The Rise of the Service Economy

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Buera and Kaboski (2012), AER

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Review

Buera and Kaboski(2012); Skills are speciallized, and specialization plays an important role in **the decision between home production and market production of services.**

- The increase in the consumption of moer skill-intensive wants causes the rise in the importance of market services.
- This leads to increase in the quantity and price of services.
- The higher wage amounts to a higher opportunity cost of home production, high-skilled workers to purchase even wider range in service market.

Review

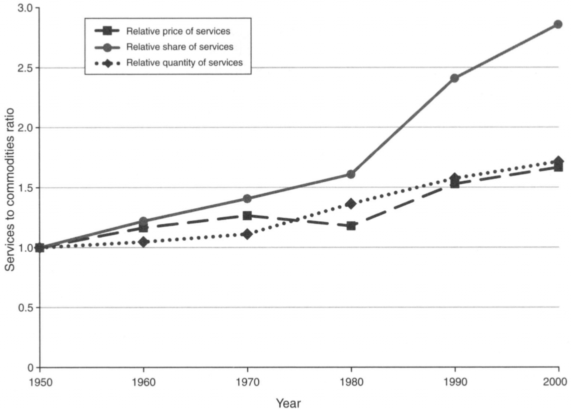


FIGURE 2. GROWTH OF RELATIVE PRICE AND QUANTITY OF SERVICES

- By making relative term using value of 1950 as a denominator, we can see:
- The relative quantity and price of services have increased dramatically.

Review

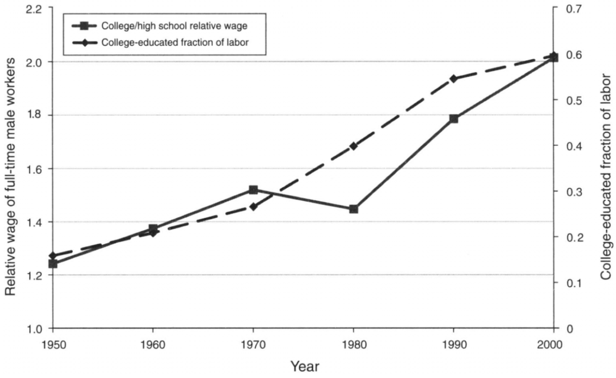


FIGURE 5. GROWTH OF COLLEGE PREMIUM AND FRACTION COLLEGE-EDUCATED

- Relative Wage; w_{college} increases from 125% in 1950 to 200% of $w_{\text{high school}}$ by 2000.
- Relative Quantity; $\frac{L_{\text{college}}}{L_{\text{high school}}}$ increases from 15% to 60% during the same period.

Environment

Schooling, Preference, Technologies and Productivity

Recall: Stand-in Household Model

A represnetative household faces a range of satiable **wants** that differ in their production costs.

The household's decisions are:

- the fraction of members that are required to obtain speciallized skills (high-skilled and low-skilled labor)
- the allocation of high- and low-skilled members' time between market work and home production.

Marker services and home production services

- Marker production has a cost advantage due to use of more speciallized skills
- Home production assumed to be more customized, which gives more utility.
- Goods are produced only in the market and always the intermdiate input in the production of services.
- The final output takes the form of services whether it is market produced or not.

Fraction of Members

- The household contains members differentiated by their skill level e that are high-skilled($e = h$) and low-skilled($e = l$). ex-ante they are identical.
- The household chooses the fraction of high-skilled members f^h and the fraction of low-skilled members f^l.

$$f^h + f^l = 1$$

Schooling

- A worker can become high-skilled by using a fraction of the worker's time to learn specialized skills for the production of a particular z.
- θ is a continuous, increasing, and strictly convex function of f^h.

$$\theta'(f^h), \theta''(f^h) > 0$$

Labor Supply and Schooling

Assumption to ensure that $f^h \in (0,1)$

- $\lim_{f \to 0}[1-\theta(f^h) - f^h\theta'(f^h)] \geq 1$
- $1-\theta(1)-\theta'(1) < 0$

Labor supply of high-skilled workers:

- Total labor supply of $f^h = f^h \cdot [1-\theta(f^h)]$
- Marginal labor supply of $f^h = \frac{\partial f^h[1-\theta(f^h)]}{\partial f^h}$

Labor supply of low-skilled workers:

- Total labor supply of $f^l = f^l \cdot 1$
- Marginal labor supply of $f^l = \frac{\partial f^l}{\partial f^l} = 1$

Labor Supply and Schooling

- $f^h \to 0$, Marginal labor supply of $f^h \geq$ Marginal labor supply of f^l

$$\lim_{f \to 0} \frac{\partial f^h[1-\theta(f^h)]}{\partial f^h} \geq 1$$

- $f^h \to 1$, Marginal labor supply of f^h decreases.

$$\lim_{f \to 1} \frac{\partial f^h[1-\theta(f^h)]}{\partial f^h} < 0$$

- These conditions make household income maximization problem have interior solution.
- These are also useful for graphical analysis later.

Non-homothetic Preferences

The household holds preferences over a continuum of dircrete, satiable wants indexed by the service that satisfies them, $z \in \mathbb{R}^+$

$$C^e(z) : \mathbb{R}^+ \to \{0, 1\}$$

$$H^e(z) : \mathbb{R}^+ \to \{0, 1\}$$

Indicator Functions $C = \{C^l(z), C^h(z)\}$ and $H = \{H^l(z), H^h(z)\}$

The stand-in household holds preferences over wants and the method of satisfying those wants.

- $C^e(z) = 1$ if want z is satisfied via home production or market production for members$(e = l, h)$
- $H^e(z) = 1$ if want z is satisfied by home production for members$(e = l, h)$

Utility Function

The utility function is:

$$u(C, H) = \Sigma_{e=l,h} f^e \int_0^\infty [H^e(z) + \nu(1 - H^e(z))] C^e(z) dz \tag{1}$$

where $H(z) \leq C(z)$ and $0 < \nu < 1$ is the discounted utility for market services.

Marginal Utility from Consuming Want z

$$MU(z) = [H(z) + \nu(1 - H(z))] C(z)$$

- The marginal utility of consumption z is symmertric in e.
- The symmetry in consumption will be relaxed later.

Consumption of Want Z

There are only three cases for every want z.

$$[H(z) + \nu(1 - H(z))]C(z)$$

- $MU(H = 1, C = 1; z) = 1$; Consumption of Home production services which gives full utility 1.
- $MU(H = 0, C = 1; z) = \nu$; Consumption of market service which gives discounted utility ν.
- $MU(H = 0, C = 0; z) = 0$; neither satisfies want z.

Three Production Technologies

- Market Production of goods
- Market Production of services
- Home Production of services

Market goods are the foundation of production. As intermediates, they are inputs into both market and home-produced services (final outputs).

Market Production of goods

$$Goods: G(z) = A_l(z)L_G(z) + A_h(z)H_G(z) \tag{2}$$

- $L_G(z)$ and $H(z)$ denote the amounts of low- and high-skilled labor produced goods.
- $A_l(z)$ and $A_h(z)$ are their respective productivities, disussed in productivity section.

Goods can be produced by using only one type of labor ($L_G(z)$ or $H(z)$). We can think of perfect substitution for the other labor for type z.

Market Production of services

$$Market\ Services : S_M(z) = \min\{A_l(z)L_G(z) + A_h(z)H_G(z), G_M(z)/q\} \tag{3}$$

- $S_M(z)$ produces value-added with the identical linear labor technology.
- The q units of input $G_M(z)$ and one effecitve unit of labor are combined as Leontief manner
- We cannot produce services without q units of manufactured inputs!

Leontief fashion implies:

$$A_l(z)L_G(z) + A_h(z)H_G(z) = G_M(z)/q$$

$$q : 1 = G_M(z) : A_l(z)L_G(z) + A_h(z)H_G(z)$$

Home Production services

$$\textit{Non-market Services}: \mathbb{S}_N(z) = \min\{A_l(z)n(z), g_N(z)/q\} \tag{4}$$

where $n(z)$ is the nonmarket time devoted to the home production of service z.

- For each service z, an alternative nonmarket technology is available to home-produce services.
- This technology differs in only one way:
- all labor has the identical, low-skilled productivity in home production.

Leontief fashion implies:

$$q : 1 = g_N(z) : A_l(z)n(z)$$

Productivity

low-skilled productivity : $A_l(z) = Az^{-\lambda_l}$
high-skilled productivity : $A_h(z) = A\phi \max\{z^{-\lambda_l}, z^{-\lambda_h}\}$

Three parametric assumptions:

- $\phi > 1$; high-skilled labor has an absolute productivity advantage over low-skilled labor in all market product
- $\lambda_l > 0$, $\lambda_h \geq 0$; high z goods are more complex in the sense that they require more resources to be produce
- $\lambda_l \geq \lambda_h$; high-skilled labor has a (weak) comparative advantage in more complex output (i.e., $z > 1$) on the market.

The parameter $A > 0$ is common across technologies and skill levels and there- fore captures neutral labor-augmenting productivity.

Competitive Equilibrium

Definition

A competitive equilibrium is given by price functions $P_G(z)$,$P_S(z)$ and wages w_h and w_l; the fraction of people who attain schooling f^h (and $f^l = 1$ - f^h); (skill-specific) consumption decisions $C^e(z)$ and $H^e(z)$, (skill-specific) home production labor allocations n^e and market labor allocations $L_G(z), L_M(z), H_G(z)$, and $H_m(z)$, such that...

Competitive Equilibrium

Household's Utility Maximization

schooling, consumption decisions, and home production labor allocation of low and high-skilled members maximize:

$$u(C,H) = \Sigma_{e=l,h} f^e \int_0^\infty [H^e(z) + \nu(1 - H^e(z))] C^e(z) dz$$

subject to a common budget constraint and the home production constraints:

$$\Sigma_{e=l,h} f^e \int_0^\infty C^e(z) H^e(z) \frac{z^{\lambda_l}}{A} dz = \Sigma_{e=l,h} f^e n^e \tag{5}$$

Competitive Equilibrium

Labor Market Clearing

$$\int_0^\infty [L_G(z) + L_M(z)]dz + n^l = f^l \tag{6}$$

where $L_G(z)$ and $L_M(z)$ denote respectively the low-skilled value-added and goods production labor used to produce type z service.

$$\int_0^\infty [H_G(z) + H_M(z)]dz + n^h = f^h[1 - \theta(f^h)] \tag{7}$$

$H_G(z)$ and $H_M(z)$ denote each labor in a similar way.

Competitive Equilibrium

Goods and Service Market Clearing

$$\forall z \quad G(z) = \Sigma_{e=l,h} f^e C^e(z) H^e(z) q + G_M(z) \tag{8}$$

where $\Sigma_{e=l,h} f^e C^e(z) H^e(z)$ indicates the demand of home production services z, therefore, home production needs q units of manufactured goods; total supply equals to total demand of goods in service z.

$$\forall z \quad S(z) = \Sigma_{e=l,h} f^e C^e(z)[1 - H^e(z)] \tag{9}$$

which means total supply of market services equals to total demand of market services z.

Competitive Equilibrium

Profit Maximization: price-taking

The representative firms in the market services and goods sectors maximize profits taking as given wages w^e, prices of goods output/intermediate $P_G(z)$, and prices of services output $P_S(z)$.

- Since production functions are constant return to scale, see equation (2),(3),(4),
- Buera and Kaboski(2012) model represnetative firms for each z in the market services and goods sectors.
- The price $P_G(z) = 1$ are normalized as the numeraire.

Equilibrium Price

Given market production technologies (2) and (3), we can easily solve for prices using *zero profit conditions*:

$$P_G(z) = \min\left\{\frac{w_l}{A_l(z)}, \frac{w_h}{A_h(z)}\right\} \tag{10}$$

$$P_S(z) = qP_G(z) + \min\left\{\frac{w_l}{A_l(z)}, \frac{w_h}{A_h(z)}\right\} \tag{11}$$

$$P_G(1) = \min\left\{\frac{w_l}{A_l(1)}, \frac{w_h}{A_h(1)}\right\} = 1$$

Therfore, $\frac{w_l}{A_l(1)} = \frac{w_h}{A_h(1)} = 1$ and skill premium(relative wage): $w \equiv \frac{w_h}{w_l} = \frac{A_h(1)}{A_l(1)} = \phi$ when $z = 1$

Comparative Advantage and Labor Allocation

According to equation (10),(11), Firms always use the lowest per unit labor cost between low-skilled and high-skilled labor to produce goods to produce service z.

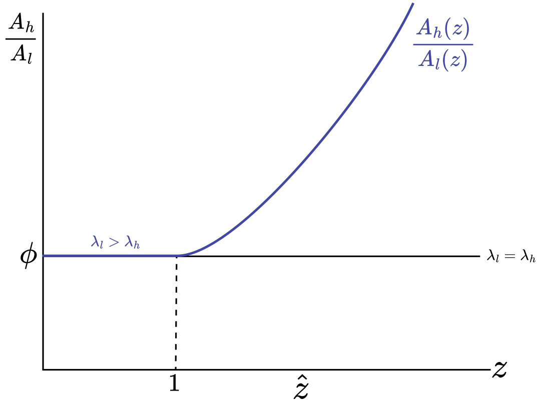
$$\frac{w_l}{A_l(z)} \gtreqless \frac{w_h}{A_h(z)}, \quad \frac{A_h(z)}{A_l(z)} \gtreqless w \tag{12}$$

high(low)-skilled labor has a (weak) comparative advantage in producing good z if the relative productivity of high-skilled to low-skilled labor for producing good z is (weakly) larger(lower) than the skill premium.

Theorem

Given any relative wage $w \geq \phi$, there exist **a threshold complexity** $\hat{z}$ such that for all $z > \hat{z}$ the cost of production using high-skilled workers is strictly lower than that using low-skilled workers.

Graphical Analysis : Relative Productivity



- $\forall z \quad \frac{A_h(z)}{A_l(z)} = \phi \quad$ *when* $\lambda_l = \lambda_h$
- when $\lambda_l > \lambda_h$
- *for* $z \leq 1 \quad \frac{A_h(z)}{A_l(z)} = \phi$
- *for* $z > 1 \quad \frac{A_h(z)}{A_l(z)} = \phi z^{\lambda_l - \lambda_h}$

Graphical Analysis : Cost Effective Labor Allocation

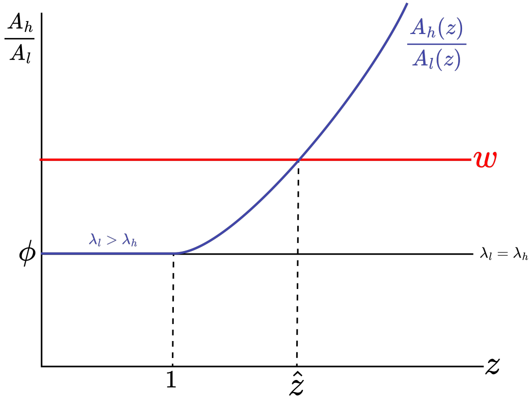


Figure: Relative productivity and Relative wage

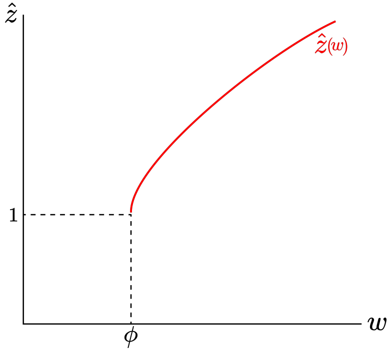
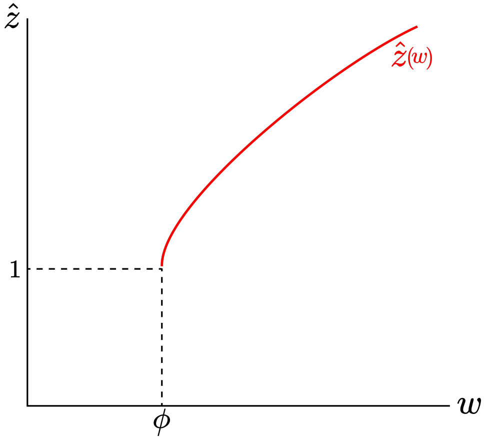


Figure: z-threshold

Graphical Analysis : threshold complexity $\hat{z}$



$$\hat{z}(w) = \left[\frac{w}{\phi}\right]^{\frac{1}{\lambda_l - \lambda_h}} \tag{13}$$

when inequality (12) are equal, we can get the threshold complexity $\hat{z}$

Symmetry of consumption:

The symmetry of the problem with respect to the consumption allocation clearly implies that the stand-in household assigns the same consumption for high- and low- skilled individuals, so we introduce the simplified notation $\mathcal{H}(z) \equiv \mathcal{H}^l(z) = \mathcal{H}^h(z)$ and $\mathcal{C}(z) \equiv \mathcal{C}^l(z) = \mathcal{C}^h(z)$

Total Expenditure

The total expenditure on goods used in home productuion is:

$$C_G \equiv \int_0^{\infty} C^e(z)H^e(z)qp_G(z)dz \tag{14}$$

where $C^e(z)H^e(z)q$ refers to the quantity of goods for home production services z.

The total expenditure on market services is:

$$C_S \equiv \int_0^{\infty} C^e(z)[(1 - H^e(z))]p_S(Z)dz \tag{15}$$

Optimal Policy functions

The optimal policy functions $f^e, n^e, e = l, h, \mathcal{H}(z)$, and $\mathcal{C}(z)$, and the associated expenditure in goods and services, C_G and C_S, maximize

$$\int_0^\infty [\mathcal{H}(z) + \nu(1 - \mathcal{H}(z))]\mathcal{C}(z)dz \tag{16}$$

subject to

$$C_G + C_S = \sum_{e=l,h} f^e w^e [1 - \theta(f^h)\mathcal{I}(e) - n^e] \tag{17}$$

$$\int_0^\infty \mathcal{C}(z)\mathcal{H}(z)\frac{z^{\lambda_l}}{A}dz = \sum_{e=l,h} f^e n^e \tag{18}$$

$\mathcal{I}(e)$ is an indicator function that equals 1 if $e = h$ and 0 if $e = l$.

PROPOSITION 1:

Equilibrium consumption decisions are characterized by thresholds $\underline{z} \leq \overline{z}$ such that

$$\mathcal{C}(z) = \begin{cases} 1 & \textit{if } z \leq \overline{z} \\ 0 & \textit{if } z > \overline{z} \end{cases} \quad \text{and} \quad \mathcal{H}(z) = \begin{cases} 1 & \textit{if } z \leq \underline{z} \\ 0 & \textit{if } z > \underline{z} \end{cases} \tag{19}$$

Household Consumption decision is simplified into choosing these thresholds.

Specification of Proposition 1

	$z \in (0, \underline{z}\,]$	$z \in (\underline{z}, \overline{z}\,]$	$z \in (\overline{z}, \infty)$
$\mathcal{C}(z)$	1	1	0
$\mathcal{H}(z)$	1	0	0
$\mathcal{C}(z)\mathcal{H}(z)$	1	0	0
$\mathcal{C}(z)[1 - \mathcal{H}(z)]$	0	1	0

This table summarizes the household consumption decision for each want z.

From the table, we can see that:

(i) $\mathcal{C}(z)\mathcal{H}(z) =$ consumption of home production services

(ii) $\mathcal{C}(z)[1 - \mathcal{H}(z)] =$ consumption of market services

Household Consumption decision

Equilibrium consumption decisions are characterized by thresholds z and z

$$\int_0^{\infty} [\mathcal{H}(z) + \nu(1 - \mathcal{H}(z))]\mathcal{C}(z)dz$$

$$= \int_0^{\underline{z}} [\mathcal{H}(z)+\nu(1-\mathcal{H}(z))]\mathcal{C}(z)dz + \int_{\underline{z}}^{\overline{z}} [\mathcal{H}(z)+\nu(1-\mathcal{H}(z))]\mathcal{C}(z)dz + \int_{\overline{z}}^{\infty} [\mathcal{H}(z)+\nu(1-\mathcal{H}(z))]\mathcal{C}(z)dz$$

$$= \int_0^{\underline{z}} dz + \int_{\underline{z}}^{\overline{z}} \nu dz + 0$$

Household Consumption decision is simplified into choosing these thresholds.

$$u(\underline{z}, \overline{z}) = \underline{z} + \nu(\overline{z} - \underline{z}) \tag{20}$$

Income Maximization

Rewrite the household problem to income maximization problem:

$$\max_{\{f^h, f^l, n^h, n^l\}} f^h w^h [1 - \theta(f^h) - n^h] + f^l w^l [1 - n^l] \tag{21}$$

subject to

$$f^h n^h + f^l n^l = \int_0^\infty \mathcal{C}(z)\mathcal{H}(z)\frac{z^{\lambda_l}}{A} dz \tag{22}$$

$$f^h + f^l = 1 \tag{23}$$

now we find optimal policy functions f^h, f^l, n^h, and n^l $(n^h, n^l \geq 0)$ and choose allocation of C_G and C_S

Home Production Constraint with Proposition 1

$$\int_0^{\infty} \mathcal{C}(z)\mathcal{H}(z)\frac{z^{\lambda_l}}{A}dz = \int_0^{\underline{z}} \frac{\mathcal{C}(z)\mathcal{H}(z)}{Az^{-\lambda_l}}dz + \int_{\underline{z}}^{\overline{z}} \frac{\mathcal{C}(z)\mathcal{H}(z)}{Az^{-\lambda_l}}dz + \int_{\overline{z}}^{\infty} \frac{\mathcal{C}(z)\mathcal{H}(z)}{Az^{-\lambda_l}}dz$$

$$= \int_0^{\underline{z}} \frac{1}{A_l(z)}dz = \frac{1}{A(\lambda_l+1)}[z^{\lambda_l+1}]_0^{\underline{z}} = \underline{z}^{\lambda_l+1}/A(\lambda_l+1)$$

home production constraint becomes

$$f^h n^h + f^l n^l = \underline{z}^{\lambda_l+1}/A(\lambda_l+1)$$

Income Maximization

we can rewrite equation (23) as a household full-time labor income from market production and its opportunity cost from home production.

$$w^h f^h[1-\theta(f^h)] + w^l f^l - w^h f^h n^h - w^l f^l n^l$$

substitute out the last term using equation 20 and $f^l = 1 - f^h$,

$$w^h f^h[1-\theta(f^h)] + w^l(1-f^h) - w^h f^h n^h - w^l(\underline{z}^{\lambda_l+1}/A(\lambda_l+1) - f^h n^h)$$

rearranging term by binding n^h

$$\max_{\{f^h, n^h\}} w^h f^h[1-\theta(f^h)] + w^l(1-f^h) - w^l(\underline{z}^{\lambda_l+1}/A(\lambda_l+1)) - (w^h - w^l) f^h n^h$$

Income Maximization

Now Optimization problem is reduced to find policy function f^h and n^h

$$\max_{\{f^h, n^h\}} w^h f^h[1-\theta(f^h)] + w^l(1-f^h) - w^l(\underline{z}^{\lambda_l+1}/A(\lambda_l+1)) - (w^h - w^l)f^h n^h$$

we can easily see that n^h maximizes above equation when $n^h = 0$ since $w_h > w_l$ and $f^h > 0$

$$\max_{\{f^h\}} w^h f^h[1-\theta(f^h)] + w^l[1 - f^h - (\underline{z}^{\lambda_l+1}/A(\lambda_l+1))]$$

Income Maximization

$$\max_{\{f^h\}} w^h f^h[1-\theta(f^h)] + w^l[1-f^h-(\underline{z}^{\lambda_l+1}/A(\lambda_l+1))]$$

Differentiate with respect to f^h and set it to zero,

$$w^h \frac{\partial f^h[1-\theta(f^h)]}{\partial f^h} - w^l = 0$$

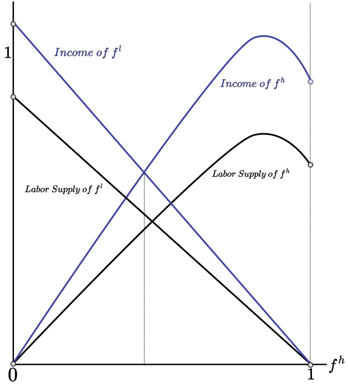
First Order Condition

$$w[1-\theta(f^h) - f^h\theta'(f^h)] = 1 \tag{24}$$

where $w = \frac{w^h}{w^l} > 1$ is the relative wage of high-skilled labor.

Graphical Analysis : Income Maximization Problem

What does FOC mean? Let's look at the graph.



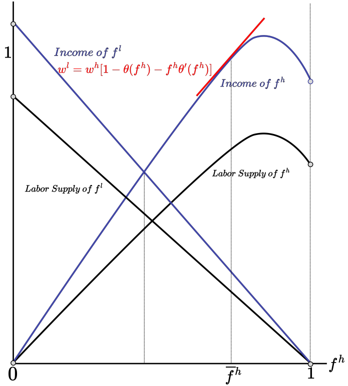
high-skilled labor follows:

- Labor supply of f^h : $f^h[1-\theta(f^h)]$.
- Income of f^h : $w^h f^h[1-\theta(f^h)]$.

low-skilled labor follows:

- Labor supply of f^l : $1 - f^h - (\underline{z}^{\lambda_l+1}/A(\lambda_l+1))$.
- Income of f^l : $w^l[1 - f^h - (\underline{z}^{\lambda_l+1}/A(\lambda_l+1))]$.
- Both labor income and labor supply are decreasing in f^h.

Graphical Analysis : Income Maximization Problem



FOC means: :

$$w^h \frac{\partial f^h[1-\theta(f^h)]}{\partial f^h} + w^l \frac{\partial[1 - f^h - (\underline{z}^{\lambda_l+1}/A(\lambda_l+1))]}{\partial f^h} = 0$$

$$w^h \frac{\partial f^h[1-\theta(f^h)]}{\partial f^h} - w^l \frac{\partial[f^l - (\underline{z}^{\lambda_l+1}/A(\lambda_l+1))]}{\partial f^l} = 0$$

$$\frac{MLS_{f^l}}{MLS_{f^h}} = \frac{w^h}{w^l} = w; \;\; \textit{skill premium}$$

marginal income of f^h = marginal income of f^l

- when marginal income of f^h is equal to marginal income of f^l, the household maximizes its income.

Graphical Analysis : Income Maximization Problem

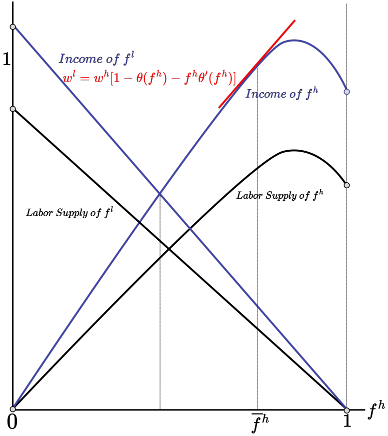


Figure: Labor Supply and Income by skilled fraction

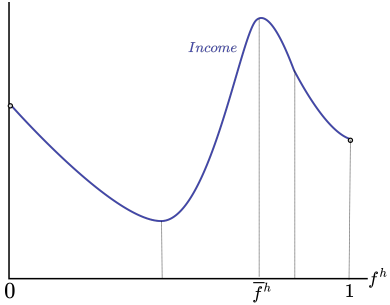


Figure: Household Income

Market Participation of low-skilled workers

Assumption

$$\frac{\phi f_0^h(1-\theta(f_0^h))}{1-f_0^h} < q \tag{25}$$

where f_0^h solves $\phi[1-\theta(f_0^h)-\theta'(f_0^h)f_0^h]$

The effective supply of skilled workers is not too large relative to the manufactruing requirements, q.

Aggregate non-market services at lower productivity level

There's no market service prodcution at lower productivity level A_0 and non-market services are produced only, which is composed of market goods and value-added home production.

$$\int_0^{\underline{z}(A_0)} g_N(z)dz = q \int_0^{\underline{z}(A_0)} A_l(z)n(z)dz$$

Since, in equilibrium, only low-skilled workers supply non-market labor,

$$\int_0^{\underline{z}(A_0)} g_N(z)dz = q \int_0^{\underline{z}(A_0)} A_l(z)n^l(z)dz$$

$$\int_0^{\underline{z}(A_0)} [A_l(z)L_G(z) + A_h(z)H_G(z)]dz = q \int_0^{\underline{z}(A_0)} A_l(z)n^l(z)dz$$

Aggregate non-market services at lower productivity level

Dividing both sides by $\int_0^{\underline{z}(A_0)} A_l(z)[L_G(z) + n^l(z)]dz$,

$$\int_0^{\underline{z}(A_0)} \frac{A_l(z)L_G(z)}{A_l(z)[L_G(z)+n^l(z)]}dz + \int_0^{\underline{z}(A_0)} \frac{A_h(z)H_G(z)}{A_l(z)[L_G(z)+n^l(z)]}dz = q\int_0^{\underline{z}(A_0)} \frac{A_l(z)n^l(z)}{A_l(z)[L_G(z)+n^l(z)]}dz$$

Since, at lower productivity A_0, $\frac{A_h(z)}{A_l(z)} = w = \phi$

$$\int_0^{\underline{z}(A_0)} \frac{L_G(z)}{[L_G(z)+n^l(z)]}dz + \phi\int_0^{\underline{z}(A_0)} \frac{H_G(z)}{[L_G(z)+n^l(z)]}dz = q\int_0^{\underline{z}(A_0)} \frac{n^l(z)}{[L_G(z)+n^l(z)]}dz$$

Rewriting the equation above using labor market clearing conditions,
$\int_0^{\underline{z}(A_0)} H_G(z)dz = f_0^h(1-\theta(f_0^h))$ and $\int_0^{\underline{z}(A_0)} L_G(z)dz + n_l = 1 - f_0^h$

Aggregate non-market services at lower productivity level

$$\frac{\phi f_0^h(1-\theta(f_0^h))}{1-f_0^h} = q\frac{n_l}{1-f_0^h} - \frac{1-f^h-n_l}{1-f_0^h}$$

$$\frac{\phi f_0^h(1-\theta(f_0^h))}{1-f_0^h} = q - \left(\frac{1+q}{1-f_0^h}\right)[(1-f_0^h)-n_l]$$

if $1-f_h = n_l$, then holds the equality and there's no market participation of low-skilled workers.
If $1-f_h > n_l$, then our assumption holds and low-skilled workers participate in the market labor.

PROPOSITION 2:

There exist productivity thresholds A_1 and A_2, such that:

(i) Consumption thresholds satisfy $\frac{\partial \underline{z}}{\partial A}\frac{1}{\underline{z}} = \frac{\partial \overline{z}}{\partial A}\frac{1}{\overline{z}}$ for $A < A_1$, and $\frac{\partial \overline{z}}{\partial A}\frac{1}{\overline{z}} > \frac{\partial \underline{z}}{\partial A}\frac{1}{\underline{z}}$ for $A \geq A_1$

(ii) The supply and price of skills satisfy $\frac{\partial f^h}{\partial A} = \frac{\partial w}{\partial A} = 0$ for $A < A_2$, and $\frac{\partial f^h}{\partial A} > 0$, $\frac{\partial w}{\partial A} > 0$ for $A \geq A_2$.

Proposition 2 with graphical representation

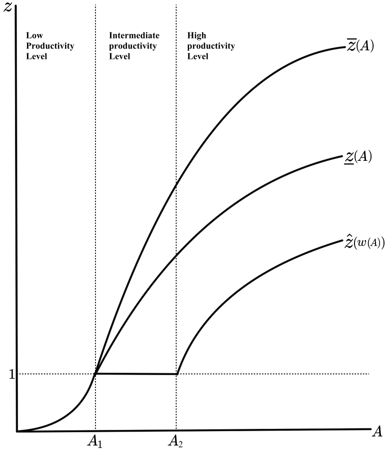


Figure: When Assumption

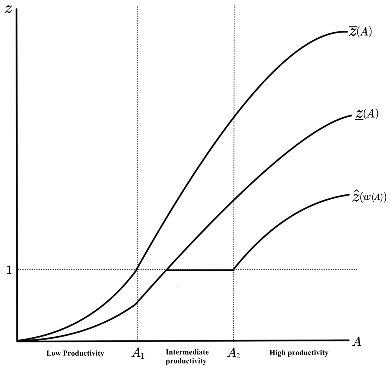


Figure: Without Holding Assumption

PROPOSITION 3:

Assume $A > A_2$ and $A_0 = A$. Then $\partial[P_S(A, A_0)/P_G(A, A_0)]/\partial A > 0$