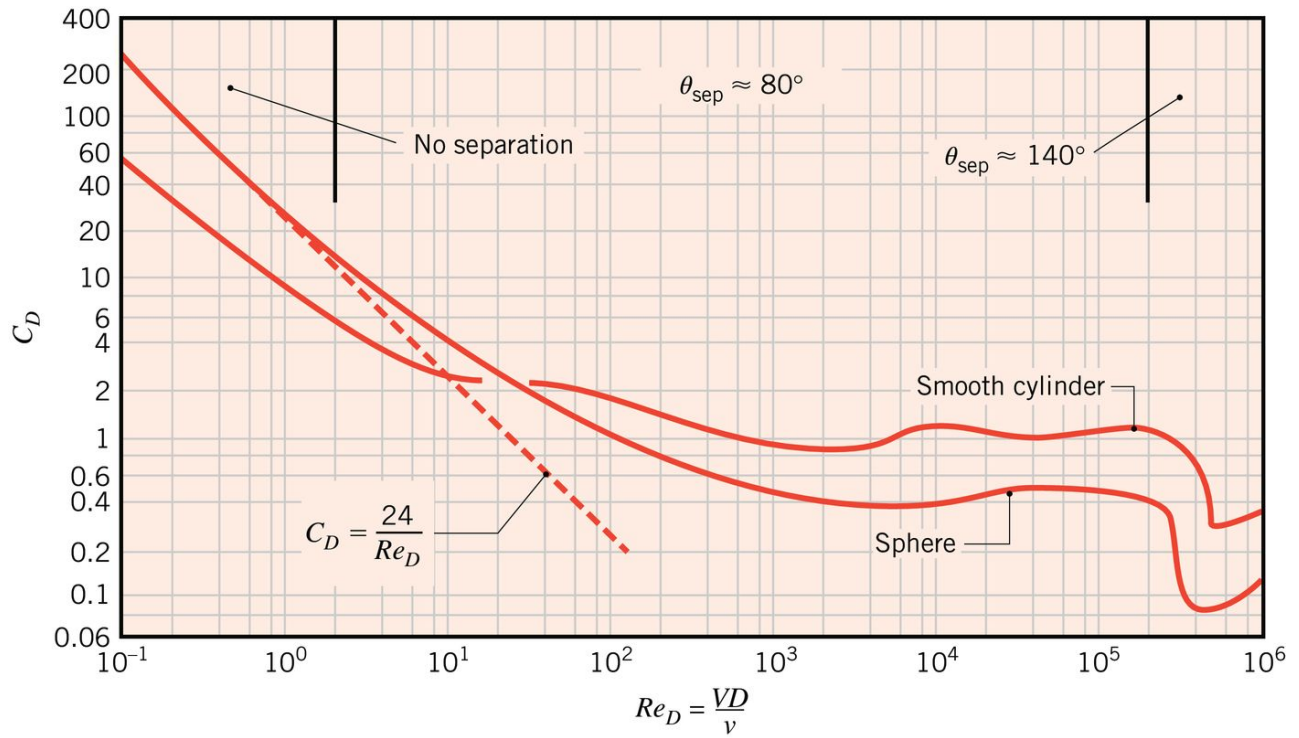
$$C_4 = [1 + (D/L)^2]^{1/2}$$

$$V = (\pi/20) D^2 L$$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$$

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Quiz 4 Formula Sheet



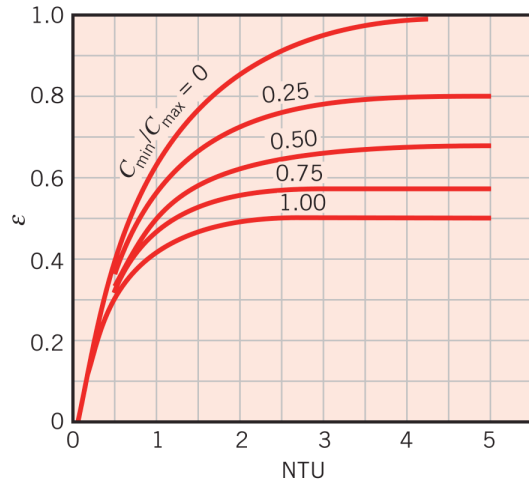
ENGT 320 Applied Thermal Systems
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Flow Arrangement	Relation
Parallel ow	$\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}$
Counterow	$\varepsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]} \quad (C_r < 1)$ $\varepsilon = \frac{NTU}{1 + NTU} \quad (C_r = 1)$
Shell-and-tube	
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2 \left\{ 1 + C_r + (1 + C_r^2)^{1/2} \times \frac{1 + \exp[-(NTU)_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(NTU)_1(1 + C_r^2)^{1/2}]} \right\}^{-1}$
n shell passes ($2n, 4n, . . .$ tube passes)	$\varepsilon = \left[\left(\frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - 1 \right] \left[\left(\frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - C_r \right]^{-1}$
Cross-ow (single pass)	
Both fluids unmixed	$\varepsilon = 1 - \exp \left[\left(\frac{1}{C_r} \right) (NTU)^{0.22} \{ \exp[-C_r(NTU)^{0.78}] - 1 \} \right]$
C_{\max} (mixed), C_{\min} (unmixed)	$\varepsilon = \left(\frac{1}{C_r} \right) (1 - \exp \{ -C_r [1 - \exp(-NTU)] \})$
C_{\min} (mixed), C_{\max} (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1} \{ 1 - \exp[-C_r(NTU)] \})$
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-NTU)$

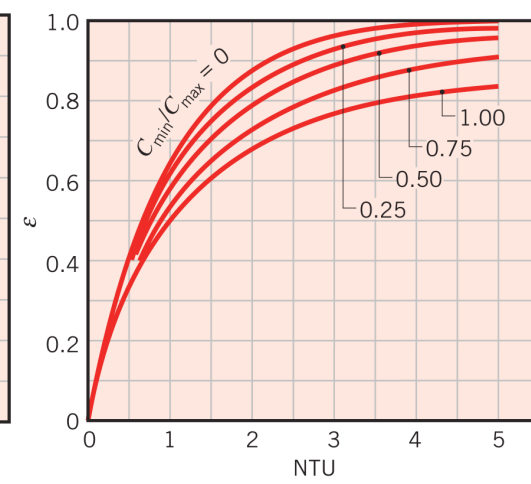
Flow Arrangement	Relation
Parallel ow	$NTU = - \frac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$
Counterow	$NTU = \frac{1}{C_r - 1} \ln \left(\frac{\varepsilon - 1}{\varepsilon C_r - 1} \right) \quad (C_r < 1)$ $NTU = \frac{\varepsilon}{1 - \varepsilon} \quad (C_r = 1)$
Shell-and-tube	
One shell pass (2, 4, . . . tube passes)	$(NTU)_1 = -(1 + C_r^2)^{-1/2} \ln \left(\frac{E - 1}{E + 1} \right)$ $E = \frac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$
n shell passes ($2n, 4n, . . .$ tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \frac{F - 1}{F - C_r} \quad F = \left(\frac{\varepsilon C_r - 1}{\varepsilon - 1} \right)^{1/n} \quad NTU = n(NTU)_1$
Cross-ow (single pass)	
C_{\max} (mixed), C_{\min} (unmixed)	$NTU = - \ln \left[1 + \left(\frac{1}{C_r} \right) \ln(1 - \varepsilon C_r) \right]$
C_{\min} (mixed), C_{\max} (unmixed)	$NTU = - \left(\frac{1}{C_r} \right) \ln [C_r \ln(1 - \varepsilon) + 1]$
All exchangers ($C_r = 0$)	$NTU = - \ln(1 - \varepsilon)$

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Quiz 4 Formula Sheet

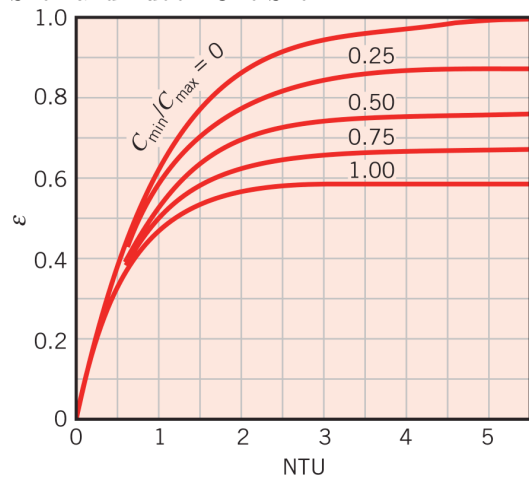
Parallel Flow



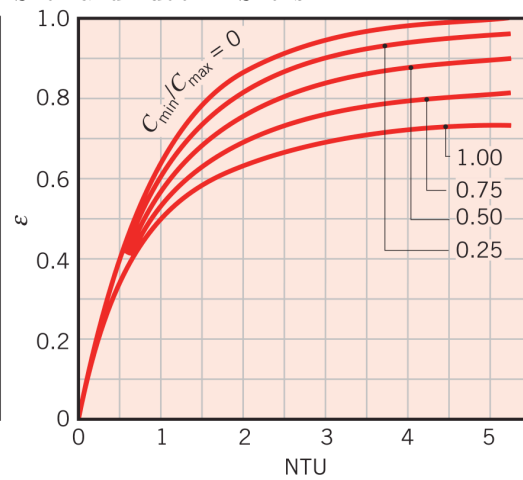
Counter Flow



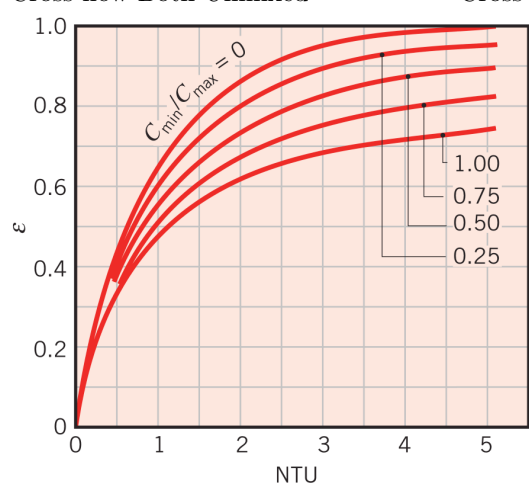
Shell and Tube - One Shell



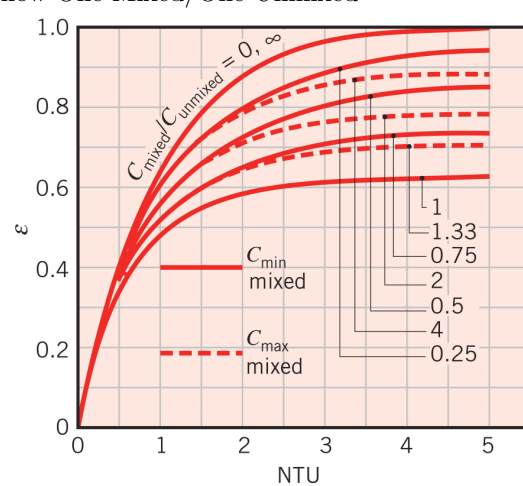
Shell and Tube - 2 Shells



Cross-flow Both Unmixed



Cross-flow One Mixed/One Unmixed



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Table 1: Contact Resistance for vacuum interfaces, $R''_{t,c} \times 10^4 \left(\frac{m^2 K}{W} \right)$

Material	100 (kPa)	10,000 (kPa)
Stainless Steel	6-25	0.7-4.0
Copper	1-10	0.1-0.5
Magnesium	1.5-3.5	0.2-0.4
Aluminum	1.5-5.0	0.2-0.4

Table 2: Contact Resistance for aluminum ($10\mu m$ surface roughness, $10kPa$ contact pressure)

Interfacial Fluid	$R''_{t,c} \times 10^4 \left(\frac{m^2 K}{W} \right)$
Air	2.75
Helium	1.05
Hydrogen	0.720
Silicone Oil	0.525
Glycerine	0.265

Table 3: Fin Tip Conditions

Case	Tip Condition	Temperature Distribution $\frac{\theta}{\theta_b}$	Fin Heat Transfer Rate q_f
A	Convection	$\frac{\cosh m(L-x) + \left(\frac{h}{mk}\right) \sinh m(L-x)}{\cosh mL + \left(\frac{h}{mK}\right) \sinh mL}$	$M \frac{\sinh mL + \left(\frac{h}{mk}\right) \cosh mL}{\cosh mL + \left(\frac{h}{mk}\right) \sinh mL}$
B	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Constant Temperature	$\frac{\left(\frac{\theta_L}{\theta_b}\right) \sinh mx + \sinh m(L-x)}{\sinh mL}$	$M \frac{\cosh mL - \frac{\theta_L}{\theta_b}}{\sinh mL}$
D	Infinite Fin ($L \rightarrow \infty$)	e^{-mx}	M

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Table 4: Energy Balance Method Case Summary

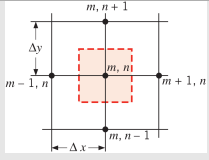
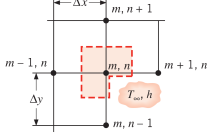
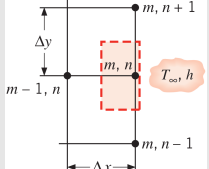
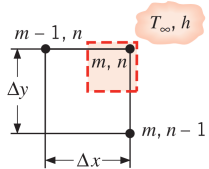
Case	Diagram	Equation
1		$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0$
2		$2(T_{m-1,n} - T_{m,n+1}) + (T_{m+1,n} + T_{m,n-1}) + 2\frac{h\Delta x}{k}T_{\infty} - 2\left(3 + \frac{h\Delta x}{k}\right)T_{m,n} = 0$
3		$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + 2\frac{h\Delta x}{k}T_{\infty} - 2\left(2 + \frac{h\Delta x}{k}\right)T_{m,n} = 0$
4		$(T_{m,n-1} + T_{m-1,n}) + 2\frac{h\Delta x}{k}T_{\infty} - 2\left(1 + \frac{h\Delta x}{k}\right)T_{m,n} = 0$

Table 5: Cylinder In Cross Flow

Re_D	C	m
0.4 – 4	0.989	0.330
4 – 40	0.911	0.385
40 – 4000	0.683	0.466
4000 – 40,000	0.193	0.618
40,000 – 400,000	0.027	0.805

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Table 6: Various Geometries In Cross Flow



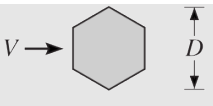
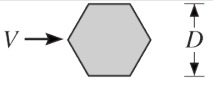

<i>Geometry</i>	Re_D	C	m
	6000 – 60,000	0.304	0.59
	5000 – 60,000	0.158	0.66
	5200 – 20,400 20,400 – 105,000	0.164 0.039	0.638 0.78
	4500 – 90,700	0.150	0.638
	10,000 – 50,000 7,000 – 80,000	0.667 0.191	0.500 0.667

Table 7: Noncircular Tubes





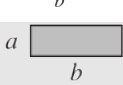
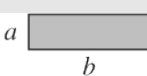
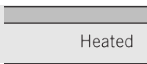


Cross Section	$\frac{b}{a}$	Uniform q_s''	Uniform T_s	fRe_{D_h}
		4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	∞	8.23	7.54	96
	∞	5.39	4.86	96
		3.11	2.49	53

Table 8: Fouling Factors

<i>Fluid</i>	R_f'' (m^2K/W)
Seawater and treated boiler feedwater (below $50^\circ C$)	0.001
Seawater and treated boiler feedwater (above $50^\circ C$)	0.002
River water (below $50^\circ C$)	0.0002 - 0.001
Fuel Oil	0.0009
Refrigerating Liquids	0.0002
Steam (nonoil bearing)	0.0001