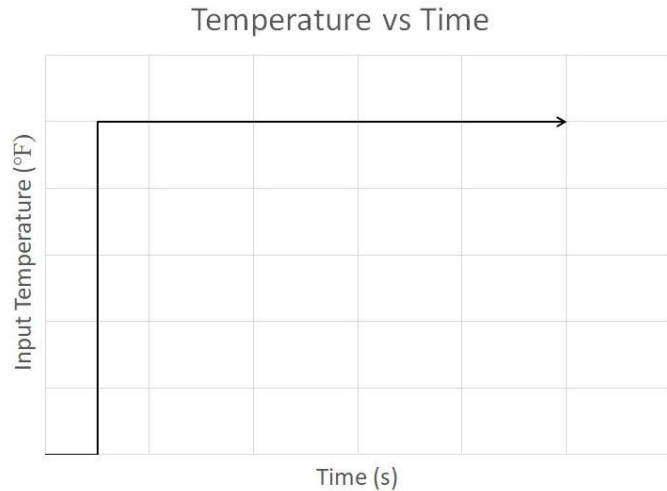


# Dynamic Measurements

# Dynamic Measurements

It often takes a measurement system time to reach equilibrium, even for static measurands (like when taking your temperature).



The situation is further complicated when the input temperature changes with time.

# Dynamic Measurements

Heat transfer and energy absorption by thermometer bulb

$$q = hA(T_{mouth} - T_{bulb}) = mc \frac{dT}{dt}$$

where

- A = bulb surface area
- c = heat capacity of bulb
- h = heat transfer coefficient
- m = mass of bulb

Rearranging,

$$T_{mouth} - T_{bulb} = \frac{mc}{hA} \frac{dT}{dt}$$

**Time constant** - As the mass of the bulb decreases, the response time decreases



# Dynamic Measurements

$$\tau = \text{time constant} = \frac{mc}{hA}$$

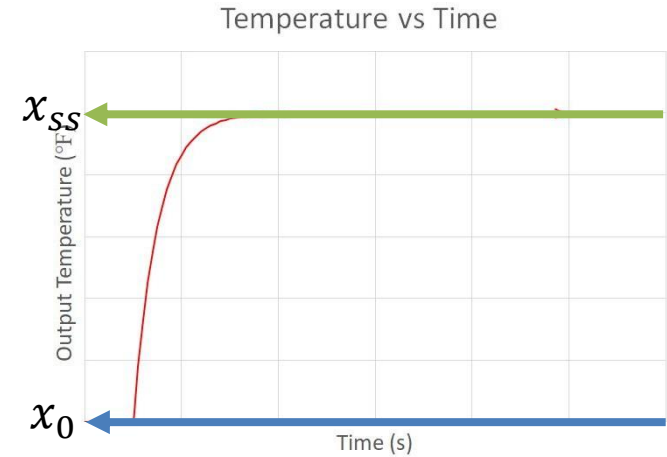
The thermometer is an example of a 1st order system. These systems have energy storage effects.

The solution to a 1st order system is

$$\frac{x(t) - x_{ss}}{x_0 - x_{ss}} = e^{\frac{-t}{\tau}}$$

where

- $x$  is the response (in the thermometer example it is temperature)
- $x_0$  and  $x_{ss}$  are the initial and steady state value of the system
- $t$  is time
- $\tau$  is the time constant



# Example

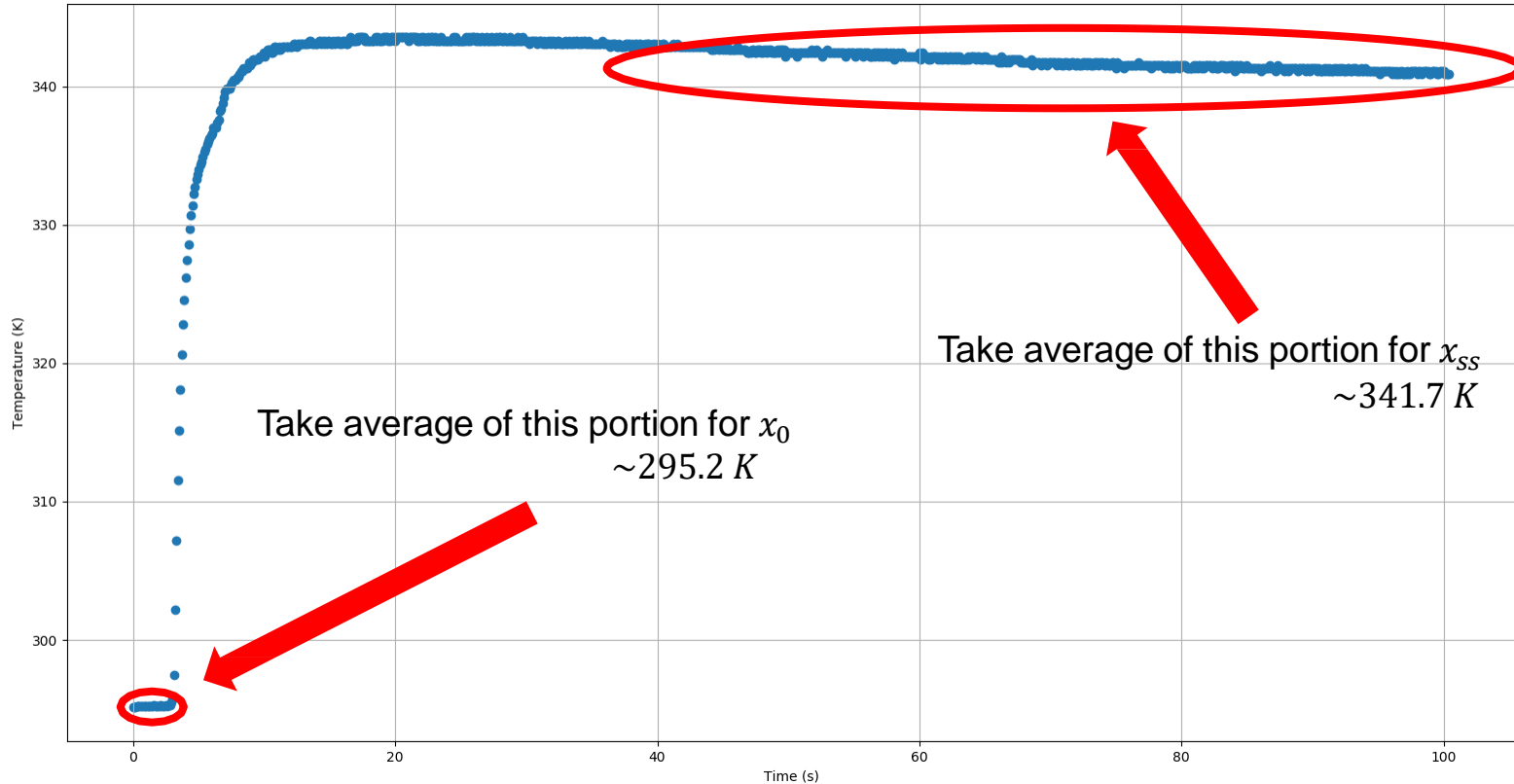
Thermistor data [here](#)

ADC equation:  $V_2 = aR \frac{5V}{1023}$

Voltage divider equation:  $R_t = R_2 \left( \frac{V_s}{V_2} - 1 \right)$

Manufacturer's Temperature equation:  $T = \frac{B}{\ln \left( \frac{R_t}{R_1} e^{\frac{B}{T_1}} \right)}$

## Example (contd.)



## Example (contd.)

Special point in 1st order response equation.  $\frac{x(t) - x_{ss}}{x_0 - x_{ss}} = e^{\frac{-t}{\tau}}$

when  $t = \tau$

$$\text{then } \frac{x(\tau) - x_{ss}}{x_0 - x_{ss}} = e^{-1} \approx 0.3679$$

This is useful!

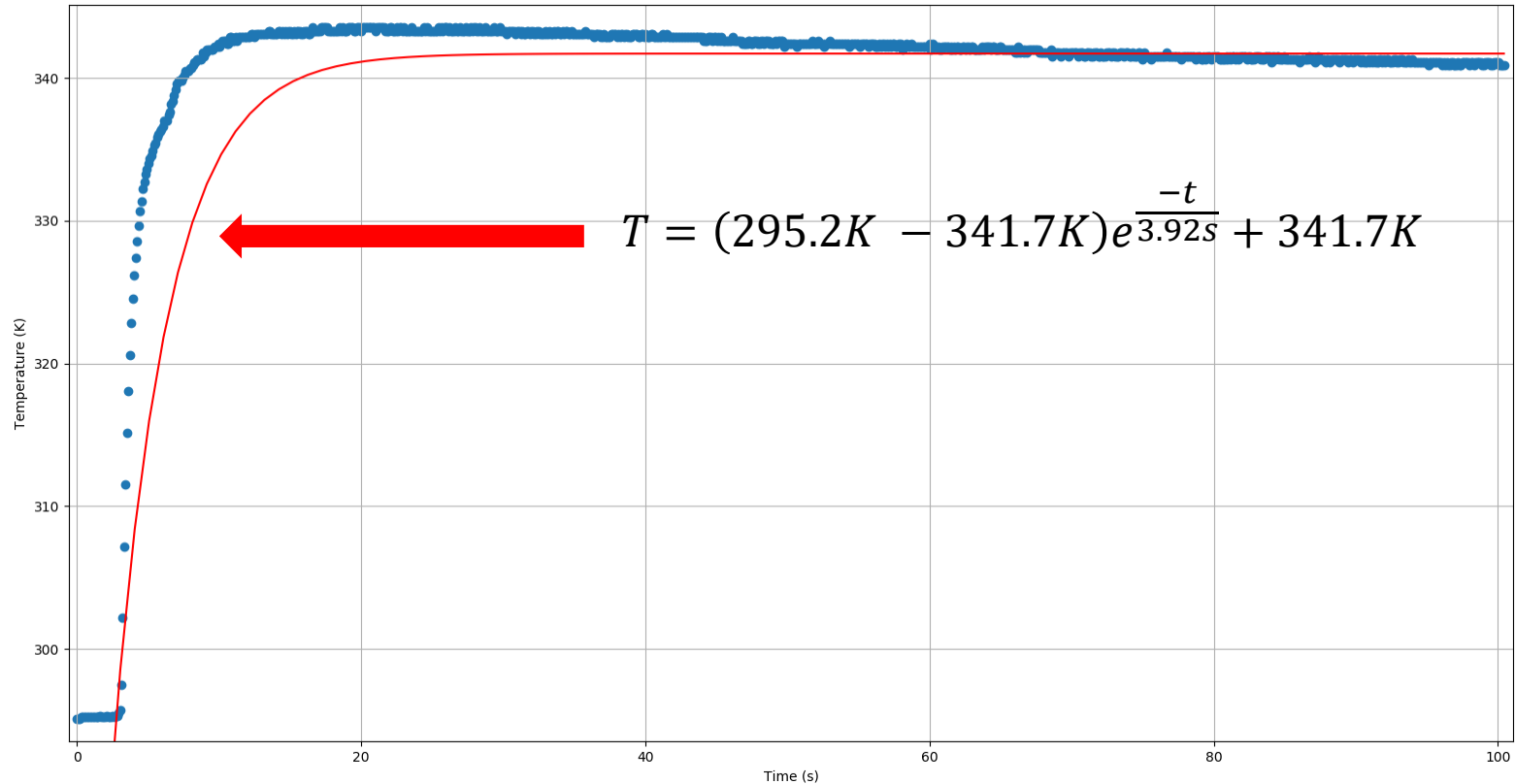
$$x(\tau) \approx 0.3679(x_0 - x_{ss}) + x_{ss}$$

$$x(\tau) \approx 0.3679(295.2K - 341.7K) + 341.7K$$

$$x(\tau) \approx 324.6K$$

Lookup  $T = 324.6K$  and find  $t$ . This give you  $\tau$ .  $\tau \approx 3.92s$

## Example (contd.)





# Dynamic Measurements

General differential equation form of 1st order system subject to a step input

$$\tau \dot{x}(t) + x(t) = x_{ss} \Phi(t)$$

where  $\Phi(t)$  is known as the [Heaviside function](#). It is a unit step function.

At  $t = 0$ ,

$$\tau \dot{x}(0) + x(0) = x_{ss} \Phi(0)$$

$$\tau \dot{x}(0) + x_0 = x_{ss}$$

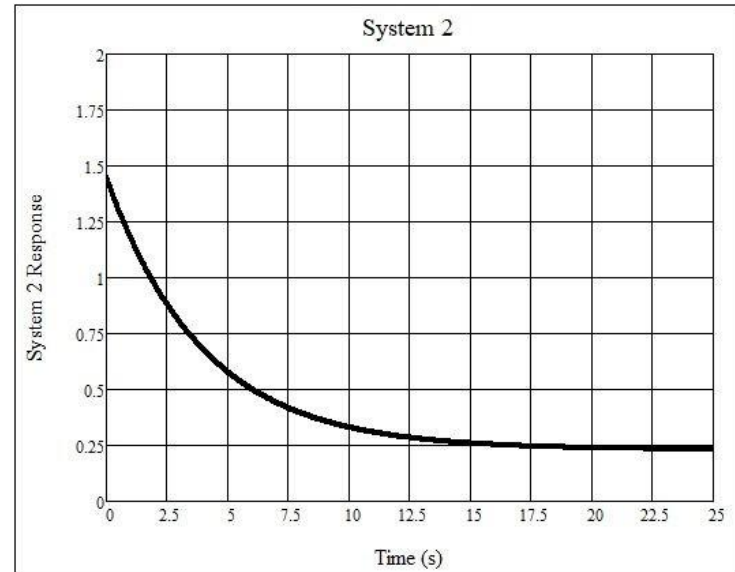
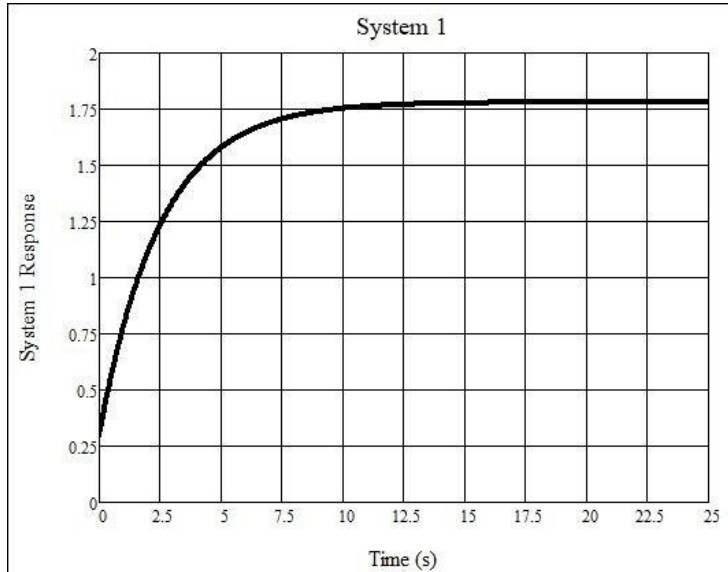
Solving for the time constant,

$$\tau = \frac{x_{ss} - x_0}{\dot{x}(0)} \quad \text{This is useful!}$$

- $x_{ss}$  - steady state value
- $x_0$  - initial value
- $\dot{x}(0)$  - slope at  $t = 0$

# Example

The 1st order responses of two different systems subject to a step input are shown below. Which has the higher time constant? What are the response equations for each system?



# System 1

From the plot

$$x_0 \approx 0.3$$

$$x_{ss} \approx 1.8$$

For 1st Order system

$$\frac{x(t) - x_{ss}}{x_0 - x_{ss}} = e^{\frac{-t}{\tau}}$$

When  $t = \tau$

$$x(\tau) = (x_0 - x_{ss})e^{-1} + x_{ss}$$

$$x(\tau) \approx (0.3 - 1.8)0.3679 + 1.8 = 1.25$$

On the plot where  $x(\tau) \approx 1.25$  then  $t \approx 2.5s$

So  $\tau_1 \approx 2.5s$  and  $x_1(t) \approx (0.3 - 1.8)e^{\frac{-t}{2.5s}} + 1.8$

# System 2

From the plot

$$x_0 \approx 1.4$$

$$x_{ss} \approx 0.25$$

$$\dot{x}(0) \approx \frac{x_2 - x_1}{t_2 - t_1} \approx \frac{0.5 - 1.4}{2.5 - 0} \approx -0.36$$

For 1st Order system

$$\tau_2 = \frac{x_{ss} - x_0}{\dot{x}(0)} \approx \frac{0.25 - 1.4}{-0.36} \approx 3.19s$$

$$x_2(t) \approx (1.4 - 0.25)e^{\frac{-t}{3.19s}} + 0.25$$

System 2 has a larger time constant.