pf of Thm 4.2

$\bigcirc\!\!\!\!\!\times \quad \phi_{LR}(x) = \begin{cases} 1, & \text{if } t(x) > t_o \\ 0, & \text{o.w} \end{cases}$

t_o is determined by $W_{LR}(\theta_o) = \alpha$

(1) Because MLR, ϕ_{LR} doesn't depend on θ_1, ~~we can~~ $\rightarrow$ by N-P lemma, We can say that

$\qquad W_{LR}(\theta_1) \geq W_{\phi'}(\theta_1)$ for all θ_1.

$\qquad$ for all ϕ' with $W_{\phi'}(\theta_o) \boxed{\leq \alpha}$.
$\qquad\qquad\qquad\qquad\qquad\qquad \rightarrow$ not $=$

(2) $W_{LR}(\theta)$ is non-dec. monotone fn of θ.

$\qquad$ pf: $\bigcirc\!\!\!\!\!\times$ Let $\theta_2 < \theta_1$

$\qquad$ Considering $H_o^*: \theta = \theta_2$ vs $H_1^*: \theta = \theta_1$

$\qquad$ Let $\beta = W_{LR}(\theta_2)$

$\qquad$ Let $\phi^*(x) = \beta$ for all $x \in \Theta$,
$\qquad$ $W_{\phi^*}(\theta) = \beta$ for all θ.
$\qquad$ By N-P Lemma, $W_{LR}(\theta_1) \geq W_{\phi'}(\theta_1) = \beta$

We therefore prove that

$$W_{LR}(\theta) \le \alpha W_{LR}(\theta_0) = \alpha$$

for all $\theta \le \theta_0$.

(3) For any test ϕ' with $W_{\phi'}(\theta) \le \alpha$

for all $\theta \le \theta_0$, we immediately

have $W_{\phi'}(\theta_0) \le \alpha$,

By N-P lemma,

$$W_{LR}(\theta_1) \ge W_{\phi'}(\theta_1) \text{ for}$$

all $\theta_1 > \theta_0$.

Putting all (1), (2), (3) together.

(1) $W_{LR}(\theta) \le \alpha$ for all $\theta \le \theta_0$

(2) $W_{LR}(\theta) \ge W_{\phi'}(\theta)$ for all $\theta > \theta_0$.

and for all ϕ' with "$W_{\phi'}(\theta) \le \alpha$

when $\theta \le \theta_0$."

That is, ϕ_{LR} is UMP test for

$H_0: \theta \le \theta_0$ vs $H_1: \theta > \theta_0$.