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1 Simple Max Matching

```
bool dfs(int u) {
    if (mx[u] == T) return false;
    mx[u] = T;
    for(int v : ke[u]) {
        if (!my[v] || dfs(my[v])) {
            my[v] = u;
            return true;
        }
    }
    return false;
}

int main() {
    For(i,1,n) {
        T++;
        res += dfs(i);
    }
```

```
    // choose my & i
}
```

2 Konig

```
void konig(){
   queue<int> qu;

   f1(i,m) if (!Assigned[i]) qu.push(i);
   f1(i,n) if (!Assigned[N-i]) qu.push(N-i);

   while (qu.size()){
      int u=qu.front(); qu.pop();
      for (int i=0; int v=a[u][i]; i++)
      if (!(Choosed[v]++)) qu.push(Assigned[v]);
   }

   f1(i,m) if (Assigned[i] && !Choosed[i] && !Choosed[Assigned[i]])
   Choosed[i]=true;
}
```

3 Hopcroft Karp Max Matching algorithm

```
// Worse Case: E * sqrt(v)
const int MAXN = 50005, MAXM = 50005;
vector<int> gph[MAXN];
int dis[MAXN], l[MAXN], r[MAXM], vis[MAXN];
void clear() {
    for (int i = 0; i < MAXN; i++)
        gph[i].clear();
}
void add_edge(int l, int r) {
    gph[l].push_back(r);
}
bool bfs(int n) {
    queue<int> que;
    bool ok = 0;
    memset(dis, 0, sizeof(dis));
    for (int i = 0; i < n; i++) {
        if (l[i] == -1 && !dis[i]) {
            que.push(i);
            dis[i] = 1;
        }
    }
    while (!que.empty()) {
        int x = que.front();
        que.pop();
        for (auto &i : gph[x]) {
            if (r[i] == -1)
                ok = 1;
            else if (!dis[r[i]]) {
                dis[r[i]] = dis[x] + 1;
                que.push(r[i]);
            }
        }
    }
    return ok;
}
bool dfs(int x) {
    for (auto &i : gph[x]) {
        if (r[i] == -1 || (!vis[r[i]] && dis[r[i]] == dis[x] + 1 &&
                           dfs(r[i]))) {
            vis[r[i]] = 1;
            l[x] = i;
            r[i] = x;
            return 1;
        }
    }
    return 0;
}
int match(int n) {
    memset(l, -1, sizeof(l));
    memset(r, -1, sizeof(r));
    int ret = 0;
    while (bfs(n)) {
        memset(vis, 0, sizeof(vis));
        for (int i = 0; i < n; i++)
            if (l[i] == -1 && dfs(i))
                ret++;
    }
    return ret;
}
bool chk[MAXN + MAXM];
void rdfs(int x, int n) {
    if (chk[x])
        return;
```

```
    chk[x] = 1;
    for (auto &i : gph[x]) {
        chk[i + n] = 1;
        rdfs(r[i], n);
    }
}
vector<int> getcover(int n, int m) {
    // solve min. vertex cover
    match(n);
    memset(chk, 0, sizeof(chk));
    for (int i = 0; i < n; i++)
        if (l[i] == -1)
            rdfs(i, n);
    vector<int> v;
    for (int i = 0; i < n; i++)
        if (!chk[i])
            v.push_back(i);
    for (int i = n; i < n + m; i++)
        if (chk[i])
            v.push_back(i);
    return v;
}
```

4 Max matching min cost

```
// numbered from 0. i -> mx[i]
const int V = 1000, INF = 1e9;
int g[V][V], mx[V], my[V], fx[V], fy[V], d[V], ar[V], tr[V], p;
int slack(int u, int v) {
    return g[u][v] - fx[u] - fy[v];
}
int augment(int s) {
    queue<int> q;
    q.push(s);
    fill_n(tr, p, -1);
    for(int i = 0; i < p; ++i) d[i] = slack(s, i), ar[i] = s;
    while(true) {
        while(!q.empty()) {
            int u = q.front();
            q.pop();
            for(int v = 0; v < p; ++v) if(tr[v] == -1) {
                    int w = slack(u, v);
                    if(w == 0) {
                        tr[v] = u;
                        if(my[v] == -1) return v;
                        q.push(my[v]);
                    }
                    if(d[v] > w) d[v] = w, ar[v] = u;
                }
        }
        int delta = INF;
        for(int v = 0; v < p; ++v) if(tr[v] == -1) delta =
                    min(delta, d[v]);
        fx[s] += delta;
        for(int v = 0; v < p; ++v)
            if(tr[v] == -1) d[v] -= delta;
            else fx[my[v]] += delta, fy[v] -= delta;
        for(int v = 0; v < p; ++v) if(tr[v] == -1 && d[v] == 0) {
                tr[v] = ar[v];
                if(my[v] == -1) return v;
                q.push(my[v]);
            }
    }
}
void maxMatchMinCost() {
    fill_n(mx, p, -1);
    fill_n(my, p, -1);
    for(int i = 0; i < p; ++i) fx[i] = *min_element(g[i], g[i]+p);
    for(int s = 0; s < p; ++s) {
        int f = augment(s);
        while(f != -1) {
            int x = tr[f], nx = mx[x];
            mx[x] = f;
            my[f] = x;
            f = nx;
        }
    }
}
```

5 Ganeral Matching

```
class MatchingGraph {
public:
    vector <vector<int> > adj;
    vector <bool> blossom;
    vector <int> parent;
```

```
    vector <int> base;
    vector <int> match;
    int n;
    MatchingGraph() {
        n = 0;
    }
    void addEdge(int x, int y) {
        adj[x].push_back(y);
        adj[y].push_back(x);
    }
    void clearGraph() {
        int i;
        for (i=0; i<SZ(adj); ++i)
            adj[i].clear();
        fill(blossom.begin(),blossom.end(),false);
        fill(parent.begin(),parent.end(),-1);
        for (i=0; i<n; ++i)
            base[i] = i;
        for (i=0; i<n; ++i)
            match[i] = -1;
    }
    void setN(int newn) {
        n = newn;
        adj.resize(n);
        blossom.resize(n);
        base.resize(n);
        match.resize(n);
        parent.resize(n);
        clearGraph();
    }
    int lca(int x, int y) {
        vector <bool> fy;
        fy.resize(n);
        fill(fy.begin(),fy.end(),false);
        while (true) {
            x = base[x];
            fy[x] = true;
            if (match[x] == -1)
                break;
            x = parent[match[x]];
        }
        while (true) {
            y = base[y];
            if (fy[y])
                return y;
            y = parent[match[y]];
        }
        return -1;
    }
    void path(int now, int child, int curbase) {
        while (base[now] != curbase) {
            blossom[base[now]] = blossom[base[match[now]]] = true;
            parent[now] = child;
            child = match[now];
            now = parent[match[now]];
        }
    }
    int augmentPath(int x) {
        int i,j;
        for (i=0; i<n; ++i)
            base[i] = i;
        for (i=0; i<n; ++i)
            parent[i] = -1;
        queue <int> bfs;
        vector <bool> sudah;
        sudah.resize(n);
        fill(sudah.begin(),sudah.end(),false);
        sudah[x] = true;
        bfs.push(x);
        while (!bfs.empty()) {
            int now = bfs.front();
            bfs.pop();
            for (i=0; i<SZ(adj[now]); ++i) {
                int next = adj[now][i];
                if (base[next]==base[now] || match[next] == now);
                else if (next == x || (match[next]!=-1 &&
                                       parent[match[next]]!=-1)) {
                    int curbase = lca(now,next);
                    fill(blossom.begin(),blossom.end(),false);
                    path(now,next,curbase);
                    path(next,now,curbase);
                    for (j = 0; j < n; ++j)
                        if (blossom[j]) {
                            base[j] = curbase;
                            if (!sudah[j]) {
                                sudah[j] = true;
                                bfs.push(j);
                            }
```

```
                    }
                } else if (parent[next]==-1) {
                    parent[next] = now;
                    if (match[next] == -1)
                        return next;
                    sudah[match[next]] = true;
                    bfs.push(match[next]);
                }
            }
        }
        return -1;
    }
    int edmondsMatch() {
        int i;
        int res = 0;
        for (i=0; i<n; ++i) {
            if (match[i]==-1) {
                int x = augmentPath(i);
                while (x>=0) {
                    int p = parent[x];
                    int pp = match[p];
                    match[x] = p;
                    match[p] = x;
                    x = pp;
                }
            }
        }
        for (i=0; i<n; ++i)
            if (match[i]!=-1)
                ++res;
        return res >> 1;
    }
};
```

6 Stable Marriage

```
/* Numbered from 0
 * For man i, L[i] = list of women in order of decreasing preference
 * For women j, R[j][i] = index of man i in j-th women's list of
     preference
 * OUTPUTS:
 *      - L2R[]:    the mate of man i (always between 0 and n-1)
 *      - R2L[]:    the mate of woman j (or -1 if single)
 * COMPLEXITY: M^2
 */

#define MAXM 1024
#define MAXW 1024
int m;
int L[MAXM][MAXW], R[MAXW][MAXM];
int L2R[MAXM], R2L[MAXW];
int p[MAXM];
void stableMarriage() {
  static int p[128];
  memset(R2L, -1, sizeof R2L);
  memset(p, 0, sizeof p);
  // Each man proposes...
  for (int i = 0; i < m; i++) {
    int man = i;
    while (man >= 0) { // propose until success
      int wom;
      while (1) {
        wom = L[man][p[man]++];
        if (R2L[wom] < 0 || R[wom][man] > R[wom][R2L[wom]]) break;
      }
      int hubby = R2L[wom];
      R2L[L2R[man] = wom] = man;
      man = hubby; // remarry the dumped guy
    }
  }
}
```

7 Dinic MaxFlow

```
class DinicFlow {
private:
    vector<int> dist, head, work;
    vector<int> point, flow, capa, next;
    int n, m;

    bool bfs(int s, int t) {
        For(i, 1, n) dist[i] = -1;
        queue<int> q;
        dist[s] = 0;
        q.push(s);
```

```
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int i = head[u]; i >= 0; i = next[i])
                if (flow[i] < capa[i] && dist[point[i]] < 0) {
                    dist[point[i]] = dist[u] + 1;
                    q.push(point[i]);
                }
        }
        return dist[t] >= 0;
    }

    int dfs(int s, int t, int f) {
        if (s == t) return f;
        for (int &i = work[s]; i >= 0; i = next[i])
            if (flow[i] < capa[i] && dist[point[i]] == dist[s] + 1) {
                int d = dfs(point[i], t, min(f, capa[i] - flow[i]));
                if (d > 0) {
                    flow[i] += d;
                    flow[i ^ 1] -= d;
                    return d;
                }
            }
        return 0;
    }

public:
    DinicFlow(int n = 0) {
        this->n = n;
        this->m = 0;
        dist.assign(n + 7, 0);
        head.assign(n + 7, -1);
        work.assign(n + 7, 0);
    }

    void addEdge(int u, int v, int c1, int c2 = 0) {
        point.push_back(v);
        capa.push_back(c1);
        flow.push_back(0);
        next.push_back(head[u]);
        head[u] = m++;
        point.push_back(u);
        capa.push_back(c2);
        flow.push_back(0);
        next.push_back(head[v]);
        head[v] = m++;
    }

    int maxFlow(int s, int t) {
        int totFlow = 0;
        while (bfs(s, t)) {
            For(i, 1, n) work[i] = head[i];
            while (true) {
                int d = dfs(s, t, cmax);
                if (d == 0) break;
                totFlow += d;
            }
        }
        return totFlow;
    }
```

8 Mincost MaxFlow SPFA

```
Min Cost Max Flow - SPFA
Index from 0
edges cap changed during find flow
Lots of double comparison --> likely to fail for double
Example:
MinCostFlow mcf(n);
mcf.addEdge(u, v, cap, cost);
cout << mcf.minCostFlow() << endl;


template<class Flow=int, class Cost=int>
struct MinCostFlow {
    const Flow INF_FLOW = 1000111000;
    const Cost INF_COST = 1000111000111000LL;

    int n, t, S, T;
    Flow totalFlow;
    Cost totalCost;
    vector<int> last, visited;
    vector<Cost> dis;
    struct Edge {
        int to;
        Flow cap;
        Cost cost;
```

```
        int next;
        Edge(int to, Flow cap, Cost cost, int next) :
                to(to), cap(cap), cost(cost), next(next) {}
    };
    vector<Edge> edges;

    MinCostFlow(int n) : n(n), t(0), totalFlow(0), totalCost(0), last(n,
        -1), visited(n, 0), dis(n, 0) {
        edges.clear();
    }

    int addEdge(int from, int to, Flow cap, Cost cost) {
        edges.push_back(Edge(to, cap, cost, last[from]));
        last[from] = t++;
        edges.push_back(Edge(from, 0, -cost, last[to]));
        last[to] = t++;
        return t - 2;
    }

    pair<Flow, Cost> minCostFlow(int _S, int _T) {
        S = _S; T = _T;
        SPFA();
        while (1) {
            while (1) {
                REP(i,n) visited[i] = 0;
                if (!findFlow(S, INF_FLOW)) break;
            }
            if (!modifyLabel()) break;
        }
        return make_pair(totalFlow, totalCost);
    }

private:
    void SPFA() {
        REP(i,n) dis[i] = INF_COST;
        priority_queue< pair<Cost,int> > Q;
        Q.push(make_pair(dis[S]=0, S));
        while (!Q.empty()) {
            int x = Q.top().second;
            Cost d = -Q.top().first;
            Q.pop();
            // For double: dis[x] > d + EPS
            if (dis[x] != d) continue;
            for(int it = last[x]; it >= 0; it = edges[it].next)
                if (edges[it].cap > 0 && dis[edges[it].to] > d + edges[it
                    ].cost)
                    Q.push(make_pair(-(dis[edges[it].to] = d + edges[it].
                        cost), edges[it].to));
        }
        Cost disT = dis[T]; REP(i,n) dis[i] = disT - dis[i];
    }

    Flow findFlow(int x, Flow flow) {
        if (x == T) {
            totalCost += dis[S] * flow;
            totalFlow += flow;
            return flow;
        }
        visited[x] = 1;
        Flow now = flow;
        for(int it = last[x]; it >= 0; it = edges[it].next)
            // For double: fabs(dis[edges[it].to] + edges[it].cost - dis[x
                ]) < EPS
            if (edges[it].cap && !visited[edges[it].to] && dis[edges[it].
                to] + edges[it].cost == dis[x]) {
                Flow tmp = findFlow(edges[it].to, min(now, edges[it].cap))
                    ;
                edges[it].cap -= tmp;
                edges[it ^ 1].cap += tmp;
                now -= tmp;
                if (!now) break;
            }
        return flow - now;
    }

    bool modifyLabel() {
        Cost d = INF_COST;
        REP(i,n) if (visited[i])
            for(int it = last[i]; it >= 0; it = edges[it].next)
                if (edges[it].cap && !visited[edges[it].to])
                    d = min(d, dis[edges[it].to] + edges[it].cost - dis[i
                        ]);

        // For double: if (d > INF_COST / 10)      INF_COST = 1e20
        if (d == INF_COST) return false;
        REP(i,n) if (visited[i])
            dis[i] += d;
        return true;
```

```
    }
};
```

9 Upper Lower

```
- For each edge in original flow:
  - Add edge with cap = upper bound - lower bound.
- Add source s, sink t.
- Let M[v] = (sum of lowerbounds of ingoing edges to v) - (sum of lower
    bounds of outgoing edges from v).
- For all v, if M[v] > 0, add (s, v, M), else add (v, t, -M).
- If all outgoing edges from S are full --> feasible flow exists, it is
    flow + lower bounds.

Feasible flow in network with upper + lower constraint, with source & sink
    :
- Add edge (t, s) with capacity [0, INF].
- Check feasible in network without source & sink.

Max flow with both upper + lower constraints, source s, sink t: add edge (
    t, s, +INF).
- Binary search lower bound, check whether feasible flow exists WITHOUT
    source / sink
```

10 Alternative Tree

```
int n, m, l, q, t, res, test,
    a[maxn], tin[maxn], tout[maxn], mark[maxn], terror[maxn], f[maxn][20];
vector<int> adj[maxn], _adj[maxn];
stack<int> stk;
void visit(const int &u) {
    tin[u] = ++t;
    for(int i = 1; i <= l; ++i) f[u][i] = f[f[u][i-1]][i-1];
    for(auto v : adj[u])
        if (v != f[u][0]) {
            f[v][0] = u;
            visit(v);
        }
    tout[u] = ++t;
}
bool anc(const int &u, const int &v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}
int lca(int u, int v) {
    if (anc(u,v)) return u;
    if (anc(v,u)) return v;
    for(int i = l; i >= 0; --i)
        if (!anc(f[u][i],v)) u = f[u][i];
    return f[u][0];
}
void query() {
    cin >> m;
    for(int i = 1; i <= m; ++i) {
        cin >> a[i];
        _adj[a[i]].clear();
        mark[a[i]] = test;
        terror[a[i]] = test;
    }
    sort(a+1,a+m+1,cmp);
    for(int i = 1; i < m; ++i) {
        int tmp = lca(a[i],a[i+1]);
        if (mark[tmp] < test) {
            mark[tmp] = test;
            a[++m] = tmp;
            _adj[tmp].clear();
        }
    }
    // sort theo tin
    sort(a+1,a+m+1,cmp);
    while (!stk.empty()) stk.pop();
    stk.push(a[1]);
    for(int i = 2; i <= m; ++i) {
        while (tout[stk.top()] < tout[a[i]]) stk.pop();
        _adj[stk.top()].push_back(a[i]);
        stk.push(a[i]);
    }
    res = 0;
    check(a[1]);
    cout << res << "\n";
}
int main() {
    l = log2(n);
    cin >> q;
    f[1][0] = 1;
    visit(1);
    for(test = 1; test <= q; ++test) query();
```

```
}
```

11 Max Clique

```
class MaxClique {
public:
    static const int MV = 210;
    int V;
    int el[MV][MV/30+1];
    int dp[MV];
    int ans;
    int s[MV][MV/30+1];
    vector<int> sol;
    void init(int v) {
        V = v; ans = 0;
        FZ(el); FZ(dp);
    }
    /* Zero Base */
    void addEdge(int u, int v) {
        if(u > v) swap(u, v);
        if(u == v) return;
        el[u][v/32] |= (1<<(v%32));
    }
    bool dfs(int v, int k) {
        int c = 0, d = 0;
        for(int i=0; i<(V+31)/32; i++) {
            s[k][i] = el[v][i];
            if(k != 1) s[k][i] &= s[k-1][i];
            c += __builtin_popcount(s[k][i]);
        }
        if(c == 0) {
            if(k > ans) {
                ans = k;
                sol.clear();
                sol.push_back(v);
                return 1;
            }
            return 0;
        }
        for(int i=0; i<(V+31)/32; i++) {
            for(int a = s[k][i]; a ; d++) {
                if(k + (c-d) <= ans) return 0;
                int lb = a&(-a), lg = 0;
                a ^= lb;
                while(lb!=1) {
                    lb = (unsigned int)(lb) >> 1;
                    lg ++;
                }
                int u = i*32 + lg;
                if(k + dp[u] <= ans) return 0;
                if(dfs(u, k+1)) {
                    sol.push_back(v);
                    return 1;
                }
            }
        }
        return 0;
    }
    int solve() {
        for(int i=V-1; i>=0; i--) {
            dfs(i, 1);
            dp[i] = ans;
        }
        return ans;
    }
};
```

12 Euler Path

```
 NOTES:
 - When choosing starting vertex (for calling find_path), make sure deg[
     start] > 0.
 - If find Euler path, starting vertex must have odd degree.
 - Check no solution: SZ(path) == nEdge + 1.
 If directed:
 - Edge --> int
 - add_edge(int a, int b) { adj[a].push_back(b); }
 - Check for no solution:
 - - for all u, |in_deg[u] - out_deg[u]| <= 1
 - - At most 1 vertex with in_deg[u] - out_deg[u] = 1
 - - At most 1 vertex with out_deg[u] - in_deg[u] = 1 (start vertex)
 - - BFS from start vertex, all vertices u with out_deg[u] > 0 must be
     visited
struct Edge {
    int to;
    list<Edge>::iterator rev;
```

```
    Edge(int to) :to(to) {}
};
const int MN = 100111;
list<Edge> adj[MN];
vector<int> path; // our result
void find_path(int v) {
    while(adj[v].size() > 0) {
        int vn = adj[v].front().to;
        adj[vn].erase(adj[v].front().rev);
        adj[v].pop_front();
        find_path(vn);
    }
    path.push_back(v);
}
void add_edge(int a, int b) {
    adj[a].push_front(Edge(b));
    auto ita = adj[a].begin();
    adj[b].push_front(Edge(a));
    auto itb = adj[b].begin();
    ita->rev = itb;
    itb->rev = ita;
}
```

13 Interection of two paths

```
int intersect(int a, int b, int c, int d){
 if(lca(b,c)!=c) return 0;
 int z = lca(b,d);
 if(lv[c]<lv[a]){
 if(lca(a,z)==a) return dist(z,a);
        }else{
 if(lca(c,z)==c) return dist(c,z);}
 return 0;
}
```

14 Tree ISO

```
namespace TreeISO {
typedef vector<vector<int>> vvi;
typedef vector<int> vi;
typedef pair<vi, int> pvii;
const int MAXN = 4010;
#define ii pair<int, int>
int N;
vvi edges[2], levels[2];
int ts[MAXN], label[2][MAXN], parent[2][MAXN];
vi centroid[2];
int findCentroid(const int tID, const int u, const int p) {
  int children = 0, curr;
  for (auto &e : edges[tID][u]) {
    if (e != p) {
      curr = findCentroid(tID, e, u);
      if (curr > (N >> 1))
        break;
      children += curr;
    }//if
  }//for
  if (N - children - 1 <= (N >> 1))
    centroid[tID].push_back(u);
  return ts[u] = children + 1;
}//findCentroid
int setLevels(const int tID, const int u, const int p, const int d) {
  parent[tID][u] = p;
  levels[tID][d].push_back(u);
  int mx = d;
  for (auto &e : edges[tID][u])
    if (e != p)
      mx = max(mx, setLevels(tID, e, u, d + 1));
  return mx;
}//setLevels
bool isoCheck(const int lvl) {
  for (int it = lvl; it >= 0; it--) {
    vector<pvii> order[2];
    for (int i = 0; i < 2; i++) {
      for (auto &u : levels[i][it]) {
        order[i].push_back(pvii(vi(), u));
        for (auto &e : edges[i][u])
          if (e != parent[i][u])
            order[i].back().first.push_back(label[i][e]);
      }//for
    }//for
    if ((int) order[0].size() != ((int) order[1].size()))
      return 0;
    for (int i = 0; i < 2; i++) {
      for (int j = 0; j < (int) order[0].size(); j++)
        sort(order[i][j].first.begin(), order[i][j].first.end());
```

```
      sort(order[i].begin(), order[i].end());
    }//for
    int labelID = 0;
    for (int i = 0; i < (int) order[0].size(); i++) {
      if (order[0][i].first != order[1][i].first)
        return 0;
      if (i && order[0][i].first == order[0][i - 1].first) {
        label[0][order[0][i].second] = label[1][order[1][i].second] =
            labelID;
        continue;
      }//if
      label[0][order[0][i].second] = label[1][order[1][i].second] = ++
          labelID;
    }//for
  }//for
  return 1;
}//isoCheck
int checkISO(int _N, vector<ii> _edges) {
  N = _N;
  int u, v;
  int T = 1;
  while (T--) {
    int cur = 0;
    memset(ts, 0, sizeof(int) * (N + 2));
    for (int i = 0; i < 2; i++) {
      edges[i].assign(N + 5, vi());
      levels[i].assign(N + 5, vi());
      memset(label[i], 0, sizeof(int) * (N + 2));
      memset(parent[i], 0, sizeof(int) * (N + 2));
      centroid[i].clear();
      for (int j = 0; j < N - 1; j++) {
        int u = _edges[cur].first;
        int v = _edges[cur].second;
        cur++;
        edges[i][u].push_back(v);
        edges[i][v].push_back(u);
      }//for
      findCentroid(i, edges[i][0].empty() ? 1 : 0, -1);
    }//for
    if (edges[0][0].empty())
      N++;
    if ((int) centroid[0].size() != (int) centroid[1].size()) {
      return 0;
    }//if
    if ((int) centroid[0].size() == 2) {
      for (int i = 0; i < 2; i++) {
        for (int j = 0; j < 2; j++) {
          edges[i][centroid[i][j]].erase(std::remove(edges[i][centroid[i][
              j]].begin(),
                                                     edges[i][centroid[i][
                                                         j]].end(),
                                                     centroid[i][!j]),
                                         edges[i][centroid
                                             [i][j]].end());
          edges[i][centroid[i][j]].push_back(N);
          edges[i][N].push_back(centroid[i][j]);
        }//for
        centroid[i][0] = N;
      }//for
    }//if
    int d[2];
    for (int i = 0; i < 2; i++)
      d[i] = setLevels(i, centroid[i][0], -1, 0);
    if (d[0] != d[1]) {
      return 0;
    }//if
    if (d[0] >= 0)
      return isoCheck(d[0] - 1) ? 1 : 0;
  }//while
  return 0;
}//main
}
```

15 Centroid

```
void findCentroid(int u, int par, int Size) {
  nChild[u] = 1;
  bool pre = true;
  for (int i = 0; i < a[u].size(); i++) {
    int v = a[u][i];
    if (v != par && ok[v]) {
      findCentroid(v, u, Size);
      if (nChild[v] > Size / 2) pre = false;
      nChild[u] += nChild[v];
    }
  }
  if (pre && nChild[u] >= Size / 2)
```

```
    centroid = u;
}
```

16 Aho Corasick

```
const int NODE = (int) 1e6 + 1;
const int NC = 26;
int nextNode[NODE][NC];
int chr[NODE];
int parent[NODE];
int prefix[NODE];
int numNodes;
set<int> match[NODE];
int getPrefix(int);
int go(int u, int c) {
    if (nextNode[u][c] != -1) return nextNode[u][c];
    if (u == 0) return 0;
    return nextNode[u][c] = go(getPrefix(u), c);
}
int getPrefix(int u) {
    if (prefix[u] != -1) return prefix[u];
    if (u == 0 || parent[u] == 0) return prefix[u] = 0;
    return prefix[u] = go(getPrefix(parent[u]), chr[u]);
}
void add(const string &s, int id) {
    int u = 0;
    for (int i = 0; i < (int) s.size(); ++i) {
        int c = s[i] - 'A';
        if (nextNode[u][c] == -1) {
            nextNode[u][c] = numNodes;
            fill(nextNode[numNodes], nextNode[numNodes] + NC, -1);
            chr[numNodes] = c;
            parent[numNodes] = u;
            prefix[numNodes] = -1;
            match[numNodes].clear();
            match[numNodes].insert(-1);
            ++numNodes;
        }
        u = nextNode[u][c];
    }
    match[u].insert(id);
}
set<int>& getMatch(int u) {
    if (match[u].count(-1) == 0) return match[u];
    const set<int> &foo = getMatch(getPrefix(u));
    match[u].insert(foo.begin(), foo.end());
    match[u].erase(-1);
    return match[u];
}
void init() {
    fill(nextNode[0], nextNode[0] + NC, -1);
    numNodes = 1;
}
```

17 Suffix Array

```
struct SuffixArray {
    string a;
    int N, m;
    vector<int> SA, LCP, x, y, w, c;
    SuffixArray(string _a, int m) : a(" " + _a), N(a.length()), m(m),
            SA(N), LCP(N), x(N), y(N), w(max(m, N)), c(N) {
        a[0] = 0;
        DA();
        kasaiLCP();
        #define REF(X) { rotate(X.begin(), X.begin()+1, X.end()); X.
            pop_back(); }
        REF(SA); REF(LCP);
        a = a.substr(1, a.size());
        for(int i = 0; i < (int) SA.size(); ++i) --SA[i];
        #undef REF
    }
    inline bool cmp (const int a, const int b, const int l) { return (y[a]
         == y[b] && y[a + l] == y[b + l]); }
    void Sort() {
        for(int i = 0; i < m; ++i) w[i] = 0;
        for(int i = 0; i < N; ++i) ++w[x[y[i]]];
        for(int i = 0; i < m - 1; ++i) w[i + 1] += w[i];
        for(int i = N - 1; i >= 0; --i) SA[--w[x[y[i]]]] = y[i];
    }
    void DA() {
        for(int i = 0; i < N; ++i) x[i] = a[i], y[i] = i;
        Sort();
        for(int i, j = 1, p = 1; p < N; j <<= 1, m = p) {
            for(p = 0, i = N - j; i < N; i++) y[p++] = i;
            for (int k = 0; k < N; ++k) if (SA[k] >= j) y[p++] = SA[k] - j
```

```
                ;
            Sort();
            for(swap(x, y), p = 1, x[SA[0]] = 0, i = 1; i < N; ++i)
                x[SA[i]] = cmp(SA[i - 1], SA[i], j) ? p - 1 : p++;
        }
    }
    void kasaiLCP() {
        for (int i = 0; i < N; i++) c[SA[i]] = i;
        for (int i = 0, j, k = 0; i < N; LCP[c[i++]] = k)
            if (c[i] > 0) for (k ? k-- : 0, j = SA[c[i] - 1]; a[i + k] ==
                a[j + k]; k++);
            else k = 0;
    }
};

int main() {
    SuffixArray sa ("mississippi", 256);
    for (int i = 0; i < sa.N - 1; ++i) cout << sa.SA[i] << ' '; cout << '\
        n';
    for (int i = 0; i < sa.N - 1; ++i) cout << sa.LCP[i] << ' '; cout << '
        \n';
    // 10 7 4 1 0 9 8 6 3 5 2
    // 0 1 1 4 0 0 1 0 2 1 3
    return 0;
}
```

18 Manacher

```
void manacher() {
    memset(p,0,sizeof p);
    int center = 0, right = 0,mi;
    for (int i = 1; i < n; i++) {
        mi = 2 * center - i;
        if (right > i) p[i] = min(right - i, p[mi]);
        while (a[i+(1+p[i])] == a[i-(1+p[i])]) p[i]++;
        //printf("%d:%d\n",i,p[i]);
        if (i + p[i] > right) {
            right = i+p[i];
            center = i;
        }
    }
}
```

19 DP knuth

```
http://codeforces.com/blog/entry/8219
Original Recurrence:
  dp[i][j] = min(dp[i][k] + dp[k][j]) + C[i][j]   for k = i+1..j-1
Necessary & Sufficient Conditions:
  A[i][j-1] <= A[i][j] <= A[i+1][j]
  with A[i][j] = smallest k that gives optimal answer
Also applicable if the following conditions are met:
  1. C[a][c] + C[b][d] <= C[a][d] + C[b][c] (quadrangle inequality)
  2. C[b][c] <= C[a][d]                     (monotonicity)
  for all a <= b <= c <= d
To use:
  Calculate dp[i][i] and A[i][i]

  FOR(len = 1..n-1)
    FOR(i = 1..n-len) {
      j = i + len
      FOR(k = A[i][j-1]..A[i+1][j])
        update(dp[i][j])
    }
// OPTCUT
#include "../template.h"
const int MN = 2011;
int a[MN], dp[MN][MN], C[MN][MN], A[MN][MN];
int n;
void solve() {
    cin >> n; FOR(i,1,n) { cin >> a[i]; a[i] += a[i-1]; }
    FOR(i,1,n) FOR(j,i,n) C[i][j] = a[j] - a[i-1];

    FOR(i,1,n) dp[i][i] = 0, A[i][i] = i;

    FOR(len,1,n-1)
        FOR(i,1,n-len) {
            int j = i + len;
            dp[i][j] = 2000111000;
            FOR(k,A[i][j-1],A[i+1][j]) {
                int cur = dp[i][k-1] + dp[k][j] + C[i][j];
                if (cur < dp[i][j]) {
                    dp[i][j] = cur;
                    A[i][j] = k;
                }
            }
        }
```

```
    cout << dp[1][n] << endl;
}
```

20 Convex Hull

```
struct Point {
    long long x, y;
    bool operator < (const Point &v) const {
        return x == v.x ? y < v.y : x < v.x;
    }
    long long cross(const Point &p, const Point &q) const {
        return (p.x - x) * (q.y - y) - (p.y - y) * (q.x - x);
    }
};
vector<Point> convexHull(vector<Point> p) {
    sort(p.begin(), p.end());
    int k = 0, n = p.size();
    vector<Point> poly (2 * n);
    for(int i = 0; i < n; ++i) {
        while(k >= 2 && poly[k-2].cross(poly[k-1], p[i]) < 0) --k;
        poly[k++] = p[i];
    }
    for(int i = n-2, t = k+1; i >= 0; --i) {
        while(k >= t && poly[k-2].cross(poly[k-1], p[i]) < 0) --k;
        poly[k++] = p[i];
    }
    poly.resize(min(n, max(0, k - 1)));
    return poly;
}
```

21 Geometry 2D

```
// Circle Circle Intersection
// zz: pairs of points
zz circleLine(double r, double a, double b, double c){
        zz res = zz(ii(-1e9 - 1, -1e9 - 1), ii(-1e9 - 1, -1e9 - 1));
        double x0 = -a*c/(a*a+b*b), y0 = -b*c/(a*a+b*b);
        if (c*c > r*r*(a*a+b*b) + eps)
            return res;
        else if (abs (c*c - r*r*(a*a+b*b)) < eps) {
            res.first = ii(x0, y0);
            return res; }
        else {
            double d = r*r - c*c/(a*a+b*b);
            double mult = sqrt (d / (a*a+b*b));
            double ax, ay, bx, by;
            ax = x0 + b * mult; bx = x0 - b * mult;
            ay = y0 - a * mult; by = y0 + a * mult;
            res.first = ii(ax, ay); res.second = ii(bx, by);
            return res; } }
zz circleCircleIntersection(Circle c1, Circle c2) {
        zz res = zz(ii(-1e9 - 1, -1e9 - 1), ii(-1e9 - 1, -1e9 - 1));
    if (dist(ii(c1.x, c1.y), ii(c2.x, c2.y)) < eps) {
        if (abs(c1.r - c2.r) < eps)
            return res;
        return res; }
    double dx = c2.x - c1.x; double dy = c2.y - c1.y;
    double A = -2 * dx; double B = -2 * dy;
    double C = dx * dx + dy * dy + c1.r * c1.r - c2.r * c2.r;
    res = circleLine(c1.r, A, B, C);
    res.first = ii(res.first.first + c1.x, res.first.second + c1.y);
    res.second = ii(res.second.first + c1.x, res.second.second + c1.y);
    return res;
}
///// 2 segments intersection
bool onSegment(Point p, Point q, Point r)  // q lies on (p, r)
int orientation(Point p, Point q, Point r){
   int val = (q.y - p.y) * (r.x - q.x) - (q.x - p.x) * (r.y - q.y);
    if (val == 0) return 0;  // colinear
    return (val > 0)? 1: 2;}
bool doIntersect(Point p1, Point q1, Point p2, Point q2){
        int o1 = orientation(p1, q1, p2); int o2 = orientation(p1, q1, q2)
            ;
    int o3 = orientation(p2, q2, p1); int o4 = orientation(p2, q2, q1);
    if (o1 != o2 && o3 != o4) return true;
    if (o1 == 0 && onSegment(p1, p2, q1)) return true;
    if (o2 == 0 && onSegment(p1, q2, q1)) return true;
    if (o3 == 0 && onSegment(p2, p1, q2)) return true;
    if (o4 == 0 && onSegment(p2, q1, q2)) return true;
    return false;}
 ////
```

22 Geometry 3D

```
typedef double T;
```

```
struct p3 {
    T x,y,z;
    p3 operator+(p3 p) {return {x+p.x, y+p.y, z+p.z};}
    p3 operator-(p3 p) {return {x-p.x, y-p.y, z-p.z};}
    p3 operator*(T d) {return {x*d, y*d, z*d};}
    p3 operator/(T d) {return {x/d, y/d, z/d};} //only for floating-point
    bool operator==(p3 p) {return tie(x,y,z) == tie(p.x,p.y,p.z);}
    bool operator!=(p3 p) {return !operator==(p);}
    T operator|(p3 v, p3 w) {return v.x*w.x + v.y*w.y + v.z*w.z;} //dot
        product
    p3 operator*(p3 v, p3 w) { //cross product
        return {v.y*w.z - v.z*w.y, v.z*w.x - v.x*w.z, v.x*w.y - v.y*w.x};
    }}
T sq(p3 v) {return v|v;}
double abs(p3 v) {return sqrt(sq(v));}
p3 unit(p3 v) {return v/abs(v);}
double angle(p3 v, p3 w) {
    double cosTheta = (v|w) / abs(v) / abs(w);
    return acos(max(-1.0, min(1.0, cosTheta)));}
T orient(p3 p, p3 q, p3 r, p3 s) {return (q-p)*(r-p)|(s-p);} // S vs plane
     PQR
struct plane {
    p3 n; T d; // From normal n and offset d
    plane(p3 n, T d) : n(n), d(d) {} // From normal n and point P
    plane(p3 n, p3 p) : n(n), d(n|p) {} // From three non-collinear points
         P,Q,R
    plane(p3 p, p3 q, p3 r) : plane((q-p)*(r-p), p) {}
    // - these work with T = int
    T side(p3 p) {return (n|p)-d;}
    double dist(p3 p) {return abs(side(p))/abs(n);}
    plane translate(p3 t) {return {n, d+(n|t)};}
    // - these require T = double
    plane shiftUp(double dist) {return {n, d + dist*abs(n)};}
    p3 proj(p3 p) {return p - n*side(p)/sq(n);}
    p3 refl(p3 p) {return p - n*2*side(p)/sq(n);}};
struct line3d {
    p3 d, o;
    // From two points P, Q
    line3d(p3 p, p3 q) : d(q-p), o(p) {}
    // From two planes p1, p2 (requires T = double)
    line3d(plane p1, plane p2) {
        d = p1.n*p2.n;
        o = (p2.n*p1.d - p1.n*p2.d)*d/sq(d);
    }
    // - these work with T = int
    double sqDist(p3 p) {return sq(d*(p-o))/sq(d);}
    double dist(p3 p) {return sqrt(sqDist(p));}
    bool cmpProj(p3 p, p3 q) {return (d|p) < (d|q);}
    // - these require T = double
    p3 proj(p3 p) {return o + d*(d|(a-o))/sq(d);}
    p3 refl(p3 p) {return proj(p)*2 - p;}
    p3 inter(plane p) {return o - d*p.side(o)/(p.n|d);}};
double dist(line l1, line l2) {
    p3 n = l1.d*l2.d;
    if (n == zero) return l1.dist(l2.o);
    return abs((l2.o-l1.o)|n)/abs(n);}
p3 closestOnL1(line l1, line l2) {
    p3 n2 = l2.d*(l1.d*l2.d);
    return l1.o + l1.d*((l2.o-l1.o)|n2)/(l1.d|n2);}
double smallAngle(p3 v, p3 w) {
    return acos(min(abs(v|w)/abs(v)/abs(w), 1.0));}
double angle(plane p1, plane p2) {
    return smallAngle(p1.n, p2.n);}
bool isParallel(plane p1, plane p2) {
    return p1.n*p2.n == zero;}
bool isPerpendicular(plane p1, plane p2) {
    return (p1.n|p2.n) == 0;}
double angle(line3d l1, line3d l2) {
    return smallAngle(l1.p, l2.d);}
bool isParallel(line3d l1, line3d l2) {
    return l1.d*l2.d == zero;}
bool isPerpendicular(line3d l1, line3d l2) {
    return (l1.d|l2.d) == 0;}
double angle(plane p, line3d l) {
    return M_PI/2 - smallAngle(p.n, l.d);}
bool isParallel(plane p, line3d l) {
    return (p.n|l.d) == 0;}
bool isPerpendicular(plane p, line3d l) {
    return p.n*l.d == zero;}
line3d perpThrough(plane p, p3 o) {return line(o, o+p.n);}
plane perpThrough(line3d l, p3 o) {return plane(l.d, o);}
```

23 C++ tricks

```
int __builtin_clz(int x);//number of leading zero
int __builtin_ctz(int x);//number of trailing zero
int __builtin_clzll(long long x);//number of leading zero
```

```
int __builtin_ctzll(long long x);//number of trailing zero
int __builtin_popcount(int x);// number of 1-bits in x
int __builtin_popcountll(long long x);//number of 1-bits i
```

24 FFT

```
const double PI = acos(-1.0);
typedef complex<double> Complex;
#define MASK(i) (1LL<<(i))
#define BIT(x,i) (((x) >> (i)) & 1)
#define LOG 17
Complex fftRoot[MASK(LOG)], invRoot[MASK(LOG)];
#define REP(i, n) for (int i = 0, _n = (n); i < _n; i = i + 1)
void initFFT(void) {
    REP(i, MASK(LOG)) {
        double alpha = 2 * PI / MASK(LOG) * i;
        fftRoot[i] = Complex(cos(alpha), sin(alpha));
        invRoot[i] = Complex(cos(-alpha), sin(-alpha));
    }
}
unsigned roundUp(unsigned v) {
    --v;
    REP(i, 5) v |= v >> MASK(i);
    return v + 1;
}
int reverse(int num, int lg) {
    int res = 0;
    REP(i, lg) if (BIT(num, i)) res |= MASK(lg - i - 1);
    return res;
}
vector<Complex> fft(vector<Complex> a, bool invert) {
    int n = a.size(), lg = 0;
    while (MASK(lg) < n) lg++;
    vector<Complex> roots(n);
    REP(i, n) roots[i] = invert ? invRoot[MASK(LOG) / n * i] :
                         fftRoot[MASK(LOG) / n * i];
    REP(i, n) {
        int rev = reverse(i, lg);
        if (i < rev) swap(a[i], a[rev]);
    }
    for (int len = 2; len <= n; len <<= 1)
        for (int i = 0; i < n; i += len)
            for (int j = 0; j < (len >> 1); j++) {
                Complex u = a[ i + j], v = a[i + j + (len >> 1)] *
                                           roots[n / len * j];
                a[i + j] = u + v;
                a[i + j + (len >> 1)] = u - v;
            }
    if (invert) REP(i, n) a[i] /= n;
    return a;
}
vector<long long> multiply(const vector<int> &a, const vector<int>
                           &b) {
    int n = roundUp(size(a) + size(b) - 1);
    vector<Complex> pa (n), pb (n);
    for(int i = 0; i < size(a); ++i) pa[i] = a[i];
    for(int i = 0; i < size(b); ++i) pb[i] = b[i];
    pa = fft(pa, false);
    pb = fft(pb, false);
    for(int i = 0; i < n; ++i) pa[i] *= pb[i];
    pa = fft(pa, true);
    vector<long long> res (n);
    for(int i = 0; i < n; ++i) res[i] = round(real(pa[i]));
    return res;
}
```

25 FFT mod

```
struct cp {
  double x, y;
  cp(double x = 0, double y = 0) : x(x), y(y) {}
  cp operator+(const cp& rhs) const { return cp(x + rhs.x, y + rhs.y); }
  cp operator-(const cp& rhs) const { return cp(x - rhs.x, y - rhs.y); }
  cp operator*(const cp& rhs) const {
    return cp(x * rhs.x - y * rhs.y, x * rhs.y + y * rhs.x);
  }
  cp operator!() const { return cp(x, -y); }
} rts[maxf + 1];
cp fa[maxf], fb[maxf];
cp fc[maxf], fd[maxf];
int bitrev[maxf];
void fftinit() {
  int k = 0;
  while ((1 << k) < maxf) k++;
  bitrev[0] = 0;
  for (int i = 1; i < maxf; i++) {
```

```
    bitrev[i] = bitrev[i >> 1] >> 1 | ((i & 1) << k - 1);
  }
  double PI = acos((double)-1.0);
  rts[0] = rts[maxf] = cp(1, 0);
  for (int i = 1; i + i <= maxf; i++) {
    rts[i] = cp(cos(i * 2 * PI / maxf), sin(i * 2 * PI / maxf));
  }
  for (int i = maxf / 2 + 1; i < maxf; i++) {
    rts[i] = !rts[maxf - i];
  }
}
void dft(cp a[], int n, int sign) {
  static int isinit;
  if (!isinit) {
    isinit = 1;
    fftinit();
  }
  int d = 0;
  while ((1 << d) * n != maxf) d++;
  for (int i = 0; i < n; i++) {
    if (i < (bitrev[i] >> d)) {
      swap(a[i], a[bitrev[i] >> d]);
    }
  }
  for (int len = 2; len <= n; len <<= 1) {
    int delta = maxf / len * sign;
    for (int i = 0; i < n; i += len) {
      cp *x = a + i, *y = a + i + (len >> 1), *w = sign > 0 ? rts : rts +
          maxf;
      for (int k = 0; k + k < len; k++) {
        cp z = *y * *w;
        *y = *x - z, *x = *x + z;
        x++, y++, w += delta;
      }
    }
  }
  if (sign < 0) {
    for (int i = 0; i < n; i++) {
      a[i].x /= n;
      a[i].y /= n;
    }
  }
}
void multiply(int a[], int b[], int na, int nb, long long c[], int dup =
    0) {
  int n = na + nb - 1;
  while (n != (n & -n)) n += n & -n;
  for (int i = 0; i < n; i++) fa[i] = fb[i] = cp();
  for (int i = 0; i < na; i++) fa[i] = cp(a[i]);
  for (int i = 0; i < nb; i++) fb[i] = cp(b[i]);
  dft(fa, n, 1);
  if (dup) {
    for (int i = 0; i < n; i++) fb[i] = fa[i];
  } else {
    dft(fb, n, 1);
  }
  for (int i = 0; i < n; i++) fa[i] = fa[i] * fb[i];
  dft(fa, n, -1);
  for (int i = 0; i < n; i++) c[i] = (long long)floor(fa[i].x + 0.5);
}
void multiply(int a[], int b[], int na, int nb, int c[], int mod = (int)1
    e9 + 7,
              int dup = 0) {
  int n = na + nb - 1;
  while (n != (n & -n)) n += n & -n;
  for (int i = 0; i < n; i++) fa[i] = fb[i] = cp();
  static const int magic = 15;
  for (int i = 0; i < na; i++)
    fa[i] = cp(a[i] >> magic, a[i] & (1 << magic) - 1);
  for (int i = 0; i < nb; i++)
    fb[i] = cp(b[i] >> magic, b[i] & (1 << magic) - 1);
  dft(fa, n, 1);
  if (dup) {
    for (int i = 0; i < n; i++) fb[i] = fa[i];
  } else {
    dft(fb, n, 1);
  }
  for (int i = 0; i < n; i++) {
    int j = (n - i) % n;
    cp x = fa[i] + !fa[j];
    cp y = fb[i] + !fb[j];
    cp z = !fa[j] - fa[i];
    cp t = !fb[j] - fb[i];
    fc[i] = (x * t + y * z) * cp(0, 0.25);
    fd[i] = x * y * cp(0, 0.25) + z * t * cp(-0.25, 0);
  }
  dft(fc, n, -1), dft(fd, n, -1);
  for (int i = 0; i < n; i++) {
```

```
    long long u = ((long long)floor(fc[i].x + 0.5)) % mod;
    long long v = ((long long)floor(fd[i].x + 0.5)) % mod;
    long long w = ((long long)floor(fd[i].y + 0.5)) % mod;
    c[i] = ((u << magic) + v + (w << magic + magic)) % mod;
  }
}
vector<int> multiply(vector<int> a, vector<int> b, int mod = (int)1e9 + 7)
    {
  static int fa[maxf], fb[maxf], fc[maxf];
  int na = a.size(), nb = b.size();
  for (int i = 0; i < na; i++) fa[i] = a[i];
  for (int i = 0; i < nb; i++) fb[i] = b[i];
  multiply(fa, fb, na, nb, fc, mod, a == b);
  int k = na + nb - 1;
  vector<int> res(k);
  for (int i = 0; i < k; i++) res[i] = fc[i];
  return res;
}
int fpow(int a, int k, int p) {
  if (!k) return 1;
  int res = a, t = a;
  k--;
  while (k) {
    if (k & 1) res = (long long)res * t % p;
    t = (long long)t * t % p;
    k >>= 1;
  }
  return res;
}
vector<int> invert(vector<int> a, int n, int mod) {
  assert(a[0] != 0);
  vector<int> x(1, fpow(a[0], mod - 2, mod));
  while (x.size() < n) {
    vector<int> tmp(a.begin(), a.begin() + min(a.size(), 2 * x.size()));
    vector<int> nx = multiply(multiply(x, x, mod), tmp, mod);
    x.resize(2 * x.size());
    for (int i = 0; i < x.size(); i++) {
      x[i] += x[i];
      x[i] -= nx[i];
      if (x[i] < 0) x[i] += mod;
      if (x[i] >= mod) x[i] -= mod;
    }
  }
  x.resize(n);
  return x;
}
pair<vector<int>, vector<int> > divmod(vector<int> a, vector<int> b, int
    mod) {
  int n = a.size(), m = b.size();
  if (n < m) {
    return make_pair(vector<int>(), a);
  }
  reverse(a.begin(), a.end());
  reverse(b.begin(), b.end());
  vector<int> rb = invert(b, n - m + 1, mod);
  vector<int> d = multiply(a, rb, mod);
  reverse(a.begin(), a.end());
  reverse(b.begin(), b.end());
  while (d.size() > n - m + 1) d.pop_back();
  reverse(d.begin(), d.end());
  vector<int> r = multiply(d, b, mod);
  while (r.size() >= m) r.pop_back();
  for (int i = 0; i < m; i++) {
    r[i] = a[i] - r[i];
    if (r[i] < 0) r[i] += mod;
  }
  return make_pair(d, r);
}
vector<int> chirpz_transform(vector<int> a, int z, int k, int mod) {
  int n = a.size();
  vector<int> x;
  vector<int> y;
  int iz = fpow(z, mod - 2, mod);
  for (int i = 0; i < n; i++) {
    x.push_back((long long)a[i] * fpow(z, (long long)i * i, mod) % mod);
  }
  for (int i = 1 - n; i < k; i++) {
    y.push_back(fpow(iz, (long long)i * i, mod));
  }
  vector<int> r = FFT::multiply(x, y, mod);
  vector<int> res(k);
  for (int i = 0; i < k; i++) {
    res[i] = (long long)r[i + n - 1] * fpow(z, (long long)i * i, mod) %
        mod;
  }
  return res;
}
}  // namespace FFT
```

26 NTT

```
const int MODULO = 998244353;
const int ROOT = 3; // Primitive root
void fft(vector<int> &a, bool invert) {
    int n = a.size();
    assert((n & (n - 1)) == 0);
    int lg = __builtin_ctz(n);
    for (int i = 0; i < n; ++i) {
        int j = 0;
        for (int k = 0; k < lg; ++k) if ((i&1<<k)!=0) j |= 1 <<
                        (lg-k-1);
        if (i < j) swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len *= 2) {
        int wlen = power(ROOT, (MODULO - 1) / len);
        if (invert) wlen = inverse(wlen);
        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; ++j) {
                int u = a[i + j];
                int v = 1LL * a[i + j + len / 2] * w % MODULO;
                a[i + j] = (u + v) % MODULO;
                a[i + j + len / 2] = (u - v + MODULO) % MODULO;
                w = 1LL * w * wlen % MODULO;
            }
        }
    }
    if (invert) {
        int mul = inverse(n);
        for (auto &x : a) x = 1LL * x * mul % MODULO;
    }
}
```

```
    998244353 = 119 * 2^23 + 1. Primitive root: 3.
    985661441 = 235 * 2^22 + 1. Primitive root: 3.
    1012924417 = 483 * 2^21 + 1. Primitive root: 5
```

27 Gauss

```
// INPUT:    a[][] = an nxn matrix
//           b[][] = an nxm matrix
// OUTPUT:   X      = an nxm matrix (stored in b[][])
//           A^{-1} = an nxn matrix (stored in a[][])
//           returns determinant of a[][]
const double EPS = 1e-10;
typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
T GaussJordan(VVT &a, VVT &b) {
    const int n = a.size();
    const int m = b[0].size();
    VI irow(n), icol(n), ipiv(n);
    T det = 1;
    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++) if (!ipiv[j])
            for (int k = 0; k < n; k++) if (!ipiv[k])
                if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j;
                    pk = k; }
        if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl
            ; exit(0); }
        ipiv[pk]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        if (pj != pk) det *= -1;
        irow[i] = pj;
        icol[i] = pk;
        T c = 1.0 / a[pk][pk];
        det *= a[pk][pk];
        a[pk][pk] = 1.0;
        for (int p = 0; p < n; p++) a[pk][p] *= c;
        for (int p = 0; p < m; p++) b[pk][p] *= c;
        for (int p = 0; p < n; p++) if (p != pk) {
            c = a[p][pk];
            a[p][pk] = 0;
            for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
            for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
        }
    }
    for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
        for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
    }
    return det;
}
```

28 Simplex

```
// Two-phase simplex algorithm for solving linear programs of the form
//
//     maximize     c^T x
//     subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments.  Then, call Solve(x).

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
    int m, n;
    VI B, N;
    VVD D;

    LPSolver(const VVD &A, const VD &b, const VD &c) :
        m(b.size()), n(c.size()), B(m), N(n + 1), D(m + 2, VD(n + 2)) {
            for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j
                ] = A[i][j];
            for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i
                ][n + 1] = b[i]; }
            for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
            N[n] = -1; D[m + 1][n] = 1;
        }

    void Pivot(int r, int s) {
        for (int i = 0; i < m + 2; i++) if (i != r)
            for (int j = 0; j < n + 2; j++) if (j != s)
                D[i][j] -= D[r][j] * D[i][s] / D[r][s];
        for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] /= D[r][s];
        for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] /= -D[r][s];
        D[r][s] = 1.0 / D[r][s];
        swap(B[r], N[s]);
    }

    bool Simplex(int phase) {
        int x = phase == 1 ? m + 1 : m;
        while (true) {
            int s = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 && N[j] == -1) continue;
                if (s == -1 || D[x][j] < D[x][s] || (D[x][j] == D[x][s] &&
                     N[j] < N[s])) s = j;
            }
            if (D[x][s] > -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {
                if (D[i][s] < EPS) continue;
                if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][
                    s] || ((D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][
                    s]) && B[i] < B[r])) r = i;
            }
            if (r == -1) return false;
            Pivot(r, s);
        }
    }

    DOUBLE Solve(VD &x) {
        int r = 0;
        for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
        if (D[r][n + 1] < -EPS) {
            Pivot(r, n);
            if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -
                numeric_limits<DOUBLE>::infinity();
            for (int i = 0; i < m; i++) if (B[i] == -1) {
                int s = -1;
                for (int j = 0; j <= n; j++)
                    if (s == -1 || D[i][j] < D[i][s] || (D[i][j] == D[i][s
                        ] && N[j] < N[s])) s = j;
                Pivot(i, s);
            }
```

```
        }
        if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
        x = VD(n);
        for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
        return D[m][n + 1];
    }
};

int main() {

    const int m = 4;
    const int n = 3;
    DOUBLE _A[m][n] = {
        { 6, -1, 0 },
        { -1, -5, 0 },
        { 1, 5, 1 },
        { -1, -5, -1 }
    };
    DOUBLE _b[m] = { 10, -4, 5, -5 };
    DOUBLE _c[n] = { 1, -1, 0 };

    VVD A(m);
    VD b(_b, _b + m);
    VD c(_c, _c + n);
    for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

    LPSolver solver(A, b, c);
    VD x;
    DOUBLE value = solver.Solve(x);

    cerr << "VALUE: " << value << endl; // VALUE: 1.29032
    cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
    for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
    cerr << endl;
    return 0;
}
```

29 Chinese Remainder

```
// Solve linear congruences equation:
// - a[i] * x = b[i] MOD m[i] (mi don't need to be co-prime)
// Tested:
// - https://open.kattis.com/problems/generalchineseremainder
bool linearCongruences(const vector<ll> &a, const vector<ll> &b,
        const vector<ll> &m, ll &x, ll &M) {
    ll n = a.size();
    x = 0; M = 1;
    REP(i, n) {
        ll a_ = a[i] * M, b_ = b[i] - a[i] * x, m_ = m[i];
        ll y, t, g = extgcd(a_, m_, y, t);
        if (b_ % g) return false;
        b_ /= g; m_ /= g;
        x += M * (y * b_ % m_);
        M *= m_;
    }
    x = (x + M) % M;
    return true;
}
```

30 Primitive Root

```
int generator(int p) {
    vector<int> fact;
    int phi = p-1, n = phi;
    for (int i=2; i*i<=n; ++i) if (n % i == 0) {
        fact.push_back(i);
        while (n % i == 0) n /= i;
    }
    if (n > 1) fact.push_back(n);
    for (int res=2; res<=p; ++res) {
        bool ok = true;
        for (size_t i=0; i<fact.size() && ok; ++i)
            ok &= powmod (res, phi / fact[i], p) != 1;
        if (ok)  return res;
    }
    return -1;
}
```

31 Range Prime Counting

```
// Primes up to 10^12 can be counted in ~1 second.
const int MAXN = 1000005; // MAXN is the maximum value of sqrt(N) +
2
bool prime[MAXN];
int prec[MAXN];
```

```
vector<int> P;
void init() {
    prime[2] = true;
    for (int i = 3; i < MAXN; i += 2) prime[i] = true;
    for (int i = 3; i*i < MAXN; i += 2) {
        if (prime[i]) {
            for (int j = i*i; j < MAXN; j += i+i) prime[j] = false;
        }
    }
    for(int i=1; i<MAXN; i++) {
        if (prime[i]) P.push_back(i);
        prec[i] = prec[i-1] + prime[i];
    }
}
lint rec(lint N, int K) {
    if (N <= 1 || K < 0) return 0;
    if (N <= P[K]) return N-1;
    if (N < MAXN && 1ll * P[K]*P[K] > N) return N-1 - prec[N] +
                prec[P[K]];
    const int LIM = 250;
    static int memo[LIM*LIM][LIM];
    bool ok = N < LIM*LIM;
    if (ok && memo[N][K]) return memo[N][K];
    lint ret = N/P[K] - rec(N/P[K], K-1) + rec(N, K-1);
    if (ok) memo[N][K] = ret;
    return ret;
}
lint count_primes(lint N) { //less than or equal to
    if (N < MAXN) return prec[N];
    int K = prec[(int)sqrt(N) + 1];
    return N-1 - rec(N, K) + prec[P[K]];
}
```

32 Knight's shortest path

```
int KSP(int x,int y) {
    if (x < y) swap(x, y);
    if (x == 1 && y == 0) return 3;
    if (x == 2 && y == 2) return 4;
    int d = x - y;
    if (y > d) return 2*((y-d+2)/3)+d;
    return d-2*((d-y)/4);
}
```

33 Extended Euclid

```
 Gia su ket qua la (x0. y0), ho nghiem la  (x_0 + k * b / d, y_0 - k * a/d
     )
 Phuong trinh ax + by = d co nghiem khi va chi khi d chia het cho gcd(a, b
     )
 a x + b y = gcd(a, b)
int extgcd(int a, int b, int &x, int &y) {
    int g = a; x = 1; y = 0;
    if (b != 0) g = extgcd(b, a % b, y, x), y -= (a / b) * x;
    return g;
}
```

34 Factorial Mod

```
int factmod (int n, int p) { // n!, excluding p^k of course
    int res = 1;
    while (n > 1) {
        res = (res * ((n/p) % 2 ? p-1 : 1)) % p;
        for (int i=2; i<=n%p; ++i)
            res = (res * i) % p;
        n /= p;
    }
    return res % p;
}
```

35 Sqrt Mod

```
// Jacobi Symbol (m/n), m,n >= 0  and n is odd
#define NEGPOW(e) ((e) % 2 ? -1 : 1)
int jacobi(int a, int m) {
    if (a == 0) return m == 1 ? 1 : 0;
    if (a % 2)  return NEGPOW((a-1)*(m-1)/4)*jacobi(m%a, a);
    else return NEGPOW((m*m-1)/8)*jacobi(a/2, m);
}
int invMod(int a, int m) {
    int x, y;
    if (extgcd(a, m, x, y) == 1) return (x + m) % m;
    else                                  return 0; // unsolvable
}
// No solution when: n(p-1)/2 = -1 mod p
int sqrtMod(int n, int p) { //find x: x2 = n (mod p) p is prime
```

```
    int S, Q, W, i, m = invMod(n, p);
    for (Q = p - 1, S = 0; Q % 2 == 0; Q /= 2, ++S);
    do { W = rand() % p; } while (W == 0 || jacobi(W, p) != -1);
    for (int R = powMod(n, (Q+1)/2, p), V = powMod(W, Q, p); ;) {
        int z = R * R * m % p;
        for (i = 0; i < S && z % p != 1; z *= z, ++i);
        if (i == 0) return R;
        R = (R * powMod(V, 1 << (S-i-1), p)) % p;
    }
}
int powMod (int a, int b, int p) {
    int res = 1;
    while (b)
        if (b & 1)
            res = int (res * 1ll * a % p),  --b;
        else
            a = int (a * 1ll * a % p),  b >>= 1;
    return res;
}
```

36 Interval line

```
class IntervalLineTree {
 private:
  int n;
  Node *root;
  void update(Node *node, int lo, int hi, Line &line) {
    int mid = (lo + hi) >> 1;
    if (line.getY(lo) <= line.getY(hi) && line.getY(hi) <= node->line.getY
        (hi))
      return;

    if (line.getY(lo) >= node->line.getY(lo) &&
        line.getY(hi) >= node->line.getY(hi)) {
      node->line = line;
      return;
    }
    // Todo: add left and right note
    if (line.getY(lo) <= node->line.getY(lo) &&
        line.getY(mid) <= node->line.getY(mid)) {
      update(node->rightNode, mid + 1, hi, line);
      return;
    }
    if (line.getY(lo) >= node->line.getY(lo) &&
        line.getY(mid) >= node->line.getY(mid)) {
      update(node->rightNode, mid + 1, hi, node->line);
      node->line = line;
      return;
    }
    if (line.getY(mid+1) <= node->line.getY(mid+1) &&
        line.getY(hi) <= node->line.getY(hi)) {
      update(node->leftNode, lo, mid, line);
    }
    if (line.getY(mid + 1) >= node->line.getY(mid + 1) &&
        line.getY(hi) >= node->line.getY(hi)) {
      update(node->leftNode, lo, mid, node->line);
      node->line = line;
    }
  }
  long long get(Node *node, int lo, int hi, int pos) {
    if (lo > pos || hi < pos) return 0;
    long long res = node->line.getY(pos);
    if (lo == hi) return res;
    int mid = (lo + hi) >> 1;
    if (node->leftNode != NULL)
      res = max(res, get(node->leftNode, lo, mid, pos));
    if (node->rightNode != NULL) {
      res = max(res, get(node->rightNode, mid + 1, hi, pos));
    }
    return res;
  }
 public:
  IntervalLineTree(int _n) {
    n = _n;
    root = new Node();
  }
  void update(Line &line) { update(root, 1, n, line); }
  long long get(int pos) { return get(root, 1, n, pos); }
};
```

37 BIT 2D

```
class BIT2D {
 public:
  vector<int> nodes[maxn];
  vector<int> f[maxn];
```

```
  void fakeUpdate(int u, int v) {
    for (int x = u; x <= n; x += x & -x)
      nodes[x].push_back(v);
  }
  void update(int u, int v) {
    for (int x = u; x <= n; x += x & -x)
      for (int y = lower_bound(nodes[x].begin(), nodes[x].end(), v) -
          nodes[x].begin() + 1; y <= nodes[x].size();
           y += y & -y)
        f[x][y]++;
  }
  int get(int u, int v) {
    int res = 0;
    for (int x = u; x > 0; x -= x & -x)
      for (int y = upper_bound(nodes[x].begin(), nodes[x].end(), v) -
          nodes[x].begin(); y > 0; y -= y & -y)
        res += f[x][y];
    return res;
  }
  void prepare(vector<pair<int, int>> queries) {
    reverse(queries.begin(), queries.end());
    for (auto query : queries) {
      fakeUpdate(query.first, query.second);
    }
    reverse(queries.begin(), queries.end());
    for (int i = 1; i <= n; i++) {
      nodes[i].push_back(inf);
      sort(nodes[i].begin(), nodes[i].end());
      f[i].resize(((int) nodes[i].size()) + 3);
    }
  }
} bit2D;
```

38 Heavy-Light Decomposition

```
void hld(int u) {
  //Neu chuoi hien tai chua co dinh dau dinh gan goc nhat thi dat u lam
      dinh dau cua no
  if (chainHead[nChain] == 0) chainHead[nChain] = u;
  //Gan chuoi hien tai cho u
  chainInd[u] = nChain;
  //Giai thich ben duoi
  posInBase[u] = ++nBase;
  // Bien luu dinh con dac biet
  int mxVtx = -1;
  // Tim dinh con dac biet trong so nhung dinh con cua u
  for (int i = 0; i < adj[u].size(); i++) {
    int v = adj[u][i];
    if (v != parent[u]) {
      if (mxVtx == -1 || nChild[v] > nChild[mxVtx]) {
        mxVtx = v;
      }
    }
  }
  //Neu tim ra dinh con dac biet (u khong phai la dinh la) thi di chuyen
      den dinh do
  if (mxVtx > -1)
    hld(mxVtx);
  // Sau khi di het mot chuoi thi tang nChain len va bat dau mot chuoi moi
  for (int i = 0; i < adj[u].size(); i++) {
    int v = adj[u][i];
    if (v != parent[u] && v != mxVtx) {
      nChain++;
      hld(v);
    }
  }
}
void update(int u, int a) {
  // uchain chuoi hien tai cua u
  // achain chuoi hien tai cua a
  int uchain = chainInd[u], achain = chainInd[a];
  while (1) {
    // Neu u va a cung nam tren mot chuoi thi update doan tu u den a va
        ket thuc
    if (uchain == achain) {
      updateIntervalTree(..., posInBase[a], posInBase[u], ...);
      break;
    }
    // Neu u va a khong nam tren cung mot chuoi thi update doan tu u den
        dinh dau cua chuoi hien tai
    updateIntervalTree(..., posInBase[chainHead[uchain]], posInBase[u],
        ...);
    // Nhay len dinh cha cua dinh dau hien tai
    u = parent[chainHead[uchain]];
    uchain = chainInd[u];
  }
}
```

$$\pi(x) = \lfloor x \rfloor - \sum_{i=1}^{a} \left\lfloor \frac{x}{p_i} \right\rfloor + \sum_{1 \le i \le j \le a} \left\lfloor \frac{x}{p_i p_j} \right\rfloor - \ldots + \frac{1}{2}(b+a-2)(b-a+1) - \sum_{a < i \le b} \pi\left(\frac{x}{p_i}\right) - \sum_{i=a+1}^{c} \sum_{j=i}^{b_i} \left[\pi\left(\frac{x}{p_i p_j}\right) - (j-1)\right], a = \pi\left(x^{1/4}\right), b = \pi\left(x^{1/2}\right), b_i = \pi\left(\sqrt{x/p_i}\right), c = \pi\left(x^{1/3}\right)$$

$$C_n = \binom{2n}{n} - \binom{2n}{n+1} = \frac{1}{n+1}\binom{2n}{n}; C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i} = \frac{2(2n+1)}{n+2} C_n$$

$C = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440$

Number of permutations of length n with k cycles:

$$s(n+1, k) = ns(n, k) + s(n, k-1)$$

Number of ways to partition a set of n labelled objects into k nonempty subsets:

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n = kS(n-1, k) + S(n, k-1)$$

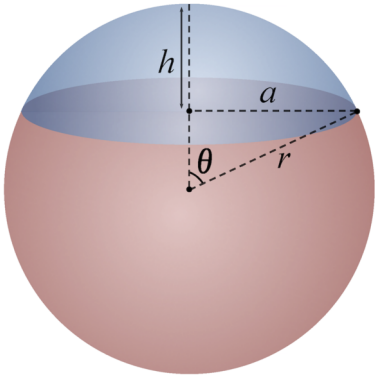
$$H_n = \sum_{k=1}^{n} \frac{1}{k} \approx \ln n + \gamma + \frac{1}{2n} - \frac{1}{12n^2} + \frac{1}{120n^4} - \frac{1}{252n^6} + \ldots$$

$$\frac{1}{2(n+1)} < H_n - \ln n - \gamma < \frac{1}{2n}; \frac{1}{24(n+1)^2} < H_n - \ln\left(n + \frac{1}{2}\right) - \gamma < \frac{1}{24n^2}$$

$$\gamma = 0.57721566490153286060651209008240243104215933593992$$

Sphere: $V = \frac{4}{3}\pi r^3; A = 4\pi r^2$

$$V = \frac{\pi h}{6}\left(3a^2 + h^2\right); A = 2\pi r h = 2\pi r^2 (1 - \cos\theta) = \pi\left(a^2 + h^2\right); r = \frac{a^2 + h^2}{2h}$$



Maximum Flows with Edge Demands: $c'(s' \to v) = \sum_{u \in V} d(u \to v)$, $c'(v \to t') = \sum_{w \in V} d(v \to w)$, $c'(u \to v) = c(u \to v) - d(u \to v)$, $c'(t \to s) = \infty$. If feasible: $c_f(u \to v) = c(u \to v) - f(u \to v)$ if $u \to v \in E$; $f(v \to u) - d(v \to u)$ if $v \to u \in E$, 0 otherwise.