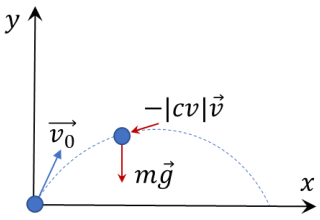
AME 556 – Robot Dynamics and Control – HW2

1. Consider the problem of throwing a ball. The mass of the ball is m. Throw the ball with an initial velocity $\vec{v}_0$. While moving with a velocity $\vec{v}$, it is affected by a drag force of $\vec{F}_{drag} = -|cv|\vec{v}$, where c is the drag coefficient.

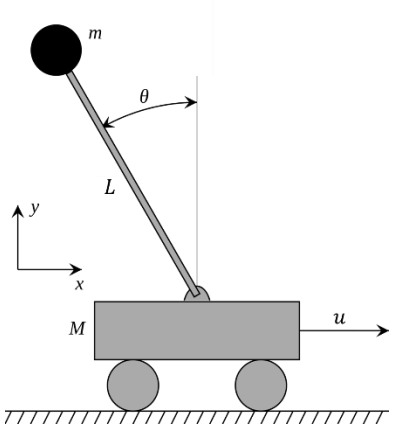


   a. Derive the dynamics of the system.

   b. Given $\vec{v}_0 = [1; 2](m/s)$, $m = 0.5\ (kg)$, $g = 9.81\ (m/s^2)$, $c = 0.05$.
   Use MATLAB ode solver (https://www.mathworks.com/help/matlab/ref/ode45.html) to simulate the dynamics of the system in 2 seconds.
   You can refer to the following MATLAB demo example [here].
   Report your code and:
      - Plot the trajectory of the ball.
      - Animate the motion of the ball, save the simulation video, and attach a viewable link of the video to your HW solution.

   c. Repeat part (b) for the following cases:
      i. $\vec{v}_0 = [2; 1](m/s)$, $c = 0.05$.
      ii. $\vec{v}_0 = [1; 2](m/s)$, $c = 0.5$.

2. Consider the following cart-pole system.



   a. Derive the dynamic of the robot using Euler-Lagrange Equation. Assume that the total mass of the pole concentrated at the end of the pole.
   b. Write a MATLAB function to derive symbolic functions of $D(q), N(q, \dot{q})$ in the equation of motion of the robot dynamics:

$$D(q)\ddot{q} + \underbrace{C(q,\dot{q})\dot{q} + g(q)}_{N(q,\dot{q})} = \tau.$$

Report your code and the result from MATLAB.

Hints:

- Use symbolic Jacobian: (https://www.mathworks.com/help/symbolic/jacobian.html)
- $\frac{\partial f}{\partial q} = jacobian(f, q)$
- If f depends on q and $\dot{q}$: $\frac{df(q,\dot{q})}{dt} = \frac{\partial f(q,\dot{q})}{\partial q}\dot{q} + \frac{\partial f(q,\dot{q})}{\partial \dot{q}}\ddot{q}$
- If f only depends on q: $\frac{df(q)}{dt} = \frac{\partial f(q)}{\partial q}\dot{q}$
- You can use the simplify function in MATLAB to compress your answer: https://www.mathworks.com/help/symbolic/simplify.html .

c. Use MATLAB ode solver to simulate the system in 2 seconds with the following parameters:
   - $M = 1\ kg, m = 0.2\ kg, L = 0.3\ m.$
   - initial conditions: $x_0 = 0; \theta_0 = \pi/6; \dot{x}_0 = 0; \dot{\theta}_0 = 0.$
   - zero control input: $u = 0.$

   Report your code and:

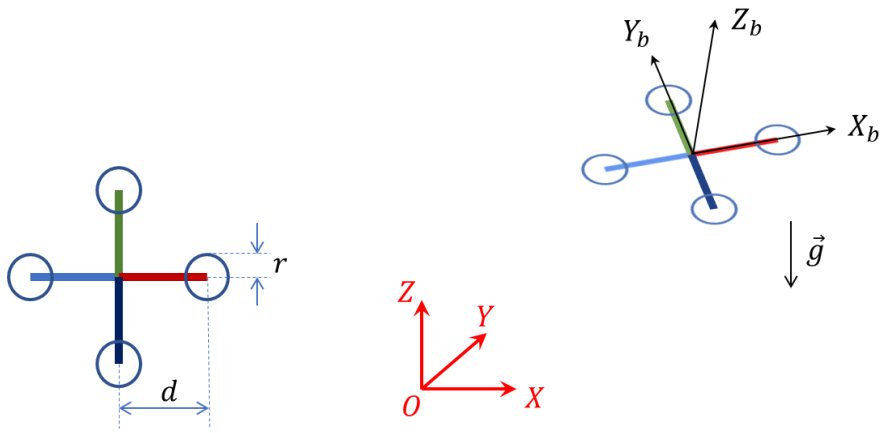
   - Plot $x(t), \theta(t)$ over time.
   - Animate the robot motion, save the simulation video, and attach a viewable link of the video to your HW solution.

   Hints:

   - Copy and paste the result you have from 2b to formulate the dynamics function for the ode solver. The ode solver will be slow if you use symbolic functions inside the ode solver.

d. Use Mujoco to solve problem 2c.
   - Plot $x(t), \theta(t)$ over time.
   - Compare the result with the one you have from problem 2c by plotting the joint angle over time of the two results for each joint in the same plot.
   - Animate the robot motion, save the simulation video, and attach a viewable link of the video to your HW solution.
   - You don't need to report your code for this part.

3. Consider the following UAV model.



Use MATLAB to simulate the UAV in 2 seconds given the following parameters:

- $d = 0.2\ m, r = 0.05\ m$
- total mass of the UAV: $m = 0.5\ kg$
- moment of inertia of the UAV wrt the body frame:
  $I_{xb} = I_{yb} = 0.01\ (kg.m^2), I_{zb} = 0.05\ (kg.m^2)$

- zero control inputs (motor thrusts): $F = [0; 0; 0; 0]$.
- initial conditions:

$$[x; y; z] = [0.5; 0.5; 1]\ (m);$$

$$[\dot{x}, \dot{y}, \dot{z}] = [0; 0; 0](m/s);$$

$$[Roll, Pitch, Yaw] = \left[\frac{\pi}{6}; \frac{\pi}{8}; \frac{\pi}{4}\right](rad);$$

$$\omega_b = [0; -0.1; 0.1](rad/s).$$

Report your code and:

- Plot COM's positions (x, y, z) and body orientation's Euler angles (Roll-Pitch-Yaw) over time.
  (Hint: You will need to simulate the Rotation matrix for the robot dynamics and then convert it to Euler angles. You can use the function *rotm2eul* in MATLAB.)
- Animate the robot motion, save the simulation video, and attach a viewable link of the video to your HW solution. Please refer to the above figure for the expected drawing of the UAV.