# Design sensitivity in the presence of uncertainties

Jiannan Yang

ISVR Seminar, 25th January 2022

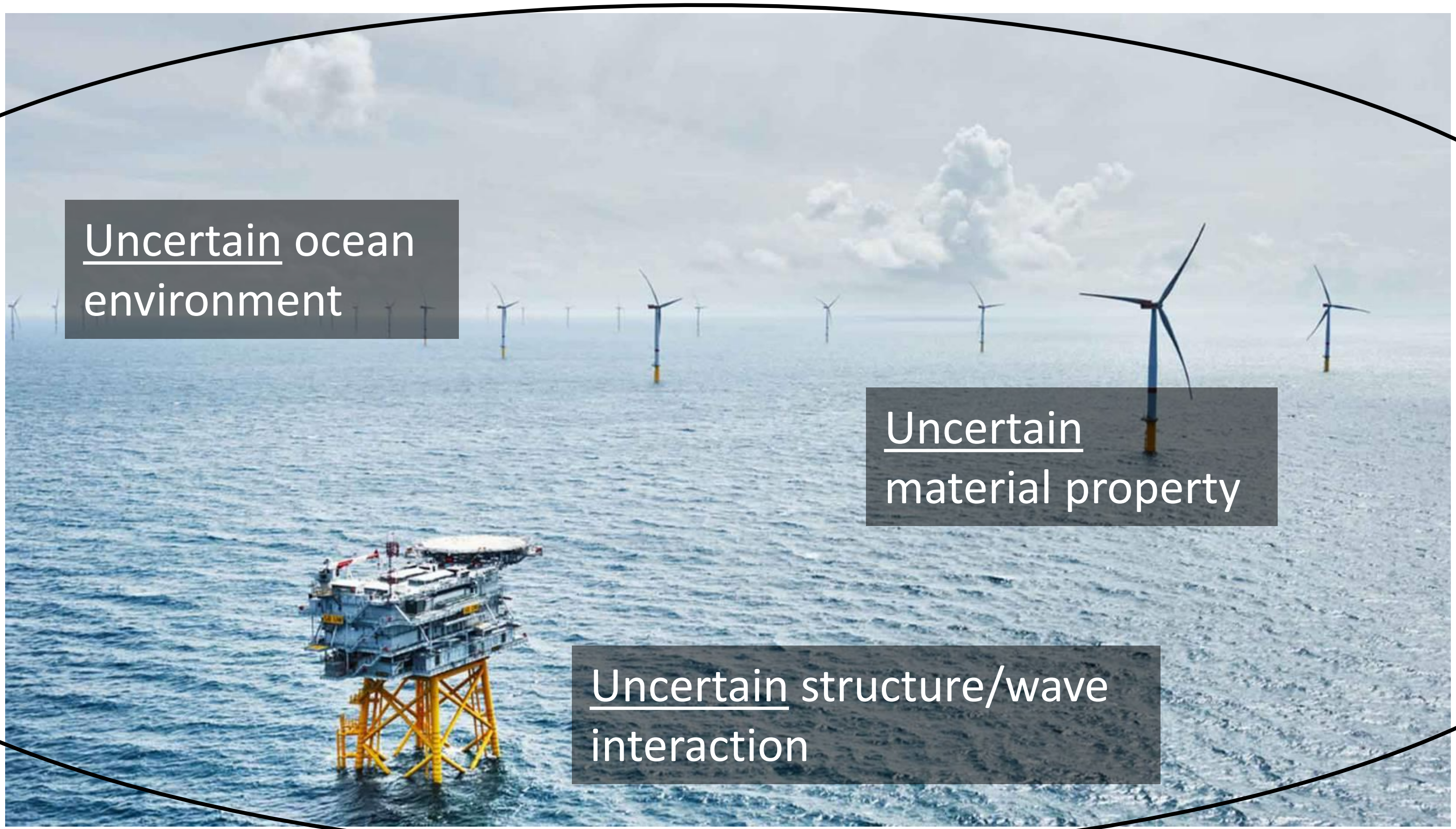
UNIVERSITY OF CAMBRIDGE

# Design in the presence of uncertainties

Uncertainties

Uncertain Input

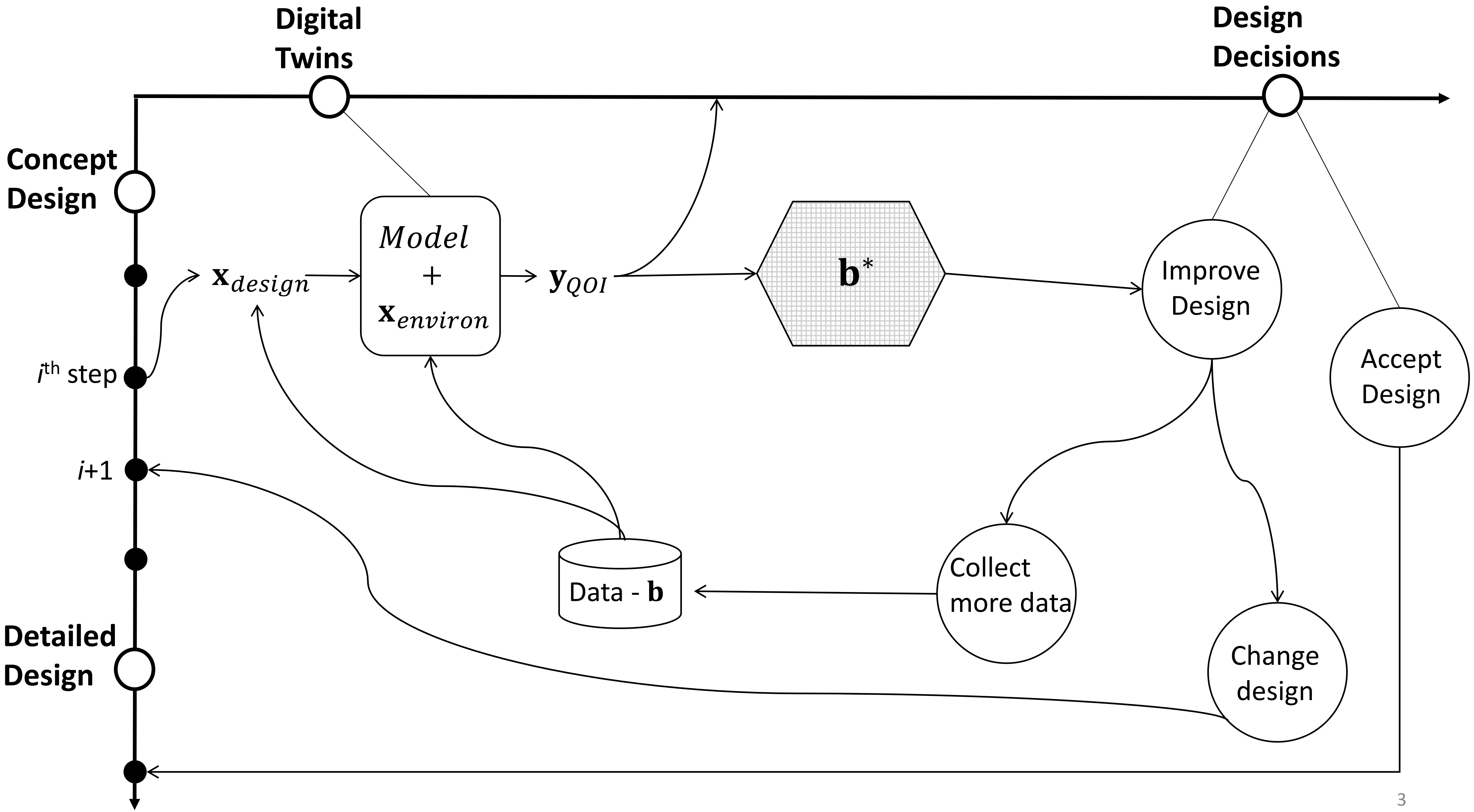
$\mathbf{X}$



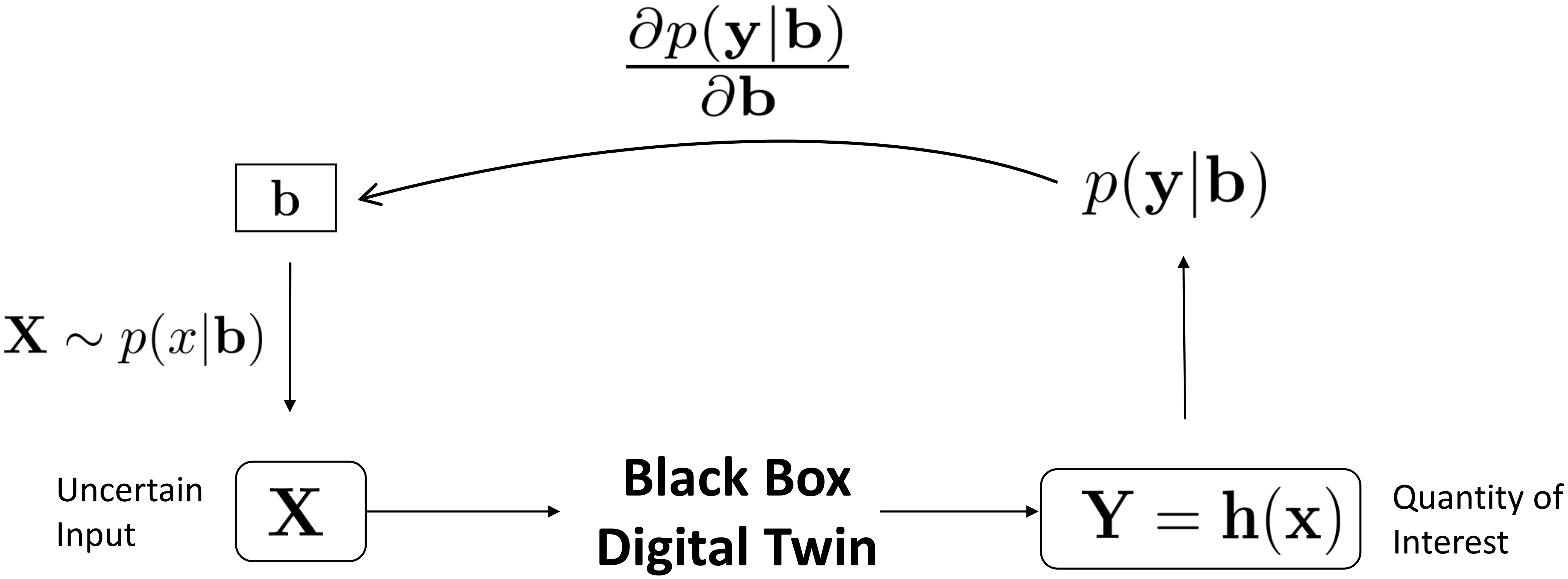

Design Quantity of Interest

$\mathbf{Y}$

Sensitivities


Digital Twins
Design Decisions
Concept Design
$\mathbf{x}_{design}$
$Model + \mathbf{x}_{environ}$
$\mathbf{y}_{QOI}$
$\mathbf{b}^*$
Improve Design
Accept Design
$i^{th}$ step
$i+1$
Data - $\mathbf{b}$
Collect more data
Change design
Detailed Design

# Design Sensitivity



# Design Sensitivity

$$\frac{\partial p(y|b)}{\partial b}$$

This is a random variable because *y* is random

$$\mathbb{E}\left[\frac{\partial p}{\partial b}\frac{b}{p}\right] = \int \frac{\partial p}{\partial b}\frac{b}{p} p dy$$
$$= b\frac{\partial}{\partial b}\int p dy = 0$$

Normalize

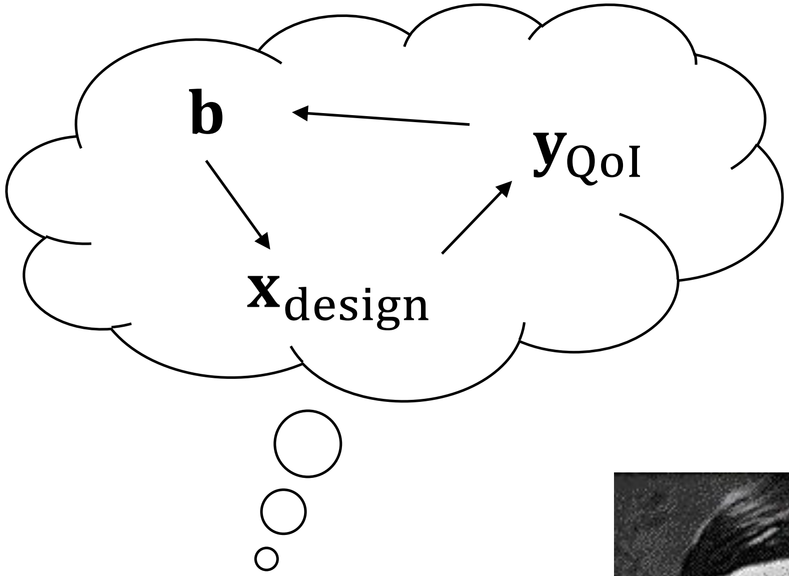
$$\frac{\partial p(y)}{\partial b}\frac{b}{p(y)}$$

mean $\mathbb{E}_Y\left[\frac{\partial p}{\partial b}\frac{b}{p}\right]$ Mean value is zero !

variance $\mathbb{E}_Y\left[\left(\frac{\partial p}{\partial b}\frac{b}{p}\right)^2\right] = b^2\mathbb{E}_Y\left[\left(\frac{\partial \ln p}{\partial b}\right)^2\right]$ We need to take a closer look at the variance!

(scalar notation is used here to keep things simple)

# Design Sensitivity




Is variance a good metric?

$$b^2 \mathbb{E}_Y \left[ \left( \frac{\partial \ln p}{\partial b} \right)^2 \right]$$

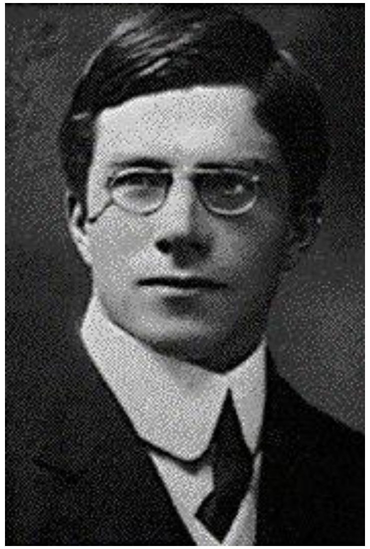
$$\downarrow$$

$$\mathbb{E}_Y \left[ \frac{b_j \partial \ln p}{\partial b_j} \frac{b_k \partial \ln p}{\partial b_k} \right] \xrightarrow{F_{jk}} \mathbf{F}$$

Change to covariance notation

Principal sensitivities

$$\operatorname{cov} \left( \frac{b_j \partial \ln p}{\partial b_j}, \frac{b_k \partial \ln p}{\partial b_k} \right) \longrightarrow \mathbf{F}\mathbf{q}_i = \lambda_i \mathbf{q}_i$$



Ronald Fisher 1912.jpg

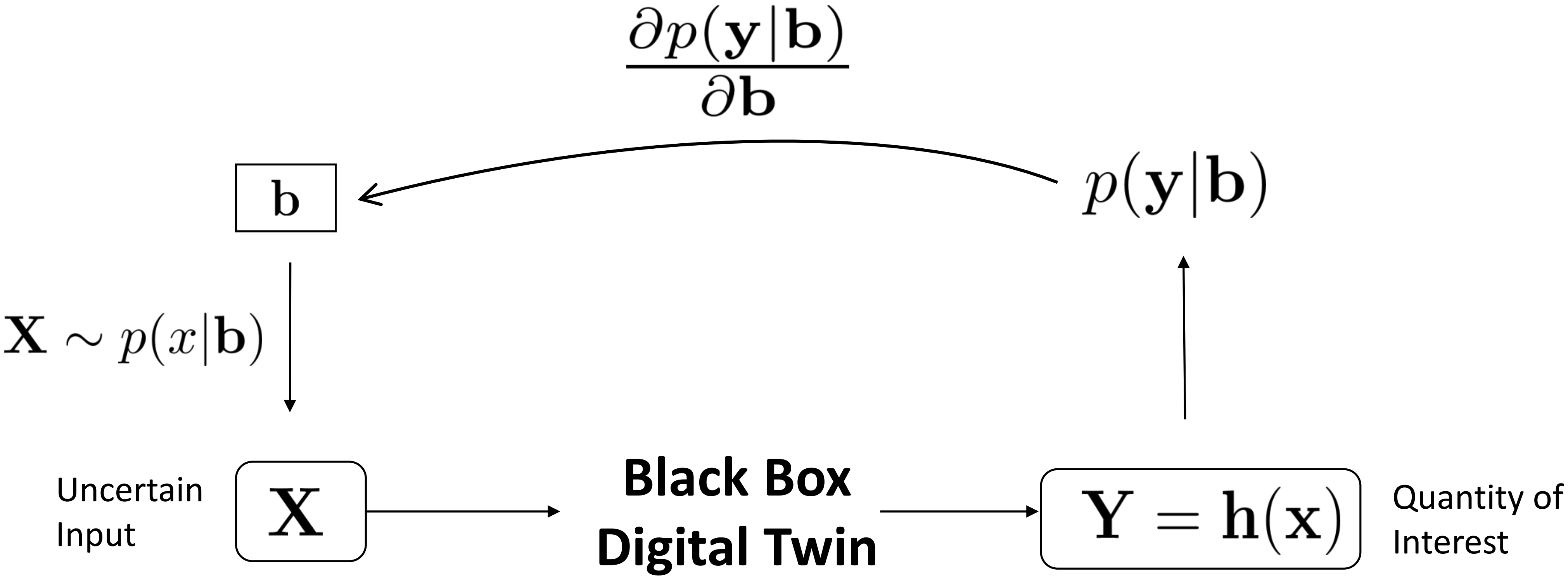
# Design Entropy

Quantity of Interest $\boxed{\mathbf{Y} = \mathbf{h}(\mathbf{x})} \xrightarrow{p(\mathbf{y}|\mathbf{b})} H = -\int p(\mathbf{y}|\mathbf{b}) \ln p(\mathbf{y}|\mathbf{b}) \mathrm{d}\mathbf{y}$

$$
\begin{aligned}
\Delta H &\equiv KL\left[p(\mathbf{y}|\mathbf{b}) || p(\mathbf{y}|\mathbf{b}+\Delta\mathbf{b})\right] \\
&= \int p(\mathbf{y}|\mathbf{b}) \ln\left[\frac{p(\mathbf{y}|\mathbf{b})}{p(\mathbf{y}|\mathbf{b}+\Delta\mathbf{b})}\right] \mathrm{d}\mathbf{y} \\
&\approx \frac{1}{2}\Delta\mathbf{b}^{\mathsf{T}} \int \frac{1}{p} \nabla p^{\mathsf{T}} \nabla p \,\mathrm{d}\mathbf{y} \Delta\mathbf{b} \\
&= \frac{1}{2}\Delta\mathbf{b}^{\mathsf{T}} \mathbf{F} \Delta\mathbf{b}
\end{aligned}
$$

Taylor expansion of the perturbed density function

# Design Sensitivity



# Reliability Sensitivity

Require information about the specifics: $g(\cdot)$ and $z$

$$\frac{\partial P_\mathrm{f}}{\partial \mathbf{b}}$$

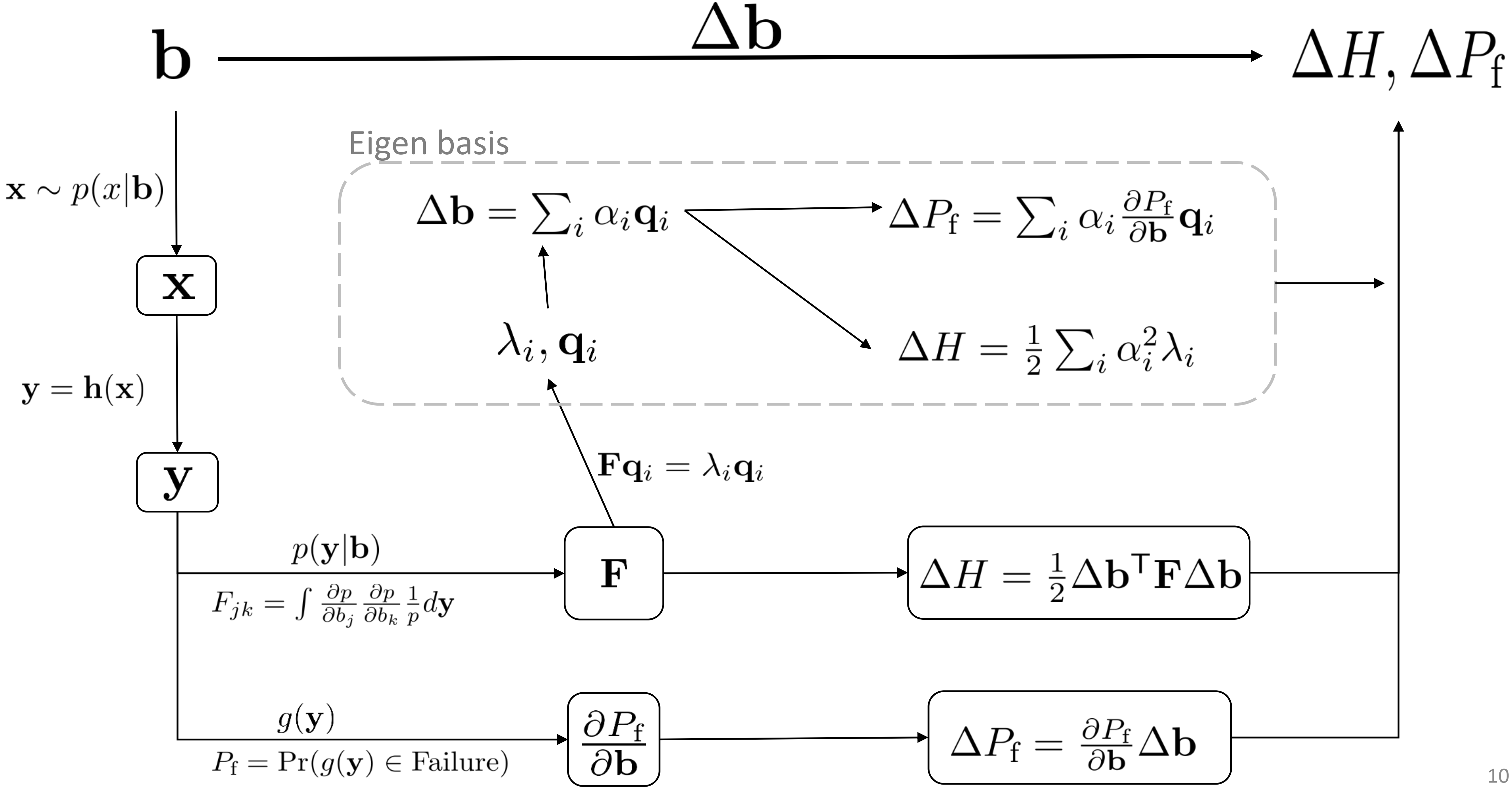
$$\boxed{\mathbf{b}}$$

$$P_\mathrm{f}\left(\mathbf{b}, z\right) = \int \mathrm{H}\left(g\left(\mathbf{y}\right) - z\right) p(\mathbf{x}|\mathbf{b}) \mathrm{d}\mathbf{x}$$

$$\mathbf{X} \sim p(x|\mathbf{b})$$

Uncertain Input $\mathbf{X}$ → **Black Box Digital Twin** → $\mathbf{Y} = \mathbf{h}(\mathbf{x})$ Quantity of Interest

# Mathematical framework for sensitivity



# Likelihood Ratio Method

$$F_{jk} = \int \frac{\partial p}{\partial b_j} \frac{\partial p}{\partial b_k} \frac{1}{p} d\mathbf{y}$$

$$\frac{\partial p(\mathbf{y}|\mathbf{b})}{\partial \mathbf{b}} \qquad\qquad p(\mathbf{y}|\mathbf{b})$$

Obtained at the same time in a single run

# Likelihood Ratio Method

$$p(\mathbf{y}|\mathbf{b}) = \int \delta\left[\mathbf{y} - \mathbf{h}(\mathbf{x})\right] p(\mathbf{x}|\mathbf{b})\,\mathrm{d}\mathbf{x}$$
$$= \mathbb{E}_X\left[\delta(\mathbf{y} - \mathbf{h}(\mathbf{x}))\right]$$

Free of charge to get gradient!

$$\frac{\partial p(\mathbf{y}|\mathbf{b})}{\partial b_j} = \int \delta\left[\mathbf{y} - \mathbf{h}(\mathbf{x})\right] \frac{\partial \ln p(\mathbf{x}|\mathbf{b})}{\partial b_j} p(\mathbf{x}|\mathbf{b})\,\mathrm{d}\mathbf{x}$$
$$= \mathbb{E}_X\left[\delta(\mathbf{y} - \mathbf{h}(\mathbf{x})) \frac{\partial \ln p(x|b)}{\partial b_j}\right]$$

Because this term is often available analytically

$$\frac{\partial \ln p(x|\mu,\sigma)}{\partial \mu} = \frac{x-\mu}{\sigma}$$
$$\frac{\partial \ln p(x|\mu,\sigma)}{\partial \sigma} = \frac{(x-\mu)^2 - \sigma^2}{\sigma^3}$$

e.g. Gaussian

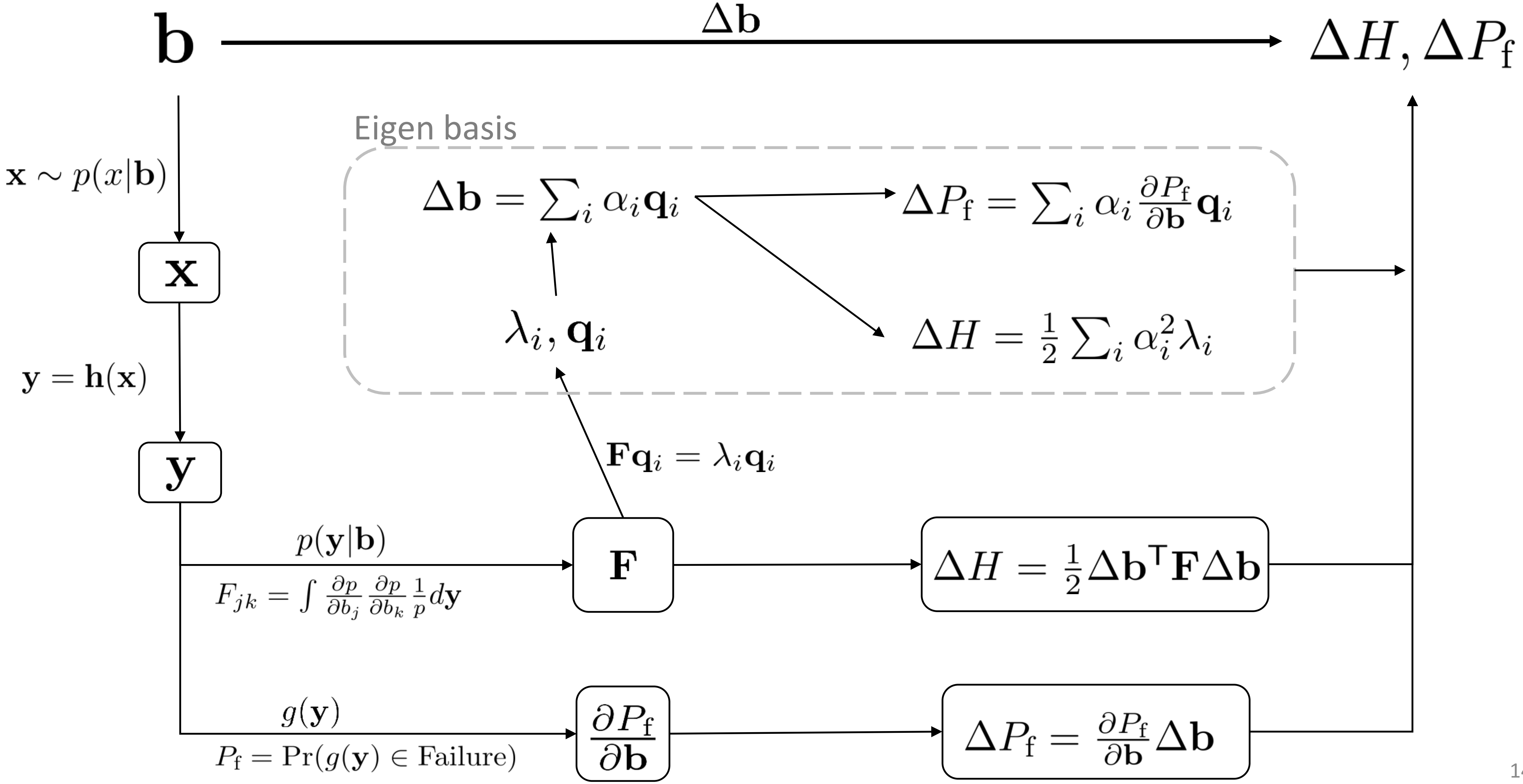
# Likelihood Ratio Method

$$P_{\mathrm{f}}\left(\mathbf{b}, z\right) = \int \mathrm{H}\left(g\left(\mathbf{y}\right) - z\right) p(\mathbf{x}|\mathbf{b})\mathrm{d}\mathbf{x}$$

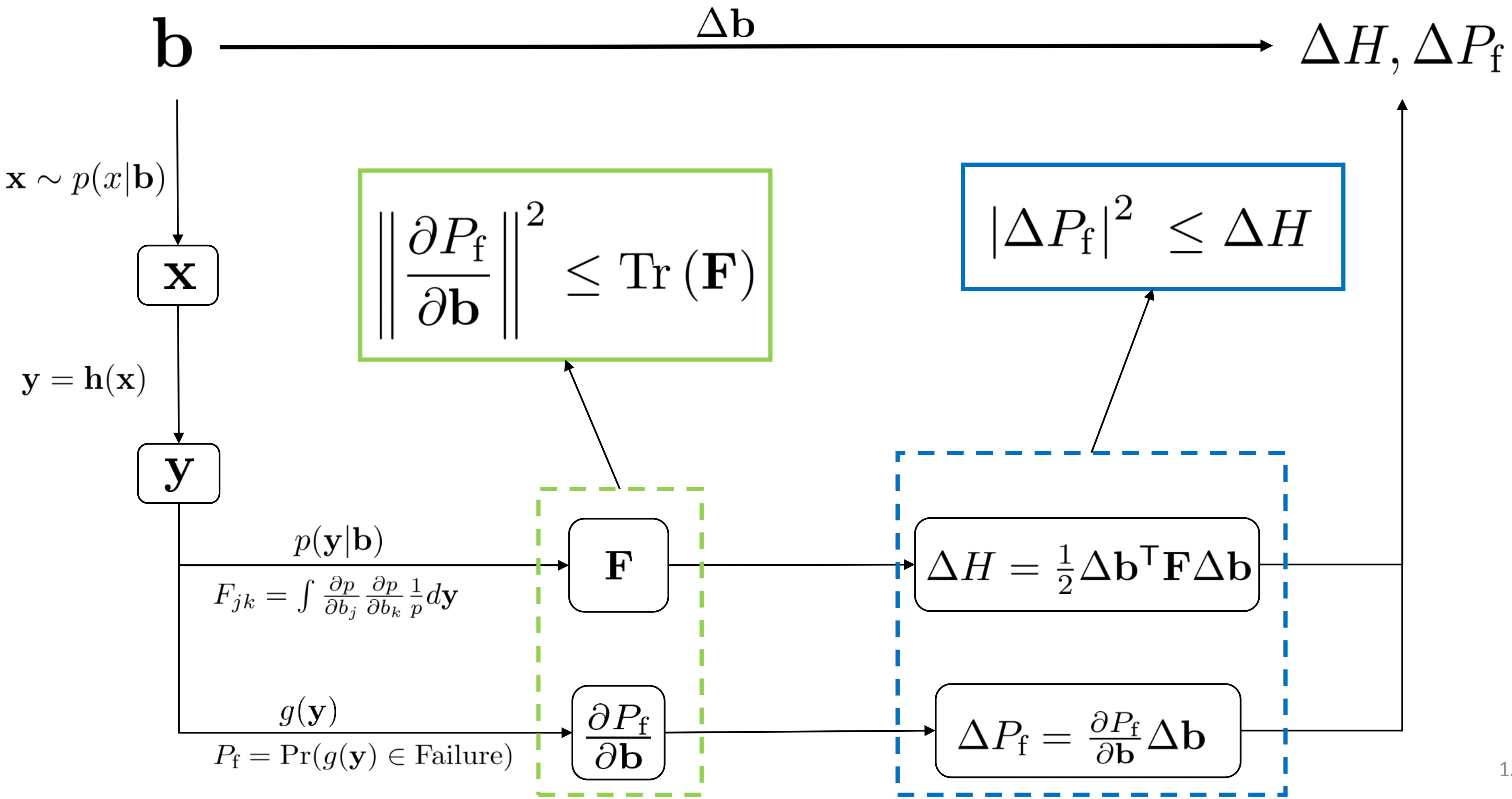
Free of charge to get gradient!

$$\frac{\partial P_{\mathrm{f}}\left(\mathbf{b}, z\right)}{\partial b_j} = \int \mathrm{H}\left(g\left(\mathbf{y}\right) - z\right) \frac{\partial \ln p\left(\mathbf{x}|\mathbf{b}\right)}{\partial b_j} p\left(\mathbf{x}|\mathbf{b}\right) \mathrm{d}\mathbf{x}$$

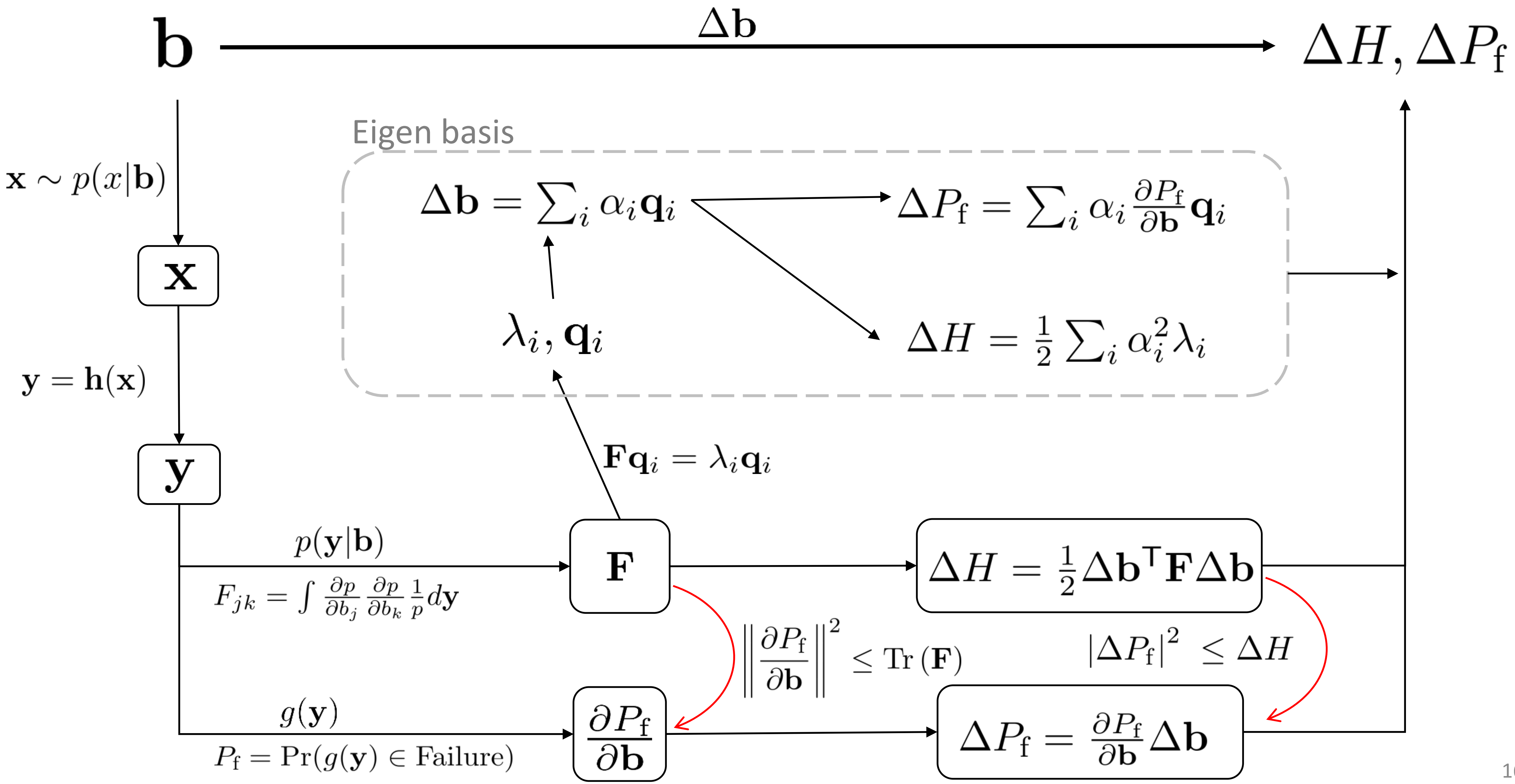
# Mathematical framework for sensitivity



# Sensitivity Bound



# Mathematical framework for sensitivity

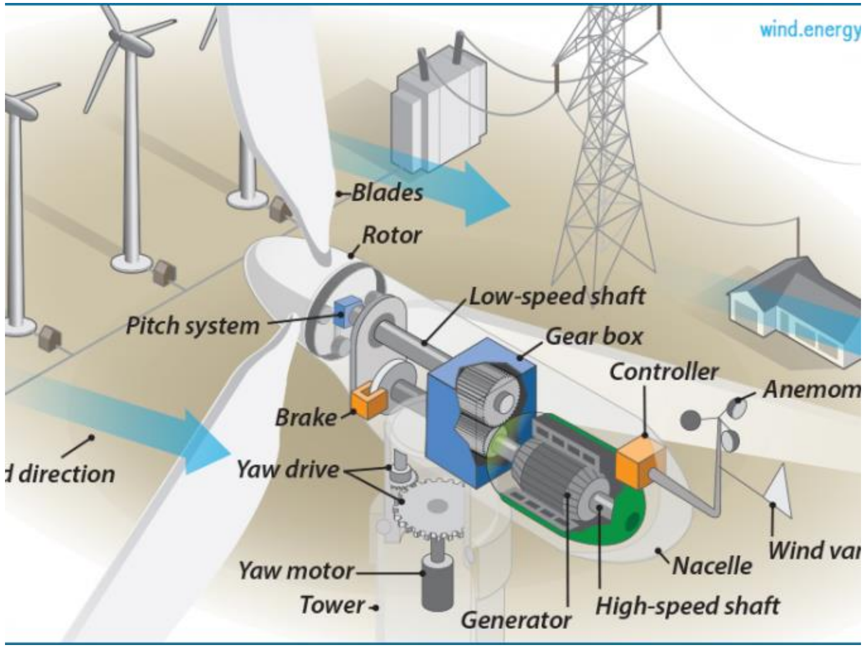


# Design in the presence of uncertainties

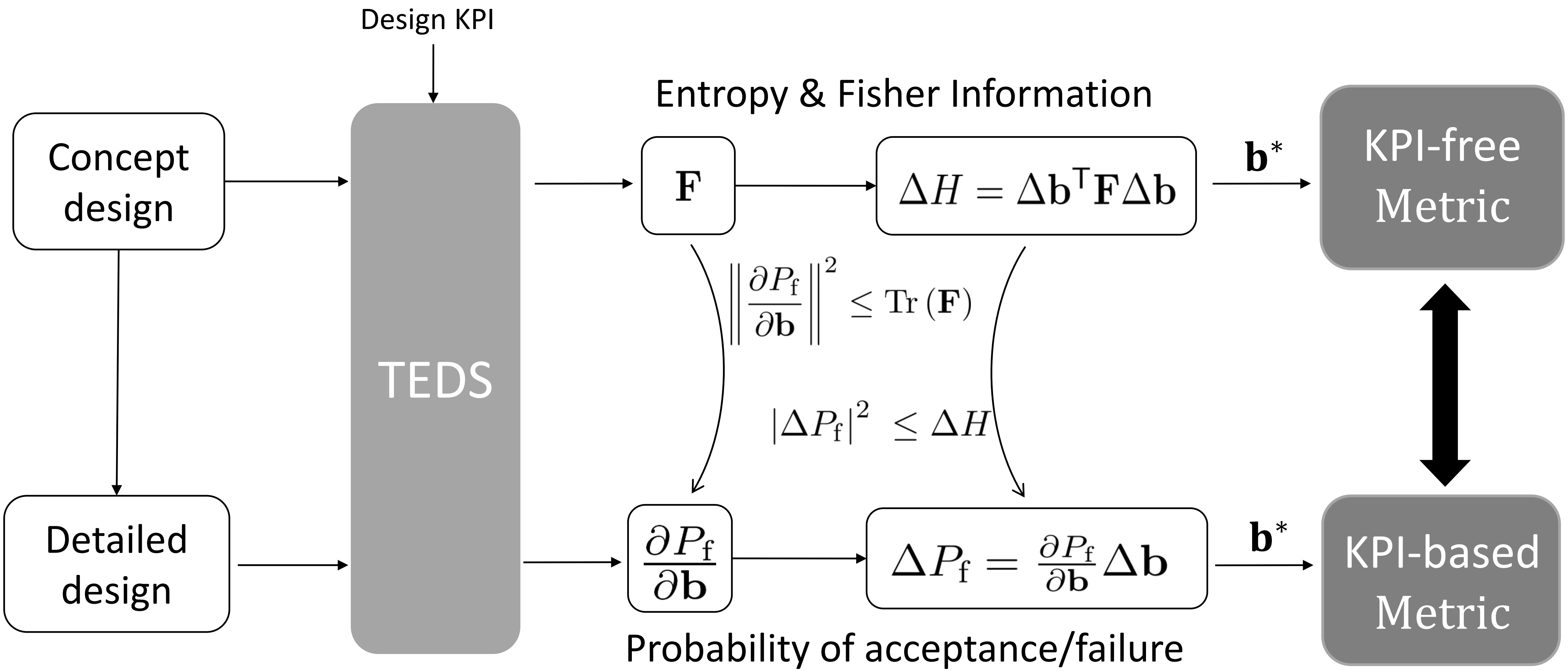
Concept design → Detailed design

High level inclusion of uncertainty

Key performance indicator (KPI)



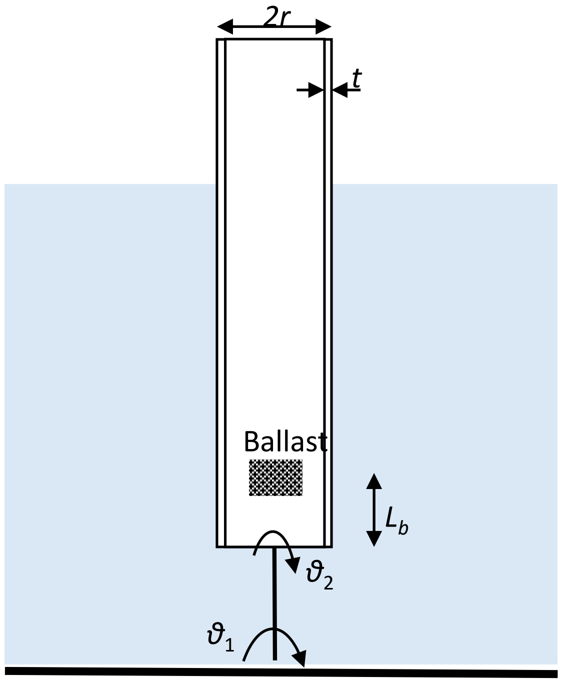
# Toolbox for Engineering Design Sensitivity (TEDS)



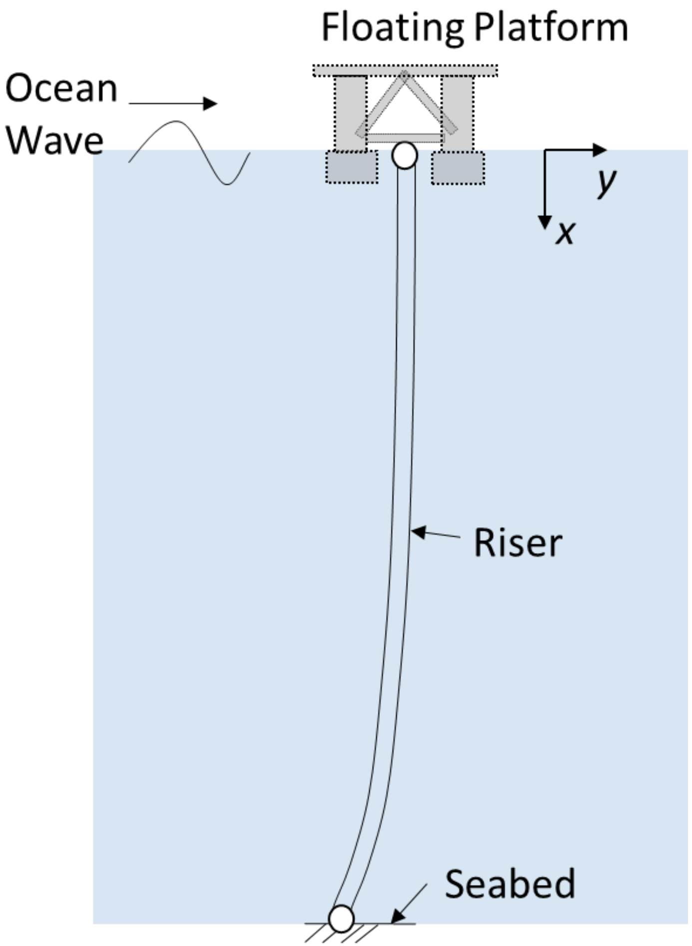

KPI: Key Performance Indicator

# Example application of TEDS

Example 1
Benchmark Case




Example 2
Design Case



# Free vibration

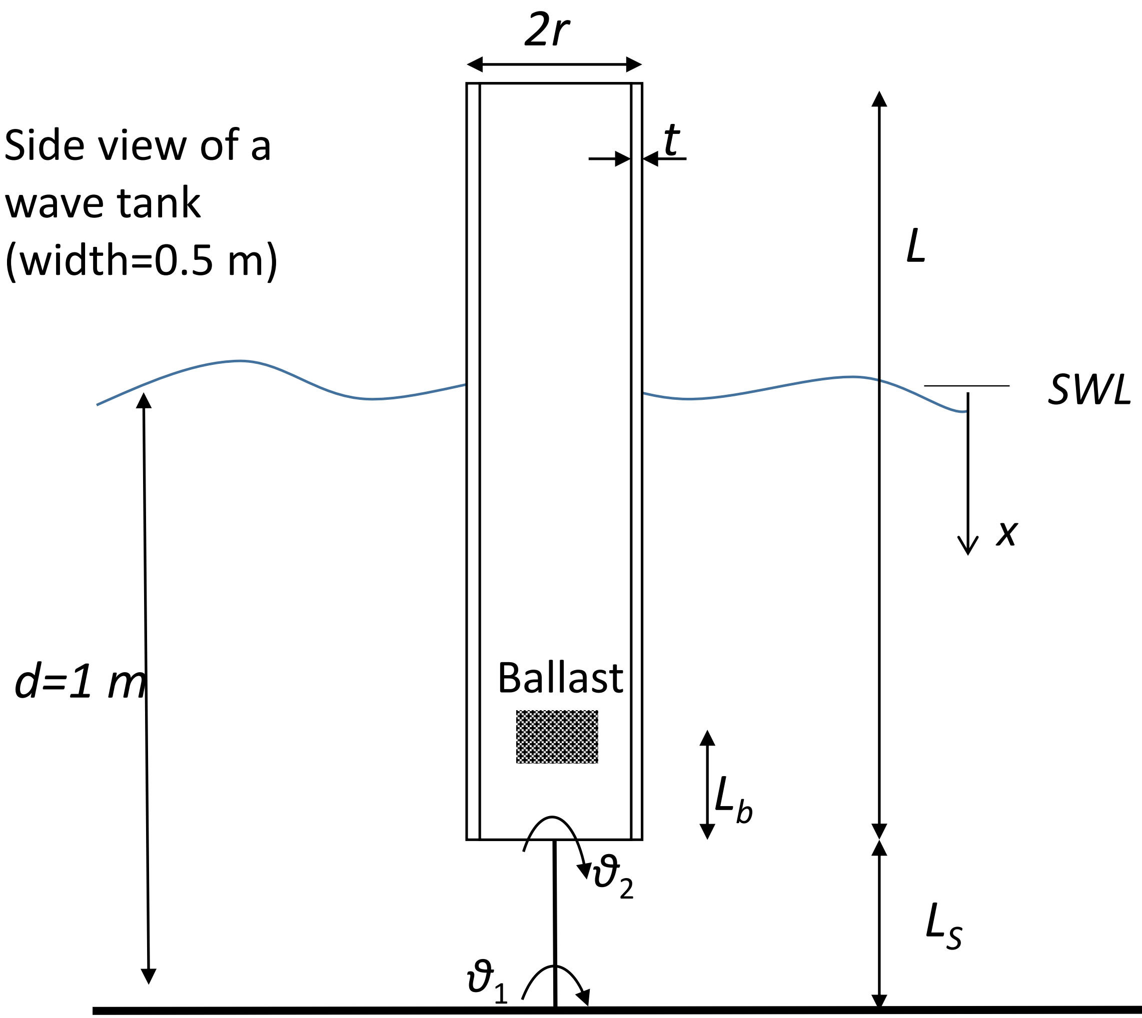
Design Variables

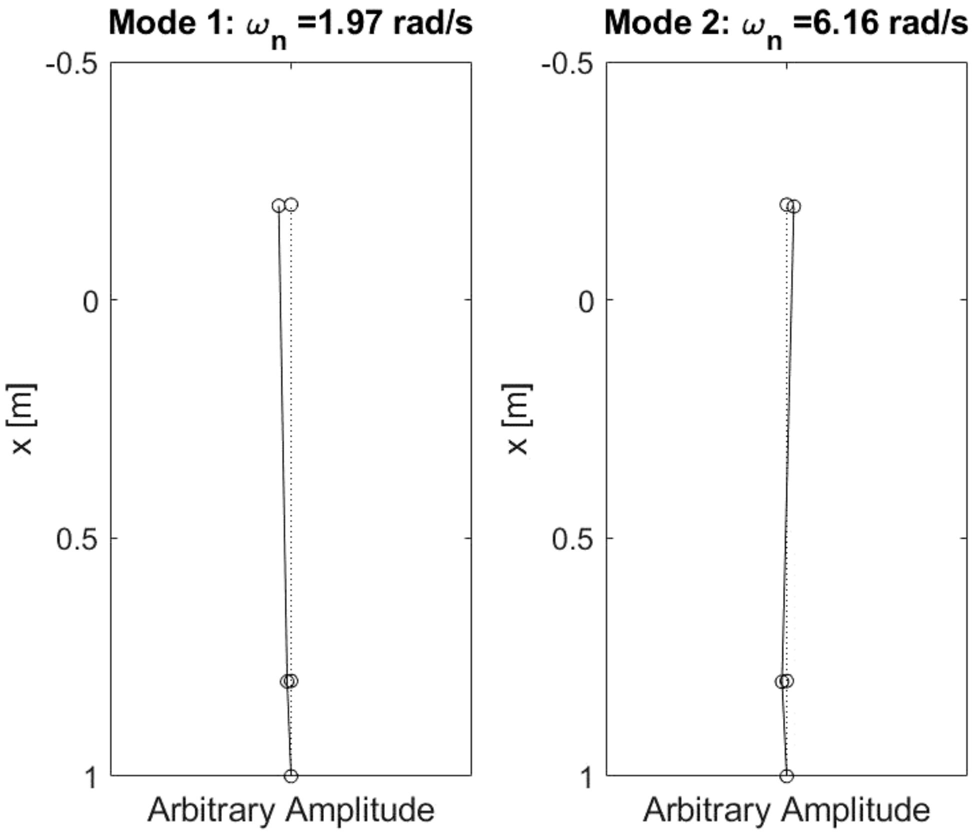
rho = 1180
rho_f = 1025
L = 1
L_S = 0.2000
L_b = 0.1500
r = 0.0450
t = 0.0350
mb = 3

**Natural Modes**

Two Modes:
Example mode shape & natural frequencies

Side view of a wave tank (width=0.5 m)

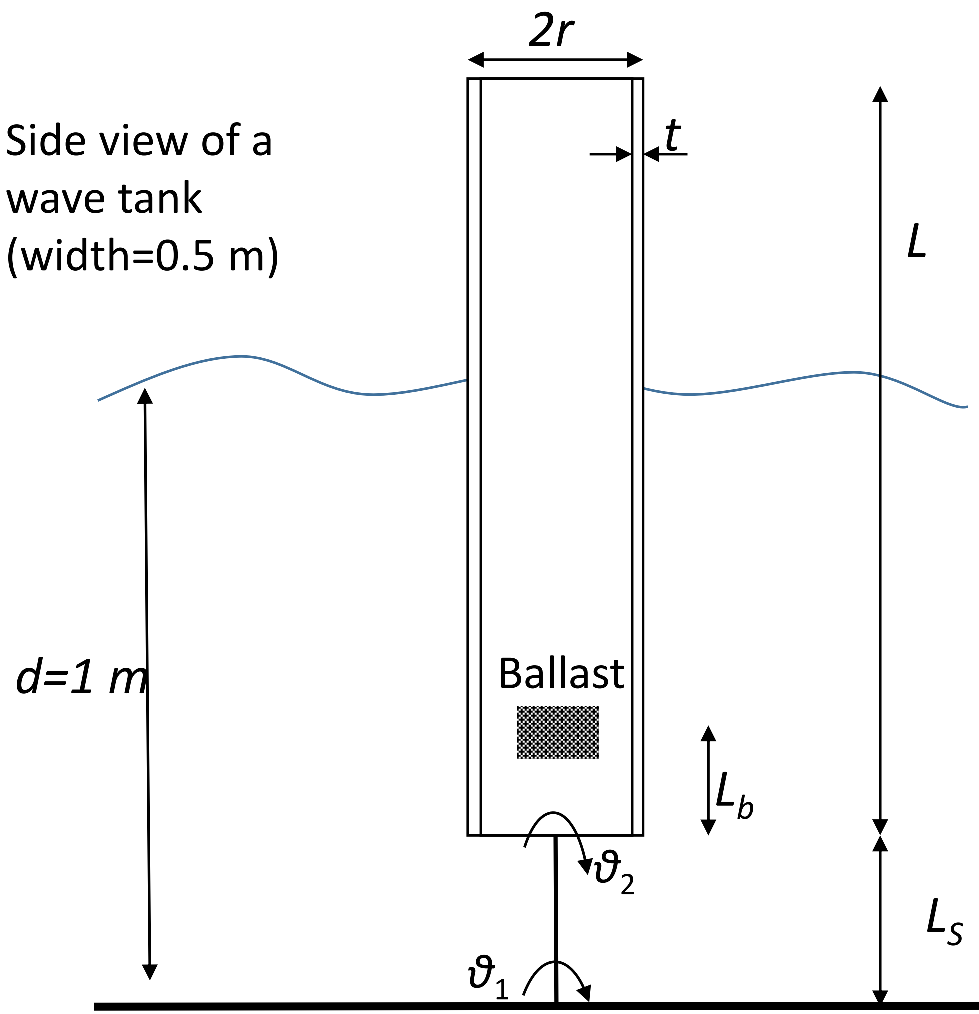


# Natural Frequency Sensitivity

| rho | rho_f | L | L_s | L_b | r | t | Mb | Ca |
|---|---|---|---|---|---|---|---|---|
| 1180 | 1025 | 1 | 0.2 | 0.15 | 0.045 | 0.003 | 3 | 1 |

Nominal Values




Analytical Sensitivity

$$\mathbf{K}\mathbf{x} = \lambda \mathbf{M}\mathbf{x}$$

$$\frac{\partial \lambda}{\partial b_i} = \mathbf{x}^{\mathsf{T}} \left[ \frac{\partial \mathbf{K}}{\partial b_i} - \lambda \frac{\partial \mathbf{M}}{\partial b_i} \right] \mathbf{x}$$

⇩

$$\frac{\partial \omega}{\partial b_i} = \frac{1}{2\omega} \frac{\partial \lambda}{\partial b_i}$$

⇩ Normalise

$$r = \frac{\partial \omega}{\partial b_i} \frac{b_i}{\omega}$$

$$\mu$$

⇩

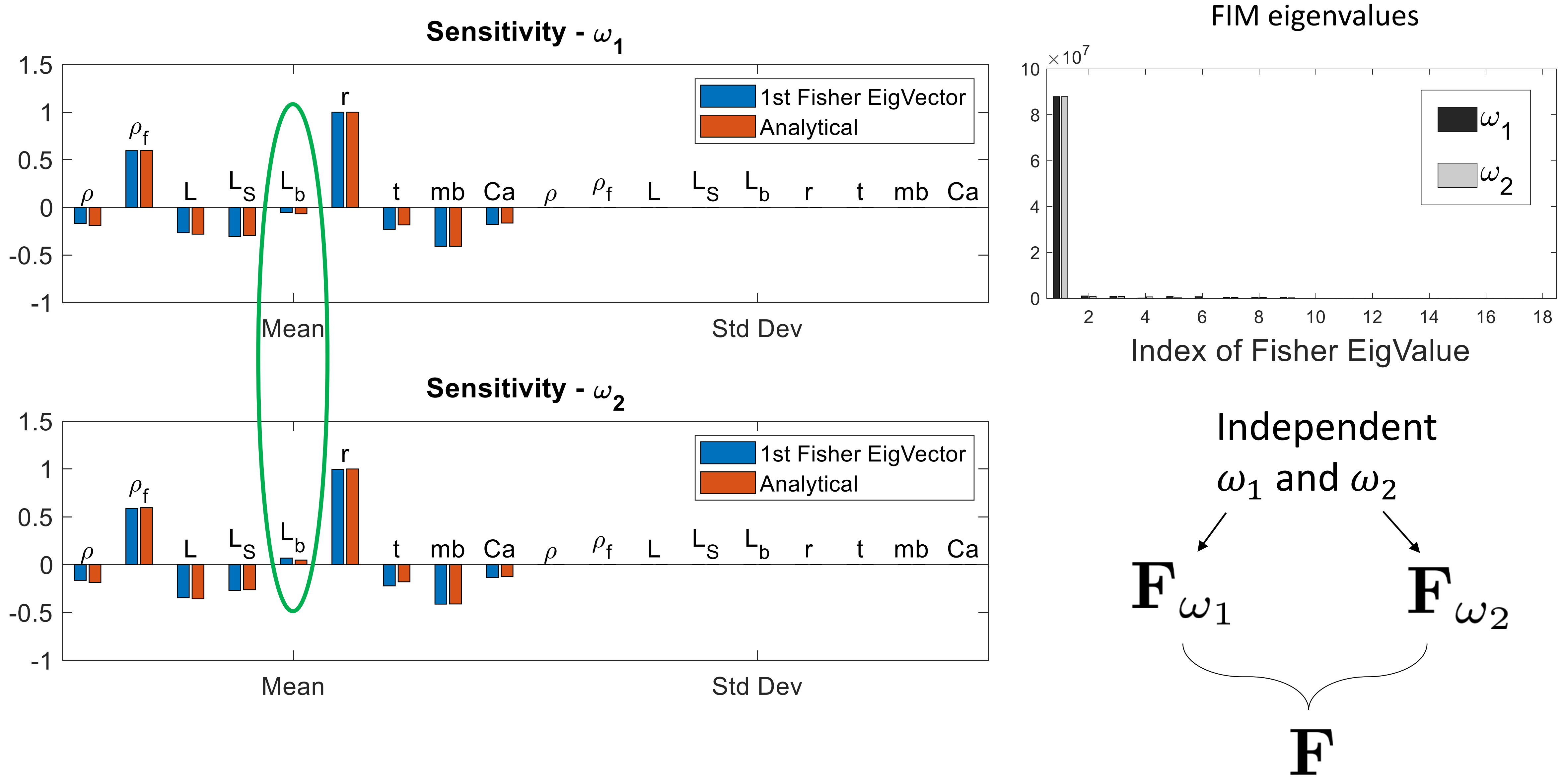
$$\mathcal{N}(\mu, \epsilon \to 0)$$

⇩

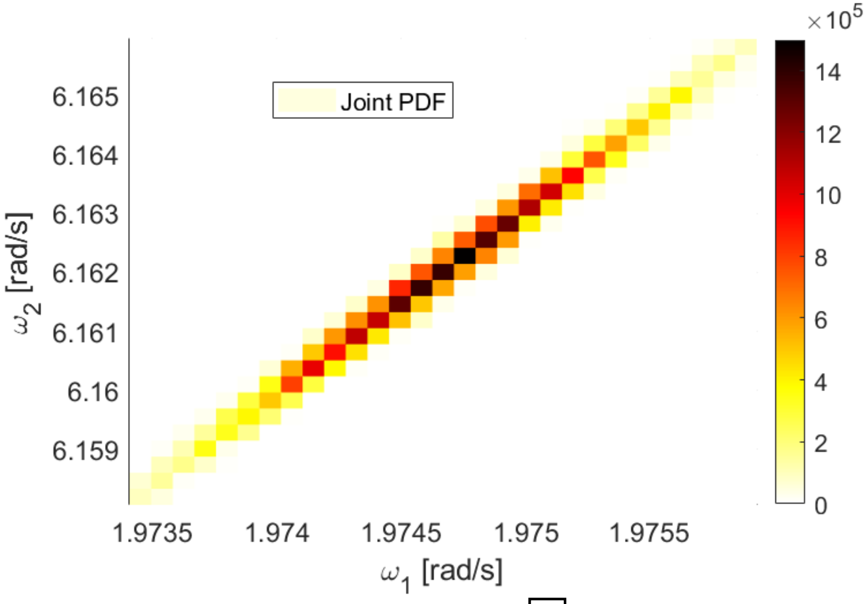
TEDS

⇩

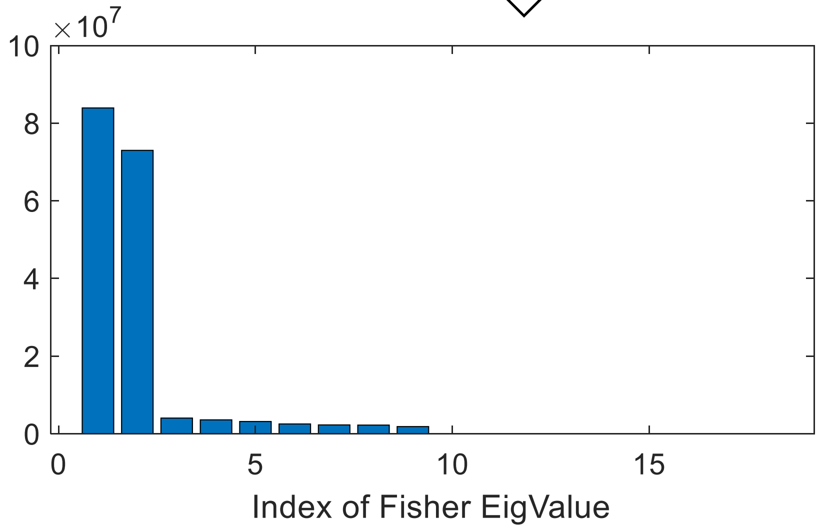
Fisher Sensitivity

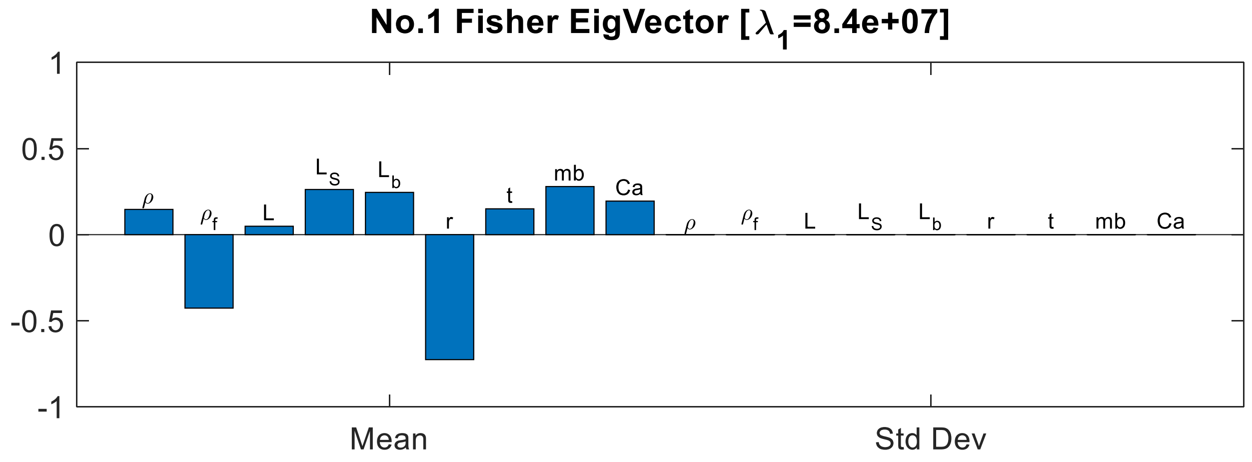


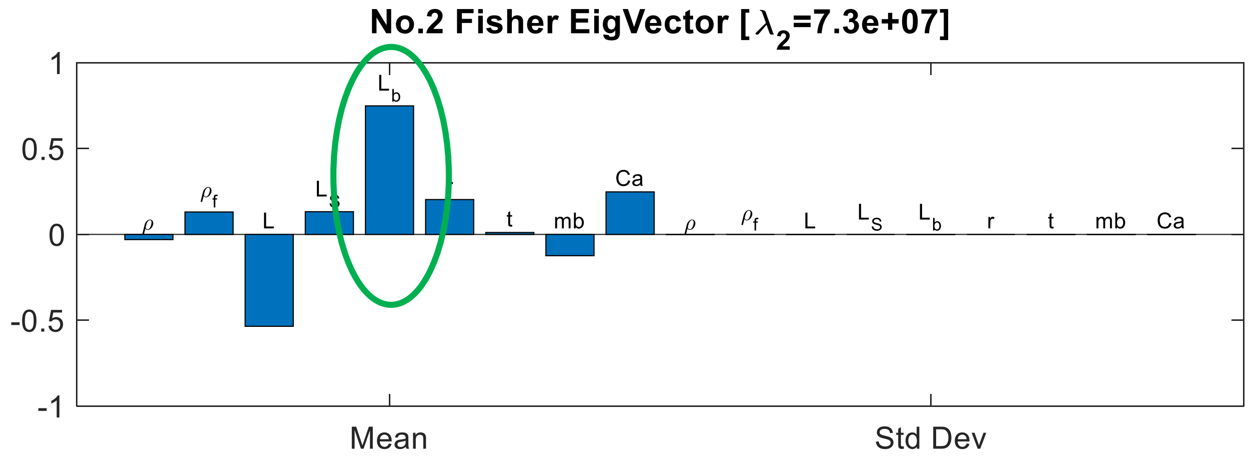
Sensitivity - ω1
1st Fisher EigVector
Analytical
ρ ρf L Ls Lb r t mb Ca ρ ρf L Ls Lb r t mb Ca
Mean
Std Dev
Sensitivity - ω2
1st Fisher EigVector
Analytical
ρ ρf L Ls Lb r t mb Ca ρ ρf L Ls Lb r t mb Ca
Mean
Std Dev
FIM eigenvalues
×10^7
ω1
ω2
Index of Fisher EigValue
Independent
ω1 and ω2
F ω1
F ω2
F




$$F_{jk} = \int \frac{\partial p}{\partial b_j} \frac{\partial p}{\partial b_k} \frac{1}{p} d\mathbf{y}$$



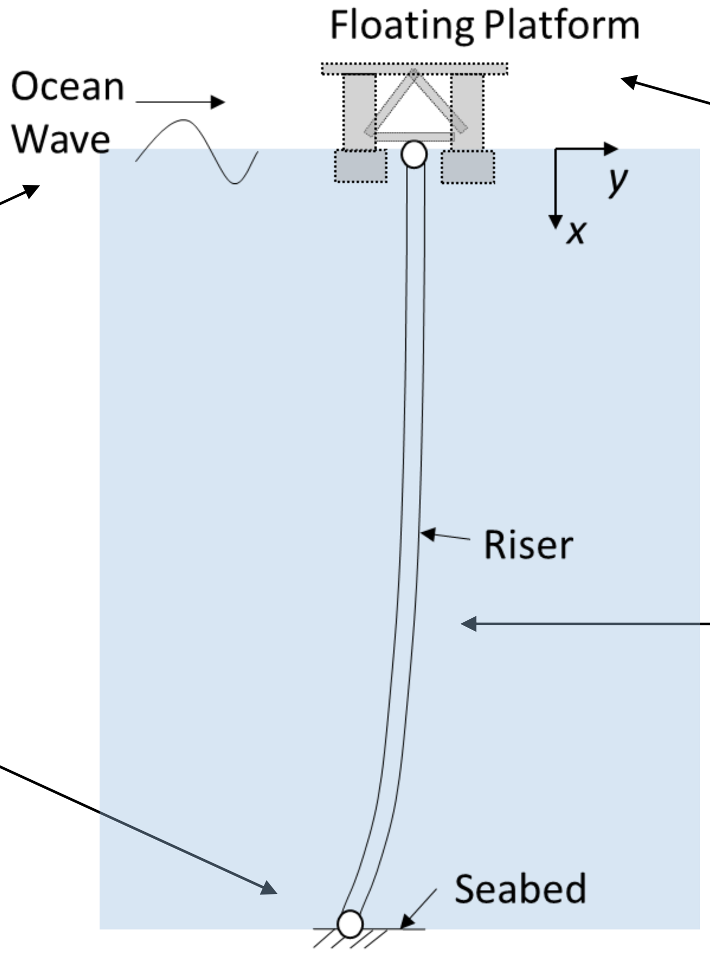



# Example 2 - offshore marine riser

Uncertain ocean wave heights which can be stochastic by nature, and uncertain structure/fluid interaction coefficients due to a lack of complete underlying physics

Uncertain force exerted by the floating platform onto the riser, depending on operation condition




Uncertain soil conditions at the sea bed

Uncertain material property, probably due to manufacturing

A marine riser is a conduit that transfers subsea oil to a surface platform. This example with a marine riser highlights the ubiquitous role of uncertainties for engineering design.

# Parameters

Mean and standard deviation values for the random input variables (Gaussian)

<u>Types of Uncertainty</u>

Wave Interaction

Material

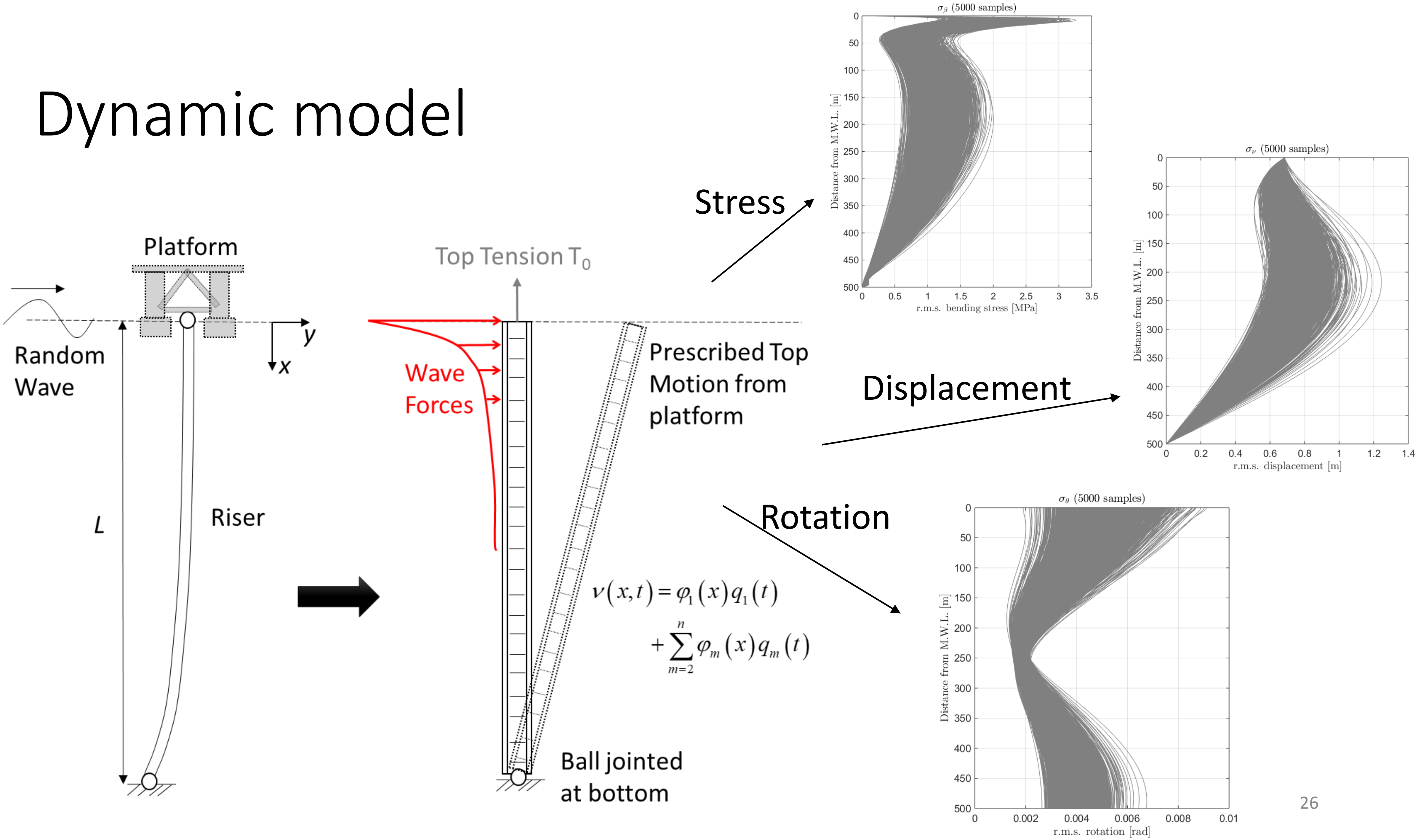
Platform interaction

Fatigue

| Random Variable | | Mean | Standard deviation |
|---|---|---|---|
| Morison's equation added mass coefficient | $C_a$ [-] | 1.5 | 0.3 |
| Morison's equation drag coefficient | $C_d$ [-] | 1.1 | 0.22 |
| Marine riser steel density | $\rho$ [kg/m$^{-3}$] | 7840 | 392 |
| Marine riser Young's modulus | E [GPa] | 200 | 10 |
| Riser internal oil density | $\rho_o$ [kg/m$^{-3}$] | 920 | 92 |
| Marine riser top tension | $T_0$ [kN] | 4905 | 490.5 |
| Material S-N curve coefficients | $\alpha$ [GPa] | 199 | 19.9 |
| | $\delta$ [-] | 3 | 0.3 |

S-N law $N(s) = \alpha s^{-\delta}$

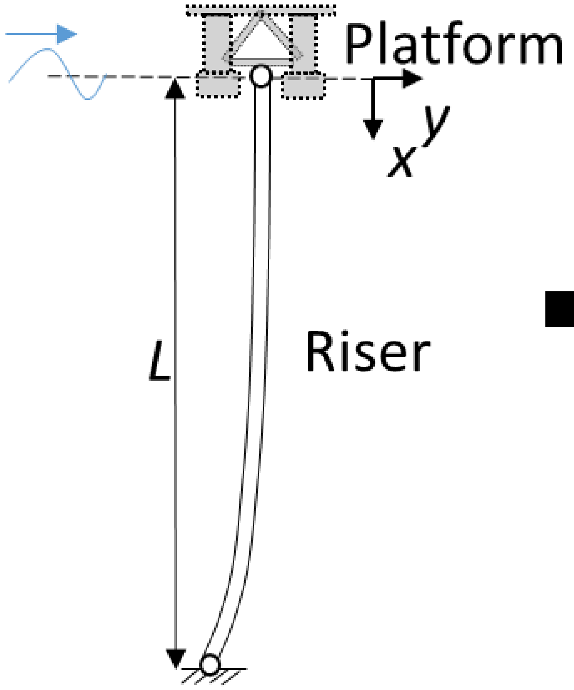
# Dynamic model

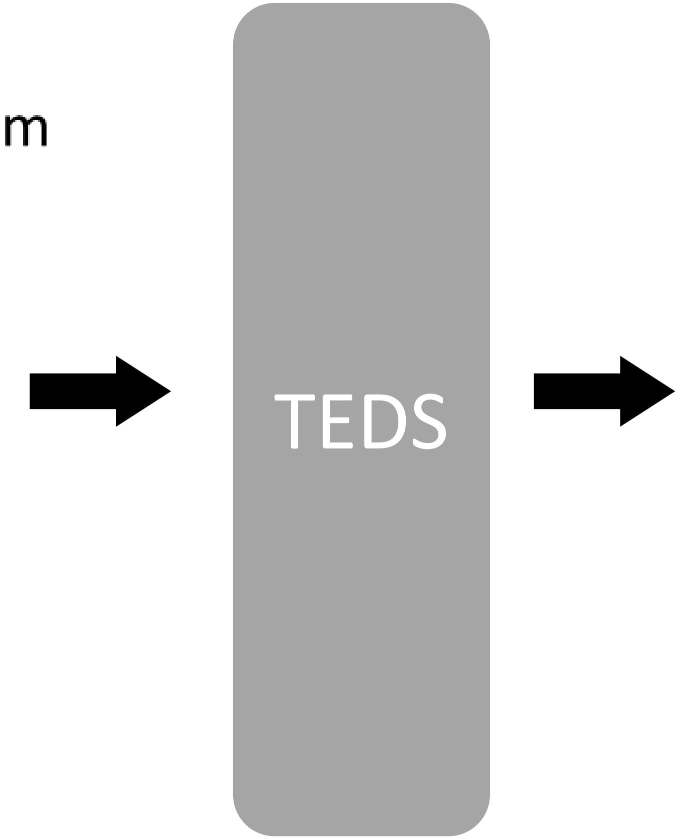



$$v(x,t) = \varphi_1(x) q_1(t) + \sum_{m=2}^{n} \varphi_m(x) q_m(t)$$

# Example 2 - offshore marine riser

Large uncertainty about KPI, various QoI considered

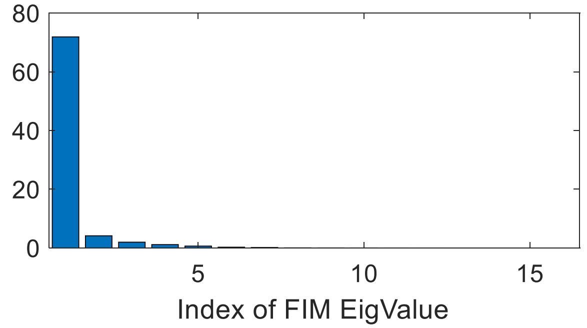


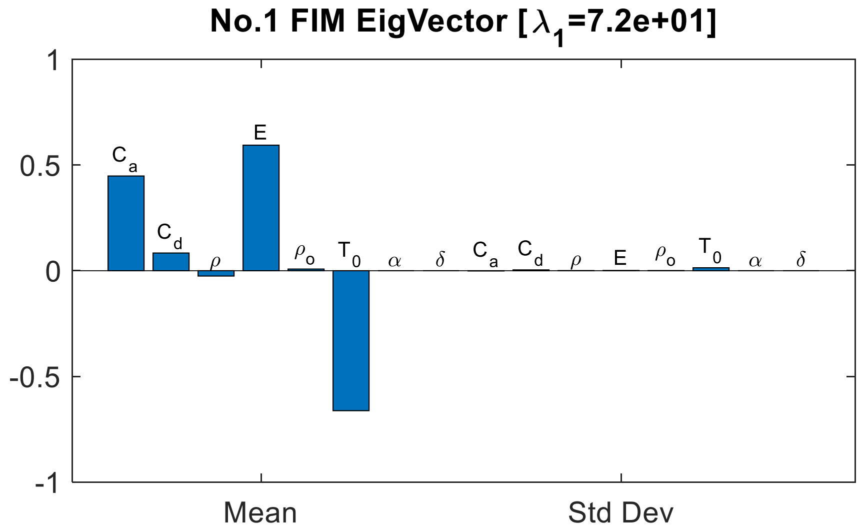

| | Scenario – 1 | Scenario - 2 |
|---|---|---|
| Design QoI | Stress | Stress, displacement and rotation |
| Design KPI | Fatigue | Fatigue |

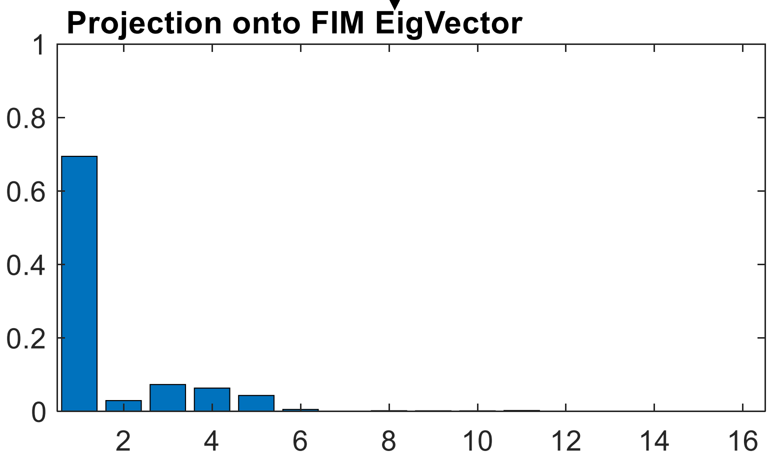
QoI: Quantity of Interest
KPI: Key Performance Indicator

# Scenario - 1



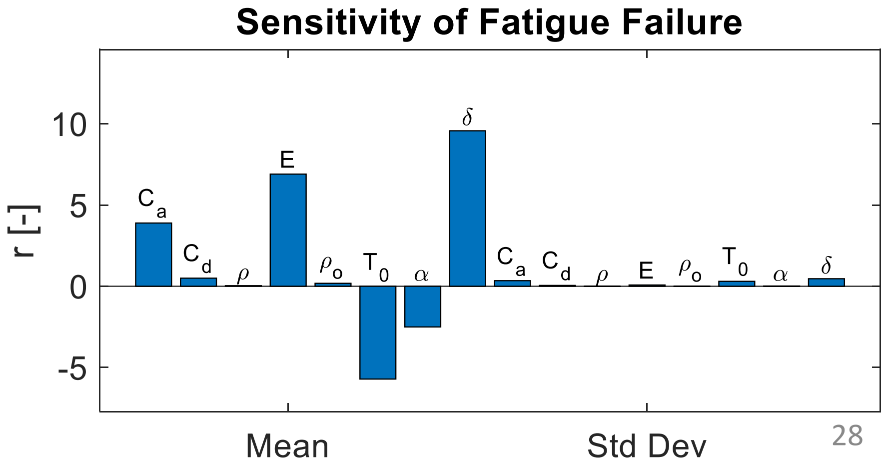

KPI-free Sensitivity



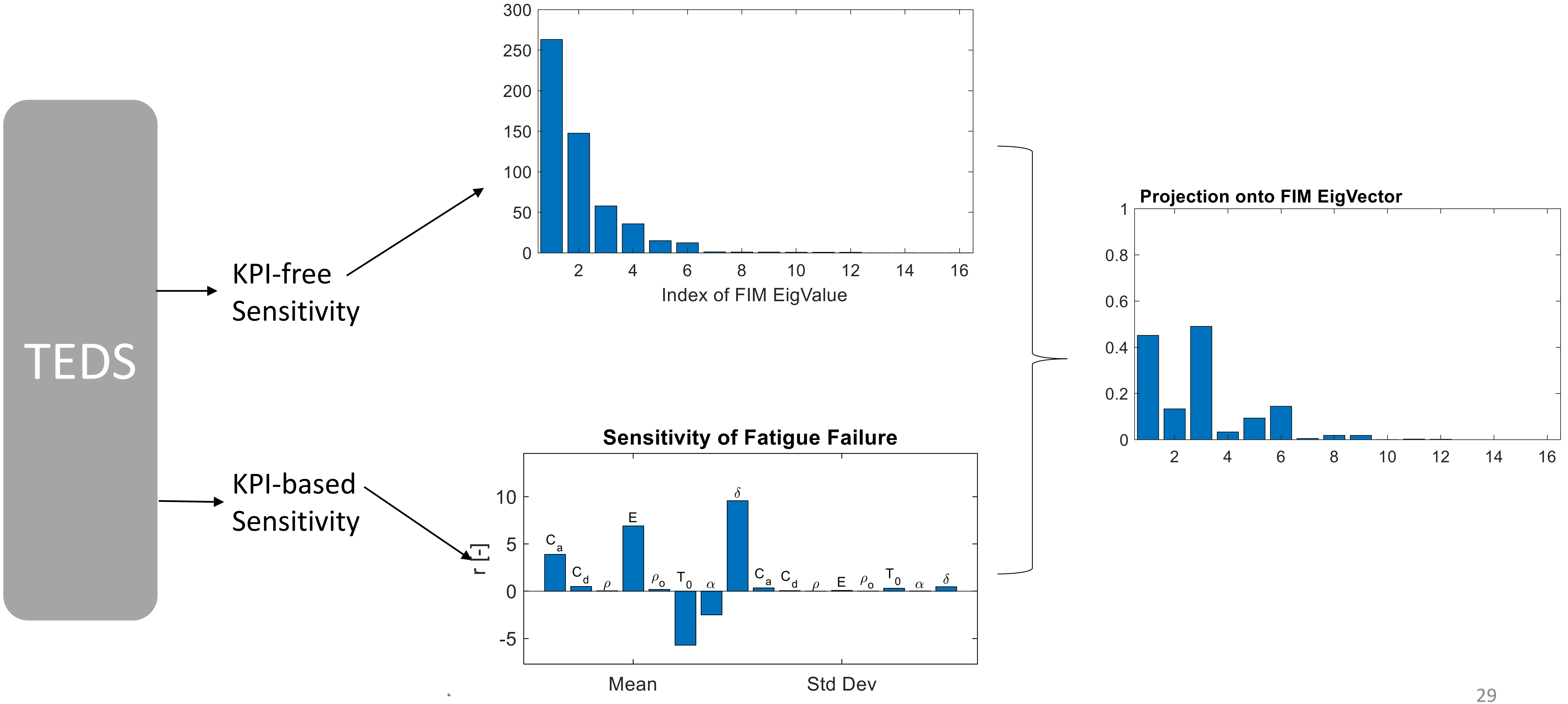

TEDS

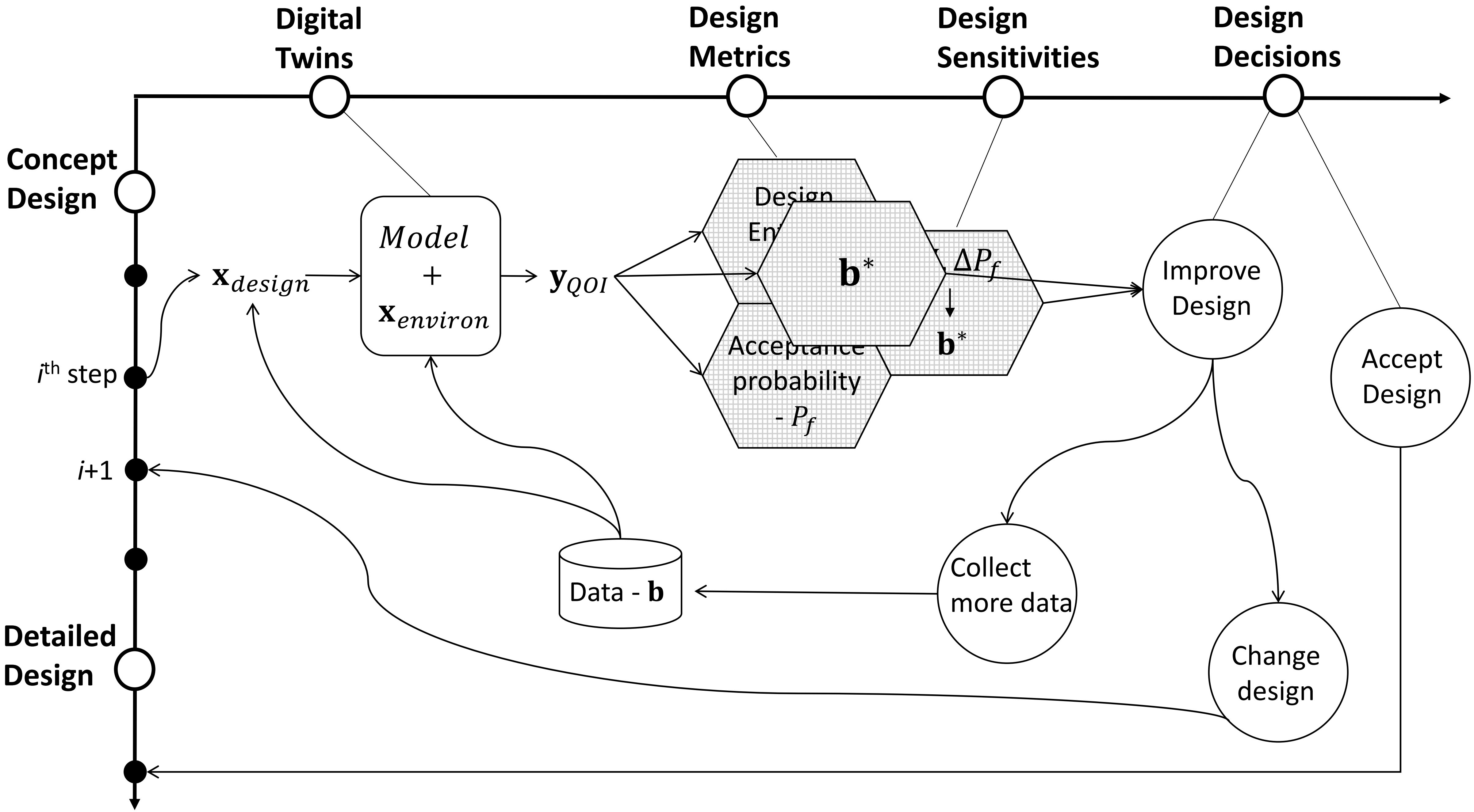
KPI-based Sensitivity




S-N law $N(s) = \alpha s^{-\delta}$

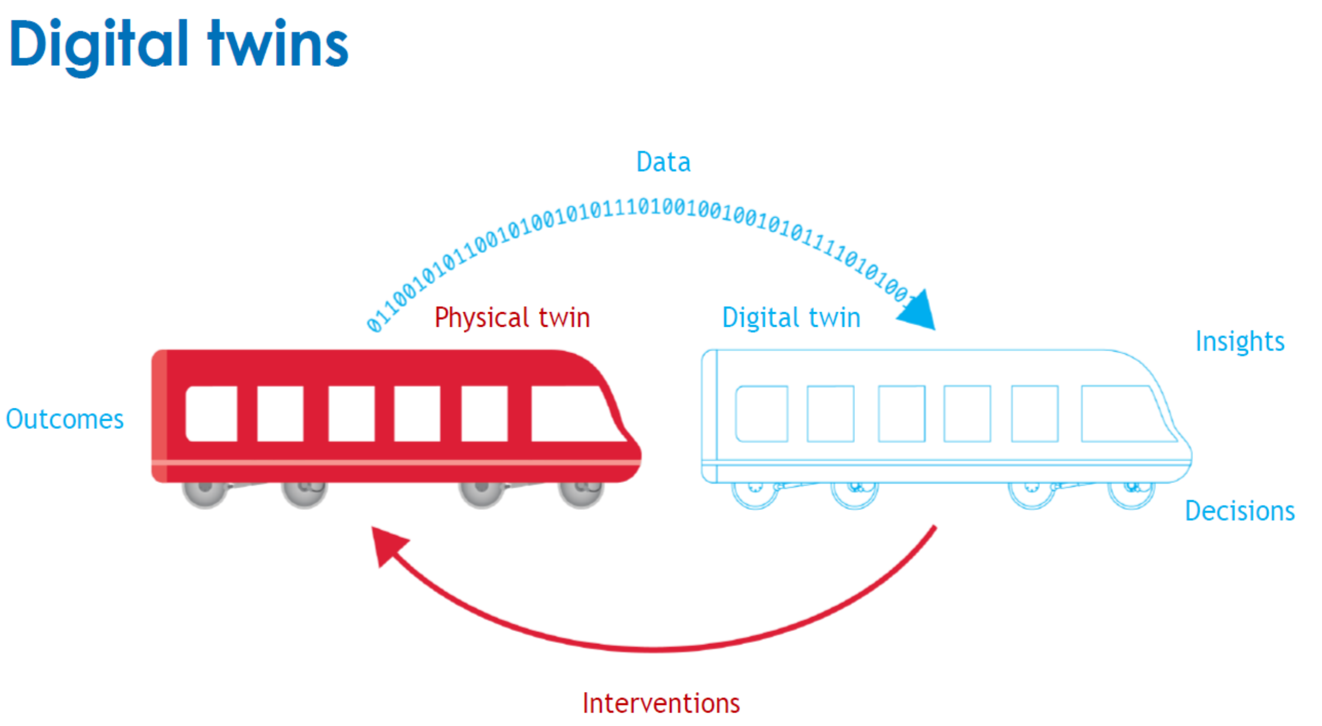
# Scenario - 2


TEDS
KPI-free Sensitivity
KPI-based Sensitivity
300
250
200
150
100
50
0
2
4
6
8
10
12
14
16
Index of FIM EigValue
Sensitivity of Fatigue Failure
r [-]
10
5
0
-5
C a
C d
ρ
E
ρ o
T 0
α
δ
Mean
Std Dev
Projection onto FIM EigVector
1
0.8
0.6
0.4
0.2



Digital Twins
Design Metrics
Design Sensitivities
Design Decisions
Concept Design
$i^{th}$ step
$i$+1
Detailed Design
$\mathbf{x}_{design}$
$Model$ + $\mathbf{x}_{environ}$
$\mathbf{y}_{QOI}$
$\mathbf{b}^*$
$\Delta P_f$
$\mathbf{b}^*$
probability
- $P_f$
Improve Design
Accept Design
Collect more data
Change design
Data - $\mathbf{b}$

# Metric Toolbox for Digital Twins




https://www.cdbb.cam.ac.uk/

Metric Toolbox

Decisions

# Acknowledgement

- Collaborators: Prof. Robin Langley, Dr. Arnau Razquin and Dr. Luis Andrade

- This work has been funded by the Engineering and Physical Sciences Research Council through the award of a Programme Grant “Digital Twins for Improved Dynamic Design”, Grant No. EP/R006768.

Any questions/comments:

jy419@cam.ac.uk

UNIVERSITY OF CAMBRIDGE