ENPM673_Project3

Curule

1. Calibration

Find matching points

Using SIFT feature detections to find matching features.

Estimate the Fundamental matrix

Calculate the Fundamental matrix with the following equations

$$x_i^{'T} F x_i = 0$$

$$\begin{bmatrix} x_i^{'} & y_i^{'} & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

The error of the above equation should be minimize for every corresponding features in the two pictures.

Transform the matrix into the following structure

$$x_i x_i^{'} f_{11} + x_i y_i^{'} f_{21} + x_i f_{31} + y_i x_i^{'} f_{12} + y_i y_i^{'} f_{22} + y_i f_{32} + x_i^{'} f_{13} + y_i^{'} f_{23} + f_{33} = 0$$

$$\begin{bmatrix} x_i x_i^{'} & y_i x_i^{'} & x_i^{'} & x_i y_i^{'} & y_i y_i^{'} & y_i^{'} & x_i & y_i & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}$$

We need at least 8 corresponding feature points to solve the above equation.
The eight equations that comes from 8 feature points will stack together and form an A matrix.
The F matrix can be solve by finding the SVD decomposition ($U\Sigma V^T$) of the A matrix and choose the column of V cooresponding to the smallest singular value.

Because there will be noise and error inside the extracted feature points, we can apply RANSAC on finding the F matrix with smallest error on $x_i^{'T} F x_i = 0$ using only good feature points.

The RANSAC parameter I used:

Outlier ratio: 0.4

Error threshold: 0.05

Desired probability to have good model: 0.95

The calculated F matrix will very likely to be a rank 3 matrix due to the noise in the feature points.

We need to set the smallest singular value of the F matrix to be zero and reconstruct F with its SVD to make F matrix a rank 2 matrix.

The final computed F matrix

```
Fundamental Matrix:
[[ 7.18739891e-10  2.86794968e-06 -1.63175652e-03]
 [-1.86874402e-06  8.99338661e-07 -6.42966180e-01]
 [ 1.22380650e-03  6.41514827e-01  4.18388645e-01]]
```

Estimate the Essential matrix

The Essential matrix can be calculated by the follow equation:

$$E = K^T F K$$

K is the intrinsic matrix of the camera

Same with computing the F matrix, the resulting Essential matrix will be a rank 3 matrix.

We take the same approach to set the smalles singular value of Essential matrix to be zero and reconstruct it.

The final Essential matrix

```
Essential Matrix:
[[ 2.22189290e-03  8.86590143e+00 -8.35433467e-02]
 [-5.77698432e+00  2.78019101e+00 -1.13282083e+03]
 [ 3.38779278e-01  1.13373235e+03  3.24072679e-02]]
```

Find T and R from Essential matrix

The Essential matrix can be decomposed into a skew-symmetric matrix T and a orthogonal matrix R.

$$E = [T_x]R$$

The T is the skew-symmetric form of the translation vector and the R is the Rotation matrix.

$$T_x = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t2 & t_1 & 0 \end{bmatrix}$$

There are several different methods to do the decomposition.

I use the algebraic solution by Hartley & Zisserman.

Step 1. Find the SVD of the Essential matrix

$$E = U\Sigma V^T$$

Step 2. Decompose the Σ

We know that the scalor doesn't matter, so we can set the Σ to be a diagonal matrix with the diagonal of [1, 1, 0].

Next, define ZW so that ZW = Σ

$$Z = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$ZW = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Z is a skew-symmetric matrix and W is a rotation matrix

Step 3. Reconfigure $E = U\Sigma V^T$

$$E = UZU^TUWV^T$$

This equation can be seperate into two parts.
The first part UZU^T remains a skew-symmatric matrix and the second part UWV^T remains a rotation matrix.
Therefore, UZU^T is the T matrix we want to find and UWV^T is the R matrix we want to find.

Step 4. Choose the correct T and R from different ZW

There are a total of four different configurations that can be made from $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$
\begin{aligned}
ZW &= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
-Z^T W &= -\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \\
-ZW^T &= -\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \\
Z^T W^T &= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Only one of them is the correct configuration.
We need to test the calculated R and T by using them to calculate the position of points on image2 in the image1 frame.

$$p = Rp^{'} + T$$

The correct configuration is the one that has positive positions in image1 frame for every feature points on image2.

The final Rotation matrix and Translation vector.

```
Rotation matrix
[[ 9.99996275e-01 -2.34398707e-04  2.71950875e-03]
 [ 2.31026653e-04  9.99999204e-01  1.24019765e-03]
 [-2.71979729e-03 -1.23956476e-03  9.99995533e-01]]
Translation
[[ 9.99969423e-01]
 [-7.39695141e-05]
 [-7.81968042e-03]]
```

2. Rectification

Step 1. Calculate the line equation for every line

The epipolar line equation on one camera can be obtained by the product of Fundemantal matrix with points on the other camera frame.

$$l_2 = Fx_1$$
$$l_1 = F^T x_2$$

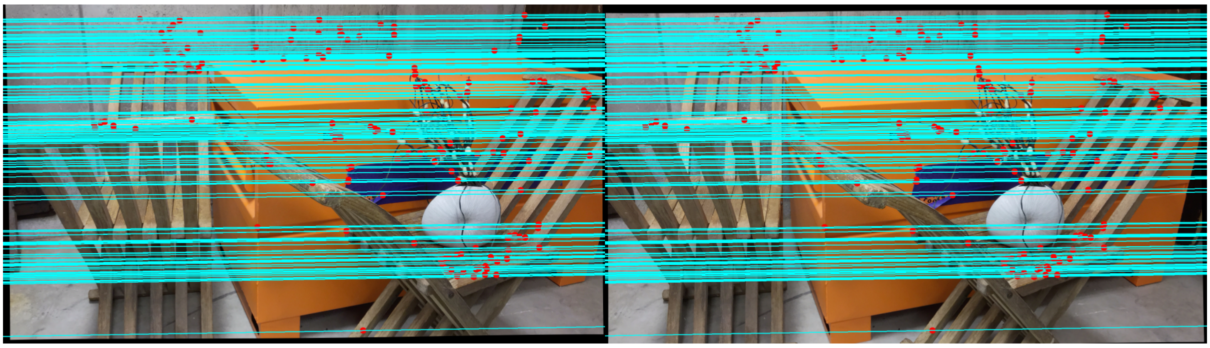
Step 2. Calculate the homography H for rectified image

```
H for left:
[[-4.00397065e-01  3.01074726e-02  5.25567050e+01]
 [-1.23344950e-02 -3.79547460e-01  1.26844177e+01]
 [-1.74191176e-05  1.95831084e-06 -3.63272983e-01]]
H for right:
[[ 1.04426213e+00 -8.00452850e-03 -3.81691987e+01]
 [ 3.25789952e-02  9.99779651e-01 -3.11568472e+01]
 [ 4.61369854e-05 -3.53651448e-07  9.55899466e-01]]
```

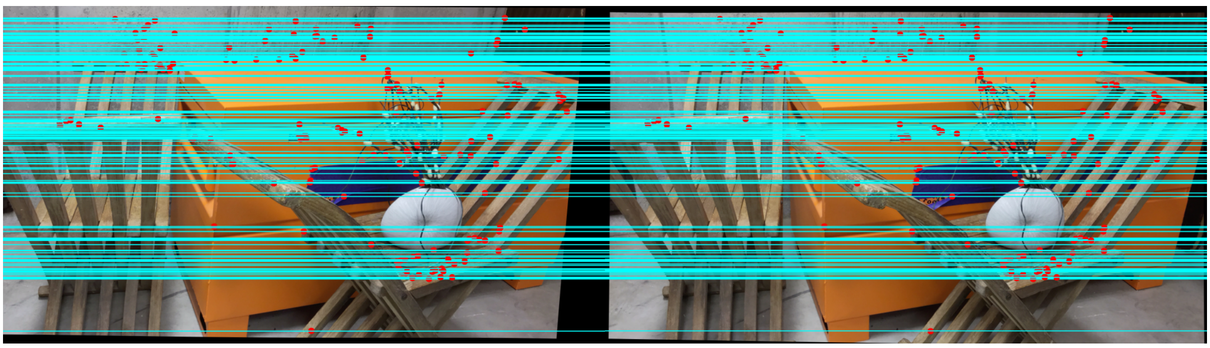
Step 3. Transform all the feature points to the new rectified image

Step 4. Draw the horizontal epipolar line

Epipolar line before rectification



Epipolar line after rectification



3. Correspondence

We can obtain the corrspondence points through matching sliding windows methond with SSD (sum of square differences).

$$SSD = \sum (f - g)^2$$

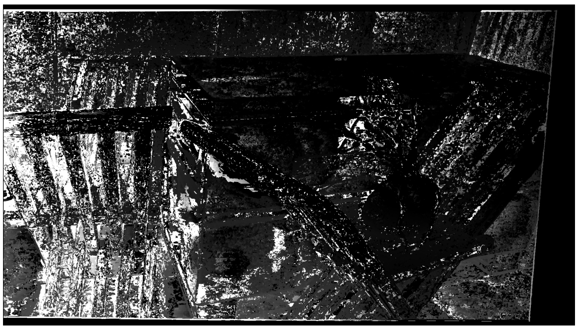
After finding all the SSD of a horizontal line , select the window pairs that has the smallest SSD to be corrspondence windows. The center pixels of these windows will be the correspondence points. The disparity can be calculated by the difference of the x value of these points.

Instead of only calcuating the SSD of windows on the epipolar line, I calculates SSD windows on all the picture to obtain a more complete disparity map.

A darker pixel on the disparity map has a smaller disparity which represent a further point to the camera.

A lighter pixel on the disparity map has a larger disparity which represents a closer point to the camera.

Disparity image



Disparity heatmap

4. Compute Depth Image

The depth can be calculate through the disparity map.

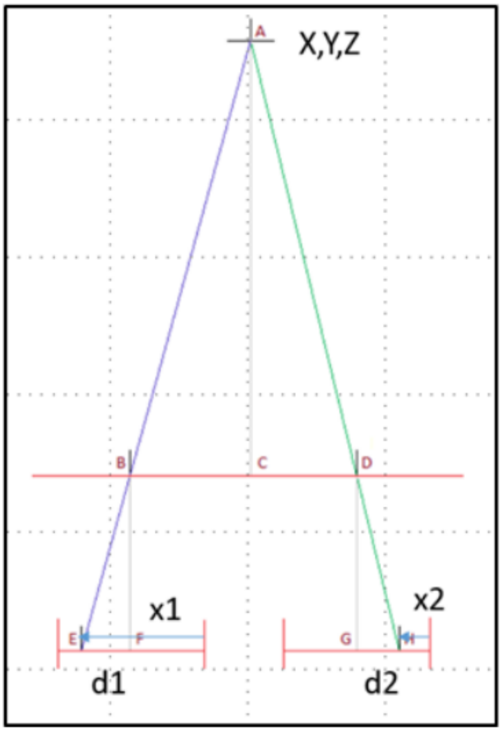
If both cameras have the same size sensor, and are already in a rectified pose, then the following equation can be use to calculate the distance of the target point.

$$\frac{x_1 - x_2}{f} = \frac{T}{Z}$$

x1 - x2 is the disparity
f is the focal length

T is the baseline

Z is the distance of the target point

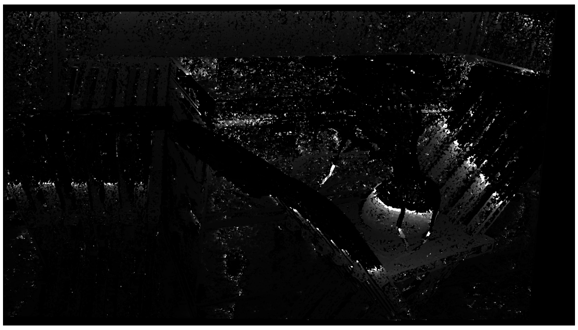


Because the resulting image is very dark, I also apply the histogram equalization on the depth and heat map for a clear view.

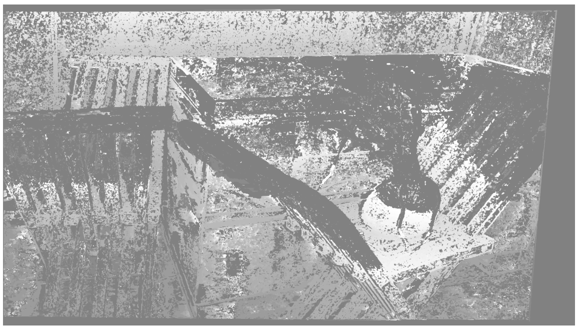
A darker pixel on the depth and heat map represents a closer point to the camera.

A lighter pixel on the depth and heat map represents a further point to the camera.

Depth image

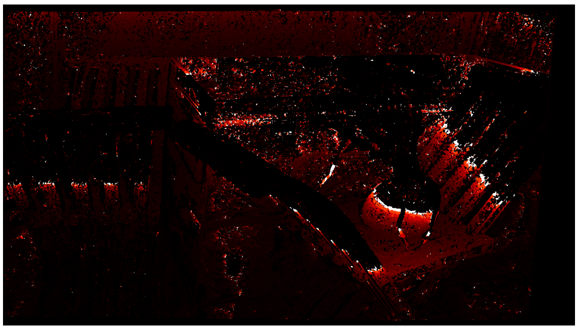


Depth image after histogram equalization



Depth heatmap

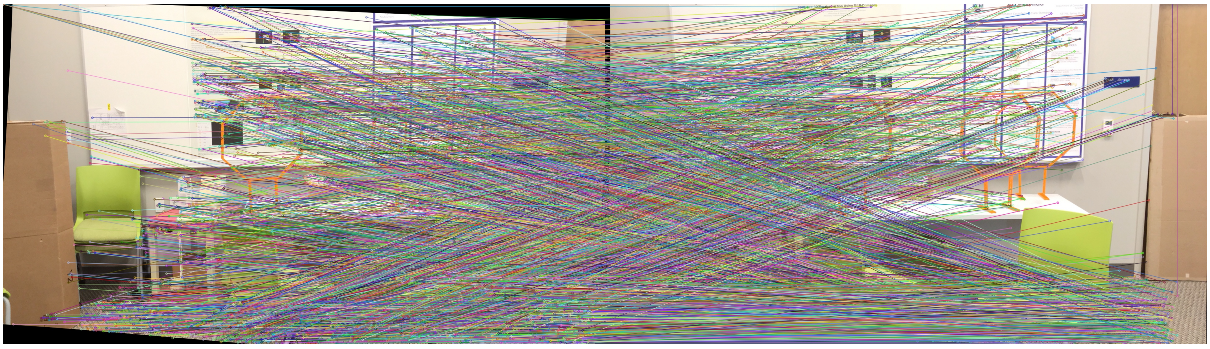
Depth heatmap after histogram equalization



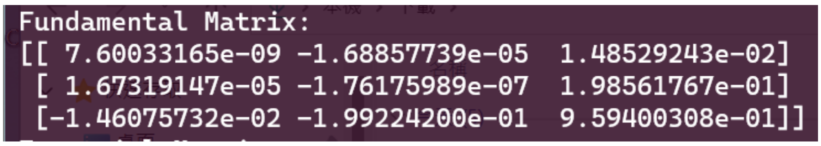
Octagon

1. Calibration

Find matching points

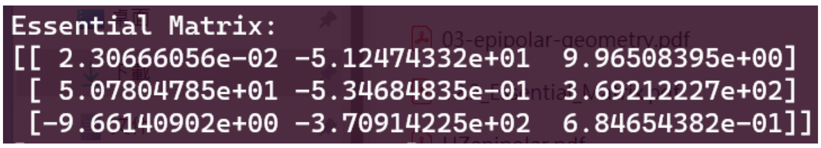


Estimate the Fundamental matrix



```
Fundamental Matrix:
[[ 7.60033165e-09 -1.68857739e-05  1.48529243e-02]
 [ 1.67319147e-05 -1.76175989e-07  1.98561767e-01]
 [-1.46075732e-02 -1.99224200e-01  9.59400308e-01]]
```

Estimate the Essential matrix



```
Essential Matrix:
[[ 2.30666056e-02 -5.12474332e+01  9.96508395e+00]
 [ 5.07804785e+01 -5.34684835e-01  3.69212227e+02]
 [-9.66140902e+00 -3.70914225e+02  6.84654382e-01]]
```

Find T and R from Essential matrix

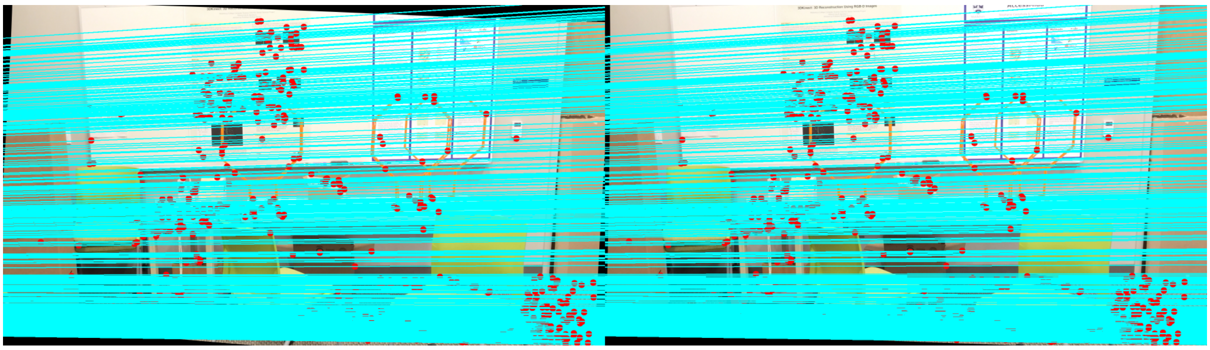
```
Rotation matrix
[[ 9.99999753e-01  4.55783871e-04  5.34416403e-04]
 [-4.55684921e-04  9.99999879e-01 -1.85263381e-04]
 [-5.34500779e-04  1.85019810e-04  9.99999840e-01]]
Translation
[[-0.9902477 ]
 [ 0.02647327]
 [ 0.1367796 ]]
```

2. Rectification

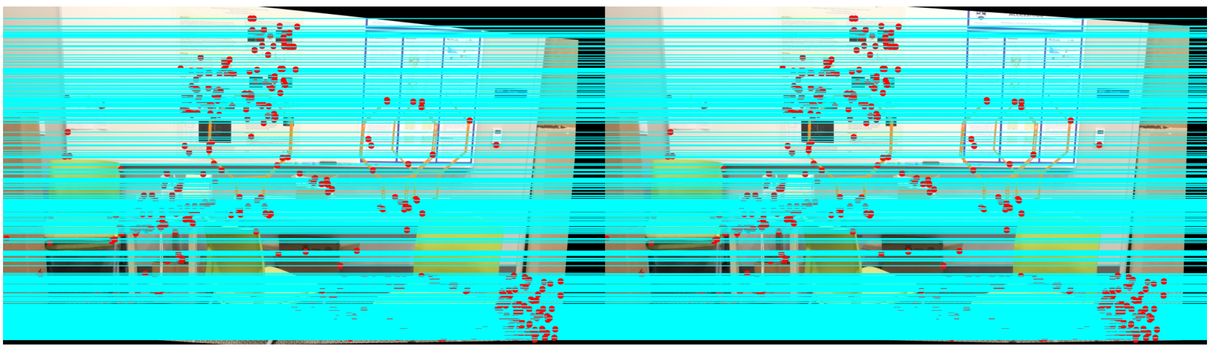
Homography

```
H for left:
[[-5.37670105e-01 -6.59886945e-03 -1.32092732e+02]
 [ 1.02426981e-01 -7.00296471e-01 -1.00147654e+02]
 [ 1.69933685e-04  4.69183661e-07 -8.68820887e-01]]
H for right:
[[ 7.66163024e-01  1.18952473e-02  2.18060063e+02]
 [-1.46989384e-01  9.97838398e-01  1.42277074e+02]
 [-2.43454659e-04 -3.77981354e-06  1.23575757e+00]]
```

Epipolar line before rectification

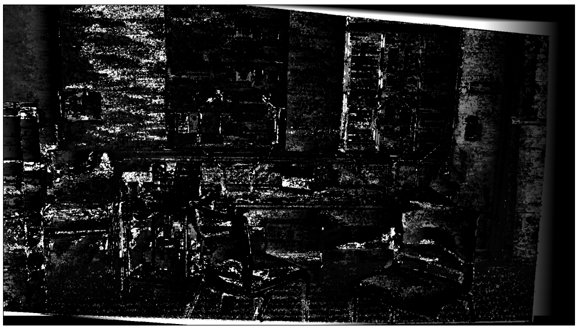


Epipolar line after rectification



3. Correspondence

Disparity image



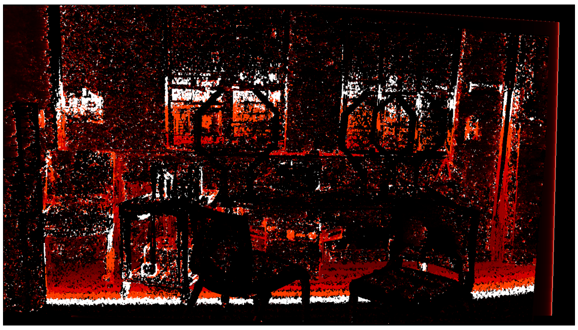
Disparity heatmap

4. Compute Depth Image

Depth image

Depth image after histogram equalization

Depth heatmap



Depth heatmap after histogram equalization

Pendulum

1. Calibration

Find matching points

Estimate the Fundamental matrix

```
Fundamental Matrix:
[[ 6.22596778e-09 -8.54025674e-06 -4.09371484e-03]
 [ 8.61944583e-06  1.34980378e-07  4.81028699e-01]
 [ 4.50729750e-03 -4.82232976e-01  7.32137742e-01]]
```

Estimate the Essential matrix

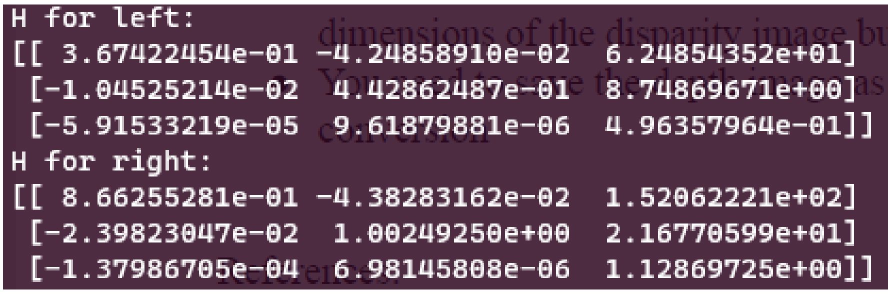
```
Essential Matrix:
[[ 1.86132398e-02 -2.55320703e+01 -1.52365324e+01]
 [ 2.57688151e+01  4.03539215e-01  8.26423119e+02]
 [ 1.60194064e+01 -8.28297484e+02 -5.74716857e-02]]
```

Find T and R from Essential matrix

```
Rotation matrix
[[ 9.99999514e-01  9.13338829e-04 -3.70247303e-04]
 [-9.13412513e-04  9.99999563e-01 -1.98893440e-04]
 [ 3.70065484e-04  1.99231532e-04  9.99999912e-01]]
Translation
[[-0.9993559 ]
 [-0.0184227 ]
 [ 0.03079593]]
```

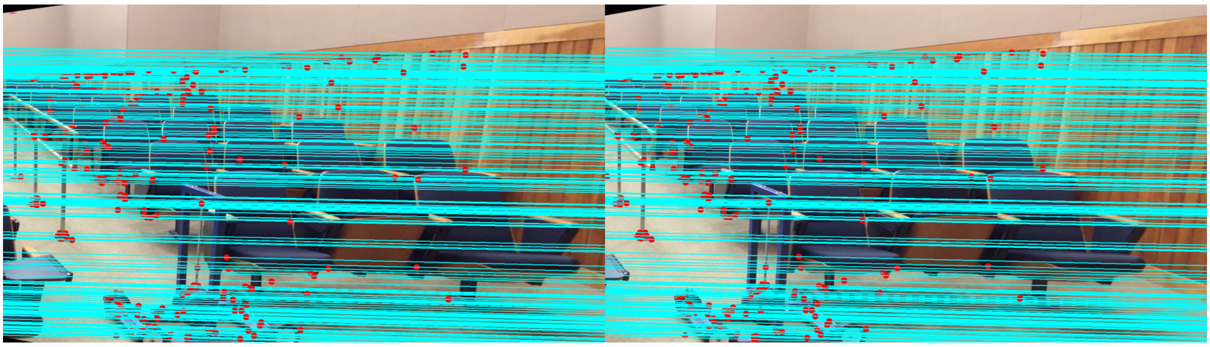
2. Rectification

Homography

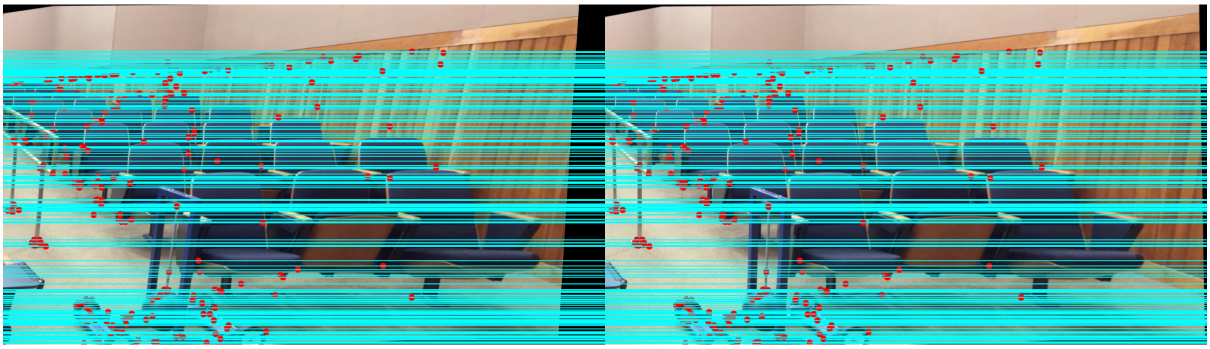


```
H for left:
[[ 3.67422454e-01 -4.24858910e-02  6.24854352e+01]
 [-1.04525214e-02  4.42862487e-01  8.74869671e+00]
 [-5.91533219e-05  9.61879881e-06  4.96357964e-01]]
H for right:
[[ 8.66255281e-01 -4.38283162e-02  1.52062221e+02]
 [-2.39823047e-02  1.00249250e+00  2.16770599e+01]
 [-1.37986705e-04  6.98145808e-06  1.12869725e+00]]
```

Epipolar line before rectification



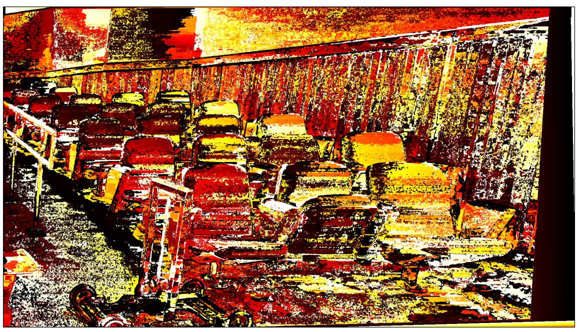
Epipolar line after rectification



3. Correspondence

Disparity image

Disparity heatmap

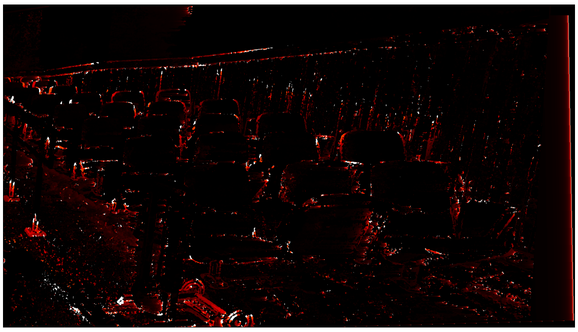


4. Compute Depth Image

Depth image

Depth image after histogram equalization

Depth heatmap



Depth heatmap after histogram equalization