ENGT 320 Applied Thermal Systems
Quiz 5 Formula Sheet

$$m = \rho \forall$$
$$\dot{\forall} = \frac{\forall}{t}$$
$$q = mc_p\Delta T$$
$$q = -kA\frac{dT}{dt}$$
$$q = hA\left(T_s - T_\infty\right)$$
$$q = \varepsilon\sigma A\left(T_s^4 - T_{sur}^4\right)$$
$$\sigma = 5.67\times10^{-8}\frac{W}{m^2K^4}$$
$$\Delta E_{sys} = \Sigma E_{in} - \Sigma E_{out}$$
$$\frac{dE_{sys}}{dt} = \Sigma\dot{E}_{in} - \Sigma\dot{E}_{out}$$
$$q'' = \frac{q}{A}$$
$$\hat{q}'' = -k_x\frac{\partial T}{\partial x}\hat{i} - k_y\frac{\partial T}{\partial y}\hat{j} - k_z\frac{\partial T}{\partial z}\hat{k}$$
$$k = \frac{9\gamma-5}{4}\frac{c_v}{\pi d^2}\sqrt{\frac{M_w k_B T}{N_A\pi}}$$
$$k_B = 1.381\times10^{-23}\frac{J}{K}$$
$$N_A = 6.022\times10^{23}$$
$$\alpha = \frac{k}{\rho c_p}$$
$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}dx$$
$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y}dy$$
$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z}dz$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = 0$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$
$$T\left(0,t\right) = T_s$$
$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_s''$$
$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0$$
$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h\left(T_\infty - T\left(0,t\right)\right)$$

$$\Delta U = N\left(V^- - V^+\right)$$
$$q = \frac{\Delta U}{\Delta t} = \frac{N}{\Delta t}\Delta V$$
$$q = -IV = \frac{-V^2}{R} = -I^2R$$
$$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$
$$S_{AB} = S_B - S_A = \frac{-\Delta V}{\Delta T}$$
$$C_1 = \frac{T_{s,2} - T_{s,1}}{L}$$
$$C_2 = T_{s,1}$$
$$T\left(x\right) = \left(T_{s,2} - T_{s,1}\right)\frac{x}{L} + T_{s,1}$$
$$q = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right)$$
$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{kA}$$
$$R = \frac{V}{I}$$
$$R_{t,conv} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$

$$R_{t,rad} = \frac{T_s - T_{sur}}{q} = \frac{1}{h_rA}$$
$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right)$$
$$q = UA\Delta T$$
$$U = \frac{1}{R_{tot}A}$$
$$R_{tot} = \Sigma R_t = \frac{1}{UA}$$
$$R_{t,c}'' = \frac{T_A - T_B}{q''}$$
$$q = \frac{k_{eff}A}{L}\left(T_1 - T_2\right)$$
$$k_{eff,min} = \frac{1}{\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}}$$
$$k_{eff,max} = \varepsilon k_f + \left(1-\varepsilon\right)k_s$$
$$R_{t,cond} = \frac{\ln\frac{r_2}{r_1}}{2\pi kL}$$
$$R_{t,conv} = \frac{1}{h2\pi rL}$$

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)\left(T - T_\infty\right) = 0$$

$$A_s = Px$$
$$\frac{d^2T}{dx^2} - \left(\frac{hP}{kA_c}\right)\left(T - T_\infty\right)$$
$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$
$$\theta = T\left(x\right) - T_\infty$$
$$m^2 = \frac{hP}{kA_c}$$
$$P = 2w + 2t$$
$$A_c = wt$$
$$P = \pi D$$
$$A_c = \frac{\pi}{4}D^2$$
$$\theta\left(x\right) = C_1e^{mx} + C_2e^{-mx}$$
$$\theta_b = T_b - T_\infty$$
$$M = \sqrt{hPKA_c}\theta_b$$
$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b}$$
$$\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$$
$$R_{t,f} = \frac{\theta_b}{q_f}$$
$$R_{t,b} = \frac{1}{hA_{c,b}}$$
$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}$$
$$\eta_f = \frac{q_f}{q_{max}} = \frac{q_f}{hA_f\theta_b}$$
$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{hA_t\theta_b}$$
$$A_t = NA_f + A_b$$
$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b$$

$$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$$
$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$$

$$\frac{\partial^2 T}{\partial x^2}\Big|_{m,n} \approx \frac{\frac{\partial T}{\partial x}\Big|_{m+\frac{1}{2},n} - \frac{\partial T}{\partial x}\Big|_{m-\frac{1}{2},n}}{\Delta x}$$
$$\frac{\partial T}{\partial x}\Big|_{m+\frac{1}{2},n} \approx \frac{T\Big|_{m+1,n} - T\Big|_{m,n}}{\Delta x}$$
$$\frac{\partial T}{\partial x}\Big|_{m-\frac{1}{2},n} \approx \frac{T\Big|_{m,n} - T\Big|_{m-1,n}}{\Delta x}$$
$$\frac{\partial^2 T}{\partial x^2}\Big|_{m,n} \approx \frac{T_{m+1,n}+T_{m-1,n}-2T_{m,n}}{(\Delta x)^2}$$
$$\frac{\partial^2 T}{\partial y^2}\Big|_{m,n} \approx \frac{T_{m+1,n}+T_{m-1,n}-2T_{m,n}}{(\Delta y)^2}$$
$$\sum_{i=1}^{4} q_{(i)\to(m,n)} + \dot{q}\,(\Delta x \cdot \Delta y \cdot 1) = 0$$

$$q_{(m-1,n)\to(m,n)} = k\,(\Delta y \cdot 1)\,\frac{T_{m-1,n}-T_{m,n}}{\Delta x}$$

$$q_{(m+1,n)\to(m,n)} = k\,(\Delta y \cdot 1)\,\frac{T_{m+1,n}-T_{m,n}}{\Delta x}$$

$$q_{(m,n+1)\to(m,n)} = k\,(\Delta x \cdot 1)\,\frac{T_{m,n+1}-T_{m,n}}{\Delta y}$$

$$q_{(m,n-1)\to(m,n)} = k\,(\Delta x \cdot 1)\,\frac{T_{m,n-1}-T_{m,n}}{\Delta y}$$

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} + \frac{\dot{q}\cdot 1\cdot(\Delta x)^2}{k} - 4T_{m,n} = 0$$

$$AT = C$$
$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix}$$
$$T = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix}$$
$$C = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{bmatrix}$$
$$T = A^{-1}C$$

$$\frac{u(y)}{u_\infty} = 0.99$$
$$C_f = \frac{\tau_s}{\rho u_\infty^2/2}$$
$$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$
$$\frac{T_s - T(y)}{T_s - T_\infty} = 0.99$$
$$q_s'' = -k\frac{\partial T}{\partial y}\Big|_{y=0}$$
$$q_s'' = h\,(T_s - T_\infty)$$
$$h = \frac{-k\frac{\partial T}{\partial y}\Big|_{y=0}}{T_s - T_\infty}$$
$$q = \int_{A_s} q''\,dA_s = (T_s - T_\infty)\int_{A_s} h\,dA_s$$
$$\overline{q}'' = \overline{h}\,(T_s - T_\infty)$$
$$\overline{q} = \overline{q}''A_s = \overline{h}A_s\,(T_s - T_\infty)$$
$$\overline{h} = \frac{1}{A_s}\int_{A_s} h\,dA_s$$
$$\overline{h} = \frac{1}{L}\int_0^L h\,dx$$
$$Re_x = \frac{\rho u_\infty x}{\mu}$$
$$x^* = \frac{x}{L}$$
$$y^* = \frac{y}{L}$$
$$u^* = \frac{u}{V}$$
$$\mathrm{v}^* = \frac{\mathrm{v}}{V}$$
$$T^* = \frac{T - T_s}{T_\infty - T_s}$$
$$p^* = \frac{p_\infty}{\rho V^2}$$
$$u^*\frac{\partial u^*}{\partial x^*} + \mathrm{v}^*\frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$
$$Re_L = \frac{VL}{\nu}$$
$$\nu = \frac{\mu}{\rho}$$
$$u^*\frac{\partial T^*}{\partial x^*} + \mathrm{v}^*\frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr}\frac{\partial^2 T^*}{\partial y^{*2}}$$
$$Pr = \frac{\nu}{\alpha}$$
$$Nu = \frac{hL}{k} = \frac{\partial T^*}{\partial y^*}\Big|_{y^*=0}$$
$$\overline{Nu} = \frac{\overline{h}L}{k}$$
$$T_f = \frac{T_s + T_\infty}{2}$$

Isothermal Flat Plate

$$\frac{\delta}{\delta_t} \approx Pr^{1/3}$$

Laminar:

$$\delta = \frac{5}{\sqrt{\frac{u_\infty}{\nu x}}} = \frac{5x}{\sqrt{Re_x}}$$
$$C_{f,x} = \frac{\tau_{s,x}}{\frac{\rho u_\infty^2}{2}} = 0.664 Re_x^{-1/2}$$
$$Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3}$$
$$(Pr \gtrsim 0.6)$$
$$\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$$
$$\overline{Nu}_x = \frac{\overline{h}_x x}{k} = 0.664 Re_x^{1/2} Pr^{1/3}$$
$$(Pr \gtrsim 0.6)$$

Turbulent:

$$\delta = 0.37 x Re_x^{-1/2}$$
$$C_{f,x} = 0.0592 Re_x^{-1/5}$$
$$\left(Re_{x,c} \lesssim Re_x \lesssim 10^8\right)$$

$$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$$
$$(0.6 \lesssim Pr \lesssim 60)$$

$$A = 0.037 Re_{x,c}^{4/5} - 0.664 Re_{x,c}^{1/2}$$
$$\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - \frac{2A}{Re_L}$$
$$\left(Re_{x,c} \lesssim Re_L \lesssim 10^8\right)$$

$$\overline{Nu}_L = \left(0.037 Re_L^{4/5} - A\right) Pr^{1/3}$$
$$(0.6 \lesssim Pr \lesssim 60)$$
$$\left(Re_{x,c} \lesssim Re_L \lesssim 10^8\right)$$

$$Re_D = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$
$$C_D = \frac{F_D}{A_f\left(\frac{\rho V^2}{2}\right)}$$

Cylinder

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = C Re_D^m Pr^{1/3}$$
$$(Pr \gtrsim 0.7)$$

See Table 5

. . . .

Various Geometries

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = CRe_D^m Pr^{1/3}$$

$$(Pr \gtrsim 0.7)$$

See Table 6

. . . .

. . . .

Sphere

$$A_s = \pi D^2$$

$$C_D = \frac{25}{Re_D}$$

$$(Re_D \lesssim 0.5)$$

$$\overline{Nu}_D = 2 + \left(0.4Re_D^{1/2} + 0.06Re_D^{2/3}\right) Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4}$$

$$(0.71 \lesssim Pr \lesssim 380)$$

$$\left(3.5 \lesssim Re_D \lesssim 7.6 \times 10^4\right)$$

$$\left(1.0 \lesssim \frac{\mu}{\mu_s} \lesssim 3.2\right)$$

. . . .

$$m = \rho u_m A_c$$

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$$

$$\left(\frac{x_{fd,t}}{D}\right) \approx 0.05 Re_D Pr$$

$$f = \frac{-\frac{dP}{dx}D}{\rho\frac{u_m^2}{2}}$$

$$f = \frac{64}{Re_D}$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{e/D}{3.7} + \frac{2.51}{Re_D\sqrt{f}}\right]$$

$$dq_{conv} = mc_p\left[(T_m + dT_m) - T_m\right] = mc_p dT_m$$

$$dq_{conv} = q_s^{''} P dx$$

$$\frac{dT_m}{dx} = \frac{q_s^{''}P}{mc_p} = \frac{P}{mc_p}h\left(T_s - T_m\right)$$

$$T_m(x) = T_{m,i} + \frac{q_s^{''}P}{mc_p}x$$

$$\Delta T = T_s - T_m$$

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{mc_p}h\Delta T$$

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = e^{-\frac{Px}{mc_p}\overline{h}}$$

$$\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$$

$$\Delta T_o = T_s - T_{m,o}$$

$$\Delta T_i = T_s - T_{m,i}$$

$$q_{conv} = \overline{h}A_s\Delta T_{lm}$$

. . . .

Circular Tubes

Fully dev. lam. w/ $q^{''} = \mathbb{C}$:

$$Nu_D = 4.36$$

Fully dev. lam. w/ $T_s = \mathbb{C}$:

$$Nu_D = 3.66$$

Fully dev. turb. w/ small ΔT:

$$Nu_D = 0.023Re_D^{4/5}Pr^n$$

$$n = 0.4, \quad T_s > T_m$$

$$n = 0.3, \quad T_s < T_m$$

$$(0.6 \lesssim Pr \lesssim 160)$$

$$(Re_D \gtrsim 10,000)$$

$$(L/D \gtrsim 10)$$

Fully dev. turb. w/ large ΔT:

$$Nu_D = 0.027Re_D^{4/5}Pr^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$(0.7 \lesssim Pr \lesssim 16,700)$$

$$(Re_D \gtrsim 10,000)$$

$$(L/D \gtrsim 10)$$

. . . .

. . . .

Noncircular Tubes

$$D_h = \frac{4A_c}{P}$$

$$Nu_D = \frac{hD_h}{k}$$

See Table 7

. . . .

$$\frac{1}{UA} = \frac{1}{U_cA_c} = \frac{1}{U_hA_h} = \frac{1}{(hA)_c} + R_w + \frac{1}{(hA)_h}$$

$$\frac{1}{UA} = \frac{1}{(\eta_o hA)_c} + \frac{R_{f,c}^{''}}{(\eta_o A)_c} + R_w + \frac{R_{f,h}^{''}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h}$$

$$q = m_h\left(i_{h,i} - i_{h,o}\right)$$

$$q = m_c\left(i_{c,i} - i_{c,o}\right)$$

$$q = m_h c_{p,h}\left(T_{h,i} - T_{h,o}\right)$$

$$q = m_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)$$

$$q = UA\Delta T_m$$

$$q = UA\Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln\frac{\Delta T_2}{\Delta T_1}} = \frac{\Delta T_1 - \Delta T_2}{\ln\frac{\Delta T_1}{\Delta T_2}}$$

$$\Delta T_1 = T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i}$$

$$\Delta T_2 = T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o}$$

$$\Delta T_1 = T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o}$$

$$\Delta T_2 = T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i}$$

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right)$$

$$\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})}$$

$$\varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})}$$

$$q = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right)$$

$$NTU = \frac{UA}{C_{min}}$$

$$\lambda = \frac{c}{\nu}$$

$$c_o = 2.998 \times 10^8 \frac{m}{s}$$

$$E = \varepsilon\sigma T^4$$

$$J = E + \rho G$$

$$q_{rad}^{''} = J - G$$

$$q_{rad}^{''} = \varepsilon\sigma T^4 - \alpha G$$

$$G = G_{ref} + G_{abs} + G_{tr}$$

$$\rho + \alpha + \tau = 1$$

$$\rho + \alpha = 1$$

$$d\omega = \frac{dA_n}{r^2}$$

$$dA_n = r^2\sin\theta d\theta d\phi$$

$$d\omega = \sin\theta d\theta d\phi$$

$$\omega_{hemi} = \int_0^{2\pi}\int_0^{\pi/2}\sin\theta d\theta d\phi = 2\pi sr$$

$$I_{\lambda,e}(\lambda,\theta,\phi) = \frac{dq}{(dA_1\cos\theta)d\omega d\lambda}$$

$$dq_\lambda = \frac{dq}{d\lambda} = I_{\lambda,e}(\lambda,\theta,\phi)\,dA_1\cos\theta d\omega$$

$$dq_\lambda^{''} = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta d\omega = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\omega$$

ENGT 320 Applied Thermal Systems
Quiz 5 Formula Sheet

$$E_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

$$E = \int_0^\infty E_\lambda(\lambda)d\lambda$$

$$E_\lambda(\lambda) = \pi I_{\lambda,e}(\lambda)$$

$$E = \pi I_e$$

$$G_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

$$G = \int_0^\infty G_\lambda(\lambda)d\lambda$$

$$J_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

$$J = \int_0^\infty J_\lambda(\lambda)d\lambda$$

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,T}(\lambda,T) = \frac{C_1}{\lambda^5\left[e^{(C_2/\lambda T)}-1\right]}$$

$$C_1 = 3.742\times10^8 W\mu m^4/m^2$$

$$C_2 = 1.439\times10^4\mu mK$$

$$\lambda_{max}T = C_3 = 2898\mu mK$$

$$E_b = \pi I_b = \int_0^\infty E_{\lambda,b}d\lambda = \sigma T^4$$

$$F_{(\lambda_1-\lambda_2)} = F_{0-\lambda_2} - F_{0-\lambda_1} = \frac{\int_0^{\lambda_2}E_{\lambda,b}d\lambda - \int_0^{\lambda_1}E_{\lambda,b}d\lambda}{\sigma T^4}$$

$$F_{(0-\lambda)} = \frac{\int_0^\lambda E_{\lambda,b}d\lambda}{\sigma T^4}$$

$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j}\frac{\cos\theta_i\cos\theta_j}{\pi R^2}dA_jdA_i$$

$$F_{ji} = \frac{1}{A_j}\int_{A_i}\int_{A_j}\frac{\cos\theta_i\cos\theta_j}{\pi R^2}dA_jdA_i$$

$$A_iF_{ij} = A_jF_{ji}$$

$$\sum_{j=1}^N F_{ij} = 1$$

$$J_i = E_{bi}$$

$$q_{ij} = q_{i\to j} - q_{j\to i}$$

$$q_{ij} = A_iF_{ij}E_{bi} - A_jF_{ji}E_{bj}$$

$$q_{ij} = A_iF_{ij}\sigma\left(T_i^4 - T_j^4\right)$$

$$q_i = \sum_{j=1}^N A_iF_{ij}\sigma\left(T_i^4 - T_j^4\right)$$

$$q_i = A_i(J_i - G_i)$$

$$q_i = A_i(E_i - \alpha_iG_i)$$

$$q_i = \frac{E_{bi}-J_i}{(1-\varepsilon_i)/\varepsilon_iA_i}$$

$$q_i = \sum_{j=1}^N A_iF_{ij}(J_i - J_j) = \sum_{j=1}^N \frac{J_i-J_j}{(A_iF_{ij})^{-1}}$$

$$q_1 = -q_2 = q_{12} = \frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1A_1}+\frac{1}{A_1F_{12}}+\frac{1-\varepsilon_2}{\varepsilon_2A_2}}$$

$$q_1 = -q_2 = \frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1A_1}+\frac{1}{A_1F_{12}+\left[(1/A_1F_{1R})+(1/A_2F_{2R})\right]^{-1}}+\frac{1-\varepsilon_1}{\varepsilon_1A_1}}$$

$$\frac{J_1-J_R}{1/A_1F_{1R}} = \frac{J_R-J_2}{1/A_2F_{2R}}$$

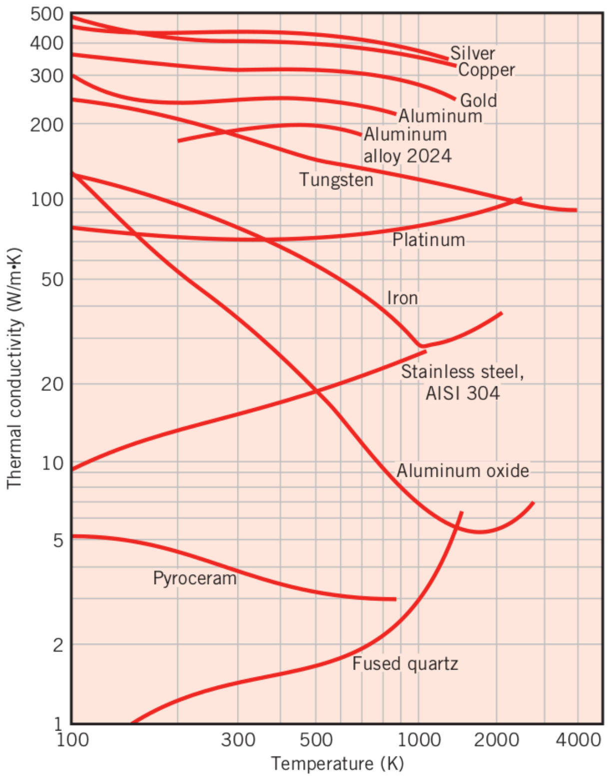
$$T_R = \left(\frac{J_R}{\sigma}\right)^{1/4}$$

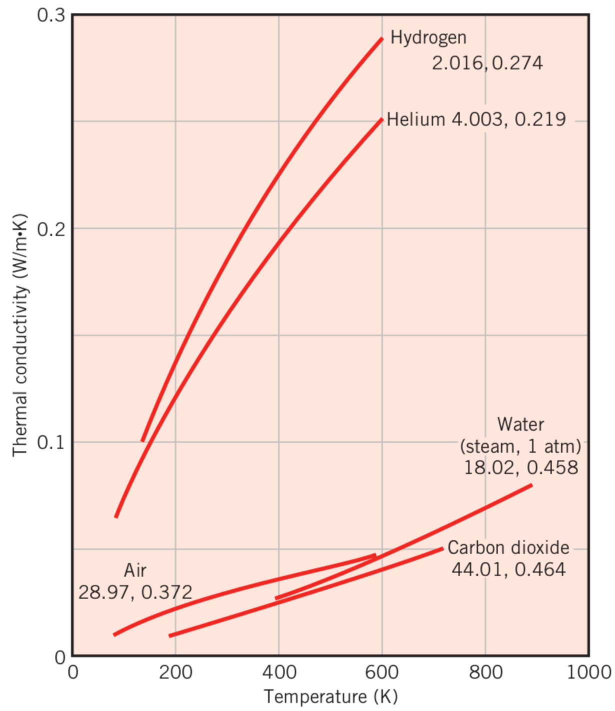
$$q_{i,ext} = q_{i,rad} + q_{i,conv} + q_{i,cond}$$

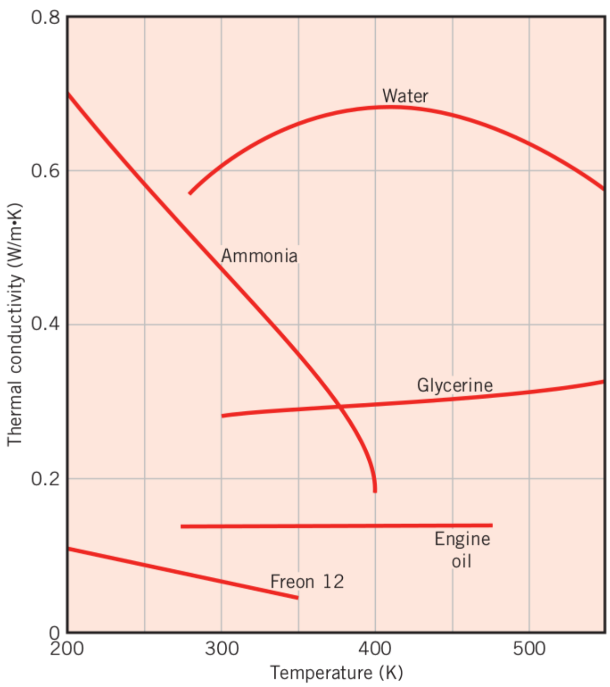
Lecture Note 14 formulas not needed

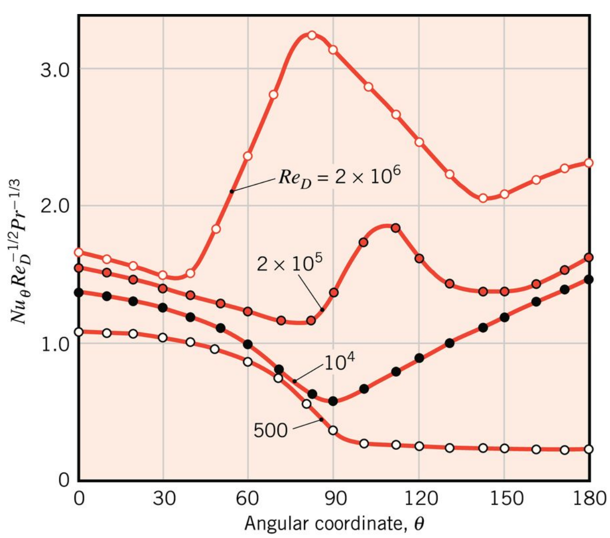
$$F_{ij} = \frac{q_{i\to j}}{A_iJ_i}$$

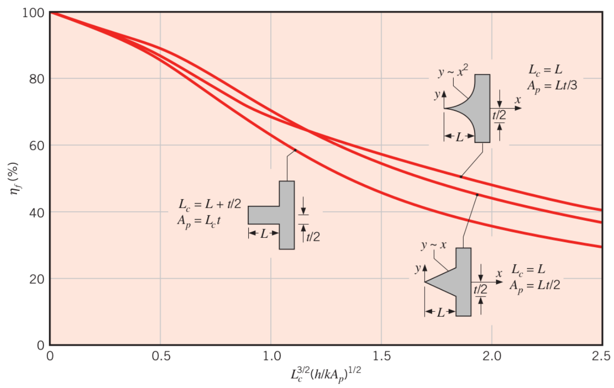
$$dq_{i\to j} = I_i\cos\theta_idA_id\omega_{j-i} = J_i\frac{\cos\theta_i\cos\theta_j}{\pi R^2}dA_jdA_i$$

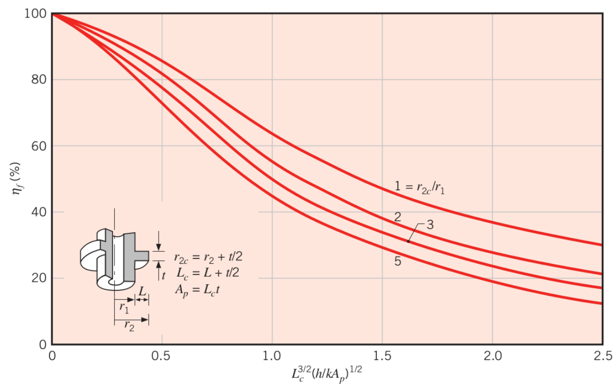

Silver
Copper
Gold
Aluminum
Aluminum alloy 2024
Tungsten
Platinum
Iron
Stainless steel, AISI 304
Aluminum oxide
Pyroceram
Fused quartz
Thermal conductivity (W/m•K)
Temperature (K)


Hydrogen 2.016, 0.274
Helium 4.003, 0.219
Water (steam, 1 atm) 18.02, 0.458
Carbon dioxide 44.01, 0.464
Air 28.97, 0.372
Thermal conductivity (W/m•K)
Temperature (K)


Water
Ammonia
Glycerine
Engine oil
Freon 12
Thermal conductivity (W/m•K)
Temperature (K)


$Re_D = 2 \times 10^6$
2×10^5
10^4
500
$Nu_\theta Re_D^{-1/2} Pr^{-1/3}$
Angular coordinate, θ





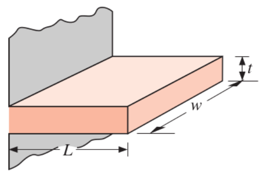
Straight Fins

Rectangular

$A_f = 2wL_c$

$L_c = L + (t/2)$

$A_p = tL$

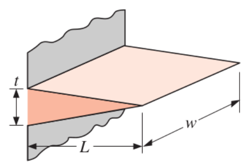


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$A_f = 2w[L^2 + (t/2)^2]^{1/2}$

$A_p = (t/2)L$



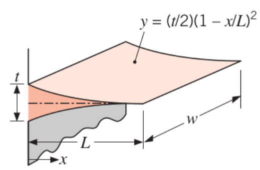
$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$

Parabolic

$A_f = w[C_1 L + (L^2/t)\ln(t/L + C_1)]$

$C_1 = [1 + (t/L)^2]^{1/2}$

$A_p = (t/3)L$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

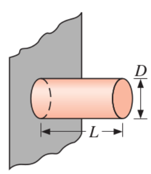
Pin Fins

Rectangular

$A_f = \pi D L_c$

$L_c = L + (D/4)$

$V = (\pi D^2/4)L$

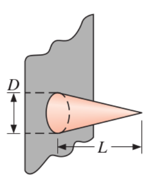


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$

$V = (\pi/12)D^2 L$



$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$

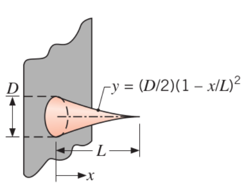
Parabolic

$A_f = \frac{\pi L^3}{8D}\left\{C_3 C_4 - \frac{L}{2D}\ln[(2DC_4/L) + C_3]\right\}$

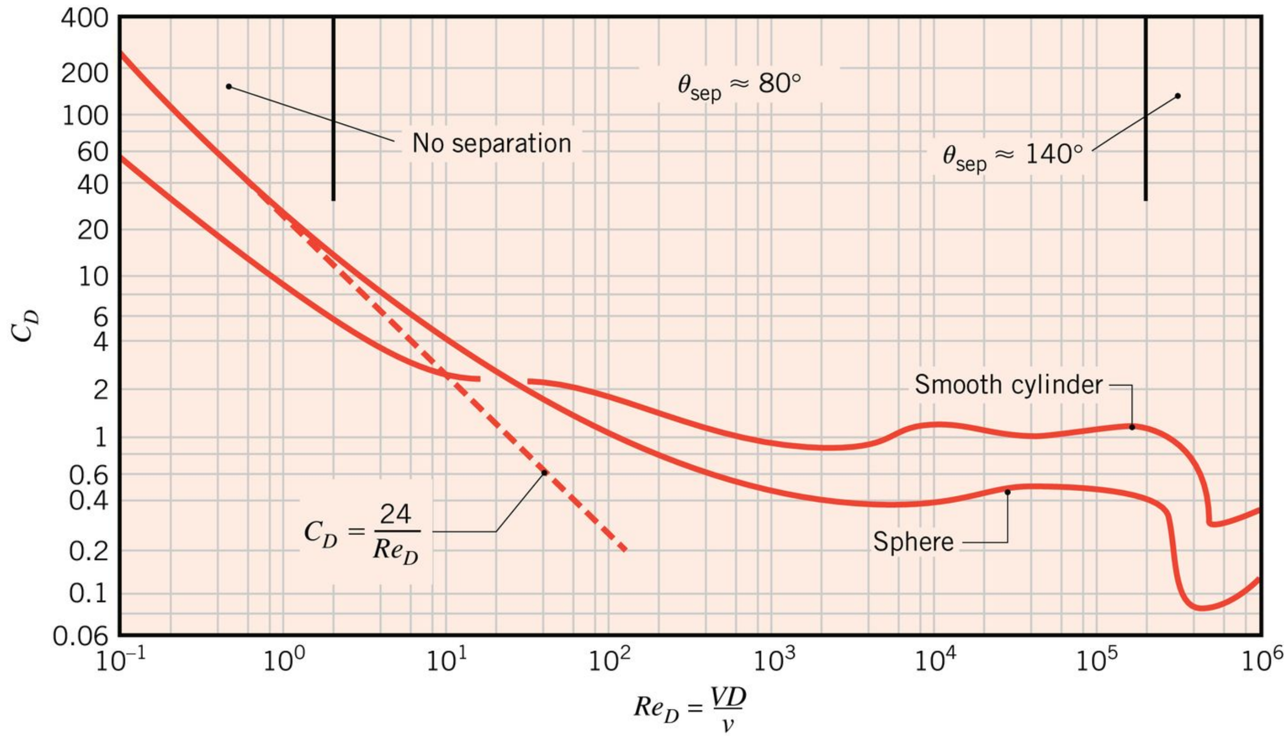
$C_3 = 1 + 2(D/L)^2$

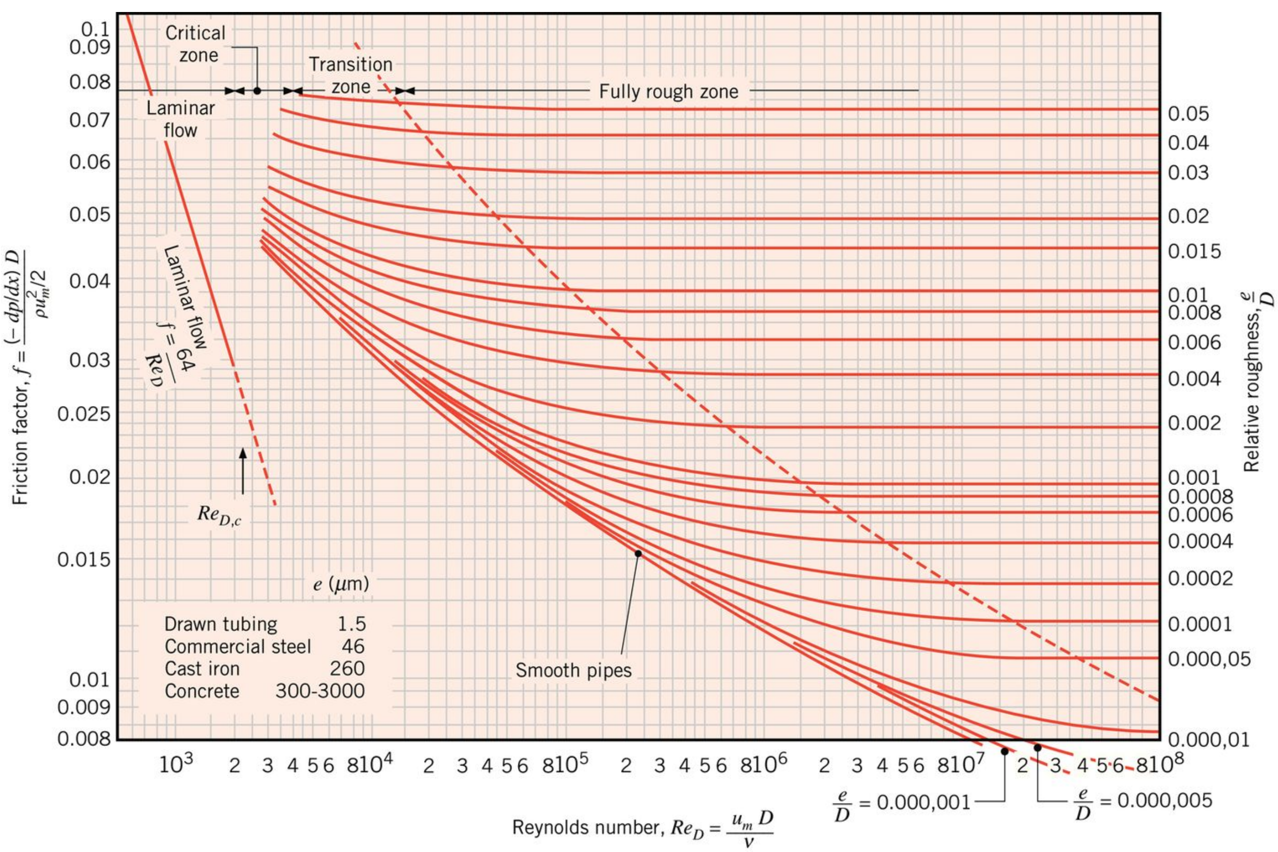
$C_4 = [1 + (D/L)^2]^{1/2}$

$V = (\pi/20)D^2 L$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$$

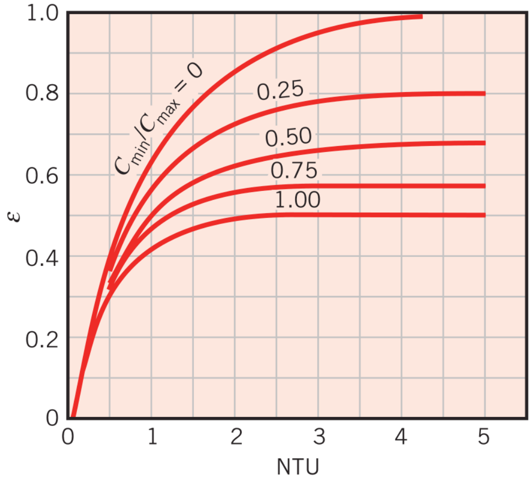

$\theta_{sep} \approx 80°$
No separation
$\theta_{sep} \approx 140°$
C_D
Smooth cylinder
Sphere
$C_D = \frac{24}{Re_D}$
$Re_D = \frac{VD}{\nu}$


Critical zone
Transition zone
Fully rough zone
Laminar flow
Laminar flow $f = \frac{64}{Re_D}$
$Re_{D,c}$
Friction factor, $f = \frac{(-dp/dx)\,D}{\rho u_m^2/2}$
Relative roughness, $\frac{e}{D}$
e (μm)
Drawn tubing 1.5
Commercial steel 46
Cast iron 260
Concrete 300-3000
Smooth pipes
$\frac{e}{D}$ = 0.000,001
$\frac{e}{D}$ = 0.000,005
Reynolds number, $Re_D = \frac{u_m D}{\nu}$

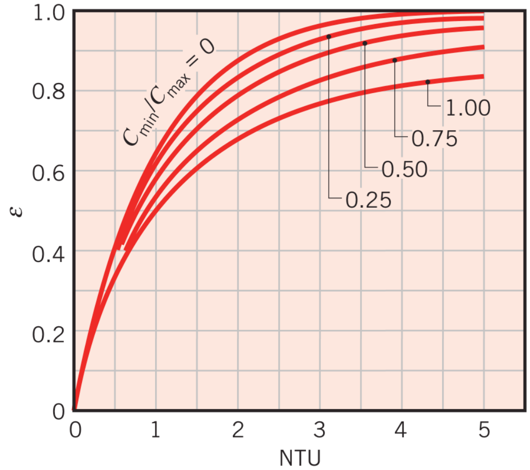
Flow Arrangement	Relation	
Parallel ow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 + C_r)]}{1 + C_r}$	
Counterow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 - C_r)]}{1 - C_r \exp[-\mathrm{NTU}(1 - C_r)]}$	$(C_r < 1)$
	$\varepsilon = \dfrac{\mathrm{NTU}}{1 + \mathrm{NTU}}$	$(C_r = 1)$
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$	
n shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$	
Cross-ow (single pass)		
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(\mathrm{NTU})^{0.22}\{\exp[-C_r(\mathrm{NTU})^{0.78}] - 1\}\right]$	
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\mathrm{NTU})]\})$	
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\mathrm{NTU})]\})$	
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-\mathrm{NTU})$	

Flow Arrangement	Relation	
Parallel ow	$\mathrm{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$	
Counterow	$\mathrm{NTU} = \dfrac{1}{C_r - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right)$	$(C_r < 1)$
	$\mathrm{NTU} = \dfrac{\varepsilon}{1 - \varepsilon}$	$(C_r = 1)$
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$(\mathrm{NTU})_1 = -(1 + C_r^2)^{-1/2}\ln\left(\dfrac{E - 1}{E + 1}\right)$ $E = \dfrac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$	
n shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F - 1}{F - C_r} \quad F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$	
Cross-ow (single pass)		
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\mathrm{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1 - \varepsilon C_r)\right]$	
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r \ln(1 - \varepsilon) + 1]$	
All exchangers ($C_r = 0$)	$\mathrm{NTU} = -\ln(1 - \varepsilon)$	

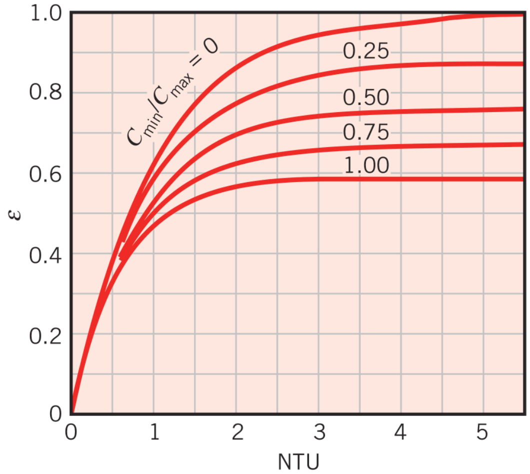
Parallel Flow



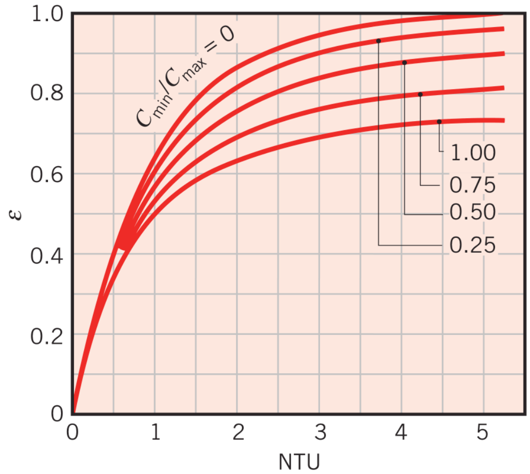
Counter Flow



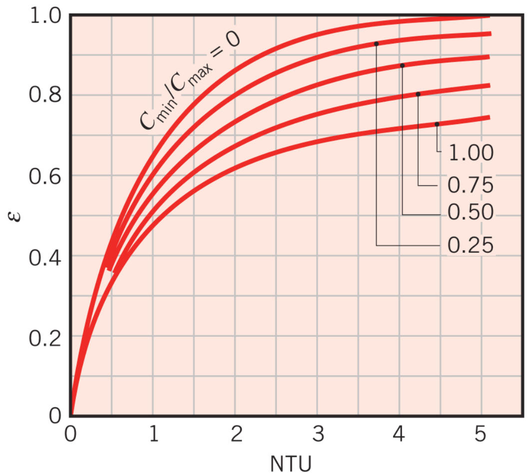
Shell and Tube - One Shell



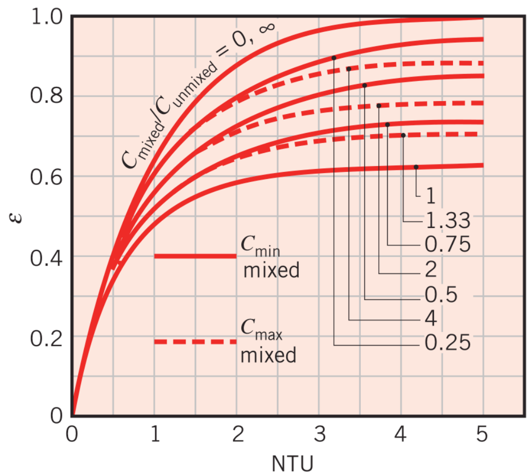
Shell and Tube - 2 Shells



Cross-flow Both Unmixed

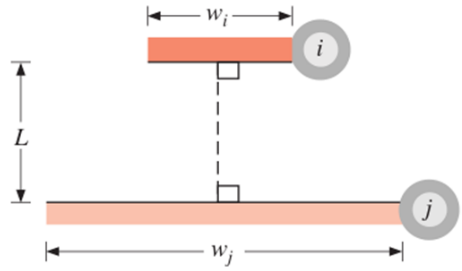


Cross-flow One Mixed/One Unmixed



Geometry	Relation

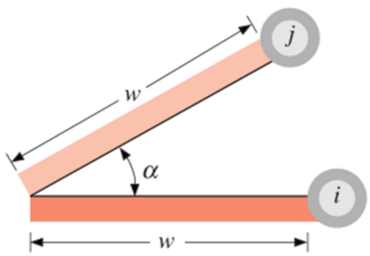
Parallel Plates with Midlines Connected by Perpendicular



$$F_{ij} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$$

$$W_i = w_i/L, \; W_j = w_j/L$$

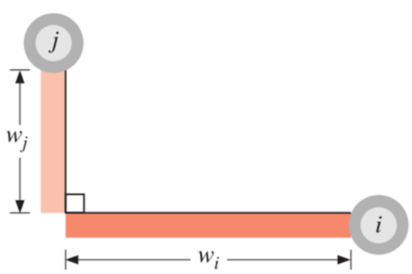
Inclined Parallel Plates of Equal Width and a Common Edge



$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$

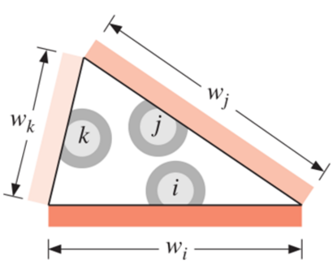
Geometry	Relation

Perpendicular Plates with a Common Edge



$$F_{ij} = \frac{1 + (w_j/w_i) - [1 + (w_j/w_i)^2]^{1/2}}{2}$$

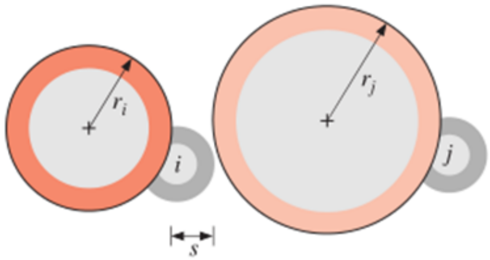
Three-Sided Enclosure



$$F_{ij} = \frac{w_i + w_j - w_k}{2w_i}$$

Geometry	Relation

Parallel Cylinders of Different Radii

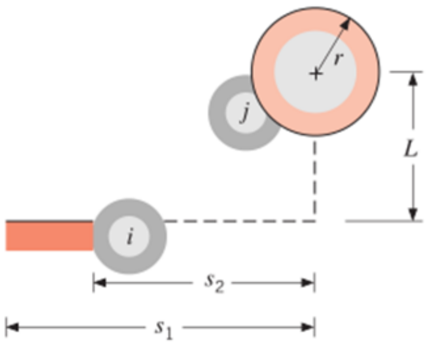


$$F_{ij} = \frac{1}{2\pi}\left\{\pi + [C^2 - (R+1)^2]^{1/2} - [C^2 - (R-1)^2]^{1/2} + (R-1)\cos^{-1}\left[\left(\frac{R}{C}\right) - \left(\frac{1}{C}\right)\right] - (R+1)\cos^{-1}\left[\left(\frac{R}{C}\right) + \left(\frac{1}{C}\right)\right]\right\}$$

$R = r_j/r_i,\ S = s/r_i$

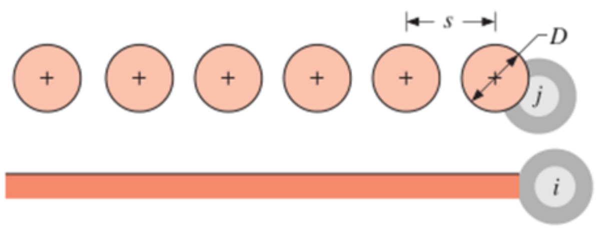
$C = 1 + R + S$

Cylinder and Parallel Rectangle

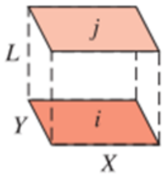
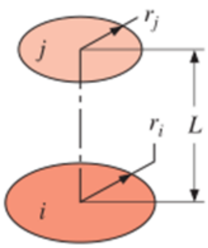
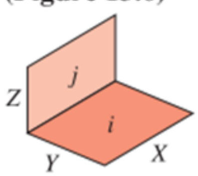


$$F_{ij} = \frac{r}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right]$$

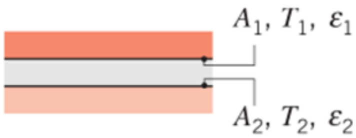
Innite Plane and Row of Cylinders



$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \left(\frac{D}{s}\right)\tan^{-1}\left[\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}\right]$$

Geometry	Relation
Aligned Parallel Rectangles **(Figure 13.4)**  	$\bar{X} = X/L, \bar{Y} = Y/L$ $F_{ij} = \frac{2}{\pi \bar{X} \bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$
Coaxial Parallel Disks **(Figure 13.5)**  	$R_i = r_i/L, R_j = r_j/L$ $S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$
Perpendicular Rectangles with a Common Edge **(Figure 13.6)**  	$H = Z/X, W = Y/X$ $F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$

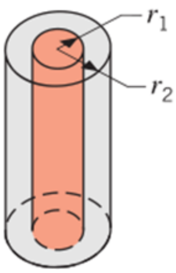
Large (Innite) Parallel Planes



$$A_1 = A_2 = A$$
$$F_{12} = 1$$

$$q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

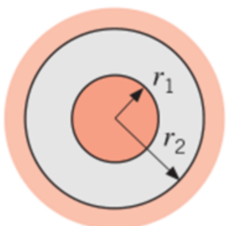
Long (Innite) Concentric Cylinders



$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$
$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

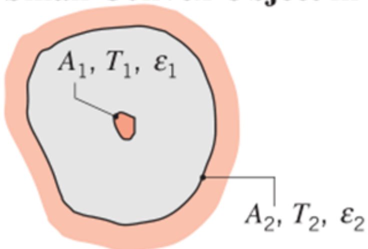
Concentric Spheres



$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$
$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2}$$

Small Convex Object in a Large Cavity



$$\frac{A_1}{A_2} \approx 0$$
$$F_{12} = 1$$

$$q_{12} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

Table 1: Contact Resistance for vacuum interfaces, $R''_{t,c} \times 10^4 \left(\frac{m^2K}{W}\right)$

Material	$100\,(kPa)$	$10,000\,(kPa)$
Stainless Steel	6-25	0.7-4.0
Copper	1-10	0.1-0.5
Magnesium	1.5-3.5	0.2-0.4
Aluminum	1.5-5.0	0.2-0.4

Table 2: Contact Resistance for aluminum ($10\mu m$ surface roughness, $10kPa$ contact pressure)

Interfacial Fluid	$R''_{t,c} \times 10^4 \left(\frac{m^2K}{W}\right)$
Air	2.75
Helium	1.05
Hydrogen	0.720
Silicone Oil	0.525
Glycerine	0.265

Table 3: Fin Tip Conditions

Case	Tip Condition	Temperature Distribution $\frac{\theta}{\theta_b}$	Fin Heat Transfer Rate q_f
A	Convection	$\frac{\cosh m(L-x)+\left(\frac{h}{mk}\right)\sinh m(L-x)}{\cosh mL+\left(\frac{h}{mK}\right)\sinh mL}$	$M\frac{\sinh mL+\left(\frac{h}{mk}\right)\cosh mL}{\cosh mL+\left(\frac{h}{mk}\right)\sinh mL}$
B	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	$M\tanh mL$
C	Constant Temperature	$\frac{\left(\frac{\theta_L}{\theta_b}\right)\sinh mx+\sinh m(L-x)}{\sinh mL}$	$M\frac{\cosh mL-\frac{\theta_L}{\theta_b}}{\sinh mL}$
D	Infinite Fin ($L \to \infty$)	e^{-mx}	M

Table 4: Energy Balance Method Case Summary

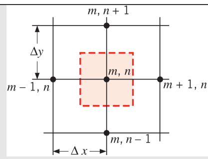
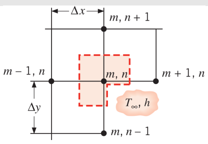
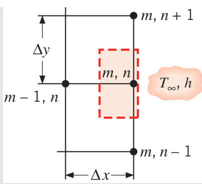
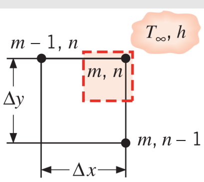
Case	Diagram	Equation
1		$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0$
2		$2\left(T_{m-1,n} - T_{m,n+1}\right) + \left(T_{m+1,n} + T_{m,n-1}\right) + 2\frac{h\Delta x}{k}T_\infty - 2\left(3 + \frac{h\Delta x}{k}\right)T_{m,n} = 0$
3		$\left(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}\right) + 2\frac{h\Delta x}{k}T_\infty - 2\left(2 + \frac{h\Delta x}{k}\right)T_{m,n} = 0$
4		$\left(T_{m,n-1} + T_{m-1,n}\right) + 2\frac{h\Delta x}{k}T_\infty - 2\left(1 + \frac{h\Delta x}{k}\right)T_{m,n} = 0$

Table 5: Cylinder In Cross Flow

Re_D	C	m
$0.4 - 4$	0.989	0.330
$4 - 40$	0.911	0.385
$40 - 4000$	0.683	0.466
$4000 - 40,000$	0.193	0.618
$40,000 - 400,000$	0.027	0.805

Table 6: Various Geometries In Cross Flow

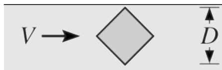
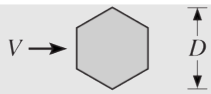
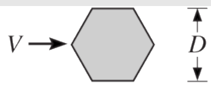
Geometry		Re_D	C	m
		$6000 - 60,000$	0.304	0.59
		$5000 - 60,000$	0.158	0.66
		$5200 - 20,400$ $20,400 - 105,000$	0.164 0.039	0.638 0.78
		$4500 - 90,700$	0.150	0.638
	Front Back	$10,000 - 50,000$ $7,000 - 80,000$	0.667 0.191	0.500 0.667

Table 7: Noncircular Tubes

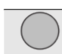
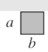
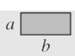
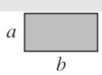
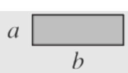
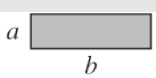
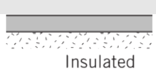
Cross Section	$\frac{b}{a}$	Uniform q_s''	Uniform T_s	fRe_{D_h}
		4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	∞	8.23	7.54	96
	∞	5.39	4.86	96
		3.11	2.49	53

Table 8: Fouling Factors

Fluid	$R_f^{''}\left(m^2K/W\right)$
Seawater and treated boiler feedwater (below $50°C$)	0.001
Seawater and treated boiler feedwater (above $50°C$)	0.002
River water (below $50°C$)	0.0002 - 0.001
Fuel Oil	0.0009
Refrigerating Liquids	0.0002
Steam (nonoil bearing)	0.0001

Table 9: Blackbody Radiation Functions

$\lambda T\,(\mu m K)$	$F_{(0\rightarrow\lambda)}$	$I_{\lambda,b}\,(\lambda,T)\,/\sigma T^5\,(\mu m K sr)^{-1}$	$\frac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
200	0.000000	$3.754247E-28$	0.000000
400	0.000000	$4.905624E-14$	0.000000
600	0.000000	$1.040747E-09$	0.000014
800	0.000016	$9.912998E-08$	0.001372
1000	0.000321	$1.185188E-06$	0.016409
1200	0.002134	$5.239683E-06$	0.072542
1400	0.007790	$1.344175E-05$	0.186098
1600	0.019719	$2.491377E-05$	0.344926
1800	0.039342	$3.755736E-05$	0.519974
2000	0.066730	$4.934325E-05$	0.683147
2200	0.100890	$5.896440E-05$	0.816350
2400	0.140257	$6.588541E-05$	0.912170
2600	0.183121	$7.012745E-05$	0.970900
2800	0.227890	$7.202172E-05$	0.997125
2898	0.250055	$7.222935E-05$	1.000000
3000	0.273229	$7.202283E-05$	0.997141
3500	0.382909	$6.665930E-05$	0.922884
4000	0.480865	$5.780330E-05$	0.800274
4500	0.564303	$4.850820E-05$	0.671586
5000	0.633726	$4.007866E-05$	0.554881
5500	0.690883	$3.291336E-05$	0.455678
6000	0.737789	$2.701020E-05$	0.373950
6500	0.776320	$2.221845E-05$	0.307610
7000	0.808075	$1.835199E-05$	0.254079
7500	0.834367	$1.523509E-05$	0.210927
8000	0.856251	$1.271746E-05$	0.176070
8500	0.874569	$1.067637E-05$	0.147812
9000	0.889989	$9.013893E-06$	0.124795
9500	0.903044	$7.652747E-06$	0.105951
10000	0.914157	$6.532242E-06$	0.090437
20000	0.985554	$6.232529E-07$	0.008629
30000	0.995291	$1.404562E-07$	0.001945
40000	0.997918	$4.738457E-08$	0.000656
50000	0.998904	$2.015856E-08$	0.000279
75000	0.999663	$4.185570E-09$	0.000058
100000	0.999855	$1.357391E-09$	0.000019