NP 引理 设总体 X 的密度函数或质量函数为 $p(x;\theta)$，θ 为未知参数，似然函数为 $L(\theta)=\prod_{i=1}^{n}p(x_i;\theta)$。给定 $\alpha\in(0,1)$，设 $W_0=\left\{(X_1,X_2,\cdots,X_n):\dfrac{L(\theta_1)}{L(\theta_0)}\geq\lambda_0\right\}$满足

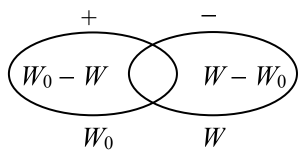
$$P\{(X_1,X_2,\cdots,X_n)\in W_0\mid\theta=\theta_0\}=\int\cdots\int_{W_0}L(\theta_0)dx_1\cdots dx_n=\alpha,$$

则对任何拒绝域 $W\subset R^n$，只要 $P\{(X_1,X_2,\cdots,X_n)\in W\mid\theta=\theta_0\}\leq\alpha$，则必有

$$P\{(X_1,X_2,\cdots,X_n)\notin W_0\mid\theta=\theta_1\}\leq P\{(X_1,X_2,\cdots,X_n)\notin W\mid\theta=\theta_1\}。$$

即 W_0 是所有显著水平为 α 的拒绝域中犯第二类错误的概率 β 最小的一个。

证明：设 W 是任一满足 $P\{(X_1,\cdots,X_n)\in W\mid\theta=\theta_0\}\leq\alpha$ 的拒绝域，则



$$\begin{aligned}
&P\{(X_1,\cdots,X_n)\in W_0\mid\theta=\theta_1\}-P\{(X_1,\cdots,X_n)\in W\mid\theta=\theta_1\}\\
&=\int_{W_0}\cdots\int L(\theta_1)dx_1\cdots dx_n-\int_{W}\cdots\int L(\theta_1)dx_1\cdots dx_n\\
&=\int_{W_0-W}\cdots\int L(\theta_1)dx_1\cdots dx_n-\int_{W-W_0}\cdots\int L(\theta_1)dx_1\cdots dx_n\\
&\geq\int_{W_0-W}\cdots\int \lambda_0L(\theta_0)dx_1\cdots dx_n-\int_{W-W_0}\cdots\int \lambda_0L(\theta_0)dx_1\cdots dx_n
\end{aligned}$$

（注：在 W_0 之中 $L(\theta_1)\geq\lambda_0L(\theta_0)$，在 W_0 之外 $L(\theta_1)<\lambda_0L(\theta_0)$）

$$=\lambda_0\left[\int_{W_0-W}\cdots\int L(\theta_0)dx_1\cdots dx_n-\int_{W-W_0}\cdots\int L(\theta_0)dx_1\cdots dx_n\right]$$

$$=\lambda_0\left[\int_{W_0}\cdots\int L(\theta_0)dx_1\cdots dx_n-\int_{W}\cdots\int L(\theta_0)dx_1\cdots dx_n\right]$$

$$=\lambda_0[\alpha-P\{(X_1,\cdots,X_n)\in W\mid\theta=\theta_0\}]\geq 0,$$

即 $P\{(X_1,\cdots,X_n)\in W_0\mid\theta=\theta_1\}\geq P\{(X_1,\cdots,X_n)\in W\mid\theta=\theta_1\}$，

故 $P\{(X_1,\cdots,X_n)\notin W_0\mid\theta=\theta_1\}\leq P\{(X_1,\cdots,X_n)\notin W\mid\theta=\theta_1\}$。