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Optimal lockdown policy for vaccination during COVID-19 pandemic

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A R T I C L E I N F O

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A B S T R A C T

As the COVID-19 spreads across the world, many nations impose lockdown measures at the early stage of the pandemic to prevent the spread of the disease. Controversy surrounds the lockdown as it is a choice between economic freedom and public health. The ultimate solution to a pandemic is to vaccinate a massive population to achieve herd immunity. However, the whole vaccination programme is a long and complicated process. The virus and the vaccine will coexist for quite a long time. How to gradually ease the lockdown based on vaccination progress is an important question, as both economic and epidemiological issues are involved. In this paper, we extend the classic SIR model to find optimal decision to balance between economy and public health in the process of vaccination rollout. The model provides an approach of vaccine value estimation. Our results provide scientific suggestion for policymakers to make important decisions on how to gradually relax the strength for the lockdown over the entire vaccination cycle.

1. Introduction

Since reaching over 100 million infections and 3 million deaths, the outbreak of Coronavirus (COVID-19) is one of the most devastating pandemics in human history. We are still in the midst of the global outbreak, which has led the world to severe economic decline and caused devastating damage to the many nations (Sułkowski, 2020). The global economy has been divided into its pre-pandemic and post-pandemic phases (Zhang et al., 2020). The financial markets are significantly influenced by COVID-19 with substantially increased volatility (Albulescu, 2020) and reduced market efficiency (Wang and Wang, 2020). In an attempt to curb the spread of the disease, the governments impose a lockdown measure at the early stage of the pandemic. Without vaccination, lockdown is more effective at mitigating the effects of COVID-19 exposure than other measures like travel bans and economic stimulus packages (Narayan et al., 2020). Several countries are in and out of the lockdown many times, which has a considerable impact on financial markets. Lockdown increases stock market volatility and liquidity (Baig et al., 2020). The stringency of the lockdown has a two-way effect on the financial markets (Aggarwal et al., 2020).

Lockdown is a dilemma between economic freedom and public health. If countries do not release lockdown as soon as possible, it will lead to economic stagnation. On the other hand, several countries open too early to prevent a recession, which triggers the numerous waves of the outbreaks. Some countries have recently gone back and forth several times from lifting the ban to the reinstatement of the lockdown. For example, the UK has been through the third period to enforce a mandatory lockdown. Therefore whether and how to impose and release lockdown should take into account the public welfare and economic benefits.

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The ultimate solution to a pandemic is to vaccinate massive population to achieve herd immunity. Recent research into COVID-19 and vaccine production has made significant progress. The current vaccine research has been in the development and production process. The regulators have approved a range of vaccines, and many countries are in the process of a mass vaccination rollout. Despite the expense and safety issues with vaccines (Forni and Mantovani, 2021), a successful vaccination programme is only known to be a conclusive milestone that will end the pandemic and result in a robust economic recovery. Vaccines significantly mitigate the adverse impact of previous pandemics (Gong et al., 2020). Forecasts suggest that there will be high economic costs in the absence of global vaccinations of COVID-19 (Çakmaklı et al., 2021). The vaccine is also an essential factor driving stock market prices (Acharya et al., 2020).

From an economic point of view, vaccination is possibly a powerful way to restore economic and social normality. Lockdown only slows down the spread of epidemic while vaccination prevents people from having the disease, thus free people for working and consumption. Many people assume that vaccination would protect them, and lockdown should be relaxed instantly after vaccination. However, there are billions of people in the world waiting to receive the vaccine and the production of vaccine is still limited. The whole vaccination rollout is long and complicated. In the future, we will live with the virus and the vaccine for quite some time. Considering the vaccination progress and the quarantine cost, lockdown will be gradually lifted when vaccination is available.

How to gradually ease the lockdown based on the vaccine progress is an important question as of this time. There is a need for a quantitative model to address the above question that policymakers may face in reality. We extend the classic SIR model (Harko et al., 2014; Kermack and McKendrick, 1927) and incorporate an equilibrium framework to study the optimal lockdown policy during the pandemic period. The model can be used to study the role of lockdown policy in the pandemic by tracking vaccination progress. Our results suggest that it is crucial to gradually relax the strength for the lockdown policy during vaccination. Our model also provides an optimal estimation on a process to release the lockdown.

Several studies have indicated that incorporating vaccination and lockdown is beneficial. Garriga et al. (2020), Alvarez et al. (2020), Acemoglu et al. (2020) include the vaccination in epidemiological models to study the optimal lockdown control policy. It assumes the arrival of vaccination as a Poisson process and once the vaccination is available, people get vaccination immediately. However, the second assumption is an over-simplification. In reality, due to the limited productivity and speed of vaccination rollout, it often takes not a short time to get vaccination for massive population.Different from previous works, we assume the vaccination is already available at the beginning and focus on the the impact of the vaccination on the lockdown policy.

The expenditure on the vaccine is a problem for policymakers to consider when there are various kinds of vaccines available in the market. We provide an approach for vaccine value estimation based on utility indifference. The estimation result can be a reference for policymakers to choose different vaccines. Under the parameter setting in our simulation, the estimated value of the vaccine is approximately at the middle of vaccine prices in the current market.

This paper continues as follows, in Section 2, we describe the SIR-Lockdown model with vaccination. In Section 3, we discuss the parameter estimation of the model. We present the numerical results of the SIR-Lockdown model with vaccination in Section 4. Section 5 makes conclusions.

2. Model

Epidemiological models have been widely studied in literature to analyse the dynamics of the pandemic (Kucharski et al., 2020; Liu et al., 2020; Wang et al., 2020; Wu et al., 2020) and search for optimal lockdown policy (Alvarez et al., 2020; Gonzalez-Eiras and Niepelt, 2020; Acemoglu et al., 2020). However, there is less discussion on the optimal lockdown policy during vaccination, which is carried out in many countries and is hopefully the best way out of COVID-19 pandemic. In this section, we extend the canonical SIR model by including the effect of vaccine to balance the economic welfare and public health during the outbreak of COVID-19. We also introduce an estimation of vaccine value based on our model.

2.1. Epidemiology model

As shown in the classic SIR model (Kermack and McKendrick, 1927; Hethcote, 1989), we classify people into three categories according to Harko et al. (2014):

- Susceptible (S) are those who have not been tested positive to the virus.
- Infectious (I) are those who are tested positive to the virus;
- Recovered (R) are those who have been tested positive to the virus and now recovered;

In a certain unit of time, we assume that all susceptible people are subjects to be infected with some probability in direct contact with infectious people, and infectious people will recover with a constant probability of π_r or die with another constant probability π_d. All infection happens via direct contact between susceptible people and infected ones into three types of activities: purchasing and/or consumption of goods and services, working with other people, and other daily activities. Vaccination is an effective method of preventing the spread of infectious diseases.

When there is no vaccination, people become immune to the virus only through getting infected and then recovered. We now include vaccination into the SIR model, with which people can become immune by a safe vaccination.

We assume that at each unit time period, a fix amount δ_v of susceptible people get vaccination, and hence swith to the class of recovered from the next unit time period.

We use the following equation (1–5) to describe our extended SIR model for the transition among four groups, i.e., the Susceptible, the Infected, the Recover, and the Death outcome.

$$T_t = \pi_{s1}\left(S_t C_t^s\right)\left(I_t C_t^i\right) + \pi_{s2}\left(S_t N_t^s\right)\left(I_t N_t^i\right) + \pi_{s3} S_t I_t, \tag{1}$$

$$S_{t+1} = S_t - T_t - \delta_v, \tag{2}$$

$$I_{t+1} = I_t + T_t - (\pi_r + \pi_d) I_t, \tag{3}$$

$$R_{t+1} = R_t + \pi_r I_t + \delta_v, \tag{4}$$

$$D_{t+1} = D_t + \pi_d I_t. \tag{5}$$

In this system of equations, S_t, I_t, R_t and D_t represents the number of people in categories of Susceptible, Infectious, Recovery and Death respectively at time t. We use (C_t^s, N_t^s) to model the (average) consumption behaviour and working hours of susceptible people. Similarly we use (C_t^i, N_t^i) for infectious people and (C_t^r, N_t^r) for recovered people. T_t in Eq. (1) models the number of newly infectious people in the time period t to $t+1$ through direct contact between infectious and susceptible people. There are three types of direct contact, which are for consumption, for working, and for other purpose of daily life. We use 3 constant parameters to describe the contribution of the 3 types of contact in the infection rate, π_{s1} for the purchasing/consuming, π_{s2} for the working, and π_{s3} for other contact.

2.2. Preference model

In this section, we introduce the preference model of the three categories of people (S,I,R)and analyse their rational behaviour in the epidemiology model.

2.2.1. Behaviour of individuals in different categories

We study the rational behaviour of all people who maximise their own welfare by choosing proper consumption and working hours like in a normal time.[1] We use the following utility function to model the utility from consumption and working of an individual,

$$u(c,n) = \ln c - \frac{\theta}{2} n^2 \tag{6}$$

where c is the consumption, and n is the working hours. In this utility, the first term measures the utility from consumption, the second term measures the disutility from working, and θ is the weight between this two terms.

Denote by A the average wage per hour of a person, then the labor income of an individual with working hour n is $A * n$, which will be the upper bound of the consumption, i.e. $c \leq An$.

The parameter θ is not emperially easy to estimate. We fix it by assuming that the prevailing behaviour of consumption and working are optimal. Denote by n_0 the full working hours in a unit time before the spread of the virus, which is officially guided by the government. It is natural that n_0 is set optimally for individuals. If a person follows the full working hours n_0 optimally, then her labor income will be An_0. Since the utility function is strictly increasing in the consumption, all labor income should be consumed up, hence the optimal consumption c_0 should be $c_0 = An_0$. By the assumed optimality of n_0, we have $\frac{\partial u(c_0,n_0)}{\partial n} = \frac{1}{An_0} - \theta n_0 = 0$, by which we choose

$$\theta = 1/\left(An_0^2\right)$$

Given a terminal time T, the total utility of a flow of consumption and working hours $\{(c_\tau, n_\tau)\}_{\tau=t,\cdots,T}$ is defined by

$$U(c_., n_.) = \sum_{t=\tau}^{T} \beta^\tau u(c_\tau, n_\tau) \tag{7}$$

To contain the spreading of the virus, governments need to apply a lockdown policy to reduce direct contacts between people, which will impose stricter constraints on their behaviour. In this paper, we study the lockdown policy by a constraint on the ratio $L \in [0,1]$ of the working hour in the full working capacity, i.e., given the full working hours n_0, the maximal working hour cannot exceed $n_0 * L$. We suppose the government cannot easily identify individuals into their categories so that the lockdown constraint on the working hours is the same for all people who are interested in their own total utility. We formulate the decision making problem for each category with a given lockdown policy $L_.$, and then study the lockdown policy-making problem for the government. It turns out that the optimal behaviour for infected and recovered people are quite straightforward, to consume and work as much as they are allowed to do. While for susceptible people, their optimal working hour may be strictly less than being allowed to do. We summarise our this properties on these optimal behaviour (the proof is deferred to the supplementary).

[1] We implicitly assume that the virus does not change people's rationality and preference.

- At time t with state X_t and the lockdown policy $\{L_\tau : \tau \in [t, T]\}$, the optimal behaviour (c^r, n^r) for recovered people is

$$c_\tau^{r*} = An_0L_\tau, n_\tau^{r*} = n_0L_\tau, \quad \tau = t, \cdots, T.$$

- Given the lockdown policy $L_\cdot$, the optimal behaviour (c^i, n^i) for infectious people is

$$c_\tau^{i*} = A\phi n_0L_\tau, n_\tau^{i*} = n_0L_\tau, \quad \tau = t, \cdots, T. \tag{8}$$

- Given the lockdown policy $L_\cdot$, the optimal (c^{s*}, n^{s*}) for susceptible people must satisfies

$$c_\tau^{s*} = An_\tau^{s*}, \quad \tau = t, \cdots, T. \tag{9}$$

Given the trivial optimal behaviour $(c_\cdot^{i*}, n_\cdot^{i*})$ for infected people and $(c_\cdot^{r*}, n_\cdot^{r*})$ for recovered people, we denote by $J^{i*}(t, L_\cdot)$ and $J^{i*}(t, L_\cdot)$ the optimal total utility from t to T for these two types of people respectively. For a susceptible people, since she may be infected or vaccinated, her total utility will involve the optimal total utilities of infected people and recovered people. Suppose her behaviour flow is described by $(c_\cdot^s, n_\cdot^s)$, then, her total utility $J^s(c_\cdot^s, n_\cdot^s; X_t, L_\cdot)$ should satisfy

$$\begin{aligned} J^s\left(c_\cdot^s, n_\cdot^s; t, X_t, L_\cdot\right) &= u\left(c_t^s, n_t^s\right) + \beta\left(1 - \frac{\delta_v}{S_t}\right)\tau_t J^{i*}(t+1, L_\cdot) \\ &+\beta\left(1 - \frac{\delta_v}{S_t}\right)(1-\tau_t)J^s\left(c_\cdot^s, n_\cdot^s; t+1, X_{t+1}, L_\cdot\right) \\ &+\beta\frac{\delta_v}{S_t}J^{r*}(t+1, L_\cdot). \end{aligned} \tag{10}$$

Hence the optimal behaviour for susceptible people can be formuated as

$$\text{Maximise}_{n_\cdot^s} J^s\left(An_\cdot^{s*}, n_\cdot^s; t, X_t, L_\cdot\right) \tag{11}$$

since the optimal consumption $c_\tau^{s*} = An_\tau^{s*}$ is trivially determined by the working hour $n_\cdot^s$

2.2.2. Optimal control of the policymaker

With the optimal behaviour in each category under a given lockdown policy $L_\cdot$, we can easily formulate the optimal policy-making problem into an optimal control problem.

Suppose we start the lockdown problem from some time t_0 with the contamination state X_{t_0} being given by $S_{t_0} = s, I_{t_0} = i$ and $R_{t_0} = r$, then the optimal lockdown policy should be the optimal control for

$$\max_{L_\cdot} J^0(L_\cdot; t, X_t) = \sum_{t=t_0}^{T} \beta^{t-t_0}\left[S_t u\left(c_t^{s*}, n_t^{s*}\right) + I_t u\left(c_t^{i*}, n_t^{i*}\right) + R_t u\left(c_t^{r*}, n_t^{r*}\right)\right], \tag{12}$$

where (c_t^{ca*}, n_t^{ca*}) are the optimal consumption and working hours for people in category ca (ca can be s, i or r), which are all determined in previous optimisation problems.

In previous objective J^0, we remove all cases of death without any penalty. In reality, since death of disease has a strong negative impact to a household as well as to the society, regulators should not ignore any death case. We correct the previous objective by introducing a heavy penalty, and dedine the new objective

$$J^\lambda(L_\cdot; t, X_t) = \sum_{\tau=t}^{T} \beta^{\tau-t}\left[S_\tau u\left(c_\tau^{s*}, n_\tau^{s*}\right) + I_\tau u\left(c_\tau^{i*}, n_\tau^{i*}\right) + R_\tau u\left(c_\tau^{r*}, n_\tau^{r*}\right) - \lambda D_\tau u\left(c_\tau^{r*}, n_\tau^{r*}\right)\right], \tag{13}$$

where the parameter $\lambda \geq 0$ describes how serious we are on death from the virus. In this new objective, we measure the the cost of a death by a multiple of the optimal utility for a recovered people, and the multiple $\lambda > 0$ can be viewed as the severity of death in the government's view. When $\lambda = 0$, J^λ reduces to our previous objective J^0.

With this new objective, the problem for a regulator to solve is

$$\begin{aligned} \max_{L_\cdot} \quad & J^\lambda(L_\cdot; t, X_t), \\ s.t. \quad & L_t \in [0, 1] \ \ \forall t \in [0, T]. \end{aligned} \tag{14}$$

Table 1
Model parameters.

parameter	value
π_1	1.244887×10^{-6}
π_2	1.0336×10^{-4}
π_3	0.01759
π_d	0.00233
π_r	0.38656
n_0	36.9
ϕ	0.8
θ	$1/(36.9)^2$
δ_v	$1/104$

2.3. A model application: value of vaccine

We introduce a method for value estimation of vaccination based on the proposed model. It is based on utility indifference, i.e, the aggregated expected utility of the whole population would be the same w/o the vaccination at the end of the epidemic. Then the gap between the accumulated aggregated utility w/o vaccination can be regarded as the utility of the vaccine value. Denote the value of vaccination as v, the overall percentage of the population that accepted the vaccination as N_v, accumulated aggregated utility as V_v and V_n from start to the end of the epidemic w/o vaccination. Then by utility indifference, we have

$$\log(vN_v) = V_v - V_n \tag{15}$$

For the utility function of vaccination, since we use log in our model as the utility function of consumption as in Eq. (6), we also apply it here as the utility function of vaccination value. By Eq. (15), we get the estimation of vaccine value v.

3. Solving the model

3.1. Solving scheme

In Problem (14), or its reduced version (12), the optimal decisions of individuals in all three categories are involved. Fortunately, the optimal decisions of recovered and infectious people are trivial due to our good structure of the model, which leaves us to tackle the optimal decision Problem (11) for susceptible people before the Problem (14).

We start our solving scheme by tackling the Problem (11) with a given lockdown policy $L_\cdot$. Because of the lockdown constraint, it is almost hopeless for us to get an explicit solution. We solve this optimal control problem numerically in the same was as in Eichenbaum et al. (2020). In this approach, the optimal control at each time step is regarded as the static optimisation with two constraints from the consumption budget and the lockdown policy on the working hours, and solutions are obtained by solving the corresponding KKT condition.[2]

With the optimal control $(c_\cdot^{s*}, n_\cdot^{s^*})$ as functions of the lockdown policy $L_\cdot$, we deal with the optimal control problem (14) as an optimisation over the high dimension space $[0,1]^T$ by the gradient-based interior-point method used in the Matlab function `fmincon`. Although we have no theoretical proof on the convergence of our scheme, our numerical results show the convergence of our scheme.

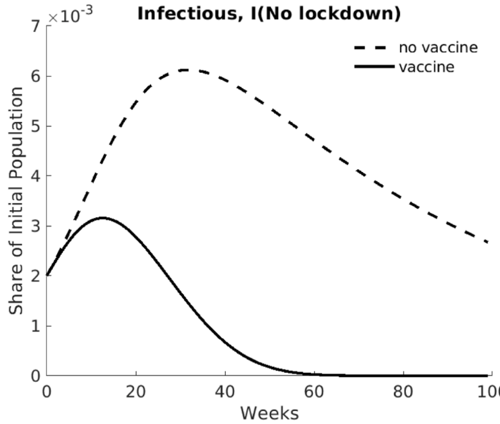
Parts of our code in our scheme are from Eichenbaum et al. (2020).

3.2. Model parameters

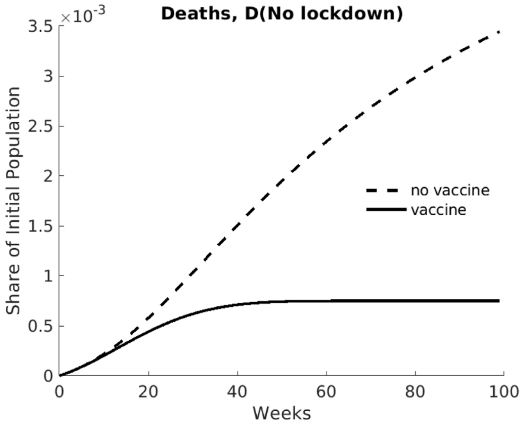
In this subsection, we specify the parameters in our model for simulation. The parameter estimation is based on COVID data in the UK. Detailed parameter estimation process refers to Fu et al. (2020)

For the simplicity of the epidemiology model, we set the starting population as 1, and all quantities for numbers of people can be regarded as the percentage of the total population at the initial time $t = 0$. Furthermore, we set the unit time period as 1 week, which is long enough to avoid delay in recording data, and is frequent enough to show the changes of the epedemic. Table 1 lists the commonly used parameters and values in our numerical experiments.

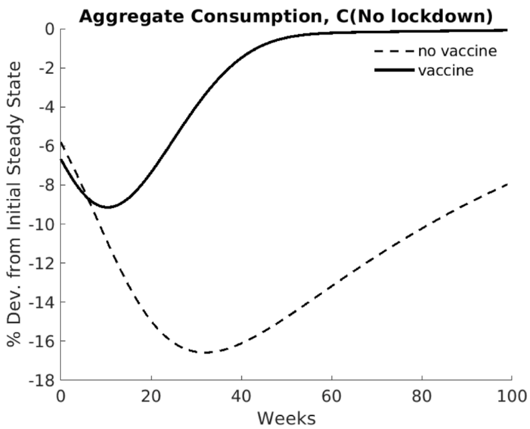
[2] In fact, when we use the numerical scheme proposed in Eichenbaum et al. (2020) to our problem, the derivative used in the KKT condition is not correct due to the absence of a complicated term from the term in Eq. (10). We decide to ignore this absence due to the following two reasons: (1) if we recover this complicated term, the calculation will be extremely complicated; (2) from real data in the COVID-19 pandemic, we know the coefficient in the third term $\beta\tau_t)$ is very close to 0, which is also observed in our numerical results.



(a) Infectious



(b) Deaths



(c) Aggregated Consumption

Fig. 1. Effect of Vaccination on COVID-19 without Lockdown.

4. Numerical results

In this section, we present the result of our numerical experiments under the parameter setting in Section 3.2. By comparing these numerical experiments, we claim the necessity of lockdown when vaccination is available.

For every experiment presented in this section, the initial state is set as $(S, I, R) = (0.998, 0.002, 0)$. The epidemic stops when the whole population become recovered. As we set $\delta_v = 1/104$ in simulation, it takes around 100 weeks to end the simulation without the lockdown control assuming vaccination is available. All lockdown in this section refer to the optimal lockdown that maximises the utility of the whole population defined in Section 2.

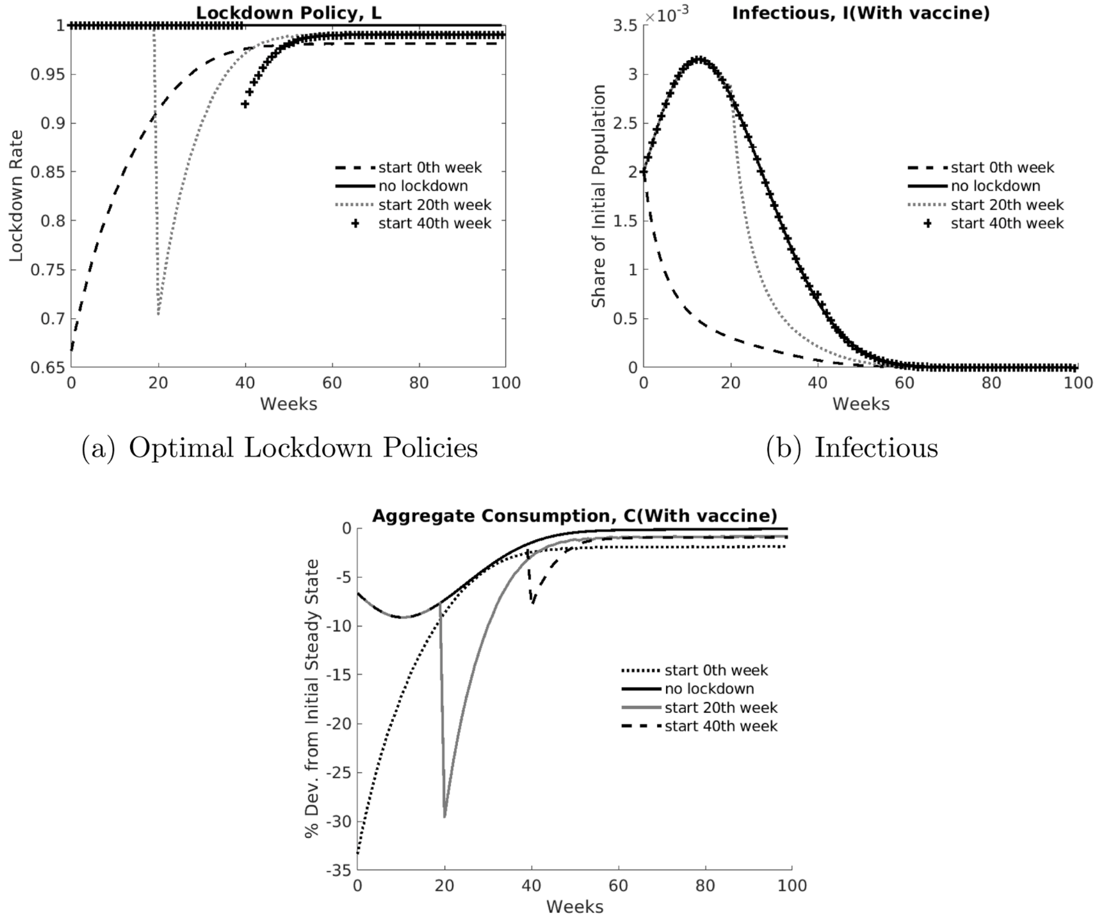
4.1. Effect of vaccination without lockdown

First, we compare the effect of the vaccine on the development of the pandemic in the absence of lockdown. As shown in Fig. 1(a) vaccination significantly reduces the infection between the susceptible and suppresses the growth of the infectious. Fig. 1(b) also indicates that the number of death is significantly reduced by the vaccine, which speeds up the ending of the epidemic by converting the population of susceptible to the recovered. As more people become recovered, they will not cut back their working hours and consumption, therefore the aggregated consumption increases in Fig. 1(c).

4.2. Effect of optimal lockdown with vaccination

In this section we analyse the effect of optimal lockdown on the tendency of COVID-19 assuming vaccination is available. We apply the solving scheme with parameters in Section 3 to obtain the optimal lockdown policy as shown in Fig. 2(a).

It is illustrated in Fig. 2(b) that by restricting work hours and decreasing consumption, the optimal lockdown policy significantly



(a) Optimal Lockdown Policies (b) Infectious

(c) Aggregated Consumption

Fig. 2. Effect of Optimal Lockdown on COVID-19 with Vaccination.

Table 2
Reduction of Expected Utility of the Susceptible (S), the Infectious (I), and the Recovered (R).

People Category	S	I	R
Rational Susceptible With Lockdown	3.8	152	2.2
Rational Susceptible Without Lockdown	10.6	149.9	0
Irrational Susceptible With Lockdown	3.8	152	2.2
Irrational Susceptible Without Lockdown	28.1	149.9	0

Table 3
Reduction of Accumulated Utility of the Susceptible (S), the Infectious (I), and the Recovered (R).

Initial Infectious Population	0.002	0.005	0.01
Rational Susceptible	3.9	5.9	7.1
Irrational Susceptible	8.7	17.2	25.4

reduces the risk of interaction between the susceptible and the infectious while slowing down the spread of the disease. For the recovered and infectious, optimal lockdown restricts their working hours and consumption. For the susceptible, if there is no lockdown, the susceptible would cut back their working hours and consumption to reduce their probability of becoming infectious. The optimal lockdown forces the susceptible to reduce even more of their working hours and consumption.

The optimal lockdown policies maximise the utility of the overall population with cost on consumption loss. Fig. 2(c) presents a clear difference between the aggregated consumption of the lockdown with no lockdown case. It is necessary to lockdown at the start of

Table 4
Vaccine company and price.

Company	Price(£)
AstraZeneca	2.2-2.9
Johnson & Johnson	7.3
Novavax	11.6
Pfizer	15
Moderna	25

the epidemic when vaccination is available. The lockdown benefits the susceptible people and slight detriments the infectious and recovered in terms of expected utility (see Table 2). In reality, susceptible people are not likely to be perfectly rational that maximise their utility exactly due to various reasons. For example, people may want to consume more for their mental needs during the isolation in the pandemic. In an extreme case, the susceptible being irrational that consume and work as usual time (see Table 2), then the optimal lockdown becomes even more necessary in benefiting the utility of the susceptible. As the susceptible people dominate the population at the start, it is reasonable to optimise their utility in the view of policymakers to maximise the utility of the whole population.

Table 3 shows the lockdown with more severe the initial situation, i.e., the larger the infectious population, the more necessary to impose lockdown. In the extreme case that susceptible people do not cut back their working hours and consumption, the lockdown will become more effective.

4.3. *Early/Late start of lockdown*

We have shown that lockdown is appropriate at the early stage of vaccination. However, in fact, it takes time for policymakers to complete regulations for the lockdown. Here we discuss if it is necessary to lockdown in the middle of the vaccination process. We focus on the impact of the delayed lockdown on the the vaccination process.

As shown in Fig. 2(b) the earlier the lockdown starts, the more effective it will be. Fig. 2(c) implies that when optimal lockdown policy starts from week 0 or 20, there is a notable difference on aggregated consumption with the no control case. However the lockdown policy has marginal influence after week 40.

The optimal lockdown level reduces when it starts late as shown in Fig. 2(a). This might because with a stricter lockdown, the benefit of lockdown on the utility of susceptible will be surpassed by the detriment of the lockdown on the recovered. Also, the stricter lockdown makes the overall utility even worse with the decreasing susceptible and increasing recovered population.

The results in Fig. 2(c) are useful for the policymakers to access the situation and decide how and when to start or relax the lockdown policy. The working hours and consumption of the susceptible returning to normal is a sign of releasing lockdown as shown in Fig. 2(c). As the infectious population decreases, the probability of being infected drops. Consequently, susceptible people will become less afraid of being infected, which raises their working hours and consumption back to normality.

4.4. *Vaccine value estimation*

In this subsection, we apply the vaccine value estimation approach in Section 2.3 under the parameter setting of our simulation in Section 3.2. To get the vaccine value estimation by Eq. (15), we specify how to estimate V_v and V_n in Eq. (15) in our simulation. According to results in Section 4.1, at 100th week, the epidemic ends in the case with vaccination, for which V_v is available. However, the epidemic is far from ending in the case without vaccination. We assume the vaccination is available at week 100th that force an ending of the epidemic, the accumulated aggregated utility, in this case is an estimation of V_n. Then by Eq. (15) we can get the estimation of v.

Starting with infectious population 0.002, the estimated value of vaccine is 11.5. Our estimation is about at the middle of the available vaccine prices in the market (Table[3] 4).

Need to mention that the estimation of vaccine value is based on our model, which is an over-simplification of reality, for instance, the model does not include people's saving behaviour. The estimated vaccine value can be used as a reference for policymakers in the vaccine spending.

5. Conclusion

In this paper, we study the relation between vaccination and lockdown policy and address the question on how to gradually relax the strength for the lockdown when vaccination is available. We extend the canonical epidemiological model SIR to find an optimal decision making with the aim to balance between economy and public health. In the model, people in different health statuses take different decisions on their working hours and consumption to maximise their own utility, while policymakers control the lockdown

[3] Data Source: https://www.bmj.com/content/371/bmj.m4670 https://www.healthline.com/health-news/how-much-will-it-cost-to-get-a-covid-19-vaccine

rate to maximise the overall welfare. This leads to a two phases optimisation problem. The model can also provide an estimation of vaccine value. Our results show that although vaccination can effectively reduce the deaths and infections caused by the COVID-19, lockdown is still necessary at the beginning of the vaccination rollout. The larger the infectious population, the more rigorous the optimal lockdown are required during vaccination. The results provides scientific suggestions for policymakers to make decisions on when to start and release the lockdown policy during the whole vaccination cycle.

CRediT authorship contribution statement

Yuting Fu: Data curation, Writing - original draft. **Hanqing Jin:** Supervision, Methodology, Writing - review & editing. **Haitao Xiang:** Conceptualization, Methodology. **Ning Wang:** Supervision, Methodology, Writing - review & editing.

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Supplementary material

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