2021 复变函数期中测试

姓名 _____ 学号 专业

				总成绩	
		_	三	鼠脱霜	
			_	10.7%	

填空题(本题共10小题,每小题3分,满分30分. 把答案填在前面空白处):

$$i^i = e^{i k i}$$

1.
$$i^i =$$
 $e^{i | L |}$
2. $Im(i + \overline{z}) \stackrel{3 |}{=} 4$ 所描述的曲线方程为______.

3.
$$\sqrt[4]{-2i} = \sqrt{|-1|} (\omega) + (-\frac{1}{2} + 1)$$

4.
$$Ln(-2) = \frac{\ln 2 + \pi i}{\ln 2 + \pi i}$$
. 它的主值 $\ln(-2) = \ln 2 + \pi i$.

5. 已知
$$f(z) = x^2 + iy^2$$
,则 $f'(1+i)$ 美国工作。

6.
$$\int_{C} \frac{\overline{z}}{|z|} dz = \underbrace{\text{Tri}}_{, \pm 0}, \pm 0 \text{ and } z = 4.$$

7.
$$\int_0^z z \cos z dz = \underline{i \sin i + \cos i + 1}.$$

7.
$$\int_{0}^{1} z \cos z dz = \underline{i \sin + \cos i - 1}.$$
8. 已知 c 为 $|z| = 2$, $f(z) = \int_{C} \frac{\xi^{2} - 1}{\xi - z} d\xi$, 则 $f'(2i) = \underline{ }$.

9. 级数
$$\sum_{n=1}^{+\infty} \frac{n!}{n^n} z^n$$
 收敛半径为 $\sum_{n=1}^{+\infty} \frac{n!}{n^n} z^n$ 收敛半径为 $\sum_{n=1}^{+\infty} \frac{n!}{n^n} z^n$ 收敛半径为 $\sum_{n=1}^{+\infty} \frac{n!}{(n+1)^{n+1}} z^n$ $\sum_{n=1}^{+\infty} \frac{n!}{(n+1)^{n+1}} z^n$

10. 幂级数展开
$$\sin(2z^3) = \frac{1}{(2z^3)^{h+1}}$$
 2.h+1

计算下列各题(第1题6分,其他每小题8分,共46分)

1. 函数
$$f(z) = (x^2 - y^2 - x) + i(2xy - y^2)$$
 在何处可导? 在何处解析? $\frac{\partial U}{\partial x} = \frac{\partial U}{\partial y} = -\frac{\partial U}{\partial y}$ つ $\frac{\partial U}{\partial y} = -\frac{\partial U}{\partial y}$ つ $\frac{\partial U}{\partial y} = -\frac{\partial U}{\partial y}$

2. 豆知
$$u+v=x^2-y^2+2xy$$
 , 试确定解析函数 $f(z)=u+iv$.

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y}$$

$$\frac{\partial U}{\partial x} = \frac{\partial U}{\partial y} \qquad \frac{\partial U}{\partial y} = -\frac{\partial U}{\partial x} \qquad \frac{\partial U}{\partial y} = 2x + yy$$

1/是山的井城湖和函数.

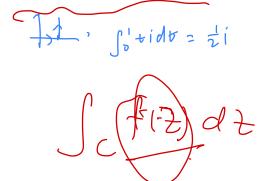
$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0 \qquad \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial y^2} > 0$$

$$U-U = 2xy + y^2 + (ply)$$

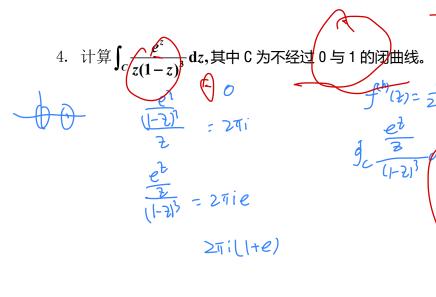
$$2x-2y = \frac{\partial U}{\partial x} - \frac{\partial U}{\partial x}$$

$$U-\dot{y} = x^2 - 2xy + b(xy)$$

- (1) C 为从原点(0,0)到(1,1)的直线段; ∫ to ひにないが) (2) C 为从原点(0,0)到(1,0)再到(1,1)的直线段. マナジ







1 D ff

 $\sum_{n=1}^{\infty} \frac{n^2}{n!} z^n$ 的收敛半径,以及和函数.

$$\lim_{h \to 0} \frac{(n+1)^{\frac{1}{2}}}{(n+1)!} / \frac{n^{\frac{1}{2}}}{n!} = \frac{n!}{(n+1)!} = \frac{1}{h+1} = 0.$$

6. 试求
$$f(z) = \frac{1}{1+z^2}$$
以 $z = i$ 为中心的洛朗级数。

- 三.证明题(每题8分,共24分)
- (1) 叙述解析函数关于柯西黎曼方程的充分必要条件,并证明。

(2)证明 C-R 条件的极坐标形式为:

$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}.$$

(3)叙述并证明柯西积分公式。