

ENGT 320 Applied Thermal Systems
Quiz 4 Formula Sheet

$$m = \rho V$$

$$\dot{V} = \frac{V}{t}$$

$$q = mc_p \Delta T$$

$$q = -kA \frac{dT}{dt}$$

$$q = hA (T_s - T_{\infty})$$

$$q = \varepsilon \sigma A (T_s^4 - T_{sur}^4)$$

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

$$\Delta E_{sys} = \Sigma E_{in} - \Sigma E_{out}$$

$$\frac{dE_{sys}}{dt} = \Sigma \dot{E}_{in} - \Sigma \dot{E}_{out}$$

$$q'' = \frac{q}{A}$$

$$\hat{q}'' = -k_x \frac{\partial T}{\partial x} \hat{i} - k_y \frac{\partial T}{\partial y} \hat{j} - k_z \frac{\partial T}{\partial z} \hat{k}$$

$$k = \frac{9\gamma-5}{4} \frac{c_v}{\pi d^2} \sqrt{\frac{M_w k_B T}{N_A \pi}}$$

$$k_B = 1.381 \times 10^{-23} \frac{J}{K}$$

$$N_A = 6.022 \times 10^{23}$$

$$\alpha = \frac{k}{\rho c_p}$$

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = 0$$

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) +$$

$$\frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} +$$

$$\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(kr^2 \frac{\partial T}{\partial r} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$T(0, t) = T_s$$

$$-k \frac{\partial T}{\partial x} \Big|_{x=0} = q_s''$$

$$\frac{\partial T}{\partial x} \Big|_{x=0} = 0$$

$$-k \frac{\partial T}{\partial x} \Big|_{x=0} = h(T_{\infty} - T(0, t))$$

$$\Delta U = N(V^- - V^+)$$

$$q = \frac{\Delta U}{\Delta t} = \frac{N}{\Delta t} \Delta V$$

$$q = -IV = \frac{-V^2}{R} = -I^2 R$$

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$

$$S_{AB} = S_B - S_A = \frac{-\Delta V}{\Delta T}$$

$$C_1 = \frac{T_{s,2} - T_{s,1}}{L}$$

$$C_2 = T_{s,1}$$

$$T(x) = (T_{s,2} - T_{s,1}) \frac{x}{L} + T_{s,1}$$

$$q = \frac{kA}{L} (T_{s,1} - T_{s,2})$$

$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{kA}$$

$$R = \frac{V}{I}$$

$$R_{t,conv} = \frac{T_s - T_{\infty}}{q} = \frac{1}{hA}$$

$$R_{t,rad} = \frac{T_s - T_{sur}}{q} = \frac{1}{h_r A}$$

$$h_r = \varepsilon \sigma (T_s + T_{sur}) (T_s^2 + T_{sur}^2)$$

$$q = U A \Delta T$$

$$U = \frac{1}{R_{tot} A}$$

$$R_{tot} = \Sigma R_t = \frac{1}{U A}$$

$$R_{t,c}'' = \frac{T_A - T_B}{q''}$$

$$q = \frac{k_{eff} A}{L} (T_1 - T_2)$$

$$k_{eff,min} = \frac{1}{\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}}$$

$$k_{eff,max} = \varepsilon k_f + (1 - \varepsilon) k_s$$

$$R_{t,cond} = \frac{\ln \frac{r_2}{r_1}}{2\pi k L}$$

$$R_{t,conv} = \frac{1}{h 2\pi r L}$$

$$\frac{d^2 T}{dx^2} + \left(\frac{1}{A_c} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \left(\frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_{\infty}) = 0$$

$$A_s = Px$$

$$\frac{d^2 T}{dx^2} - \left(\frac{hP}{kA_c} \right) (T - T_{\infty})$$

$$\frac{d^2 \theta}{dx^2} - m^2 \theta = 0$$

$$\theta = T(x) - T_{\infty}$$

$$m^2 = \frac{hP}{kA_c}$$

$$P = 2w + 2t$$

$$A_c = wt$$

$$P = \pi D$$

$$A_c = \frac{\pi}{4} D^2$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta_b = T_b - T_{\infty}$$

$$M = \sqrt{h P K A_c} \theta_b$$

$$\varepsilon_f = \frac{q_f}{h A_{c,b} \theta_b}$$

$$\varepsilon_f = \sqrt{\frac{kP}{h A_c}}$$

$$R_{t,f} = \frac{\theta_b}{q_f}$$

$$R_{t,b} = \frac{1}{h A_{c,b}}$$

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}$$

$$\eta_f = \frac{q_f}{q_{max}} = \frac{q_f}{h A_f \theta_b}$$

$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{h A_t \theta_b}$$

$$A_t = N A_f + A_b$$

$$q_t = N \eta_f h A_f \theta_b + h A_b \theta_b$$

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$\eta_o = 1 - \frac{NA_f}{A_t} (1 - \eta_f)$ $R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$	$C_{f,x} = \frac{\tau_{s,x}}{\frac{\rho u_\infty^2}{2}} = 0.664 Re_x^{-1/2}$ $Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3}$ $(Pr \gtrsim 0.6)$ $\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$ $\overline{Nu}_x = \frac{\overline{h}_x x}{k} = 0.664 Re_x^{1/2} Pr^{1/3}$ $(Pr \gtrsim 0.6)$	<p style="text-align: center;">Various Geometries</p> $\overline{Nu}_D = \frac{\overline{h}D}{k} = C Re_D^m Pr^{1/3}$ $(Pr \gtrsim 0.7)$ <p style="text-align: center;">See Table 5</p>
<hr/> $\frac{u(y)}{u_\infty} = 0.99$ $C_f = \frac{\tau_s}{\rho u_\infty^2 / 2}$ $\tau_s = \mu \left. \frac{\partial u}{\partial y} \right _{y=0}$ $\frac{T_s - T(y)}{T_s - T_\infty} = 0.99$ $q_s'' = -k \left. \frac{\partial T}{\partial y} \right _{y=0}$ $q_s'' = h (T_s - T_\infty)$ $h = \frac{-k \left. \frac{\partial T}{\partial y} \right _{y=0}}{T_s - T_\infty}$ $q = \int_{A_s} q'' dA_s = (T_s - T_\infty) \int_{A_s} h dA_s$ $\overline{q}'' = \overline{h} (T_s - T_\infty)$ $\overline{q} = \overline{q}'' A_s = \overline{h} A_s (T_s - T_\infty)$ $\overline{h} = \frac{1}{A_s} \int_{A_s} h dA_s$ $\overline{h} = \frac{1}{L} \int_0^L h dx$ $Re_x = \frac{\rho u_\infty x}{\mu}$ $x^* = \frac{x}{L}$ $y^* = \frac{y}{L}$ $u^* = \frac{u}{V}$ $v^* = \frac{v}{V}$ $T^* = \frac{T - T_s}{T_\infty - T_s}$ $p^* = \frac{p - p_\infty}{p_\infty}$ $u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$ $Re_L = \frac{VL}{\nu}$ $\nu = \frac{\mu}{\rho}$ $u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$ $Pr = \frac{\nu}{\alpha}$ $Nu = \frac{hL}{k} = \left. \frac{\partial T^*}{\partial y^*} \right _{y^*=0}$ $\overline{Nu} = \frac{\overline{h}L}{k}$ $T_f = \frac{T_s + T_\infty}{2}$	<p style="text-align: center;">Turbulent:</p> $\delta = 0.37 x Re_x^{-1/2}$ $C_{f,x} = 0.0592 Re_x^{-1/5}$ $(Re_{x,c} \lesssim Re_x \lesssim 10^8)$ $Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$ $(0.6 \lesssim Pr \lesssim 60)$ $A = 0.037 Re_{x,c}^{4/5} - 0.664 Re_{x,c}^{1/2}$ $\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - \frac{2A}{Re_L}$ $(Re_{x,c} \lesssim Re_L \lesssim 10^8)$ $\overline{Nu}_L = \left(0.037 Re_L^{4/5} - A \right) Pr^{1/3}$ $(0.6 \lesssim Pr \lesssim 60)$ $(Re_{x,c} \lesssim Re_L \lesssim 10^8)$ $Re_D = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$ $C_D = \frac{F_D}{A_f \left(\frac{\rho V^2}{2} \right)}$	<p style="text-align: center;">Sphere</p> $A_s = \pi D^2$ $C_D = \frac{25}{Re_D}$ $(Re_D \lesssim 0.5)$ $\overline{Nu}_D = 2 + \left(0.4 Re_D^{1/2} + 0.06 Re_D^{2/3} \right) Pr^{0.4} \left(\frac{\mu}{\mu_s} \right)^{1/4}$ $(0.71 \lesssim Pr \lesssim 380)$ $(3.5 \lesssim Re_D \lesssim 7.6 \times 10^4)$ $\left(1.0 \lesssim \frac{\mu}{\mu_s} \lesssim 3.2 \right)$ <p style="text-align: center;">.....</p> $m = \rho u_m A_c$ $Re_D = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$ $\left(\frac{x_{fd,t}}{D} \right) \approx 0.05 Re_D Pr$ $f = \frac{-\frac{dP}{dx}}{\rho \frac{u_m^2}{2}}$ $f = \frac{64}{Re_D}$ $\frac{1}{\sqrt{f}} = -2.0 \log \left[\frac{e/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right]$ $dq_{conv} = mc_p [(T_m + dT_m) - T_m] = mc_p dT_m$
<p style="text-align: center;">Isothermal Flat Plate</p> $\frac{\delta}{\delta_t} \approx Pr^{1/3}$ <p style="text-align: center;">Laminar:</p> $\delta = \frac{5}{\sqrt{\frac{u_\infty}{\nu x}}} = \frac{5x}{\sqrt{Re_x}}$	<p style="text-align: center;">Cylinder</p> $\overline{Nu}_D = \frac{\overline{h}D}{k} = C Re_D^m Pr^{1/3}$ $(Pr \gtrsim 0.7)$ <p style="text-align: center;">See Table 4</p>	$dq_{conv} = q_s'' P dx$ $\frac{dT_m}{dx} = \frac{q_s'' P}{mc_p} = \frac{P}{mc_p} h (T_s - T_m)$ $T_m(x) = T_{m,i} + \frac{q_s'' P}{mc_p} x$ $\Delta T = T_s - T_m$ $\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{mc_p} h \Delta T$ $\frac{T_s - T_m(x)}{T_s - T_{m,i}} = e^{-\frac{Px}{\frac{P}{mc_p} h}}$ $\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln \left(\frac{\Delta T_o}{\Delta T_i} \right)}$ $\Delta T_o = T_s - T_{m,o}$

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$$\Delta T_i = T_s - T_{m,i}$$

$$q_{conv} = \bar{h} A_s \Delta T_{lm}$$

$$q = m_h (i_{h,i} - i_{h,o})$$

$$q = m_c (i_{c,i} - i_{c,o})$$

Circular Tubes

Fully dev. lam. w/ $q'' = \mathbb{C}$:

$$Nu_D = 4.36$$

$$q = m_h c_{p,h} (T_{h,i} - T_{h,o})$$

$$q = m_c c_{p,c} (T_{c,o} - T_{c,i})$$

$$q = U A \Delta T_m$$

$$q = U A \Delta T_{lm}$$

Fully dev. lam. w/ $T_s = \mathbb{C}$:

$$Nu_D = 3.66$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$$

$$\Delta T_1 = T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i}$$

$$\Delta T_2 = T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o}$$

Fully dev. turb. w/ small ΔT :

$$Nu_D = 0.023 Re_D^{4/5} Pr^n$$

$$n = 0.4, \quad T_s > T_m$$

$$n = 0.3, \quad T_s < T_m$$

$$(0.6 \lesssim Pr \lesssim 160)$$

$$(Re_D \gtrsim 10,000)$$

$$(L/D \gtrsim 10)$$

$$\Delta T_1 = T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o}$$

$$\Delta T_2 = T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i}$$

$$q_{max} = C_{min} (T_{h,i} - T_{c,i})$$

$$\varepsilon = \frac{C_h (T_{h,i} - T_{h,o})}{C_{min} (T_{h,i} - T_{c,i})}$$

$$\varepsilon = \frac{C_c (T_{c,o} - T_{c,i})}{C_{min} (T_{h,i} - T_{c,i})}$$

$$q = \varepsilon C_{min} (T_{h,i} - T_{c,i})$$

$$NTU = \frac{UA}{C_{min}}$$

Fully dev. turb. w/ large ΔT :

$$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_s} \right)^{0.14}$$

$$(0.7 \lesssim Pr \lesssim 16,700)$$

$$(Re_D \gtrsim 10,000)$$

$$(L/D \gtrsim 10)$$

Noncircular Tubes

$$D_h = \frac{4A_c}{P}$$

$$Nu_D = \frac{h D_h}{k}$$

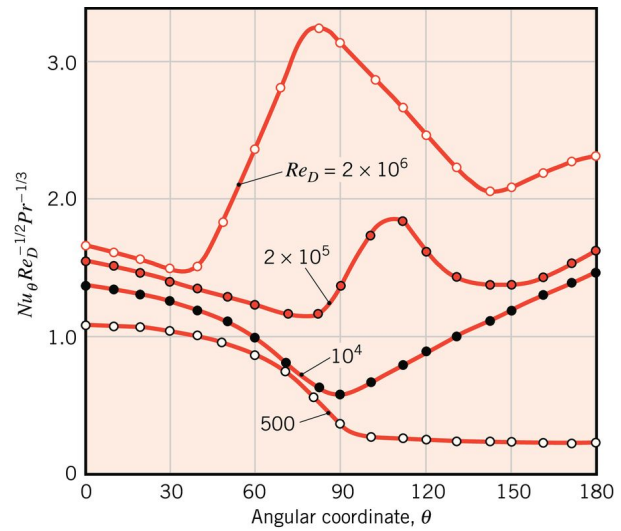
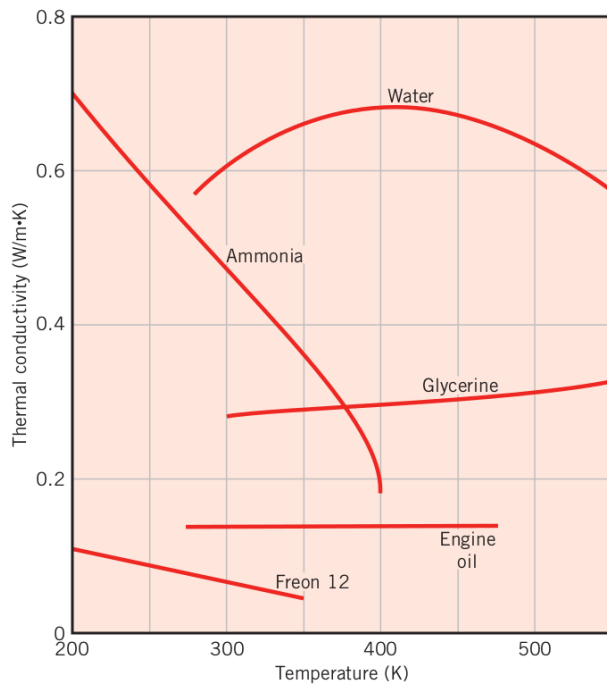
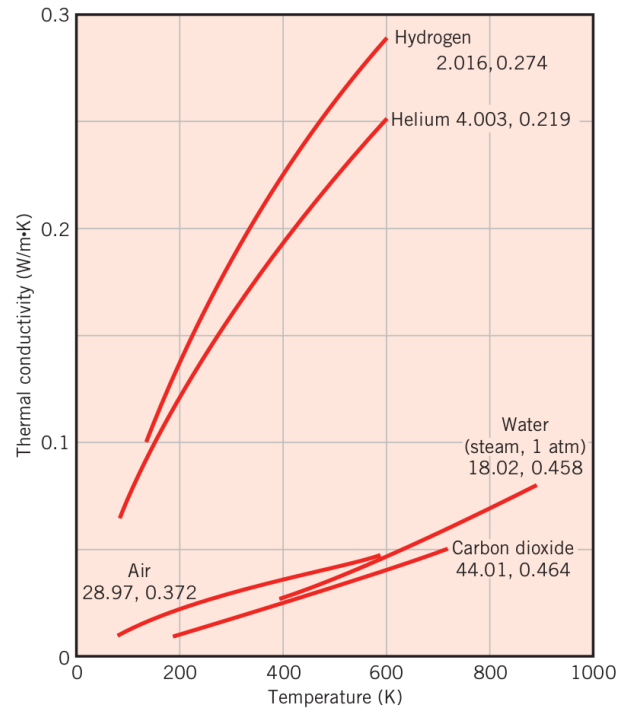
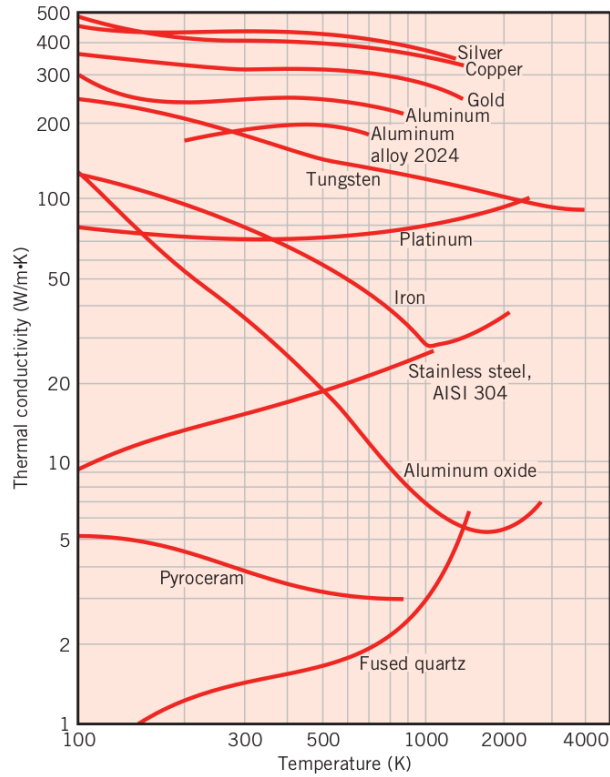
See Table 6

$$\frac{1}{UA} = \frac{1}{U_c A_c} = \frac{1}{U_h A_h} = \frac{1}{(hA)_c} + R_w + \frac{1}{(hA)_h}$$

$$\frac{1}{UA} = \frac{1}{(\eta_o h A)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o h A)_h}$$

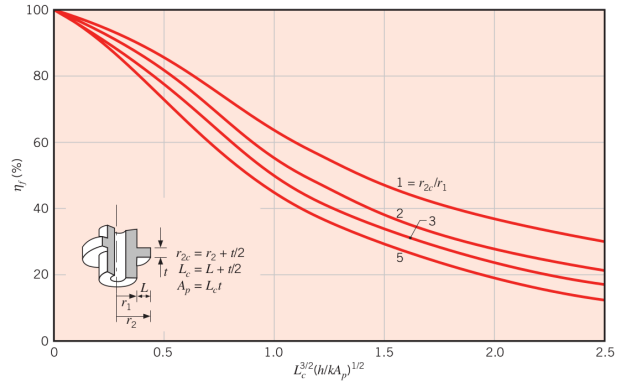
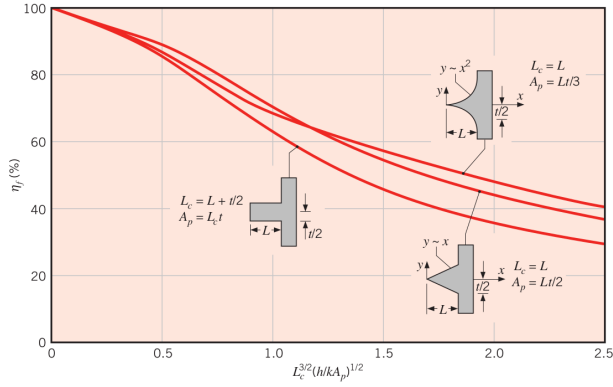
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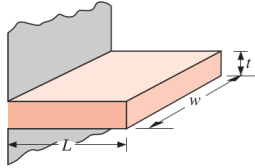
Straight Fins

Rectangular

$$A_f = 2wL_c$$

$$L_c = L + (t/2)$$

$$A_p = tL$$

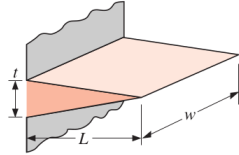


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$$A_f = 2w[L^2 + (t/2)^2]^{1/2}$$

$$A_p = (t/2)L$$



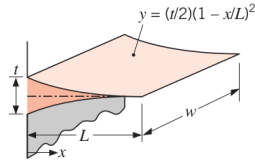
$$\eta_f = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$$

Parabolic

$$A_f = w[C_1L + (L^2/t)\ln(t/L + C_1)]$$

$$C_1 = [1 + (t/L)^2]^{1/2}$$

$$A_p = (t/3)L$$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

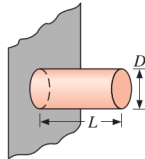
Pin Fins

Rectangular

$$A_f = \pi DL_c$$

$$L_c = L + (D/4)$$

$$V = (\pi D^2/4)L$$

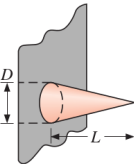


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$$A_f = \frac{\pi D}{2} [L^2 + (D/2)^2]^{1/2}$$

$$V = (\pi/12)D^2L$$



$$\eta_f = \frac{2}{mL} \frac{I_2(2mL)}{I_1(2mL)}$$

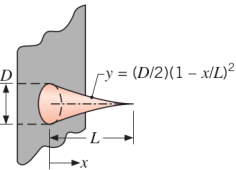
Parabolic

$$A_f = \frac{\pi L^3}{8D} \{C_3C_4 - \frac{L}{2D} \ln [(2DC_4/L) + C_3]\}$$

$$C_3 = 1 + 2(D/L)^2$$

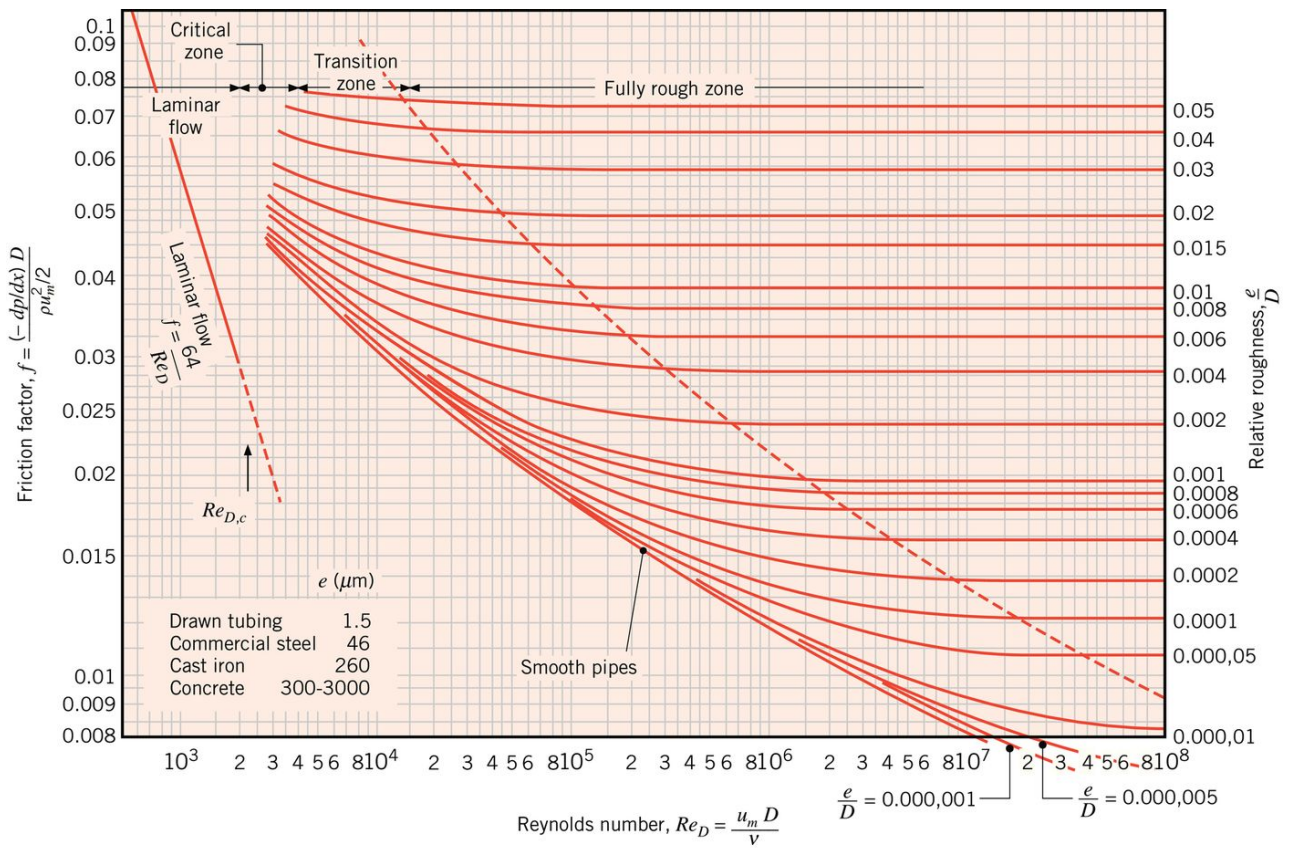
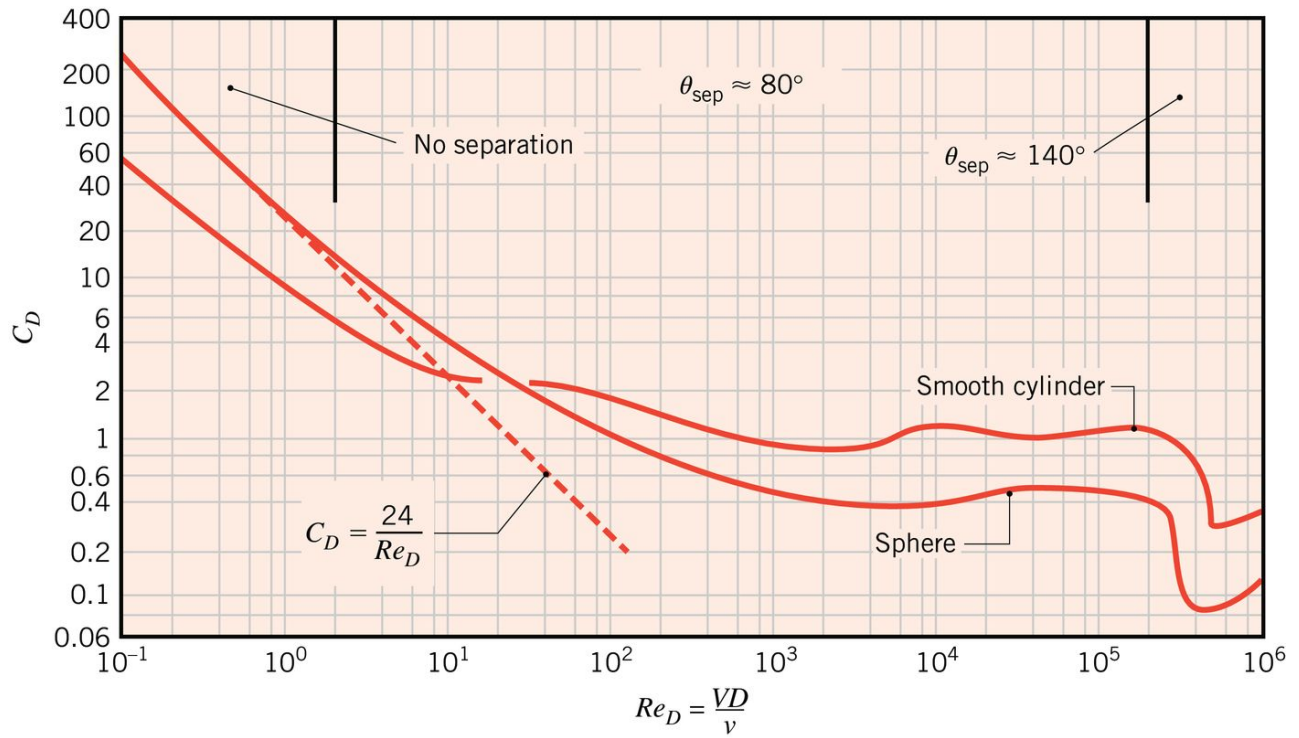
$$C_4 = [1 + (D/L)^2]^{1/2}$$

$$V = (\pi/20)D^2L$$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$$

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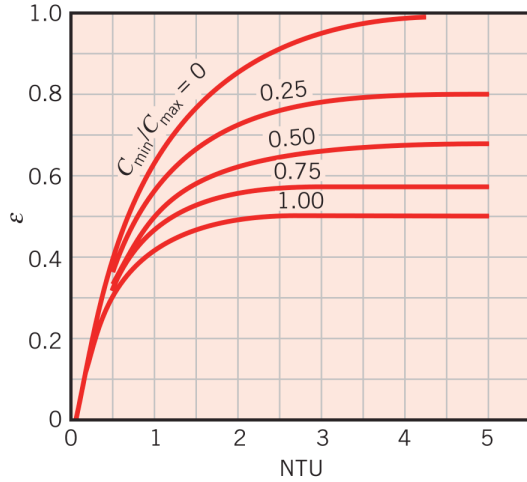
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Flow Arrangement	Relation
Parallel ow	$\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}$
Counterow	$\varepsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]} \quad (C_r < 1)$ $\varepsilon = \frac{NTU}{1 + NTU} \quad (C_r = 1)$
Shell-and-tube	
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2 \left\{ 1 + C_r + (1 + C_r^2)^{1/2} \times \frac{1 + \exp[-(NTU)_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(NTU)_1(1 + C_r^2)^{1/2}]} \right\}^{-1}$
n shell passes ($2n, 4n, . . .$ tube passes)	$\varepsilon = \left[\left(\frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - 1 \right] \left[\left(\frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - C_r \right]^{-1}$
Cross-ow (single pass)	
Both fluids unmixed	$\varepsilon = 1 - \exp \left[\left(\frac{1}{C_r} \right) (NTU)^{0.22} \{ \exp[-C_r(NTU)^{0.78}] - 1 \} \right]$
C_{\max} (mixed), C_{\min} (unmixed)	$\varepsilon = \left(\frac{1}{C_r} \right) (1 - \exp \{ -C_r [1 - \exp(-NTU)] \})$
C_{\min} (mixed), C_{\max} (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1} \{ 1 - \exp[-C_r(NTU)] \})$
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-NTU)$

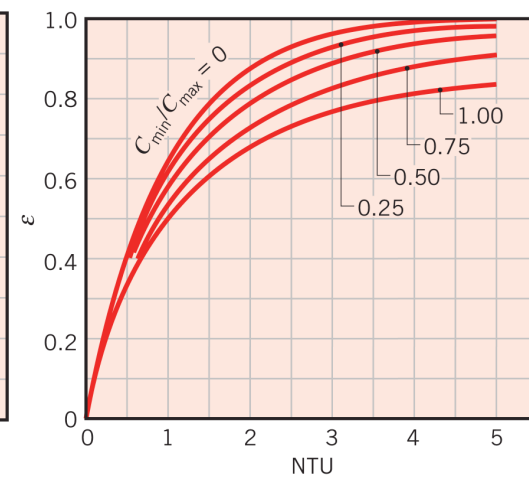
Flow Arrangement	Relation
Parallel ow	$NTU = - \frac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$
Counterow	$NTU = \frac{1}{C_r - 1} \ln \left(\frac{\varepsilon - 1}{\varepsilon C_r - 1} \right) \quad (C_r < 1)$ $NTU = \frac{\varepsilon}{1 - \varepsilon} \quad (C_r = 1)$
Shell-and-tube	
One shell pass (2, 4, . . . tube passes)	$(NTU)_1 = -(1 + C_r^2)^{-1/2} \ln \left(\frac{E - 1}{E + 1} \right)$ $E = \frac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$
n shell passes ($2n, 4n, . . .$ tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \frac{F - 1}{F - C_r} \quad F = \left(\frac{\varepsilon C_r - 1}{\varepsilon - 1} \right)^{1/n} \quad NTU = n(NTU)_1$
Cross-ow (single pass)	
C_{\max} (mixed), C_{\min} (unmixed)	$NTU = - \ln \left[1 + \left(\frac{1}{C_r} \right) \ln(1 - \varepsilon C_r) \right]$
C_{\min} (mixed), C_{\max} (unmixed)	$NTU = - \left(\frac{1}{C_r} \right) \ln [C_r \ln(1 - \varepsilon) + 1]$
All exchangers ($C_r = 0$)	$NTU = - \ln(1 - \varepsilon)$

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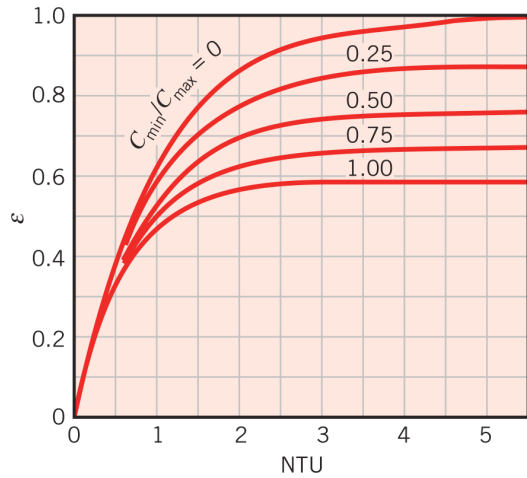
Parallel Flow



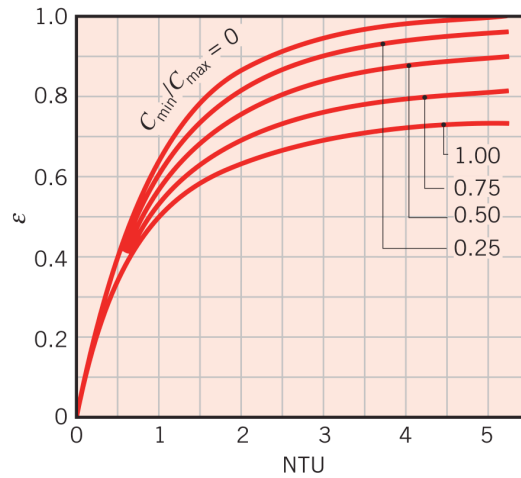
Counter Flow



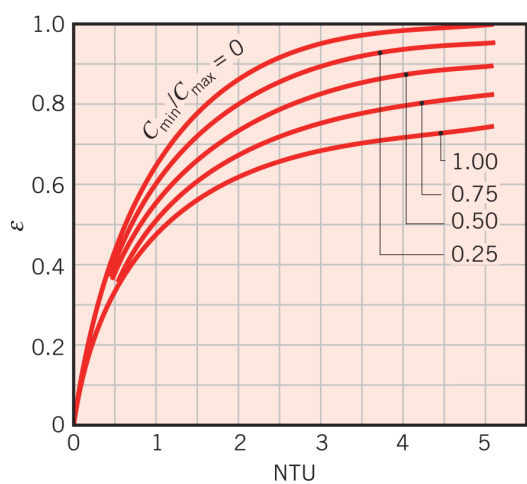
Shell and Tube - One Shell



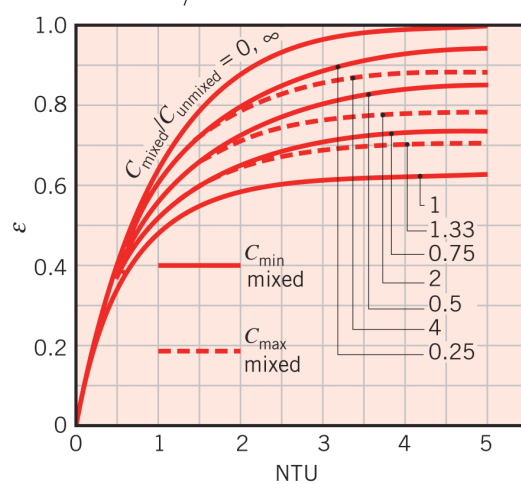
Shell and Tube - 2 Shells



Cross-flow Both Unmixed



Cross-flow One Mixed/One Unmixed



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Table 1: Contact Resistance for vacuum interfaces, $R''_{t,c} \times 10^4 \left(\frac{m^2 K}{W} \right)$

Material	100 (kPa)	10,000 (kPa)
Stainless Steel	6-25	0.7-4.0
Copper	1-10	0.1-0.5
Magnesium	1.5-3.5	0.2-0.4
Aluminum	1.5-5.0	0.2-0.4

Table 2: Contact Resistance for aluminum ($10\mu m$ surface roughness, $10kPa$ contact pressure)

Interfacial Fluid	$R''_{t,c} \times 10^4 \left(\frac{m^2 K}{W} \right)$
Air	2.75
Helium	1.05
Hydrogen	0.720
Silicone Oil	0.525
Glycerine	0.265

Table 3: Fin Tip Conditions

Case	Tip Condition	Temperature Distribution $\frac{\theta}{\theta_b}$	Fin Heat Transfer Rate q_f
A	Convection	$\frac{\cosh m(L-x) + \left(\frac{h}{mk}\right) \sinh m(L-x)}{\cosh mL + \left(\frac{h}{mK}\right) \sinh mL}$	$M \frac{\sinh mL + \left(\frac{h}{mk}\right) \cosh mL}{\cosh mL + \left(\frac{h}{mk}\right) \sinh mL}$
B	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Constant Temperature	$\frac{\left(\frac{\theta_L}{\theta_b}\right) \sinh mx + \sinh m(L-x)}{\sinh mL}$	$M \frac{\cosh mL - \frac{\theta_L}{\theta_b}}{\sinh mL}$
D	Infinite Fin ($L \rightarrow \infty$)	e^{-mx}	M

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Table 4: Cylinder In Cross Flow

Re_D	C	m
0.4 – 4	0.989	0.330
4 – 40	0.911	0.385
40 – 4000	0.683	0.466
4000 – 40,000	0.193	0.618
40,000 – 400,000	0.027	0.805

Table 5: Various Geometries In Cross Flow






Geometry	Re_D	C	m
$V \rightarrow$  $\begin{matrix} \uparrow D \\ \downarrow \end{matrix}$	6000 – 60,000	0.304	0.59
$V \rightarrow$  $\begin{matrix} \uparrow D \\ \downarrow \end{matrix}$	5000 – 60,000	0.158	0.66
$V \rightarrow$  $\begin{matrix} \uparrow D \\ \downarrow \end{matrix}$	5200 – 20,400 20,400 – 105,000	0.164 0.039	0.638 0.78
$V \rightarrow$  $\begin{matrix} \uparrow D \\ \downarrow \end{matrix}$	4500 – 90,700	0.150	0.638
$V \rightarrow$  $\begin{matrix} \uparrow D \\ \downarrow \end{matrix}$ Front Back	10,000 – 50,000 7,000 – 80,000	0.667 0.191	0.500 0.667

Table 6: Noncircular Tubes





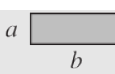
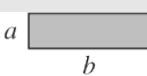



Cross Section	$\frac{b}{a}$	Uniform q_s''	Uniform T_s	fRe_{D_h}
		4.36	3.66	64
 $\begin{matrix} a \\ b \end{matrix}$	1.0	3.61	2.98	57
 $\begin{matrix} a \\ b \end{matrix}$	1.43	3.73	3.08	59
 $\begin{matrix} a \\ b \end{matrix}$	2.0	4.12	3.39	62
 $\begin{matrix} a \\ b \end{matrix}$	3.0	4.79	3.96	69
 $\begin{matrix} a \\ b \end{matrix}$	4.0	5.33	4.44	73
 Heated	∞	8.23	7.54	96
 Insulated	∞	5.39	4.86	96
		3.11	2.49	53

Table 7: Fouling Factors

<i>Fluid</i>	R_f'' (m^2K/W)
Seawater and treated boiler feedwater (below $50^\circ C$)	0.001
Seawater and treated boiler feedwater (above $50^\circ C$)	0.002
River water (below $50^\circ C$)	0.0002 - 0.001
Fuel Oil	0.0009
Refrigerating Liquids	0.0002
Steam (nonoil bearing)	0.0001