

Assignment 2: ALDA

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1 Reading the Data

We are going to perform the analyses on two datasets: the original email dataset, and a sample dataset that contains resting state data from 91 participants.

```
> cell = read.csv("cell_withitems_complete.csv", header = TRUE, sep = ",")
> alda_sample = read.csv("alda_sample.csv", header = TRUE, sep = ",")
> head(cell)
```

X	ID	Days	ItemNo	Memorablity	Accuracy	Messages	
1	1	103	17	1	11.0000000	1	19
2	2	103	14	2	7.4285717	1	6
3	3	103	68	3	30.7142870	1	11
4	4	103	279	4	21.0000000	0	3
5	5	103	52	5	30.0000000	1	4
6	6	103	203	6	0.5714285	0	2

	RecallBeforeHint	Vividness	GuessedMonth	NumItems	Month
1	1	10	1	30	1
2	3	10	1	30	1
3	1	9	3	30	3
4	5	5	9	30	10
5	1	10	2	30	2
6	4	9	7	30	7

	TimeJudgmentDistance
1	0
2	0
3	0
4	1
5	0
6	0

```
> head(alda_sample)
```

ID	group	time	CON	CON_SAL	CON_SMN7	DAN	DAN_CON	
1	6	PD	4	0.1929	0.0708	0.0088	0.1619	0.1156
2	6	PD	5	0.1949	0.0862	0.0040	0.1677	0.1173
3	6	PD	6	0.1811	0.0916	-0.0037	0.2153	0.1636
4	29	PD	4	0.1594	0.0438	-0.0253	0.1749	0.0161
5	29	PD	5	0.0881	0.0446	-0.0288	0.1356	0.0377
6	34	CTRL	4	0.1372	0.0113	-0.0792	0.1659	0.0045

	DAN_SAL	DAN_SMN7	DMN6	DMN6_CON	DMN6_DAN	DMN6_SAL
1	0.0739	0.0041	0.1775	-0.1162	-0.1184	-0.1264
2	0.0812	0.0204	0.1512	-0.0709	-0.1177	-0.1214
3	0.1133	0.0155	0.1733	-0.0994	-0.1215	-0.1622

4	0.0621	0.0121	0.2145	0.0199	-0.1355	-0.0815	
5	0.0277	0.0444	0.1338	0.0311	-0.0819	-0.0237	
6	-0.0075	0.0432	0.1250	0.0493	-0.0893	-0.0539	
	DMN6_SMN7	SAL	SAL_SMN7	SMN7	wave	date	Exclude
1	-0.0257	0.2097	0.0695	0.0352	1	2011-01-07	keep
2	-0.0342	0.2036	0.0453	0.0504	2	2011-12-19	keep
3	-0.0295	0.2024	0.0443	0.0135	3	2012-11-12	keep
4	-0.0998	0.2337	0.0552	0.1104	1	2011-10-03	keep
5	-0.0655	0.1229	0.0465	0.0761	2	2012-10-16	keep
6	-0.0655	0.1432	0.0072	0.1226	1	2011-10-27	keep
	RSNdata	RSNexclude	RSNexcludeDevDem	CogDate_0			
1	1	keep	keep	2006-12-08			
2	1	keep	keep	2006-12-08			
3	1	keep	keep	2006-12-08			
4	1	keep	exclude	2007-11-27			
5	1	keep	exclude	2007-11-27			
6	1	keep	keep	2008-04-17			
	CCIRtrio_MR_date_0	Dur_PDsx_0	PIBpos18	Neuro_Dx			
1	<NA>	NA	PIB-	iPD			
2	<NA>	NA	PIB-	iPD			
3	<NA>	NA	PIB-	iPD			
4	<NA>	NA	PIB-	iPD			
5	<NA>	NA	PIB-	iPD			
6	<NA>	NA	PIB-	HC			
	NeuroCDR_0	NeuroCDR_1	NeuroCDR_2	NeuroCDR_3	NeuroCDR_4		
1	PD CDR=0	PD CDR=.5	PD CDR=.5	PD CDR=0	PD CDR=.5		
2	PD CDR=0	PD CDR=.5	PD CDR=.5	PD CDR=0	PD CDR=.5		
3	PD CDR=0	PD CDR=.5	PD CDR=.5	PD CDR=0	PD CDR=.5		
4	PD CDR=0	PD CDR=0	<NA>	PD CDR=.5	PD CDR=.5		
5	PD CDR=0	PD CDR=0	<NA>	PD CDR=.5	PD CDR=.5		
6	HC CDR=0	<NA>	<NA>	<NA>	HC CDR=0		
	NeuroCDR_5	NeuroCDR_6	NeuroCDR_7	NeuroCDR_8	YOB	Sex	
1	PD CDR=.5	PD CDR=.5	PD CDR=.5	PD CDR=0.5	1937	female	
2	PD CDR=.5	PD CDR=.5	PD CDR=.5	PD CDR=0.5	1937	female	
3	PD CDR=.5	PD CDR=.5	PD CDR=.5	PD CDR=0.5	1937	female	
4	PD CDR=1	PD CDR=.5	<NA>	<NA>	1935	male	
5	PD CDR=1	PD CDR=.5	<NA>	<NA>	1935	male	
6	<NA>	HC CDR=0	<NA>	<NA>	1944	female	
	Ethnicity	Education	APOEs	APOE4	Orig_MCBP_30to60	Asyn	
1	caucasian	14	33	0	-0.084801180	NA	
2	caucasian	14	33	0	-0.084801180	NA	
3	caucasian	14	33	0	-0.084801180	NA	
4	caucasian	14	23	0	0.001760821	3356.47	
5	caucasian	14	23	0	0.001760821	3356.47	
6	caucasian	12	33	0	0.105326328	NA	
	Abeta	Total_Tau	week	Cog_rest_datediff			
1	NA	NA	0.0000000	NA			
2	NA	NA	0.9479452	NA			
3	NA	NA	1.8493151	NA			
4	1273.076	357.214	0.0000000	NA			
5	1273.076	357.214	1.0383562	NA			
6	NA	NA	0.0000000	NA			

In the CELL data, the DV is TimeJudgmentDistance, and the IV is Days. The question we're trying to ask is, whether the estimate of the month of the email is farther or closer to the actual month depending on how many days have passed since the email was written. In the resting state data, we will use DMN6 as a dependent variable, and time as the independent variable, and the research question is whether DMN6 increases or decreases as a function of time.

2 Linear Model for All Subjects

We first run a linear model for all the subjects:

CELL data

```
> library(lme4)
> cell_lm1 = lm(data = cell, TimeJudgmentDistance ~ Days)
> summary(cell_lm1)
```

Call:
`lm(formula = TimeJudgmentDistance ~ Days, data = cell)`

Residuals:

Min	1Q	Median	3Q	Max
-3.4125	-1.4466	-0.7086	0.3640	10.3797

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.6125055	0.1550466	3.950	8.45e-05 ***
Days	0.0078212	0.0008296	9.428	< 2e-16 ***

Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.557 on 845 degrees of freedom
Multiple R-squared: 0.09518, Adjusted R-squared: 0.09411
F-statistic: 88.89 on 1 and 845 DF, p-value: < 2.2e-16

The average intercept is 0.6125 and indicates the distance between the actual month and the participant's estimate at day 0, i.e. if the email was sent on the day of the test, the estimate is 0.6125 months off. The average slope is 0.0078, and indicates that for every 1-day change, the distance will increase by 0.0078 months. This is a weak but positive association.

Sample Data

```
> sample_lm1 = lm(data = alda_sample, DMN6 ~ time)
> summary(sample_lm1)
```

Call:
`lm(formula = DMN6 ~ time, data = alda_sample)`

Residuals:

Min	1Q	Median	3Q	Max
-0.20666	-0.07126	-0.01532	0.05281	0.32706

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.2171926	0.0092565	23.464	<2e-16 ***
time	-0.0003761	0.0036769	-0.102	0.919

Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.09631 on 223 degrees of freedom

Multiple R-squared: 4.691e-05, Adjusted R-squared: -0.004437
F-statistic: 0.01046 on 1 and 223 DF, p-value: 0.9186

The average intercept is 0.2172 and indicates the value of the DMN6 measure and time 0. The average slope is -0.00037, and indicates that for every 1-year change in time, the value of DMN6 decreases by 0.0003 units. This is a very weak negative association.

3 MLM with Random Intercept

CELL Data

```
> cell_mlm1 = lmer(data = cell, TimeJudgmentDistance ~ 1 + (1|ID))
> summary(cell_mlm1)
```

Linear mixed model fit by REML ['lmerMod']
Formula: TimeJudgmentDistance ~ 1 + (1 | ID)
Data: cell

REML criterion at convergence: 4076.6

Scaled residuals:

Min	1Q	Median	3Q	Max
-0.8575	-0.5833	-0.2994	0.0677	4.2154

Random effects:

Groups	Name	Variance	Std.Dev.
ID	(Intercept)	0.1551	0.3939
Residual		7.0646	2.6579

Number of obs: 847, groups: ID, 44

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	1.8150	0.1098	16.52

Note that the intercept-only model only takes into account the differences across subjects. We can also calculate the ICC for this model:

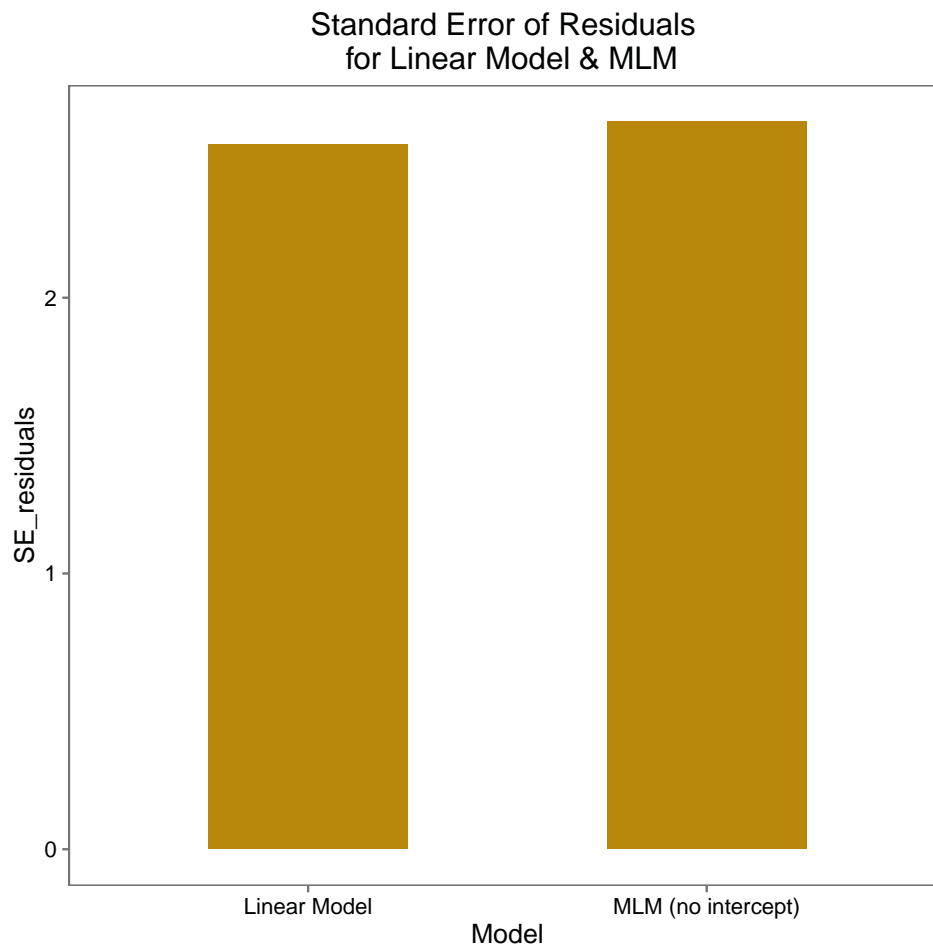
```
> cell_ICC = 0.1551/(0.1551+7.0646)
> print(cell_ICC)
```

```
[1] 0.02148289
```

Below, we compare the residual standard deviation from MLM1 to the residual standard error from the linear model.

```
> library(broom)
> library(ggplot2)
> library(ggthemes)
> cell_mlm1_fitted = augment(cell_mlm1, cell)
> cell_lm_fitted = augment(cell_lm1, cell)
> cell_residual_plot = matrix(nrow = 2, ncol = 2)
> colnames(cell_residual_plot) = c("Model", "SE_residuals")
> cell_residual_plot = as.data.frame(cell_residual_plot)
```

```
> cell_residual_plot$Model = c("Linear Model", "MLM (no intercept)")
> cell_residual_plot$SE_residuals = c(sd(cell_lm_fitted$resid), sd(cell_mlm1_fitted$resid))
> ggplot(cell_residual_plot, aes(x = Model, y = SE_residuals)) +
+   geom_bar(fill = "darkgoldenrod", stat = "identity", width = 0.5) +
+   theme_few()+
+   ggtitle("Standard Error of Residuals \n for Linear Model & MLM")
```



Note that the the residual SE is slightly larger in MLM1 (i.e. the intercept-only model), which means that there is more unexplained variance in the MLM than in the linear model.

Sample Data

```
> sample_mlm1 = lmer(data = alda_sample, DMN6 ~ 1 + (1|ID))
> summary(sample_mlm1)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: DMN6 ~ 1 + (1 | ID)
Data: alda_sample
```

```
REML criterion at convergence: -514.7
```

```
Scaled residuals:
```

```
      Min       1Q   Median       3Q      Max
-2.83141 -0.53002 -0.05154  0.45287  2.99901
```

```
Random effects:
```

Groups	Name	Variance	Std.Dev.
ID	(Intercept)	0.005944	0.07710
Residual		0.002759	0.05253

Number of obs: 225, groups: ID, 91

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	0.213057	0.008841	24.1

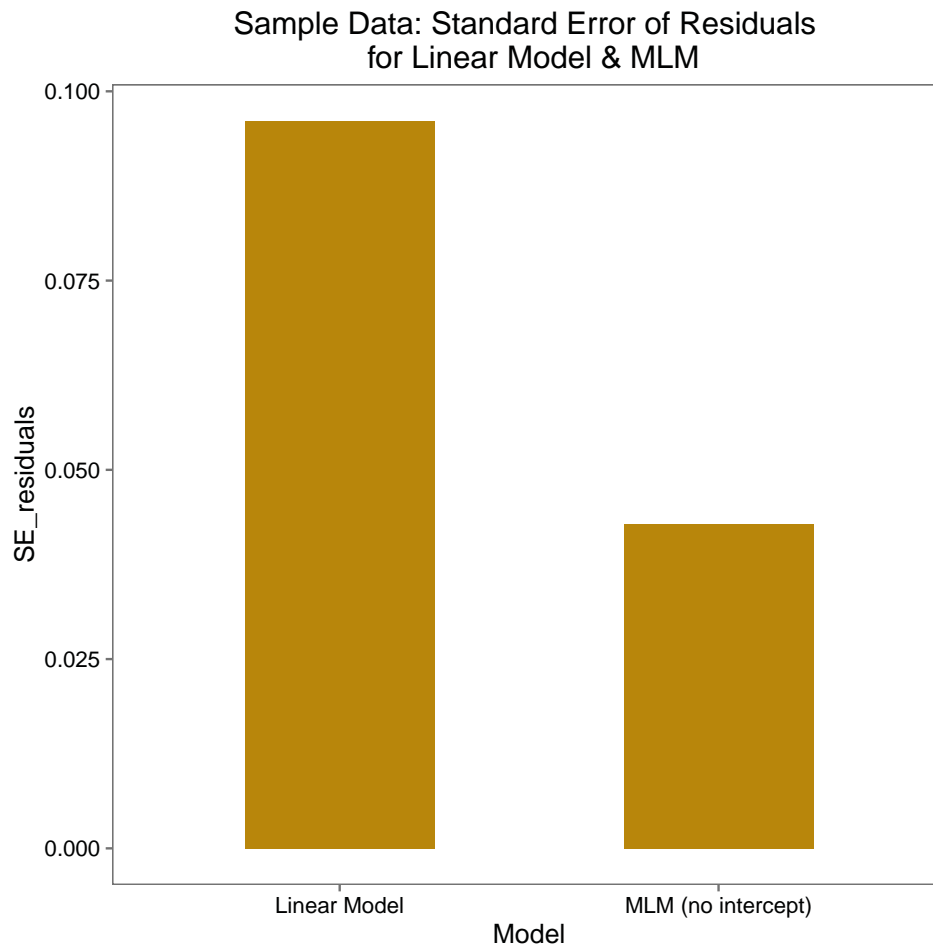
The ICC for this model is:

```
> sample_ICC = 0.005944/(0.005944 + 0.002759)
> print(sample_ICC)
```

```
[1] 0.6829829
```

Below, we compare the residual standard deviation from MLM1 to the residual standard error from the linear model.

```
> sample_mlm1_fitted = augment(sample_mlm1, alda_sample)
> sample_lm_fitted = augment(sample_lm1, alda_sample)
> sample_residual_plot = matrix(nrow = 2, ncol = 2)
> colnames(sample_residual_plot) = c("Model", "SE_residuals")
> sample_residual_plot = as.data.frame(sample_residual_plot)
> sample_residual_plot$Model = c("Linear Model", "MLM (no intercept)")
> sample_residual_plot$SE_residuals = c(sd(sample_lm_fitted$.resid), sd(sample_mlm1_fitted$.resid))
> ggplot(sample_residual_plot, aes(x = Model, y = SE_residuals)) +
+   geom_bar(fill = "darkgoldenrod", stat = "identity", width = 0.5) +
+   theme_few()+
+   ggtitle("Sample Data: Standard Error of Residuals \n for Linear Model & MLM")
```



In this case, the MLM residual standard error is much lower than the SE from the linear model.

4 Fixed Slope

CELL Data

We introduce a fixed slope term for Days in the model:

```
> cell_mlm2 = lmer(data = cell, TimeJudgmentDistance ~ Days + (1|ID))
> summary(cell_mlm2)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: TimeJudgmentDistance ~ Days + (1 | ID)
Data: cell
```

```
REML criterion at convergence: 4002.1
```

```
Scaled residuals:
```

Min	1Q	Median	3Q	Max
-1.4830	-0.5791	-0.2552	0.1900	4.2185

```
Random effects:
```

Groups	Name	Variance	Std.Dev.
ID	(Intercept)	0.1891	0.4349
Residual		6.3506	2.5200

```
Number of obs: 847, groups: ID, 44
```

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	0.6100832	0.1668470	3.657
Days	0.0078864	0.0008239	9.572

Correlation of Fixed Effects:

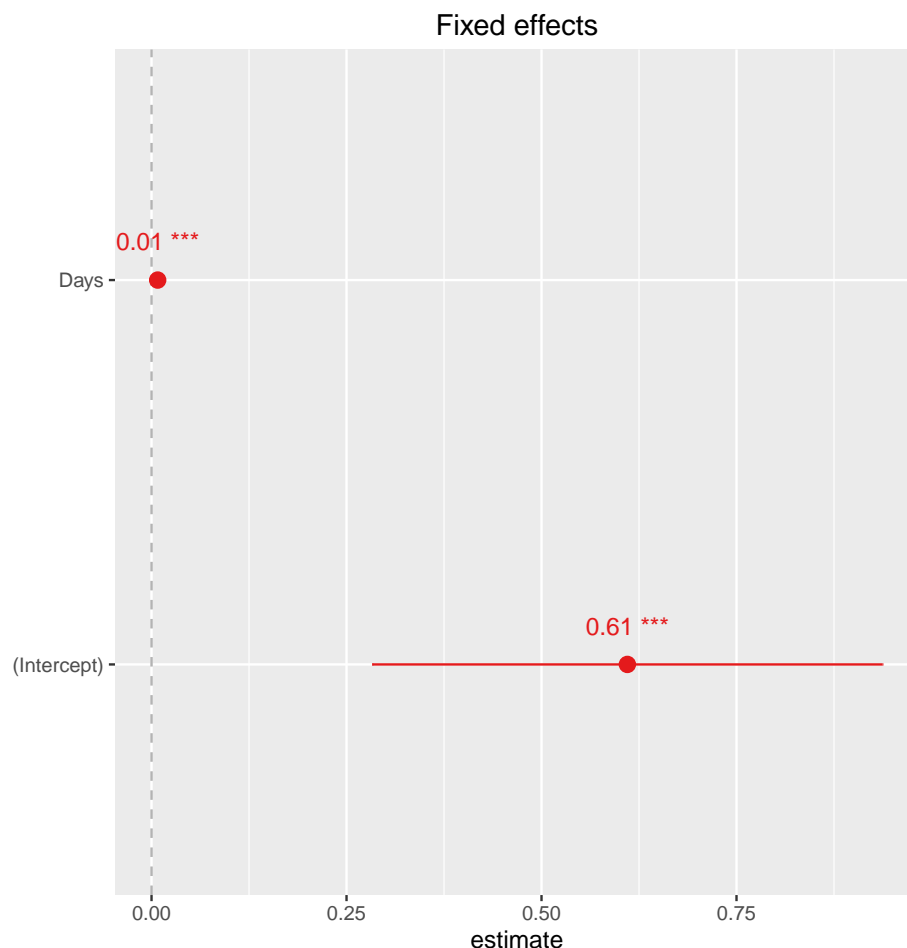
(Intr)
Days -0.754

Now, we have a fixed estimate of the change in TimeJudgmentDistance as a function of Days. This estimate is different from the previous estimate because now the regression line that we're fitting also has a slope. The intercept-only model fit parallel lines with slope 0 for each subject. This model fits lines with slope = 0.0078 for each subject. Note that the lines are still parallel, since this effect is fixed.

The residual standard error is now 6.35, in comparison to 7.06. Thus adding a predictor has reduced the residual standard error, or the unexplained variance in the model.

Below, we show the fixed effects estimates and the CIs around them using the sjPlot package (we can also use the confint function and manually make the plots):

```
> library(sjPlot)
> sjp.lmer(cell_mlm2, type = "fe")
```



Sample Data

```
> sample_mlm2 = lmer(data = alda_sample, DMN6 ~ time + (1|ID))
> summary(sample_mlm2)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: DMN6 ~ time + (1 | ID)
Data: alda_sample
```

```
REML criterion at convergence: -507.6
```

```
Scaled residuals:
```

Min	1Q	Median	3Q	Max
-2.7370	-0.4612	-0.0904	0.4127	3.1761

```
Random effects:
```

Groups	Name	Variance	Std.Dev.
ID	(Intercept)	0.006036	0.07769
Residual		0.002699	0.05195

```
Number of obs: 225, groups: ID, 91
```

```
Fixed effects:
```

	Estimate	Std. Error	t value
(Intercept)	0.220655	0.009857	22.385
time	-0.004297	0.002413	-1.781

```
Correlation of Fixed Effects:
```

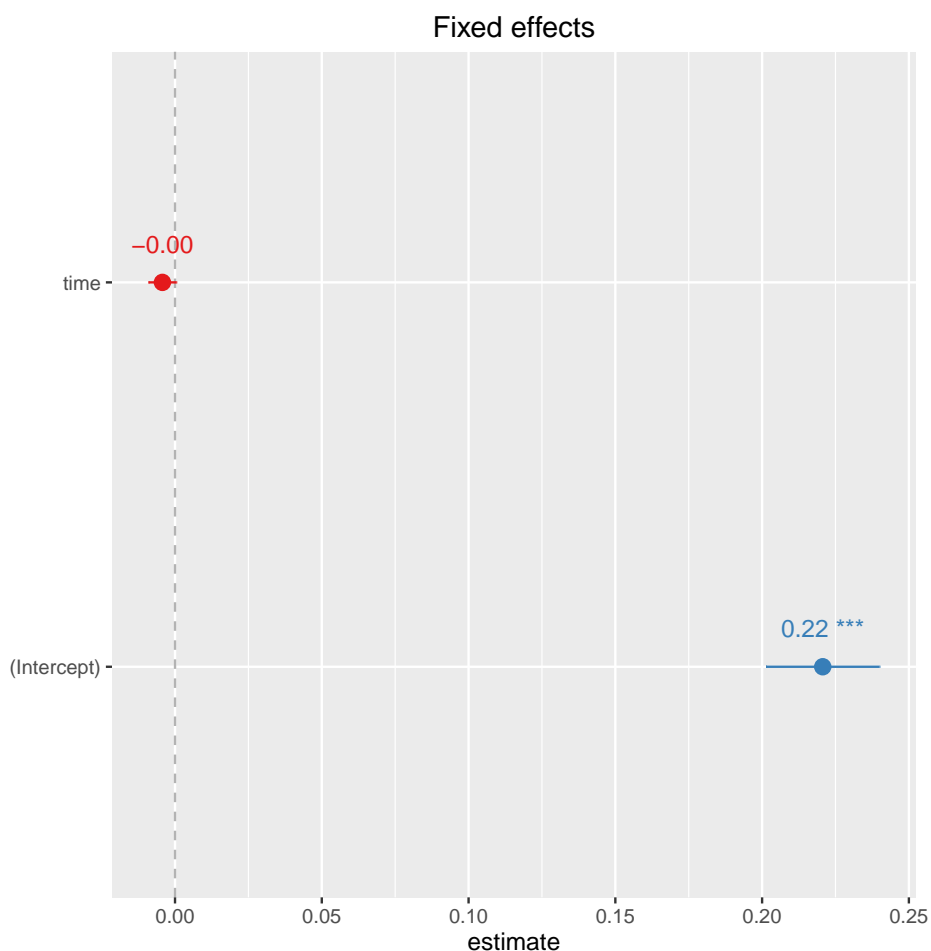
	(Intr)
time	-0.434

Just as in the previous case, this fixed effect estimate represents the slope of the regression line, and since there is only a fixed term for time and a random intercept for subject, each subject has the same slope but different intercepts i.e. the model produces a set of parallel lines with the slope = -0.00429.

The residual standard error is now 0.0026, and it was 0.0027 earlier. Thus, adding the new predictor has only slightly reduced the error variance.

Below, we plot the fixed effect:

```
> sjp.lmer(sample_mlm2, type = "fe")
```



5 Random Slope

CELL Data

We introduce a random slope term for Days in the model:

```
> cell_mlm3_1 = lmer(data = cell, TimeJudgmentDistance ~ Days + (Days|ID))
> summary(cell_mlm3_1)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: TimeJudgmentDistance ~ Days + (Days | ID)
Data: cell
```

```
REML criterion at convergence: 3966
```

```
Scaled residuals:
```

Min	1Q	Median	3Q	Max
-2.2453	-0.5440	-0.2062	0.1900	4.1650

```
Random effects:
```

Groups	Name	Variance	Std.Dev.	Corr
ID	(Intercept)	7.344e-02	0.270989	
	Days	3.124e-05	0.005589	-1.00
	Residual	5.877e+00	2.424338	

```
Number of obs: 847, groups: ID, 44
```

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	0.596585	0.153988	3.874
Days	0.008164	0.001184	6.898

Correlation of Fixed Effects:

(Intr)

Days -0.739

convergence code: 0

Model failed to converge with max|grad| = 0.013951 (tol = 0.002, component 1)

Model is nearly unidentifiable: very large eigenvalue

- Rescale variables?

> ##model fails to converge. we rescale Days.

>

> cell\$zDays = scale(cell\$Days, scale = TRUE, center = TRUE)

> cell_mlm3_2 = lmer(data = cell, TimeJudgmentDistance ~ zDays + (zDays|ID))

> summary(cell_mlm3_2)

Linear mixed model fit by REML ['lmerMod']

Formula: TimeJudgmentDistance ~ zDays + (zDays | ID)

Data: cell

REML criterion at convergence: 3956.7

Scaled residuals:

Min	1Q	Median	3Q	Max
-2.2453	-0.5440	-0.2062	0.1900	4.1650

Random effects:

Groups	Name	Variance	Std.Dev.	Corr
ID	(Intercept)	0.3478	0.5897	
	zDays	0.3507	0.5922	1.00
Residual		5.8774	2.4243	

Number of obs: 847, groups: ID, 44

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	1.8538	0.1242	14.927
zDays	0.8650	0.1254	6.898

Correlation of Fixed Effects:

(Intr)

zDays 0.551

Introducing the random slope term allows each subject to have a different slope and intercept. In comparison to the previous model the residual variance is lower and therefore, we can say that more variance is being explained by this model. Hence, we keep the random slope. We can also compare the two models to see if the second model is a better fit:

> anova(cell_mlm2, cell_mlm3_2)

Data: cell

Models:

```

cell_mlm2: TimeJudgmentDistance ~ Days + (1 | ID)
cell_mlm3_2: TimeJudgmentDistance ~ zDays + (zDays | ID)
      Df      AIC      BIC logLik deviance Chisq Chi Df
cell_mlm2    4 3995.1 4014.1 -1993.5   3987.1
cell_mlm3_2  6 3963.7 3992.1 -1975.8   3951.7 35.407    2
      Pr(>Chisq)
cell_mlm2
cell_mlm3_2 2.049e-08 ***
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

This test tells us that the mlm3 fits the data better.

Sample Data

```

> sample_mlm3 = lmer(data = alda_sample, DMN6 ~ time + (time|ID))
> summary(sample_mlm3)

```

```

Linear mixed model fit by REML ['lmerMod']
Formula: DMN6 ~ time + (time | ID)
Data: alda_sample

```

REML criterion at convergence: -512.8

Scaled residuals:

	Min	1Q	Median	3Q	Max
	-2.43325	-0.42316	-0.04733	0.34412	2.72929

Random effects:

Groups	Name	Variance	Std.Dev.	Corr
ID	(Intercept)	0.005864	0.07658	
	time	0.000180	0.01342	-0.09
Residual		0.002135	0.04620	

Number of obs: 225, groups: ID, 91

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	0.222121	0.009504	23.372
time	-0.005727	0.002721	-2.105

Correlation of Fixed Effects:

	(Intr)
time	-0.381

Just as before, the random slope term introduces a slope for each subject. The residual variance now is 0.002135, in comparison to 0.002699, which shows a slight decrease in the unexplained variance. We can also statistically test if this model is a better fit than the previous one:

```

> anova(sample_mlm2, sample_mlm3)

```

Data: alda_sample

Models:

sample_mlm2: DMN6 ~ time + (1 | ID)

sample_mlm3: DMN6 ~ time + (time | ID)

```

      Df      AIC      BIC logLik deviance  Chisq Chi Df
sample_mlm2  4 -517.47 -503.8 262.73  -525.47
sample_mlm3  6 -518.40 -497.9 265.20  -530.40 4.9342      2
      Pr(>Chisq)
sample_mlm2
sample_mlm3    0.08483 .
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

This shows that the new model is only a marginally better fit than the previous model. Thus, the random slopes do not add a lot, but we will still keep them.

6 Correlation between Intercept and Slope

The correlation between the slope and intercept tells us something about how a predictor might influence these variables, and essentially the regression line. For example, if the correlation between the slope and intercept is positive, then the predictor will move both in the same direction. On the other hand, if the slope and intercept have a negative correlation, then they will move in the opposite direction with the increase in the predictor.

For the CELL data, we see that the correlation between the slope and intercept is positive, i.e. the slope and intercept move in the same direction. For the sample data, the correlation is negative, i.e. the slope and intercept move in opposite directions.

7 Density Plot of Random Effects

Below is the density plot for the random effects:

```

> library(merTools)
> cell_re.sim = RESim(cell_mlm3_2)
> sample_re.sim = RESim(sample_mlm3)
> cell_fe.sim = FESim(cell_mlm3_2)
> sample_fe.sim = FESim(sample_mlm3)

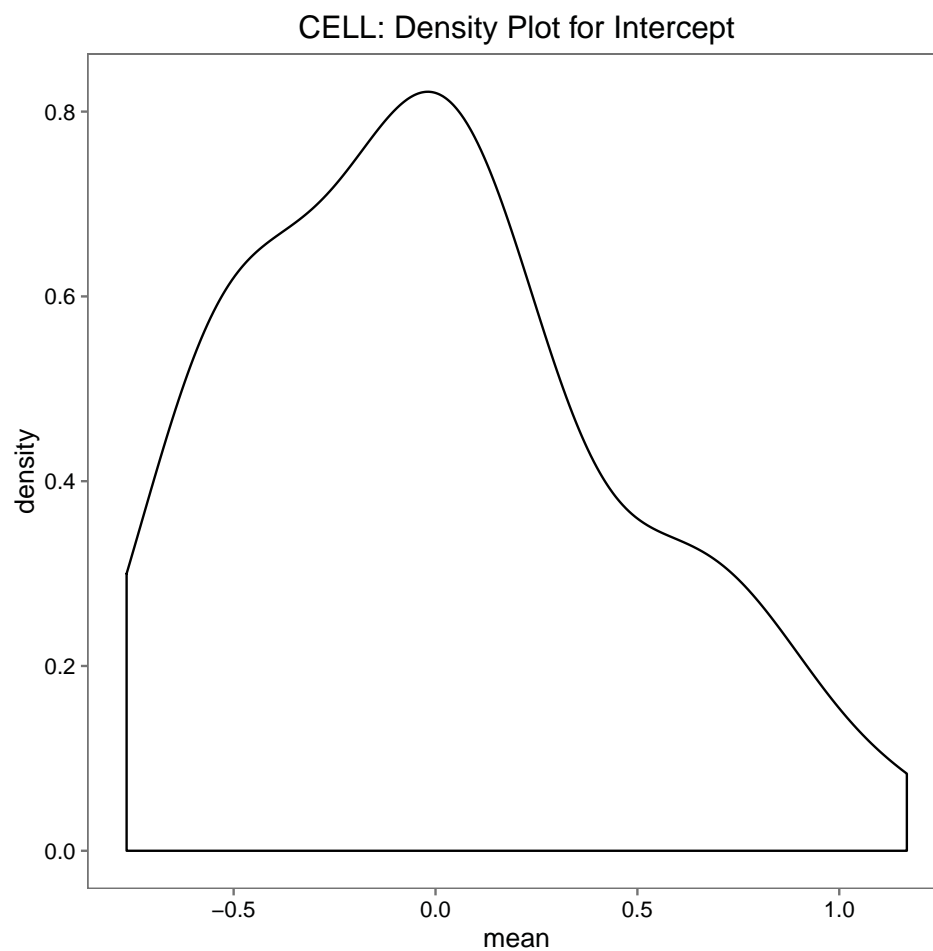
```

CELL data

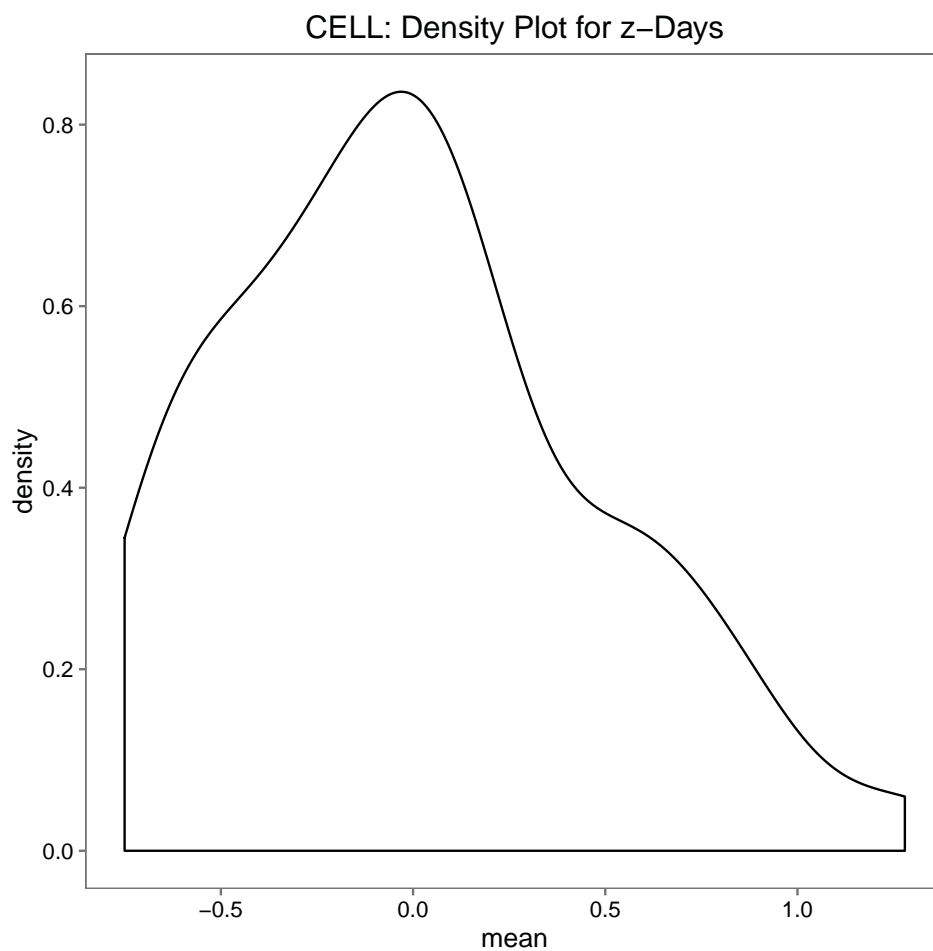
```

> library(ggplot2)
> cell_g1 <- cell_re.sim %>%
+   filter(term == "(Intercept)")
> ggplot(cell_g1, aes(mean)) +
+   geom_density() +
+   theme_few()+
+   ggtitle("CELL: Density Plot for Intercept")

```

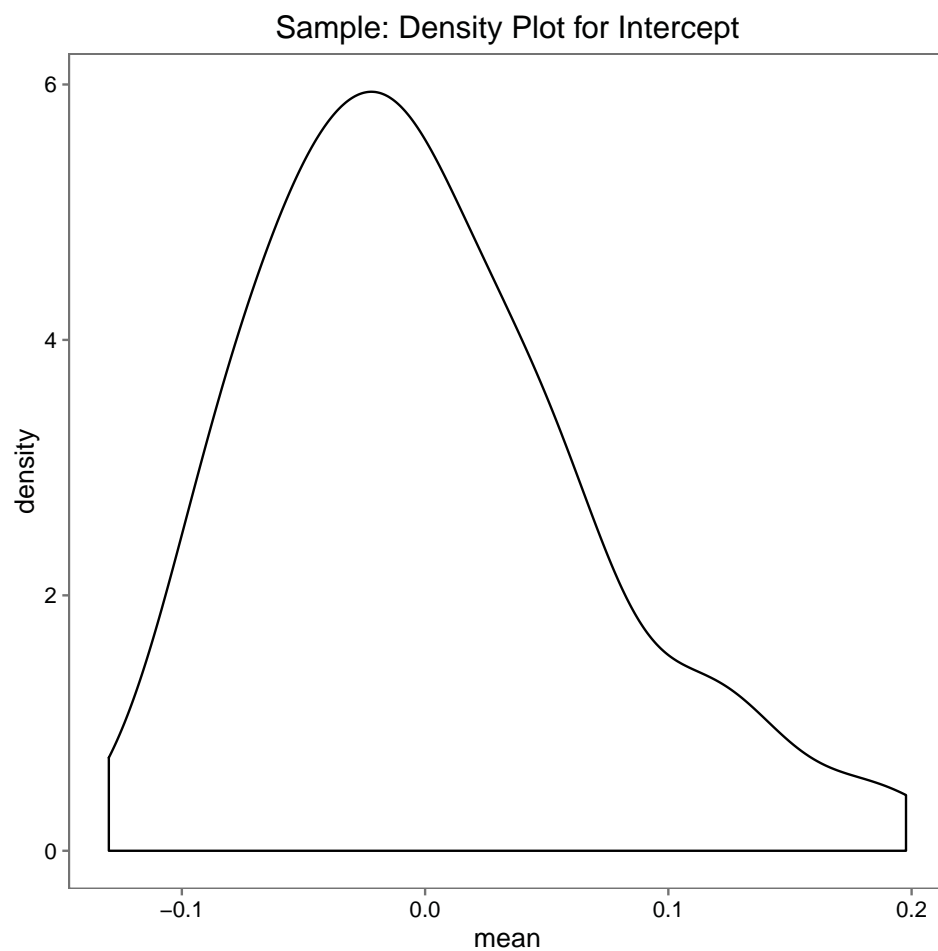


```
> cell_g2 <- cell_re.sim %>%  
+   filter(term == "zDays")  
> ggplot(cell_g2, aes(mean)) +  
+   geom_density() +  
+   theme_few()+  
+   ggtitle("CELL: Density Plot for z-Days")
```

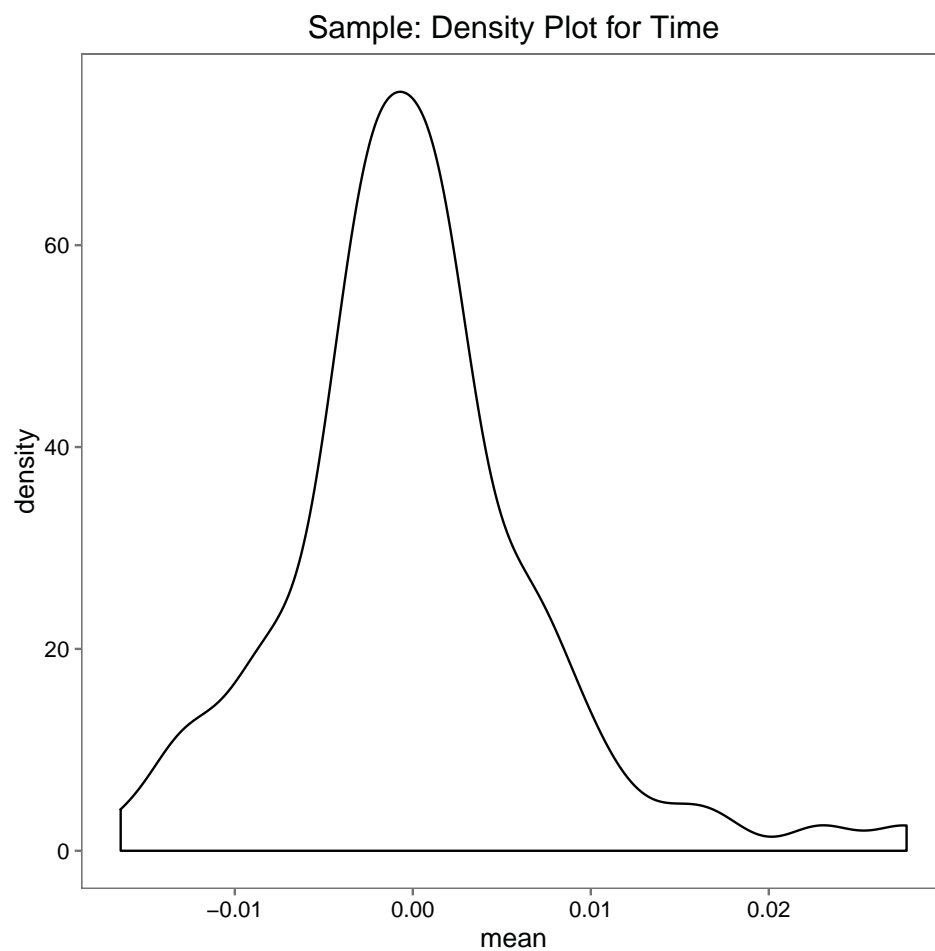


Sample data

```
> library(ggplot2)
> library(ggthemes)
> sample_g1 <- sample_re.sim %>%
+   filter(term == "(Intercept)")
> ggplot(sample_g1, aes(mean)) +
+   geom_density() +
+   theme_few() +
+   ggtitle("Sample: Density Plot for Intercept")
```



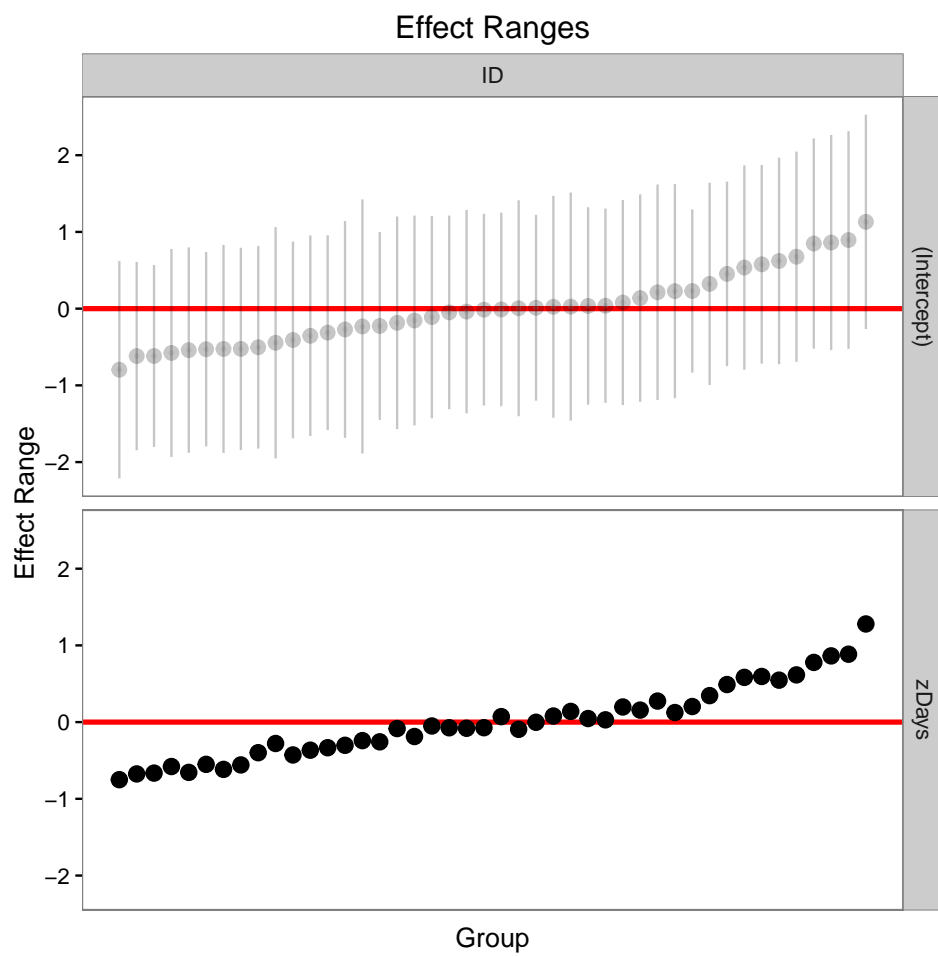
```
> sample_g2 <- sample_re.sim %>%  
+   filter(term == "time")  
> ggplot(sample_g2, aes(mean)) +  
+   geom_density() +  
+   theme_few()+  
+   ggtitle("Sample: Density Plot for Time")
```



8 Caterpillar Plot of Random Effects

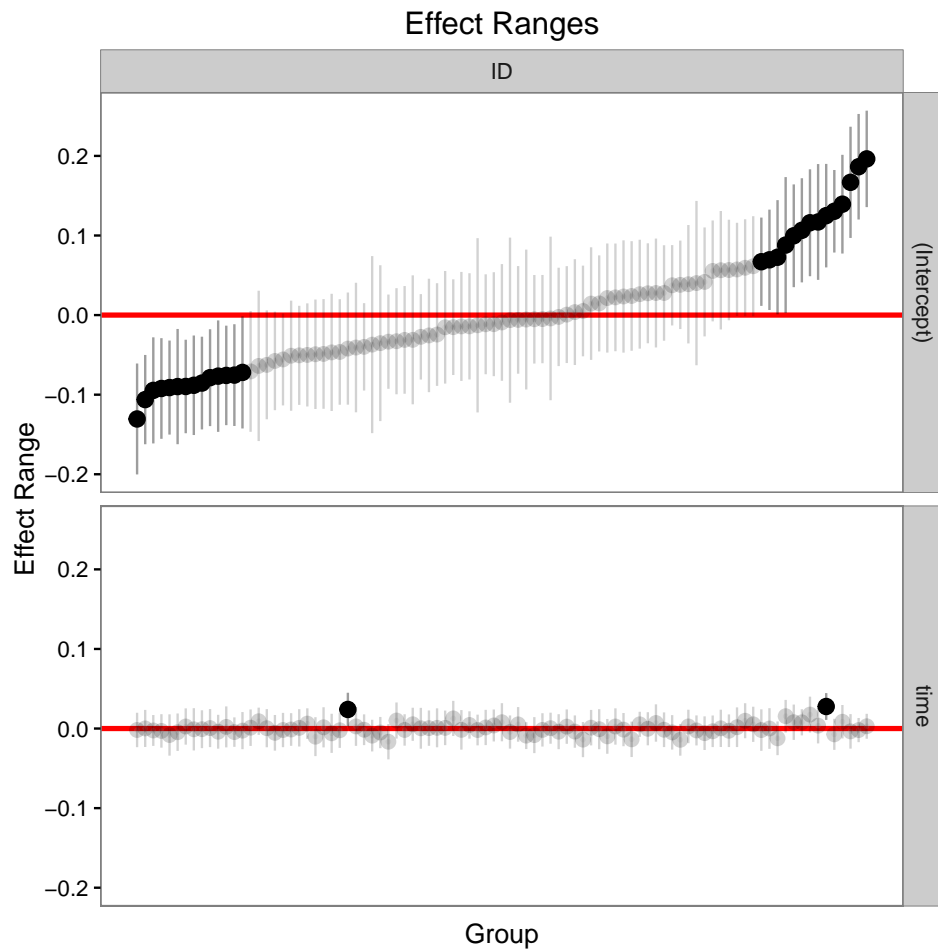
CELL data

```
> cell_p1 = plotREsim(cell_re.sim)
> cell_p1
```



Sample data

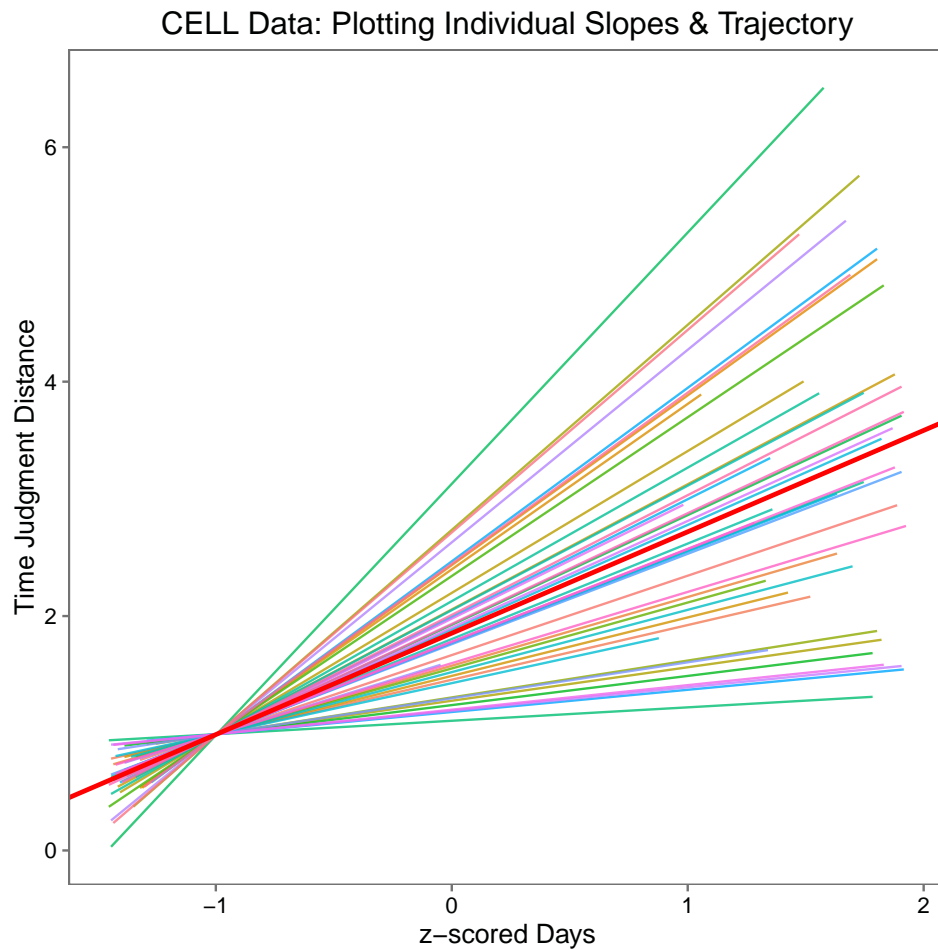
```
> sample_p1 = plotREsim(sample_re.sim)
> sample_p1
```



9 Plotting Trajectories

CELL data

```
> library(broom)
> cell_fittedvalues = augment(cell_mlm3_2, cell)
> cell_fittedvalues$ID = as.factor(cell_fittedvalues$ID)
> ggplot(cell_fittedvalues, aes(x = zDays, y = .fitted)) +
+   geom_line(aes(color = ID), show.legend = F, alpha = 0.8 ) +
+   geom_abline(slope = 0.8650, intercept = 1.8538,
+               color = "red", size = 1)+
+   theme_few()+
+   xlab("z-scored Days") + ylab("Time Judgment Distance") +
+   ggtitle("CELL Data: Plotting Individual Slopes & Trajectory")
```



Sample data

```
> library(broom)
> sample_fittedvalues = augment(sample_mlm3, alda_sample)
> sample_fittedvalues$ID = as.factor(sample_fittedvalues$ID)
> ggplot(sample_fittedvalues, aes(x = time, y = .fitted)) +
+   theme_few() +
+   geom_line(aes(color = ID), show.legend = F, alpha = 0.8 ) +
+   geom_abline(slope = -0.005727, intercept = 0.222121,
+               color = "red", size = 1) +
+   xlab("Time") + ylab("DMN6") +
+   ggtitle("Sample Data: Plotting Individual Slopes & Trajectory")
>
```

Sample Data: Plotting Individual Slopes & Trajectory

