Data Analysis and Simulation for Ratio and Regression Estimation

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Contents

1 Functions and packages for Analyzing Data

```
## ydata --- observations of the variable of interest
## xdata --- observations of the auxilliary variable
## N --- population size
## xbarU --- population mean of auxilliary variable

## the output is the estimate mean or total (est.total=TRUE)
srs_reg_est <- function (ydata, xdata, xbarU, N = Inf, est.total = FALSE)
{
  n <- length (ydata)
```

```r
  lmfit <- lm (ydata ~ xdata)
  Bhat <- lmfit$coefficients
  efit <- lmfit$residuals
  SSe <- sum (efit^2) / (n - 2)
  yhat_reg <- Bhat[1] + Bhat[2] * xbarU
  se_yhat_reg <- sqrt ((1-n/N) * SSe / n)
  mem <- qt (0.975, df = n - 2) * se_yhat_reg
  output <- c(yhat_reg, se_yhat_reg, yhat_reg - mem, yhat_reg + mem)

  if (est.total) {
      if(!is.finite(N)) stop("N must be finite for estimating population total" )
      output <- output * N
  }

  names (output) <- c("Est.", "S.E.", "ci.low", "ci.upp" )
  output
}


## ydata --- observations of the variable of interest
## xdata --- observations of the auxilliary variable
## N --- population size

## the output is the ratio of ybarU/xbarU
srs_ratio_est <- function (ydata, xdata, N = Inf)
{
  n <- length (xdata)
  xbar <- mean (xdata)
  ybar <- mean (ydata)
  B_hat <- ybar / xbar
  d <- ydata - B_hat * xdata
  var_d <- sum (d^2) / (n - 1)
  sd_B_hat <- sqrt ((1 - n/N) * var_d / n) / xbar
  mem <- qt (0.975, df = n - 1) * sd_B_hat
  output <- c (B_hat, sd_B_hat, B_hat - mem, B_hat + mem )


  names (output) <- c("Est.", "S.E.", "ci.low", "ci.upp" )
  output
}


## sdata --- a vector of original survey data
## N --- population size
## to find total, multiply N to the estimate returned by this function

srs_mean_est <- function (sdata, N = Inf)
{
    n <- length (sdata)
    ybar <- mean (sdata)
    se.ybar <- sqrt((1 - n / N)) * sd (sdata) / sqrt(n)
    mem <- qt (0.975, df = n - 1) * se.ybar
    c (Est. = ybar, S.E. = se.ybar, ci.low = ybar - mem, ci.upp = ybar + mem)
```

```
}

## to find total, multiply N to the estimate returned by this function
## for poststratification, use nh = n * Nh/N
str_mean_estimate <- function (ybarh, sh, nh, Nh)
{
    N <- sum (Nh)
    Pi_h <- Nh/N
    ybar <- sum(ybarh * Pi_h)
    seybar <- sqrt(sum((1-nh/Nh)*Pi_h^2*sh^2/nh))
    mem <- 1.96 * seybar
    c(Est. = ybar, S.E. = seybar, ci.low = ybar - mem, ci.upp = ybar + mem)
}
```

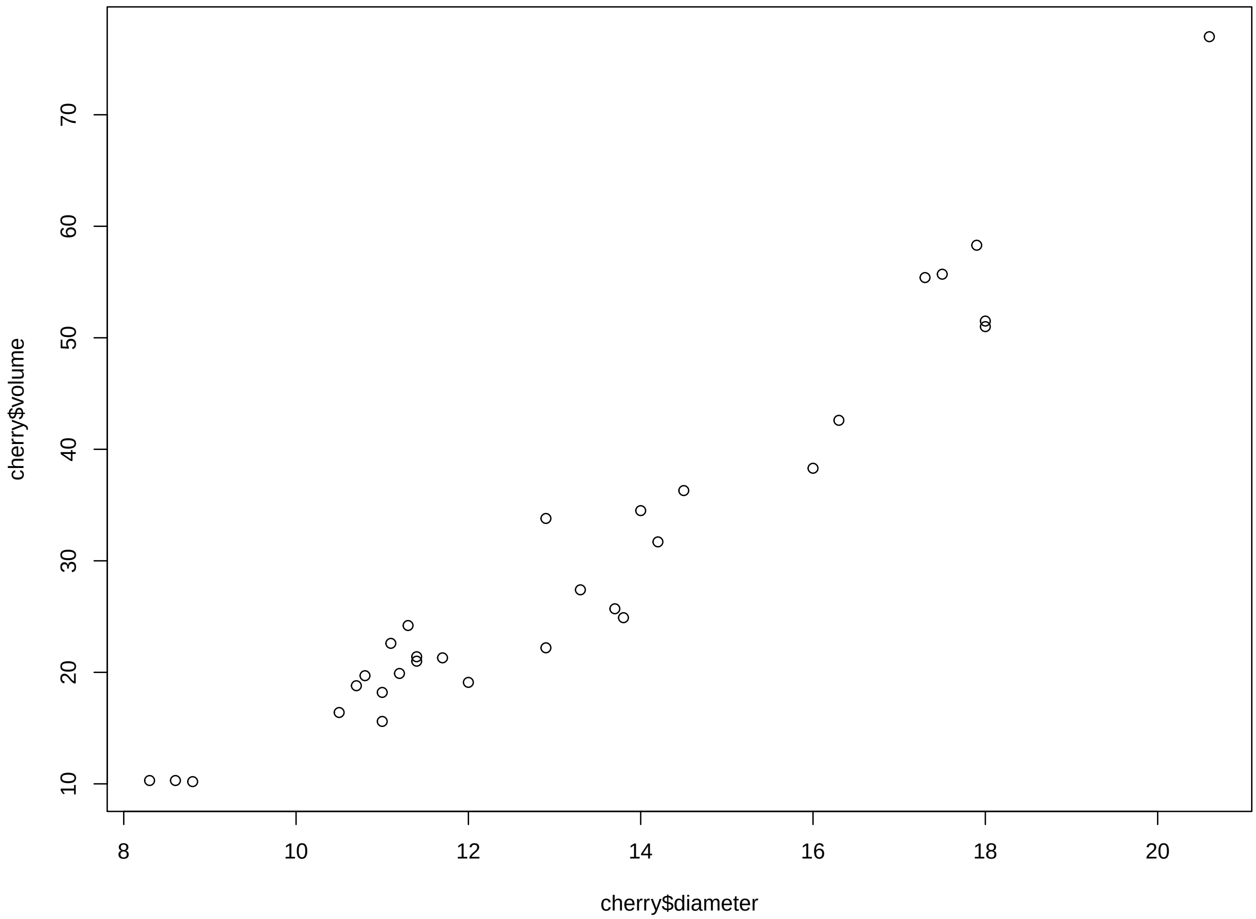
2 Ratio Estimation for cherry.csv dataset

2.1 Importing data

```
cherry <- read.csv ("data/cherry.csv", header = T)
cherry
```

```
##    diameter height volume
## 1       8.3     70   10.3
## 2       8.6     65   10.3
## 3       8.8     63   10.2
## 4      10.5     72   16.4
## 5      10.7     81   18.8
## 6      10.8     83   19.7
## 7      11.0     66   15.6
## 8      11.0     75   18.2
## 9      11.1     80   22.6
## 10     11.2     75   19.9
## 11     11.3     79   24.2
## 12     11.4     76   21.0
## 13     11.4     76   21.4
## 14     11.7     69   21.3
## 15     12.0     75   19.1
## 16     12.9     74   22.2
## 17     12.9     85   33.8
## 18     13.3     86   27.4
## 19     13.7     71   25.7
## 20     13.8     64   24.9
## 21     14.0     78   34.5
## 22     14.2     80   31.7
## 23     14.5     74   36.3
## 24     16.0     72   38.3
## 25     16.3     77   42.6
## 26     17.3     81   55.4
## 27     17.5     82   55.7
## 28     17.9     80   58.3
## 29     18.0     80   51.5
## 30     18.0     80   51.0
## 31     20.6     87   77.0
```

```
plot (cherry$volume ~ cherry$diameter)
```




```
summary (lm(cherry$volume ~ 0+cherry$diameter))
```

```
##
## Call:
## lm(formula = cherry$volume ~ 0 + cherry$diameter)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -11.104  -8.470  -6.199   1.883  27.129
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## cherry$diameter   2.4209     0.1253   19.32   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.493 on 30 degrees of freedom
## Multiple R-squared:  0.9256, Adjusted R-squared:  0.9231
## F-statistic: 373.1 on 1 and 30 DF,  p-value: < 2.2e-16
```

2.2 SRS estimate

```r
N <- 2967
## estimating the mean of volume
srs_mean_volume <- srs_mean_est(cherry$volume, N = N)
srs_mean_volume
```

```
##      Est.      S.E.    ci.low    ci.upp
## 30.170968  2.936861 24.173098 36.168837
```

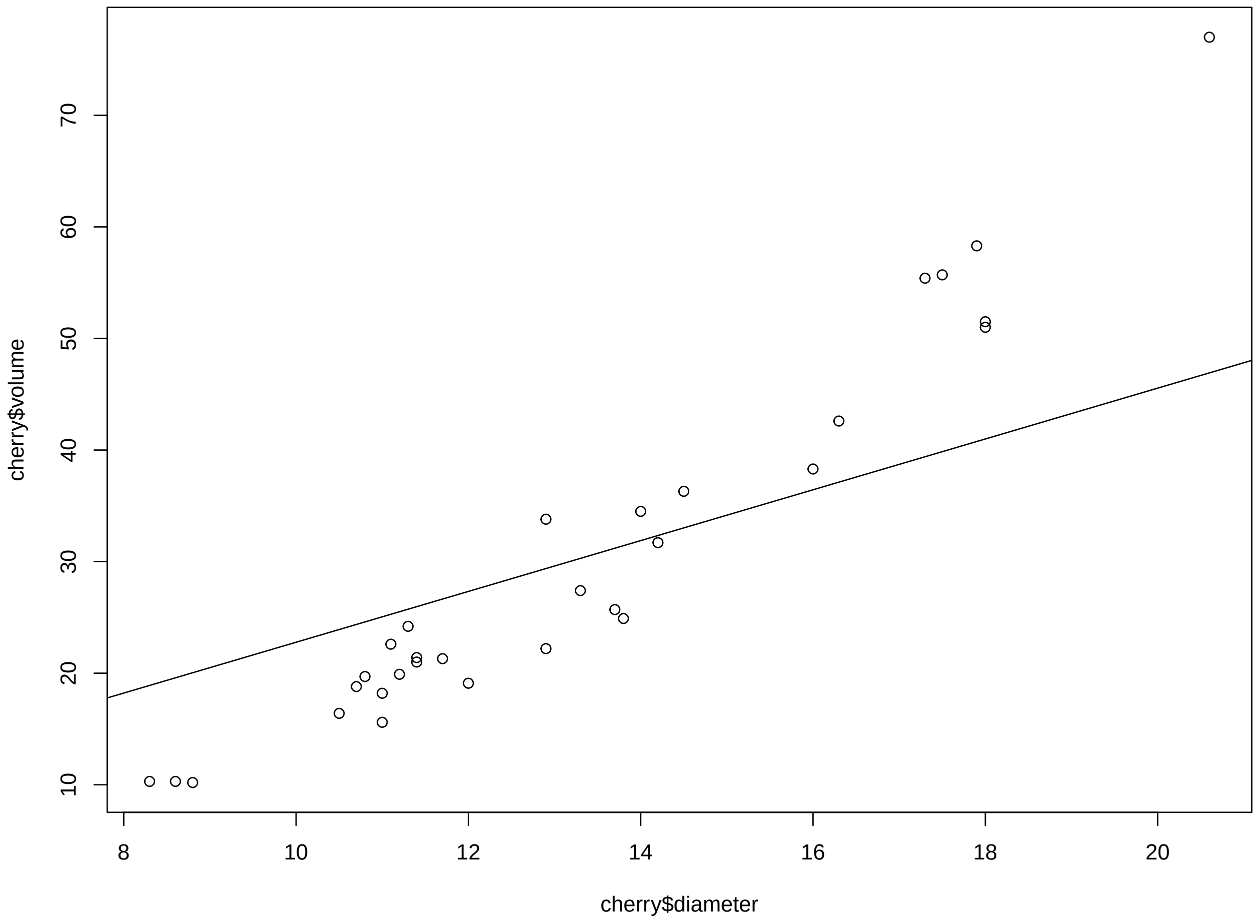
```r
## estimating the total of volume
srs_total_volume <- srs_mean_est(cherry$volume, N = N) * N
srs_total_volume
```

```
##       Est.       S.E.     ci.low     ci.upp
##  89517.261   8713.665  71721.583 107312.940
```

2.3 Step-by-step calculation with Ratio Estimation

2.3.1 Estimating B and calculating residuals

```r
## input
ydata <- cherry$volume
xdata <- cherry$diameter
N <- 2967

## calculation
n <- length (xdata)
xbar <- mean (xdata)
ybar <- mean (ydata)
B_hat <- ybar / xbar ## ratio estimate


plot (cherry$volume ~ cherry$diameter)
abline (a = 0, b = B_hat)
```




```
d <- ydata - B_hat * xdata ## errors
data.frame (cherry, d = d)
```

```
##    diameter height volume          d
## 1       8.3     70   10.3 -8.6018505
## 2       8.6     65   10.3 -9.2850499
## 3       8.8     63   10.2 -9.8405162
## 4      10.5     72   16.4 -7.5119795
## 5      10.7     81   18.8 -5.5674458
## 6      10.8     83   19.7 -4.8951790
## 7      11.0     66   15.6 -9.4506452
## 8      11.0     75   18.2 -6.8506452
## 9      11.1     80   22.6 -2.6783784
## 10     11.2     75   19.9 -5.6061115
## 11     11.3     79   24.2 -1.5338447
## 12     11.4     76   21.0 -4.9615778
## 13     11.4     76   21.4 -4.5615778
## 14     11.7     69   21.3 -5.3447772
## 15     12.0     75   19.1 -8.2279766
## 16     12.9     74   22.2 -7.1775749
## 17     12.9     85   33.8  4.4224251
## 18     13.3     86   27.4 -2.8885074
## 19     13.7     71   25.7 -5.4994400
## 20     13.8     64   24.9 -6.5271731
## 21     14.0     78   34.5  2.6173606
```

```
## 22      14.2      80    31.7 -0.6381057
## 23      14.5      74    36.3  3.2786949
## 24      16.0      72    38.3  1.8626978
## 25      16.3      77    42.6  5.4794984
## 26      17.3      81    55.4 16.0021670
## 27      17.5      82    55.7 15.8467008
## 28      17.9      80    58.3 17.5357682
## 29      18.0      80    51.5 10.5080351
## 30      18.0      80    51.0 10.0080351
## 31      20.6      87    77.0 30.0869735
```

2.3.2 Estimating SE of B

```
## estimating S^2_e
var_d <- sum (d^2) / (n - 1) ## variance of errors
sd_B_hat <- sqrt ((1 - n/N) * var_d / n) / xbar ## SE for B
mem <- qt (0.975, df = n - 1) * sd_B_hat ## margin error for B

## output
output_B <- c (B_hat, sd_B_hat, B_hat - mem, B_hat + mem )
names (output_B) <- c("Est.", "S.E.", "ci.low", "ci.upp" )
output_B
```

```
##     Est.     S.E.   ci.low   ci.upp
## 2.277331 0.130786 2.010231 2.544432
```

2.3.3 Estimating the mean volume of wood

```
mean_diameters <- 41835/N
output_B * mean_diameters
```

```
##      Est.      S.E.    ci.low    ci.upp
## 32.110603  1.844097 28.344455 35.876750
```

2.3.4 Estimating the total volume of wood

```
t_diameters <- 41835
output_B * t_diameters
```

```
##       Est.       S.E.     ci.low     ci.upp
##  95272.159   5471.434  84097.999 106446.318
```

2.4 Ratio estimation with a function

2.4.1 Estimating the ratio of volume to diameter

```
B_v2d <- srs_ratio_est (ydata = cherry$volume, xdata = cherry$diameter,  N = 2967)
B_v2d
```

```
##     Est.     S.E.   ci.low   ci.upp
## 2.277331 0.130786 2.010231 2.544432
```

2.4.2 Estimating the mean of volume

```
xbarU <- 41835/N
ratio_mean_volume <- srs_ratio_est (ydata = cherry$volume, xdata = cherry$diameter, N = 2967) * xbarU
ratio_mean_volume
```

```
##      Est.      S.E.    ci.low    ci.upp
## 32.110603  1.844097 28.344455 35.876750
```

2.4.3 Estimating the total of volume

```
total_diameters <- 41835
srs_ratio_est (ydata = cherry$volume, xdata = cherry$diameter, N = 2967) * total_diameters
```

```
##       Est.       S.E.     ci.low     ci.upp
##  95272.159   5471.434  84097.999 106446.318
```

2.4.4 Percentage of Variance Reduction

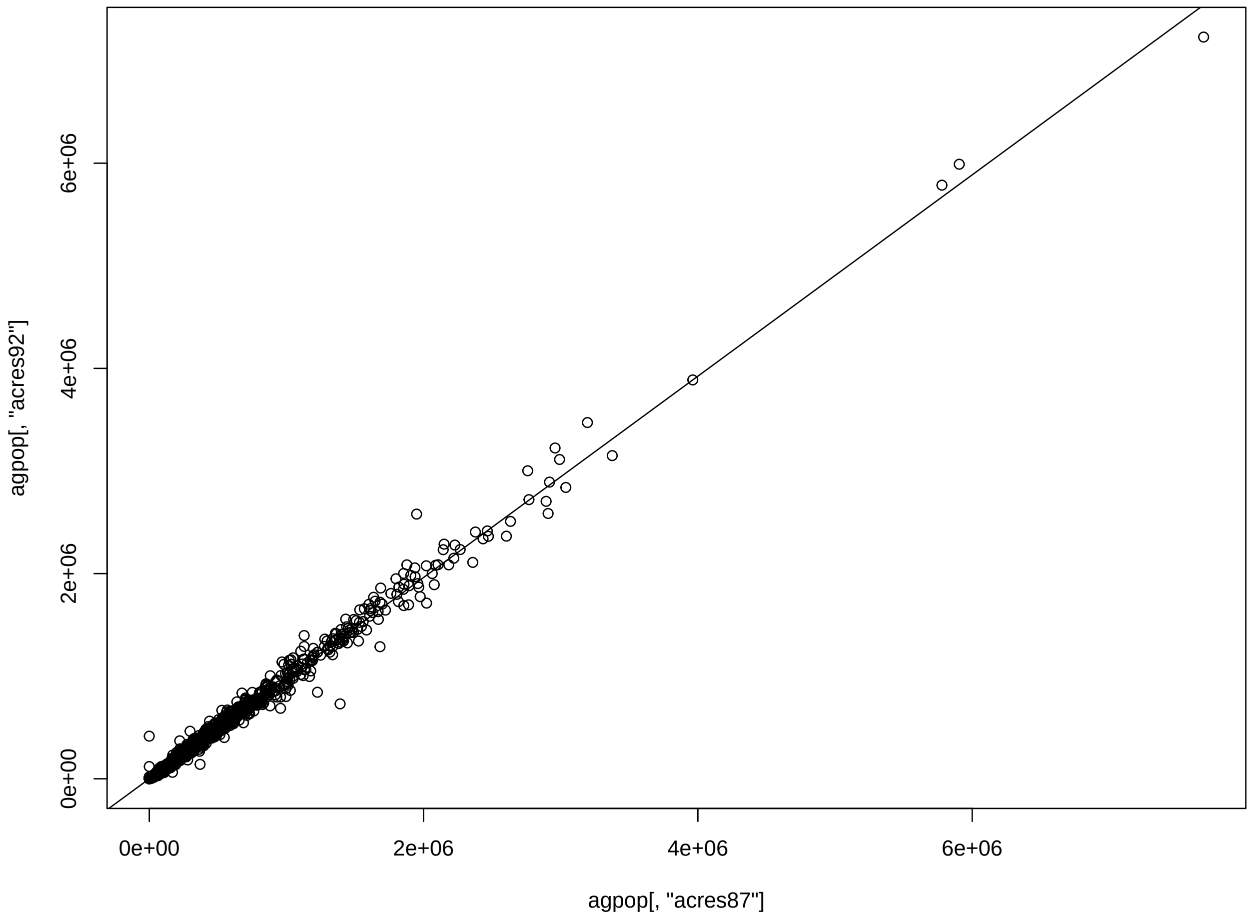
```
cat(1-(ratio_mean_volume[2]/srs_mean_volume[2])^2)
```

```
## 0.6057237
```

3 Simulation study with agpop.csv

3.1 Information of population data

```
agpop <- read.csv ("data/agpop.csv")
agpop <- agpop[agpop$acres92 != -99, ] ## remove those counties with na

# sample size
n <- 300
# population size
N <- nrow (agpop)

# true values that we want to estimate
tyU <- sum (agpop [,"acres92"])
# suppose known for ratio estimate
txU <- sum (agpop [,"acres87"])
B <- tyU/txU
plot (agpop [, "acres87"], agpop [, "acres92"])
abline(a=0,b=B)
```




```
# expected reduction of variance of ratio estimate to srs

1-var(agpop$acres92-B*agpop$acres87)/var(agpop$acres92)
```

```
## [1] 0.9917355
```

```
# linear model output

summary(lm(agpop$acres92 ~ agpop$acres87))
```

```
##
## Call:
## lm(formula = agpop$acres92 ~ agpop$acres87)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -642399   -8256    -692    5593  656458
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -2.121e+03  8.648e+02  -2.453   0.0142 *
## agpop$acres87  9.878e-01  1.626e-03 607.388   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 38560 on 3057 degrees of freedom
```

```
## Multiple R-squared:  0.9918, Adjusted R-squared:  0.9918
## F-statistic: 3.689e+05 on 1 and 3057 DF,  p-value: < 2.2e-16
```

3.2 Simulation Studies

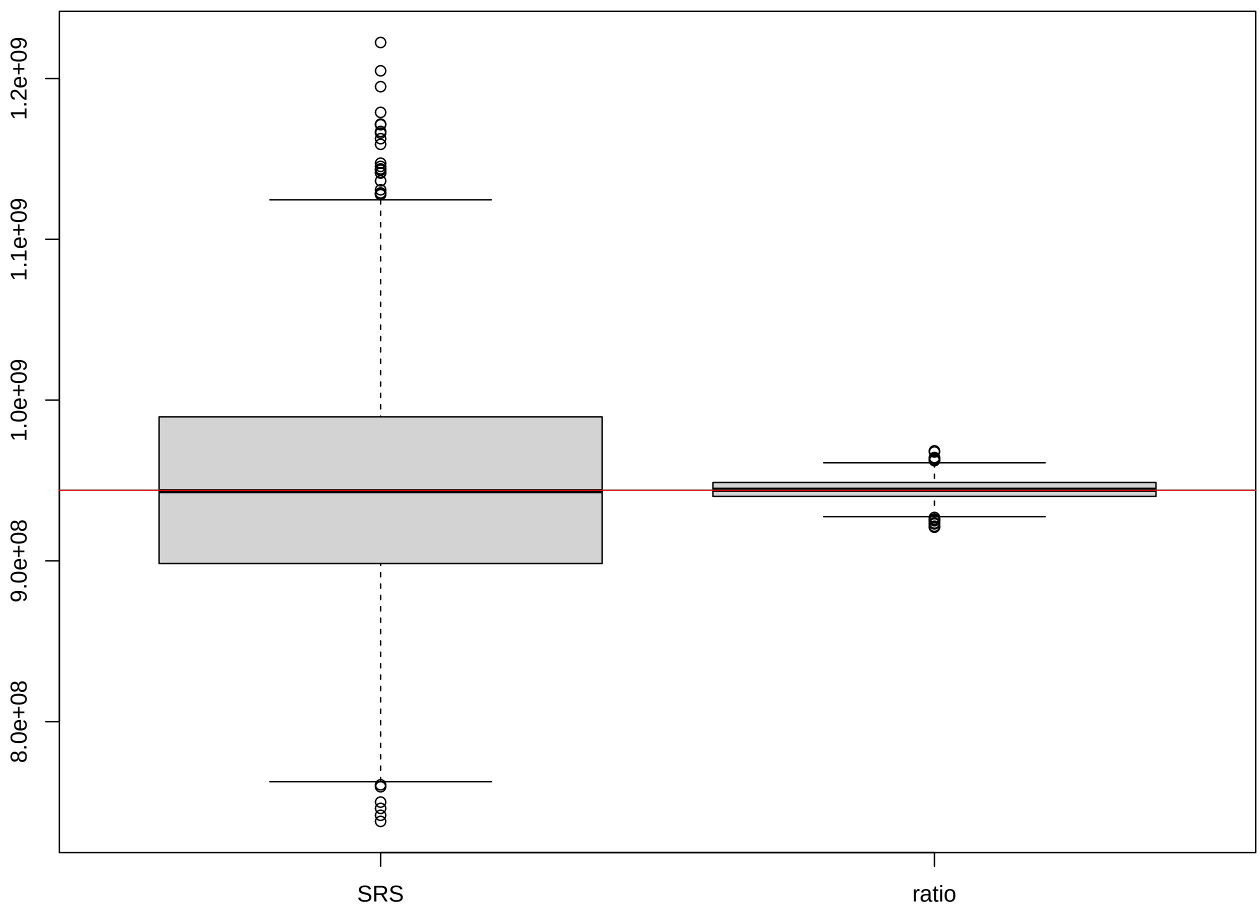
```r
nsim <- 2000
sim_rat <- data.frame (
    SRS = rep (0, nsim), ratio = rep (0, nsim), B = rep (0, nsim))

for (i in 1:nsim)
{
  srs <- sample (N,n)
  y_srs <- agpop[srs, "acres92"]
  x_srs <- agpop[srs, "acres87"]

  # SRS estimate
  sim_rat$SRS [i] <- mean (y_srs) * N
  # ratio estimate
  sim_rat$B [i] <- mean (y_srs) / mean (x_srs)
  sim_rat$ratio [i] <-  sim_rat$B [i] * txU
}

sim_rat[1:100,]
```

```
##            SRS     ratio         B
## 1    997053081 934306832 0.9710204
## 2   1065801250 951001769 0.9883714
## 3    835637359 931777408 0.9683916
## 4    884044821 951427148 0.9888135
## 5    880539645 943610775 0.9806899
## 6   1038293550 955806763 0.9933652
## 7   1050994875 948744078 0.9860250
## 8    999050883 945070978 0.9822075
## 9    973774254 927508035 0.9639545
## 10   951140223 945957356 0.9831287
## 11   974541380 951541108 0.9889319
## 12   822151870 944472183 0.9815852
## 13   875595893 942586881 0.9796258
## 14   980499129 945854177 0.9830215
## 15   887177533 949015276 0.9863068
## 16  1058595552 950113290 0.9874480
## 17   908437674 936252521 0.9730426
## 18   980882534 945537497 0.9826924
## 19   992678089 953797760 0.9912772
## 20   793868376 945437712 0.9825887
## 21   945405965 941259409 0.9782462
## 22  1077907437 949627404 0.9869430
## 23   944331328 940662800 0.9776261
## 24   970130414 939476165 0.9763929
## 25   912212011 949921619 0.9872488
## 26   972037496 928869117 0.9653690
## 27   848683158 941614735 0.9786155
## 28   926324443 941342212 0.9783322
## 29   994427939 944925640 0.9820565
```

```
## 30   980420635 943131921 0.9801923
## 31   956148642 946075889 0.9832519
## 32  1067539599 952222516 0.9896401
## 33   999368407 939036174 0.9759356
## 34   930882118 946738211 0.9839403
## 35   969850281 949696261 0.9870146
## 36  1022753779 943666232 0.9807476
## 37  1005968047 942469679 0.9795040
## 38  1000634874 957676965 0.9953089
## 39   905901682 938684616 0.9755702
## 40   974597604 941970080 0.9789848
## 41   995913827 938938387 0.9758340
## 42   925841508 939565430 0.9764856
## 43   922415387 942889909 0.9799408
## 44   966149217 947563939 0.9847985
## 45   880993346 933157007 0.9698254
## 46   965450562 940360021 0.9773115
## 47   850433559 938762615 0.9756513
## 48   992446991 944688720 0.9818103
## 49  1124187241 948224019 0.9854845
## 50   898817609 953493348 0.9909609
## 51   918075666 934568778 0.9712926
## 52  1003255509 944029236 0.9811249
## 53  1003074416 949718049 0.9870372
## 54   934130093 952604644 0.9900372
## 55   829425703 942124929 0.9791457
## 56  1034330585 944032569 0.9811283
## 57   837945568 944621232 0.9817401
## 58   964280841 943332564 0.9804008
## 59   891258279 939092406 0.9759940
## 60   935410346 943022380 0.9800784
## 61   916769860 945824192 0.9829903
## 62   967796590 936099248 0.9728833
## 63   876059118 944828035 0.9819550
## 64   919904907 953540135 0.9910095
## 65   959845189 943638134 0.9807184
## 66   889462228 938553493 0.9754339
## 67   986029311 963282641 1.0011348
## 68  1089592236 942858335 0.9799079
## 69   935389942 938554630 0.9754351
## 70   920966380 941595907 0.9785959
## 71   896231194 938550563 0.9754309
## 72   852172641 943691058 0.9807734
## 73   943744051 936538925 0.9733402
## 74   925211589 948052813 0.9853065
## 75  1026150452 946625225 0.9838229
## 76   857475611 954560147 0.9920696
## 77   834654503 950628408 0.9879833
## 78   939971865 952861179 0.9903038
## 79   903424993 940538883 0.9774973
## 80   916022934 947762520 0.9850048
## 81  1000677649 952605358 0.9900380
## 82   877705054 951710525 0.9891080
## 83   893412029 938276471 0.9751460
```

```
## 84  1065275082 953094514 0.9905464
## 85   898496903 943423631 0.9804954
## 86   972319740 951041871 0.9884130
## 87   913383333 937864874 0.9747183
## 88   927686727 946263199 0.9834466
## 89   950826604 932124909 0.9687527
## 90  1005642722 948648633 0.9859258
## 91   892428061 956421777 0.9940044
## 92   963164010 952398185 0.9898227
## 93   943115232 953176268 0.9906313
## 94   901763467 940211787 0.9771574
## 95   867930763 936382171 0.9731773
## 96   962163952 943769857 0.9808553
## 97   913067624 933171928 0.9698409
## 98  1017688299 951132578 0.9885073
## 99   936666962 940080974 0.9770214
## 100 1012506547 952241122 0.9896594
```

```
boxplot (sim_rat [, 1:2])
abline (h = tyU, col = "red")
```




```
sim_var <- sapply (sim_rat[,1:2], var)

ratio_sim_var <- sim_var/sim_var[1]
percentage_reduction <- 1-ratio_sim_var
data.frame (sim_var, ratio_sim_var,percentage_reduction)
```

```
##             sim_var ratio_sim_var percentage_reduction
## SRS   5.022863e+15   1.000000000            0.0000000
## ratio 4.171088e+13   0.008304204            0.9916958
```

4 Estimating Domain Means and Post-stratification with agsrs.csv example

4.1 Estimating Domain Means

```
agsrs <- read.csv("data/agsrs.csv")
unique (agsrs$region)
```

```
## [1] "S"  "W"  "NC" "NE"
```

```
region_agsrs <- data.frame(rbind(
    NC = srs_mean_est(agsrs$acres92[agsrs$region=="NC"], N = 3078),
    NE = srs_mean_est(agsrs$acres92[agsrs$region=="NE"], N = 3078),
    S = srs_mean_est(agsrs$acres92[agsrs$region=="S"], N = 3078),
    W = srs_mean_est(agsrs$acres92[agsrs$region=="W"], N = 3078))
)
region_agsrs
```

```
##        Est.     S.E.    ci.low   ci.upp
## NC 323416.4 31184.34 261320.48 385512.4
## NE 106549.0 30391.81  42427.89 170670.1
## S  246185.8 24088.58 198610.98 293760.7
## W  518977.6 73466.42 370818.47 667136.8
```

4.2 Post-stratifiation Analysis

```
nh <- tapply (1:nrow(agsrs), agsrs[,"region"], length)
n <- sum (nh)
n
```

```
## [1] 300
```

```
sh <- tapply (agsrs[, "acres92"], agsrs[,"region"], sd)
ybarh <- tapply (agsrs[, "acres92"], agsrs[,"region"], mean)
# create a vector with external information
Nh <- c(NC = 1054, NE = 220, S= 1382, W = 422)
```

Create nh proportional to Nh, instead of using observed nh

```
nh_post <- Nh/sum(Nh)* n

## regional summary for stratified sampling estimation
data.frame(cbind(ybarh, sh, nh, prop_obs=nh/sum(nh), Nh, nh_post, prop_post = nh_post/sum(nh_post)))
```

```
##       ybarh       sh  nh  prop_obs   Nh  nh_post  prop_post
## NC 323416.4 278970.0  78 0.2600000 1054 102.7290 0.34243015
## NE 106549.0 129320.2  18 0.0600000  220  21.4425 0.07147498
## S  246185.8 312941.3 160 0.5333333 1382 134.6979 0.44899285
## W  518977.6 490842.0  44 0.1466667  422  41.1306 0.13710201
```

```
## strata mean estimate
str_mean_estimate (ybarh, sh, nh_post, Nh)
```

```
##      Est.      S.E.    ci.low    ci.upp
## 300051.69  17760.26 265241.58 334861.79
```

```
## compare with SRS estimate

srs_mean_est(agsrs$acres92, N = 3087)
```

```
##      Est.      S.E.    ci.low    ci.upp
## 297897.05  18901.41 260700.40 335093.69
```

```
## true mean
mean(agpop$acres92)
```

```
## [1] 308582.4
```

5 Estimating Domain Means and Post-stratification with teacher example

5.1 Importing Data

```
teacher <- read.csv ("data/college_teacher.csv")
t(table(teacher))
```

```
##       teacher
## gender   0   1
##      1 156  84
##      2 120  40
```

```
t(prop.table(table(teacher),2))
```

```
##       teacher
## gender    0    1
##      1 0.65 0.35
##      2 0.75 0.25
```

5.2 Domain Summary

```
n <- nrow (teacher)
yh <- tapply (teacher$teacher, INDEX = teacher$gender, FUN = mean); yh
```

```
##    1    2
## 0.35 0.25
```

```
sh <- tapply (teacher$teacher, INDEX = teacher$gender, FUN = sd); sh
```

```
##         1         2
## 0.4779664 0.4343722
```

```
nh_obs <- tapply (teacher$teacher, INDEX = teacher$gender, FUN = length)
ph_obs <- nh_obs/sum (nh_obs); ph_obs
```

```
##   1   2
## 0.6 0.4
```

```
data.frame(yh, sh, nh_obs, ph_obs)
```

```
##     yh        sh nh_obs ph_obs
```

```
## 1 0.35 0.4779664    240    0.6
## 2 0.25 0.4343722    160    0.4
```

5.3 Estimating Domain Means

```
Nh <- c(3000, 1000)
rbind(
    female=srs_mean_est(subset(teacher, gender==1)$teacher, N=3000),
    male = srs_mean_est(subset(teacher, gender==2)$teacher, N=1000)
)
```

```
##         Est.       S.E.    ci.low    ci.upp
## female 0.35 0.02959277 0.2917040 0.4082960
## male   0.25 0.03147326 0.1878404 0.3121596
```

5.4 Post-stratification Analysis

```
Nh <- c(3000, 1000)
ph_post <- Nh/sum (Nh); ph_post
```

```
## [1] 0.75 0.25
```

Create nh proportional to Nh, instead of using observed nh

```
nh_post <- Nh/sum (Nh) * n

## show the differences
data.frame(p_hat = yh, sh, nh_obs, prop_obs=ph_obs, nh_post, prop_post=ph_post)
```

```
##   p_hat        sh nh_obs prop_obs nh_post prop_post
## 1  0.35 0.4779664    240      0.6     300      0.75
## 2  0.25 0.4343722    160      0.4     100      0.25
```

```
## poststratification estimation of mean
str_mean_estimate (yh, sh, nh_post, Nh)
```

```
##       Est.       S.E.     ci.low     ci.upp
## 0.32500000 0.02217306 0.28154080 0.36845920
```

5.5 Analysis with SRS

```
srs_mean_est (teacher$teacher, N=sum(Nh))
```

```
##       Est.       S.E.     ci.low     ci.upp
## 0.31000000 0.02196545 0.26681751 0.35318249
```

6 Regression Estimation for the cherry.csv dataset

6.1 Importing data

```
cherry <- read.csv ("data/cherry.csv", header = T)
ydata <- cherry$volume
xdata <- cherry$diameter
t_diameters <- 41835
```

```r
xbarU <- t_diameters/2967
N <- 2967
```

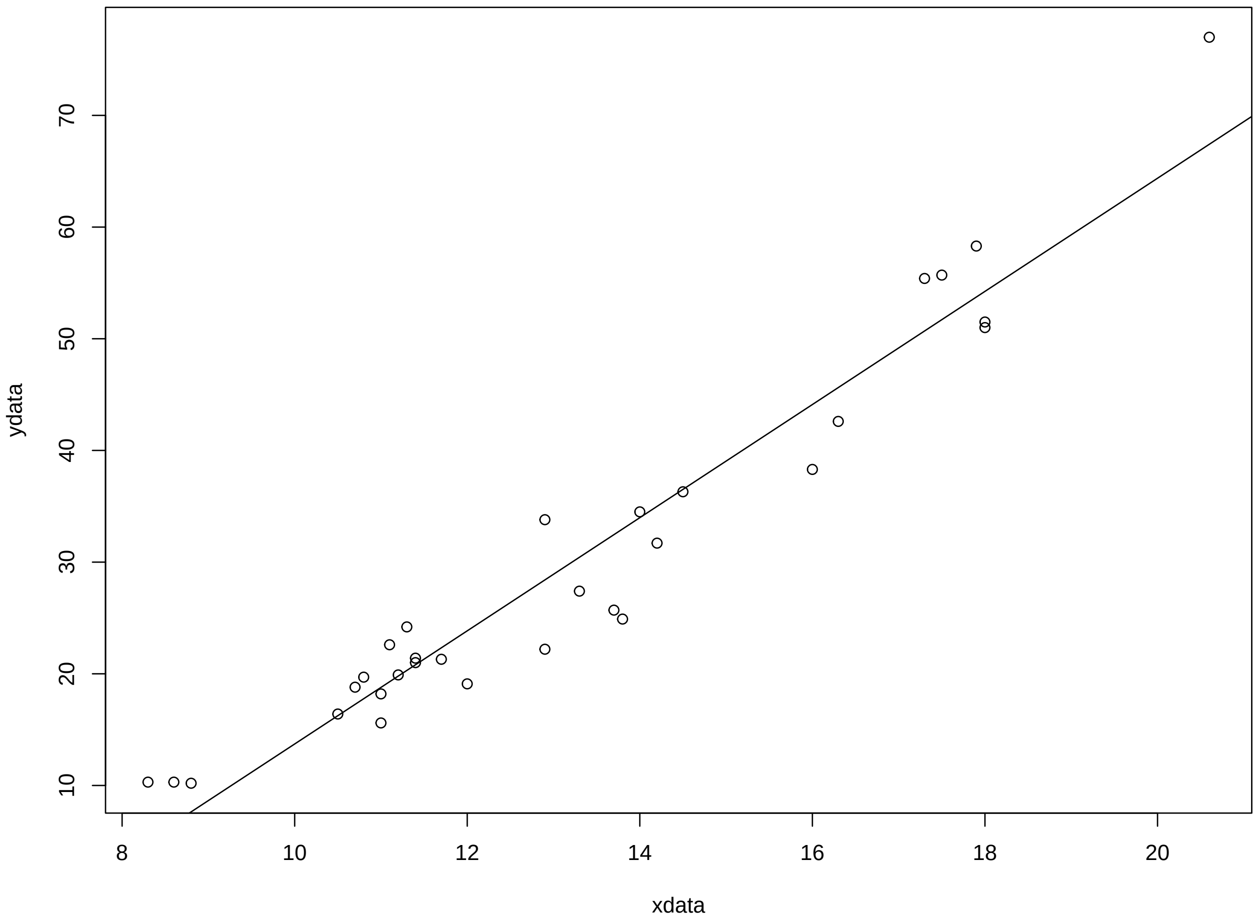
6.2 Step-by-step calculation

6.2.1 Fitting a linear regression model

```r
n <- length (ydata)
lmfit <- lm (ydata ~ xdata)
summary (lmfit)
```

```
##
## Call:
## lm(formula = ydata ~ xdata)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -8.065 -3.107  0.152  3.495  9.587
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -36.9435     3.3651  -10.98 7.62e-12 ***
## xdata         5.0659     0.2474   20.48  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.252 on 29 degrees of freedom
## Multiple R-squared:  0.9353, Adjusted R-squared:  0.9331
## F-statistic: 419.4 on 1 and 29 DF,  p-value: < 2.2e-16
```

```r
plot (xdata, ydata)
abline (lmfit)
```




```
Bhat <- lmfit$coefficients
efit <- ydata - (Bhat[1] + Bhat[2] * xdata)
data.frame (cherry, residual=efit) ## for visualization
```

```
##    diameter height volume   residual
## 1       8.3     70   10.3  5.1968508
## 2       8.6     65   10.3  3.6770939
## 3       8.8     63   10.2  2.5639226
## 4      10.5     72   16.4  0.1519667
## 5      10.7     81   18.8  1.5387954
## 6      10.8     83   19.7  1.9322098
## 7      11.0     66   15.6 -3.1809615
## 8      11.0     75   18.2 -0.5809615
## 9      11.1     80   22.6  3.3124528
## 10     11.2     75   19.9  0.1058672
## 11     11.3     79   24.2  3.8992815
## 12     11.4     76   21.0  0.1926959
## 13     11.4     76   21.4  0.5926959
## 14     11.7     69   21.3 -1.0270610
## 15     12.0     75   19.1 -4.7468179
## 16     12.9     74   22.2 -6.2060887
## 17     12.9     85   33.8  5.3939113
## 18     13.3     86   27.4 -3.0324313
## 19     13.7     71   25.7 -6.7587739
## 20     13.8     64   24.9 -8.0653595
```

```
## 21      14.0     78   34.5  0.5214692
## 22      14.2     80   31.7 -3.2917021
## 23      14.5     74   36.3 -0.2114590
## 24      16.0     72   38.3 -5.8102436
## 25      16.3     77   42.6 -3.0300006
## 26      17.3     81   55.4  4.7041430
## 27      17.5     82   55.7  3.9909717
## 28      17.9     80   58.3  4.5646292
## 29      18.0     80   51.5 -2.7419565
## 30      18.0     80   51.0 -3.2419565
## 31      20.6     87   77.0  9.5868168
```

```
SSe <- sum (efit^2) / (n - 2)
SSe
```

```
## [1] 18.0794
```

```
## SSe can be obtained directly from lm fitting output
anova(lmfit)
```

```
## Analysis of Variance Table
##
## Response: ydata
##           Df Sum Sq Mean Sq F value    Pr(>F)
## xdata      1 7581.8  7581.8  419.36 < 2.2e-16 ***
## Residuals 29  524.3    18.1
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

6.2.2 Estiamte the mean

```
yhat_reg <- Bhat[1] + Bhat[2] * xbarU
se_yhat_reg <- sqrt ((1-n/N) * SSe / n)
mem <- qt (0.975, df = n - 2) * se_yhat_reg
output <- c(yhat_reg, se_yhat_reg, yhat_reg - mem, yhat_reg + mem)
names (output) <- c("Est.", "S.E.", "ci.low", "ci.upp" )
output
```

```
##       Est.       S.E.     ci.low     ci.upp
## 34.4856287  0.7596795 32.9319097 36.0393476
```

6.2.3 Estiamte the total

```
output * N
```

```
##       Est.       S.E.     ci.low     ci.upp
## 102318.860   2253.969  97708.976 106928.744
```

6.3 Regression estimation Using the function

6.3.1 Estimating the mean

```
reg_mean_volume <- srs_reg_est(ydata = cherry$volume, xdata = cherry$diameter,
                  xbarU=t_diameters/2967, N = 2967)
reg_mean_volume
```

```
##       Est.       S.E.     ci.low     ci.upp
## 34.4856287  0.7596795 32.9319097 36.0393476
```

6.3.2 Estimating the total

```r
t_diameters <- 41835
srs_reg_est(ydata = cherry$volume, xdata = cherry$diameter,
                 xbarU=t_diameters/2967, N = 2967, est.total = TRUE)
```

```
##       Est.       S.E.     ci.low     ci.upp
## 102318.860   2253.969  97708.976 106928.744
```

6.3.3 Percentage of Variance Reduction

```r
cat(1-(reg_mean_volume[2]/srs_mean_volume[2])^2)
```

```
## 0.9330895
```

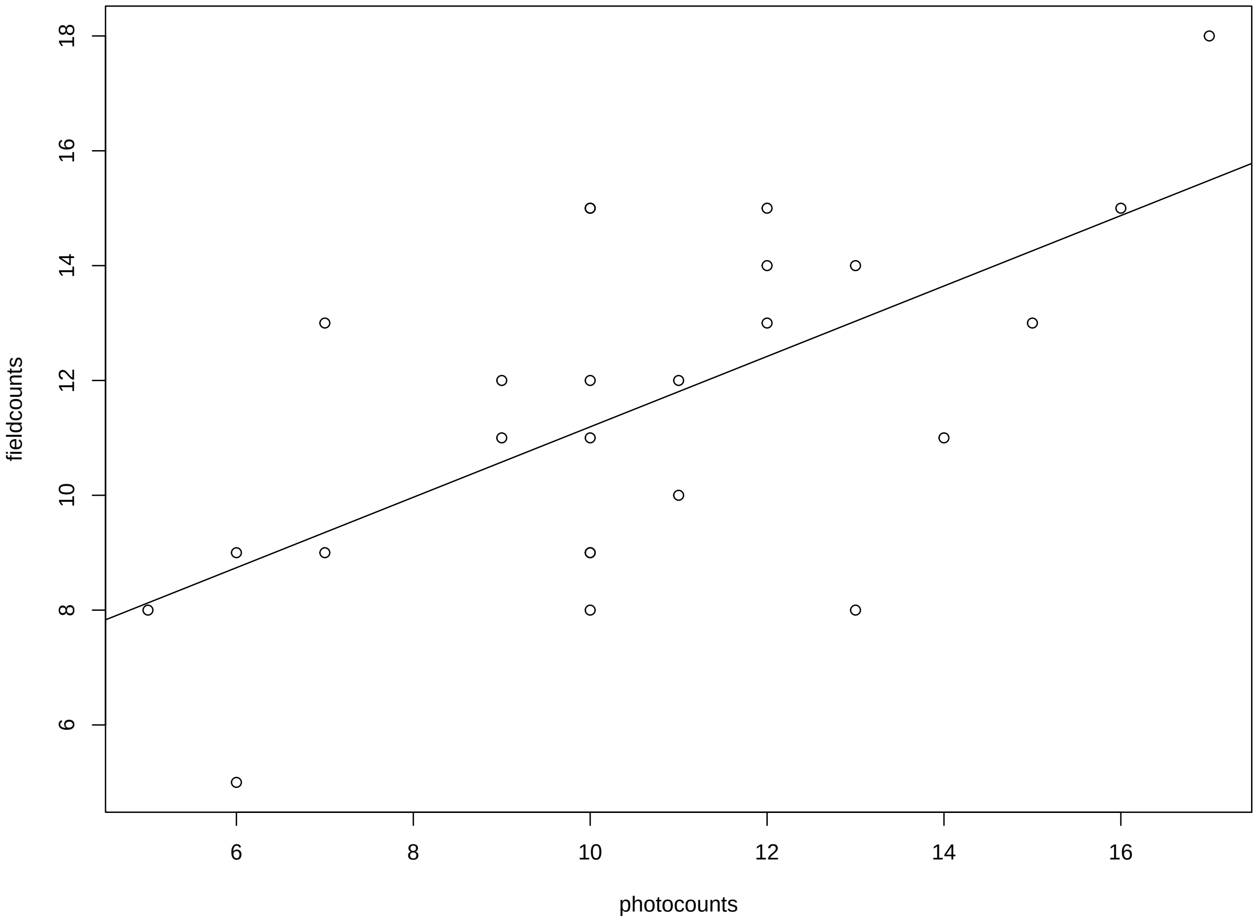
7 Regression estimation for photo counts of dead trees

To estimate the number of dead trees in an area, we divide the area into 100 square plots and count the number of dead trees on a photograph of each plot. Photo counts can be made quickly, but sometimes a tree is misclassified or not detected. So we select an SRS of 25 of the plots for field counts of dead trees. We know that the population mean number of dead trees per plot from the photo count is 11.3.

```r
photocounts <- c (10,12,7, 13,13, 6,17, 16, 15, 10, 14, 12, 10, 5,12, 10,
10, 9, 6, 11, 7, 9, 11, 10, 10)

fieldcounts <- c (15, 14, 9, 14, 8, 5, 18, 15, 13, 15, 11, 15, 12, 8, 13,
9, 11, 12, 9, 12, 13, 11, 10, 9, 8)


lmfit <- lm (fieldcounts ~ photocounts)
summary (lmfit)
```

```
##
## Call:
## lm(formula = fieldcounts ~ photocounts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.0319 -1.8053  0.1947  1.4212  3.8080
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   5.0593     1.7635   2.869 0.008676 **
## photocounts   0.6133     0.1601   3.832 0.000854 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.406 on 23 degrees of freedom
## Multiple R-squared:  0.3896, Adjusted R-squared:  0.3631
## F-statistic: 14.68 on 1 and 23 DF,  p-value: 0.0008538
```

```
plot (photocounts, fieldcounts)
abline (lmfit)
```




```
# estimate for the mean number of dead trees per plot
srs_reg_est (ydata = fieldcounts, xdata = photocounts, xbarU = 11.3, N = 100)
```

```
##        Est.       S.E.     ci.low     ci.upp
## 11.9892920  0.4167579 11.1271625 12.8514215
```

```
# estimate for the total number of dead trees in the area
srs_reg_est (ydata = fieldcounts, xdata = photocounts, xbarU = 11.3, N = 100, est.total = TRUE)
```

```
##        Est.       S.E.     ci.low     ci.upp
## 1198.92920   41.67579 1112.71625 1285.14215
```

```
# compare to simple estimate
srs_mean_est (sdata = fieldcounts, N = 100)
```

```
##        Est.       S.E.     ci.low     ci.upp
## 11.5600000  0.5222069 10.4822180 12.6377820
```

```
# compare to ratio estimate
srs_ratio_est (ydata = fieldcounts, xdata = photocounts, N = 100) * 11.3
```

```
##        Est.       S.E.     ci.low     ci.upp
## 12.3233962  0.5121485 11.2663736 13.3804189
```