```
Muc luc
```

```
1 Simple Max Matching
2 Konig
3 Hopcroft Karp Max Matching algorithm
4 Max matching min cost
   Ganeral Matching
   Dinic MaxFlow
   Two sat
  Alternative Tree
   Aho Corasick
10 Suffix Array
11 Suffix Array O(n)
12 Manacher
13 Convex Hull
14 Geometry's tricks
15 FFT
16 NTT
17 Primitive Root
18 Range Prime Counting
19 Knight's shortest path
    Simple Max Matching
```

```
bool dfs(int u) {
    if (mx[u] == T) return false;
    mx[u] = T;
    for(int v : ke[u]) {
        if (!my[v] || dfs(my[v])) {
            my[v] = u;
    }
}
```

```
return true;
1
        return false;
1
    int main() {
        For(i,1,n) {
2
            T++;
            res += dfs(i);
 3
 5
        // choose my & i
5
        Konig
7
    void konig(){
       queue < int > qu;
8
       f1(i,m) if (!Assigned[i]) qu.push(i);
       f1(i,n) if (!Assigned[N-i]) qu.push(N-i);
10
10
       while (qu.size()){
          int u=qu.front(); qu.pop();
10
          for (int i=0; int v=a[u][i]; i++)
          if (!(Choosed[v]++)) qu.push(Assigned[v]);
11
       }
12
       f1(i,m) if (Assigned[i] && !Choosed[i] && !Choosed[Assigned[i]])
12
       Choosed[i]=true;
13
13
       Hopcroft Karp Max Matching algorithm
    // Worse Case: E * sqrt(v)
    const int MAXN = 50005, MAXM = 50005;
    vector < int > gph[MAXN];
    int dis[MAXN], 1[MAXN], r[MAXM], vis[MAXN];
    void clear() {
        for (int i = 0; i < MAXN; i++)
            gph[i].clear();
```

```
void add_edge(int 1, int r) {
    gph[1].push_back(r);
}
bool bfs(int n) {
    queue < int > que;
    bool ok = 0;
    memset(dis, 0, sizeof(dis));
    for (int i = 0; i < n; i++) {
        if (1[i] == -1 && !dis[i]) {
            que.push(i);
            dis[i] = 1;
        }
    }
    while (!que.empty()) {
        int x = que.front();
        que.pop();
        for (auto &i : gph[x]) {
            if (r[i] == -1)
                ok = 1;
            else if (!dis[r[i]]) {
                dis[r[i]] = dis[x] + 1;
                que.push(r[i]);
            }
    return ok;
bool dfs(int x) {
    for (auto &i : gph[x]) {
        if (r[i] == -1 || (!vis[r[i]] && dis[r[i]] == dis[x] + 1 &&
                            dfs(r[i]))) {
            vis[r[i]] = 1;
            l[x] = i;
            r[i] = x;
            return 1;
        }
    }
    return 0;
int match(int n) {
    memset(1, -1, sizeof(1));
    memset(r, -1, sizeof(r));
    int ret = 0;
    while (bfs(n)) {
```

```
memset(vis, 0, sizeof(vis));
        for (int i = 0; i < n; i++)
            if (1[i] == -1 && dfs(i))
                ret++;
   return ret:
bool chk[MAXN + MAXM];
void rdfs(int x, int n) {
    if (chk[x])
        return;
    chk[x] = 1;
    for (auto &i : gph[x]) {
        chk[i + n] = 1;
        rdfs(r[i], n);
   }
vector<int> getcover(int n, int m) {
    // solve min. vertex cover
    match(n):
    memset(chk, 0, sizeof(chk));
   for (int i = 0; i < n; i++)
        if (1[i] == -1)
            rdfs(i, n);
    vector < int > v;
   for (int i = 0; i < n; i++)
        if (!chk[i])
            v.push_back(i);
    for (int i = n; i < n + m; i++)
        if (chk[i])
            v.push_back(i);
    return v;
    Max matching min cost
// numbered from 0. i \rightarrow mx[i]
const int V = 1000, INF = 1e9;
int g[V][V], mx[V], my[V], fx[V], fy[V], d[V], ar[V], tr[V], p;
int slack(int u, int v) {
    return g[u][v] - fx[u] - fy[v];
int augment(int s) {
```

```
queue < int > q;
    q.push(s);
    fill_n(tr, p, -1);
    for(int i = 0; i < p; ++i) d[i] = slack(s, i), ar[i] = s;
    while(true) {
        while(!q.empty()) {
            int u = q.front();
            q.pop();
            for(int v = 0; v < p; ++v) if(tr[v] == -1) {
                    int w = slack(u, v);
                    if(w == 0) {
                        tr[v] = u;
                        if(my[v] == -1) return v;
                        q.push(my[v]);
                    if(d[v] > w) d[v] = w. ar[v] = u:
                }
        int delta = INF:
        for (int v = 0; v < p; ++v) if (tr[v] == -1) delta =
                    min(delta, d[v]);
        fx[s] += delta;
        for(int v = 0; v < p; ++v)
            if(tr[v] == -1) d[v] -= delta;
            else fx[my[v]] += delta, fy[v] -= delta;
        for(int v = 0; v < p; ++v) if(tr[v] == -1 && d[v] == 0) {
                tr[v] = ar[v];
                if(my[v] == -1) return v;
                q.push(my[v]);
            }
    }
}
void maxMatchMinCost() {
    fill_n(mx, p, -1);
    fill_n(my, p, -1);
    for(int i = 0; i < p; ++i) fx[i] = *min_element(g[i], g[i]+p);
    for(int s = 0; s < p; ++s) {
        int f = augment(s);
        while(f != -1) {
            int x = tr[f], nx = mx[x];
            mx[x] = f;
            mv[f] = x;
            f = nx;
```

5 Ganeral Matching

```
class MatchingGraph {
public:
    vector <vector <int> > adj;
    vector <bool> blossom;
    vector <int> parent;
    vector <int> base;
    vector <int> match:
    int n;
    MatchingGraph() {
        n = 0:
    void addEdge(int x, int y) {
        adj[x].push_back(y);
        adj[y].push_back(x);
    void clearGraph() {
        int i;
        for (i=0; i < SZ(adj); ++i)
            adj[i].clear();
        fill(blossom.begin(),blossom.end(),false);
        fill(parent.begin(),parent.end(),-1);
        for (i=0; i < n; ++i)
            base[i] = i;
        for (i=0; i<n; ++i)
            match \lceil i \rceil = -1:
    }
    void setN(int newn) {
        n = newn;
        adj.resize(n);
        blossom.resize(n);
        base.resize(n);
        match.resize(n);
        parent.resize(n);
        clearGraph();
    int lca(int x, int y) {
        vector <bool> fy;
        fy.resize(n);
```

```
fill(fy.begin(),fy.end(),false);
    while (true) {
        x = base[x]:
        fy[x] = true;
        if (match[x] == -1)
            break:
        x = parent[match[x]];
    }
    while (true) {
        v = base[v];
        if (fy[y])
            return y;
        y = parent[match[y]];
    return -1;
}
void path(int now, int child, int curbase) {
    while (base[now] != curbase) {
        blossom[base[now]] = blossom[base[match[now]]] = true;
        parent[now] = child;
        child = match[now]:
        now = parent[match[now]];
int augmentPath(int x) {
    int i, j;
    for (i=0; i < n; ++i)
        base[i] = i;
    for (i=0; i < n; ++i)
        parent[i] = -1;
    queue <int> bfs;
    vector <bool> sudah;
    sudah.resize(n);
    fill(sudah.begin(), sudah.end(), false);
    sudah[x] = true;
    bfs.push(x);
    while (!bfs.empty()) {
        int now = bfs.front();
        bfs.pop();
        for (i=0; i < SZ(adj[now]); ++i) {
            int next = adj[now][i];
            if (base[next] == base[now] || match[next] == now);
             else if (next == x || (match[next]!=-1 &&
                                    parent[match[next]]!=-1)) {
```

```
int curbase = lca(now,next);
                     fill(blossom.begin(),blossom.end(),false);
                     path (now, next, curbase);
                     path (next, now, curbase);
                     for (j = 0; j < n; ++j)
                         if (blossom[j]) {
                             base[j] = curbase;
                             if (!sudah[j]) {
                                 sudah[j] = true;
                                 bfs.push(j);
                             }
                } else if (parent[next] == -1) {
                     parent[next] = now;
                     if (match[next] == -1)
                         return next;
                     sudah[match[next]] = true;
                     bfs.push(match[next]);
            }
        return -1;
    int edmondsMatch() {
        int i:
        int res = 0;
        for (i=0; i<n; ++i) {
            if (match[i]==-1) {
                int x = augmentPath(i);
                while (x>=0) {
                     int p = parent[x];
                     int pp = match[p];
                     match[x] = p;
                     match[p] = x;
                     x = pp;
            }
        for (i=0; i<n; ++i)
            if (match[i]!=-1)
                ++res;
        return res >> 1;
   }
};
```

6 Dinic MaxFlow

```
class DinicFlow {
private:
    vector <int > dist, head, work;
    vector < int > point , flow , capa , next;
    int n, m;
    bool bfs(int s, int t) {
        For(i, 1, n) dist[i] = -1;
        queue < int > q;
        dist[s] = 0;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int i = head[u]; i >= 0; i = next[i])
                 if (flow[i] < capa[i] && dist[point[i]] < 0) {</pre>
                     dist[point[i]] = dist[u] + 1;
                     q.push(point[i]);
                }
        }
        return dist[t] >= 0;
    }
    int dfs(int s. int t. int f) {
        if (s == t) return f;
        for (int &i = work[s]; i >= 0; i = next[i])
            if (flow[i] < capa[i] && dist[point[i]] == dist[s] + 1) {</pre>
                 int d = dfs(point[i], t, min(f, capa[i] - flow[i]));
                if (d > 0) {
                     flow[i] += d;
                     flow[i ^ 1] -= d;
                     return d:
                }
            }
        return 0;
    }
public:
    DinicFlow(int n = 0) {
        this -> n = n;
        this -> m = 0:
        dist.assign(n + 7, 0);
```

```
head.assign(n + 7, -1);
        work.assign(n + 7, 0);
   }
    void addEdge(int u, int v, int c1, int c2 = 0) {
        point.push_back(v);
        capa.push_back(c1);
        flow.push_back(0);
        next.push_back(head[u]);
        head[u] = m++;
        point.push_back(u);
        capa.push_back(c2);
        flow.push_back(0);
        next.push_back(head[v]);
        head[v] = m++;
    int maxFlow(int s, int t) {
        int totFlow = 0;
        while (bfs(s, t)) {
            For(i, 1, n) work[i] = head[i];
            while (true) {
                int d = dfs(s, t, cmax);
                if (d == 0) break;
                totFlow += d;
            }
        }
        return totFlow;
    Two sat
int n, m, g[maxn];
bool cx[maxn]:
vector <int> listV, ke[maxn], K[maxn];
int cal(int x) {
    if (x\%2 == 0) return x - 1;
    else return x + 1:
void add(int u. int v) {
```

ke[u].pb(v);

```
K[v].pb(u);
void dfs(int u) {
    cx[u] = true;
    for(int v : ke[u])
        if (!cx[v]) dfs(v):
    listV.pb(u);
}
void dfs(int u, int x) {
    g[u] = x;
    for(int v : K[u])
        if (g[v] == 0) dfs(v,x);
}
int main() {
    cin >> m >> n;
    n += n:
    For(i,1,m) {
        int u, v;
        cin >> u >> v;
        u *= 2:
        v *= 2;
        if (u < 0) u = cal(abs(u));
        if (v < 0) v = cal(abs(v));
        add(cal(u),v);
        add(cal(v),u);
    listV.pb(0);
    For(i,1,n)
    if (!cx[i]) dfs(i);
    int ng = 0;
    Ford(i,n,1) {
        int u = listV[i];
        if (g[u] == 0) dfs(u, ++ng);
    for(int i = 2; i <= n; i += 2)
        if (g[i] == g[i-1]) NO;
    YES;
    vector <int> result;
    for(int i = 2; i <= n; i += 2)
        if (g[i] > g[i-1]) result.pb(i>>1);
}
```

8 Alternative Tree

```
int n, m, 1, q, t, res, test,
    a[maxn], tin[maxn], tout[maxn], mark[maxn], terror[maxn], f[maxn][20];
vector < int > adj[maxn], _adj[maxn];
stack<int> stk:
void visit(const int &u) {
    tin[u] = ++t;
    for (int i = 1; i \le 1; i + i) f[u][i] = f[f[u][i-1]][i-1];
   for(auto v : adj[u])
        if (v != f[u][0]) {
            f[v][0] = u:
            visit(v);
    tout[u] = ++t;
bool anc(const int &u, const int &v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
int lca(int u, int v) {
    if (anc(u,v)) return u;
    if (anc(v,u)) return v;
   for(int i = 1; i >= 0; --i)
        if (!anc(f[u][i],v)) u = f[u][i];
   return f[u][0];
void query() {
    cin >> m;
    for(int i = 1; i <= m; ++i) {
        cin >> a[i];
        _adj[a[i]].clear();
        mark[a[i]] = test;
        terror[a[i]] = test;
    sort(a+1,a+m+1,cmp);
    for(int i = 1: i < m: ++i) {
        int tmp = lca(a[i],a[i+1]);
        if (mark[tmp] < test) {</pre>
            mark[tmp] = test;
            a[++m] = tmp;
```

```
_adj[tmp].clear();
    }
    // sort theo tin
    sort(a+1,a+m+1,cmp);
    while (!stk.empty()) stk.pop();
    stk.push(a[1]);
    for(int i = 2; i <= m; ++i) {
        while (tout[stk.top()] < tout[a[i]]) stk.pop();</pre>
        _adj[stk.top()].push_back(a[i]);
        stk.push(a[i]);
    }
    res = 0;
    check(a[1]);
    cout << res << "\n";
int main() {
    1 = log 2(n);
    cin >> q;
    f[1][0] = 1;
    visit(1);
    for(test = 1; test <= q; ++test) query();</pre>
    Aho Corasick
const int NODE = (int) 1e6 + 1;
const int NC = 26:
int nextNode[NODE][NC];
int chr[NODE];
int parent[NODE];
int prefix[NODE];
int numNodes;
set < int > match [NODE];
int getPrefix(int);
int go(int u, int c) {
    if (nextNode[u][c] != -1) return nextNode[u][c];
    if (u == 0) return 0:
```

return nextNode[u][c] = go(getPrefix(u), c);

```
int getPrefix(int u) {
    if (prefix[u] != -1) return prefix[u];
    if (u == 0 || parent[u] == 0) return prefix[u] = 0;
    return prefix[u] = go(getPrefix(parent[u]), chr[u]);
}
void add(const string &s, int id) {
    int u = 0;
    for (int i = 0; i < (int) s.size(); ++i) {
        int c = s[i] - 'A';
        if (nextNode[u][c] == -1) {
            nextNode[u][c] = numNodes;
            fill(nextNode[numNodes], nextNode[numNodes] + NC, -1);
            chr[numNodes] = c;
            parent[numNodes] = u;
            prefix[numNodes] = -1;
            match[numNodes].clear();
            match[numNodes].insert(-1);
            ++numNodes:
        u = nextNode[u][c];
    match[u].insert(id);
set < int > & getMatch(int u) {
    if (match[u].count(-1) == 0) return match[u];
    const set <int> &foo = getMatch(getPrefix(u));
    match[u].insert(foo.begin(), foo.end());
    match[u].erase(-1):
    return match[u];
void init() {
    fill(nextNode[0], nextNode[0] + NC, -1);
    numNodes = 1;
    Suffix Array
struct SuffixArray {
```

```
const int L;
    string s:
    vector<vector<int> > P:
    vector<pair<pair<int,int>,int> > M;
    SuffixArray(const string &s) : L(s.length()), s(s), P(1,
                vector < int > (L, 0)), M(L) {
        for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
        for (int skip = 1, lv = 1; skip < L; skip *= 2, lv++) {
            P.push_back(vector<int>(L, 0));
            for (int i = 0; i < L; i++)
                M[i] = make_pair(make_pair(P[lv-1][i], i + skip < L
                                           P[1v-1][i + skip] : -1000), i
                                               );
            sort(M.begin(), M.end());
            for (int i = 0; i < L; i++)
                P[lv][M[i].se] = (i > 0 && M[i].fi == M[i-1].fi)?
                                 P[lv][M[i-1].se] : i:
    vector < int > GetSuffixArrav() {
        return P.back():
// returns the length of the longest common prefix of s[i...L-1]
    and s[j...L-1]
    int LongestCommonPrefix(int i, int j) {
        int len = 0:
        if (i == j) return L - i;
        for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
            if (P[k][i] == P[k][i]) {
                i += 1 << k:
                i += 1 << k;
                len += 1 << k;
            }
        return len:
    }
};
      Suffix Array O(n)
#include <bits/stdc++.h>
#define FOR(i.a.b) for (int i=(a), b=(b): i <= b: i=i+1)
#define REP(i,n) for (int i=0, n=(n):i < n:i=i+1)
```

```
#define MASK(i) (1LL <<(i))
#define BIT(x,i) (((x)>>(i))&1)
#define tget(i) BIT(t[(i) >> 3], (i) & 7)
#define tset(i, b) { if (b) t[(i) >> 3] |= MASK((i) & 7); else t[(i) >> 3]
     &= ~MASK((i) & 7); }
#define chr(i) (cs == sizeof(int) ? ((int *)s)[i] : ((unc *)s)[i])
#define isLMS(i) ((i) > 0 && tget(i) && !tget((i) - 1))
typedef unsigned char unc;
class SuffixArray {
    public:
    int *sa, *lcp, *rank, n;
    void getbuckets (unc s[], vector < int > &bkt, int n, int k, int cs, bool
        end) {
        FOR(i, 0, k) bkt[i] = 0;
        REP(i, n) bkt[chr(i)]++:
        int sum = 0;
       FOR(i, 0, k) {
            sum += bkt[i]:
            bkt[i] = end ? sum : sum - bkt[i]:
    void inducesal(vector < unc > &t, int sa[], unc s[], vector < int > &bkt,
        int n, int k, int cs, bool end) {
        getbuckets(s, bkt, n, k, cs, end);
        REP(i, n) {
            int j = sa[i] - 1;
            if (j >= 0 && !tget(j)) sa[bkt[chr(j)]++] = j;
       }
   }
    void induces as (vector < unc > &t, int sa[], unc s[], vector < int > &bkt,
        int n, int k, int cs, bool end) {
        getbuckets(s, bkt, n, k, cs, end);
        FORD(i, n - 1, 0) {
            int j = sa[i] - 1;
            if (j >= 0 && tget(j)) sa[--bkt[chr(j)]]=j;
       }
    void build(unc s[], int sa[], int n, int k, int cs) {
        vector < unc > t = vector < unc > (n / 8 + 1, 0);
        tset(n - 2, 0);
        tset(n - 1, 1):
```

```
FORD(i, n - 3, 0) tset(i, chr(i) < chr(i+1) || (chr(i) == chr(i+1)
     && tget(i+1))):
vector<int> bkt = vector<int> (k + 1, 0):
getbuckets(s, bkt, n, k, cs, true);
REP(i, n) sa[i] = -1;
REP(i, n) if (isLMS(i)) sa[--bkt[chr(i)]] = i;
inducesal(t, sa, s, bkt, n, k, cs, false);
inducesas(t, sa, s, bkt, n, k, cs, true);
bkt.clear();
int n1 = 0;
REP(i, n) if (isLMS(sa[i])) sa[n1++] = sa[i];
FOR(i, n1, n - 1) sa[i] = -1;
int name = 0;
int prev = -1;
REP(i, n1) {
    int pos = sa[i];
   bool diff = false:
    REP(d, n) {
        if (prev < 0 || chr(prev + d) != chr(pos + d) || tget(prev
             + d) != tget(pos + d)) {
            diff = true:
            break;
       }
        else if (d > 0 && (isLMS(prev + d) || isLMS(pos + d)))
   }
    if (diff) {
        name++;
        prev = pos;
   }
    sa[n1 + pos / 2] = name - 1;
}
i = n - 1;
FORD(i, n - 1, n1) if (sa[i] >= 0) sa[j--] = sa[i];
int *sa1 = sa;
int *s1 = sa + n - n1;
if (name < n1) build((unc *)s1, sa1, n1, name-1, sizeof(int));</pre>
else REP(i, n1) sa1[s1[i]] = i;
bkt.assign(k + 1, 0);
getbuckets(s, bkt, n, k, cs, true);
i = 0;
REP(i, n) if (isLMS(i)) s1[j++] = i;
REP(i, n1) sa1[i] = s1[sa1[i]];
FOR(i, n1, n - 1) sa[i] = -1:
```

```
FORD(i, n1 - 1, 0) {
            i = sa[i]:
            sa[i] = -1:
            sa[--bkt[chr(j)]] = j;
        inducesal(t, sa, s, bkt, n, k, cs, false);
        inducesas(t, sa, s, bkt, n, k, cs, true);
        bkt.clear():
        t.clear();
    void calc_lcp(void) {
        FOR(i,1,n) rank[sa[i]] = i;
        int h = 0;
        REP(i, n) if (rank[i] < n) {</pre>
            int j = sa[rank[i] + 1];
            while (s[i + h] == s[j + h]) h++;
            lcp[rank[i]] = h;
            if (h > 0) h - -:
        }
    SuffixArray() {
        n = 0:
        sa = lcp = rank = NULL;
        s = NULL;
    SuffixArray(string ss) {
        n = ss.size();
        sa = new int[n + 7];
        lcp = new int [n + 7];
        rank = new int \lceil n + 7 \rceil:
        s = (unc *)ss.c_str();
        build(s, sa, n + 1, 256, sizeof(char));
        calc_lcp();
   }
};
//Sorted suffices are SA[1] to SA[N]. The values of SA[1], SA[2], ..., SA[
    N] are 0, 1, ..., N-1
//The\ longest\ common\ prefix\ of\ SA[i]\ and\ SA[i+1]\ is\ LCP[i]
int main(void) {
    string s = "mississippi";
    SuffixArray suffixArray(s);
    FOR(i, 1, 11) printf("%d %s %d\n", suffixArray.sa[i], s.substr(
```

```
suffixArray.sa[i]).c_str(), suffixArray.lcp[i]);
}
     Manacher
void manacher() {
    memset(p,0,sizeof p);
    int center = 0, right = 0, mi;
    for (int i = 1; i < n; i++) {
        mi = 2 * center - i:
        if (right > i) p[i] = min(right - i, p[mi]);
        while (a[i+(1+p[i])] == a[i-(1+p[i])]) p[i]++;
        //printf("%d:%d\n",i,p[i]);
        if (i + p[i] > right) {
            right = i+p[i];
            center = i;
        }
    }
      Convex Hull
struct Point {
    long long x, y;
    bool operator < (const Point &v) const {
        return x == v.x ? y < v.y : x < v.x;
    long long cross(const Point &p, const Point &q) const {
        return (p.x - x) * (q.y - y) - (p.y - y) * (q.x - x);
    }
};
vector<Point> convexHull(vector<Point> p) {
    sort(p.begin(), p.end());
    int k = 0, n = p.size();
    vector < Point > poly (2 * n);
    for(int i = 0; i < n; ++i) {
        while (k \ge 2 \&\& poly[k-2].cross(poly[k-1], p[i]) < 0) --k;
        poly[k++] = p[i];
    }
    for(int i = n-2, t = k+1; i \ge 0; --i) {
        \label{eq:while(k >= t && poly[k-2].cross(poly[k-1], p[i]) < 0) --k;}
        poly[k++] = p[i];
```

```
poly.resize(min(n, max(0, k - 1)));
   return poly;
    Geometry's tricks
const double eps = 1e-9;
bool equal(const double &x, const double &y) {
    return fabs(x - y) <
           eps;
struct Point {
    double x, y;
   Point(double x = 0, double y = 0): x(x), y(y) {}
    Point operator + (const Point &p) const {
        return \{x + p.x, y +
               р.у
              };
   Point operator - (const Point &p) const {
        return {x - p.x, y -
               р.у
              };
   Point operator * (double t) const {
       return {x * t, y * t};
   double operator * (const Point &p) const {
        return x * p.x + y *
               р.у;
    double operator % (const Point &p) const {
        return x * p.y - y *
               p.x;
    bool operator == (const Point &p) const {
        return equal(x, p.x)
               && equal(y, p.y);
    double operator ~ () const {
        return sqrt(*this **this);
   }
```

```
};
struct Comparator {
    Point a, b;
    Comparator (Point a, Point b): a(a), b(b) {}
    bool operator () (const Point &p, const Point &q) {
        return (p-a) * (b-a) < (q-a) * (b-a);
    }
};
bool between (double x, double 1, double r) {
    if (1 > r) swap(1, r);
    return x + eps > 1 && x - eps < r;
}
bool inside(Point q, const vector < Point > &p) {
    int n = p.size();
    for (int i = 0; i < n; i++) {
        int j = i + 1 < n ? i + 1 : 0;
        if (fabs((q - p[i]) % (p[j] - p[i])) > eps) continue;
        if ((q - p[i]) * (p[j] - p[i]) < -eps) continue;
        if ((q - p[j]) * (p[i] - p[j]) < -eps) continue;
        return true:
    }
    int fl = 0;
    for (int i = 0; i < n; i++) {
        int j = i + 1 < n ? i + 1 : 0;
        Point a = p[i], b = p[j];
        if (equal(a.x, b.x)) continue;
        if (a.x > b.x) swap(a, b);
        if (q.x < a.x - eps) continue;
        if (q.x > b.x - eps) continue;
        if ((q - a) \% (b - a) > 0) fl ^= 1;
    }
    return fl;
void intersect(Point p, Point q, Point a, Point b, vector<Point>
               &ints) {
    double na = (a - p) \% (q - p), nb = (b - p) \% (q - p);
    if (na * nb > eps) return;
    if (equal(na, nb)) return;
    ints.push_back(a + (b - a) * (na / (na - nb)));
void intersectCircleLine() {
    double r, a, b, c;
    double x0 = -a*c/(a*a+b*b), y0 = -b*c/(a*a+b*b);
    if (c*c > r*r*(a*a+b*b)+EPS) puts ("no points");
```

```
else if (abs (c*c - r*r*(a*a+b*b)) < EPS) {
        puts ("1 point");
        cout << x0 << ', ' << y0 << '\n';
   } else {
        double d = r*r - c*c/(a*a+b*b);
        double mult = sqrt (d / (a*a+b*b));
        double ax, ay, bx, by;
        ax = x0 + b * mult;
        bx = x0 - b * mult;
        av = v0 - a * mult;
        by = y0 + a * mult;
        puts ("2 points");
        cout << ax << ', ', << ay << '\n' << bx << ', ' << by << '\n';
   }
    FFT
15
const double PI = acos(-1.0);
typedef complex < double > Complex;
#define MASK(i) (1LL<<(i))
#define BIT(x,i) (((x) >> (i)) & 1)
#define LOG 17
Complex fftRoot[MASK(LOG)], invRoot[MASK(LOG)];
#define REP(i, n) for (int i = 0, n = (n); i < n; i = i + 1)
void initFFT(void) {
    REP(i, MASK(LOG)) {
        double alpha = 2 * PI / MASK(LOG) * i;
        fftRoot[i] = Complex(cos(alpha), sin(alpha));
        invRoot[i] = Complex(cos(-alpha), sin(-alpha));
   }
unsigned roundUp(unsigned v) {
    REP(i, 5) v = v >> MASK(i);
    return v + 1;
int reverse(int num, int lg) {
    int res = 0:
    REP(i, lg) if (BIT(num, i)) res \mid= MASK(lg - i - 1);
    return res;
vector < Complex > fft(vector < Complex > a, bool invert) {
```

```
int n = a.size(), lg = 0;
    while (MASK(lg) < n) lg++;
    vector < Complex > roots(n);
    REP(i, n) roots[i] = invert ? invRoot[MASK(LOG) / n * i] :
                          fftRoot[MASK(LOG) / n * i];
    REP(i, n) {
        int rev = reverse(i, lg);
        if (i < rev) swap(a[i], a[rev]);</pre>
    }
    for (int len = 2; len <= n; len <<= 1)
        for (int i = 0; i < n; i += len)
            for (int j = 0; j < (len >> 1); j++) {
                 Complex u = a[i + j], v = a[i + j + (len >> 1)] *
                                            roots[n / len * j];
                a[i + j] = u + v;
                a[i + j + (len >> 1)] = u - v;
    if (invert) REP(i, n) a[i] /= n;
    return a;
vector<long long> multiply(const vector<int> &a, const vector<int>
    int n = roundUp(size(a) + size(b) - 1);
    vector < Complex > pa (n), pb (n);
    for(int i = 0; i < size(a); ++i) pa[i] = a[i];
    for(int i = 0; i < size(b); ++i) pb[i] = b[i];
    pa = fft(pa, false);
    pb = fft(pb, false);
    for(int i = 0; i < n; ++i) pa[i] *= pb[i];
    pa = fft(pa, true);
    vector<long long> res (n);
    for(int i = 0; i < n; ++i) res[i] = round(real(pa[i]));</pre>
    return res;
}
      NTT
16
const int MODULO = 998244353;
const int ROOT = 3: // Primitive root
void fft(vector<int> &a, bool invert) {
    int n = a.size();
    assert((n & (n - 1)) == 0):
    int lg = __builtin_ctz(n);
```

```
for (int i = 0; i < n; ++i) {
        for (int k = 0; k < lg; ++k) if ((i&1 << k)!=0) j |= 1 <<
                        (lg-k-1);
        if (i < j) swap(a[i], a[j]);</pre>
   }
   for (int len = 2; len <= n; len *= 2) {
        int wlen = power(ROOT, (MODULO - 1) / len);
        if (invert) wlen = inverse(wlen);
        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; ++j) {
                int u = a[i + j];
                int v = 1LL * a[i + j + len / 2] * w % MODULO;
                a[i + j] = (u + v) % MODULO;
                a[i + j + len / 2] = (u - v + MODULO) % MODULO;
                w = 1LL * w * wlen % MODULO:
           }
        }
   }
    if (invert) {
        int mul = inverse(n);
        for (auto &x : a) x = 1LL * x * mul % MODULO;
   }
    998244353 = 119 * 2^23 + 1. Primitive root: 3.
    985661441 = 235 * 2^2 + 1. Primitive root: 3.
    1012924417 = 483 * 2^21 + 1. Primitive root: 5
    Primitive Root
int generator(int p) {
    vector < int > fact;
    int phi = p-1, n = phi;
   for (int i=2; i*i<=n; ++i) if (n % i == 0) {
        fact.push_back(i);
        while (n \% i == 0) n /= i;
   if (n > 1) fact.push_back(n);
   for (int res=2; res<=p; ++res) {
        bool ok = true;
        for (size_t i=0; i<fact.size() && ok; ++i)
            ok &= powmod (res, phi / fact[i], p) != 1;
```

```
if (ok) return res;
}
return -1;
}
```

18 Range Prime Counting

```
// Primes up to 10^12 can be counted in ~1 second.
const int MAXN = 1000005; // MAXN is the maximum value of sqrt(N) +
bool prime [MAXN];
int prec[MAXN];
vector < int > P;
void init() {
    prime[2] = true;
    for (int i = 3; i < MAXN; i += 2) prime[i] = true;
    for (int i = 3; i*i < MAXN; i += 2) {
        if (prime[i]) {
            for (int j = i*i; j < MAXN; j += i+i) prime[j] = false;</pre>
        }
    for(int i=1; i<MAXN; i++) {</pre>
        if (prime[i]) P.push_back(i);
        prec[i] = prec[i-1] + prime[i];
    }
}
lint rec(lint N, int K) {
    if (N <= 1 || K < 0) return 0;
    if (N <= P[K]) return N-1;
    if (N < MAXN && 111 * P[K]*P[K] > N) return N-1 - prec[N] +
                prec[P[K]];
    const int LIM = 250;
    static int memo[LIM*LIM][LIM];
    bool ok = N < LIM*LIM;
    if (ok && memo[N][K]) return memo[N][K];
    lint ret = N/P[K] - rec(N/P[K], K-1) + rec(N, K-1);
    if (ok) memo[N][K] = ret;
    return ret;
}
lint count_primes(lint N) { //less than or equal to
    if (N < MAXN) return prec[N];</pre>
    int K = prec[(int)sqrt(N) + 1];
    return N-1 - rec(N, K) + prec[P[K]];
```

19 Knight's shortest path

```
int KSP(int x,int y) {
   if (x < y) swap(x, y);
   if (x == 1 && y == 0) return 3;
   if (x == 2 && y == 2) return 4;
   int d = x - y;
   if (y > d) return 2*((y-d+2)/3)+d;
   return d-2*((d-y)/4);
}
```