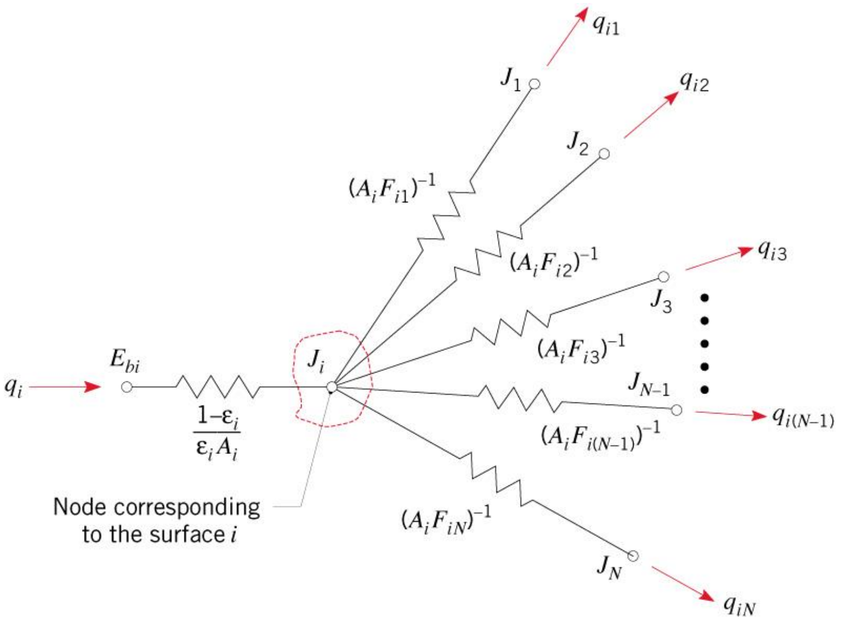
# Radiation Exchange



# Basic Concepts

- Enclosures consist of two or more surfaces that envelop a region of space (typically gas-filled) and between which there is radiation transfer. Virtual, as well as real, surfaces may be introduced to form an enclosure.

- A nonparticipating medium within the enclosure neither emits, absorbs, nor scatters radiation and hence has no effect on radiation exchange between the surfaces.

- Each surface of the enclosure is assumed to be isothermal, opaque, diffuse and gray, and to be characterized by uniform radiosity and irradiation.

# The View Factor (also Configuration or Shape Factor)

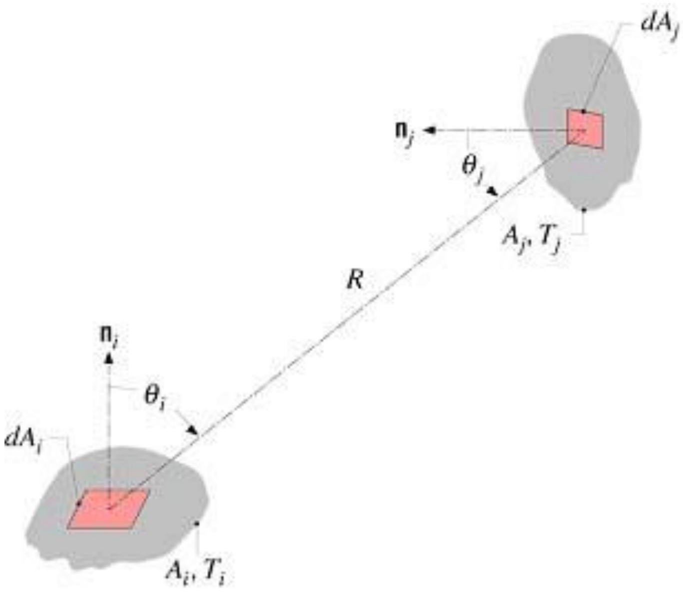
- The view factor, $F_{ij}$, is a geometrical quantity corresponding to the fraction of the radiation leaving surface *i* that is intercepted by surface *j*.

$$F_{ij} = \frac{q_{i \to j}}{A_i J_i}$$

- The view factor integral provides a general expression for $F_{ij}$. Consider exchange between diffusely-emitting and reflecting differential areas $dA_i$ and $dA_j$ :

$$dq_{i \to j} = I_i \cos\theta_i dA_i d\omega_{j-i} = J_i \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j dA_i$$

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j dA_i$$

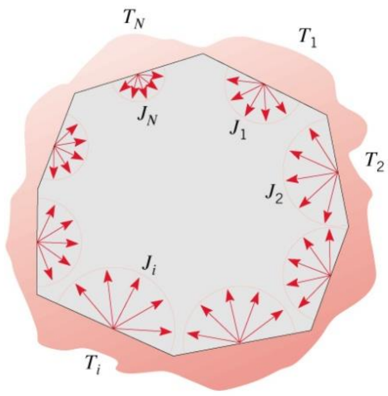


# View Factor Relations

- Reciprocity Relation. With

$$F_{ji} = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j dA_i$$

$$A_i F_{ij} = A_j F_{ji}$$




- Summation Rule for Enclosures.

$$\sum_{j=1}^{N} F_{ij} = 1$$

- Two-Dimensional Geometries (see Table 15.1 on next several slides)

Table 15.1

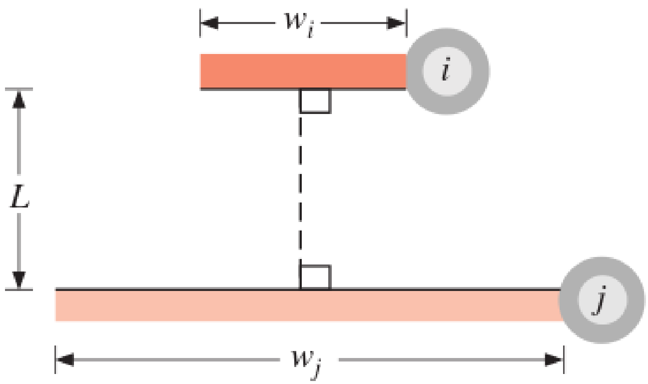
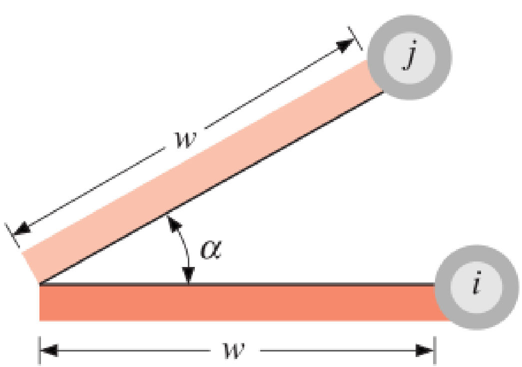
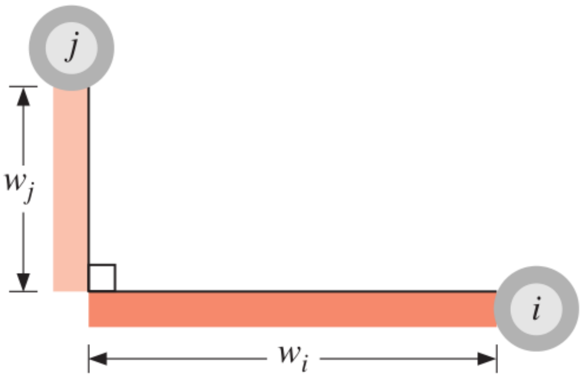
| Geometry | Relation |
|---|---|
| **Parallel Plates with Midlines Connected by Perpendicular**   | $F_{ij} = \dfrac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$<br>$W_i = w_i/L,\ W_j = w_j/L$ |
| **Inclined Parallel Plates of Equal Width and a Common Edge**   | $F_{ij} = 1 - \sin\left(\dfrac{\alpha}{2}\right)$ |

Table 15.1 (contd.)

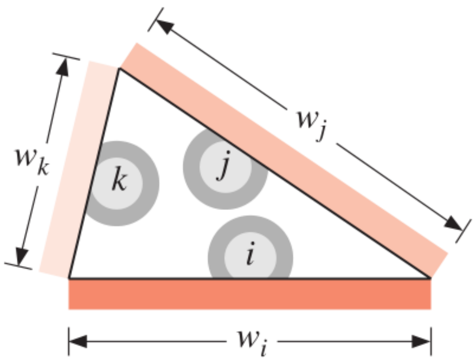
| Geometry | Relation |
|---|---|

**Perpendicular Plates with a Common Edge**




$$F_{ij} = \frac{1 + (w_j/w_i) - [1 + (w_j/w_i)^2]^{1/2}}{2}$$

**Three-Sided Enclosure**

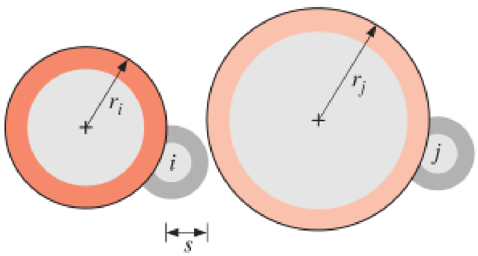



$$F_{ij} = \frac{w_i + w_j - w_k}{2w_i}$$

Table 15.1 (contd.)

| Geometry | Relation |
|---|---|

**Parallel Cylinders of Different Radii**

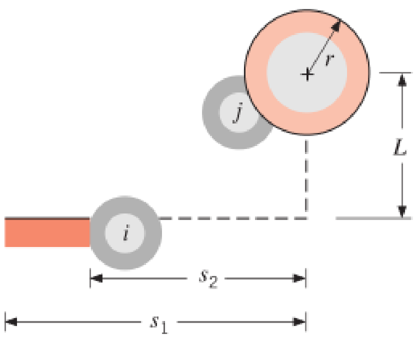



$$F_{ij} = \frac{1}{2\pi}\Big\{\pi + [C^2 - (R+1)^2]^{1/2} - [C^2 - (R-1)^2]^{1/2} + (R-1)\cos^{-1}\left[\left(\frac{R}{C}\right) - \left(\frac{1}{C}\right)\right] - (R+1)\cos^{-1}\left[\left(\frac{R}{C}\right) + \left(\frac{1}{C}\right)\right]\Big\}$$

$R = r_j/r_i$, $S = s/r_i$

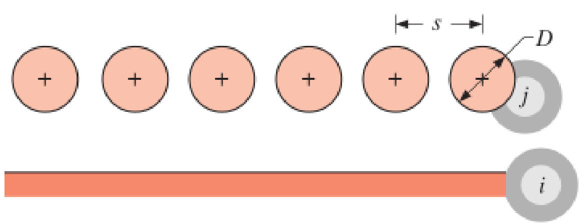
$C = 1 + R + S$

**Cylinder and Parallel Rectangle**




$$F_{ij} = \frac{r}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right]$$

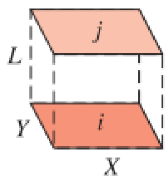
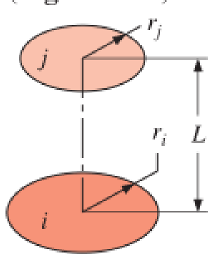
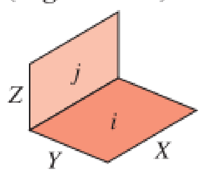
**Innite Plane and Row of Cylinders**




$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \left(\frac{D}{s}\right)\tan^{-1}\left[\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}\right]$$

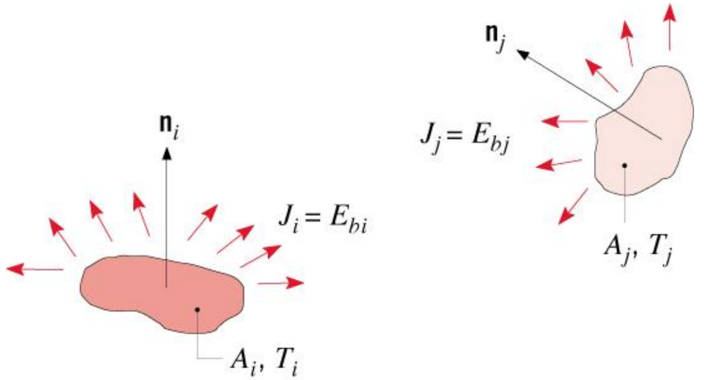
View Factor Relations (cont.)

- Three-Dimensional Geometries (Table 15.2)

| Geometry | Relation |
|---|---|
| **Aligned Parallel Rectangles** **(Figure 13.4)**   | $\bar{X} = X/L, \bar{Y} = Y/L$ <br> $F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}} \left\{ \ln \left[ \frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2} \right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2} \tan^{-1} \frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2} \tan^{-1} \frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y} \right\}$ |
| **Coaxial Parallel Disks** **(Figure 13.5)**   | $R_i = r_i/L, R_j = r_j/L$ <br> $S = 1 + \frac{1+R_j^2}{R_i^2}$ <br> $F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$ |
| **Perpendicular Rectangles with a Common Edge** **(Figure 13.6)**   | $H = Z/X, W = Y/X$ <br> $F_{ij} = \frac{1}{\pi W}\left( W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{ \frac{(1+W^2)(1+H^2)}{1+W^2+H^2} \left[ \frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)} \right]^{W^2} \times \left[ \frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)} \right]^{H^2} \right\} \right)$ |

# Blackbody Radiation Exchange

- For a blackbody, $J_i = E_{bi}$.




- Net radiative exchange between two surfaces that can be approximated as blackbodies $\longrightarrow$ *net* rate at which radiation leaves surface *i* due to its interaction with *j*

  or net rate at which surface *j* gains radiation due to its interaction with *i*

$$q_{ij} = q_{i \to j} - q_{j \to i}$$

$$q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj}$$

$$q_{ij} = A_i F_{ij} \sigma \left( T_i^4 - T_j^4 \right)$$

- Net radiation transfer from surface *i* due to exchange with all (*N*) surfaces of an enclosure:

$$q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma \left( T_i^4 - T_j^4 \right)$$

# General Radiation Analysis for Exchange between the *N* Opaque, Diffuse, Gray Surfaces of an Enclosure

$$(\varepsilon_i = \alpha_i = 1 - \rho_i)$$

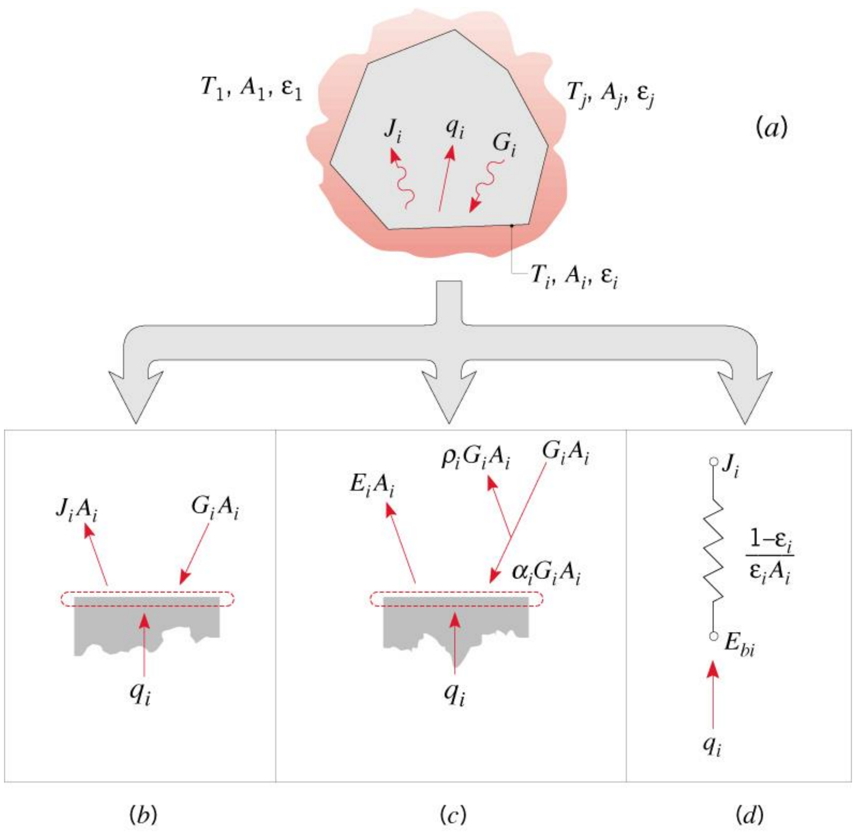
- Alternative expressions for net radiative transfer from surface *i*:

$$q_i = A_i (J_i - G_i) \rightarrow \text{Fig. (b)} \qquad (1)$$

$$q_i = A_i (E_i - \alpha_i G_i) \rightarrow \text{Fig. (c)} \qquad (2)$$

$$q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} \rightarrow \text{Fig. (d)} \qquad (3)$$

Suggests a surface radiative resistance of the form: $(1-\varepsilon_i)/\varepsilon_i A_i$



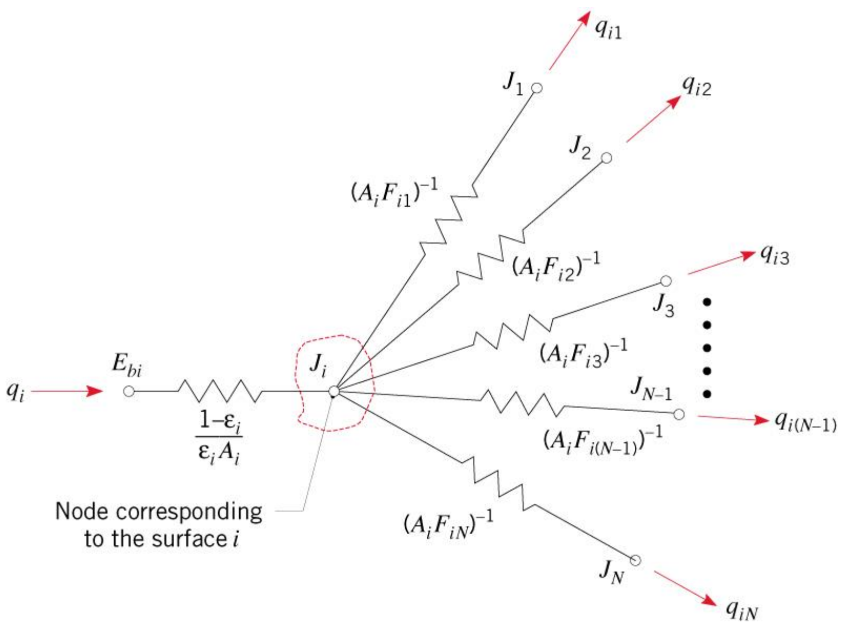
$$q_i = \sum_{j=1}^{N} A_i F_{ij}\left(J_i - J_j\right) = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad (4)$$

Suggests a space or geometrical resistance of the form: $\left(A_i F_{ij}\right)^{-1}$

- Equating Eqs. (3) and (4) corresponds to a radiation balance on surface *i*:

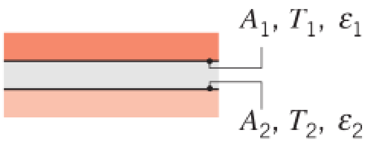
$$\frac{E_{bi} - J_i}{\left(1-\varepsilon_i\right)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad (5)$$

which may be represented by a radiation network of the form



- Special Diffuse, Gray, Two-Surface Enclosures (Table 15.3)

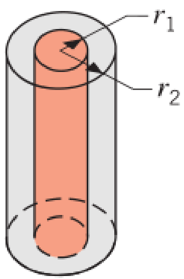
**Large (Innite) Parallel Planes**




$$A_1 = A_2 = A$$
$$F_{12} = 1$$

$$q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

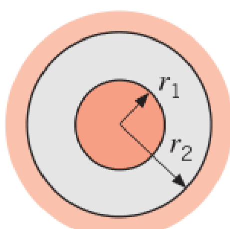
**Long (Innite) Concentric Cylinders**




$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$
$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

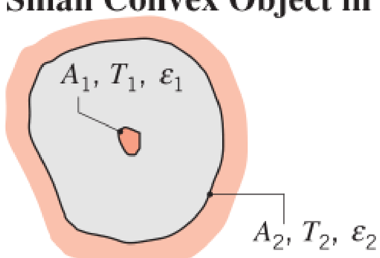
**Concentric Spheres**




$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$
$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2}$$

**Small Convex Object in a Large Cavity**




$$\frac{A_1}{A_2} \approx 0$$
$$F_{12} = 1$$

$$q_{12} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

- Methodology of an Enclosure Analysis

  - Apply Eq. (4) to each surface for which the net radiation heat rate $q_i$ is known.

  - Apply Eq. (5) to each of the remaining surfaces for which the temperature $T_i$, and hence $E_{bi}$, is known.

  - Evaluate all of the view factors appearing in the resulting equations.

  - Solve the system of $N$ equations for the unknown radiosities, $J_1, J_2, \ldots, J_N$.

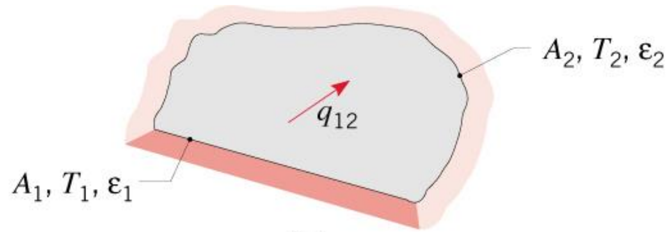
  - Use Eq. (3) to determine $q_i$ for each surface of known $T_i$ and $T_i$ for each surface of known $q_i$.

- Treatment of the virtual surface corresponding to an opening (aperture) of area $A_i$, through which the interior surfaces of an enclosure exchange radiation with large surroundings at $T_{\mathrm{sur}}$:

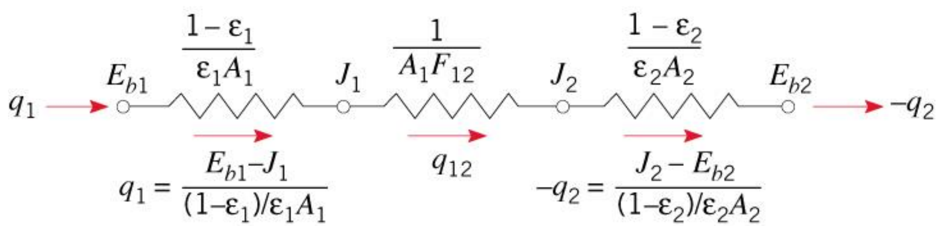
  - Approximate the opening as blackbody of area, $A_i$, temperature, $T_i = T_{\mathrm{sur}}$, and properties, $\varepsilon_i = \alpha_i = 1$.

# Two-Surface Enclosures

- Simplest enclosure for which radiation exchange is exclusively between two surfaces and a single expression for the rate of radiation transfer may be inferred from a network representation of the exchange.




(a)



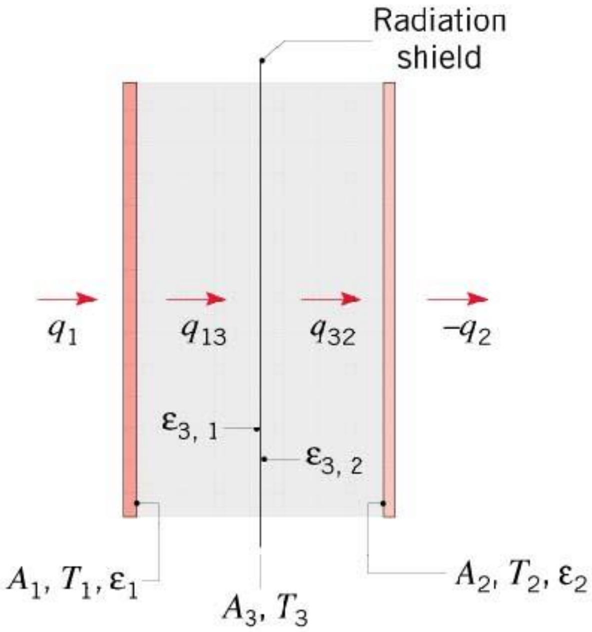

(b)

$$q_1 = -q_2 = q_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

# Radiation Shields

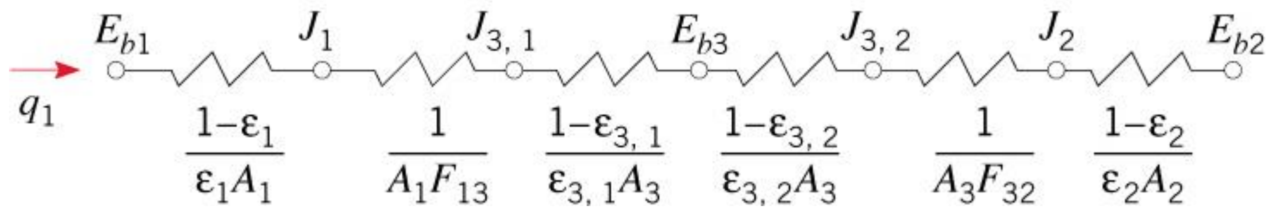
- High reflectivity (low $\alpha = \varepsilon$) surface(s) inserted between two surfaces for which a reduction in radiation exchange is desired.

- Consider use of a single shield in a two-surface enclosure, such as that associated with large parallel plates:




Note that, although rarely the case, emissivities may differ for opposite surfaces of the shield.

- Radiation Network:



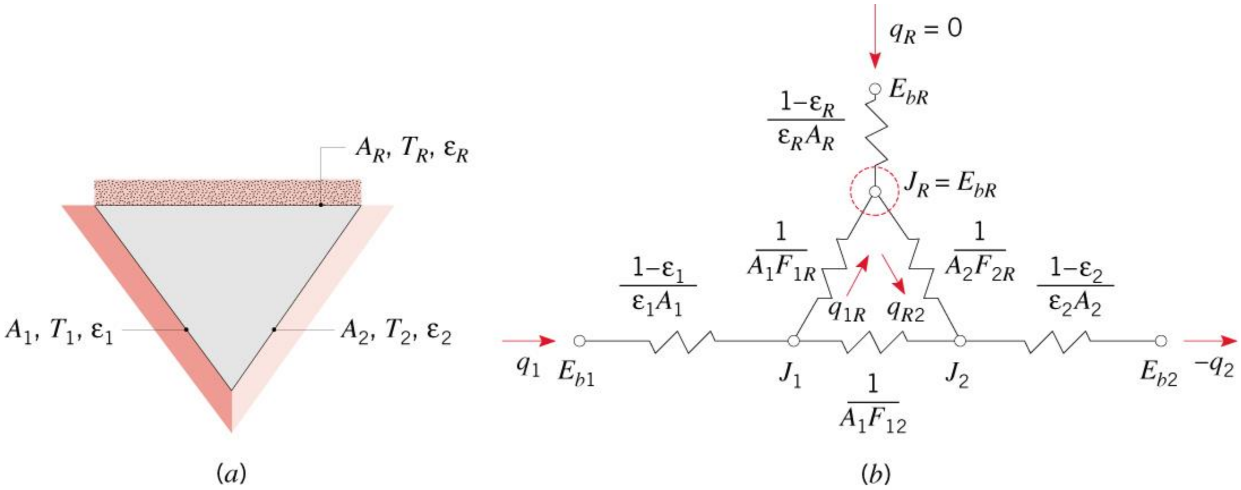

- The foregoing result may be readily extended to account for multiple shields and may be applied to long, concentric cylinders and concentric spheres, as well as large parallel plates.

# Example 1

A cryogenic fluid flows through a long tube of $20\ mm$ diameter, the outer surface of which is diffuse and gray with $\varepsilon_1 = 0.02$ and $T_1 = 77K$. This tube is concentric with a larger tube of $50\ mm$ diameter, the inner surface of which is diffuse and gray with $\varepsilon_2 = 0.05$ and $T_2 = 300K$. The space between the surfaces is evacuated. Calculate the heat gain by the cryogenic fluid per unit length of tubes. If a thin radiation shield of $35\ mm$ diameter and $\varepsilon_3 = 0.02$ (both sides) is inserted midway between the inner and outer surfaces, calculate the change (percentage) in heat gain per unit length of the tubes.

# The Reradiating Surface

- An idealization for which $G_R = J_R$. Hence, $q_R = 0$ and $J_R = E_{bR}$.
- Approximated by surfaces that are well insulated on one side and for which convection is negligible on the opposite (radiating) side.
- Three-Surface Enclosure with a Reradiating Surface:




$$q_1 = -q_2 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1F_{12} + \left[\left(1/A_1F_{1R}\right) + \left(1/A_2F_{2R}\right)\right]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

- Temperature of reradiating surface $T_R$ may be determined from knowledge of its radiosity $J_R$ . With $q_R = 0$ , a radiation balance on the surface yields

$$\frac{J_1 - J_R}{(1/A_1F_{1R})} = \frac{J_R - J_2}{(1/A_2F_{2R})}$$

$$T_R = \left(\frac{J_R}{\sigma}\right)^{1/4}$$

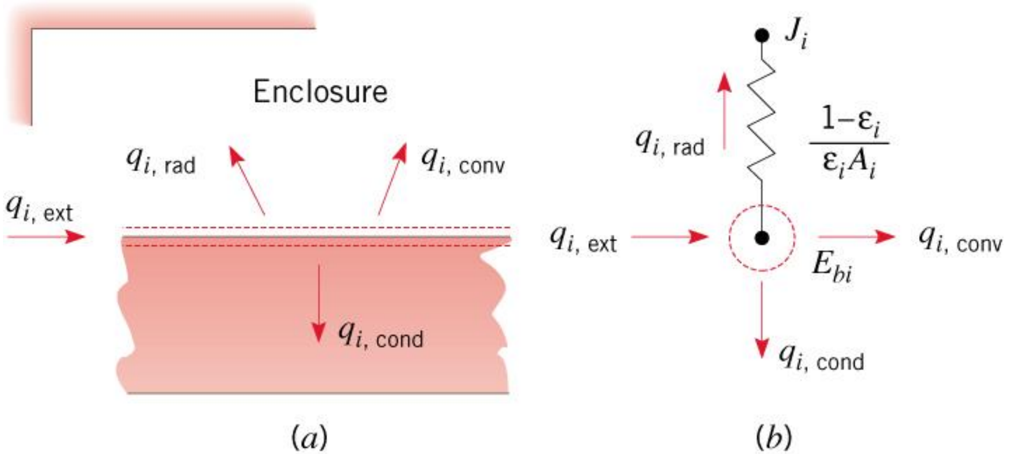
# Example 2

A paint baking oven consists of a long, triangular duct in which a heated surface is maintained at $1200K$ and another surface is insulated. Painted panels, which are maintained at $500K$, occupy the third surface. The triangle is of width $W = 1m$ on a side, and the heated and insulated surfaces have an emissivity of $0.8$. The emissivity of the panels is $0.4$. During steady-state operation, at what rate must energy be supplied to the heated side per unit length of the duct to maintain its temperature at $1200K$? What is the temperature of the insulated surface?

# Multimode Effects

- In an enclosure with conduction and convection heat transfer to or from one or more surfaces, the foregoing treatments of radiation exchange may be combined with surface energy balances to determine thermal conditions.

- Consider a general surface condition for which there is external heat addition (e.g., electrically), as well as conduction, convection and radiation.




$$q_{i,\text{ext}} = q_{i,\text{rad}} + q_{i,\text{conv}} + q_{i,\text{cond}}$$

# Example 3

Consider an air heater consisting of a semicircular tube for which the plane surface is maintained at $1000K$, and the other surface is well insulated. The tube radius is $20mm$, and both surfaces have an emissivity of 0.8. If atmospheric air flows through the tube at $0.01\frac{kg}{s}$ and $T_m = 400K$, what is the temperature of the insulated surface?