ENPM673_Midterm

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Question 1: Coins Image

1. Separating

Apply Opening to cut down the connection between coins in original image

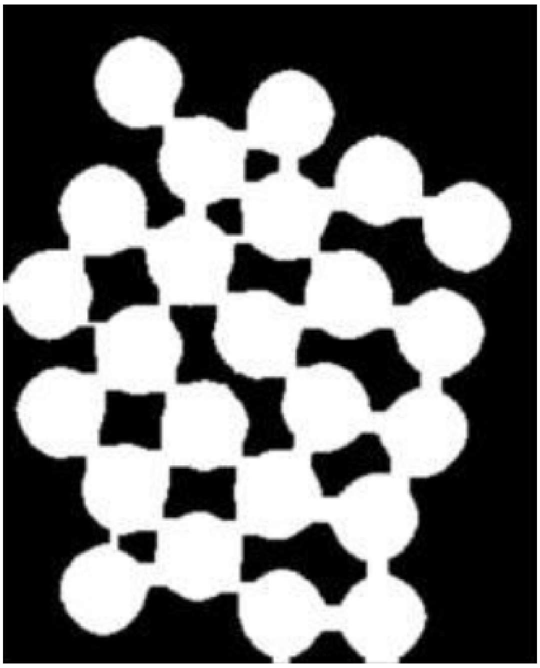
Step 1. Erosion

Using erosion with a mask bigger than the connected part in image can shrink all the coins and remove connected parts.

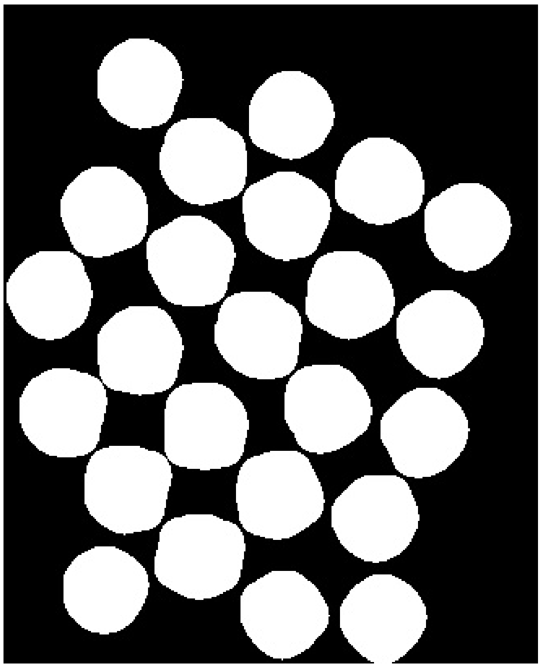
Step 2. Dilation

Using dilation with the same parameter as erosion can reconstruct orignal coins from shrinked coins.

Original image



Separated coins after opening



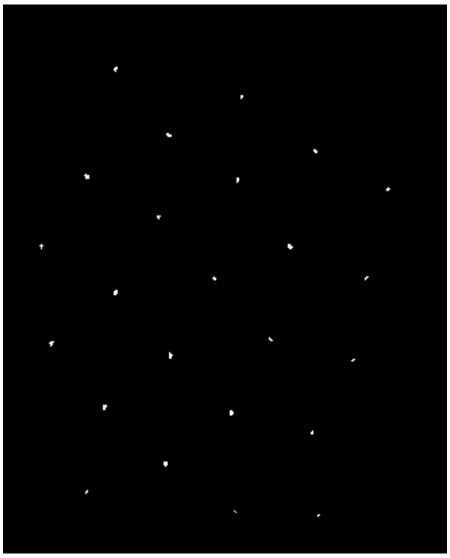
2. Counting

Apply Connected Component Labeling to find coins by finding connected parts.

Step 1. Erosion

Apply erosion with a bigger mask to make white parts smaller for faster calculating speed.

Eroded image



Step 2. Connected Component Labeling

```
label_count = 0
Traversal all the white pixels:
  Traversal neighbors of current white pixel:
    if neighbor of the current pixel is labeled and current pixel is not labeled:
       label the current pixel with neighbor label
    else
       Set neighbor label and current pixel label as equivalent
  if current pixel has no labeled neighbor:
    increase label_count
    label the current pixel with label_count

Count labels that are independent
```

Total coins: 24

Questions 2: Image Stitching

Pipeline:

Step 1. Turn image into gray scale

Step 2. Apply SIFT feature detection

Step 3. Select better maching points in result from step 2

Step 4. Homography calculation

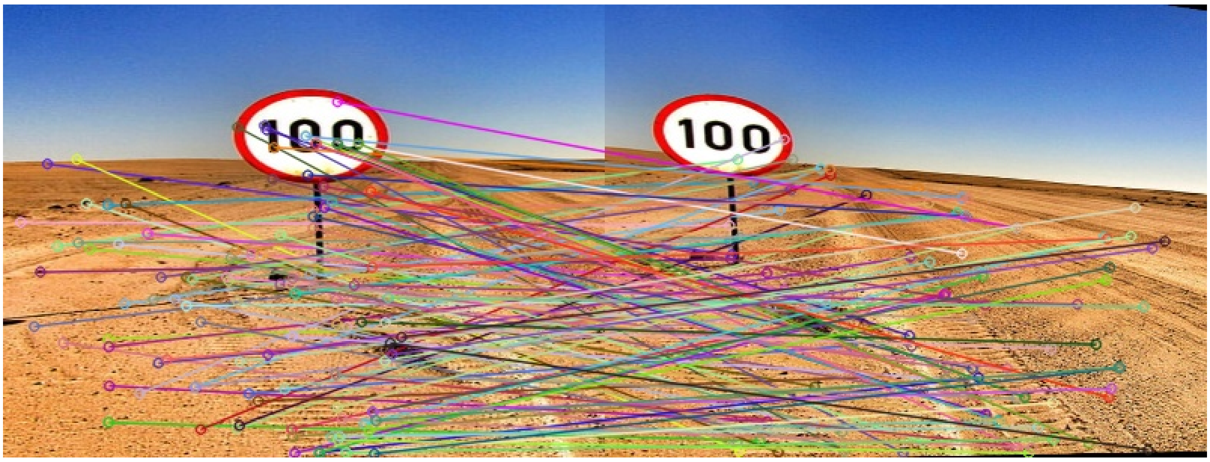
Use RANSAC to reject outlier matching points when finding homography

Step 5. Warp image

Image A

Image B

Matching points



Stitched image

Questions 3: Camera Calibration

1. Minimum number matching points

Assume that P is the mapping between the world coordinate and the image points.

$$x_i = PX_i$$

Because the image point are homogeneous coordinates, we can soften the condition to find image points that are proportional to x_i

$$x_i \propto PX_i$$
$$x_i \times PX_i = 0$$

Let $p_1^T,\ p_2^T,\ p_3^T$ be the three row vectors of P

$$PX_i = \begin{bmatrix} p_1^T X_i \\ p_2^T X_i \\ p_3^T X_i \end{bmatrix}$$

Assume $x_i = [u, v, w]^T$

$$x_i \times PX_i = \begin{bmatrix} 0 & -w & v \\ w & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} p_1^T X_i \\ p_2^T X_i \\ p_3^T X_i \end{bmatrix}$$

$$\begin{bmatrix} 0 & -wX_i^T & vX_i^T \\ wX_i^T & 0 & -uX_i^T \\ -vX_i^T & uX_i^T & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = 0$$

The third row of the matrix can be computed by the first two rows.

So two independence equation can be formed with 1 pair of matching points.

P vector has 11 degree of freedom.

Therefore, p vector can be calculated by a minimum of **6 paris** of matching points

2. Pipeline

Step 1. Find corners of the calibration board

1.1 Filter out the noise

1.2 Transform the image to binary image by thresholding

1.3 Run edge detection algorithm (Canny)

1.4 Use corner detection algorithm (Harris) or
find the intersection of lines using line detection methonds (Hough Transform)

Step 2. Match the corners in camera with real corners coordinates

Step 3. Calculate P matrix with matched points

Step 4. Find the camera position C using SVD

Step 5. Divide P with $[I \ \ -C]$ to get KR matrix

Step 6. Apply RQ factorization to the KR matrix to find K and R

3. Mathematical formation

1. Calculate P matrix using 6 pairs of matching points.

$$x = PX$$

$$\begin{bmatrix} u_i * w \\ v_i * w \\ w \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Image coordinates (u_i, v_i) can be calculated by

$$u_i = \frac{m_{11}X_i + m_{12}Y_i + m_{13}Z_i + m_{14}}{m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}}$$
$$v_i = \frac{m_{21}X_i + m_{22}Y_i + m_{23}Z_i + m_{24}}{m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}}$$

Rewrite the equations

$$u_i(m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}) = m_{11}X_i + m_{12}Y_i + m_{13}Z_i + m_{14}$$
$$v_i(m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}) = m_{21}X_i + m_{22}Y_i + m_{23}Z_i + m_{24}$$

Further transform the equations

$$u_i m_{31}X_i + u_i m_{32}Y_i + u_i m_{33}Z_i + u_i m_{34} - m_{11}X_i + m_{12}Y_i + m_{13}Z_i + m_{14} = 0$$
$$v_i m_{31}X_i + v_i m_{32}Y_i + v_i m_{33}Z_i + v_i m_{34}) - m_{21}X_i + m_{22}Y_i + m_{23}Z_i + m_{24} = 0$$

Rewrite the equations into matrix format

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_iX_i & -u_iY_i & -u_iZ_i - u_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_iX_i & -v_iY_i & -v_iZ_i - v_i \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = 0$$

Continue to stack the equations for at least 6 pair of points

$$Am = 0$$

m equals to the last column of V in SVD ($U\Sigma V^T$) equation of the A matrix.

Reshape M column vector into P matrix

2. Calculate the camera position using P matrix.

Camera position C equals to the last column of V in SVD ($U\Sigma V^T$) equation of the P matrix.

$$PC = 0$$

3. Calculate KR with C and P

$$P \begin{bmatrix} I & -C \end{bmatrix}^{-1} = KR$$

$$P \begin{bmatrix} I \\ 0 \end{bmatrix} = KR$$

4. Calculate K and R by RQ factorization

RQ factorization can be computed using given rotation method

$$M = KR$$

$$\begin{bmatrix} m11 & m12 & m13 \\ m21 & m22 & m23 \\ m31 & m32 & m33 \end{bmatrix} = KR$$

Compute R_x, R_y, R_z

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{bmatrix} R_y = \begin{bmatrix} c' & 0 & s' \\ 0 & 1 & 0 \\ -s' & 0 & c' \end{bmatrix} R_x = \begin{bmatrix} c'' & -s'' & 0 \\ s'' & c'' & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

First construct c and s to make (3,2) in MR_x equals to zero

$$c = -\frac{m_{33}}{(m_{32}^2 + m_{33}^2)^{1/2}}$$
$$s = \frac{m_{32}}{(m_{32}^2 + m_{33}^2)^{1/2}}$$

Then construct c' and s' to make (3,1) in MR_xR_y equals to zero

$$c'MR_x[3,1] - s'MR_x[3,3] = 0$$

$$c' = \frac{m'_{33}}{(m'^2_{31} + m'^2_{33})^{1/2}}$$
$$s' = \frac{m'_{31}}{(m'^2_{31} + m'^2_{33})^{1/2}}$$

Then construct c'' and s'' to make (2,1) in $MR_xR_yR_z$ equals to zero

$$c''MR_xR_y[2,1] + s''MR_xR_y[2,2] = 0$$

$$c' = \frac{m'_{22}}{(m'^2_{21} + m'^2_{22})^{1/2}}$$
$$s' = -\frac{m'_{21}}{(m'^2_{21} + m'^2_{22})^{1/2}}$$

The resulting $MR_xR_yR_z$ is a upper triangular matrix K

Multiply K by a scalar to make the last element 1

And $M = KR_z^TR_y^TR_x^T = KR$

4. Calculate intrinsic and extrinsic parameters

```
C matrix:
[[15.95734064]
 [ 7.42561435]
 [17.16026409]]
K matrix:
[[-1.61901802e+03 -1.89270966e+00  8.00113193e+02]
 [ 0.00000000e+00  1.61202594e+03  6.16150419e+02]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00]]
R matrix:
[[-0.74948643 -0.00587017  0.66199368]
 [-0.0453559   0.99806642 -0.04250013]
 [-0.66046418 -0.06187859 -0.74830349]]
P matrix
[[ 6.85071776e+02 -4.18950134e+01 -1.67042675e+03  1.80441367e+04]
 [-4.80060176e+02  1.57078244e+03 -5.29578810e+02  5.08417135e+03]
 [-6.60464181e-01 -6.18785893e-02 -7.48303485e-01  2.38398239e+01]]
---Testing---
world points:  [0 1 7]
groud truth image points:  [340 159]
Calculated by P:  [340.30827251 159.00380793]
---
world points:  [ 0 11  7]
groud truth image points:  [ 329 1041]
Calculated by P:  [ 328.68103244 1040.99604446]
```

Questions 4: K-means

K-means algorithm:

Step 1. Initialize k means

Step 2. Calculate the distances between every pixels and every means

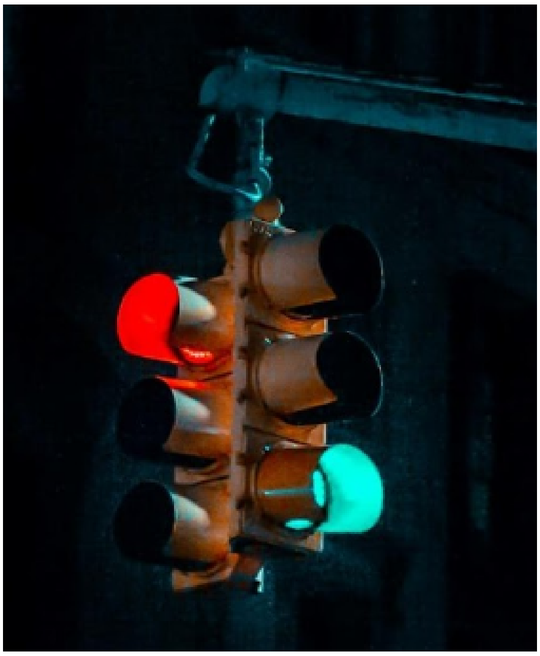
Step 3. Find the minimum distance from distances in step 2

Step 4. Categorize all the pixels by minimum distance

Step 5. Calculate new means by finding the mean of every category

Step 6. Repeat Step 2

Original image



K-means after 50 iterations