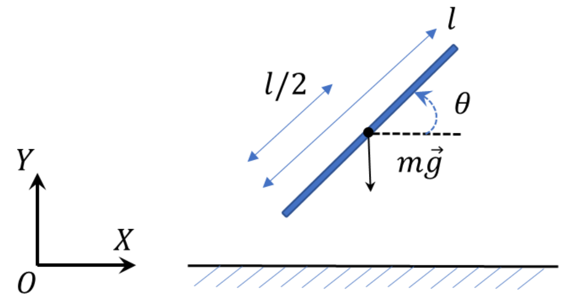
AME 556 – Robot Dynamics and Control – HW3

1. (5 points) Simulation of a free-falling bar.

   Assume that the COM of the bar is at the center of the bar and the moment of inertia of the bar around its COM is $I = \frac{1}{12} ml^2$.



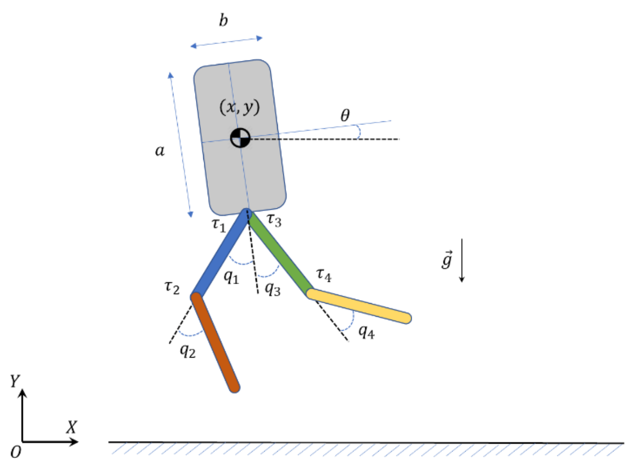
   Given $m = 0.5kg$, $l = 0.2m$, $g = 9.81\, m/s^2$ and the system starts from the initial condition of $x_0 = 0m, y_0 = 0.3m, \theta_0 = \frac{\pi}{4}, \dot{x}_0 = 0, \dot{y}_0 = 0, \dot{\theta}_0 = 0$.

   a. Simulate the system in 2 seconds using MuJoCo with the following settings:
      - Only consider the contact between the lower end of the bar and the ground.
      - $K_p^{ground} = 10^5, K_d^{ground} = 10^3$
      - Coefficient of static friction = 0.7
      - Coefficient of dynamic friction = 0.5

   b. Show plots of $x(t), y(t), \theta(t)$ of the COM position and orientation over time.

   c. Export a simulation video and attach the video link to your HW.

2. Consider the following system of a 2D biped robot.



Assume that the COM of each link is at the center of the link and the moment of inertia of the link around its COM is $I = \frac{1}{12}ml^2$.

Note that the positive direction of each angle is counterclockwise.

Given:

- Leg link: $m_i = 0.25kg, l_i = 0.22m$
- Body/trunk: $m_b = 8kg, a = 0.25m, b = 0.15m$
- Initial condition:
  - $x = 0m, y = 0.65m, \theta = 0,$
  - $q_1 = \frac{-\pi}{3}, q_3 = 0, q_2 = q_4 = \frac{\pi}{2},$
  - and all zero velocities.

Simulate the system using MuJoCo with the following settings:

- $K_p^{ground} = 10^5, K_d^{ground} = 10^3$
- Coefficient of static friction = 0.7
- Coefficient of dynamic friction = 0.5

a. (5 points) Simulate the system in 2 seconds with zero control inputs.

   i. Show plots of $x(t), y(t), \theta(t), q_i(t) (i = 1{:}4)$.

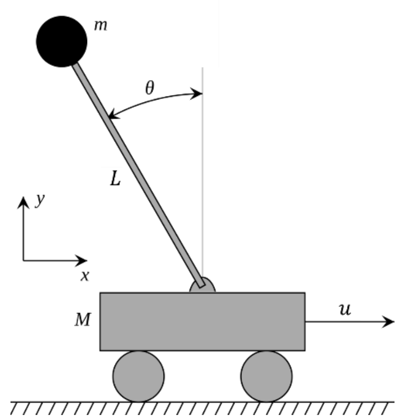
   ii. Create an animation for the system and attach the video link to your HW.

b. (5 points) Simulate the system in 2 seconds with a joint PD controller for each joint with $K_P = 50, K_D = 2$ to keep the desired joint angles at the initial condition for q_i mentioned above.

   i. Show plots of $x(t), y(t), \theta(t), q_i(t) (i = 1{:}4)$.

   ii. Create an animation for the system and attach the video link to your HW.

3. (5 points) Consider the cart-pole system in HW2:



The system dynamics is given as follows:

$$\begin{aligned}(M+m)\ddot{x} + mLsin(\theta)\dot{\theta}^2 - mLcos(\theta)\ddot{\theta} &= u \\ mL^2\ddot{\theta} - mLcos(\theta)\ddot{x} - mgLsin(\theta) &= 0\end{aligned}$$

- Given $M = 1kg, m = 0.2kg, L = 0.3m, g = 9.81\, m/s^2$.
- The system starts with the following initial conditions:

  $x_0 = 0.1m; \theta_0 = 0.1rad; \dot{x}_0 = 0\, m/s\,; \dot{\theta}_0 = 0\, rad/s.$

a. Linearize the system around the following operating point:

$$x_d = 0; \theta_d = 0; \dot{x}_d = 0; \dot{\theta}_d = 0.$$

b. Design a linear controller to stabilize the system around the operating point. Please tune your control parameter to guarantee a settling time (of 5% error) between $[0.5{:}1](s)$ and an overshoot between $[10{:}20]\%$.

   i. Prove that with your controller, the closed-loop system has Local Exponential Stability.

   ii. Simulate the system in 2 seconds.

   iii. Show plots of $x(t), \theta(t), u(t)$ over time.

   iv. Create an animation for the system and attach the video link to your HW.

Please refer to the following links for the definitions of settling time and overshoot:

- https://en.wikipedia.org/wiki/Settling_time
- https://en.wikipedia.org/wiki/Overshoot_(signal)