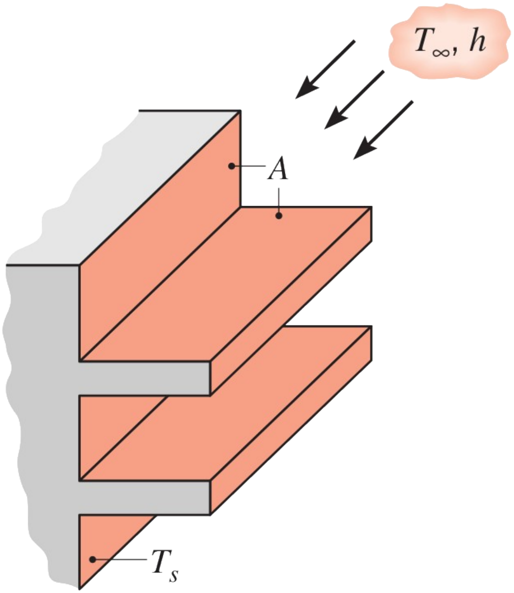
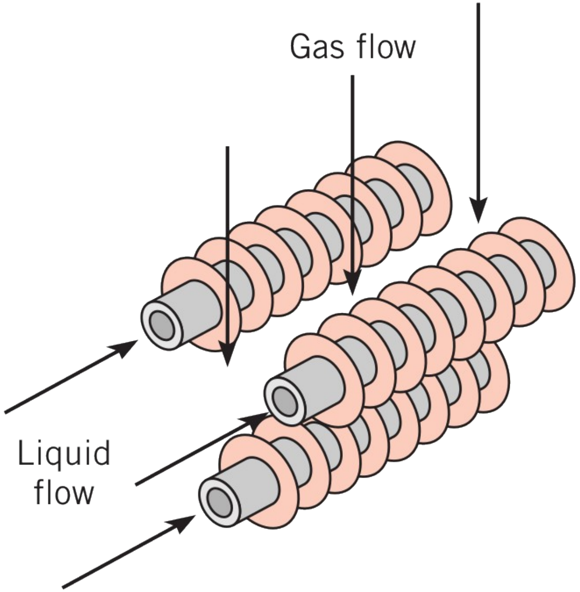
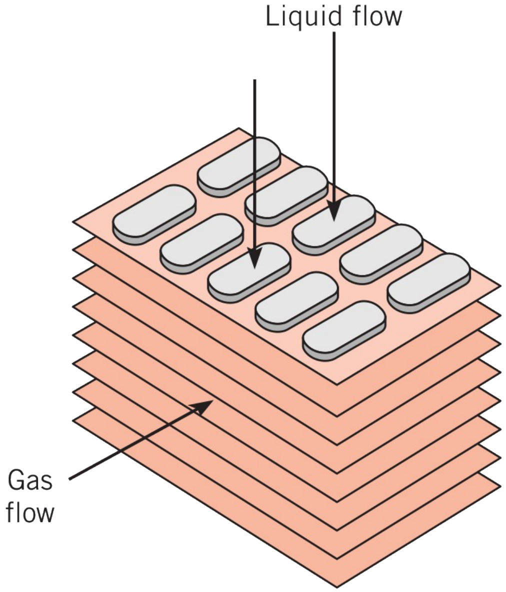
Extended Surfaces

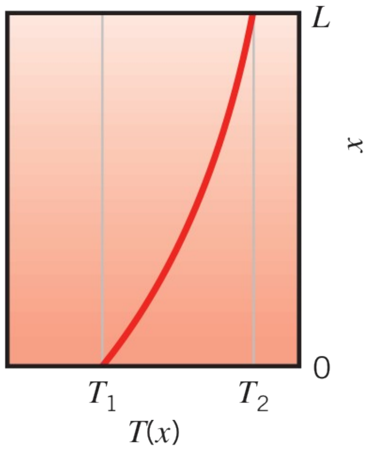


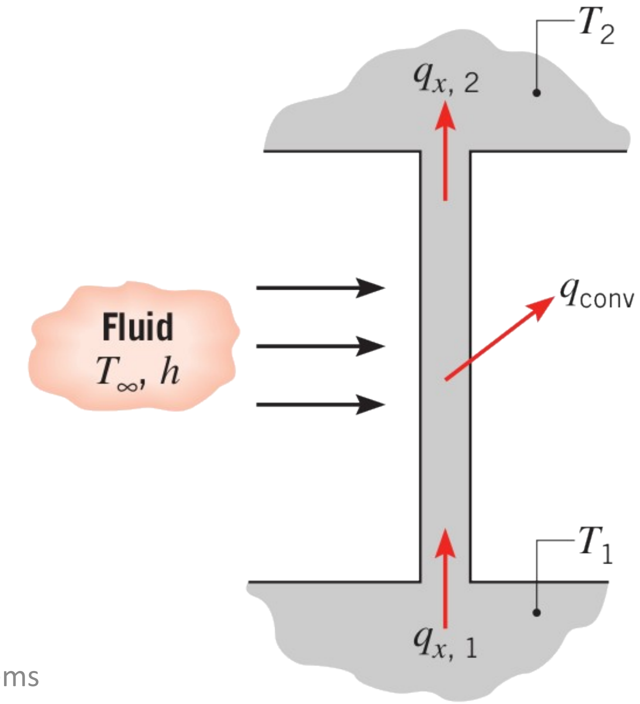




Extended Surfaces

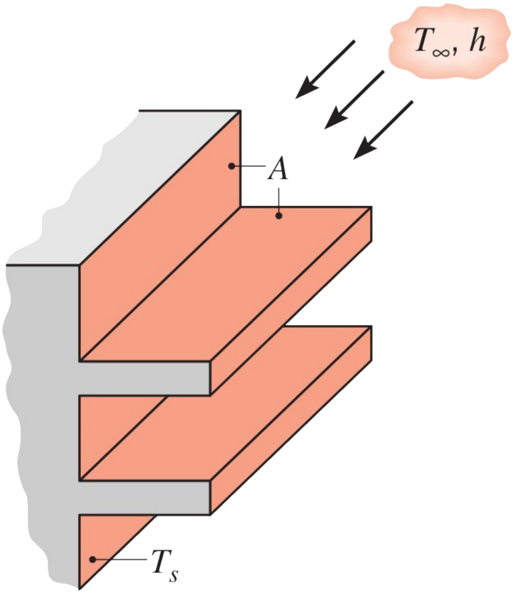
- Until now, the heat transfer from solid boundaries have been in the same direction as the heat transfer in the solid (by conduction).
- For an extended surface, the direction from the solid boundaries are perpendicular to the direction of the heat transfer in the solid.
- Consider a strut:
  - Connects two walls at different temperature.
  - Fluid flows across strut.
  - $T_1 > T_2 > T_\infty$

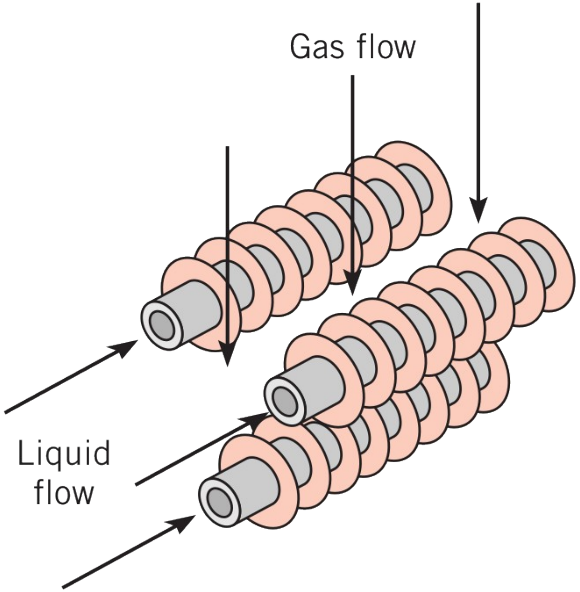


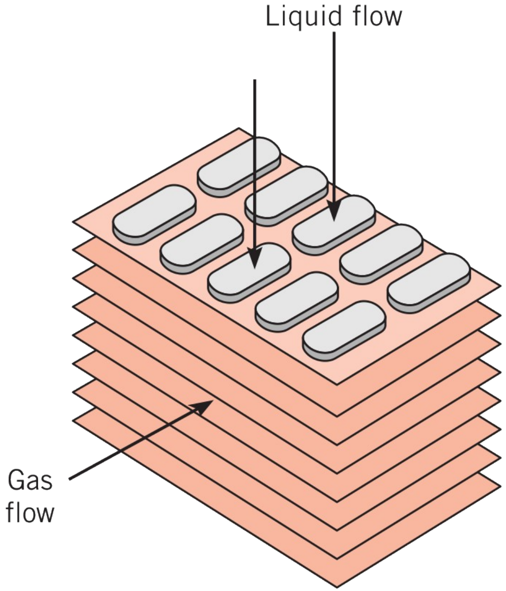


Extended Surfaces

- Main purpose is to enhance heat transfer between solid and fluid.
- An extended surface is known as a n.







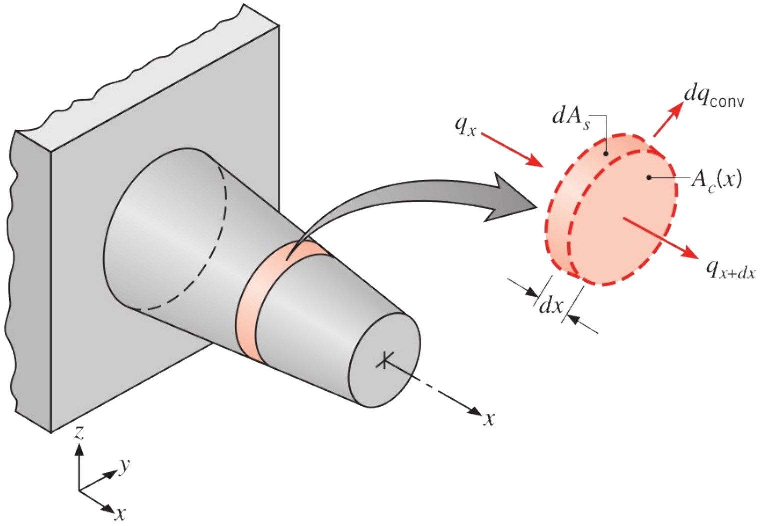
Extended Surfaces

- A 1st Law look at an extended surface shows

$$\frac{d^2T}{dx^2}+\left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx}-\left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T-T_\infty)=0$$

where A_c is the cross sectional area,

A_s is the surface area.



- If the fin has a uniform cross section
  - A_c is constant
  - $A_s = Px$
  - P is the fin perimeter

$$\frac{d^2T}{dx^2}-\left(\frac{hP}{kA_c}\right)(T-T_\infty)=0$$

- Often times this may be written as

$$\frac{d^2\theta}{dx^2}-m^2\theta=0$$

where $\theta \equiv T(x) - T_\infty$ and $m^2 \equiv hP/kA_c$

Extended Surfaces

- Consider the two fins shown to the right.
- The rectangular fin has

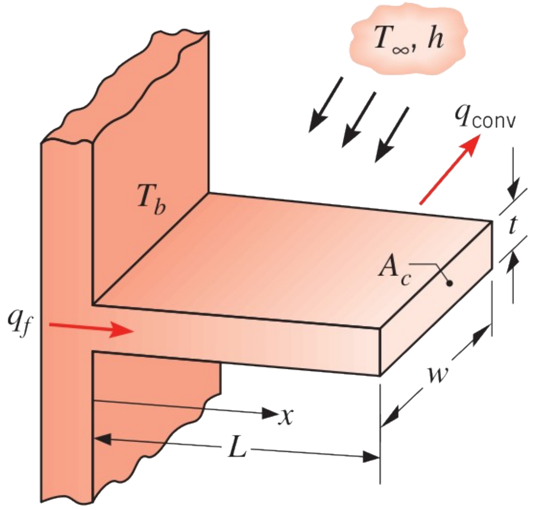
$$P = 2w + 2t$$

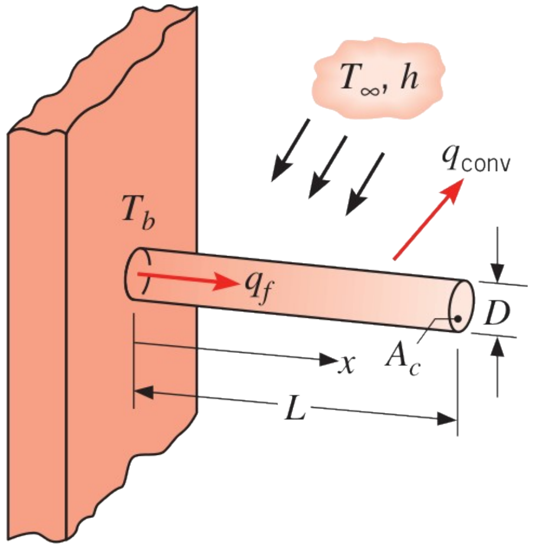
$$A_c = wt$$

- The cylindrical fin has

$$P = \pi D$$

$$A_c = \frac{\pi}{4} D^2$$





Extended Surfaces

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

- The solution to governing differential equation is

  $$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

  where C_1 and C_2 are constants determined by boundary conditions.

- 1st Boundary Condition: Constant temperature at base, T_b

  $$\theta(0) = T_b - T_\infty \equiv \theta_b$$

- 2nd Boundary Condition: At tip
  - Convection
  - Adiabatic
  - Constant Temperature
  - Infinite Long Fin

Tip Conditions

Case	Tip Condition	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection	$\dfrac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$	$M\dfrac{\sinh mL+(h/mk)\cosh mL}{\cosh mL+(h/mk)\sinh mL}$
B	Adiabatic	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M\tanh mL$
C	Constant Temperature	$\dfrac{(\theta_L/\theta_b)\sinh mx+\sinh m(L-x)}{\sinh mL}$	$M\dfrac{\cosh mL-\theta_L/\theta_b}{\sinh mL}$
D	Infinite Fin $(L \rightarrow \theta)$	e^{-mx}	M

$\theta(x)=T(x)-T_\infty$ $\qquad \theta_b=\theta(0)=T_b-T_\infty$ $\qquad m^2=\dfrac{hP}{kA_c}$ $\qquad M=\sqrt{hPkA_c}\,\theta_b$

Example 1

A very long rod $5\ mm$ in diameter has one end maintained at $100\ °C$. The surface of the rod is exposed to ambient air at $25\ °C$ with a convection heat transfer coefficient of $100\ W/m^2K$.

(a) Determine the temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?

(b) Estimate how long the rods must be for the assumption of infinite length to yield an accurate estimate of the heat loss.

Fin Performance

- A fin does not guarantee an increase of heat transfer.
- A fin adds a conduction resistance and needs to be less than the gains of the increased convection effects.
- Fin performance may be defined in terms of effectiveness.
- The effectiveness ε_f of a single n fin may be defined as

  $$\varepsilon_f = \frac{q_f}{h A_{c,b} \theta_b}$$

  where $A_{c,b}$ is the fin cross-sectional area at the base.
- Effectiveness is the ratio of the actual heat transfer over heat transfer without fin.
- Generally, fins are rarely justified unless $\varepsilon_f > 2$.

Fin Effectiveness

- For case D (infinite long fin), the effectiveness is
  $$\varepsilon_f = \sqrt{\frac{kP}{h A_c}}$$
- This illustrates that fin performance is due to:
  - Material properties
  - Geometry
- For geometry, the ratio of perimeter to cross sectional area is important.
- For this reason, thin but closely spaced fins is typically preferred.
- However, too closely spaced fins may start to impact convection.

Fin Effectiveness

- The effectiveness of a fin may also be expressed in terms of thermal resistances.

- The thermal resistance of a fin is

$$R_{t,f} = \frac{\theta_b}{q_f}$$

- The thermal resistance of the convection if the fin was not present is

$$R_{t,b} = \frac{1}{h\,A_{c,b}}$$

- The effectiveness in terms of thermal resistance is then

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}$$

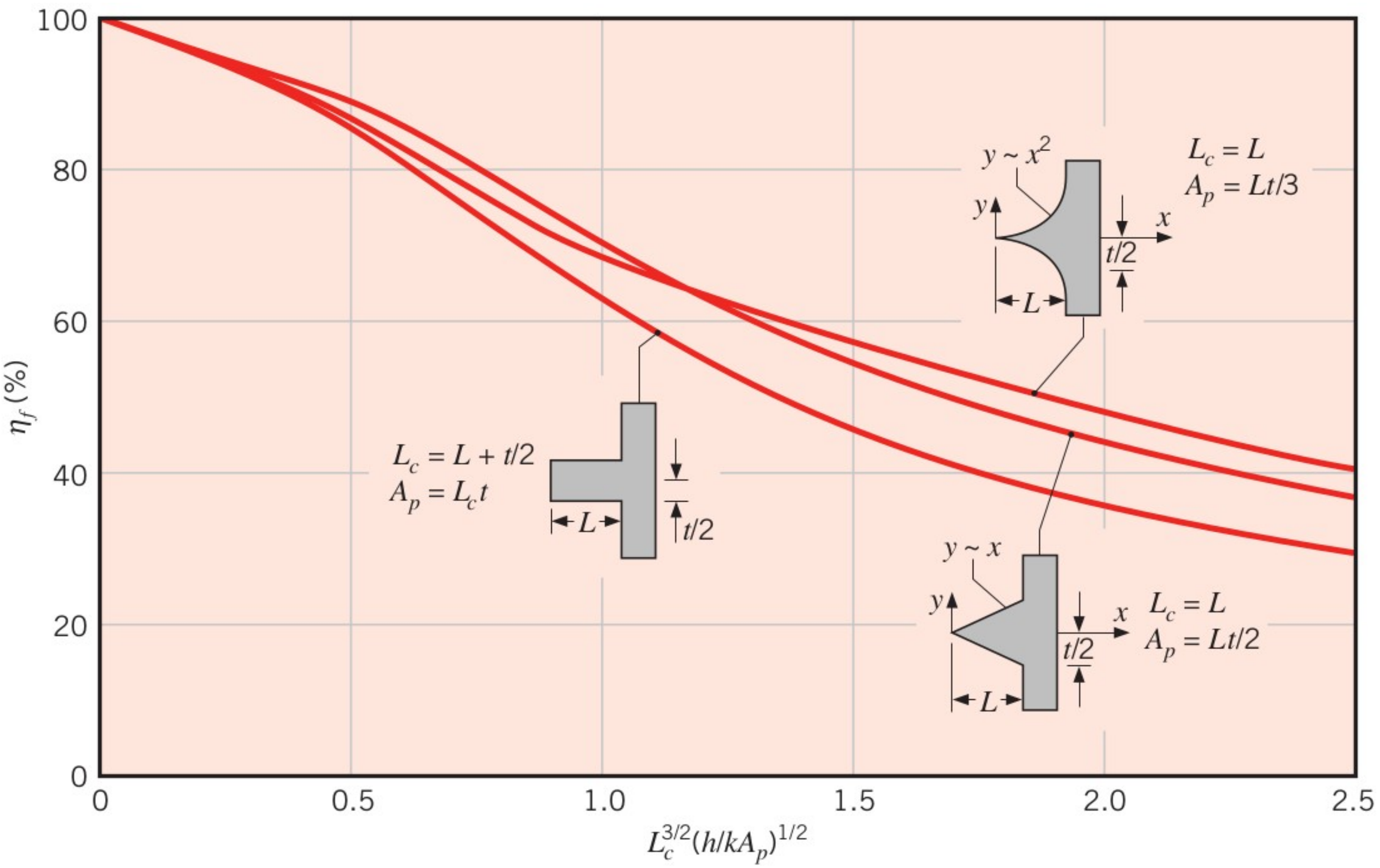
Fin Efficiency

- Another measure of fin performance is fin efficiency.
- Fin efficiency is defined in terms of the maximum heat transfer possible.
- The maximum heat transfer possible would occur if the entire fin, not just the base, was at the base temperature.
- This is written as

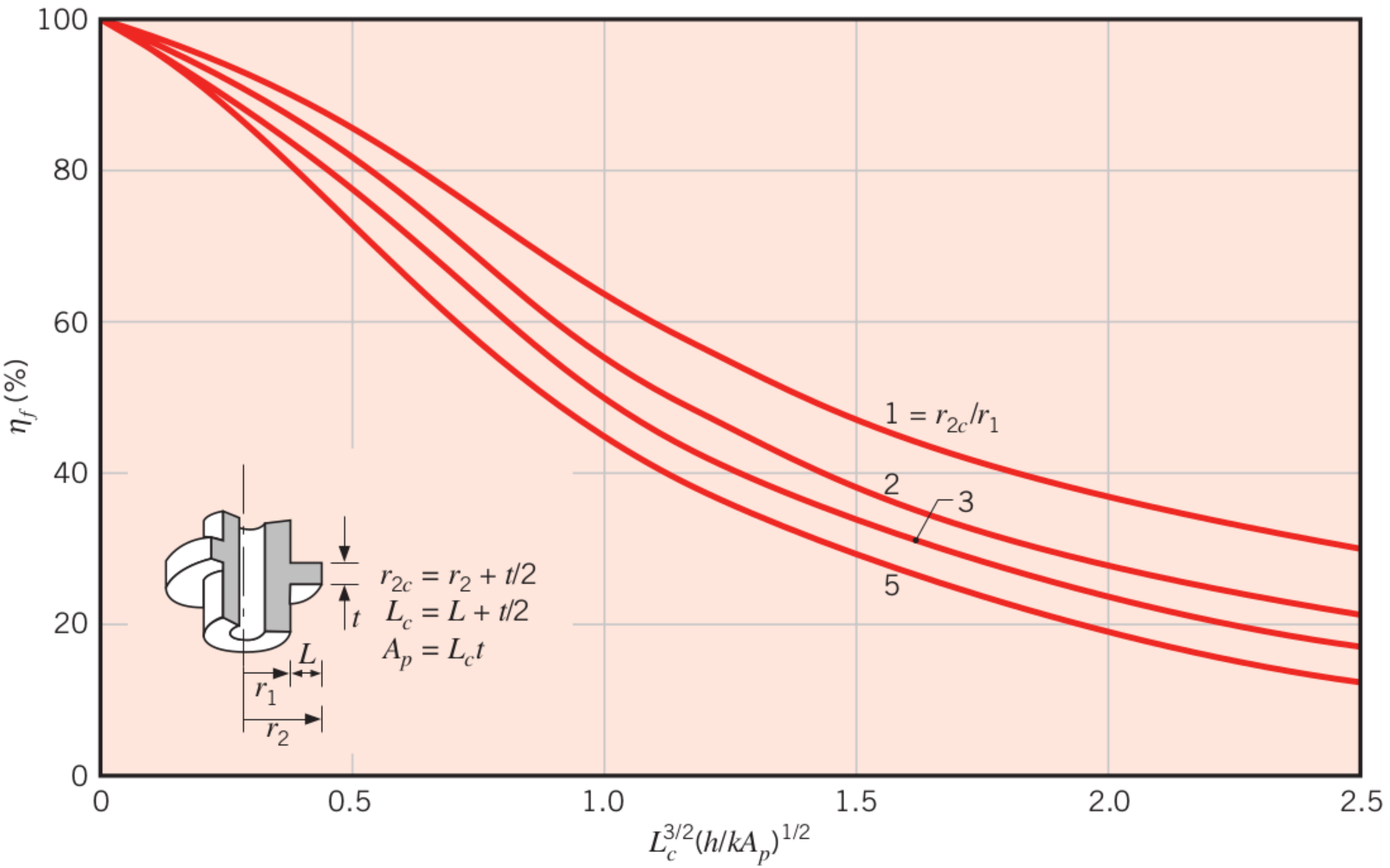
$$\eta_f \equiv \frac{q_f}{q_{max}} = \frac{q_f}{h A_f \theta_b}$$

where A_f is the surface area of the fin.

Fin Efficiency



Fin Efficiency



Fin Efficiency

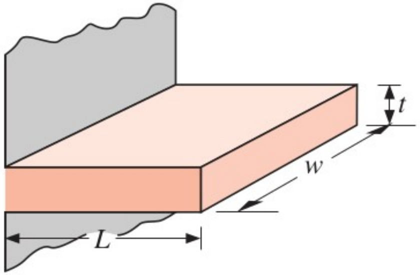
Straight Fins

Rectangular

$A_f = 2wL_c$

$L_c = L + (t/2)$

$A_p = tL$

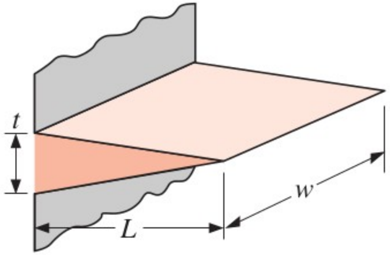


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$A_f = 2w[L^2 + (t/2)^2]^{1/2}$

$A_p = (t/2)L$



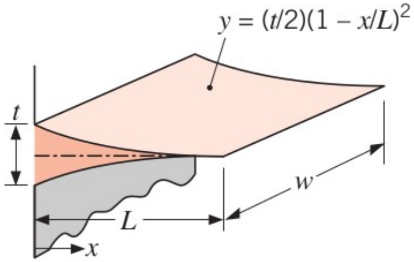
$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$

Parabolic

$A_f = w[C_1L +$
$\quad (L^2/t)\ln(t/L + C_1)]$

$C_1 = [1 + (t/L)^2]^{1/2}$

$A_p = (t/3)L$



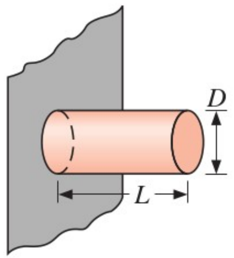
$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

Fin Efficiency

Pin Fins

Rectangular

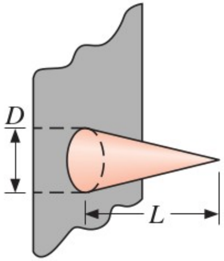
$A_f = \pi D L_c$
$L_c = L + (D/4)$
$V = (\pi D^2/4)L$



$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular

$$A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$$
$V = (\pi/12)D^2L$

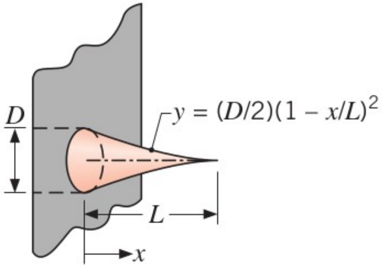


$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$

Parabolic

$$A_f = \frac{\pi L^3}{8D}\{C_3C_4 - \frac{L}{2D}\ln[(2DC_4/L) + C_3]\}$$

$C_3 = 1 + 2(D/L)^2$
$C_4 = [1 + (D/L)^2]^{1/2}$
$V = (\pi/20)D^2 L$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$$

Overall Surface Efficiency

- η_f denotes the efficiency of a single fin.
- An arrangement of fins may be described by the overall surface efficiency

  $$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{h A_t \theta_b}$$

  where q_t is the total heat rate from the surface A_t associated with both the fin and the exposed portion at the base.
- The exposed portion is known as the prime surface, A_b.
- If there are N fins in an array, then the total area is

  $$A_t = N A_f + A_b$$
- The total rate of heat transfer is

  $$q_t = N \eta_f h A_f \theta_b + h A_b \theta_b$$

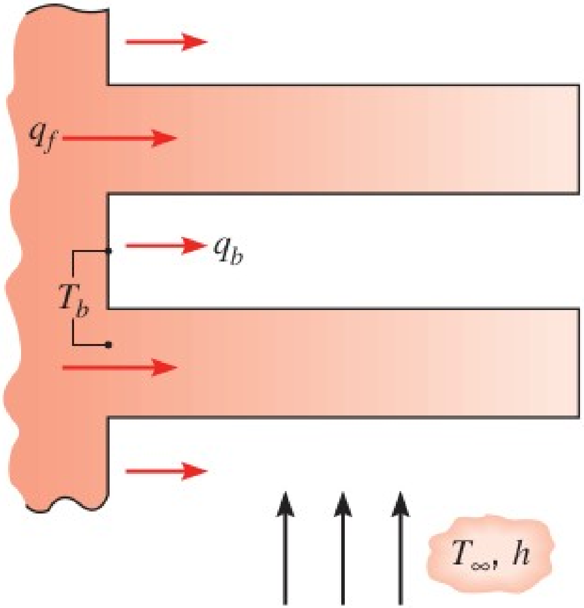
Overall Surface Efficiency

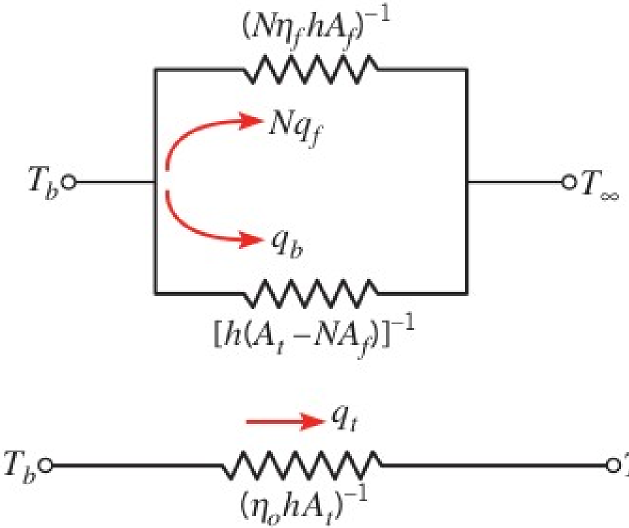
- After some manipulation, the overall surface efficiency becomes

$$\eta_o = 1 - \frac{N A_f}{A_t}(1 - \eta_f)$$

- It may also be related to thermal resistance of the fin array.

$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$$





Example 2

The engine cylinder of a motorcycle is constructed of 2024-T6 aluminum alloy is of height $H=0.15\ m$ and outside diameter $D=50\ mm$. Under typical operating conditions the outer surface of the cylinder is at a temperature of $500\ K$ and is exposed to ambient air at $300\ K$, with a convection coefficient of $50\ W/m^2K$. Annular fins are integrally cast with the cylinder to increase heat transfer to the surroundings. Consider five such fins, which are of thickness $t=6\ mm$, length $L=20\ mm$, and equally spaced. What is the increase in heat transfer due to use of the fins?

Example 3, part 1

To generate a power of $P = 9\ W$, the temperature of a fuel cell must be maintained at $T_c = 56.4\ °C$, which requires removal of $11.25\ W$ from the fuel cell and a cooling air velocity of $V = 9.4\ m/s$ for $T_\infty = 25\ °C$. To provide these convective conditions, the fuel cell is centered in a $50\ mm \times 26\ mm$ rectangular duct, with $10\ mm$ gaps between the exterior of the $50\ mm \times 50\ mm \times 6\ mm$ fuel cell and the top and bottom of the well insulated duct wall. A small fan, powered by the fuel cell, is used to circulate the cooling air. Inspection of a particular fan vendor's data sheets suggests that the ratio of the fan power consumption to the fan's volumetric flow rate is $P_f/V_f = C = 1000\ W/(m^3/s)$ for the range of $10^{-4} \leq V_f \leq 10^{-2}\ m^3/s$. Determine the net electric power produced by the fuel cell fan system, $P_{net} = P - P_f$.

Example 3, part 2

Consider the effect of attaching an aluminum ($k = 200\ W/mK$) finned heat sink, of identical top and bottom sections, onto the fuel cell body. The base of the heat sink is of thickness $t_b = 2\ mm$. Each of the N rectangular fins is of length $L_f = 8\ mm$ and thickness $t_f = 1\ mm$, and spans the entire length of the fuel cell, $L_c = 50\ mm$. With the heat sink in place, radiation losses are negligible and the convective heat transfer coefficient may be related to the size and geometry of a typical air channel by an expression of the form $h = 1.78\ k_{air}\ (L_f + a)/(L_f a)$, where a is the distance between fins. Draw an equivalent thermal circuit for part 2 and determine the total number fins needed to reduce the fan power consumption to half of the value found in part 1.