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User Guided Image Warping

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Introduction

Image warping is a branch of image processing that deals with the geometric transformation of digital images. Warping can be used for creative purposes or for the correction of image distortion. Image processing software such as Photoshop, allows users to alter images by specifying a warp at a given point. However, this process may become arduous when the user is attempting to warp an image along a desired curve. The goal of this project is to create a method that will allow the user to warp e.g. shrink, stretch or pinch a specific part of an image along any specified spline while preserving the surrounding area. Ultimately this process will be applied to images of Classical Renaissance paintings with the goal of correcting perceived anatomical imperfections.

Implementation on *The Madonna with the Long Neck*

Fig 5: Spline applied to neck and face

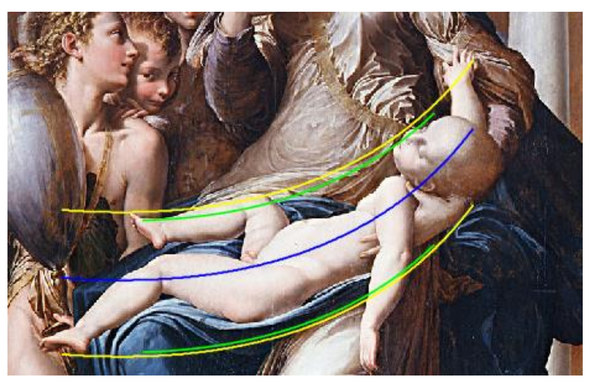

Fig 6: Spline (blue) pinched along baby, contours (yellow) and transformed splines(green)

Methodology

Block diagram

User specified sample points along spline

Parametric representation of spline → **Transform parametric equations** → **Differential equations propagate pixel displacement around spline.** → **Use resulting propagation values for bilinear interpolation in original image**

Transforming a Spline

- User takes sample points along spline
- Parametric representation of line is created.
- Least Squares is used to fit lines to the parametric equations
- The parametric lines are altered according to user's desires
- Vector fields store x and y values of pixel displacement
- $displacement(x, y) = (x', y') - (x, y)$

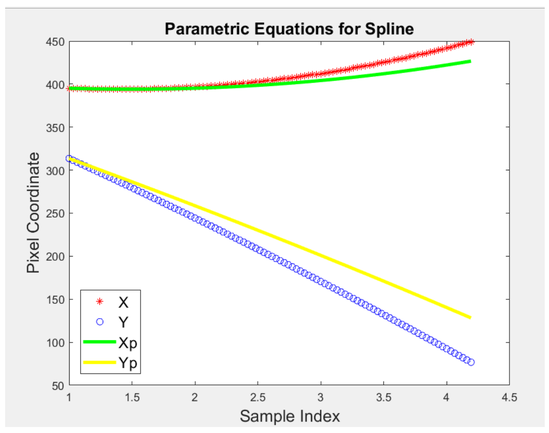

Figure 1

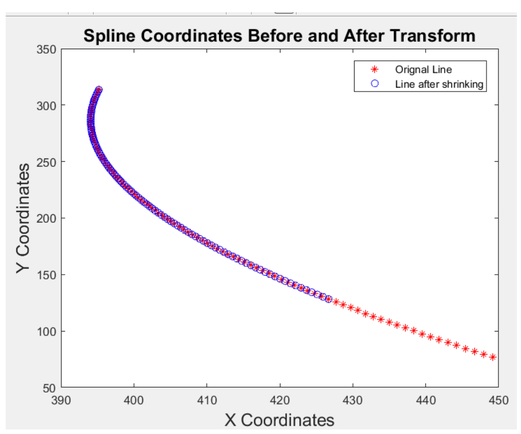

Figure 2

Using Partial Differential Equations to Fill in Vector Field

- The pixel displacements are known along the spline and at a boundary around warped area (where displacement is 0). A Partial differential equation can be used to fill in the unknown displacements.

$$\nabla^2 D(x,y) = \frac{\partial^2 D}{\partial x^2} + \frac{\partial^2 D}{\partial y^2} = f(x,y)$$

- Where D *[x,y]* is the displacement in either the horizontal or vertical direction of a pixel at position *(x,y)* after the warping process.
- $f(x,y) = 0$ for Laplace equation
- This is done for both the vertical and the horizontal displacements.

Successive Over relaxation (SOR)

- SOR is used to generate a numerical solution to the partial differential equations

$$D^{K+1}[y,x] = D^{K}[y,x] + a\frac{E^{K}[y,x]}{4}$$

- Where $E^K[Y,X]$ is error between the 2nd order Finite element approximation and the current pixel displacement, k is the iteration and a is the relations parameter.(2)

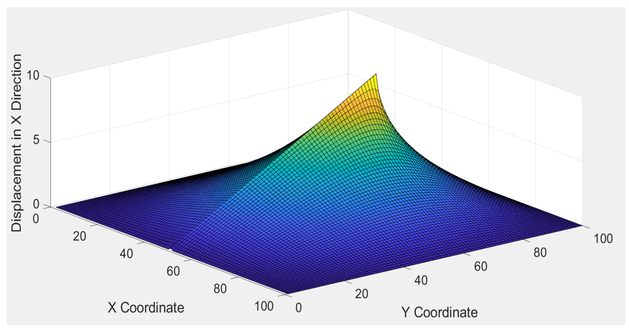

Fig 3: *Displacements around spline after PDE has been solved*

- The standard deviation of the error of between each iteration corresponds to convergence

Contour Lines

- Contour lines are added to alter the manner in which the displacement values are propagated.
- Preserves areas with a lot of detail, such as a face.

Bilinear Interpolation

- A pixel's intensity after the warp is found through interpolation.
- The input product is the original image and the output is the displacement field.

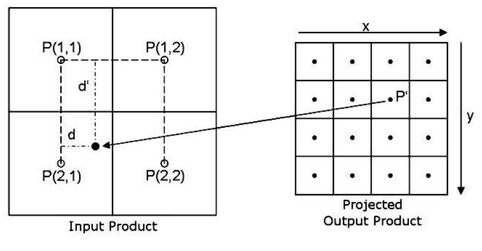

Fig 4

Results

Figure 7: Vector field(red) showing propagation of displacements around image

Fig 8: Before Warp

Fig 9: After Warp

Comments

- An appropriate force function f for Poisson would allow improvements to vector propagation. Perhaps f could correspond to pixel intensity variance and therefore preserve areas with a lot of texture.
- Computational speed depends largely on SOR method
- Decrease time for SOR convergence, by varying a
- Overall, method performs well and achieves desired goals.

References

1)George Wolberg. 1990. Digital Image Warping. IEE Computer Society Press Monograph.

2) Larry A. Glasgow. Applied Mathematics for Science and Engineering,1.

Painting used: The Madonna with the Long Neck by Parmiganino. c. 1535-1540