

Mục lục

1	Java Fast Input
2	Simple Max Matching
3	Konig
4	Hopcroft Karp Max Matching algorithm
5	Max matching min cost
6	Ganeral Matching
7	Dinic MaxFlow
8	Mincost MaxFlow SPFA
9	Upper Lower
10	Two sat
11	Alternative Tree
12	Max Clique
13	Euler Path
14	Aho Corasick
15	Suffix Array
16	Suffix Array $O(n)$
17	Manacher
18	DP knuth
19	DP divide conquer
20	Convex Hull
21	Geometry's tricks
22	FFT
23	NTT
24	Primitive Root

25	Range Prime Counting	18
26	Knight's shortest path	18
27	Extended Euclid	18
28	Factorial Mod	18
29	Sqrt Mod	18
30	Interval line	19
31	BIT 2D	20
32	Heavy-Light Decomposition	21
33	Splay Tree	22
6	1 Java Fast Input	
7	class InputReader {	
8	private final BufferedReader reader;	
8	private StringTokenizer tokenizer;	
9	public InputReader(InputStream stream) {	
10	reader = new BufferedReader(new InputStreamReader(stream));	
10	tokenizer = null;	
10	}	
11	public String nextLine() {	
12	try {	
12	return reader.readLine();	
13	} catch (IOException e) {	
13	throw new RuntimeException(e);	
13	}	
14	}	
15	public String next() {	
15	while (tokenizer == null !tokenizer.hasMoreTokens()) {	
16	tokenizer = new StringTokenizer(nextLine());	
16	}	
16	return tokenizer.nextToken();	
16	}	
17	public int nextInt() {	
17	return Integer.parseInt(next());	

```

    }
}

```

2 Simple Max Matching

```

bool dfs(int u) {
    if (mx[u] == T) return false;
    mx[u] = T;
    for(int v : ke[u]) {
        if (!my[v] || dfs(my[v])) {
            my[v] = u;
            return true;
        }
    }
    return false;
}

```

```

int main() {
    For(i,1,n) {
        T++;
        res += dfs(i);
    }
    // choose my & i
}

```

3 Konig

```

void konig(){
    queue<int> qu;

    f1(i,m) if (!Assigned[i]) qu.push(i);
    f1(i,n) if (!Assigned[N-i]) qu.push(N-i);

    while (qu.size()){
        int u=qu.front(); qu.pop();
        for (int i=0; int v=a[u][i]; i++)
            if (!(Choosed[v]++)) qu.push(Assigned[v]);
    }

    f1(i,m) if (Assigned[i] && !Choosed[i] && !Choosed[Assigned[i]])
        Choosed[i]=true;
}

```

4 Hopcroft Karp Max Matching algorithm

```

// Worse Case: E * sqrt(v)
const int MAXN = 50005, MAXM = 50005;
vector<int> gph[MAXN];
int dis[MAXN], l[MAXN], r[MAXM], vis[MAXN];
void clear() {
    for (int i = 0; i < MAXN; i++)
        gph[i].clear();
}
void add_edge(int l, int r) {
    gph[l].push_back(r);
}
bool bfs(int n) {
    queue<int> que;
    bool ok = 0;
    memset(dis, 0, sizeof(dis));
    for (int i = 0; i < n; i++) {
        if (l[i] == -1 && !dis[i]) {
            que.push(i);
            dis[i] = 1;
        }
    }
    while (!que.empty()) {
        int x = que.front();
        que.pop();
        for (auto &i : gph[x]) {
            if (r[i] == -1)
                ok = 1;
            else if (!dis[r[i]]) {
                dis[r[i]] = dis[x] + 1;
                que.push(r[i]);
            }
        }
    }
    return ok;
}
bool dfs(int x) {
    for (auto &i : gph[x]) {
        if (r[i] == -1 || (!vis[r[i]] && dis[r[i]] == dis[x] + 1 &&
            dfs(r[i]))) {
            vis[r[i]] = 1;
            l[x] = i;
            r[i] = x;
        }
    }
}

```

```

        return 1;
    }
}
return 0;
}

int match(int n) {
    memset(l, -1, sizeof(l));
    memset(r, -1, sizeof(r));
    int ret = 0;
    while (bfs(n)) {
        memset(vis, 0, sizeof(vis));
        for (int i = 0; i < n; i++)
            if (l[i] == -1 && dfs(i))
                ret++;
    }
    return ret;
}

bool chk[MAXN + MAXM];
void rdfs(int x, int n) {
    if (chk[x])
        return;
    chk[x] = 1;
    for (auto &i : gph[x]) {
        chk[i + n] = 1;
        rdfs(r[i], n);
    }
}

vector<int> getcover(int n, int m) {
    // solve min. vertex cover
    match(n);
    memset(chk, 0, sizeof(chk));
    for (int i = 0; i < n; i++)
        if (l[i] == -1)
            rdfs(i, n);
    vector<int> v;
    for (int i = 0; i < n; i++)
        if (!chk[i])
            v.push_back(i);
    for (int i = n; i < n + m; i++)
        if (chk[i])
            v.push_back(i);
    return v;
}

```

5 Max matching min cost

```

// numbered from 0. i -> mx[i]
const int V = 1000, INF = 1e9;
int g[V][V], mx[V], my[V], fx[V], fy[V], d[V], ar[V], tr[V], p;
int slack(int u, int v) {
    return g[u][v] - fx[u] - fy[v];
}

int augment(int s) {
    queue<int> q;
    q.push(s);
    fill_n(tr, p, -1);
    for(int i = 0; i < p; ++i) d[i] = slack(s, i), ar[i] = s;
    while(true) {
        while(!q.empty()) {
            int u = q.front();
            q.pop();
            for(int v = 0; v < p; ++v) if(tr[v] == -1) {
                int w = slack(u, v);
                if(w == 0) {
                    tr[v] = u;
                    if(my[v] == -1) return v;
                    q.push(my[v]);
                }
                if(d[v] > w) d[v] = w, ar[v] = u;
            }
        }

        int delta = INF;
        for(int v = 0; v < p; ++v) if(tr[v] == -1) delta = min(delta, d[v]);
        fx[s] += delta;
        for(int v = 0; v < p; ++v)
            if(tr[v] == -1) d[v] -= delta;
            else fx[my[v]] += delta, fy[v] -= delta;
        for(int v = 0; v < p; ++v) if(tr[v] == -1 && d[v] == 0) {
            tr[v] = ar[v];
            if(my[v] == -1) return v;
            q.push(my[v]);
        }
    }
}

void maxMatchMinCost() {
    fill_n(mx, p, -1);
    fill_n(my, p, -1);
}

```

```

for(int i = 0; i < p; ++i) fx[i] = *min_element(g[i], g[i]+p);
for(int s = 0; s < p; ++s) {
    int f = augment(s);
    while(f != -1) {
        int x = tr[f], nx = mx[x];
        mx[x] = f;
        my[f] = x;
        f = nx;
    }
}
}

```

6 General Matching

```

class MatchingGraph {
public:
    vector <vector<int> > adj;
    vector <bool> blossom;
    vector <int> parent;
    vector <int> base;
    vector <int> match;
    int n;
    MatchingGraph() {
        n = 0;
    }
    void addEdge(int x, int y) {
        adj[x].push_back(y);
        adj[y].push_back(x);
    }
    void clearGraph() {
        int i;
        for (i=0; i<SZ(adj); ++i)
            adj[i].clear();
        fill(blossom.begin(),blossom.end(),false);
        fill(parent.begin(),parent.end(),-1);
        for (i=0; i<n; ++i)
            base[i] = i;
        for (i=0; i<n; ++i)
            match[i] = -1;
    }
    void setN(int newn) {
        n = newn;
        adj.resize(n);
    }

```

```

        blossom.resize(n);
        base.resize(n);
        match.resize(n);
        parent.resize(n);
        clearGraph();
    }
    int lca(int x, int y) {
        vector <bool> fy;
        fy.resize(n);
        fill(fy.begin(),fy.end(),false);
        while (true) {
            x = base[x];
            fy[x] = true;
            if (match[x] == -1)
                break;
            x = parent[match[x]];
        }
        while (true) {
            y = base[y];
            if (fy[y])
                return y;
            y = parent[match[y]];
        }
        return -1;
    }
    void path(int now, int child, int curbase) {
        while (base[now] != curbase) {
            blossom[base[now]] = blossom[base[match[now]]] = true;
            parent[now] = child;
            child = match[now];
            now = parent[match[now]];
        }
    }
    int augmentPath(int x) {
        int i,j;
        for (i=0; i<n; ++i)
            base[i] = i;
        for (i=0; i<n; ++i)
            parent[i] = -1;
        queue <int> bfs;
        vector <bool> sudah;
        sudah.resize(n);
        fill(sudah.begin(),sudah.end(),false);
        sudah[x] = true;
    }

```

```

bfs.push(x);
while (!bfs.empty()) {
    int now = bfs.front();
    bfs.pop();
    for (i=0; i<SZ(adj[now]); ++i) {
        int next = adj[now][i];
        if (base[next]==base[now] || match[next] == now);
        else if (next == x || (match[next]!=-1 &&
                               parent[match[next]]!=-1)) {
            int curbase = lca(now,next);
            fill(blossom.begin(),blossom.end(),false);
            path(now,next,curbase);
            path(next,now,curbase);
            for (j = 0; j < n; ++j)
                if (blossom[j]) {
                    base[j] = curbase;
                    if (!sudah[j]) {
                        sudah[j] = true;
                        bfs.push(j);
                    }
                }
        } else if (parent[next]==-1) {
            parent[next] = now;
            if (match[next] == -1)
                return next;
            sudah[match[next]] = true;
            bfs.push(match[next]);
        }
    }
}
return -1;
}

int edmondsMatch() {
    int i;
    int res = 0;
    for (i=0; i<n; ++i) {
        if (match[i]==-1) {
            int x = augmentPath(i);
            while (x>=0) {
                int p = parent[x];
                int pp = match[p];
                match[x] = p;
                match[p] = x;
                x = pp;
            }
        }
    }
    return res;
}

```

```

    }
    }
    for (i=0; i<n; ++i)
        if (match[i]!=-1)
            ++res;
    return res >> 1;
}
};

```

7 Dinic MaxFlow

```

class DinicFlow {
private:
    vector<int> dist, head, work;
    vector<int> point, flow, capa, next;
    int n, m;

    bool bfs(int s, int t) {
        For(i, 1, n) dist[i] = -1;
        queue<int> q;
        dist[s] = 0;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int i = head[u]; i >= 0; i = next[i])
                if (flow[i] < capa[i] && dist[point[i]] < 0) {
                    dist[point[i]] = dist[u] + 1;
                    q.push(point[i]);
                }
        }
        return dist[t] >= 0;
    }

    int dfs(int s, int t, int f) {
        if (s == t) return f;
        for (int &i = work[s]; i >= 0; i = next[i])
            if (flow[i] < capa[i] && dist[point[i]] == dist[s] + 1) {
                int d = dfs(point[i], t, min(f, capa[i] - flow[i]));
                if (d > 0) {
                    flow[i] += d;
                    flow[i ^ 1] -= d;
                }
            }
    }
}

```

```

        return d;
    }
}
return 0;
}

public:
    DinicFlow(int n = 0) {
        this->n = n;
        this->m = 0;
        dist.assign(n + 7, 0);
        head.assign(n + 7, -1);
        work.assign(n + 7, 0);
    }

    void addEdge(int u, int v, int c1, int c2 = 0) {
        point.push_back(v);
        capa.push_back(c1);
        flow.push_back(0);
        next.push_back(head[u]);
        head[u] = m++;
        point.push_back(u);
        capa.push_back(c2);
        flow.push_back(0);
        next.push_back(head[v]);
        head[v] = m++;
    }

    int maxFlow(int s, int t) {
        int totFlow = 0;
        while (bfs(s, t)) {
            For(i, 1, n) work[i] = head[i];
            while (true) {
                int d = dfs(s, t, cmax);
                if (d == 0) break;
                totFlow += d;
            }
        }
        return totFlow;
    }
}

```

8 Mincost MaxFlow SPFA

```

Min Cost Max Flow - SPFA
Index from 0
edges cap changed during find flow
Lots of double comparison --> likely to fail for double
Example:
MinCostFlow mcf(n);
mcf.addEdge(u, v, cap, cost);
cout << mcf.minCostFlow() << endl;

template<class Flow=int, class Cost=int>
struct MinCostFlow {
    const Flow INF_FLOW = 1000111000;
    const Cost INF_COST = 1000111000111000LL;

    int n, t, S, T;
    Flow totalFlow;
    Cost totalCost;
    vector<int> last, visited;
    vector<Cost> dis;
    struct Edge {
        int to;
        Flow cap;
        Cost cost;
        int next;
        Edge(int to, Flow cap, Cost cost, int next) :
            to(to), cap(cap), cost(cost), next(next) {}
    };
    vector<Edge> edges;

    MinCostFlow(int n) : n(n), t(0), totalFlow(0), totalCost(0), last(n,
        -1), visited(n, 0), dis(n, 0) {
        edges.clear();
    }

    int addEdge(int from, int to, Flow cap, Cost cost) {
        edges.push_back(Edge(to, cap, cost, last[from]));
        last[from] = t++;
        edges.push_back(Edge(from, 0, -cost, last[to]));
        last[to] = t++;
        return t - 2;
    }

    pair<Flow, Cost> minCostFlow(int _S, int _T) {

```

```

    S = _S; T = _T;
    SPFA();
    while (1) {
        while (1) {
            REP(i,n) visited[i] = 0;
            if (!findFlow(S, INF_FLOW)) break;
        }
        if (!modifyLabel()) break;
    }
    return make_pair(totalFlow, totalCost);
}

private:
void SPFA() {
    REP(i,n) dis[i] = INF_COST;
    priority_queue< pair<Cost,int> > Q;
    Q.push(make_pair(dis[S]=0, S));
    while (!Q.empty()) {
        int x = Q.top().second;
        Cost d = -Q.top().first;
        Q.pop();
        // For double: dis[x] > d + EPS
        if (dis[x] != d) continue;
        for(int it = last[x]; it >= 0; it = edges[it].next)
            if (edges[it].cap > 0 && dis[edges[it].to] > d + edges[it].cost)
                Q.push(make_pair(-(dis[edges[it].to] = d + edges[it].cost), edges[it].to));
    }
    Cost disT = dis[T]; REP(i,n) dis[i] = disT - dis[i];
}

Flow findFlow(int x, Flow flow) {
    if (x == T) {
        totalCost += dis[S] * flow;
        totalFlow += flow;
        return flow;
    }
    visited[x] = 1;
    Flow now = flow;
    for(int it = last[x]; it >= 0; it = edges[it].next)
        // For double: fabs(dis[edges[it].to] + edges[it].cost - dis[x]) < EPS
        if (edges[it].cap && !visited[edges[it].to] && dis[edges[it].

```

```

        to] + edges[it].cost == dis[x]) {
            Flow tmp = findFlow(edges[it].to, min(now, edges[it].cap))
            ;
            edges[it].cap -= tmp;
            edges[it ^ 1].cap += tmp;
            now -= tmp;
            if (!now) break;
        }
    }
    return flow - now;
}

bool modifyLabel() {
    Cost d = INF_COST;
    REP(i,n) if (visited[i])
        for(int it = last[i]; it >= 0; it = edges[it].next)
            if (edges[it].cap && !visited[edges[it].to])
                d = min(d, dis[edges[it].to] + edges[it].cost - dis[i]);

    // For double: if (d > INF_COST / 10)      INF_COST = 1e20
    if (d == INF_COST) return false;
    REP(i,n) if (visited[i])
        dis[i] += d;
    return true;
}
};

```

9 Upper Lower

- For each edge in original flow:
 - Add edge with cap = upper bound - lower bound.
- Add source s, sink t.
- Let $M[v] = (\text{sum of lowerbounds of ingoing edges to } v) - (\text{sum of lower bounds of outgoing edges from } v)$.
- For all v, if $M[v] > 0$, add (s, v, M), else add (v, t, -M).
- If all outgoing edges from S are full --> feasible flow exists, it is flow + lower bounds.

Feasible flow in network with upper + lower constraint, with source & sink

```

:
- Add edge (t, s) with capacity [0, INF].
- Check feasible in network without source & sink.

```

```

Max flow with both upper + lower constraints, source s, sink t: add edge (
    t, s, +INF).
- Binary search lower bound, check whether feasible flow exists WITHOUT
    source / sink

```

10 Two sat

```

int n, m, g[maxn];
bool cx[maxn];
vector<int> listV, ke[maxn], K[maxn];

int cal(int x) {
    if (x%2 == 0) return x - 1;
    else return x + 1;
}

void add(int u, int v) {
    ke[u].pb(v);
    K[v].pb(u);
}

void dfs(int u) {
    cx[u] = true;
    for(int v : ke[u])
        if (!cx[v]) dfs(v);
    listV.pb(u);
}

void dfs(int u, int x) {
    g[u] = x;
    for(int v : K[u])
        if (g[v] == 0) dfs(v, x);
}

int main() {
    cin >> m >> n;
    n += n;
    For(i, 1, m) {
        int u, v;
        cin >> u >> v;
        u *= 2;
        v *= 2;
        if (u < 0) u = cal(abs(u));

```

```

        if (v < 0) v = cal(abs(v));
        add(cal(u), v);
        add(cal(v), u);
    }
    listV.pb(0);
    For(i, 1, n)
        if (!cx[i]) dfs(i);
    int ng = 0;
    Ford(i, n, 1) {
        int u = listV[i];
        if (g[u] == 0) dfs(u, ++ng);
    }
    for(int i = 2; i <= n; i += 2)
        if (g[i] == g[i-1]) NO;
    YES;
    vector<int> result;
    for(int i = 2; i <= n; i += 2)
        if (g[i] > g[i-1]) result.pb(i>>1);
}

```

11 Alternative Tree

```

int n, m, l, q, t, res, test,
    a[maxn], tin[maxn], tout[maxn], mark[maxn], terror[maxn], f[maxn][20];
vector<int> adj[maxn], _adj[maxn];
stack<int> stk;

void visit(const int &u) {
    tin[u] = ++t;
    for(int i = 1; i <= l; ++i) f[u][i] = f[f[u][i-1]][i-1];
    for(auto v : adj[u])
        if (v != f[u][0]) {
            f[v][0] = u;
            visit(v);
        }
    tout[u] = ++t;
}

bool anc(const int &u, const int &v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}

int lca(int u, int v) {

```



```

    if (anc(u,v)) return u;
    if (anc(v,u)) return v;
    for(int i = 1; i >= 0; --i)
        if (!anc(f[u][i],v)) u = f[u][i];
    return f[u][0];
}

void query() {
    cin >> m;
    for(int i = 1; i <= m; ++i) {
        cin >> a[i];
        _adj[a[i]].clear();
        mark[a[i]] = test;
        terror[a[i]] = test;
    }
    sort(a+1,a+m+1,cmp);
    for(int i = 1; i < m; ++i) {
        int tmp = lca(a[i],a[i+1]);
        if (mark[tmp] < test) {
            mark[tmp] = test;
            a[++m] = tmp;
            _adj[tmp].clear();
        }
    }
    // sort theo tin
    sort(a+1,a+m+1,cmp);
    while (!stk.empty()) stk.pop();
    stk.push(a[1]);
    for(int i = 2; i <= m; ++i) {
        while (tout[stk.top()] < tout[a[i]]) stk.pop();
        _adj[stk.top()].push_back(a[i]);
        stk.push(a[i]);
    }
    res = 0;
    check(a[1]);
    cout << res << "\n";
}

int main() {
    l = log2(n);
    cin >> q;
    f[1][0] = 1;
    visit(1);
    for(test = 1; test <= q; ++test) query();
}

```

```

}

```

12 Max Clique

```

class MaxClique {
public:
    static const int MV = 210;

    int V;
    int el[MV][MV/30+1];
    int dp[MV];
    int ans;
    int s[MV][MV/30+1];
    vector<int> sol;

    void init(int v) {
        V = v; ans = 0;
        FZ(el); FZ(dp);
    }

    /* Zero Base */
    void addEdge(int u, int v) {
        if(u > v) swap(u, v);
        if(u == v) return;
        el[u][v/32] |= (1<<(v%32));
    }

    bool dfs(int v, int k) {
        int c = 0, d = 0;
        for(int i=0; i<(V+31)/32; i++) {
            s[k][i] = el[v][i];
            if(k != 1) s[k][i] &= s[k-1][i];
            c += __builtin_popcount(s[k][i]);
        }
        if(c == 0) {
            if(k > ans) {
                ans = k;
                sol.clear();
                sol.push_back(v);
                return 1;
            }
            return 0;
        }
    }
}

```

```

for(int i=0; i<(V+31)/32; i++) {
    for(int a = s[k][i]; a ; d++) {
        if(k + (c-d) <= ans) return 0;
        int lb = a&(-a), lg = 0;
        a ^= lb;
        while(lb!=1) {
            lb = (unsigned int)(lb) >> 1;
            lg ++;
        }
        int u = i*32 + lg;
        if(k + dp[u] <= ans) return 0;
        if(dfs(u, k+1)) {
            sol.push_back(v);
            return 1;
        }
    }
}
return 0;

int solve() {
    for(int i=V-1; i>=0; i--) {
        dfs(i, 1);
        dp[i] = ans;
    }
    return ans;
}
};

```

13 Euler Path

NOTES:

- When choosing starting vertex (for calling find_path), make sure $\deg[\text{start}] > 0$.
- If find Euler path, starting vertex must have odd degree.
- Check no solution: $\text{SZ}(\text{path}) == \text{nEdge} + 1$.

If directed:

- Edge \rightarrow int
- add_edge(int a, int b) { adj[a].push_back(b); }
- Check for no solution:
 - for all u, $|\text{in_deg}[u] - \text{out_deg}[u]| \leq 1$
 - At most 1 vertex with $\text{in_deg}[u] - \text{out_deg}[u] = 1$

```

- - At most 1 vertex with  $\text{out\_deg}[u] - \text{in\_deg}[u] = 1$  (start vertex)
- - BFS from start vertex, all vertices u with  $\text{out\_deg}[u] > 0$  must be
    visited
struct Edge {
    int to;
    list<Edge>::iterator rev;

    Edge(int to) :to(to) {}
};

const int MN = 100111;
list<Edge> adj[MN];
vector<int> path; // our result

void find_path(int v) {
    while(adj[v].size() > 0) {
        int vn = adj[v].front().to;
        adj[vn].erase(adj[v].front().rev);
        adj[v].pop_front();
        find_path(vn);
    }
    path.push_back(v);
}

void add_edge(int a, int b) {
    adj[a].push_front(Edge(b));
    auto ita = adj[a].begin();
    adj[b].push_front(Edge(a));
    auto itb = adj[b].begin();
    ita->rev = itb;
    itb->rev = ita;
}

```

14 Aho Corasick

```

const int NODE = (int) 1e6 + 1;
const int NC = 26;

int nextNode[NODE][NC];
int chr[NODE];
int parent[NODE];
int prefix[NODE];
int numNodes;

```

```

set<int> match[NODE];

int getPrefix(int);

int go(int u, int c) {
    if (nextNode[u][c] != -1) return nextNode[u][c];
    if (u == 0) return 0;
    return nextNode[u][c] = go(getPrefix(u), c);
}

int getPrefix(int u) {
    if (prefix[u] != -1) return prefix[u];
    if (u == 0 || parent[u] == 0) return prefix[u] = 0;
    return prefix[u] = go(getPrefix(parent[u]), chr[u]);
}

void add(const string &s, int id) {
    int u = 0;
    for (int i = 0; i < (int) s.size(); ++i) {
        int c = s[i] - 'A';
        if (nextNode[u][c] == -1) {
            nextNode[u][c] = numNodes;
            fill(nextNode[numNodes], nextNode[numNodes] + NC, -1);
            chr[numNodes] = c;
            parent[numNodes] = u;
            prefix[numNodes] = -1;
            match[numNodes].clear();
            match[numNodes].insert(-1);
            ++numNodes;
        }
        u = nextNode[u][c];
    }
    match[u].insert(id);
}

set<int>& getMatch(int u) {
    if (match[u].count(-1) == 0) return match[u];
    const set<int> &foo = getMatch(getPrefix(u));
    match[u].insert(foo.begin(), foo.end());
    match[u].erase(-1);
    return match[u];
}

void init() {

```

```

    fill(nextNode[0], nextNode[0] + NC, -1);
    numNodes = 1;
}

```

15 Suffix Array

```

struct SuffixArray {
    const int L;
    string s;
    vector<vector<int>> > P;
    vector<pair<pair<int,int>,int>> > M;
    SuffixArray(const string &s) : L(s.length()), s(s), P(1,
        vector<int>(L, 0)), M(L) {
        for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
        for (int skip = 1, lv = 1; skip < L; skip *= 2, lv++) {
            P.push_back(vector<int>(L, 0));
            for (int i = 0; i < L; i++)
                M[i] = make_pair(make_pair(P[lv-1][i], i + skip < L
                    ? P[lv-1][i + skip] : -1000), i);

            sort(M.begin(), M.end());
            for (int i = 0; i < L; i++)
                P[lv][M[i].se] = (i > 0 && M[i].fi == M[i-1].fi) ?
                    P[lv][M[i-1].se] : i;
        }
    }

    vector<int> GetSuffixArray() {
        return P.back();
    }

    // returns the length of the longest common prefix of s[i...L-1]
    and s[j...L-1]
    int LongestCommonPrefix(int i, int j) {
        int len = 0;
        if (i == j) return L - i;
        for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;
                j += 1 << k;
                len += 1 << k;
            }
        }
        return len;
    }
}

```

```
};
```

16 Suffix Array O(n)

```
#include <bits/stdc++.h>
#define FOR(i,a,b) for (int i=(a),_b=(b);i<=_b;i=i+1)
#define REP(i,n) for (int i=0,_n=(n);i<_n;i=i+1)
#define MASK(i) (1LL<<(i))
#define BIT(x,i) (((x)>>(i))&1)
#define tget(i) BIT(t[(i)>>3], (i)&7)
#define tset(i, b) { if (b) t[(i)>>3] |= MASK((i)&7); else t[(i)>>3]
    &= ~MASK((i)&7); }
#define chr(i) (cs == sizeof(int) ? ((int *)s)[i] : ((unc *)s)[i])
#define isLMS(i) ((i) > 0 && tget(i) && !tget((i) - 1))

typedef unsigned char unc;
class SuffixArray {
public:
    int *sa, *lcp, *rank, n;
    unc *s;
    void getbuckets(unc s[], vector<int> &bkt, int n, int k, int cs, bool
        end) {
        FOR(i, 0, k) bkt[i] = 0;
        REP(i, n) bkt[chr(i)]++;
        int sum = 0;
        FOR(i, 0, k) {
            sum += bkt[i];
            bkt[i] = end ? sum : sum - bkt[i];
        }
    }
    void inducesal(vector<unc> &t, int sa[], unc s[], vector<int> &bkt,
        int n, int k, int cs, bool end) {
        getbuckets(s, bkt, n, k, cs, end);
        REP(i, n) {
            int j = sa[i] - 1;
            if (j >= 0 && !tget(j)) sa[bkt[chr(j)]++] = j;
        }
    }
    void inducesas(vector<unc> &t, int sa[], unc s[], vector<int> &bkt,
        int n, int k, int cs, bool end) {
        getbuckets(s, bkt, n, k, cs, end);
        FORD(i, n - 1, 0) {
            int j = sa[i] - 1;
```

```
            if (j >= 0 && tget(j)) sa[--bkt[chr(j)]] = j;
        }
    }
    void build(unc s[], int sa[], int n, int k, int cs) {
        int j;
        vector<unc> t = vector<unc>(n / 8 + 1, 0);
        tset(n - 2, 0);
        tset(n - 1, 1);
        FORD(i, n - 3, 0) tset(i, chr(i) < chr(i+1) || (chr(i) == chr(i+1)
            && tget(i+1)));
        vector<int> bkt = vector<int>(k + 1, 0);
        getbuckets(s, bkt, n, k, cs, true);
        REP(i, n) sa[i] = -1;
        REP(i, n) if (isLMS(i)) sa[--bkt[chr(i)]] = i;
        inducesal(t, sa, s, bkt, n, k, cs, false);
        inducesas(t, sa, s, bkt, n, k, cs, true);
        bkt.clear();
        int n1 = 0;
        REP(i, n) if (isLMS(sa[i])) sa[n1++] = sa[i];
        FOR(i, n1, n - 1) sa[i] = -1;
        int name = 0;
        int prev = -1;
        REP(i, n1) {
            int pos = sa[i];
            bool diff = false;
            REP(d, n) {
                if (prev < 0 || chr(prev + d) != chr(pos + d) || tget(prev
                    + d) != tget(pos + d)) {
                    diff = true;
                    break;
                }
                else if (d > 0 && (isLMS(prev + d) || isLMS(pos + d)))
                    break;
            }
            if (diff) {
                name++;
                prev = pos;
            }
            sa[n1 + pos / 2] = name - 1;
        }
        j = n - 1;
        FORD(i, n - 1, n1) if (sa[i] >= 0) sa[j--] = sa[i];
        int *sal = sa;
        int *s1 = sa + n - n1;
```

```

    if (name < n1) build((unc *)s1, sa1, n1, name-1, sizeof(int));
    else REP(i, n1) sa1[s1[i]] = i;
    bkt.assign(k + 1, 0);
    getbuckets(s, bkt, n, k, cs, true);
    j = 0;
    REP(i, n) if (isLMS(i)) s1[j++] = i;
    REP(i, n1) sa1[i] = s1[sa1[i]];
    FOR(i, n1, n - 1) sa[i] = -1;
    FORD(i, n1 - 1, 0) {
        j = sa[i];
        sa[i] = -1;
        sa[--bkt[chr(j)]] = j;
    }
    inducesal(t, sa, s, bkt, n, k, cs, false);
    inducesas(t, sa, s, bkt, n, k, cs, true);
    bkt.clear();
    t.clear();
}

void calc_lcp(void) {
    FOR(i, 1, n) rank[sa[i]] = i;
    int h = 0;
    REP(i, n) if (rank[i] < n) {
        int j = sa[rank[i] + 1];
        while (s[i + h] == s[j + h]) h++;
        lcp[rank[i]] = h;
        if (h > 0) h--;
    }
}

SuffixArray() {
    n = 0;
    sa = lcp = rank = NULL;
    s = NULL;
}

SuffixArray(string ss) {
    n = ss.size();
    sa = new int[n + 7];
    lcp = new int[n + 7];
    rank = new int[n + 7];
    s = (unc *)ss.c_str();
    build(s, sa, n + 1, 256, sizeof(char));
    calc_lcp();
}
};

```

//Sorted suffices are SA[1] to SA[N]. The values of SA[1], SA[2], ..., SA[N] are 0, 1, ..., N - 1
//The longest common prefix of SA[i] and SA[i + 1] is LCP[i]

```

int main(void) {
    string s = "mississippi";
    SuffixArray suffixArray(s);
    FOR(i, 1, 11) printf("%d %s %d\n", suffixArray.sa[i], s.substr(
        suffixArray.sa[i]).c_str(), suffixArray.lcp[i]);
}

```

17 Manacher

```

void manacher() {
    memset(p, 0, sizeof p);
    int center = 0, right = 0, mi;
    for (int i = 1; i < n; i++) {
        mi = 2 * center - i;
        if (right > i) p[i] = min(right - i, p[mi]);
        while (a[i+(1+p[i])] == a[i-(1+p[i])]) p[i]++;
        //printf("%d:%d\n", i, p[i]);
        if (i + p[i] > right) {
            right = i + p[i];
            center = i;
        }
    }
}

```

18 DP knuth

<http://codeforces.com/blog/entry/8219>

Original Recurrence:

$dp[i][j] = \min(dp[i][k] + dp[k][j]) + C[i][j] \quad \text{for } k = i+1..j-1$

Necessary & Sufficient Conditions:

$A[i][j-1] \leq A[i][j] \leq A[i+1][j]$

with $A[i][j]$ = smallest k that gives optimal answer

Also applicable if the following conditions are met:

1. $C[a][c] + C[b][d] \leq C[a][d] + C[b][c]$ (quadrangle inequality)
2. $C[b][c] \leq C[a][d]$ (monotonicity)

for all $a \leq b \leq c \leq d$

To use:

Calculate $dp[i][i]$ and $A[i][i]$

```

FOR(len = 1..n-1)
  FOR(i = 1..n-len) {
    j = i + len
    FOR(k = A[i][j-1]..A[i+1][j])
      update(dp[i][j])
  }

// OPTCUT
#include "../template.h"

const int MN = 2011;
int a[MN], dp[MN][MN], C[MN][MN], A[MN][MN];
int n;

void solve() {
  cin >> n; FOR(i,1,n) { cin >> a[i]; a[i] += a[i-1]; }
  FOR(i,1,n) FOR(j,i,n) C[i][j] = a[j] - a[i-1];

  FOR(i,1,n) dp[i][i] = 0, A[i][i] = i;

  FOR(len,1,n-1)
    FOR(i,1,n-len) {
      int j = i + len;
      dp[i][j] = 2000111000;
      FOR(k,A[i][j-1],A[i+1][j]) {
        int cur = dp[i][k-1] + dp[k][j] + C[i][j];
        if (cur < dp[i][j]) {
          dp[i][j] = cur;
          A[i][j] = k;
        }
      }
    }
  cout << dp[1][n] << endl;
}

```

19 DP divide conquer

<http://codeforces.com/blog/entry/8219>

Divide and conquer optimization:

Original Recurrence

$dp[i][j] = \min(dp[i-1][k] + C[k][j])$ for $k < j$

Sufficient condition:

```

A[i][j] <= A[i][j+1]
where A[i][j] = smallest k that gives optimal answer
How to use:
// compute i-th row of dp from L to R. optL <= A[i][L] <= A[i][R] <=
  optR
compute(i, L, R, optL, optR)
  1. special case L == R
  2. let M = (L + R) / 2. Calculate dp[i][M] and opt[i][M] using 0(
    optR - optL + 1)
  3. compute(i, L, M-1, optL, opt[i][M])
  4. compute(i, M+1, R, opt[i][M], optR)

const int MN = 4011;
const int inf = 1000111000;
int n, k, cost[MN][MN], dp[811][MN];

inline int getCost(int i, int j) {
  return cost[j][j] - cost[j][i-1] - cost[i-1][j] + cost[i-1][i-1];
}

void compute(int i, int L, int R, int optL, int optR) {
  if (L > R) return ;

  int mid = (L + R) >> 1, savek = optL;
  dp[i][mid] = inf;
  FOR(k,optL,min(mid-1, optR)) {
    int cur = dp[i-1][k] + getCost(k+1, mid);
    if (cur < dp[i][mid]) {
      dp[i][mid] = cur;
      savek = k;
    }
  }
  compute(i, L, mid-1, optL, savek);
  compute(i, mid+1, R, savek, optR);
}

void solve() {
  cin >> n >> k;
  FOR(i,1,n) FOR(j,1,n) {
    cin >> cost[i][j];
    cost[i][j] = cost[i-1][j] + cost[i][j-1] - cost[i-1][j-1] + cost[i-1][j];
  }
}

```

```

dp[0][0] = 0;
FOR(i,1,n) dp[0][i] = inf;

FOR(i,1,k) {
    compute(i, 1, n, 0, n);
}
cout << dp[k][n] / 2 << endl;
}

```

20 Convex Hull

```

struct Point {
    long long x, y;
    bool operator < (const Point &v) const {
        return x == v.x ? y < v.y : x < v.x;
    }
    long long cross(const Point &p, const Point &q) const {
        return (p.x - x) * (q.y - y) - (p.y - y) * (q.x - x);
    }
};

vector<Point> convexHull(vector<Point> p) {
    sort(p.begin(), p.end());
    int k = 0, n = p.size();
    vector<Point> poly (2 * n);
    for(int i = 0; i < n; ++i) {
        while(k >= 2 && poly[k-2].cross(poly[k-1], p[i]) < 0) --k;
        poly[k++] = p[i];
    }
    for(int i = n-2, t = k+1; i >= 0; --i) {
        while(k >= t && poly[k-2].cross(poly[k-1], p[i]) < 0) --k;
        poly[k++] = p[i];
    }
    poly.resize(min(n, max(0, k - 1)));
    return poly;
}

```

21 Geometry's tricks

```

// hash func overload
struct point{
    int x, y;
    bool operator==(const point &p)const{ return x == p.x && y == p.y; }
}

```

```

};
struct hasher {
    size_t operator()(const point &p)const{ return p.x * 2 + p.y * 3; }
};
unordered_map<point, int, hasher> hsh;
const double eps = 1e-9;
bool equal(const double &x, const double &y) {
    return fabs(x - y) <
        eps;
}
struct Point {
    double x, y;
    Point(double x = 0, double y = 0): x(x), y(y) {}
    Point operator + (const Point &p) const {
        return {x + p.x, y +
            p.y
        };
    }
    Point operator - (const Point &p) const {
        return {x - p.x, y -
            p.y
        };
    }
    Point operator * (double t) const {
        return {x * t, y * t};
    }
    double operator * (const Point &p) const {
        return x * p.x + y *
            p.y;
    }
    double operator % (const Point &p) const {
        return x * p.y - y *
            p.x;
    }
    bool operator == (const Point &p) const {
        return equal(x, p.x)
            && equal(y, p.y);
    }
    double operator ~ () const {
        return sqrt(*this **this);
    }
};
struct Comparator {
    Point a, b;
}

```

```

Comparator(Point a, Point b): a(a), b(b) {}
bool operator () (const Point &p, const Point &q) {
    return (p-a) * (b-a) < (q-a) * (b-a);
}
};
bool between(double x, double l, double r) {
    if (l > r) swap(l, r);
    return x + eps > l && x - eps < r;
}
bool inside(Point q, const vector<Point> &p) {
    int n = p.size();
    for (int i = 0; i < n; i++) {
        int j = i + 1 < n ? i + 1 : 0;
        if (fabs((q - p[i]) % (p[j] - p[i])) > eps) continue;
        if ((q - p[i]) * (p[j] - p[i]) < -eps) continue;
        if ((q - p[j]) * (p[i] - p[j]) < -eps) continue;
        return true;
    }
    int fl = 0;
    for (int i = 0; i < n; i++) {
        int j = i + 1 < n ? i + 1 : 0;
        Point a = p[i], b = p[j];
        if (equal(a.x, b.x)) continue;
        if (a.x > b.x) swap(a, b);
        if (q.x < a.x - eps) continue;
        if (q.x > b.x - eps) continue;
        if ((q - a) % (b - a) > 0) fl ^= 1;
    }
    return fl;
}
void intersect(Point p, Point q, Point a, Point b, vector<Point>
    &ints) {
    double na = (a - p) % (q - p), nb = (b - p) % (q - p);
    if (na * nb > eps) return;
    if (equal(na, nb)) return;
    ints.push_back(a + (b - a) * (na / (na - nb)));
}
void intersectCircleLine() {
    double r, a, b, c;
    double x0 = -a*c/(a*a+b*b), y0 = -b*c/(a*a+b*b);
    if (c*c > r*r*(a*a+b*b)+EPS) puts ("no points");
    else if (abs (c*c - r*r*(a*a+b*b)) < EPS) {
        puts ("1 point");
        cout << x0 << ' ' << y0 << '\n';
    }
}

```

```

    } else {
        double d = r*r - c*c/(a*a+b*b);
        double mult = sqrt (d / (a*a+b*b));
        double ax,ay,bx,by;
        ax = x0 + b * mult;
        bx = x0 - b * mult;
        ay = y0 - a * mult;
        by = y0 + a * mult;
        puts ("2 points");
        cout << ax << ' ' << ay << '\n' << bx << ' ' << by << '\n';
    }
}
}

```

22 FFT

```

const double PI = acos(-1.0);
typedef complex<double> Complex;
#define MASK(i) (1LL<<(i))
#define BIT(x,i) (((x) >> (i)) & 1)
#define LOG 17
Complex fftRoot[MASK(LOG)], invRoot[MASK(LOG)];
#define REP(i, n) for (int i = 0, _n = (n); i < _n; i = i + 1)
void initFFT(void) {
    REP(i, MASK(LOG)) {
        double alpha = 2 * PI / MASK(LOG) * i;
        fftRoot[i] = Complex(cos(alpha), sin(alpha));
        invRoot[i] = Complex(cos(-alpha), sin(-alpha));
    }
}
unsigned roundUp(unsigned v) {
    --v;
    REP(i, 5) v |= v >> MASK(i);
    return v + 1;
}
int reverse(int num, int lg) {
    int res = 0;
    REP(i, lg) if (BIT(num, i)) res |= MASK(lg - i - 1);
    return res;
}
vector<Complex> fft(vector<Complex> a, bool invert) {
    int n = a.size(), lg = 0;
    while (MASK(lg) < n) lg++;
    vector<Complex> roots(n);
}

```



```

    REP(i, n) roots[i] = invert ? invRoot[MASK(LOG) / n * i] :
        fftRoot[MASK(LOG) / n * i];

    REP(i, n) {
        int rev = reverse(i, lg);
        if (i < rev) swap(a[i], a[rev]);
    }

    for (int len = 2; len <= n; len <= 1)
        for (int i = 0; i < n; i += len)
            for (int j = 0; j < (len >> 1); j++) {
                Complex u = a[i + j], v = a[i + j + (len >> 1)] *
                    roots[n / len * j];

                a[i + j] = u + v;
                a[i + j + (len >> 1)] = u - v;
            }

    if (invert) REP(i, n) a[i] /= n;
    return a;
}

vector<long long> multiply(const vector<int> &a, const vector<int>
    &b) {
    int n = roundUp(size(a) + size(b) - 1);
    vector<Complex> pa (n), pb (n);
    for(int i = 0; i < size(a); ++i) pa[i] = a[i];
    for(int i = 0; i < size(b); ++i) pb[i] = b[i];
    pa = fft(pa, false);
    pb = fft(pb, false);
    for(int i = 0; i < n; ++i) pa[i] *= pb[i];
    pa = fft(pa, true);
    vector<long long> res (n);
    for(int i = 0; i < n; ++i) res[i] = round(real(pa[i]));
    return res;
}

```

23 NTT

```

const int MODULO = 998244353;
const int ROOT = 3; // Primitive root
void fft(vector<int> &a, bool invert) {
    int n = a.size();
    assert((n & (n - 1)) == 0);
    int lg = __builtin_ctz(n);
    for (int i = 0; i < n; ++i) {
        int j = 0;
        for (int k = 0; k < lg; ++k) if ((i & 1 << k) != 0) j |= 1 <<

```

```

        (lg-k-1);
        if (i < j) swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len *= 2) {
        int wlen = power(ROOT, (MODULO - 1) / len);
        if (invert) wlen = inverse(wlen);
        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; ++j) {
                int u = a[i + j];
                int v = 1LL * a[i + j + len / 2] * w % MODULO;
                a[i + j] = (u + v) % MODULO;
                a[i + j + len / 2] = (u - v + MODULO) % MODULO;
                w = 1LL * w * wlen % MODULO;
            }
        }
    }

    if (invert) {
        int mul = inverse(n);
        for (auto &x : a) x = 1LL * x * mul % MODULO;
    }
}

998244353 = 119 * 2^23 + 1. Primitive root: 3.
985661441 = 235 * 2^22 + 1. Primitive root: 3.
1012924417 = 483 * 2^21 + 1. Primitive root: 5

```

24 Primitive Root

```

int generator(int p) {
    vector<int> fact;
    int phi = p-1, n = phi;
    for (int i=2; i*i<=n; ++i) if (n % i == 0) {
        fact.push_back(i);
        while (n % i == 0) n /= i;
    }
    if (n > 1) fact.push_back(n);
    for (int res=2; res<=p; ++res) {
        bool ok = true;
        for (size_t i=0; i<fact.size() && ok; ++i)
            ok &= powmod (res, phi / fact[i], p) != 1;
        if (ok) return res;
    }
    return -1;
}

```

}

25 Range Prime Counting

```
// Primes up to  $10^{12}$  can be counted in ~1 second.
const int MAXN = 1000005; // MAXN is the maximum value of  $\sqrt{N}$  + 2
bool prime[MAXN];
int prec[MAXN];
vector<int> P;
void init() {
    prime[2] = true;
    for (int i = 3; i < MAXN; i += 2) prime[i] = true;
    for (int i = 3; i*i < MAXN; i += 2) {
        if (prime[i]) {
            for (int j = i*i; j < MAXN; j += i) prime[j] = false;
        }
    }
    for(int i=1; i<MAXN; i++) {
        if (prime[i]) P.push_back(i);
        prec[i] = prec[i-1] + prime[i];
    }
}

lint rec(lint N, int K) {
    if (N <= 1 || K < 0) return 0;
    if (N <= P[K]) return N-1;
    if (N < MAXN && 1ll * P[K]*P[K] > N) return N-1 - prec[N] +
        prec[P[K]];
    const int LIM = 250;
    static int memo[LIM*LIM][LIM];
    bool ok = N < LIM*LIM;
    if (ok && memo[N][K]) return memo[N][K];
    lint ret = N/P[K] - rec(N/P[K], K-1) + rec(N, K-1);
    if (ok) memo[N][K] = ret;
    return ret;
}

lint count_primes(lint N) { //less than or equal to
    if (N < MAXN) return prec[N];
    int K = prec[(int)sqrt(N) + 1];
    return N-1 - rec(N, K) + prec[P[K]];
}
```

26 Knight's shortest path

```
int KSP(int x,int y) {
    if (x < y) swap(x, y);
    if (x == 1 && y == 0) return 3;
    if (x == 2 && y == 2) return 4;
    int d = x - y;
    if (y > d) return 2*((y-d+2)/3)+d;
    return d-2*((d-y)/4);
}
```

27 Extended Euclid

Gia su ket qua la (x_0, y_0) , ho nghiem la $(x_0 + k * b / d, y_0 - k * a / d)$

Phuong trinh $ax + by = d$ co nghiem khi va chi khi d chia het cho $\gcd(a, b)$

$ax + by = \gcd(a, b)$

```
int extgcd(int a, int b, int &x, int &y) {
    int g = a; x = 1; y = 0;
    if (b != 0) g = extgcd(b, a % b, y, x), y -= (a / b) * x;
    return g;
}
```

28 Factorial Mod

```
int factmod (int n, int p) { //  $n!$ , excluding  $p^k$  of course
    int res = 1;
    while (n > 1) {
        res = (res * ((n/p) % 2 ? p-1 : 1)) % p;
        for (int i=2; i<=n%p; ++i)
            res = (res * i) % p;
        n /= p;
    }
    return res % p;
}
```

29 Sqrt Mod

```
// Jacobi Symbol  $(m/n)$ ,  $m, n \geq 0$  and  $n$  is odd
```

```

#define NEGPOW(e) ((e) % 2 ? -1 : 1)
int jacobi(int a, int m) {
    if (a == 0) return m == 1 ? 1 : 0;
    if (a % 2) return NEGPOW((a-1)*(m-1)/4)*jacobi(m%a, a);
    else return NEGPOW((m*m-1)/8)*jacobi(a/2, m);
}
int invMod(int a, int m) {
    int x, y;
    if (extgcd(a, m, x, y) == 1) return (x + m) % m;
    else return 0; // unsolvable
}
// No solution when:  $n(p-1)/2 = -1 \pmod p$ 
int sqrtMod(int n, int p) { //find x:  $x^2 = n \pmod p$  p is prime
    int S, Q, W, i, m = invMod(n, p);
    for (Q = p - 1, S = 0; Q % 2 == 0; Q /= 2, ++S);
    do { W = rand() % p; } while (W == 0 || jacobi(W, p) != -1);
    for (int R = powMod(n, (Q+1)/2, p), V = powMod(W, Q, p); ;) {
        int z = R * R * m % p;
        for (i = 0; i < S && z % p != 1; z *= z, ++i);
        if (i == 0) return R;
        R = (R * powMod(V, 1 << (S-i-1), p)) % p;
    }
}
int powMod (int a, int b, int p) {
    int res = 1;
    while (b)
        if (b & 1)
            res = int (res * 1ll * a % p), --b;
        else
            a = int (a * 1ll * a % p), b >>= 1;
    return res;
}

```

30 Interval line

```

// template Interval line Min
#define mid ((lo + hi)>>1)

```

```

class Line {
public:
    ll a, b;

    Line (ll x = cmax, ll y = cmax) {

```

```

        a = x, b = y;
    }

    ll get(int x) {
        return 1ll * val[x] * a + b;
    }
};

const Line oo = Line(cmax, cmax);

class ILTree {
    int m;
public:
    Line t[maxn*4];

    ILTree(int last = 200000) {
        m = last;
        init(1,1,m);
    }

    void init(int i, int lo, int hi) {
        t[i] = oo;
        if (lo == hi) return;
        init(i * 2, lo, mid);
        init(i * 2 + 1, mid + 1, hi);
    }

    void update(int i, int lo, int hi, int l, int r, Line d) {
        if (l > hi || r < lo) return;
        if (lo >= l && hi <= r) {
            // t[i] hoan toan nam duoi d
            if (t[i].get(lo) <= d.get(lo) && t[i].get(hi) <= d.get(hi))
                return;
            //t[i] hoan toan nam tren d thi cap nhap t[i] = d
            if (t[i].get(lo) >= d.get(lo) && t[i].get(hi) >= d.get(hi)) {
                t[i] = d;
                return;
            }
            //nua dau cua d tot hon
            if (t[i].get(lo) >= d.get(lo) && t[i].get(mid) >= d.get(mid))
            {
                update(i * 2 + 1, mid + 1, hi, l, r, t[i]);
                t[i] = d;

```

```

        return;
    }
    // nua dau cua t[i] tot hon
    if (t[i].get(lo) <= d.get(lo) && t[i].get(mid) <= d.get(mid))
    {
        update(i * 2 + 1, mid + 1, hi, l, r, d);
        return;
    }
    // nua sau cua d tot hon
    if (t[i].get(mid + 1) >= d.get(mid + 1) && t[i].get(hi) >= d.get(hi)) {
        update(i * 2, lo, mid, l, r, t[i]);
        t[i] = d;
        return;
    }
    // nua sau cua t[i] tot hon
    if (t[i].get(mid + 1) <= d.get(mid + 1) && t[i].get(hi) <= d.get(hi)) {
        update(i * 2, lo, mid, l, r, d);
        return;
    }
}
update(i * 2, lo, mid, l, r, d);
update(i * 2 + 1, mid + 1, hi, l, r, d);
}

11 get(int i, int lo, int hi, int pos) {
    if (lo > pos || hi < pos) return llmax;
    11 res = t[i].get(pos);
    if (lo == hi) return res;
    res = min(res, get(i * 2, lo, mid, pos));
    res = min(res, get(i * 2 + 1, mid + 1, hi, pos));
    return res;
}
};

#undef mid
#undef oo

31 BIT 2D

class query{
public:

```

```

    int x, y, id, type;
    void next_x(){
        if(type==1) x+=x&-x;
        else x-=x&-x;
    }
    void next_y(){
        if(type==1) y+=y&-y;
        else y-=y&-y;
    }
};
const int maxqsize=100000;
vector <query> q;
deque <int> start[maxqsize+1];
vector <query> temp[n+1];
class node{
public:
    int value, y;
    int next, prev;
    node(int y){
        this->y=y;
        value=0;
        next=prev=-1;
    }
};
vector <node> ft[n+1];
int done[n+1];
int pos[n+1];
int main() {
    // your code goes here
    for(query a: q){
        while((a.x)&&(a.x<=n)){
            temp[a.x].pb(a);
            a.next_x();
        }
    }
    FOR(i, 1, n){
        for(query a: temp[i]){
            while((a.y)&&(a.y<=n)){
                if(done[a.y]!=i){
                    done[a.y]=i;
                    pos[a.y]=ft[i].size();
                    ft[i].pb(node(a.y));
                }
                a.next_y();
            }
        }
    }
}

```

```

        }
    }
    for(node &a: ft[i]){
        int yy=a.y+(a.y&(-a.y));
        if(yy<=n){
            if(done[yy]==i) a.next=pos[yy];
        }
        yy=a.y-(a.y&(-a.y));
        if(done[yy]==i) a.prev=pos[yy];
    }
    for(query a: temp[i]){
        if(a.type==1) //up
            start[a.id].pb(pos[a.y]);
        else start[a.id].push_front(pos[a.y]);
    }
}
return 0;
}

```

32 Heavy-Light Decomposition

chainHead [c] dinh dau cua chuoì c
chainInd [u] chuoì ma dinh u nam trong

```

void hld(int u) {

    //Neu chuoì hien tai chua co dinh dau dinh gan goc nhât thì dat u lam
    //dinh dau cua no
    if (chainHead[nChain] == 0) chainHead[nChain] = u;

    //Gan chuoì hien tai cho u
    chainInd[u] = nChain;

    //Giai thich ben duoi
    posInBase[u] = ++nBase;

    // Bien luu dinh con dac biet
    int mxVtx = -1;

    // Tim dinh con dac biet trong so nhung dinh con cua u
    for (int i = 0; i < adj[u].size(); i++) {
        int v = adj[u][i];

```

```

        if (v != parent[u]) {
            if (mxVtx == -1 || nChild[v] > nChild[mxVtx]) {
                mxVtx = v;
            }
        }
    }

    //Neu tim ra dinh con dac biet (u khong phai la dinh la) thì di chuyen
    //den dinh do
    if (mxVtx > -1)
        hld(mxVtx);

    // Sau khi di het mot chuoì thì tang nChain len va bat dau mot chuoì
    //moi
    for (int i = 0; i < adj[u].size(); i++) {
        int v = adj[u][i];
        if (v != parent[u] && v != mxVtx) {
            nChain++;
            hld(v);
        }
    }
}

void update(int u, int a) {
    // uchain chuoì hien tai cua u
    // achain chuoì hien tai cua a
    int uchain = chainInd[u], achain = chainInd[a];

    while (1) {
        // Neu u va a cung nam tren mot chuoì thì update doan tu u den a
        // va ket thuc
        if (uchain == achain) {
            updateIntervalTree(..., posInBase[a], posInBase[u], ...);
            break;
        }
        // Neu u va a khong nam tren cung mot chuoì thì update doan tu u
        // den dinh dau cua chuoì hien tai
        updateIntervalTree(..., posInBase[chainHead[uchain]], posInBase[
            u], ...);

        // Nhay len dinh cha cua dinh dau hien tai
        u = parent[chainHead[uchain]];
        uchain = chainInd[u];
    }
}

```

```

    }
}

```

33 Splay Tree

```

/**
 * Problem's query: insert a[i] after a[j]
 */
struct node {
    node *par, *left, *right;
    int value, cnt;
};
int n, Q, tree[N];
node *root, *nilT;
void calc(node *x) {
    if (x != nilT)
        x->cnt = x->left->cnt + x->right->cnt + 1;
}

node *FindPosition(node *cur, int pos) {
    node *x = cur;
    while (x != nilT) {
        int ord = x->left->cnt + 1;
        if (ord == pos)
            return x;
        if (ord > pos)
            x = x->left;
        else
            x = x->right, pos -= ord;
    }
    return nilT;
}

void SetL(node *parent, node *child) {
    if (child != nilT)
        child->par = parent;
    if (parent != nilT)
        parent->left = child;
}

void SetR(node *parent, node *child) {
    if (child != nilT)
        child->par = parent;
    if (parent != nilT)
        parent->right = child;
}

```

```

}

void Uptree(node *x) {
    if (x == root)
        return;
    node *y = x->par, *z = y->par;
    if (y->left == x) {
        SetL(y, x->right);
        SetR(x, y);
    } else {
        SetR(y, x->left);
        SetL(x, y);
    }
    if (z == nilT)
        root = x, x->par = nilT;
    else if (z->left == y)
        SetL(z, x);
    else
        SetR(z, x);
    calc(y);
    calc(x);
}

void Splay(node *x) {
    while (1) {
        node *y = x->par;
        if (y == nilT)
            break;
        node *z = y->par;
        if (z != nilT) {
            if ((y == z->left) == (y->left == x))
                Uptree(y);
            else
                Uptree(x);
        }
        Uptree(x);
    }
    root = x;
}

void Split(node *r, int i, node *&r1, node *&r2) {
    if (i == 0) {
        r1 = nilT, r2 = r;
        return;
    }
    node *x = FindPosition(r, i);
    Splay(x);
}

```

```

    r2 = x->right;
    r1 = x;
    r1->right = nilT;
    calc(r1), calc(r2);
}

node *Join(node *r1, node *r2) {
    if (r1 == nilT)
        return r2;
    while (r1->right != nilT)
        r1 = r1->right;
    Splay(r1);
    SetR(r1, r2);
    calc(r1);
    return r1;
}

void Insert(int i, int val) {
    node *x = new node;
    x->value = val;
    x->par = nilT;
    node *r1, *r2;
    Split(root, i - 1, r1, r2);
    SetL(x, r1), SetR(x, r2);
    calc(x);
    root = x;
}

void Delete(int i) {
    node *x = FindPosition(root, i);
    Splay(x);
    node *r1 = x->left, *r2 = x->right;
    r1->par = nilT; r2->par = nilT;
    delete x;
    root = Join(r1, r2);
}

vector<int> arr;
void GetArray(node *x) {
    if (x == nilT)
        return;
    GetArray(x->left);
    arr.push_back(x->value);
    GetArray(x->right);
    delete x;
}

```

```

int main() {
    nilT = new node;
    nilT->value = nilT->cnt = 0;
    nilT->left = nilT->right = nilT->par = nilT;
    root = new node;
    root->value = root->cnt = 1;
    root->left = root->right = root->par = nilT;
    cin >> n >> Q;
    for (int i = 2; i <= n; i++)
        Insert(i, i);
    for (int i = 1; i <= Q; i++) {
        cin >> u >> v;
        int val = FindPosition(root, u)->value;
        Delete(u);
        Insert(v, val);
    }
    GetArray(root);
    int ans = 0;
    for (int i = 0; i < n; i++) {
        int F = 1;
        for (int x = arr[i]; x; x -= x & (-x))
            F = max(F, tree[x] + 1);
        ans = max(ans, F);
        for (int x = arr[i]; x <= n; x += x & (-x))
            tree[x] = max(tree[x], F);
    }
    cout << n - ans << endl;
}

```

/**

Geometry - Tungluu18

*/

Duong tron di qua 3 diem cho truoc Let $A = (0, 0)$ centers are $Cy(Bx^2 + By^2) - By(Cx^2 + Cy^2)/D$
and $(Bx(Cx^2 + Cy^2) - Cx(Bx^2 + By^2))/D$ where $D = 2(BxCy - ByCx)$.

Diem trong tam giac

```

bool isInside(const Vector &P) {
    Vector a = C - A, b = B - A, c = P - A;
    T under = a.x*b.y - b.x*a.y;
    T u = (c.x*b.y - c.y*b.x);

```

```

    T v = (a.x*c.y-a.y*c.x);
    return u >= 0 && v >= 0 && u+v <= under || u<=0
    && v<=0 && u+v >= under;
} //remove equalities if not want the boundary

```

Pick's theorem (Số điểm trong đa giác có đỉnh nguyên)

$I = A - B/2 + 1$

where

A is the area of a lattice polygon,
 I is number of lattice points inside it,
 B is number of lattice points on the boundary.
 Number of lattice points minus one on a line segment
 from (0, 0) and (x, y) is gcd(x, y).

Tích chap 3d: $a \times b = (aybz - azby, azbx - axbz, axby - aybx)$

Distance from line AB to P (for any dimension) : $| (A - P)x(B - P) | / (A - B)$

Khoảng cách từ điểm đến đoạn thẳng if (dot(B-A, P-A) < 0) return dist(A,P); if (dot(A-B, P-B) < 0) return dist(B,P); return fabs(cross(P,A,B) / dist(A,B)); dot - tích vô hướng, cross - tích chập

Hình chiếu: Hình chiếu của C trên đường thẳng AB [dot(AB, AC) / dot(AB, AB)] AB

Đường tròn cắt đường tròn

```

// (x1,y1,r1) and (x2,y2,r2)
a = x2 - x1;
b = y2 - y1;
$c = [(r1^2 - x1^2 - y1^2) - (r2^2 - x2^2 - y2^2)] / 2;
$d = sqrt(a^2 + b^2);$

```

if not |r1 - r2| <= d <= |r1 + r2|, return "no solution"
 if d == 0, circles are concentric, a special case

Đường tròn cắt đường thẳng

```

// (x1,y1,r1) and ax+by=c
Normalize line:
d = sqrt(a^2+b^2)
a /= d;
b /= d;
c /= d;
e = c - a*x1 - b*y1;
h = sqrt(r1^2 - e^2); // check if r1<e for circle-line test

```

```

    return (x1, y1) + (a*e, b*e) + h*(-b, a),
    (x1, y1) + (a*e, b*e) - h*(-b, a);

```

Template

double eps = 1e-8;

```

struct point2d {
    double x,y;
    point2d operator-(point2d v) { return (point2d){x-v.x, y-v.y}; }
    point2d operator+(point2d v) { return (point2d){x+v.x, y+v.y}; }
};

```

```

struct point3d {
    double x,y,z;
    point3d operator-(point3d v) { return (point3d){x-v.x, y-v.y, z-v.z}; }
};

```

```

double trip(point3d a, point3d b, point3d c) {
    return a.x * ( b.y * c.z - b.z * c.y )
        - a.y * ( b.x * c.z - b.z * c.x )
        + a.z * ( b.x * c.y - b.y * c.x );
}

```

double len(point2d a) { return sqrt(a.x*a.x+a.y*a.y); }

double len(point3d a) { return sqrt(a.x*a.x+a.y*a.y+a.z*a.z); }

```

point3d cross(point3d a, point3d b) {
    return (point3d){
        a.y*b.z - a.z*b.y,
        a.z*b.x - a.x*b.z,
        a.x*b.y - a.y*b.x
    };
}

```

double dot(point3d a, point3d b) { return a.x*b.x + a.y*b.y + a.z*b.z; }

double dot(point2d a, point2d b) { return a.x*b.x + a.y*b.y; }

```

double cross(point2d a, point2d b) {
    return a.x*b.y - a.y*b.x;
}

```

```

point2d rotate(point2d c, double angle) {
    return (point2d) {
        c.x*cos(angle)-c.y*sin(angle),

```



```

        c.x*sin(angle)+c.y*cos(angle)
    };
}

// distance AB to C
double linePointDist(point2d A, point2d B, point2d C, bool isSegment) {
    double dist = cross(B-A,C-A) / len(B-A);
    if(isSegment){
        if( dot(C-B,B-A) > eps) return len(B-C);
        if( dot(C-A,A-B) > eps) return len(A-C);
    }
    return fabs(dist);
}

struct Line { double A,B,C; }; //Ax + By = C
Line makeline(point2d a1, point2d a2)
{
    Line ret = (Line){ a2.y-a1.y, a1.x-a2.x };
    ret.C = ret.A * a1.x + ret.B * a1.y;
    return ret;
}

double dist(Line l, point2d p)
{
    return fabs(p.x * l.A + p.y * l.B - l.C)/sqrt(l.A*l.A+l.B*l.B);
}

Line rot90(Line l, point2d p)
{
    Line ret = (Line){ -l.B, l.A };
    ret.C = ret.A * p.x + ret.B * p.y;
    return ret;
}

point2d intersect(Line l1, Line l2)
{
    double det = l1.A*l2.B - l2.A*l1.B;
    if(fabs(det) < eps) det=0; //zero means parallel
    return (point2d) { (l2.B*l1.C - l1.B*l2.C)/det,
        (l1.A*l2.C - l2.A*l1.C)/det};
}

//for segment segment intersection, check additionally
//min(x1,x2) <= x <= max(x1,x2)
//min(y1,y2) <= y <= max(y1,y2)
bool segmentsIntersect( point2d A, point2d B, point2d C, point2d E )

```

```

{
    point2d in = intersect( makeline(A,B), makeline(C,E) );
    return linePointDist(A,B,in,true) < eps
    && linePointDist(C,E,in,true) < eps;
}

// get a line passing between two points
Line getmidline(point2d a, point2d b)
{
    point2d mid(a+b); mid.x/=2; mid.y/=2;
    return rot90( makeline(a,b), mid );
}

//reflect a point into it's "mirror" with respect to a line
point2d reflectPoint(Line l, point2d p)
{
    Line r = rot90(l, p);
    point2d Y=intersect(l,r);
    return Y-(p-Y);
}

```