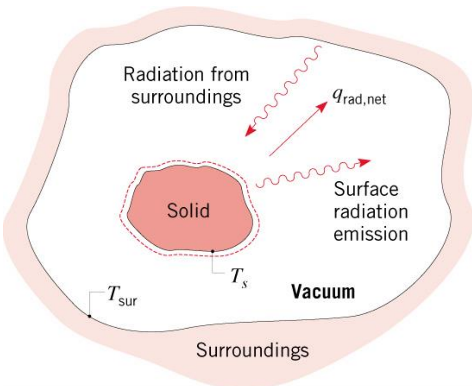
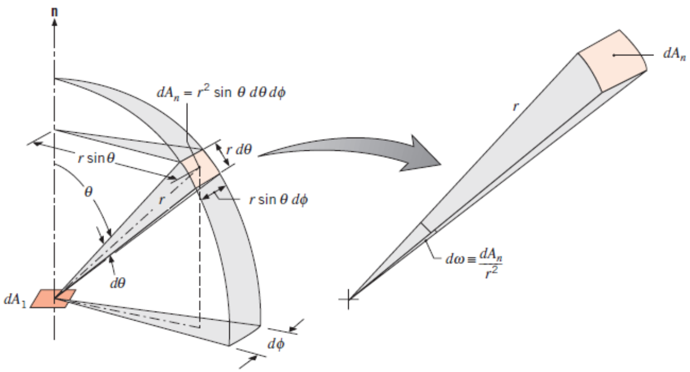
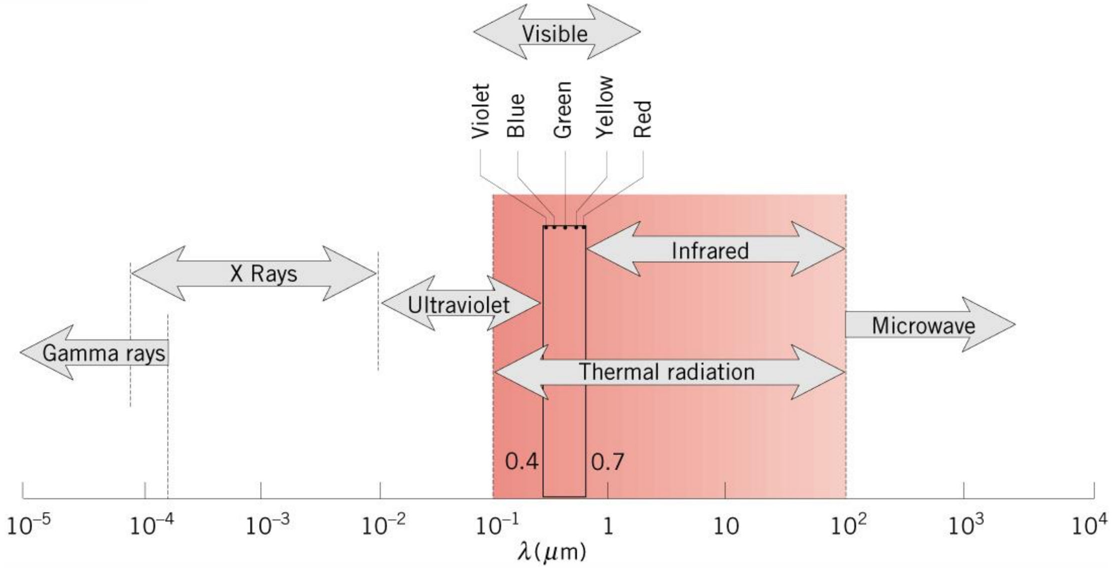
Radiation Processes and Properties







General Considerations

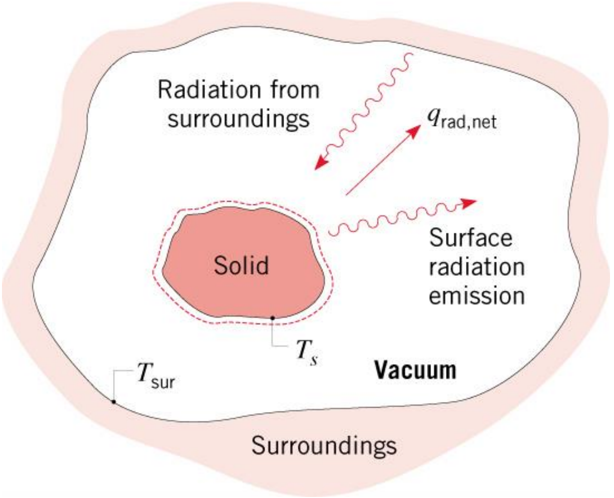
- Attention is focused on thermal radiation, whose origins are associated with emission from matter at an absolute temperature $T > 0$.

- Emission is due to oscillations and transitions of the many electrons that comprise matter, which are, in turn, sustained by the thermal energy of the matter.

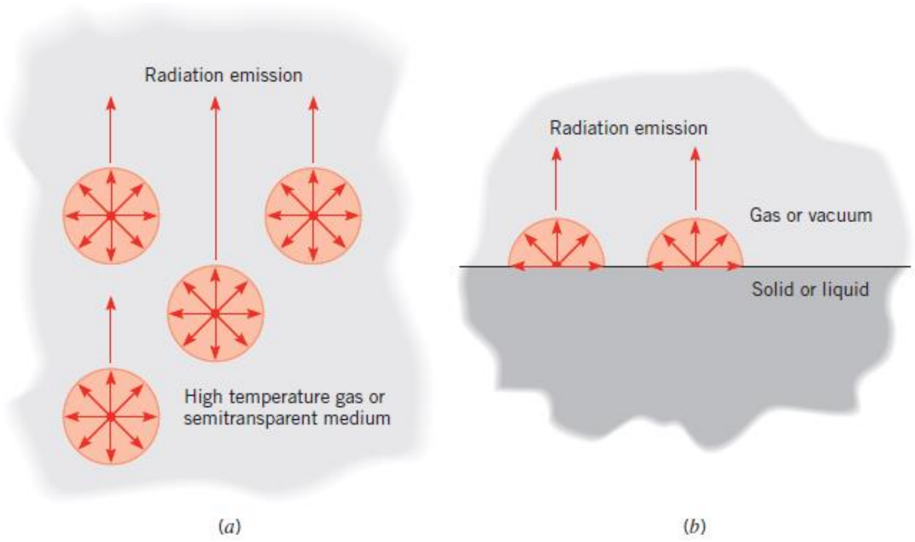
- Emission corresponds to heat transfer from the matter and hence to a reduction in its thermal energy.

- Radiation may also be intercepted and absorbed by matter, resulting in its increase in thermal energy.

- Consider a solid of temperature T_s in an evacuated enclosure whose walls are at a fixed temperature T_{sur}:
  - What changes occur if $T_s > T_{sur}$? Why?
  - What changes occur if $T_s < T_{sur}$? Why?



- Emission from a gas or a semitransparent solid or liquid is a volumetric phenomenon. Emission from an opaque solid or liquid is treated as a surface phenomenon.



For an opaque solid or liquid, emission originates from atoms and molecules within 1 μm of the surface.

- The dual nature of radiation:
  - In some cases, the physical manifestations of radiation may be explained by viewing it as particles (known as photons or quanta).
  - In other cases, radiation behaves as an electromagnetic wave.

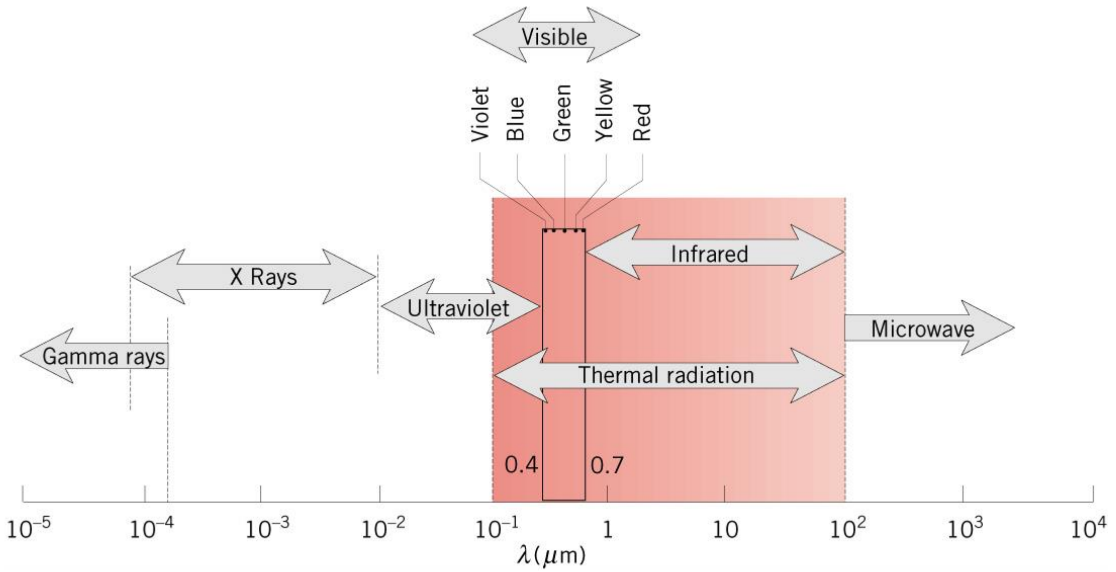
– In all cases, radiation can be characterized by a wavelength λ and frequency ν, which are related through the speed at which radiation propagates in the medium of interest:

$$\lambda = \frac{c}{\nu}$$

For propagation in a vacuum,

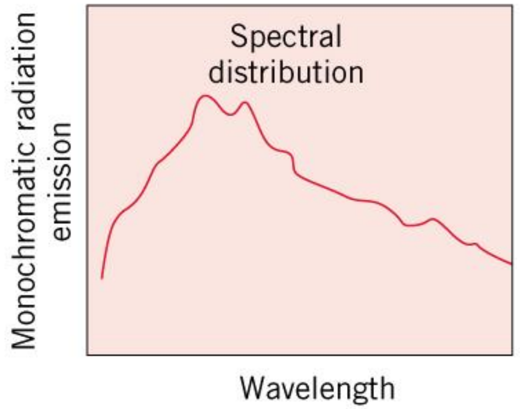
$$c = c_o = 2.998 \ x \ 10^8 \ \text{m/s}$$

The Electromagnetic Spectrum



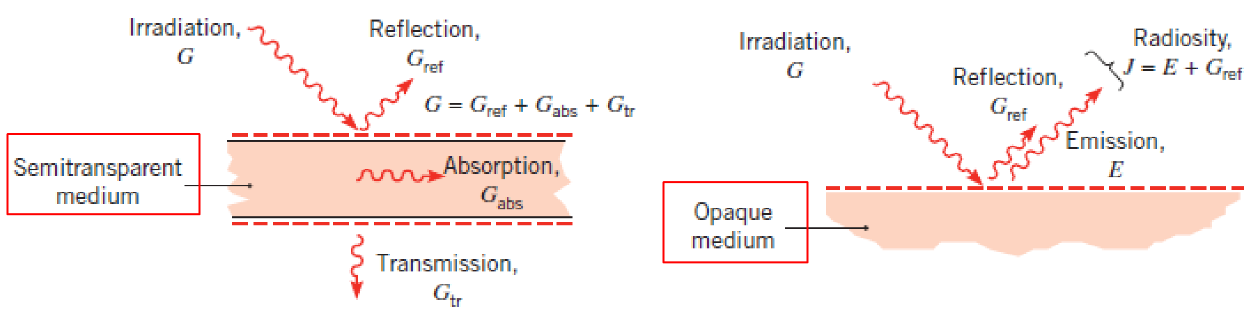
- Thermal radiation is confined to the infrared, visible and ultraviolet regions of the spectrum $\left(0.1 < \lambda < 100\mu m\right)$.

- The amount of radiation emitted by an opaque surface varies with wavelength, and we may speak of the spectral distribution over all wavelengths or of monochromatic/spectral components associated with particular wavelengths.



Radiation Heat Fluxes and Material Properties

Flux (W/m^2)	Description	Comment
Emissive power, E	Rate at which radiation is emitted from a surface per unit area	$E = \varepsilon \sigma T_s^4$
Irradiation, G	Rate at which radiation is incident upon a surface per unit area	Irradiation can be reflected, absorbed, or transmitted
Radiosity, J	Rate at which radiation leaves a surface per unit area	For an opaque surface $J = E + \rho G$
Net radiative flux, $q''_{rad} = J - G$	Net rate of radiation leaving a surface per unit area	For an opaque surface $q''_{rad} = \varepsilon \sigma T_s^4 - \alpha G$



$\rho \rightarrow$ reflectivity $\rightarrow$ fraction of irradiation (G) reflected.

$\alpha \rightarrow$ absorptivity $\rightarrow$ fraction of irradiation absorbed.

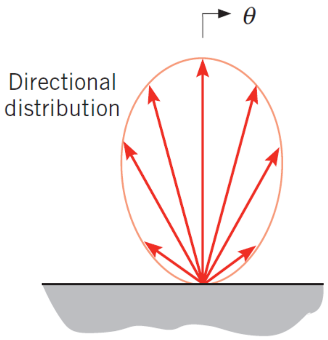
$\tau \rightarrow$ transmissivity $\rightarrow$ fraction of irradiation transmitted through the medium.

$\rho + \alpha + \tau = 1$ for any medium. $\rho + \alpha = 1$ for an opaque medium.

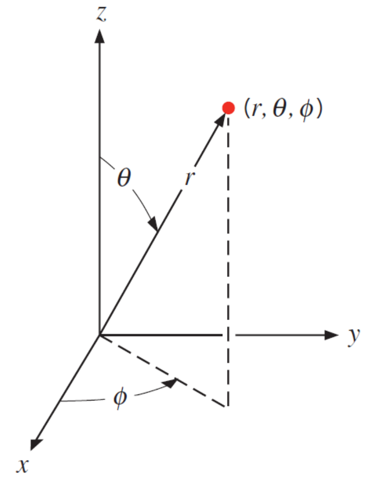
Directional Considerations and Radiation Intensity

- In general, radiation fluxes can be determined only from knowledge of the directional and spectral nature of the radiation.

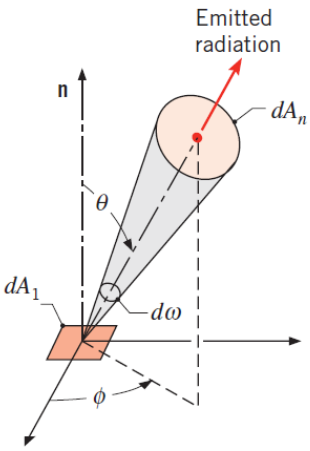
- Radiation emitted by a surface will be in all directions associated with a hypothetical hemisphere about the surface and is characterized by a directional distribution.



- Direction may be represented in a spherical coordinate system characterized by the zenith or polar angle θ and the azimuthal angle ϕ.

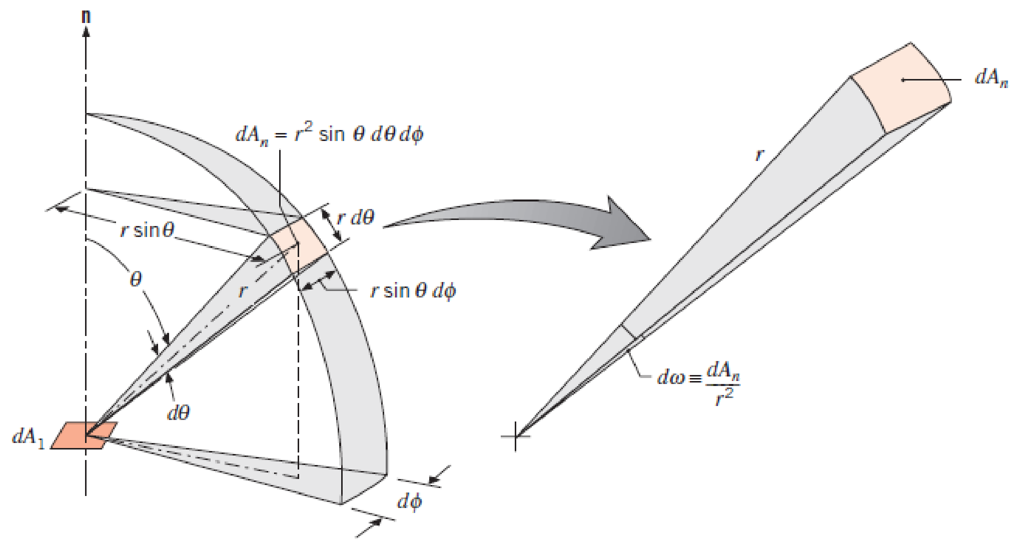


- The amount of radiation emitted from a surface, dA_1, and propagating in a particular direction, θ, ϕ, is quantified in terms of a differential solid angle associated with the direction.



$$d\omega \equiv \frac{dA_n}{r^2}$$

$dA_n \rightarrow$ unit element of surface on a hypothetical sphere and normal to the θ, ϕ direction.



$$dA_n = r^2 \sin\theta d\theta d\phi$$

$$d\omega = \frac{dA_n}{r^2} = \sin\theta d\theta d\phi$$

– The solid angle ω has units of steradians (sr).

– The solid angle associated with a complete hemisphere is

$$\omega_{\text{hemi}} = \int_0^{2\pi}\int_0^{\pi/2} \sin\theta d\theta d\phi = 2\pi \text{ sr}$$

- Spectral Intensity: A quantity used to specify the radiant heat flux $\left(\text{W/m}^2\right)$ within a unit solid angle about a prescribed direction $\left(\text{W/m}^2 \cdot \text{sr}\right)$ and within a unit wavelength interval about a prescribed wavelength $\left(\text{W/m}^2 \cdot \text{sr} \cdot \mu\text{m}\right)$.

- The spectral intensity $I_{\lambda,e}$ associated with emission from a surface element dA_1 in the solid angle $d\omega$ about θ,ϕ and the wavelength interval $d\lambda$ about λ is defined as:

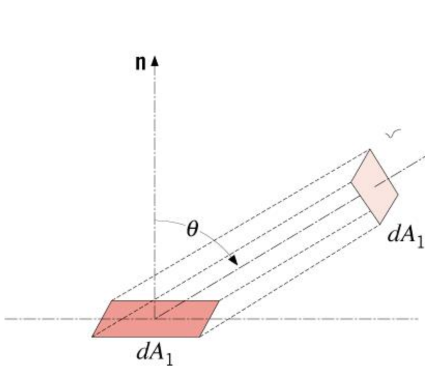
$$I_{\lambda,e}(\lambda,\theta,\phi) \equiv \frac{dq}{(dA_1\cos\theta)\cdot d\omega\cdot d\lambda}$$

- The rationale for defining the radiation flux in terms of the projected surface area $(dA_1\cos\theta)$ stems from the existence of surfaces for which, to a good approximation, $I_{\lambda,e}$ is independent of direction. Such surfaces are termed diffuse, and the radiation is said to be isotropic.

  - The projected area is how dA_1 would appear if observed along θ,ϕ .

    – What is the projected area for $\theta = 0$?

    – What is the projected area for $\theta = \pi/2$?



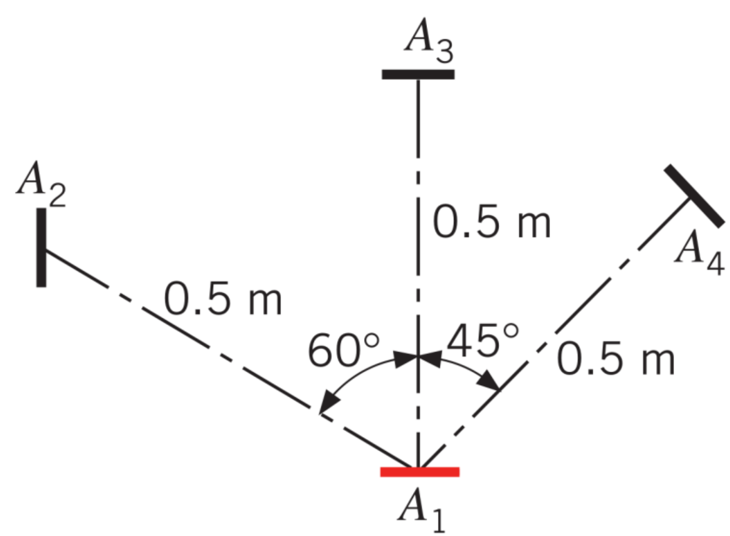
- The spectral heat rate and heat flux associated with emission from dA_1 are, respectively,

$$dq_\lambda \equiv \frac{dq}{d\lambda} = I_{\lambda,e}(\lambda,\theta,\phi)\,dA_1\cos\theta\,d\omega$$

$$dq''_\lambda = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\,d\omega = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta\,d\theta\,d\phi$$

Example 1

A small surface of area $A_1 = 10^{-3} m^2$ is known to emit diffusely, and from measurements the total intensity associated with emission in the normal direction is $I_n = 7000 \ \frac{W}{m^2 sr}$. Radiation emitted from the surface is intercepted by three other surfaces of area $A_2 = A_3 = A_4 = 10^{-3} m^2$, which are 0.5$m$ from A_1 and are oriented as shown. What is the intensity associated with emission in each of the three directions? What are the solid angles subtended by the three surfaces when viewed from A_1? What is the rate at which radiation emitted by A_1 is intercepted by the three surfaces?



Relation of Intensity to Emissive Power, Irradiation, and Radiosity

- The spectral emissive power $\left(\mathrm{W/m^2 \cdot \mu m}\right)$ corresponds to spectral emission overall possible directions.

$$E_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi) \cos\theta \sin\theta \, d\theta \, d\phi$$

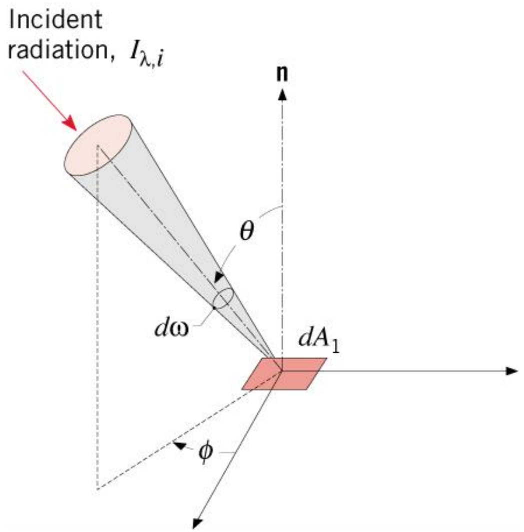
- The total emissive power $\left(\mathrm{W/m^2}\right)$ corresponds to emission over all directions and wavelengths.

$$E = \int_0^\infty E_\lambda(\lambda) \, d\lambda$$

- For a diffuse surface, emission is isotropic and

$$E_\lambda(\lambda) = \pi I_{\lambda,e}(\lambda) \qquad\qquad E = \pi I_e$$

- The spectral intensity of radiation incident on a surface, $I_{\lambda,i}$, is defined in terms of the unit solid angle about the direction of incidence, the wavelength interval $d\lambda$ about λ, and the projected area of the receiving surface, $dA_1 \cos\theta$.



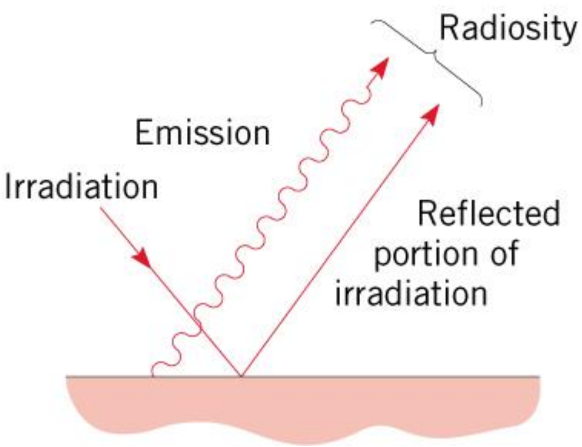
- The spectral irradiation $\left(\mathrm{W/m^2 \cdot \mu m}\right)$ is then:

$$G_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) \cos\theta \sin\theta d\theta d\phi$$

and the total irradiation $\left(\mathrm{W/m^2}\right)$ is

$$G = \int_0^\infty G_\lambda(\lambda) d\lambda$$

- The radiosity of an opaque surface accounts for all of the radiation leaving the surface in all directions and may include contributions from both reflection and emission.



- With $I_{\lambda,e+r}$ designating the spectral intensity associated with radiation emitted by the surface and the reflection of incident radiation, the spectral radiosity $\left(\mathrm{W/m^2 \cdot \mu m}\right)$ is:

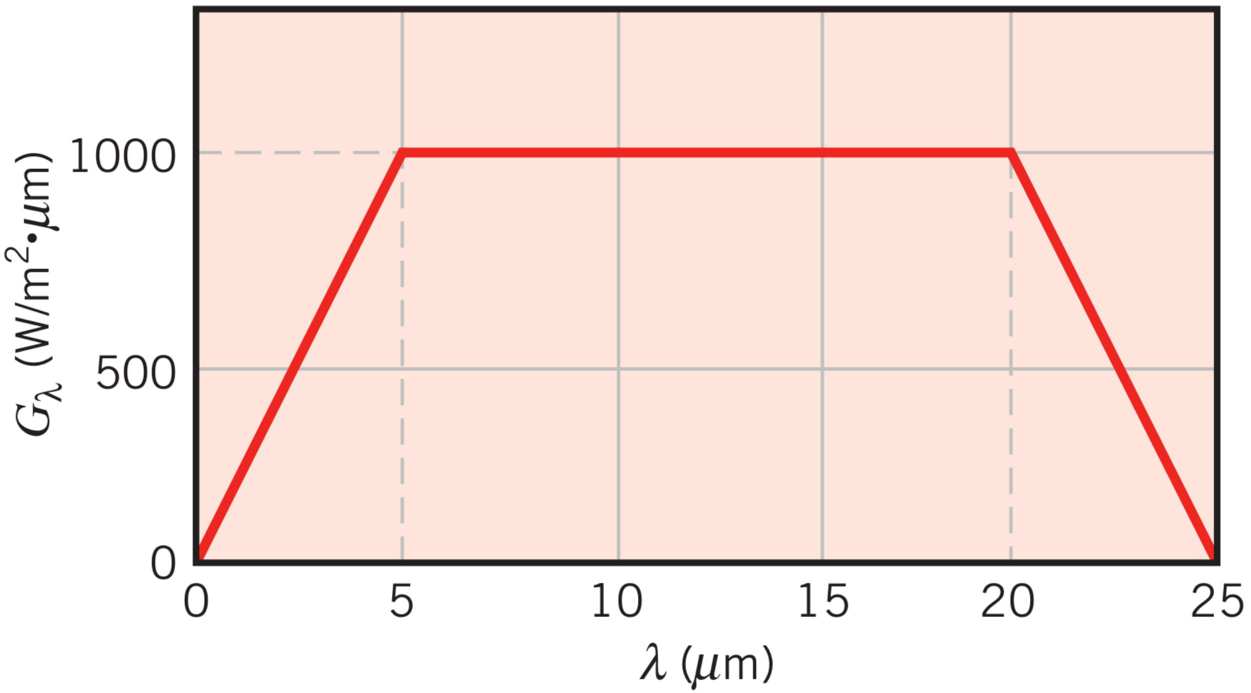
$$J_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

and the total radiosity $\left(\mathrm{W/m^2}\right)$ is

$$J = \int_0^\infty J_\lambda(\lambda) d\lambda$$

Example 2

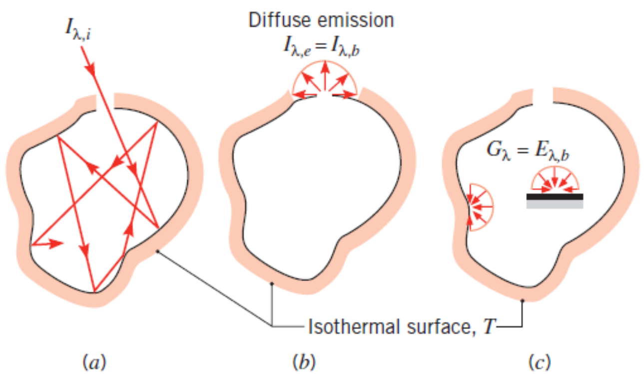
The spectral distribution of surface irradiation is shown below. What is the total irradiation?



Blackbody Radiation and Its Intensity

- The Blackbody
  - An idealization providing limits on radiation emission and absorption by matter.
    - For a prescribed temperature and wavelength, no surface can emit more radiation than a blackbody: the ideal emitter.
    - A blackbody is a diffuse emitter.
    - A blackbody absorbs all incident radiation: the ideal absorber.

- The Isothermal Cavity



(a) After multiple reflections, virtually all radiation entering the cavity is absorbed.

(b) Emission from the aperture is the maximum possible emission achievable for the temperature associated with the cavity and is diffuse.

(c) The cumulative effect of radiation emission from and reflection off the cavity wall is to provide diffuse irradiation corresponding to emission from a blackbody $\left(G_\lambda = E_{\lambda,b}\right)$ for any surface in the cavity.

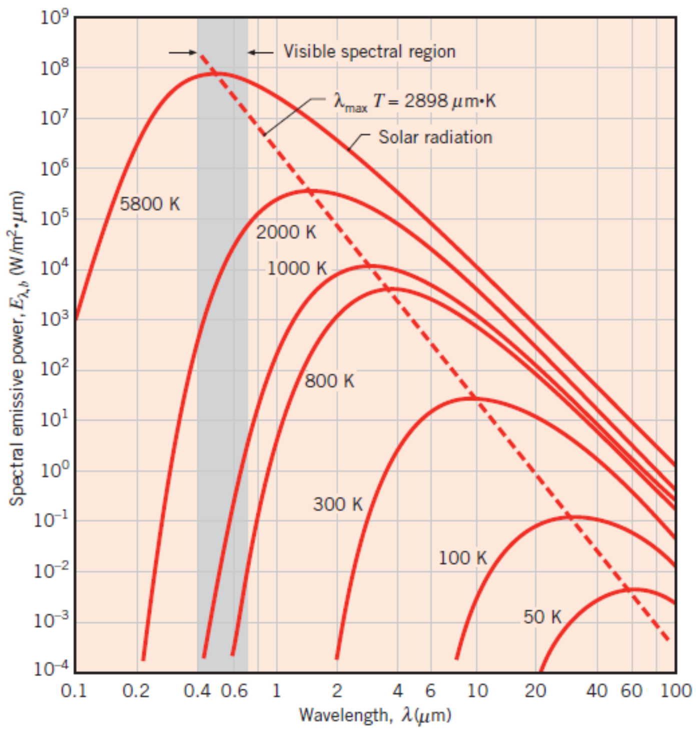
The Spectral (Planck) Distribution of Blackbody Radiation

- The spectral distribution of the blackbody emissive power (determined theoretically and confirmed experimentally) is

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]}$$

First radiation constant: $C_1 = 3.742 \times 10^8\,\mathrm{W}\cdot\mu\mathrm{m}^4/\mathrm{m}^2$

Second radiation constant: $C_2 = 1.439 \times 10^4\,\mu\mathrm{m}\cdot\mathrm{K}$



- $E_{\lambda,b}$ (and $I_{\lambda,b}$) varies continuously with λ and increases with T.
- The distribution is characterized by a maximum for which $\lambda_{\max}$ is given by Wien's displacement law:

$$\lambda_{\max} T = C_3 = 2898\ \mu\text{m} \cdot \text{K}$$

- The fractional amount of total blackbody emission appearing at lower wavelengths increases with increasing T.

The Stefan-Boltzmann Law and Band Emission

- The total emissive power of a blackbody is obtained by integrating the Planck distribution over all wavelengths.

$$E_b = \pi I_b = \int_0^\infty E_{\lambda,b} d\lambda = \sigma T^4$$

→ the Stefan-Boltzmann law, where

$$\sigma = 5.670 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4} \rightarrow \text{ the Stefan-Boltzmann constant}$$

- The fraction of total blackbody emission that is in a prescribed wavelength interval or band $(\lambda_1 < \lambda < \lambda_2)$ is

$$F_{(\lambda_1 - \lambda_2)} = F_{(0-\lambda_2)} - F_{(0-\lambda_1)} = \frac{\int_0^{\lambda_2} E_{\lambda,b} d\lambda - \int_o^{\lambda_1} E_{\lambda,b} d\lambda}{\sigma T^4}$$

where, in general,

$$F_{(0-\lambda)} = \frac{\int_0^{\lambda} E_{\lambda,b} d\lambda}{\sigma T} = f(\lambda T)$$

and numerical results are given in Table 13.1.

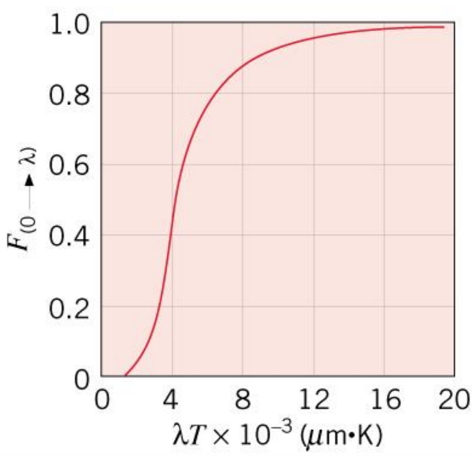


Table 13.1

Blackbody Radiation Functions

λT (μm · K)	$F_{(0\rightarrow\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ $(\mu\mathrm{m}\cdot\mathrm{K}\cdot\mathrm{sr})^{-1}$	$\dfrac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
⋮	⋮	⋮	⋮

Note ability to readily determine $I_{\lambda,b}$ and its relation to the maximum intensity from the 3rd and 4th columns, respectively.

- If emission from the sun may be approximated as that from a blackbody at 5800 K, at what wavelength does peak emission occur?
- Would you expect radiation emitted by a blackbody at 800 K to be discernible by the naked eye?
- As the temperature of a blackbody is increased, what color would be the first to be discerned by the naked eye?

Example 3

Consider a large isothermal enclosure that is maintained at a uniform temperature of 2000K. Calculate the emissive power of the radiation that emerges from a small aperture on the enclosure surface. What is the wavelength λ_1 below which 10% of the emission is concentrated? What is the wavelength λ_2 above which 10% of the emission is concentrated? Determine the maximum spectral emissive power and the wavelength at which this emission occurs. What is the irradiation incident on a small object placed inside the enclosure?