



# SCALABLE DISTRIBUTED SUBGRAPH ENUMERATION

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# OUTLINE

PROBLEM DEFINITION

ALGORITHM FRAMEWORK

TWINTWIG JOIN - VLDB15'

SEED

EXPERIMENTS

CONCLUSION

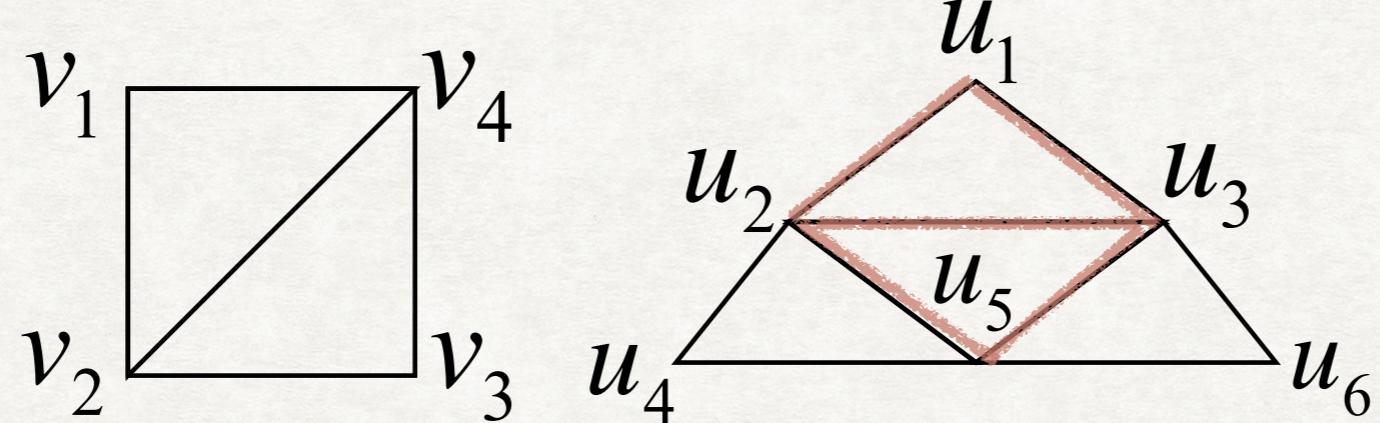
# PROBLEM

# PROBLEM DEFINITION

## SUBGRAPH ENUMERATION

- Given a data graph  $G$ , and a pattern graph  $P$ , subgraph enumeration aims to find all subgraphs  $g \subseteq G$  (matches), that are isomorphic to  $P$ .

- 



$P$

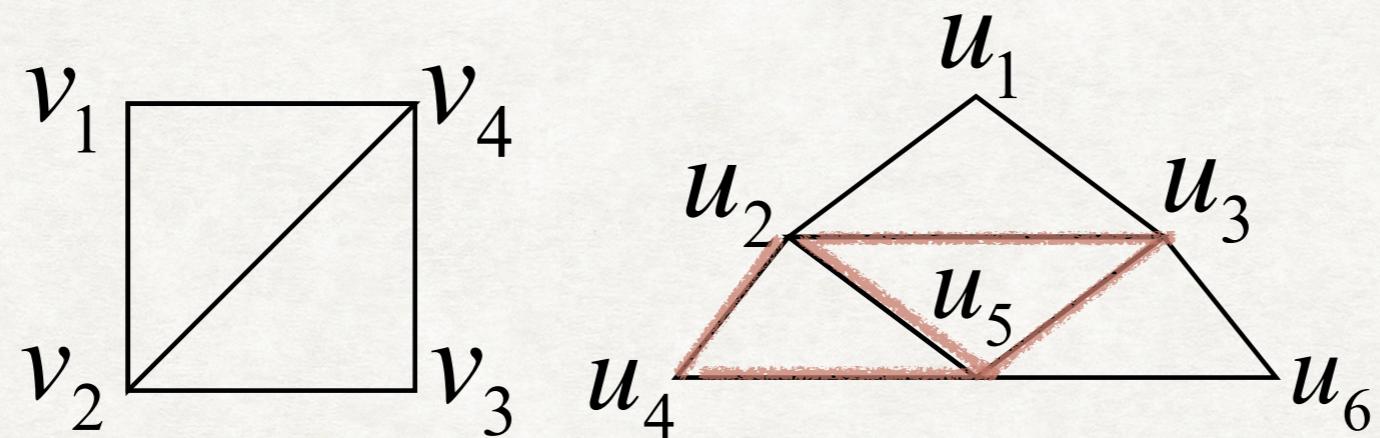
$G$

$$\begin{pmatrix} v_1 & v_2 & v_3 & v_4 \\ u_1 & u_2 & u_5 & u_3 \end{pmatrix}$$

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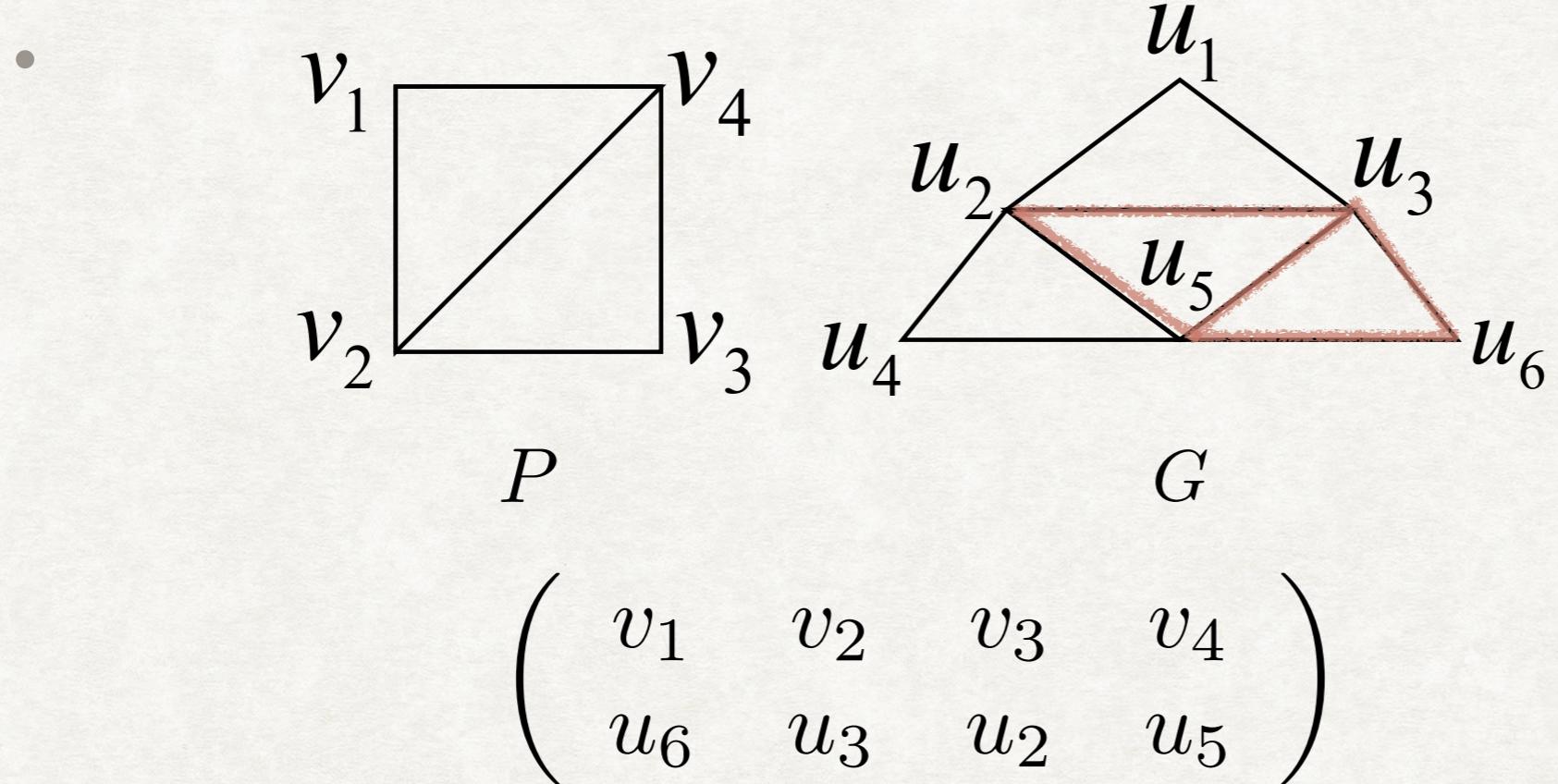
$P$                              $G$

$$\begin{pmatrix} v_1 & v_2 & v_3 & v_4 \\ u_4 & u_2 & u_3 & u_5 \end{pmatrix}$$

# PROBLEM DEFINITION

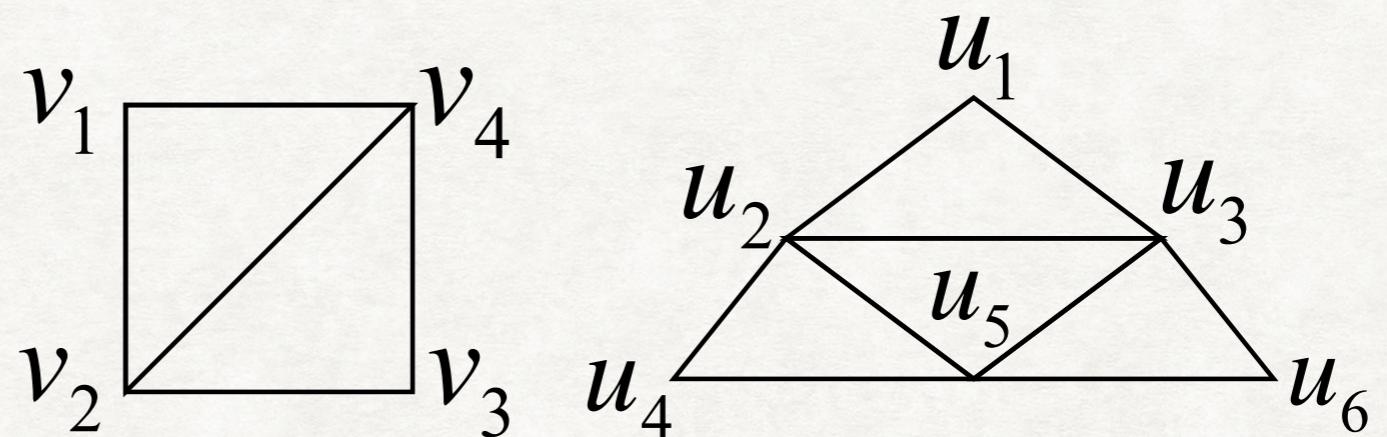
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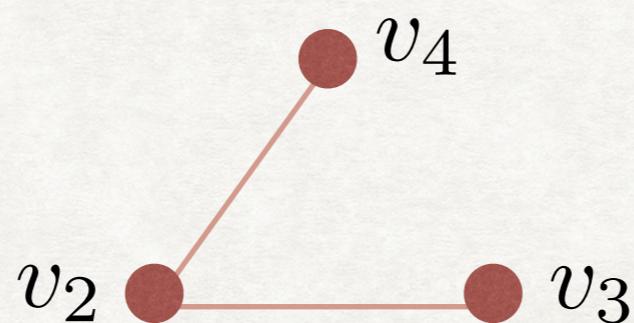
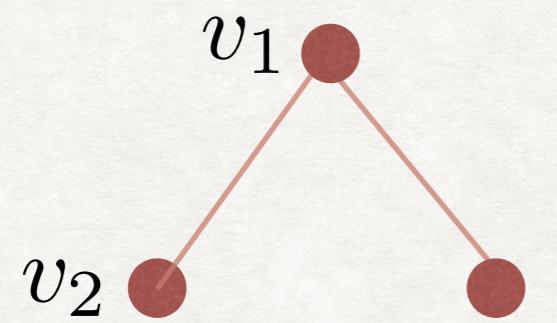
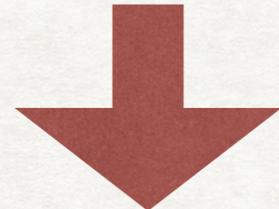


# FRAMEWORK

# PATTERN DECOMPOSITION



$$P = p_0 \cup p_1 \cup p_2$$



$p_0$

$p_1$

$p_2$

Join Units

# WHAT CAN BE JOIN UNITS

- Graph Storage  $\Phi(G) = \{G_u | u \in V(G)\}$ 
  - Stored as  $(u; G_u)$  for each data node
  - $G_u$ : Local Graph of  $u$  s.t.
    - (1) Connected
    - (2)  $u \in V(G_u)$
    - (3)  $\bigcup_{u \in V(G)} E(G_u) = E(G)$

# WHAT CAN BE JOIN UNITS

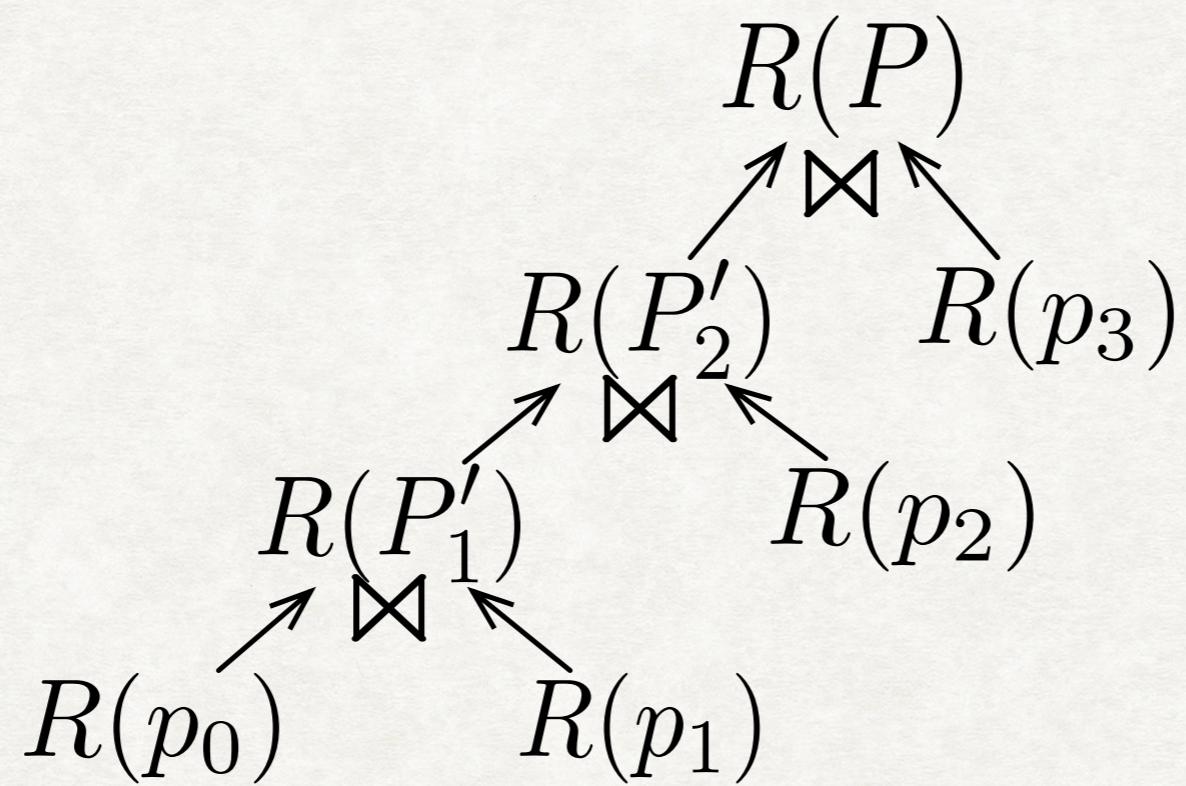
- A structure  $p$  can be a join unit iff.

$$R_G(p) = \bigcup_{u \in V(G)} R_{G_u}(p)$$

- $R_{\mathcal{G}}(p)$  stands for the matches of  $p$  in  $\mathcal{G}$

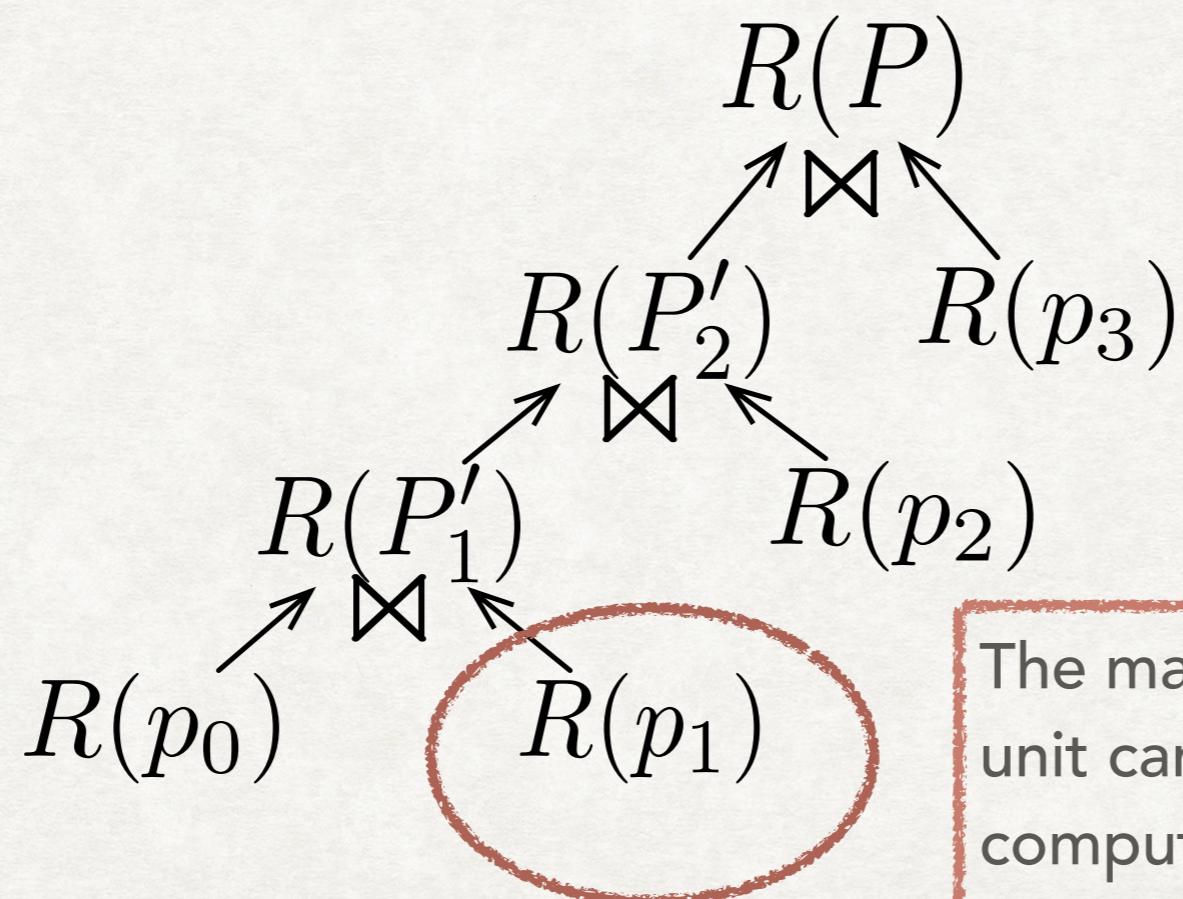
# JOIN PLAN (TREE)

- Decomposing  $P = p_0 \cup p_1 \cup p_2 \cup p_3$
- Solving:  $R(P) = R(p_0) \bowtie R(p_1) \bowtie R(p_2) \bowtie R(p_3)$



# JOIN PLAN (TREE)

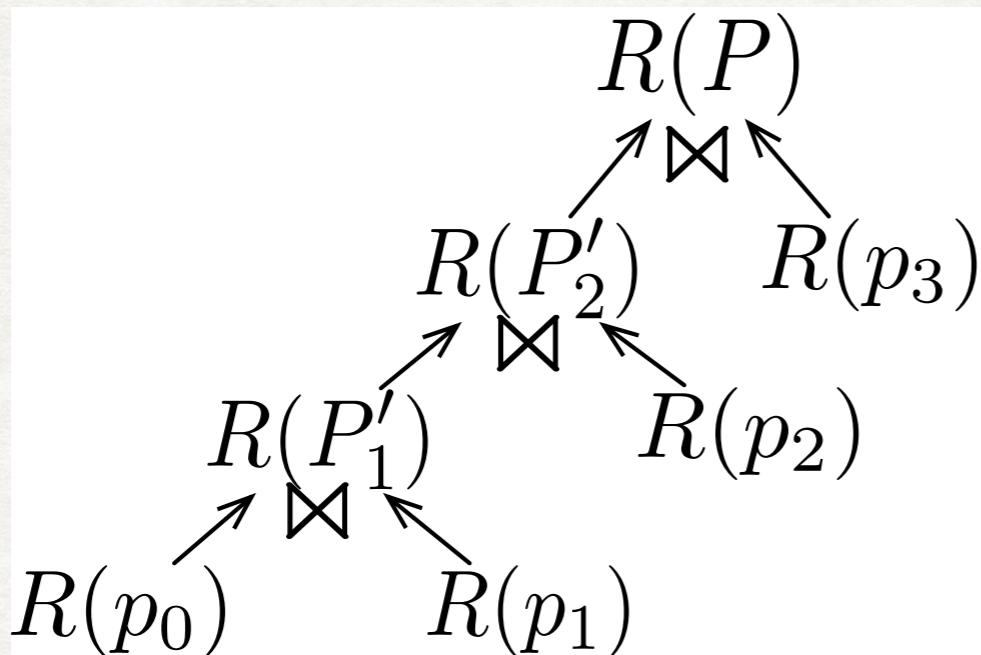
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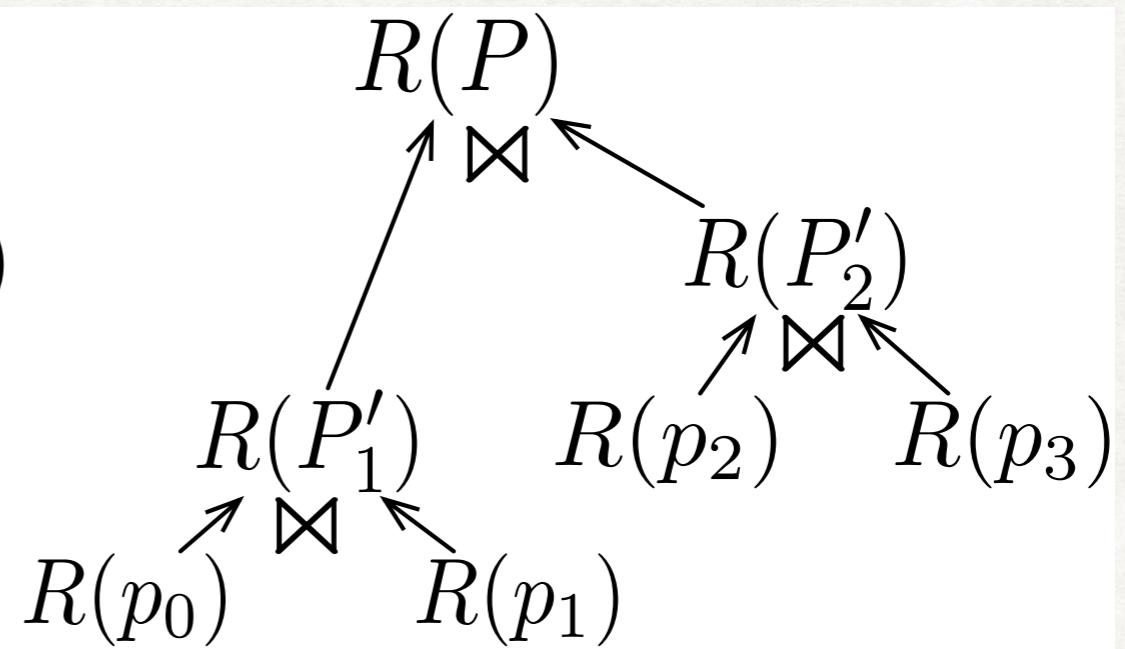
The matches of each join unit can be online computed independently in each local graph

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Left-deep tree



Bushy tree

# DESCRIBE THE ALGORITHMS

- Graph Storage mechanism
  - Determine the join units, thereafter the pattern decomposition
- Join Structure
  - Left-deep tree vs bushy tree

**TWIN TWIG  
JOIN - Vldb15'**

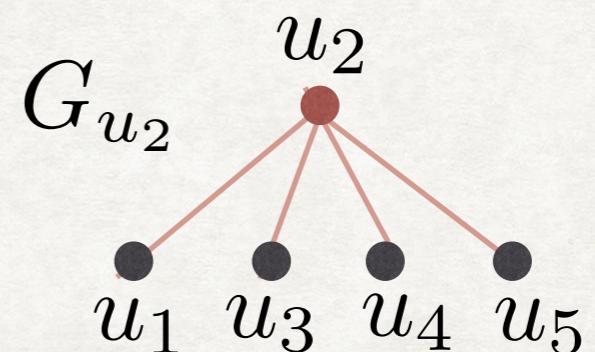
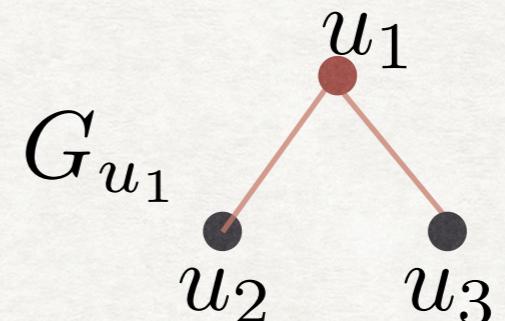
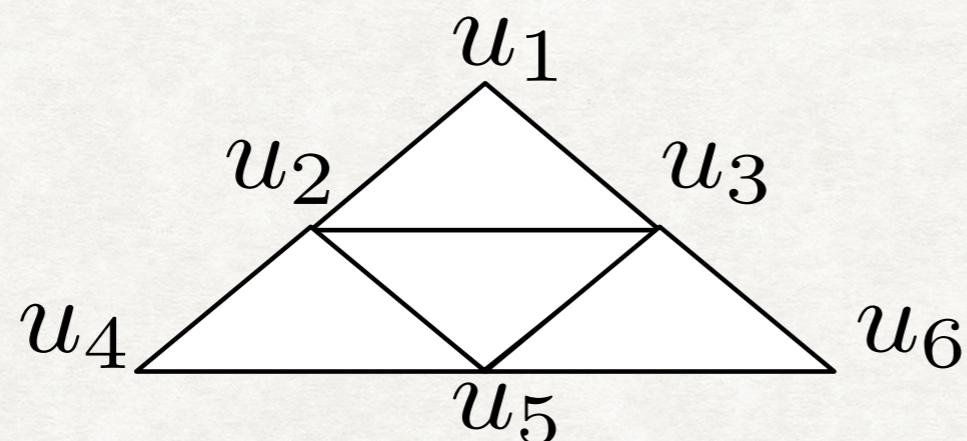
# TWINTWIG JOIN - VLDB2015

## SIMPLE GRAPH STORAGE

- The simple graph storage, each local graph  $G_u$

$$V(G_u) = \{u\} \cup \mathcal{N}(u)$$

$$E(G_u) = \{(u, u') | u' \in \mathcal{N}(u)\}$$



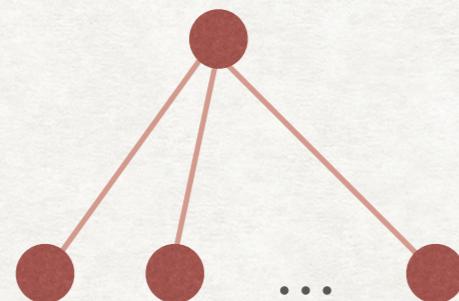
# TWINTWIG JOIN - VLDB2015

## SIMPLE GRAPH STORAGE

- The simple graph storage, where

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Star as the join unit

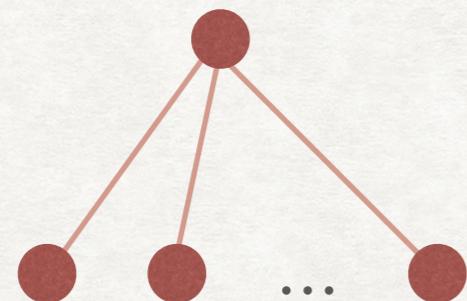
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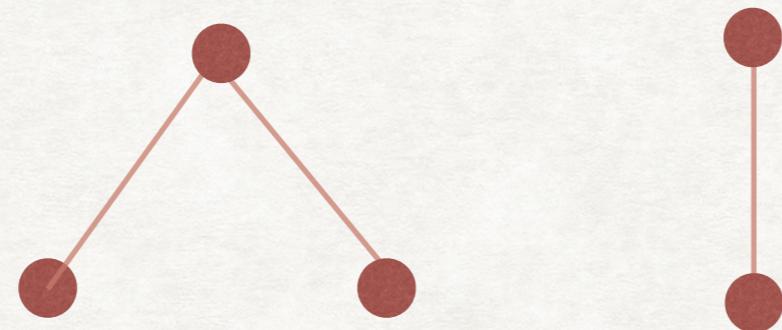
A node with degree 1,000,000  
will generate  $10^{18}$  3-stars

Star as the join unit

# TWINTWIG JOIN

## SIMPLE GRAPH STORAGE

- Using twintwigs as the join units

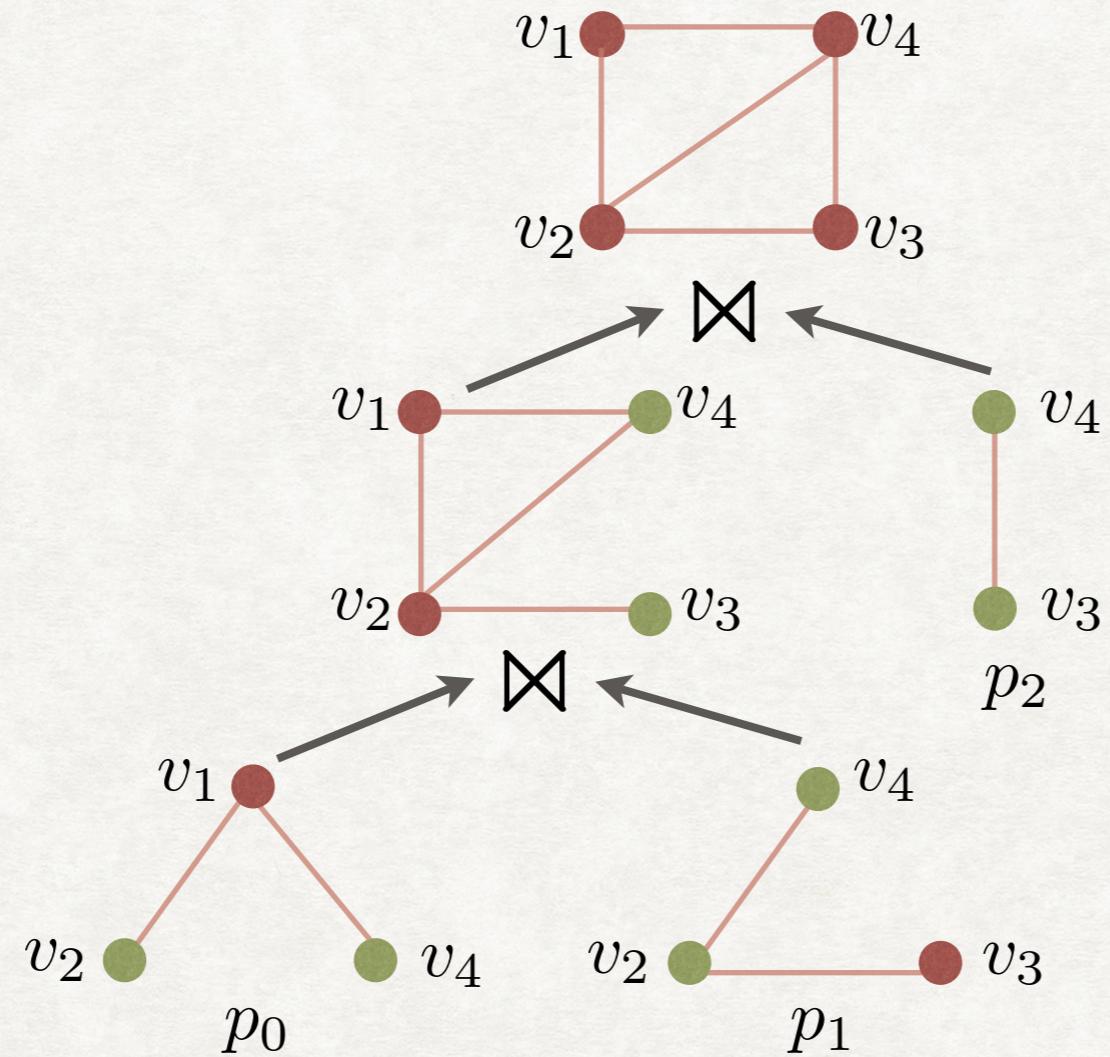


- Instance Optimality
  - Given any join plan involving general stars, we can solve it using twintwigs with at most the same (**often much less**) cost

# TWINTWIG JOIN

## LEFT-DEEP JOIN PLAN

- An optimal left-deep join plan with minimum estimated cost



# TWINTWIG JOIN

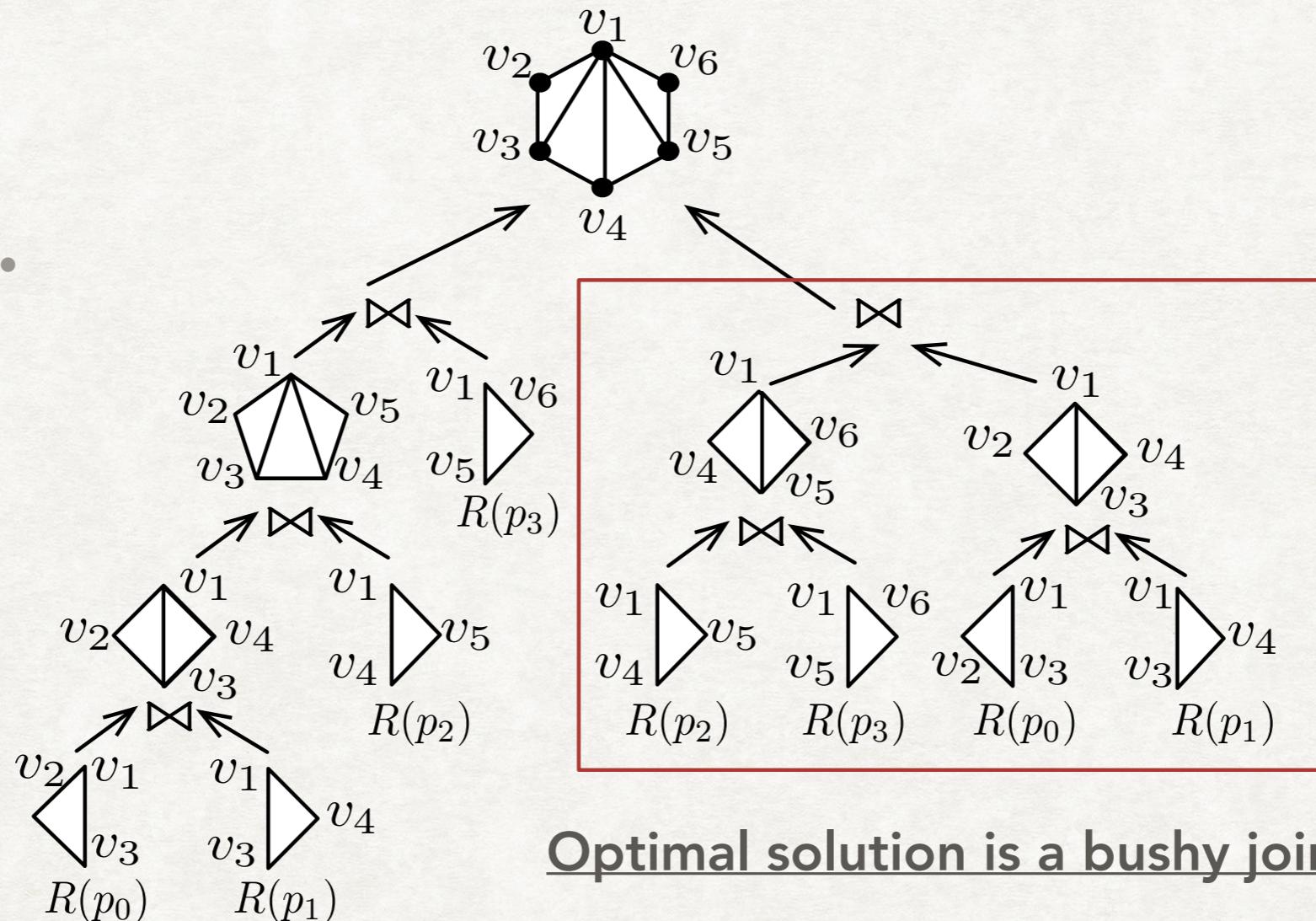
## DRAWBACKS

- Simple storage mechanism only support using star as join units, too many intermediate results
- Twintwig: confine to be at most two edges
  - The node with degree 1,000,000 still have  $10^{12}$  two-edge twintwigs
- Too many execution rounds.
  - A clique of 6 nodes (15 edges): Seven rounds of TwinTwigJoin

# TWINTWIG JOIN

## DRAWBACKS

- Left-deep join: may result in sub-optimal results



# SEED - VLDB17'

## MOTIVATIONS

- Subgraph EnumEration in Distributed Context
  - SCP (Star-Clique-Preserved) graph storage: Use star and clique as the join units
    - We can avoid using star if clique is an alternative
    - Shorter execution. The 6-clique can now be processed in one single round, instead of 7 rounds in TwinTwigJoin
  - Bushy join plan: Optimality Guarantee
  - Much better performance

**SEED**

# SEED

## SCP GRAPH STORAGE

- The SCP Graph Storage, where each local graph  $G_u^+$

$$V(G_u^+) = V(G_u) = \{u\} \cup \mathcal{N}(u)$$

$$E(G_u^+) = E(G_u) \cup$$

$$\{(u', u'') | (u', u'') \in E(G) \wedge u', u'' \in \mathcal{N}(u)\}$$

# SEED

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**NEIGHBOUR EDGES**

# SEED

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**TRIANGLE EDGES**

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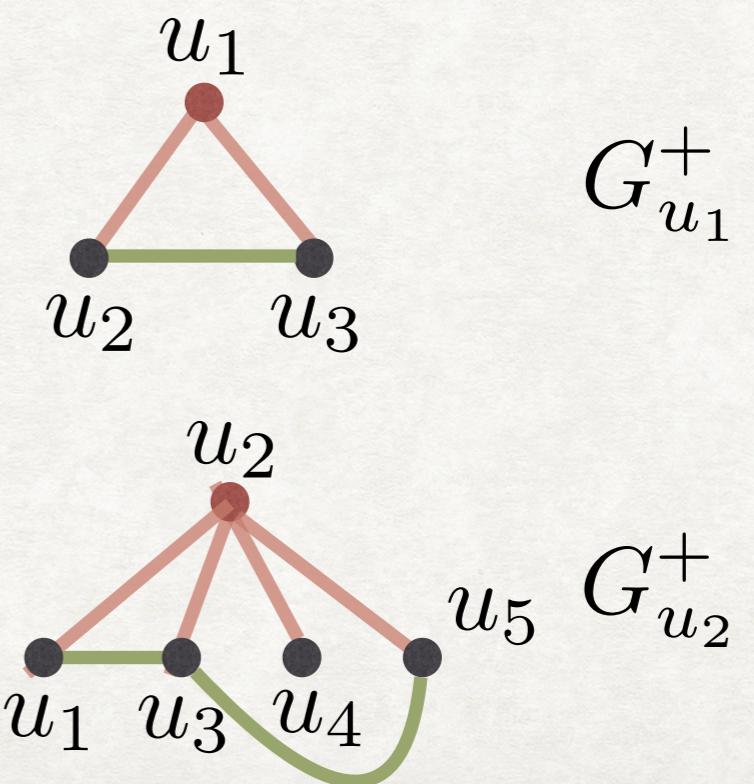
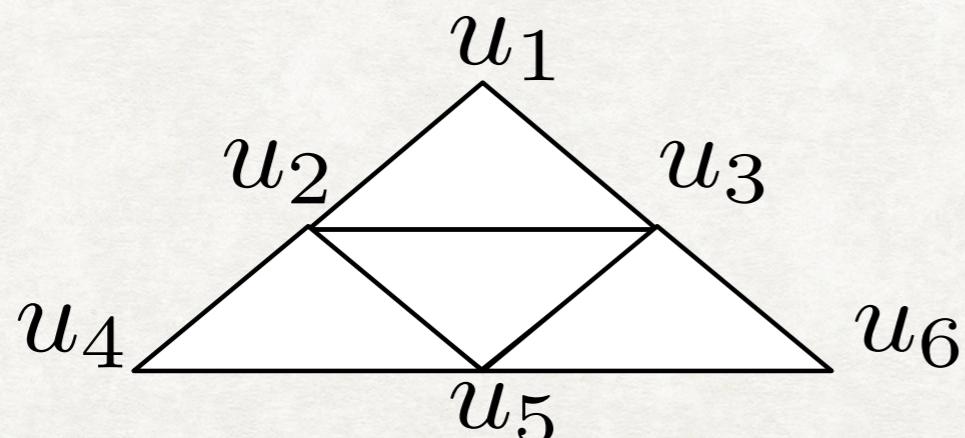
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**NEIGHBOUR EDGES**

**TRIANGLE EDGES**



# SEED

## SCP GRAPH STORAGE

- We show that SCP graph storage supports using both star and clique as the join units
- A more compact version which has bounded size for each local graph

# SEED

## OPTIMAL BUSHY JOIN PLAN

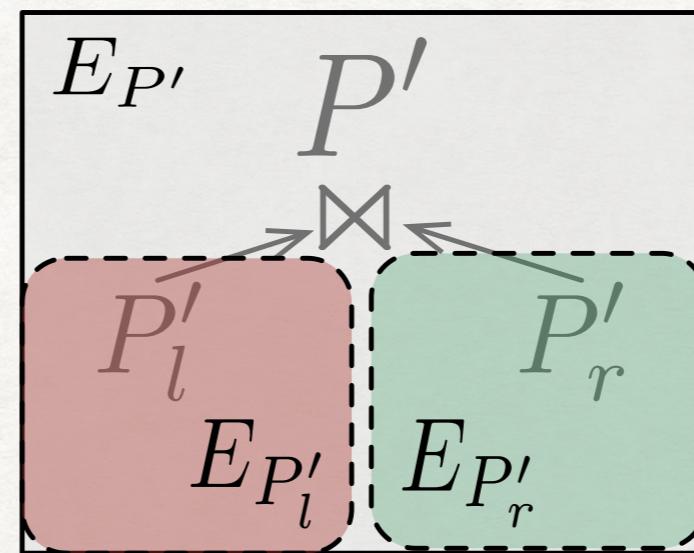
- Notations
  - $E_P$  : The join plan to solve  $P$
  - $C(E_P)$  : The cost of the join plan
  - $C(P)$  : Estimated # matches of  $P$  in  $G$
- We aim at finding a join plan for  $P$  , s.t.

$C(E_P)$  is minimised

# SEED

## OPTIMAL BUSHY JOIN PLAN

- A dynamic programming transform function
  - e.g.  $E_{P'}$ 
    - (1)  $E_{P'_l}$
    - (2)  $E_{P'_r}$
    - (3)  $R(P') = R(P'_l) \bowtie R(P'_r)$



# SEED

## OPTIMAL BUSHY JOIN PLAN

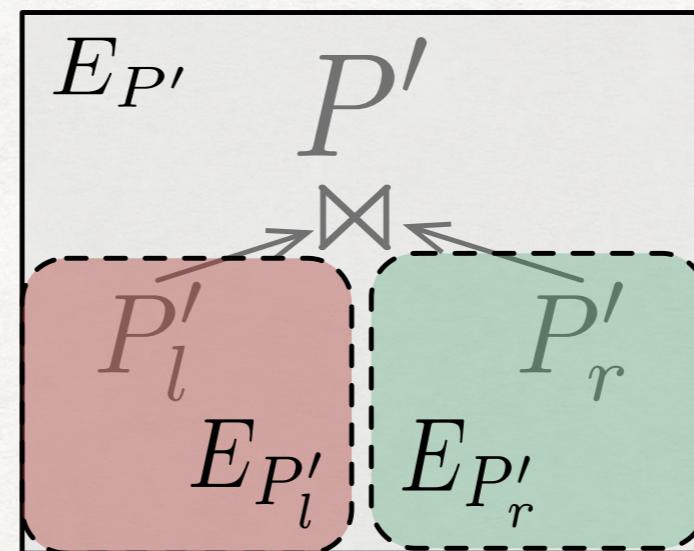
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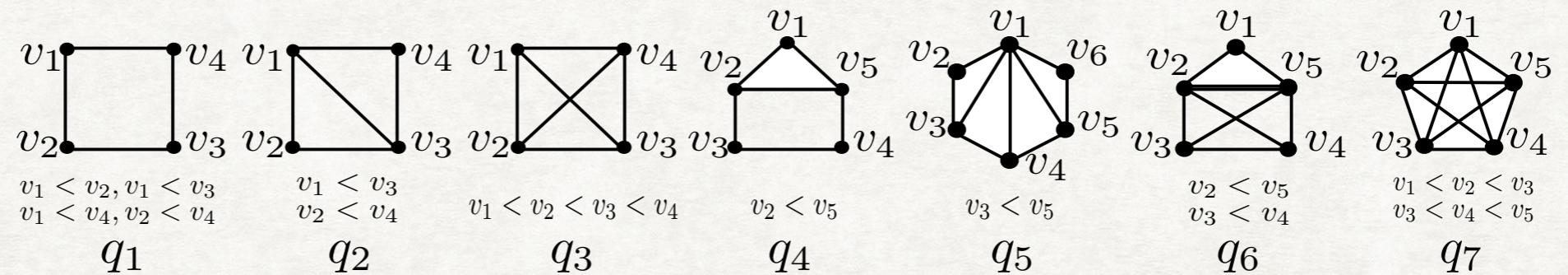
$$C(E_{P'}) = \min_{P'_l \subset P' \wedge P'_r = P' \setminus P'_l} \{C(E_{P'_l}) + C(P'_l) + C(E_{P'_r}) + C(P'_r)\}$$

# EXPERIMENTS

# EXPERIMENTS

## SETUP

- **Queries**



- **Algorithms**

- SEED+O (The most optimised SEED)
- TT (The most optimised TwinTwigJoin, VLDB 2015)
- pSgL (Shao et al. Sigmod 2014)

# EXPERIMENTS

## SETUP

- Cluster

- Amazon EC2: 1 master node, 10 slave nodes

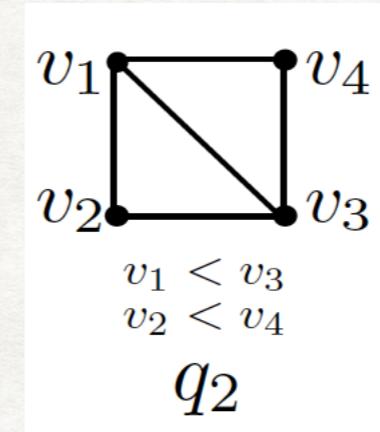
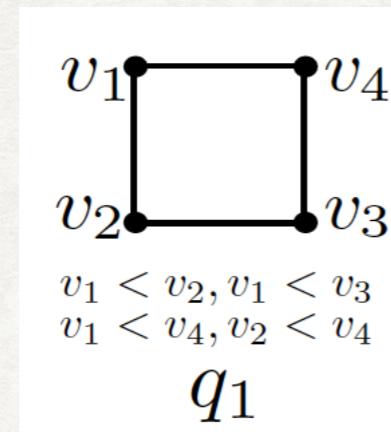
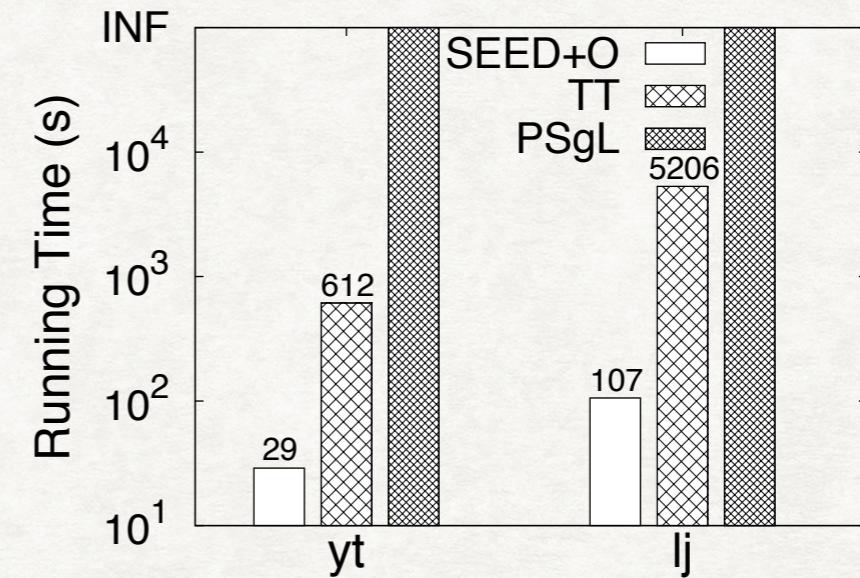
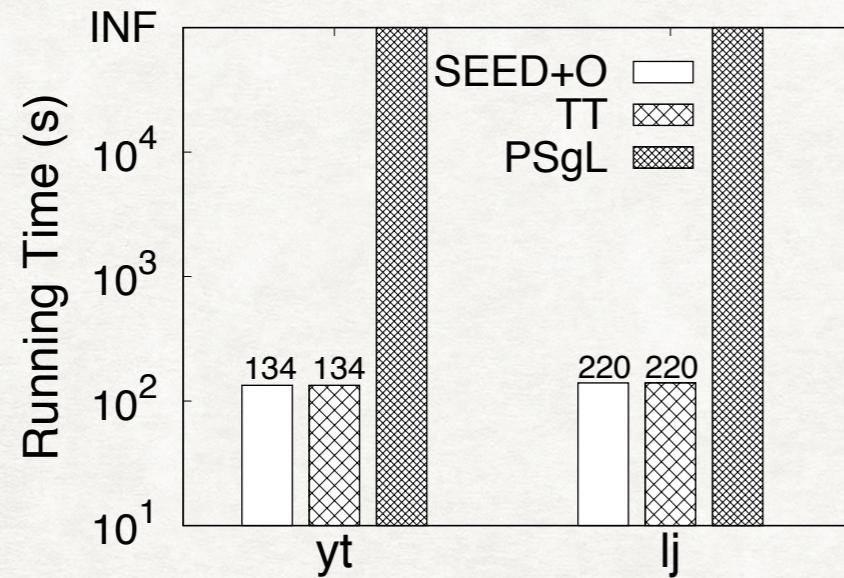
Node	Instance	vCPU	Memory	Disk
master	m3.xlarge	4	15GB	2 x 40GB SSD
slave	c3.4xlarge	16	30GB	2 x 160GB SSD

- Hadoop 2.6.2

- JVM heap space: mapper 1524MB, reducer 2848MB
  - 6 mappers and 6 reducers each machine

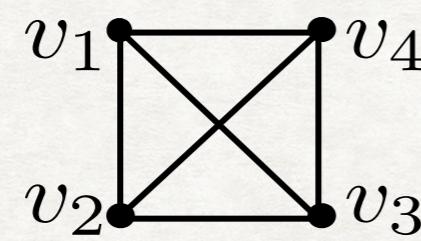
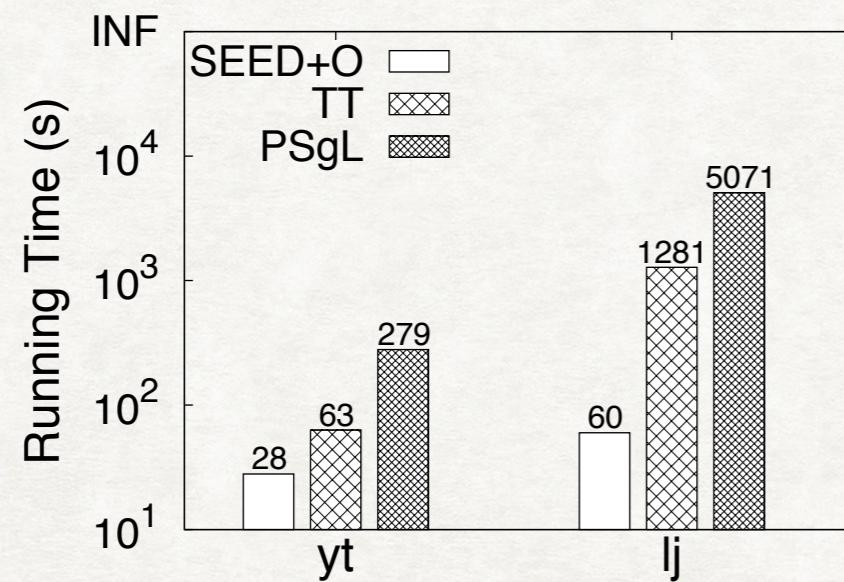
# EXPERIMENTS

## RESULTS



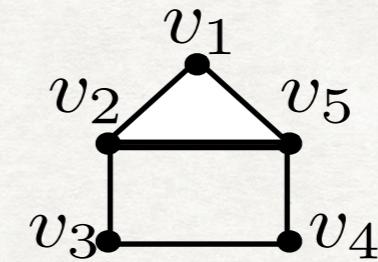
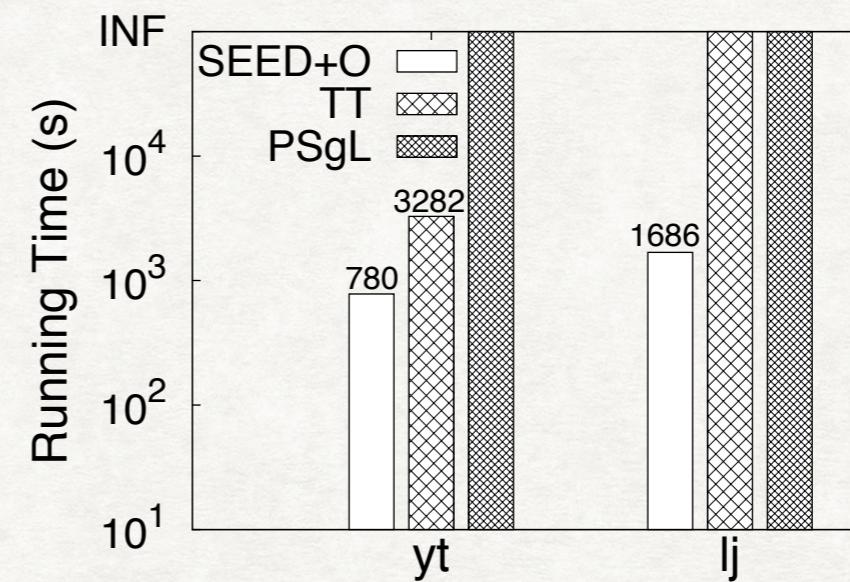
# EXPERIMENTS

## RESULTS



$v_1 < v_2 < v_3 < v_4$

$q_3$

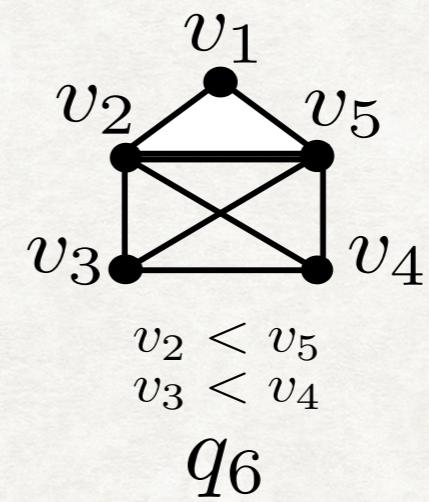
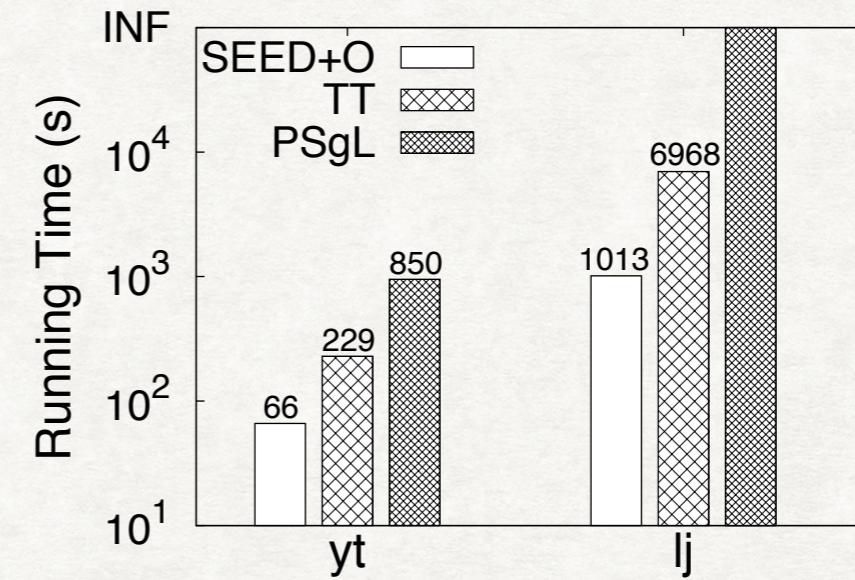
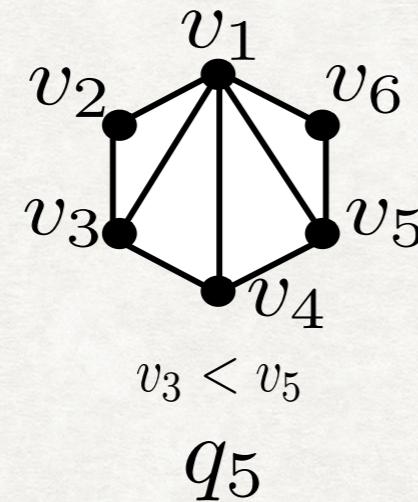
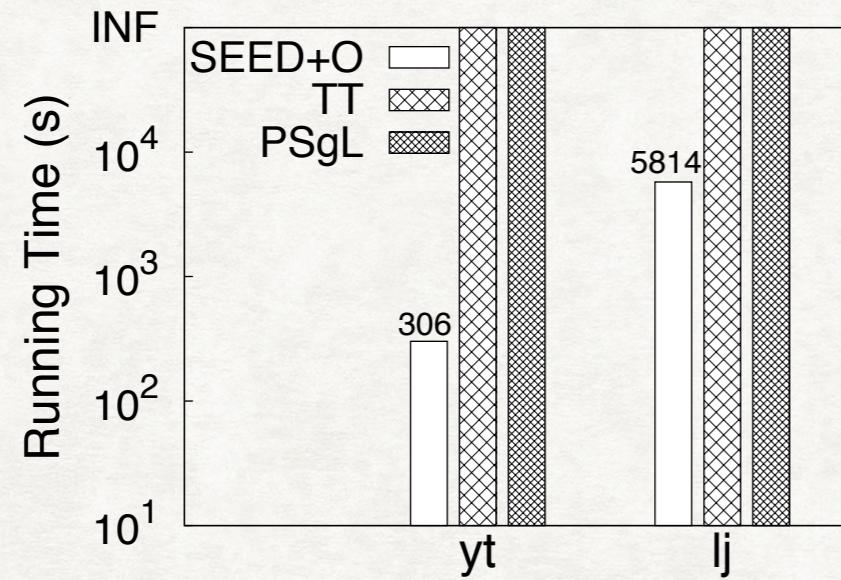


$v_2 < v_5$

$q_4$

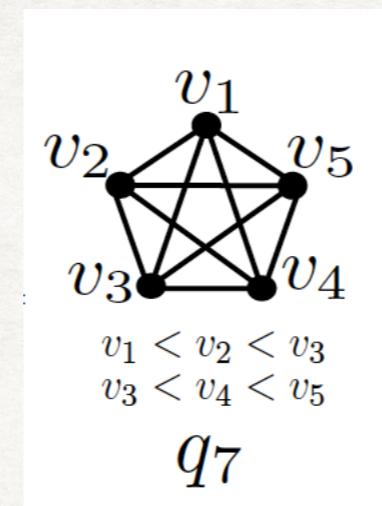
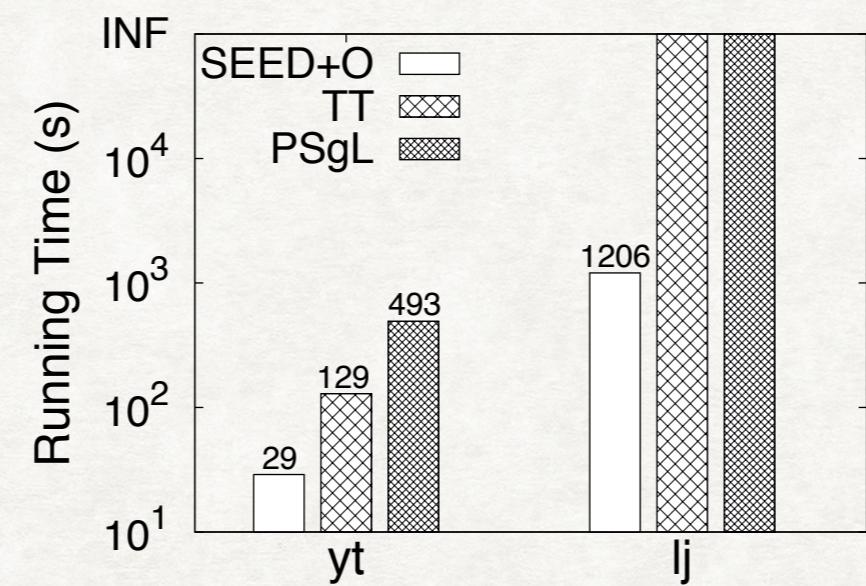
# EXPERIMENTS

## RESULTS



# EXPERIMENTS

## RESULTS



# CONCLUSION

- A general decompose-and-join framework to solve subgraph enumeration
- TwinTwigJoin = Simple graph storage (twintwigs as the join units) + Optimal left-deep join
- SEED = SCP graph storage (star and clique as the join units) + Optimal bushy join

**Q & A**

**THANK YOU!**