$$m = \rho \forall$$

$$\dot{\forall} = \frac{\forall}{t}$$

$$q = m c_p \Delta T$$

$$q = -kA \frac{dT}{dt}$$

$$q = hA (T_s - T_\infty)$$

$$q = \varepsilon \sigma A \left(T_s^4 - T_{sur}^4 \right)$$

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

$$\Delta E_{sys} = \Sigma E_{in} - \Sigma E_{out}$$

$$\frac{dE_{sys}}{dt} = \Sigma \dot{E}_{in} - \Sigma \dot{E}_{out}$$

$$q'' = \frac{q}{A}$$

$$\hat{q}'' = -k_x \frac{\partial T}{\partial x} \hat{i} - k_y \frac{\partial T}{\partial y} \hat{j} - k_z \frac{\partial T}{\partial z} \hat{k}$$

$$k = \frac{9\gamma - 5}{4} \frac{c_v}{\pi d^2} \sqrt{\frac{M_w k_B T}{N_A \pi}}$$

$$k_B = 1.381 \times 10^{-23} \frac{J}{K}$$

$$N_A = 6.022 \times 10^{23}$$

$$\alpha = \frac{k}{\rho c_p}$$

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = 0$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$
$$\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$
$$\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$
$$\frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$
$$\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$T(0, t) = T_s$$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_s''$$

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0$$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h\left(T_\infty - T(0, t)\right)$$

$$\Delta U = N \left(V^- - V^+ \right)$$

$$q = \frac{\Delta U}{\Delta t} = \frac{N}{\Delta t} \Delta V$$

$$q = -IV = \frac{-V^2}{R} = -I^2 R$$

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$

$$S_{AB} = S_B - S_A = \frac{-\Delta V}{\Delta T}$$

$$C_1 = \frac{T_{s,2} - T_{s,1}}{L}$$

$$C_2 = T_{s,1}$$

$$T(x) = (T_{s,2} - T_{s,1})\frac{x}{L} + T_{s,1}$$

$$q = \frac{kA}{L}(T_{s,1} - T_{s,2})$$

$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{kA}$$

$$R = \frac{V}{I}$$

$$R_{t,conv} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$

$$R_{t,rad} = \frac{T_s - T_{sur}}{q} = \frac{1}{h_r A}$$

$$h_r = \varepsilon \sigma (T_s + T_{sur})\left(T_s^2 + T_{sur}^2\right)$$

$$q = UA\Delta T$$

$$U = \frac{1}{R_{tot} A}$$

$$R_{tot} = \Sigma R_t = \frac{1}{UA}$$

$$R_{t,c}'' = \frac{T_A - T_B}{q''}$$

$$q = \frac{k_{eff} A}{L}(T_1 - T_2)$$

$$k_{eff,min} = \frac{1}{\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}}$$

$$k_{eff,max} = \varepsilon k_f + (1 + \varepsilon)k_s$$

$$R_{t,cond} = \frac{\ln \frac{r_2}{r_1}}{2\pi k L}$$

$$R_{t,conv} = \frac{1}{h 2\pi r L}$$

$$\frac{d^2 T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T - T_\infty) = 0$$

$$A_s = Px$$

$$\frac{d^2 T}{dx^2} - \left(\frac{hP}{kA_c}\right)(T - T_\infty)$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta = T(x) - T_\infty$$

$$m^2 = \frac{hP}{kA_c}$$

$$P = 2w + 2t$$

$$A_c = wt$$

$$P = \pi D$$

$$A_c = \frac{\pi}{4}D^2$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta_b = T_b - T_\infty$$

$$M = \sqrt{hPKA_c}\,\theta_b$$

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b}$$

$$\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$$

$$R_{t,f} = \frac{\theta_b}{q_f}$$

$$R_{t,b} = \frac{1}{hA_{c,b}}$$

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}$$

$$\eta_f = \frac{q_f}{q_{max}} = \frac{q_f}{hA_f\theta_b}$$

$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{hA_t\theta_b}$$

$$A_t = NA_f + A_b$$

$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b$$

$$\eta_o = 1 - \frac{NA_f}{A_t}(1 - \eta_f)$$

$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$$

$$\frac{u(y)}{u_\infty} = 0.99$$

$$C_f = \frac{\tau_s}{\rho u_\infty^2/2}$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$\frac{T_s - T(y)}{T_s - T_\infty} = 0.99$$

$$q_s'' = -k \frac{\partial T}{\partial y}\Big|_{y=0}$$

$$q_s'' = h(T_s - T_\infty)$$

$$h = \frac{-k \frac{\partial T}{\partial y}\big|_{y=0}}{T_s - T_\infty}$$

$$q = \int_{A_s} q'' dA_s = (T_s - T_\infty) \int_{A_s} h dA_s$$

$$\overline{q}'' = \overline{h}(T_s - T_\infty)$$

$$\overline{q} = \overline{q}'' A_s = \overline{h} A_s (T_s - T_\infty)$$

$$\overline{h} = \frac{1}{A_s} \int_{A_s} h dA_s$$

$$\overline{h} = \frac{1}{L} \int_0^L h dx$$

$$Re_x = \frac{\rho u_\infty x}{\mu}$$

$$x^* = \frac{x}{L}$$

$$y^* = \frac{y}{L}$$

$$u^* = \frac{u}{V}$$

$$v^* = \frac{v}{V}$$

$$T^* = \frac{T - T_s}{T_\infty - T_s}$$

$$p^* = \frac{p_\infty}{\rho V^2}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$Re_L = \frac{VL}{\nu}$$

$$\nu = \frac{\mu}{\rho}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$Pr = \frac{\nu}{\alpha}$$

$$Nu = \frac{hL}{k} = \frac{\partial T^*}{\partial y^*}\Big|_{y^*=0}$$

$$\overline{Nu} = \frac{\overline{h} L}{k}$$

$$T_f = \frac{T_s + T_\infty}{2}$$

....

Isothermal Flat Plate

$$\frac{\delta}{\delta_t} \approx Pr^{1/3}$$

Laminar:

$$\delta = \frac{5}{\sqrt{\frac{u_\infty}{\nu x}}} = \frac{5x}{\sqrt{Re_x}}$$

$$C_{f,x} = \frac{\tau_{s,x}}{\frac{\rho u_\infty^2}{2}} = 0.664 Re_x^{-1/2}$$

$$Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3}$$

$$(Pr \gtrsim 0.6)$$

$$\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$$

$$\overline{Nu}_x = \frac{\overline{h}_x x}{k} = 0.664 Re_x^{1/2} Pr^{1/3}$$

$$(Pr \gtrsim 0.6)$$

Turbulent:

$$\delta = 0.37 x Re_x^{-1/2}$$

$$C_{f,x} = 0.0592 Re_x^{-1/5}$$

$$\left(Re_{x,c} \lesssim Re_x \lesssim 10^8\right)$$

$$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$$

$$(0.6 \lesssim Pr \lesssim 60)$$

$$A = 0.037 Re_{x,c}^{4/5} - 0.664 Re_{x,c}^{1/2}$$

$$\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - \frac{2A}{Re_L}$$

$$\left(Re_{x,c} \lesssim Re_L \lesssim 10^8\right)$$

$$\overline{Nu}_L = \left(0.037 Re_L^{4/5} - A\right) Pr^{1/3}$$

$$(0.6 \lesssim Pr \lesssim 60)$$

$$\left(Re_{x,c} \lesssim Re_L \lesssim 10^8\right)$$

....

$$Re_D = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

$$C_D = \frac{F_D}{A_f \left(\frac{\rho V^2}{2}\right)}$$

....

Cylinder

$$\overline{Nu}_D = \frac{\overline{h} D}{k} = C Re_D^m Pr^{1/3}$$

$$(Pr \gtrsim 0.7)$$

See Table 4

....

....

Various Geometries

$$\overline{Nu}_D = \frac{\overline{h} D}{k} = C Re_D^m Pr^{1/3}$$

$$(Pr \gtrsim 0.7)$$

See Table 5

....

....

Sphere

$$C_D = \frac{25}{Re_D}$$

$$(Re_D \lesssim 0.5)$$

$$\overline{Nu}_D = 2 +$$

$$\left(0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right) Pr^{0.4} \left(\frac{\mu}{\mu_s}\right)^{1/4}$$

$$(0.71 \lesssim Pr \lesssim 380)$$

$$\left(3.5 \lesssim Re_D \lesssim 7.6 \times 10^4\right)$$

$$\left(1.0 \lesssim \frac{\mu}{\mu_s} \lesssim 3.2\right)$$

....

$$m = \rho u_m A_c$$

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$$

$$\left(\frac{x_{fd,t}}{D}\right) \approx 0.05 Re_D Pr$$

$$f = \frac{-\frac{dP}{dx} D}{\rho \frac{u_m^2}{2}}$$

$$f = \frac{64}{Re_D}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log \left[\frac{e/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}}\right]$$

$$dq_{conv} = mc_p [(T_m + dT_m) - T_m] =$$

$$mc_p dT_m$$

$$dq_{conv} = q_s'' P dx$$

$$\frac{dT_m}{dx} = \frac{q_s'' P}{mc_p} = \frac{P}{mc_p} h(T_s - T_m)$$

$$T_m(x) = T_{m,i} + \frac{q_s'' P}{mc_p} x$$

$$\Delta T = T_s - T_m$$

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{mc_p} h \Delta T$$

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = e^{-\frac{Px}{mc_p} \overline{h}}$$

$$\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$$

$$\Delta T_o = T_s - T_{m,o}$$

$$\Delta T_i = T_s - T_{m,i}$$

$$q_{conv} = \bar{h}A_s \Delta T_{lm}$$

....

Circular Tubes

Fully dev. lam. w/ $q'' = \mathbb{C}$:

$$Nu_D = 4.36$$

Fully dev. lam. w/ $T_s = \mathbb{C}$:

$$Nu_D = 3.66$$

Fully dev. turb. w/ small ΔT:

$$Nu_D = 0.023 Re_D^{4/5} Pr^n$$
$$n = 0.4, \quad T_s > T_m$$
$$n = 0.3, \quad T_s < T_m$$
$$(0.6 \lesssim Pr \lesssim 160)$$
$$(Re_D \gtrsim 10,000)$$
$$(L/D \gtrsim 10)$$

Fully dev. turb. w/ large ΔT:

$$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$
$$(0.7 \lesssim Pr \lesssim 16,700)$$
$$(Re_D \gtrsim 10,000)$$
$$(L/D \gtrsim 10)$$

....

....

Noncircular Tubes

$$D_h = \frac{4A_c}{P}$$
$$Nu_D = \frac{hD_h}{k}$$

See Table 6

....
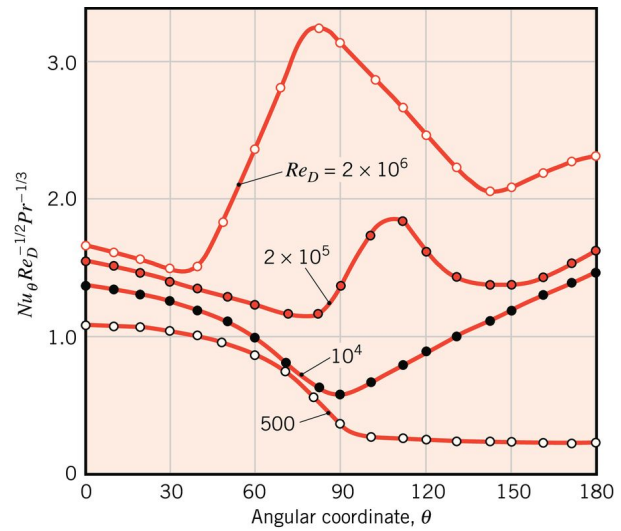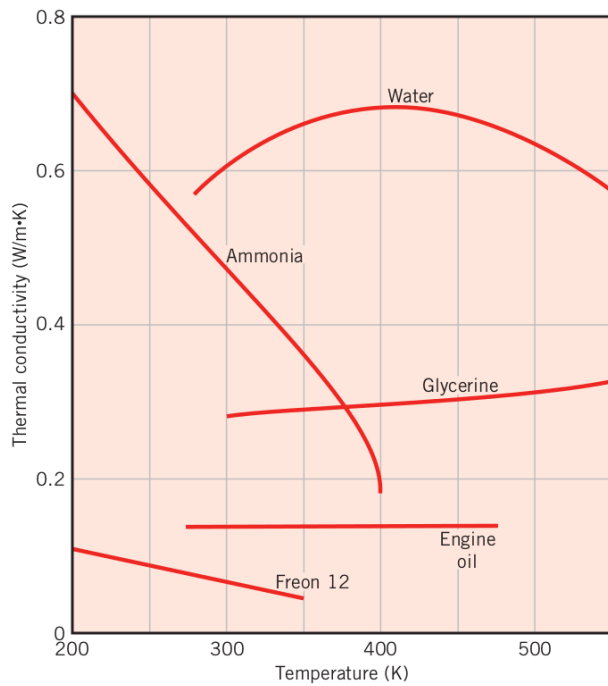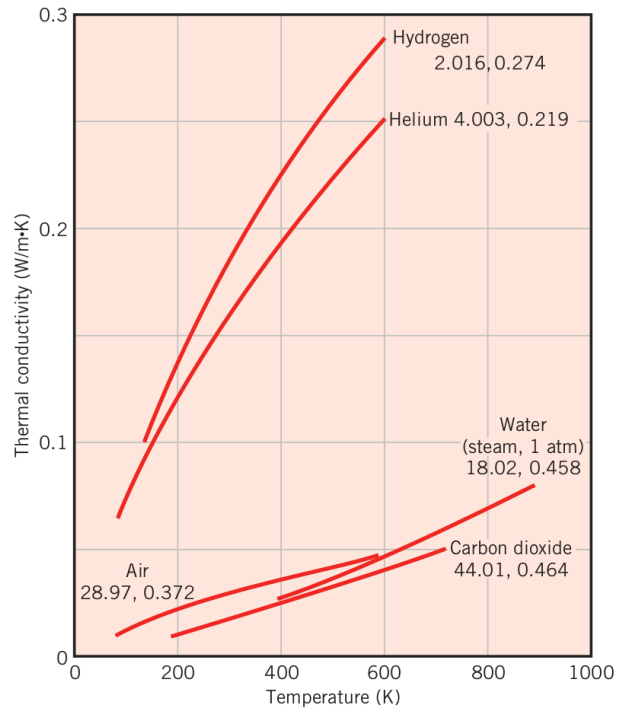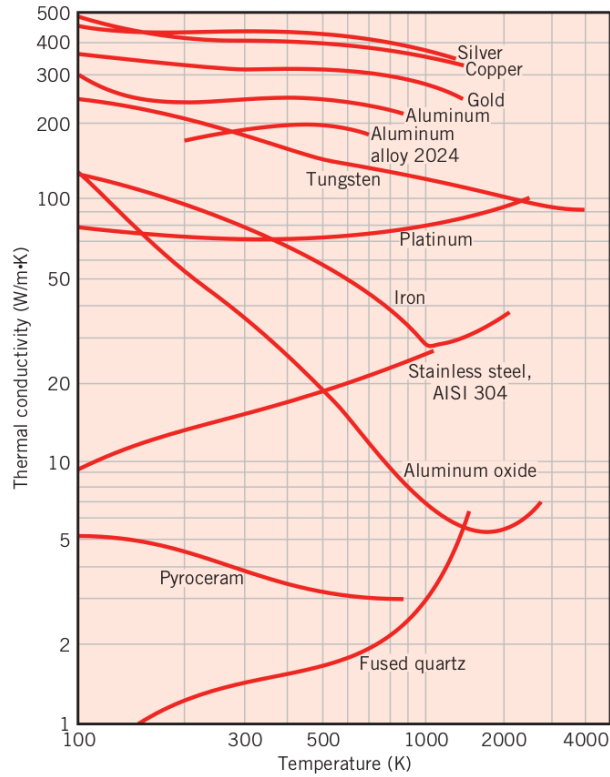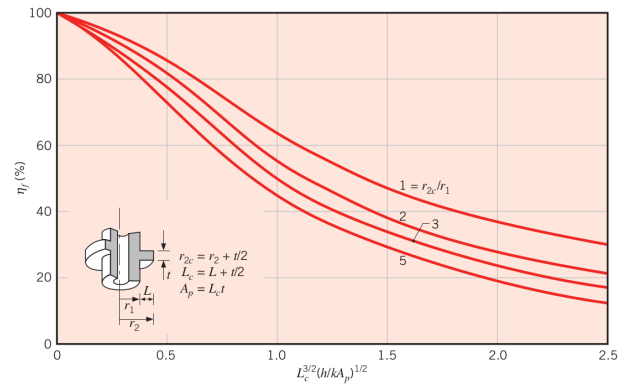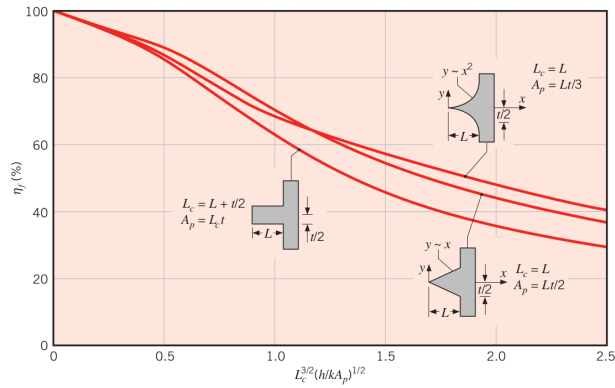
Straight Fins

Rectangular

$A_f = 2wL_c$

$L_c = L + (t/2)$

$A_p = tL$
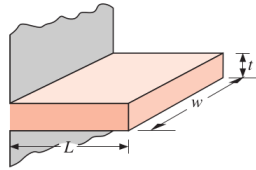
$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$A_f = 2w[L^2 + (t/2)^2]^{1/2}$

$A_p = (t/2)L$
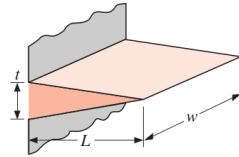
$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$

Parabolic

$A_f = w[C_1 L +$
$\qquad (L^2/t)\ln(t/L + C_1)]$

$C_1 = [1 + (t/L)^2]^{1/2}$

$A_p = (t/3)L$

$y = (t/2)(1 - x/L)^2$
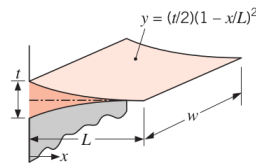
$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

Pin Fins

Rectangular

$A_f = \pi D L_c$

$L_c = L + (D/4)$

$V = (\pi D^2/4)L$
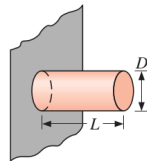
$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$A_f = \dfrac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$

$V = (\pi/12)D^2 L$
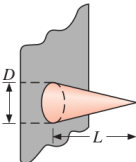
$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$

Parabolic

$A_f = \dfrac{\pi L^3}{8D}\{C_3 C_4 -$
$\qquad \dfrac{L}{2D}\ln[(2DC_4/L) + C_3]\}$

$C_3 = 1 + 2(D/L)^2$

$C_4 = [1 + (D/L)^2]^{1/2}$

$V = (\pi/20)D^2 L$
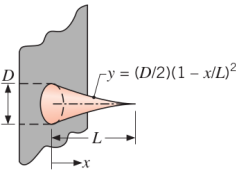
$y = (D/2)(1 - x/L)^2$

$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$$
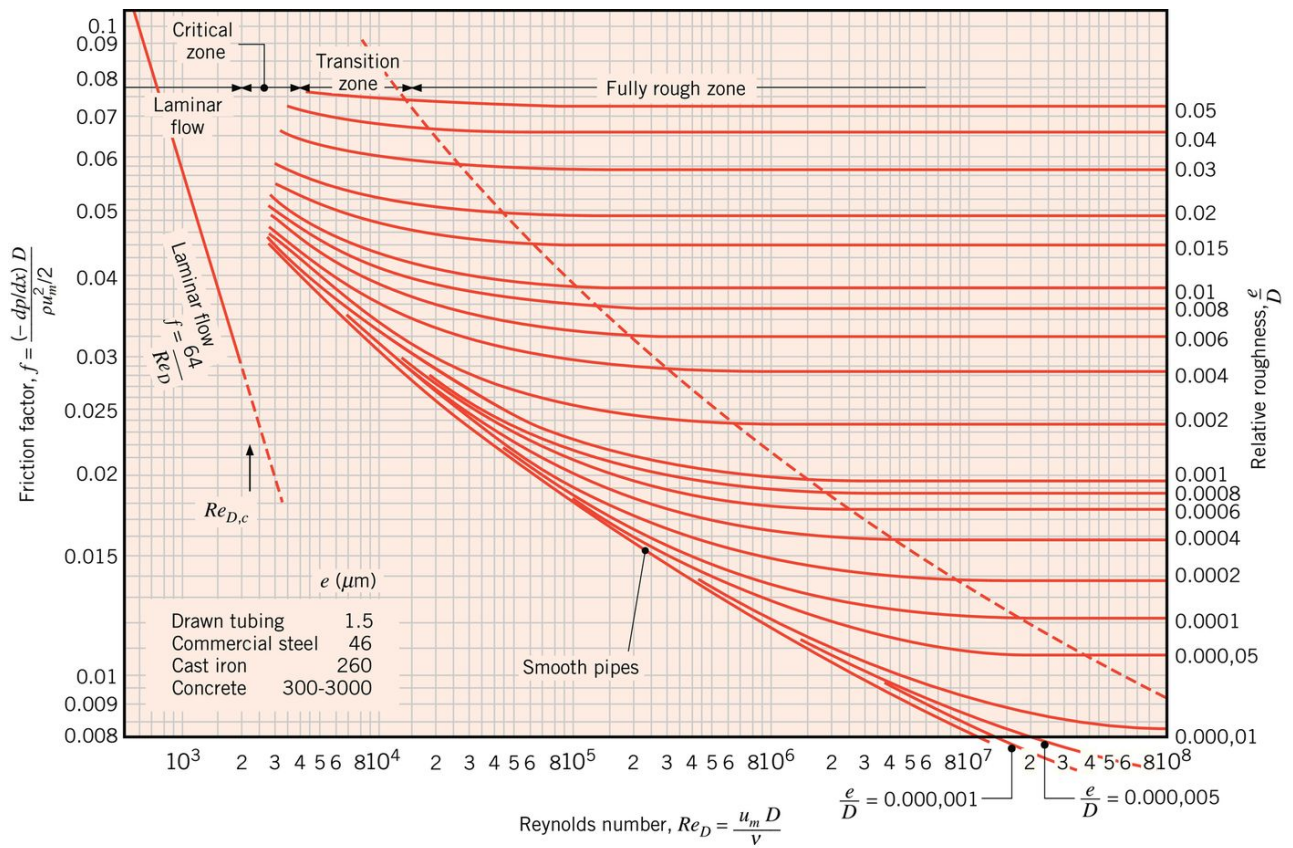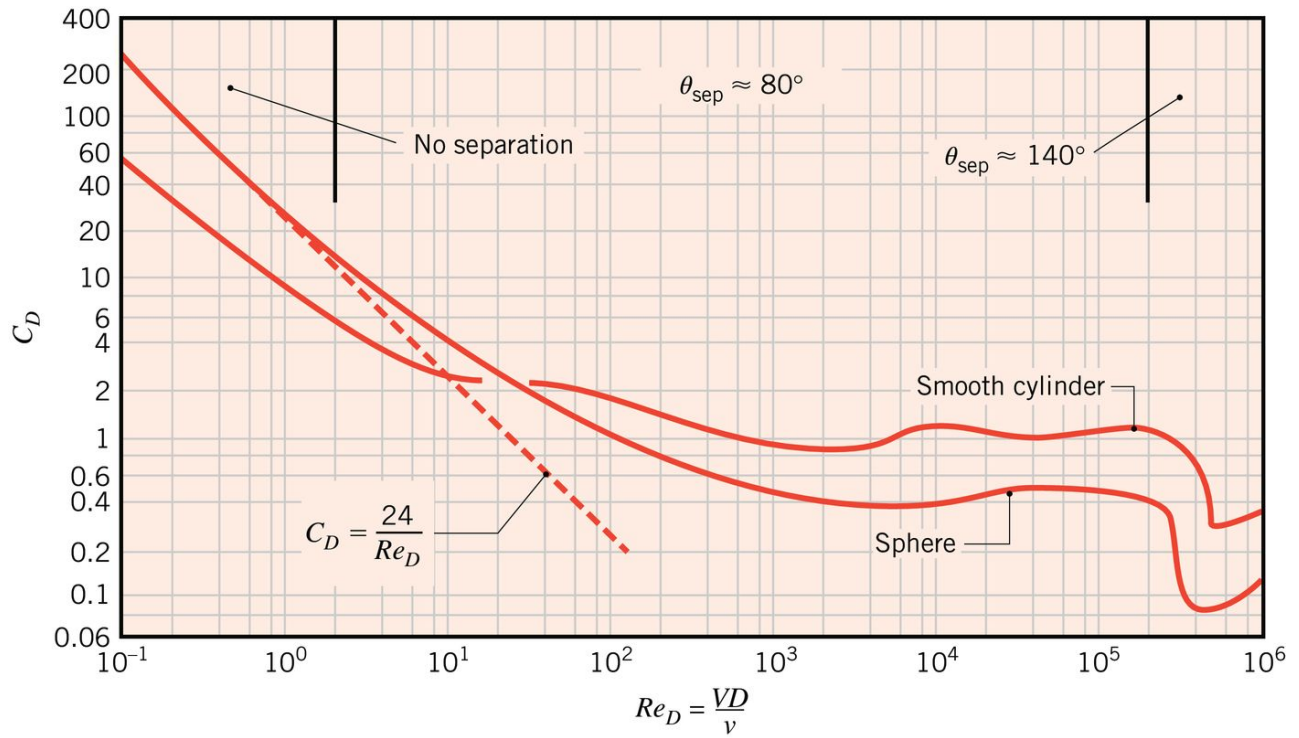
Table 1: Contact Resistance for vacuum interfaces, $R_{t,c}^{''} \times 10^4 \left(\frac{m^2 K}{W} \right)$

Material	$100\,(kPa)$	$10,000\,(kPa)$
Stainless Steel	6-25	0.7-4.0
Copper	1-10	0.1-0.5
Magnesium	1.5-3.5	0.2-0.4
Aluminum	1.5-5.0	0.2-0.4

Table 2: Contact Resistance for aluminum ($10\mu m$ surface roughness, $10 kPa$ contact pressure)

Interfacial Fluid	$R_{t,c}^{''} \times 10^4 \left(\frac{m^2 K}{W} \right)$
Air	2.75
Helium	1.05
Hydrogen	0.720
Silicone Oil	0.525
Glycerine	0.265

Table 3: Fin Tip Conditions

Case	Tip Condition	Temperature Distribution $\frac{\theta}{\theta_b}$	Fin Heat Transfer Rate q_f
A	Convection	$\frac{\cosh m(L-x) + \left(\frac{h}{mk} \right) \sinh m(L-x)}{\cosh mL + \left(\frac{h}{mK} \right) \sinh mL}$	$M \frac{\sinh mL + \left(\frac{h}{mk} \right) \cosh mL}{\cosh mL + \left(\frac{h}{mk} \right) \sinh mL}$
B	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Constant Temperature	$\frac{\left(\frac{\theta_L}{\theta_b} \right) \sinh mx + \sinh m(L-x)}{\sinh mL}$	$M \frac{\cosh mL - \frac{\theta_L}{\theta_b}}{\sinh mL}$
D	Infinite Fin $(L \to \infty)$	e^{-mx}	M

Table 4: Cylinder In Cross Flow

Re_D	C	m
$0.4 - 4$	0.989	0.330
$4 - 40$	0.911	0.385
$40 - 4000$	0.683	0.466
$4000 - 40,000$	0.193	0.618
$40,000 - 400,000$	0.027	0.805

Table 5: Various Geometries In Cross Flow
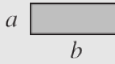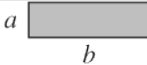
Geometry		Re_D	C	m
$V \rightarrow$ ◇ D		$6000 - 60,000$	0.304	0.59
$V \rightarrow$ ▢ D		$5000 - 60,000$	0.158	0.66
$V \rightarrow$ ⬡ D		$5200 - 20,400$ $20,400 - 105,000$	0.164 0.039	0.638 0.78
$V \rightarrow$ ⬢ D		$4500 - 90,700$	0.150	0.638
$V \rightarrow$ ▯ D	Front Back	$10,000 - 50,000$ $7,000 - 80,000$	0.667 0.191	0.500 0.667

Table 6: Noncircular Tubes

Cross Section	$\frac{b}{a}$	Uniform $q_s^{''}$	Uniform T_s	fRe_{D_h}
⬤		4.36	3.66	64
▢	1.0	3.61	2.98	57
▭	1.43	3.73	3.08	59
▭	2.0	4.12	3.39	62
▭	3.0	4.79	3.96	69
▭	4.0	5.33	4.44	73
Heated	∞	8.23	7.54	96
Insulated	∞	5.39	4.86	96
△		3.11	2.49	53