

Assignment 6: SEM

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1 Reading the Data

I am going to use the class dataset for Questions 1 and 2, and the lavaan dataset, HolzingerSwineford1939 for the multiple group analyses.

```
> library(lavaan)
> head(HolzingerSwineford1939)
```

	id	sex	ageyr	agemo	school	grade	x1	x2	x3
1	1	1	13	1	Pasteur	7	3.333333	7.75	0.375
2	2	2	13	7	Pasteur	7	5.333333	5.25	2.125
3	3	2	13	1	Pasteur	7	4.500000	5.25	1.875
4	4	1	13	2	Pasteur	7	5.333333	7.75	3.000
5	5	2	12	2	Pasteur	7	4.833333	4.75	0.875
6	6	2	14	1	Pasteur	7	5.333333	5.00	2.250

	x4	x5	x6	x7	x8	x9
1	2.333333	5.75	1.2857143	3.391304	5.75	6.361111
2	1.666667	3.00	1.2857143	3.782609	6.25	7.916667
3	1.000000	1.75	0.4285714	3.260870	3.90	4.416667
4	2.666667	4.50	2.4285714	3.000000	5.30	4.861111
5	2.666667	4.00	2.5714286	3.695652	6.30	5.916667
6	1.000000	3.00	0.8571429	4.347826	6.65	7.500000

```
> sem_long = read.csv("SEM_long.csv", header = TRUE, sep = ",")
> head(sem_long)
```

	PosAFF11	PosAFF21	PosAFF31	NegAFF11	NegAFF21
1	3.407195916	3.161366	2.691232	2.4207020	2.3531140
2	2.594690943	2.682974	2.676173	1.6652403	0.8228416
3	3.168698274	2.854090	3.277873	1.5646086	1.6909398
4	4.600159845	4.698997	4.751315	0.9352668	0.7944903
5	1.911294244	2.184561	1.661057	1.6076395	1.5046016
6	3.472353729	4.016982	4.021543	1.9754108	1.1348223

	NegAFF31	PosAFF12	PosAFF22	PosAFF32	NegAFF12	NegAFF22
1	2.338767	3.608672	2.385396	3.029975	2.283468	2.3173734
2	1.083938	3.103346	2.666753	3.251786	1.651941	1.7019345
3	1.502032	2.327999	3.061529	3.272173	1.749022	1.6237575
4	0.205178	4.254750	3.792440	3.934392	1.279030	0.8741891
5	1.209850	3.971308	4.674580	4.093508	1.019751	1.0376248
6	1.729405	3.845904	4.186825	4.316398	1.189548	1.1866801

	NegAFF32	PosAFF13	PosAFF23	PosAFF33	NegAFF13	NegAFF23
1	2.3932485	3.561401	3.506353	3.862975	2.090484	1.3276333
2	1.3462416	2.893546	2.804488	2.637762	1.257376	1.6585273

```

3 1.5763265 2.705572 2.310391 2.841122 1.699849 1.7373582
4 1.1822674 3.212826 2.775791 3.239004 1.824778 1.3016450
5 0.7776716 3.487932 2.673107 3.093565 1.316975 0.7120284
6 1.3158520 3.236481 3.256453 4.023383 1.053895 1.4449087
  NegAFF33
1 1.6680488
2 1.4298172
3 1.8542867
4 1.5356285
5 0.7083461
6 0.9600753

```

2 Testing Measurement Invariance

Configural

```

> config <- '
+ ## define latent variables
+ PosAffect1 =~ PosAFF11 + PosAFF21 + PosAFF31
+ PosAffect2 =~ PosAFF12 + PosAFF22 + PosAFF32
+ PosAffect3 =~ PosAFF13 + PosAFF23 + PosAFF33
+
+
+ ## correlated residuals across time
+ PosAFF11 ~~ PosAFF12 + PosAFF13
+ PosAFF12 ~~ PosAFF13
+ PosAFF21 ~~ PosAFF22 + PosAFF23
+ PosAFF22 ~~ PosAFF23
+ PosAFF31 ~~ PosAFF32 + PosAFF33
+ PosAFF32 ~~ PosAFF33
+ '
> config <- cfa(config, data=sem_long, meanstructure=TRUE, std.lv=TRUE)
> summary(config, standardized=TRUE, fit.measures=TRUE)

```

lavaan (0.5-23.1097) converged normally after 106 iterations

	Used	Total
Number of observations	368	369
Estimator	ML	
Minimum Function Test Statistic	8.366	
Degrees of freedom	15	
P-value (Chi-square)	0.908	

Model test baseline model:

Minimum Function Test Statistic	2617.825
Degrees of freedom	36
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	1.000
Tucker-Lewis Index (TLI)	1.006

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	NA
Loglikelihood unrestricted model (H1)	NA
Number of free parameters	39
Akaike (AIC)	NA
Bayesian (BIC)	NA

Root Mean Square Error of Approximation:

RMSEA		0.000
90 Percent Confidence Interval	0.000	0.020
P-value RMSEA <= 0.05		0.998

Standardized Root Mean Square Residual:

SRMR	0.011
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
PosAffect1 =~				
PosAFF11	92.657	4.503	20.579	0.000
PosAFF21	0.652	0.031	20.723	0.000
PosAFF31	0.681	0.031	21.954	0.000
PosAffect2 =~				
PosAFF12	0.554	0.026	21.131	0.000
PosAFF22	0.643	0.030	21.657	0.000
PosAFF32	0.642	0.027	23.351	0.000
PosAffect3 =~				
PosAFF13	0.508	0.024	21.023	0.000
PosAFF23	0.546	0.027	20.364	0.000
PosAFF33	0.538	0.026	20.777	0.000
Std.lv Std.all				
	92.657	0.872		
	0.652	0.876		
	0.681	0.907		
	0.554	0.881		
	0.643	0.894		
	0.642	0.936		
	0.508	0.887		
	0.546	0.868		
	0.538	0.879		

Covariances:

	Estimate	Std.Err	z-value	P(> z)
.PosAFF11 ~~				
.PosAFF12	0.235	1.092	0.215	0.830
.PosAFF13	0.501	1.048	0.478	0.633
.PosAFF12 ~~				
.PosAFF13	0.003	0.006	0.528	0.597

.PosAFF21 ~~				
.PosAFF22	0.006	0.008	0.653	0.514
.PosAFF23	0.005	0.008	0.618	0.537
.PosAFF22 ~~				
.PosAFF23	0.011	0.007	1.490	0.136
.PosAFF31 ~~				
.PosAFF32	0.008	0.007	1.059	0.290
.PosAFF33	0.016	0.008	2.098	0.036
.PosAFF32 ~~				
.PosAFF33	0.005	0.006	0.777	0.437
PosAffect1 ~~				
PosAffect2	0.479	0.044	10.783	0.000
PosAffect3	0.408	0.048	8.451	0.000
PosAffect2 ~~				
PosAffect3	0.451	0.046	9.831	0.000
Std.lv Std.all				
0.235	0.015			
0.501	0.036			
0.003	0.039			
0.006	0.048			
0.005	0.045			
0.011	0.108			
0.008	0.100			
0.016	0.173			
0.005	0.068			
0.479	0.479			
0.408	0.408			
0.451	0.451			

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.PosAFF11	185.500	5.539	33.488	0.000
.PosAFF21	3.105	0.039	80.038	0.000
.PosAFF31	3.248	0.039	82.950	0.000
.PosAFF12	3.253	0.033	99.150	0.000
.PosAFF22	3.126	0.038	83.334	0.000
.PosAFF32	3.274	0.036	91.558	0.000
.PosAFF13	3.302	0.030	110.547	0.000
.PosAFF23	3.094	0.033	94.373	0.000
.PosAFF33	3.280	0.032	102.804	0.000
PosAffect1	0.000			
PosAffect2	0.000			
PosAffect3	0.000			
Std.lv Std.all				
185.500	1.746			
3.105	4.172			
3.248	4.324			
3.253	5.169			
3.126	4.344			
3.274	4.773			

3.302	5.763
3.094	4.920
3.280	5.359
0.000	0.000
0.000	0.000
0.000	0.000

Variances:

	Estimate	Std.Err	z-value	P(> z)
.PosAFF11	2705.921	288.582	9.377	0.000
.PosAFF21	0.129	0.014	9.222	0.000
.PosAFF31	0.100	0.013	7.514	0.000
.PosAFF12	0.089	0.009	9.956	0.000
.PosAFF22	0.104	0.011	9.351	0.000
.PosAFF32	0.058	0.009	6.464	0.000
.PosAFF13	0.070	0.008	8.256	0.000
.PosAFF23	0.098	0.011	9.207	0.000
.PosAFF33	0.085	0.010	8.700	0.000
PosAffect1	1.000			
PosAffect2	1.000			
PosAffect3	1.000			
Std.lv	Std.all			
2705.921	0.240			
0.129	0.233			
0.100	0.178			
0.089	0.224			
0.104	0.201			
0.058	0.124			
0.070	0.213			
0.098	0.247			
0.085	0.228			
1.000	1.000			
1.000	1.000			
1.000	1.000			

Weak: Constrain Loadings

```
> weak <- '
+ ## define latent variables
+ PosAffect1 =~ L1*PosAFF11 + L2*PosAFF21 + L3*PosAFF31
+ PosAffect2 =~ L1*PosAFF12 + L2*PosAFF22 + L3*PosAFF32
+ PosAffect3 =~ L1*PosAFF13 + L2*PosAFF23 + L3*PosAFF33
+
+
+ ## free latent variances at later times (only set the scale once)
+ PosAffect2 ~~ NA*PosAffect2
+ PosAffect3 ~~ NA*PosAffect3
+
+ ## correlated residuals across time
+ PosAFF11 ~~ PosAFF12 + PosAFF13
+ PosAFF12 ~~ PosAFF13
+ PosAFF21 ~~ PosAFF22 + PosAFF23
+ PosAFF22 ~~ PosAFF23
+ PosAFF31 ~~ PosAFF32 + PosAFF33
+ PosAFF32 ~~ PosAFF33
+
+
+
```

```
+ '
> weak <- cfa(weak, data=sem_long, meanstructure=TRUE, std.lv=TRUE)
> summary(weak, standardized=TRUE, fit.measures=TRUE)
```

lavaan (0.5-23.1097) converged normally after 89 iterations

	Used	Total
Number of observations	368	369
Estimator	ML	
Minimum Function Test Statistic	425.559	
Degrees of freedom	19	
P-value (Chi-square)	0.000	

Model test baseline model:

Minimum Function Test Statistic	2617.825
Degrees of freedom	36
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.843
Tucker-Lewis Index (TLI)	0.702

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	NA
Loglikelihood unrestricted model (H1)	NA
Number of free parameters	35
Akaike (AIC)	NA
Bayesian (BIC)	NA

Root Mean Square Error of Approximation:

RMSEA	0.241
90 Percent Confidence Interval	0.222 0.261
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.186
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Parameter Estimates:

Information	Expected
Standard Errors	Standard

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
PosAffect1 =~				
PosAFF11 (L1)	0.591	0.029	20.353	0.000
PosAFF21 (L2)	0.667	0.029	22.740	0.000
PosAFF31 (L3)	0.666	0.030	22.490	0.000
PosAffect2 =~				
PosAFF12 (L1)	0.591	0.029	20.353	0.000

PosAFF22	(L2)	0.667	0.029	22.740	0.000
PosAFF32	(L3)	0.666	0.030	22.490	0.000
PosAffect3 =~					
PosAFF13	(L1)	0.591	0.029	20.353	0.000
PosAFF23	(L2)	0.667	0.029	22.740	0.000
PosAFF33	(L3)	0.666	0.030	22.490	0.000
Std.lv	Std.all				

0.591	0.006
0.667	0.891
0.666	0.890

0.564	0.886
0.636	0.892
0.636	0.932

0.487	0.872
0.549	0.870
0.549	0.886

Covariances:

	Estimate	Std.Err	z-value	P(> z)
.PosAFF11 ~~				
.PosAFF12	1.235	1.862	0.663	0.507
.PosAFF13	1.533	1.781	0.861	0.389
.PosAFF12 ~~				
.PosAFF13	0.003	0.006	0.525	0.600
.PosAFF21 ~~				
.PosAFF22	-0.001	0.010	-0.157	0.876
.PosAFF23	0.004	0.009	0.393	0.694
.PosAFF22 ~~				
.PosAFF23	0.011	0.007	1.451	0.147
.PosAFF31 ~~				
.PosAFF32	0.013	0.008	1.535	0.125
.PosAFF33	0.020	0.009	2.241	0.025
.PosAFF32 ~~				
.PosAFF33	0.005	0.006	0.818	0.414
PosAffect1 ~~				
PosAffect2	0.463	0.050	9.345	0.000
PosAffect3	0.328	0.045	7.296	0.000
PosAffect2 ~~				
PosAffect3	0.352	0.051	6.875	0.000
Std.lv	Std.all			

1.235	0.040
1.533	0.053

0.003	0.038
-------	-------

-0.001	-0.014
0.004	0.034

0.011	0.107
-------	-------

0.013	0.154
0.020	0.202

0.005	0.072
-------	-------

0.485	0.485
0.398	0.398
0.448	0.448

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.PosAFF11	185.500	5.514	33.642	0.000
.PosAFF21	3.105	0.039	79.606	0.000
.PosAFF31	3.248	0.039	83.222	0.000
.PosAFF12	3.253	0.033	97.976	0.000
.PosAFF22	3.126	0.037	84.067	0.000
.PosAFF32	3.274	0.036	92.051	0.000
.PosAFF13	3.302	0.029	113.457	0.000
.PosAFF23	3.094	0.033	94.047	0.000
.PosAFF33	3.280	0.032	101.557	0.000
PosAffect1	0.000			
PosAffect2	0.000			
PosAffect3	0.000			
Std.lv	Std.all			
185.500	1.754			
3.105	4.150			
3.248	4.338			
3.253	5.107			
3.126	4.382			
3.274	4.799			
3.302	5.914			
3.094	4.903			
3.280	5.294			
0.000	0.000			
0.000	0.000			
0.000	0.000			

Variances:

	Estimate	Std.Err	z-value	P(> z)
PosAffect2	0.912	0.095	9.612	0.000
PosAffect3	0.679	0.074	9.113	0.000
.PosAFF11	11188.266	824.813	13.565	0.000
.PosAFF21	0.115	0.020	5.647	0.000
.PosAFF31	0.117	0.020	5.696	0.000
.PosAFF12	0.087	0.009	9.870	0.000
.PosAFF22	0.104	0.011	9.527	0.000
.PosAFF32	0.061	0.009	6.935	0.000
.PosAFF13	0.075	0.008	9.201	0.000
.PosAFF23	0.097	0.010	9.292	0.000
.PosAFF33	0.082	0.010	8.568	0.000
PosAffect1	1.000			
Std.lv	Std.all			
1.000	1.000			
1.000	1.000			
11188.266	1.000			
0.115	0.206			
0.117	0.208			
0.087	0.215			
0.104	0.204			
0.061	0.131			
0.075	0.239			

	0.097	0.243
	0.082	0.215
	1.000	1.000

```

>

```

```

> anova(config, weak)

```

Chi Square Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff	Pr(>Chisq)
config	15			8.3664			
weak	19		425.5595	417.19		4	< 2.2e-16

config
weak ***

Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Strong: Constrain Intercepts, Slopes

```

> strong <- '
+ ## define latent variables
+ Pos1 =~ L1*PosAFF11 + L2*PosAFF21 + L3*PosAFF31
+ Pos2 =~ L1*PosAFF12 + L2*PosAFF22 + L3*PosAFF32
+ Pos3 =~ L1*PosAFF13 + L2*PosAFF23 + L3*PosAFF33
+
+
+ ## free latent variances at later times (only set the scale once)
+ Pos2 ~~ NA*Pos2
+ Pos3 ~~ NA*Pos3
+
+
+ ## correlated residuals across time
+ PosAFF11 ~~ PosAFF12 + PosAFF13
+ PosAFF12 ~~ PosAFF13
+ PosAFF21 ~~ PosAFF22 + PosAFF23
+ PosAFF22 ~~ PosAFF23
+ PosAFF31 ~~ PosAFF32 + PosAFF33
+ PosAFF32 ~~ PosAFF33
+
+
+ ## constrain intercepts across time
+ PosAFF11 ~ t1*1
+ PosAFF21 ~ t2*1
+ PosAFF31 ~ t3*1
+
+
+ PosAFF12 ~ t1*1
+ PosAFF22 ~ t2*1
+ PosAFF32 ~ t3*1
+
+
+ PosAFF13 ~ t1*1

```

```

+ PosAFF23 ~ t2*1
+ PosAFF33 ~ t3*1
+
+
+ ## free latent means at later times (only set the scale once)
+ Pos2 ~ NA*1
+ Pos3 ~ NA*1'
> strong <- cfa(strong, data=sem_long, meanstructure=TRUE, std.lv=TRUE)
> summary(strong, standardized=TRUE, fit.measures=TRUE)

```

lavaan (0.5-23.1097) converged normally after 165 iterations

	Used	Total
Number of observations	368	369
Estimator	ML	
Minimum Function Test Statistic	936.404	
Degrees of freedom	23	
P-value (Chi-square)	0.000	

Model test baseline model:

Minimum Function Test Statistic	2617.825
Degrees of freedom	36
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.646
Tucker-Lewis Index (TLI)	0.446

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	NA
Loglikelihood unrestricted model (H1)	NA
Number of free parameters	31
Akaike (AIC)	NA
Bayesian (BIC)	NA

Root Mean Square Error of Approximation:

RMSEA	0.329
90 Percent Confidence Interval	0.311 0.347
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.498
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Parameter Estimates:

Information	Expected
Standard Errors	Standard

Latent Variables:

Estimate	Std.Err	z-value	P(> z)
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```

Pos1 =~
  PosAFF11 (L1)    0.591    0.029    20.342    0.000
  PosAFF21 (L2)    0.666    0.029    22.725    0.000
  PosAFF31 (L3)    0.666    0.030    22.484    0.000
Pos2 =~
  PosAFF12 (L1)    0.591    0.029    20.342    0.000
  PosAFF22 (L2)    0.666    0.029    22.725    0.000
  PosAFF32 (L3)    0.666    0.030    22.484    0.000
Pos3 =~
  PosAFF13 (L1)    0.591    0.029    20.342    0.000
  PosAFF23 (L2)    0.666    0.029    22.725    0.000
  PosAFF33 (L3)    0.666    0.030    22.484    0.000
Std.lv   Std.all

    0.591    0.003
    0.666    0.890
    0.666    0.890

    0.566    0.887
    0.638    0.892
    0.637    0.933

    0.486    0.869
    0.548    0.869
    0.547    0.885

```

Covariances:

	Estimate	Std.Err	z-value	P(> z)
.PosAFF11 ~~				
.PosAFF12	-0.427	3.709	-0.115	0.908
.PosAFF13	10.469	3.625	2.888	0.004
.PosAFF12 ~~				
.PosAFF13	0.003	0.006	0.452	0.651
.PosAFF21 ~~				
.PosAFF22	-0.001	0.010	-0.149	0.881
.PosAFF23	0.004	0.009	0.398	0.691
.PosAFF22 ~~				
.PosAFF23	0.010	0.007	1.396	0.163
.PosAFF31 ~~				
.PosAFF32	0.012	0.008	1.438	0.150
.PosAFF33	0.020	0.009	2.299	0.021
.PosAFF32 ~~				
.PosAFF33	0.005	0.006	0.817	0.414
Pos1 ~~				
Pos2	0.469	0.050	9.444	0.000
Pos3	0.323	0.045	7.183	0.000
Pos2 ~~				
Pos3	0.352	0.051	6.866	0.000
Std.lv Std.all				
	-0.427	-0.007		
	10.469	0.179		
	0.003	0.033		
	-0.001	-0.013		
	0.004	0.034		

0.010	0.103
0.012	0.145
0.020	0.207
0.005	0.072
0.490	0.490
0.393	0.393
0.447	0.447

Intercepts:

		Estimate	Std.Err	z-value	P(> z)
.PosAFF11	(t1)	3.242	0.035	92.993	0.000
.PosAFF21	(t2)	3.096	0.037	82.836	0.000
.PosAFF31	(t3)	3.256	0.038	86.197	0.000
.PosAFF12	(t1)	3.242	0.035	92.993	0.000
.PosAFF22	(t2)	3.096	0.037	82.836	0.000
.PosAFF32	(t3)	3.256	0.038	86.197	0.000
.PosAFF13	(t1)	3.242	0.035	92.993	0.000
.PosAFF23	(t2)	3.096	0.037	82.836	0.000
.PosAFF33	(t3)	3.256	0.038	86.197	0.000
Pos2		0.031	0.056	0.551	0.581
Pos3		0.022	0.058	0.382	0.702
Pos1		0.000			
Std.lv	Std.all				
		3.242	0.015		
		3.096	4.140		
		3.256	4.351		
		3.242	5.077		
		3.096	4.335		
		3.256	4.765		
		3.242	5.793		
		3.096	4.912		
		3.256	5.266		
		0.032	0.032		
		0.027	0.027		
		0.000	0.000		

Variances:

		Estimate	Std.Err	z-value	P(> z)
Pos2		0.917	0.095	9.628	0.000
Pos3		0.676	0.074	9.088	0.000
.PosAFF11		44406.486	3273.692	13.565	0.000
.PosAFF21		0.116	0.020	5.677	0.000
.PosAFF31		0.117	0.020	5.700	0.000
.PosAFF12		0.087	0.009	9.861	0.000
.PosAFF22		0.104	0.011	9.540	0.000
.PosAFF32		0.061	0.009	6.925	0.000
.PosAFF13		0.077	0.008	9.274	0.000
.PosAFF23		0.098	0.010	9.327	0.000
.PosAFF33		0.083	0.010	8.571	0.000
Pos1		1.000			
Std.lv	Std.all				
		1.000	1.000		
		1.000	1.000		
44406.486		1.000			

0.116	0.207
0.117	0.208
0.087	0.214
0.104	0.203
0.061	0.130
0.077	0.245
0.098	0.246
0.083	0.216
1.000	1.000

```
> anova(weak, strong)
```

Chi Square Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff	Pr(>Chisq)
weak	19			425.56			
strong	23		936.40	510.85		4	< 2.2e-16 ***

Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Yes, there is strong measurement invariance and we can run growth models.

3 Second Order Growth Model

```
> sec.order <- '
+ ## define latent variables
+ Pos1 =~ NA*PosAFF11 + L1*PosAFF11 + L2*PosAFF21 + L3*PosAFF31
+ Pos2 =~ NA*PosAFF12 + L1*PosAFF12 + L2*PosAFF22 + L3*PosAFF32
+ Pos3 =~ NA*PosAFF13 + L1*PosAFF13 + L2*PosAFF23 + L3*PosAFF33
+
+ ## intercepts
+ PosAFF11 ~ t1*1
+ PosAFF21 ~ t2*1
+ PosAFF31 ~ t3*1
+
+ PosAFF12 ~ t1*1
+ PosAFF22 ~ t2*1
+ PosAFF32 ~ t3*1
+
+ PosAFF13 ~ t1*1
+ PosAFF23 ~ t2*1
+ PosAFF33 ~ t3*1
+
+
+ ## correlated residuals across time
+ PosAFF11 ~~ PosAFF12 + PosAFF13
+ PosAFF12 ~~ PosAFF13
+ PosAFF21 ~~ PosAFF22 + PosAFF23
+ PosAFF22 ~~ PosAFF23
+ PosAFF31 ~~ PosAFF32 + PosAFF33
+ PosAFF32 ~~ PosAFF33
+
+
+ ## latent variable intercepts
```

```

+ Pos1 ~ 0*1
+ Pos2 ~ 0*1
+ Pos3 ~ 0*1
+
+ #model constraints for effect coding
+ ## loadings must average to 1
+ L1 == 3 - L2 - L3
+ ## means must average to 0
+ t1 == 0 - t2 - t3
+
+ i =~ 1*Pos1 + 1*Pos2 + 1*Pos3
+ s =~ 0*Pos1 + 1*Pos2 + 2*Pos3 '
> fit.sec.order <- growth(sec.order, data=sem_long, missing = "ML")
> summary(fit.sec.order, fit.measures=TRUE)

```

lavaan (0.5-23.1097) converged normally after 135 iterations

Number of observations	369
Number of missing patterns	2
Estimator	ML
Minimum Function Test Statistic	934.954
Degrees of freedom	24
P-value (Chi-square)	0.000

Model test baseline model:

Minimum Function Test Statistic	2617.825
Degrees of freedom	36
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.647
Tucker-Lewis Index (TLI)	0.471

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	NA
Loglikelihood unrestricted model (H1)	NA
Number of free parameters	30
Akaike (AIC)	NA
Bayesian (BIC)	NA

Root Mean Square Error of Approximation:

RMSEA	0.321
90 Percent Confidence Interval	0.303 0.338
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.494
------	-------

Parameter Estimates:

Information
Standard Errors

Observed
Standard

Latent Variables:

		Estimate	Std.Err	z-value	P(> z)
Pos1 =~					
PosAFF11	(L1)	0.922	0.018	50.272	0.000
PosAFF21	(L2)	1.039	0.018	58.235	0.000
PosAFF31	(L3)	1.039	0.017	61.108	0.000
Pos2 =~					
PosAFF12	(L1)	0.922	0.018	50.272	0.000
PosAFF22	(L2)	1.039	0.018	58.235	0.000
PosAFF32	(L3)	1.039	0.017	61.108	0.000
Pos3 =~					
PosAFF13	(L1)	0.922	0.018	50.272	0.000
PosAFF23	(L2)	1.039	0.018	58.235	0.000
PosAFF33	(L3)	1.039	0.017	61.108	0.000
i =~					
Pos1		1.000			
Pos2		1.000			
Pos3		1.000			
s =~					
Pos1		0.000			
Pos2		1.000			
Pos3		2.000			

Covariances:

	Estimate	Std.Err	z-value	P(> z)
.PosAFF11 ~~				
.PosAFF12	-0.242	6.436	-0.038	0.970
.PosAFF13	10.274	6.282	1.635	0.102
.PosAFF12 ~~				
.PosAFF13	0.003	0.006	0.450	0.653
.PosAFF21 ~~				
.PosAFF22	-0.001	0.009	-0.148	0.883
.PosAFF23	0.004	0.009	0.396	0.692
.PosAFF22 ~~				
.PosAFF23	0.010	0.007	1.382	0.167
.PosAFF31 ~~				
.PosAFF32	0.012	0.009	1.431	0.153
.PosAFF33	0.020	0.009	2.271	0.023
.PosAFF32 ~~				
.PosAFF33	0.005	0.006	0.822	0.411
i ~~				
s	-0.060	0.023	-2.549	0.011

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.PosAFF11 (t1)	0.292	0.055	5.276	0.000
.PosAFF21 (t2)	-0.226	0.057	-3.976	0.000
.PosAFF31 (t3)	-0.066	0.054	-1.215	0.224
.PosAFF12 (t1)	0.292	0.055	5.276	0.000
.PosAFF22 (t2)	-0.226	0.057	-3.976	0.000
.PosAFF32 (t3)	-0.066	0.054	-1.215	0.224
.PosAFF13 (t1)	0.292	0.055	5.276	0.000
.PosAFF23 (t2)	-0.226	0.057	-3.976	0.000
.PosAFF33 (t3)	-0.066	0.054	-1.215	0.224

<i>Pos1</i>	0.000			
<i>Pos2</i>	0.000			
<i>Pos3</i>	0.000			
<i>i</i>	3.202	0.035	90.708	0.000
<i>s</i>	0.007	0.019	0.352	0.724

Variances:

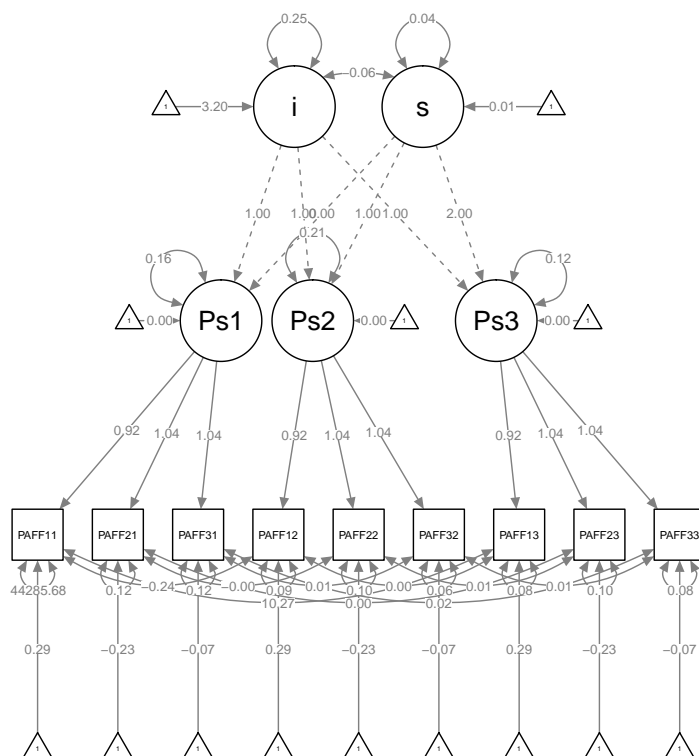
	<i>Estimate</i>	<i>Std.Err</i>	<i>z-value</i>	<i>P(> z)</i>
<i>.PosAFF11</i>	44285.676	0.001	29682467.681	0.000
<i>.PosAFF21</i>	0.116	0.020	5.673	0.000
<i>.PosAFF31</i>	0.117	0.021	5.685	0.000
<i>.PosAFF12</i>	0.087	0.009	9.901	0.000
<i>.PosAFF22</i>	0.104	0.011	9.536	0.000
<i>.PosAFF32</i>	0.061	0.009	6.917	0.000
<i>.PosAFF13</i>	0.077	0.009	8.473	0.000
<i>.PosAFF23</i>	0.098	0.011	9.278	0.000
<i>.PosAFF33</i>	0.083	0.010	8.575	0.000
<i>Pos1</i>	0.159	0.043	3.729	0.000
<i>Pos2</i>	0.208	0.023	9.233	0.000
<i>Pos3</i>	0.121	0.034	3.583	0.000
<i>i</i>	0.252	0.044	5.777	0.000
<i>s</i>	0.036	0.018	1.951	0.051

Constraints:

	<i> Slack </i>
<i>L1 - (3-L2-L3)</i>	0.000
<i>t1 - (0-t2-t3)</i>	0.000

Plotting

```
> library(semPlot)
> library(semTools)
> semPaths(fit.sec.order, whatLabels = "est", panelGroups = TRUE)
```



Comparing with Normal Growth Model

```
> simple.growth <- 'i_1 =~ 1*PosAFF11 + 1*PosAFF21 + 1*PosAFF31
+
+ s_1 =~ 0*PosAFF11 + 1*PosAFF21 + 2*PosAFF31
+
+ i_2 =~ 1*PosAFF12 + 1*PosAFF22 + 1*PosAFF32
+
+ s_2 =~ 0*PosAFF12 + 1*PosAFF22 + 2*PosAFF32
+
+ i_3 =~ 1*PosAFF13 + 1*PosAFF23 + 1*PosAFF33
+
+ s_3 =~ 0*PosAFF13 + 1*PosAFF23 + 2*PosAFF33
+ '
> fit.growth <- growth(simple.growth, data=sem_long)
> summary(fit.growth, fit.measures = TRUE)
```

lavaan (0.5-23.1097) converged normally after 169 iterations

	Used	Total
Number of observations	368	369
Estimator	ML	
Minimum Function Test Statistic	734.084	
Degrees of freedom	18	
P-value (Chi-square)	0.000	

Model test baseline model:

Minimum Function Test Statistic	2617.825
---------------------------------	----------

Degrees of freedom	36
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.723
Tucker-Lewis Index (TLI)	0.445

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	NA
Loglikelihood unrestricted model (H1)	NA
Number of free parameters	36
Akaike (AIC)	NA
Bayesian (BIC)	NA

Root Mean Square Error of Approximation:

RMSEA	0.329
90 Percent Confidence Interval	0.309 0.349
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.753
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
i_1 =~				
PosAFF11	1.000			
PosAFF21	1.000			
PosAFF31	1.000			
s_1 =~				
PosAFF11	0.000			
PosAFF21	1.000			
PosAFF31	2.000			
i_2 =~				
PosAFF12	1.000			
PosAFF22	1.000			
PosAFF32	1.000			
s_2 =~				
PosAFF12	0.000			
PosAFF22	1.000			
PosAFF32	2.000			
i_3 =~				
PosAFF13	1.000			
PosAFF23	1.000			
PosAFF33	1.000			
s_3 =~				
PosAFF13	0.000			
PosAFF23	1.000			
PosAFF33	2.000			

Covariances:

	Estimate	Std.Err	z-value	P(> z)
i_1 ~~				
s_1	7.985	5.307	1.505	0.132
i_2	0.061	0.028	2.168	0.030
s_2	-0.005	0.009	-0.627	0.531
i_3	0.042	0.025	1.643	0.100
s_3	-0.008	0.009	-0.912	0.362
s_1 ~~				
i_2	0.005	0.016	0.292	0.770
s_2	0.007	0.005	1.353	0.176
i_3	-0.005	0.014	-0.319	0.749
s_3	0.010	0.005	1.964	0.050
i_2 ~~				
s_2	0.014	0.013	1.065	0.287
i_3	0.141	0.020	7.035	0.000
s_3	-0.002	0.006	-0.383	0.702
s_2 ~~				
i_3	0.005	0.006	0.798	0.425
s_3	0.002	0.002	1.145	0.252
i_3 ~~				
s_3	0.007	0.013	0.545	0.586

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.PosAFF11	0.000			
.PosAFF21	0.000			
.PosAFF31	0.000			
.PosAFF12	0.000			
.PosAFF22	0.000			
.PosAFF32	0.000			
.PosAFF13	0.000			
.PosAFF23	0.000			
.PosAFF33	0.000			
i_1	2.098	0.044	47.440	0.000
s_1	0.109	0.025	4.354	0.000
i_2	3.223	0.033	98.490	0.000
s_2	0.020	0.010	1.969	0.049
i_3	3.263	0.030	109.830	0.000
s_3	-0.015	0.010	-1.496	0.135

Variances:

	Estimate	Std.Err	z-value	P(> z)
.PosAFF11	44700.976	3295.413	13.565	0.000
.PosAFF21	-109.366	8.471	-12.911	0.000
.PosAFF31	227.036	17.425	13.029	0.000
.PosAFF12	0.069	0.024	2.880	0.004
.PosAFF22	0.139	0.015	8.956	0.000
.PosAFF32	0.024	0.026	0.950	0.342
.PosAFF13	0.070	0.023	2.986	0.003
.PosAFF23	0.137	0.015	9.196	0.000
.PosAFF33	0.085	0.025	3.386	0.001
i_1	212.144	18.171	11.675	0.000
s_1	-117.437	9.695	-12.113	0.000
i_2	0.332	0.035	9.560	0.000
s_2	0.015	0.012	1.262	0.207
i_3	0.262	0.030	8.607	0.000

```

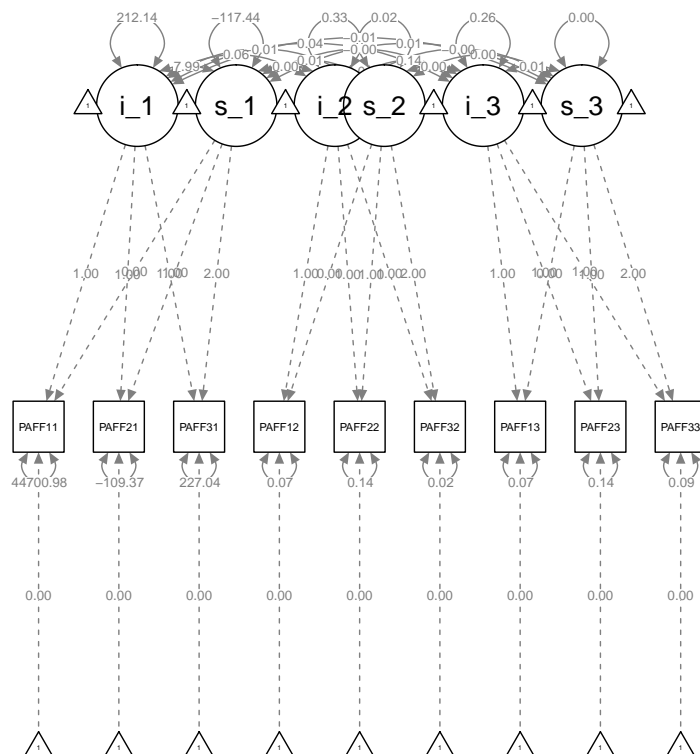
s_3          0.000    0.012    0.026    0.979

```

```

> semPaths(fit.growth, whatLabels = "est", panelGroups = TRUE)

```



```

> anova(fit.sec.order, fit.growth)

```

Chi Square Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff
fit.growth	18		734.08			
fit.sec.order	24		934.95	200.87		6

Pr(>Chisq)

```

fit.growth
fit.sec.order < 2.2e-16 ***
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

4 Multiple Groups

```

> groupmodel.1 <- ' visual =~ x1 + x2 + x3
+               textual =~ x4 + x5 + x6

```

```

+           speed    =~ x7 + x8 + x9 '
> fit.1 <- cfa(groupmodel.1,
+           data = HolzingerSwineford1939,
+           group = "sex")
> summary(fit.1, standardized=TRUE, fit.measures=TRUE)

```

lavaan (0.5-23.1097) converged normally after 59 iterations

Number of observations per group	
1	146
2	155
Estimator	ML
Minimum Function Test Statistic	105.795
Degrees of freedom	48
P-value (Chi-square)	0.000

Chi-square for each group:

1	61.395
2	44.401

Model test baseline model:

Minimum Function Test Statistic	967.228
Degrees of freedom	72
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.935
Tucker-Lewis Index (TLI)	0.903

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-3698.817
Loglikelihood unrestricted model (H1)	-3645.919
Number of free parameters	60
Akaike (AIC)	7517.633
Bayesian (BIC)	7740.060
Sample-size adjusted Bayesian (BIC)	7549.774

Root Mean Square Error of Approximation:

RMSEA	0.089
90 Percent Confidence Interval	0.066 0.113
P-value RMSEA <= 0.05	0.004

Standardized Root Mean Square Residual:

SRMR	0.063
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Group 1 [1]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
visual =~				
x1	1.000			
x2	0.511	0.175	2.924	0.003
x3	0.873	0.221	3.950	0.000
textual =~				
x4	1.000			
x5	1.159	0.103	11.204	0.000
x6	0.930	0.086	10.812	0.000
speed =~				
x7	1.000			
x8	1.480	0.304	4.863	0.000
x9	0.679	0.139	4.895	0.000
Std.lv	Std.all			
0.808	0.725			
0.413	0.329			
0.706	0.602			
0.927	0.835			
1.075	0.857			
0.863	0.818			
0.646	0.644			
0.957	0.864			
0.439	0.490			

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				
textual	0.327	0.094	3.491	0.000
speed	0.157	0.068	2.304	0.021
textual ~~				
speed	0.098	0.062	1.584	0.113
Std.lv	Std.all			
0.436	0.436			
0.301	0.301			
0.164	0.164			

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1	5.033	0.092	54.589	0.000
.x2	6.231	0.104	59.969	0.000
.x3	2.462	0.097	25.368	0.000
.x4	2.918	0.092	31.741	0.000
.x5	4.264	0.104	41.082	0.000
.x6	2.173	0.087	24.887	0.000
.x7	4.056	0.083	48.796	0.000
.x8	5.571	0.092	60.757	0.000
.x9	5.326	0.074	71.828	0.000
visual	0.000			

textual	0.000
speed	0.000
Std.lv	Std.all
5.033	4.518
6.231	4.963
2.462	2.099
2.918	2.627
4.264	3.400
2.173	2.060
4.056	4.038
5.571	5.028
5.326	5.945
0.000	0.000
0.000	0.000
0.000	0.000

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.589	0.168	3.504	0.000
.x2	1.406	0.175	8.022	0.000
.x3	0.878	0.157	5.573	0.000
.x4	0.374	0.067	5.535	0.000
.x5	0.417	0.084	4.948	0.000
.x6	0.369	0.062	5.938	0.000
.x7	0.591	0.103	5.713	0.000
.x8	0.312	0.171	1.828	0.068
.x9	0.610	0.080	7.588	0.000
visual	0.653	0.200	3.270	0.001
textual	0.860	0.147	5.855	0.000
speed	0.418	0.123	3.404	0.001
Std.lv	Std.all			
0.589	0.474			
1.406	0.892			
0.878	0.638			
0.374	0.303			
0.417	0.265			
0.369	0.331			
0.591	0.586			
0.312	0.254			
0.610	0.760			
1.000	1.000			
1.000	1.000			
1.000	1.000			

Group 2 [2]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
visual =~				
x1	1.000			
x2	0.557	0.108	5.153	0.000
x3	0.586	0.107	5.470	0.000
textual =~				
x4	1.000			
x5	1.075	0.083	12.898	0.000
x6	0.928	0.071	12.996	0.000
speed =~				

x7	1.000			
x8	1.076	0.227	4.746	0.000
x9	1.691	0.354	4.773	0.000
Std.lv	Std.all			

0.988	0.820
0.550	0.511
0.579	0.553

1.041	0.872
1.118	0.850
0.966	0.856

0.538	0.469
0.579	0.638
0.910	0.827

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				
textual	0.519	0.114	4.547	0.000
speed	0.319	0.086	3.718	0.000
textual ~~				
speed	0.202	0.068	2.973	0.003
Std.lv	Std.all			

0.504	0.504
0.599	0.599

0.361	0.361
-------	-------

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1	4.844	0.097	50.058	0.000
.x2	5.953	0.087	68.783	0.000
.x3	2.051	0.084	24.367	0.000
.x4	3.196	0.096	33.349	0.000
.x5	4.413	0.106	41.757	0.000
.x6	2.197	0.091	24.233	0.000
.x7	4.308	0.092	46.747	0.000
.x8	5.486	0.073	75.197	0.000
.x9	5.419	0.088	61.316	0.000
visual	0.000			
textual	0.000			
speed	0.000			

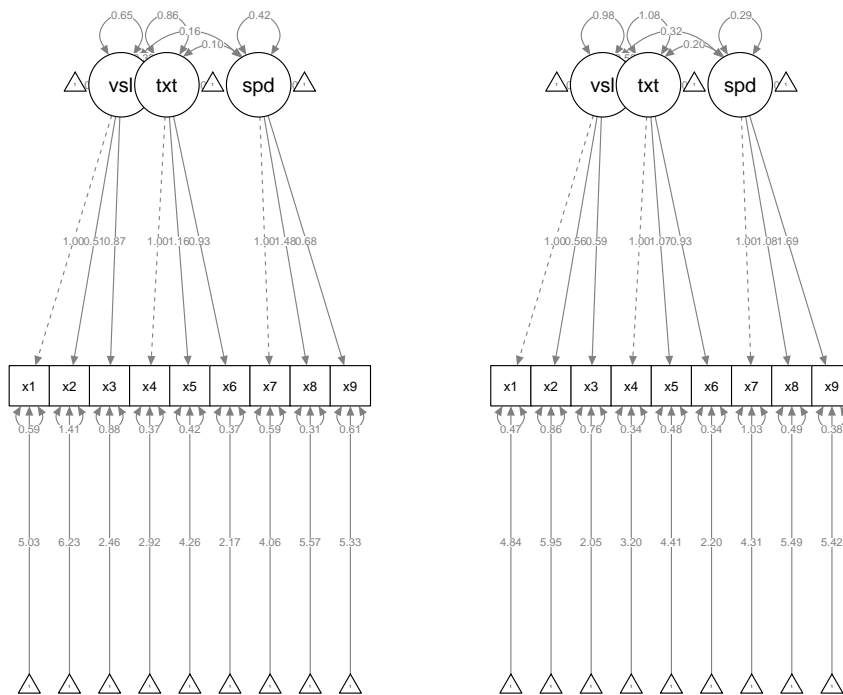
Std.lv	Std.all
4.844	4.021
5.953	5.525
2.051	1.957
3.196	2.679
4.413	3.354
2.197	1.946
4.308	3.755
5.486	6.040
5.419	4.925
0.000	0.000
0.000	0.000
0.000	0.000

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.474	0.144	3.284	0.001
.x2	0.858	0.110	7.800	0.000
.x3	0.762	0.101	7.511	0.000
.x4	0.340	0.064	5.311	0.000
.x5	0.481	0.081	5.956	0.000
.x6	0.341	0.059	5.803	0.000
.x7	1.027	0.128	8.018	0.000
.x8	0.489	0.075	6.547	0.000
.x9	0.382	0.125	3.053	0.002
visual	0.977	0.206	4.754	0.000
textual	1.083	0.165	6.560	0.000
speed	0.290	0.108	2.695	0.007
Std.lv	Std.all			
0.474	0.327			
0.858	0.739			
0.762	0.694			
0.340	0.239			
0.481	0.278			
0.341	0.268			
1.027	0.780			
0.489	0.593			
0.382	0.316			
1.000	1.000			
1.000	1.000			
1.000	1.000			

Plotting

```
> library(semPlot)
> library(semTools)
> semPaths(fit.1, whatLabels = "est", panelGroups = TRUE)
```



Constraining Parameters

4.0.1 Constraining Means

Constraining Visual Means

```
> groupmodel.2.1 <- ' visual =~ c(L1,L1)*x1 + c(L2, L2)*x2 + c(L3,L3)*x3
+           textual =~ x4 + x5 + x6
+           speed   =~ x7 + x8 + x9 '
> fit.2.1 <- cfa(groupmodel.2.1,
+               data = HolzingerSwineford1939,
+               group = "sex")
> summary(fit.2.1, standardized=TRUE, fit.measures=TRUE)
```

lavaan (0.5-23.1097) converged normally after 53 iterations

Number of observations per group

1	146
2	155

Estimator

ML

Minimum Function Test Statistic

108.223

Degrees of freedom

50

P-value (Chi-square)

0.000

Chi-square for each group:

1	62.988
2	45.235

Model test baseline model:

Minimum Function Test Statistic	967.228
Degrees of freedom	72
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.935
Tucker-Lewis Index (TLI)	0.906

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-3700.031
Loglikelihood unrestricted model (H1)	-3645.919
Number of free parameters	58
Akaike (AIC)	7516.061
Bayesian (BIC)	7731.074
Sample-size adjusted Bayesian (BIC)	7547.131

Root Mean Square Error of Approximation:

RMSEA	0.088
90 Percent Confidence Interval	0.065 0.111
P-value RMSEA <= 0.05	0.004

Standardized Root Mean Square Residual:

SRMR	0.065
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Group 1 [1]:

Latent Variables:

		Estimate	Std.Err	z-value	P(> z)
visual =~					
x1	(L1)	1.000			
x2	(L2)	0.532	0.092	5.782	0.000
x3	(L3)	0.670	0.098	6.839	0.000
textual =~					
x4		1.000			
x5		1.160	0.104	11.207	0.000
x6		0.932	0.086	10.820	0.000
speed =~					
x7		1.000			
x8		1.507	0.313	4.815	0.000
x9		0.677	0.139	4.872	0.000
Std.lv	Std.all				

0.866	0.772
0.461	0.361
0.581	0.508

0.927	0.834
1.075	0.857
0.864	0.818

0.641	0.639
0.966	0.872
0.434	0.484

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				
textual	0.359	0.095	3.793	0.000
speed	0.166	0.070	2.366	0.018
textual ~~				
speed	0.098	0.061	1.591	0.112
Std.lv Std.all				
	0.448	0.448		
	0.298	0.298		
	0.164	0.164		

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1	5.033	0.093	54.232	0.000
.x2	6.231	0.106	58.899	0.000
.x3	2.462	0.095	26.046	0.000
.x4	2.918	0.092	31.741	0.000
.x5	4.264	0.104	41.082	0.000
.x6	2.173	0.087	24.887	0.000
.x7	4.056	0.083	48.796	0.000
.x8	5.571	0.092	60.757	0.000
.x9	5.326	0.074	71.828	0.000
visual	0.000			
textual	0.000			
speed	0.000			
Std.lv Std.all				
5.033	4.488			
6.231	4.874			
2.462	2.156			
2.918	2.627			
4.264	3.400			
2.173	2.060			
4.056	4.038			
5.571	5.028			
5.326	5.945			
0.000	0.000			
0.000	0.000			
0.000	0.000			

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.508	0.149	3.402	0.001

.x2	1.422	0.176	8.072	0.000
.x3	0.968	0.132	7.322	0.000
.x4	0.375	0.067	5.563	0.000
.x5	0.417	0.084	4.956	0.000
.x6	0.368	0.062	5.928	0.000
.x7	0.597	0.104	5.770	0.000
.x8	0.294	0.175	1.677	0.093
.x9	0.614	0.081	7.620	0.000
visual	0.750	0.175	4.288	0.000
textual	0.858	0.147	5.849	0.000
speed	0.411	0.122	3.371	0.001
Std.lv	Std.all			
0.508	0.404			
1.422	0.870			
0.968	0.742			
0.375	0.304			
0.417	0.265			
0.368	0.330			
0.597	0.592			
0.294	0.239			
0.614	0.765			
1.000	1.000			
1.000	1.000			
1.000	1.000			

Group 2 [2]:

Latent Variables:

		Estimate	Std.Err	z-value	P(> z)
visual =~					
x1	(L1)	1.000			
x2	(L2)	0.532	0.092	5.782	0.000
x3	(L3)	0.670	0.098	6.839	0.000
textual =~					
x4		1.000			
x5		1.075	0.083	12.887	0.000
x6		0.929	0.072	12.993	0.000
speed =~					
x7		1.000			
x8		1.078	0.227	4.751	0.000
x9		1.688	0.353	4.785	0.000
Std.lv	Std.all				
0.958	0.800				
0.510	0.480				
0.642	0.599				
1.040	0.872				
1.118	0.850				
0.967	0.856				
0.539	0.469				
0.580	0.639				
0.909	0.826				

Covariances:

Estimate	Std.Err	z-value	P(> z)
----------	---------	---------	---------

visual	~~				
textual		0.500	0.111	4.494	0.000
speed		0.315	0.084	3.737	0.000
textual	~~				
speed		0.202	0.068	2.975	0.003
Std.lv	Std.all				
0.502	0.502				
0.611	0.611				
0.361	0.361				

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1	4.844	0.096	50.332	0.000
.x2	5.953	0.085	69.789	0.000
.x3	2.051	0.086	23.824	0.000
.x4	3.196	0.096	33.348	0.000
.x5	4.413	0.106	41.757	0.000
.x6	2.197	0.091	24.233	0.000
.x7	4.308	0.092	46.747	0.000
.x8	5.486	0.073	75.197	0.000
.x9	5.419	0.088	61.316	0.000
visual	0.000			
textual	0.000			
speed	0.000			
Std.lv	Std.all			
4.844	4.043			
5.953	5.606			
2.051	1.914			
3.196	2.679			
4.413	3.354			
2.197	1.946			
4.308	3.755			
5.486	6.040			
5.419	4.925			
0.000	0.000			
0.000	0.000			
0.000	0.000			

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.518	0.132	3.923	0.000
.x2	0.868	0.108	8.005	0.000
.x3	0.736	0.102	7.218	0.000
.x4	0.342	0.064	5.328	0.000
.x5	0.480	0.081	5.950	0.000
.x6	0.340	0.059	5.782	0.000
.x7	1.027	0.128	8.020	0.000
.x8	0.488	0.075	6.548	0.000
.x9	0.384	0.124	3.091	0.002
visual	0.918	0.186	4.930	0.000
textual	1.081	0.165	6.552	0.000
speed	0.290	0.107	2.698	0.007
Std.lv	Std.all			
0.518	0.361			
0.868	0.769			
0.736	0.641			

0.342	0.240
0.480	0.277
0.340	0.267
1.027	0.780
0.488	0.592
0.384	0.317
1.000	1.000
1.000	1.000
1.000	1.000

Now we compare the two fits:

```
> anova(fit.1,fit.2.1)
```

Chi Square Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff
fit.1	48	7517.6	7740.1	105.80		
fit.2.1	50	7516.1	7731.1	108.22	2.4282	2

Pr(>Chisq)

fit.1	
fit.2.1	0.297

Thus, constraining the means did not make the model fit worse i.e. the groups are not reliably different from each other. If they were, constraining the means to be the same would have produced a worse fit. Thus, men and women don't differ on visual ability in these data.

Constraining Textual Means

```
> groupmodel.2.2 <- ' visual =~ x1 + x2 + x3
+           textual =~ c(L1,L1)*x4 + c(L2, L2)*x5 + c(L3,L3)*x6
+           speed   =~ x7 + x8 + x9 '
> fit.2.2 <- cfa(groupmodel.2.2,
+           data = HolzingerSwineford1939,
+           group = "sex")
> summary(fit.2.2, standardized=TRUE, fit.measures=TRUE)
```

lavaan (0.5-23.1097) converged normally after 59 iterations

<i>Number of observations per group</i>	
1	146
2	155
 <i>Estimator</i>	
ML	
<i>Minimum Function Test Statistic</i>	106.329
<i>Degrees of freedom</i>	50
<i>P-value (Chi-square)</i>	0.000

Chi-square for each group:

1	61.700
2	44.629

Model test baseline model:

<i>Minimum Function Test Statistic</i>	967.228
----------------------------------------	---------

Degrees of freedom	72
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.937
Tucker-Lewis Index (TLI)	0.909

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-3699.083
Loglikelihood unrestricted model (H1)	-3645.919
Number of free parameters	58
Akaike (AIC)	7514.166
Bayesian (BIC)	7729.179
Sample-size adjusted Bayesian (BIC)	7545.236

Root Mean Square Error of Approximation:

RMSEA	0.087
90 Percent Confidence Interval	0.064 0.109
P-value RMSEA <= 0.05	0.006

Standardized Root Mean Square Residual:

SRMR	0.064
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Group 1 [1]:

Latent Variables:

		Estimate	Std.Err	z-value	P(> z)
visual =~					
x1		1.000			
x2		0.509	0.174	2.926	0.003
x3		0.872	0.220	3.970	0.000
textual =~					
x4	(L1)	1.000			
x5	(L2)	1.110	0.065	17.106	0.000
x6	(L3)	0.930	0.055	16.885	0.000
speed =~					
x7		1.000			
x8		1.480	0.304	4.863	0.000
x9		0.679	0.139	4.895	0.000
Std.lv	Std.all				
		0.809	0.726		
		0.412	0.328		
		0.705	0.601		
		0.941	0.841		
		1.046	0.845		

0.875	0.823
0.647	0.644
0.957	0.863
0.439	0.490

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				
textual	0.336	0.094	3.566	0.000
speed	0.157	0.068	2.306	0.021
textual ~~				
speed	0.100	0.063	1.587	0.113
Std.lv Std.all				
	0.441	0.441		
	0.301	0.301		
	0.164	0.164		

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1	5.033	0.092	54.589	0.000
.x2	6.231	0.104	59.969	0.000
.x3	2.462	0.097	25.368	0.000
.x4	2.918	0.093	31.494	0.000
.x5	4.264	0.102	41.652	0.000
.x6	2.173	0.088	24.706	0.000
.x7	4.056	0.083	48.796	0.000
.x8	5.571	0.092	60.757	0.000
.x9	5.326	0.074	71.828	0.000
visual	0.000			
textual	0.000			
speed	0.000			
Std.lv Std.all				
	5.033	4.518		
	6.231	4.963		
	2.462	2.099		
	2.918	2.606		
	4.264	3.447		
	2.173	2.045		
	4.056	4.038		
	5.571	5.028		
	5.326	5.945		
	0.000	0.000		
	0.000	0.000		
	0.000	0.000		

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.587	0.167	3.511	0.000
.x2	1.407	0.175	8.027	0.000
.x3	0.879	0.157	5.600	0.000
.x4	0.367	0.065	5.645	0.000
.x5	0.437	0.079	5.539	0.000
.x6	0.364	0.060	6.035	0.000
.x7	0.591	0.103	5.711	0.000
.x8	0.312	0.171	1.830	0.067

.x9	0.610	0.080	7.587	0.000
visual	0.654	0.199	3.284	0.001
textual	0.886	0.133	6.671	0.000
speed	0.418	0.123	3.404	0.001
Std.lv	Std.all			
0.587	0.473			
1.407	0.892			
0.879	0.639			
0.367	0.293			
0.437	0.285			
0.364	0.322			
0.591	0.586			
0.312	0.254			
0.610	0.760			
1.000	1.000			
1.000	1.000			
1.000	1.000			

Group 2 [2]:

Latent Variables:

		Estimate	Std.Err	z-value	P(> z)
visual =~					
x1		1.000			
x2		0.557	0.108	5.151	0.000
x3		0.586	0.107	5.467	0.000
textual =~					
x4	(L1)	1.000			
x5	(L2)	1.110	0.065	17.106	0.000
x6	(L3)	0.930	0.055	16.885	0.000
speed =~					
x7		1.000			
x8		1.076	0.227	4.744	0.000
x9		1.692	0.355	4.772	0.000
Std.lv	Std.all				
		0.988	0.820		
		0.550	0.511		
		0.580	0.553		
		1.029	0.868		
		1.143	0.858		
		0.957	0.852		
		0.538	0.469		
		0.579	0.638		
		0.911	0.828		

Covariances:

		Estimate	Std.Err	z-value	P(> z)
visual ~~					
textual		0.511	0.112	4.568	0.000
speed		0.319	0.086	3.717	0.000
textual ~~					
speed		0.200	0.067	2.984	0.003
Std.lv	Std.all				

0.503	0.503
0.599	0.599
0.362	0.362

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1	4.844	0.097	50.058	0.000
.x2	5.953	0.087	68.783	0.000
.x3	2.051	0.084	24.367	0.000
.x4	3.196	0.095	33.553	0.000
.x5	4.413	0.107	41.246	0.000
.x6	2.197	0.090	24.372	0.000
.x7	4.308	0.092	46.747	0.000
.x8	5.486	0.073	75.197	0.000
.x9	5.419	0.088	61.316	0.000
visual	0.000			
textual	0.000			
speed	0.000			
Std.lv	Std.all			
4.844	4.021			
5.953	5.525			
2.051	1.957			
3.196	2.695			
4.413	3.313			
2.197	1.958			
4.308	3.755			
5.486	6.040			
5.419	4.925			
0.000	0.000			
0.000	0.000			
0.000	0.000			

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.475	0.145	3.283	0.001
.x2	0.858	0.110	7.798	0.000
.x3	0.762	0.102	7.508	0.000
.x4	0.347	0.062	5.616	0.000
.x5	0.468	0.079	5.896	0.000
.x6	0.345	0.057	6.040	0.000
.x7	1.027	0.128	8.020	0.000
.x8	0.490	0.075	6.552	0.000
.x9	0.382	0.125	3.046	0.002
visual	0.977	0.206	4.751	0.000
textual	1.059	0.152	6.987	0.000
speed	0.289	0.107	2.694	0.007
Std.lv	Std.all			
0.475	0.327			
0.858	0.739			
0.762	0.694			
0.347	0.247			
0.468	0.264			
0.345	0.274			
1.027	0.780			
0.490	0.593			
0.382	0.315			
1.000	1.000			

1.000	1.000
1.000	1.000

Now we compare the two fits:

```
> anova(fit.1,fit.2.2)
```

Chi Square Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff
fit.1	48	7517.6	7740.1	105.80		
fit.2.2	50	7514.2	7729.2	106.33	0.53332	2

Pr(>Chisq)

fit.1	
fit.2.2	0.7659

Similar to visual, men and women do not differ on textual ability.

Constraining Speed Means

```
> groupmodel.2.3 <- ' visual =~ x1 + x2 + x3
+           textual =~ x4 + x5 + x6
+           speed   =~ c(L1,L1)*x7 + c(L2, L2)*x8 + c(L3,L3)*x9 '
> fit.2.3 <- cfa(groupmodel.2.3,
+           data = HolzingerSwineford1939,
+           group = "sex")
> summary(fit.2.3, standardized=TRUE, fit.measures=TRUE)
```

lavaan (0.5-23.1097) converged normally after 57 iterations

Number of observations per group	
1	146
2	155

Estimator	ML
Minimum Function Test Statistic	122.650
Degrees of freedom	50
P-value (Chi-square)	0.000

Chi-square for each group:

1	71.011
2	51.639

Model test baseline model:

Minimum Function Test Statistic	967.228
Degrees of freedom	72
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.919
Tucker-Lewis Index (TLI)	0.883

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-3707.244
Loglikelihood unrestricted model (H1)	-3645.919

Number of free parameters	58
Akaike (AIC)	7530.488
Bayesian (BIC)	7745.500
Sample-size adjusted Bayesian (BIC)	7561.557

Root Mean Square Error of Approximation:

RMSEA	0.098
90 Percent Confidence Interval	0.076 0.120
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.072
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Group 1 [1]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
visual =~				
x1	1.000			
x2	0.557	0.180	3.085	0.002
x3	0.951	0.227	4.192	0.000
textual =~				
x4	1.000			
x5	1.158	0.103	11.204	0.000
x6	0.929	0.086	10.805	0.000
speed =~				
x7 (L1)	1.000			
x8 (L2)	1.157	0.162	7.133	0.000
x9 (L3)	1.059	0.149	7.122	0.000
Std.lv	Std.all			
	0.776	0.696		
	0.432	0.344		
	0.737	0.629		
	0.928	0.836		
	1.075	0.857		
	0.862	0.817		
	0.606	0.606		
	0.701	0.661		
	0.642	0.683		

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				

textual	0.307	0.091	3.372	0.001
speed	0.192	0.066	2.894	0.004
textual ~~				
speed	0.104	0.061	1.688	0.091
Std.lv Std.all				
0.427 0.427				
0.409 0.409				
0.184 0.184				

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1	5.033	0.092	54.589	0.000
.x2	6.231	0.104	59.969	0.000
.x3	2.462	0.097	25.368	0.000
.x4	2.918	0.092	31.741	0.000
.x5	4.264	0.104	41.082	0.000
.x6	2.173	0.087	24.887	0.000
.x7	4.056	0.083	49.022	0.000
.x8	5.571	0.088	63.400	0.000
.x9	5.326	0.078	68.442	0.000
visual	0.000			
textual	0.000			
speed	0.000			
Std.lv Std.all				
5.033 4.518				
6.231 4.963				
2.462 2.099				
2.918 2.627				
4.264 3.400				
2.173 2.060				
4.056 4.057				
5.571 5.247				
5.326 5.664				
0.000 0.000				
0.000 0.000				
0.000 0.000				

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.640	0.153	4.188	0.000
.x2	1.390	0.174	7.969	0.000
.x3	0.832	0.156	5.327	0.000
.x4	0.372	0.068	5.513	0.000
.x5	0.417	0.084	4.937	0.000
.x6	0.370	0.062	5.950	0.000
.x7	0.632	0.096	6.619	0.000
.x8	0.635	0.107	5.944	0.000
.x9	0.472	0.084	5.605	0.000
visual	0.601	0.182	3.300	0.001
textual	0.861	0.147	5.860	0.000
speed	0.367	0.091	4.016	0.000
Std.lv Std.all				
0.640 0.515				
1.390 0.882				
0.832 0.605				
0.372 0.302				

0.417	0.265
0.370	0.333
0.632	0.633
0.635	0.564
0.472	0.534
1.000	1.000
1.000	1.000
1.000	1.000

Group 2 [2]:

Latent Variables:

		Estimate	Std.Err	z-value	P(> z)
visual =~					
x1		1.000			
x2		0.563	0.112	5.013	0.000
x3		0.586	0.112	5.248	0.000
textual =~					
x4		1.000			
x5		1.072	0.083	12.892	0.000
x6		0.927	0.071	13.006	0.000
speed =~					
x7	(L1)	1.000			
x8	(L2)	1.157	0.162	7.133	0.000
x9	(L3)	1.059	0.149	7.122	0.000
Std.lv	Std.all				
		0.986	0.818		
		0.555	0.515		
		0.578	0.552		
		1.042	0.873		
		1.117	0.849		
		0.966	0.855		
		0.632	0.548		
		0.731	0.781		
		0.669	0.637		

Covariances:

		Estimate	Std.Err	z-value	P(> z)
visual ~~					
textual		0.518	0.114	4.536	0.000
speed		0.312	0.081	3.844	0.000
textual ~~					
speed		0.212	0.072	2.953	0.003
Std.lv	Std.all				
		0.505	0.505		
		0.500	0.500		
		0.322	0.322		

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1	4.844	0.097	50.058	0.000
.x2	5.953	0.087	68.783	0.000

.x3	2.051	0.084	24.367	0.000
.x4	3.196	0.096	33.348	0.000
.x5	4.413	0.106	41.757	0.000
.x6	2.197	0.091	24.233	0.000
.x7	4.308	0.093	46.520	0.000
.x8	5.486	0.075	72.988	0.000
.x9	5.419	0.084	64.207	0.000
visual	0.000			
textual	0.000			
speed	0.000			
Std.lv	Std.all			
4.844	4.021			
5.953	5.525			
2.051	1.957			
3.196	2.679			
4.413	3.354			
2.197	1.946			
4.308	3.737			
5.486	5.863			
5.419	5.157			
0.000	0.000			
0.000	0.000			
0.000	0.000			

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.480	0.153	3.126	0.002
.x2	0.853	0.111	7.686	0.000
.x3	0.764	0.103	7.406	0.000
.x4	0.338	0.064	5.276	0.000
.x5	0.484	0.081	5.980	0.000
.x6	0.342	0.059	5.804	0.000
.x7	0.930	0.123	7.574	0.000
.x8	0.341	0.081	4.193	0.000
.x9	0.656	0.098	6.681	0.000
visual	0.972	0.212	4.591	0.000
textual	1.086	0.165	6.571	0.000
speed	0.399	0.102	3.900	0.000
Std.lv	Std.all			
0.480	0.331			
0.853	0.735			
0.764	0.696			
0.338	0.237			
0.484	0.279			
0.342	0.268			
0.930	0.700			
0.341	0.390			
0.656	0.594			
1.000	1.000			
1.000	1.000			
1.000	1.000			

Now we compare the two fits:

```
> anova(fit.1,fit.2.3)
```

Chi Square Difference Test

```

      Df      AIC      BIC  Chisq Chisq diff Df diff
fit.1   48 7517.6 7740.1 105.80
fit.2.3 50 7530.5 7745.5 122.65      16.855      2
      Pr(>Chisq)
fit.1
fit.2.3 0.0002188 ***
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

Here, we see a difference in fits, such that constraining the means for speed in men and women makes the model fit worse. Thus, men and women do in fact differ in their speeds.

4.0.2 Constraining Factor Loadings

```

> groupmodel.3 <- ' visual =~ x1 + x2 + x3
+                  textual =~ x4 + x5 + x6
+                  speed  =~ x7 + x8 + x9 '
> fit.3 <- cfa(groupmodel.3,
+              data = HolzingerSwineford1939,
+              group = "sex", group.equal = "loadings")
> summary(fit.3, standardized=TRUE, fit.measures=TRUE)

```

lavaan (0.5-23.1097) converged normally after 47 iterations

<i>Number of observations per group</i>	
1	146
2	155
<i>Estimator</i>	
	ML
<i>Minimum Function Test Statistic</i>	126.226
<i>Degrees of freedom</i>	54
<i>P-value (Chi-square)</i>	0.000

Chi-square for each group:

1	72.534
2	53.692

Model test baseline model:

<i>Minimum Function Test Statistic</i>	967.228
<i>Degrees of freedom</i>	72
<i>P-value</i>	0.000

User model versus baseline model:

<i>Comparative Fit Index (CFI)</i>	0.919
<i>Tucker-Lewis Index (TLI)</i>	0.892

Loglikelihood and Information Criteria:

<i>Loglikelihood user model (H0)</i>	-3709.032
<i>Loglikelihood unrestricted model (H1)</i>	-3645.919
<i>Number of free parameters</i>	54
<i>Akaike (AIC)</i>	7526.064

Bayesian (BIC)	7726.248
Sample-size adjusted Bayesian (BIC)	7554.990

Root Mean Square Error of Approximation:

RMSEA	0.094
90 Percent Confidence Interval	0.073 0.116
P-value RMSEA <= 0.05	0.001

Standardized Root Mean Square Residual:

SRMR	0.074
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Group 1 [1]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
visual =~				
x1	1.000			
x2 (.p2.)	0.552	0.096	5.766	0.000
x3 (.p3.)	0.694	0.103	6.747	0.000
textual =~				
x4	1.000			
x5 (.p5.)	1.110	0.065	17.092	0.000
x6 (.p6.)	0.930	0.055	16.895	0.000
speed =~				
x7	1.000			
x8 (.p8.)	1.168	0.163	7.170	0.000
x9 (.p9.)	1.037	0.145	7.157	0.000
Std.lv Std.all				
	0.851	0.757		
	0.470	0.368		
	0.591	0.519		
	0.941	0.841		
	1.045	0.845		
	0.876	0.824		
	0.614	0.614		
	0.717	0.675		
	0.636	0.674		

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				
textual	0.364	0.095	3.844	0.000
speed	0.203	0.069	2.923	0.003
textual ~~				
speed	0.105	0.063	1.680	0.093
Std.lv Std.all				

0.454	0.454
0.388	0.388
0.183	0.183

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1	5.033	0.093	54.090	0.000
.x2	6.231	0.106	58.928	0.000
.x3	2.462	0.094	26.122	0.000
.x4	2.918	0.093	31.489	0.000
.x5	4.264	0.102	41.664	0.000
.x6	2.173	0.088	24.704	0.000
.x7	4.056	0.083	49.034	0.000
.x8	5.571	0.088	63.420	0.000
.x9	5.326	0.078	68.204	0.000
visual	0.000			
textual	0.000			
speed	0.000			
Std.lv	Std.all			
5.033	4.477			
6.231	4.877			
2.462	2.162			
2.918	2.606			
4.264	3.448			
2.173	2.044			
4.056	4.058			
5.571	5.249			
5.326	5.645			
0.000	0.000			
0.000	0.000			
0.000	0.000			

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.540	0.145	3.710	0.000
.x2	1.412	0.176	8.043	0.000
.x3	0.948	0.131	7.240	0.000
.x4	0.367	0.065	5.654	0.000
.x5	0.437	0.079	5.551	0.000
.x6	0.363	0.060	6.028	0.000
.x7	0.622	0.095	6.558	0.000
.x8	0.613	0.107	5.755	0.000
.x9	0.486	0.084	5.768	0.000
visual	0.724	0.170	4.251	0.000
textual	0.886	0.133	6.670	0.000
speed	0.377	0.093	4.056	0.000
Std.lv	Std.all			
0.540	0.427			
1.412	0.865			
0.948	0.731			
0.367	0.293			
0.437	0.286			
0.363	0.321			
0.622	0.623			
0.613	0.544			
0.486	0.545			
1.000	1.000			

1.000	1.000
1.000	1.000

Group 2 [2]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
visual =~				
x1	1.000			
x2 (.p2.)	0.552	0.096	5.766	0.000
x3 (.p3.)	0.694	0.103	6.747	0.000
textual =~				
x4	1.000			
x5 (.p5.)	1.110	0.065	17.092	0.000
x6 (.p6.)	0.930	0.055	16.895	0.000
speed =~				
x7	1.000			
x8 (.p8.)	1.168	0.163	7.170	0.000
x9 (.p9.)	1.037	0.145	7.157	0.000
Std.lv Std.all				
	0.941	0.787		
	0.520	0.489		
	0.653	0.608		
	1.029	0.868		
	1.142	0.857		
	0.957	0.853		
	0.632	0.548		
	0.738	0.788		
	0.655	0.625		

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				
textual	0.484	0.108	4.474	0.000
speed	0.303	0.079	3.854	0.000
textual ~~				
speed	0.207	0.071	2.932	0.003
Std.lv Std.all				
	0.500	0.500		
	0.510	0.510		
	0.318	0.318		

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1	4.844	0.096	50.439	0.000
.x2	5.953	0.085	69.757	0.000
.x3	2.051	0.086	23.763	0.000
.x4	3.196	0.095	33.557	0.000
.x5	4.413	0.107	41.234	0.000
.x6	2.197	0.090	24.374	0.000
.x7	4.308	0.093	46.504	0.000
.x8	5.486	0.075	72.953	0.000

.x9	5.419	0.084	64.410	0.000
visual	0.000			
textual	0.000			
speed	0.000			
Std.lv	Std.all			
4.844	4.051			
5.953	5.603			
2.051	1.909			
3.196	2.695			
4.413	3.312			
2.197	1.958			
4.308	3.735			
5.486	5.860			
5.419	5.174			
0.000	0.000			
0.000	0.000			
0.000	0.000			

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.544	0.136	4.007	0.000
.x2	0.859	0.109	7.897	0.000
.x3	0.728	0.104	6.999	0.000
.x4	0.347	0.062	5.607	0.000
.x5	0.471	0.080	5.911	0.000
.x6	0.343	0.057	6.015	0.000
.x7	0.931	0.123	7.578	0.000
.x8	0.332	0.082	4.051	0.000
.x9	0.668	0.098	6.826	0.000
visual	0.886	0.185	4.793	0.000
textual	1.059	0.152	6.987	0.000
speed	0.399	0.102	3.918	0.000
Std.lv	Std.all			
0.544	0.381			
0.859	0.761			
0.728	0.631			
0.347	0.247			
0.471	0.265			
0.343	0.272			
0.931	0.700			
0.332	0.379			
0.668	0.609			
1.000	1.000			
1.000	1.000			
1.000	1.000			

Checking if this constraint reduces fit:

```
> anova(fit.1, fit.3)
```

Chi Square Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff	Pr(>Chisq)
fit.1	48	7517.6	7740.1	105.80			
fit.3	54	7526.1	7726.2	126.23	20.431	6	0.00232

fit.1

```
fit.3 **
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

And indeed it does – this means that men and women do in fact differ on some construct, as constraining the factor loadings to be the same produces a worse fit.

4.0.3 Constraining Intercepts

```
> groupmodel.4 <- ' visual =~ x1 + x2 + x3
+               textual =~ x4 + x5 + x6
+               speed   =~ x7 + x8 + x9 '
> fit.4 <- cfa(groupmodel.4,
+             data = HolzingerSwineford1939,
+             group = "sex", group.equal = "intercepts")
> summary(fit.4, standardized=TRUE, fit.measures=TRUE)
```

lavaan (0.5-23.1097) converged normally after 68 iterations

<i>Number of observations per group</i>	
1	146
2	155

<i>Estimator</i>	<i>ML</i>
<i>Minimum Function Test Statistic</i>	125.520
<i>Degrees of freedom</i>	54
<i>P-value (Chi-square)</i>	0.000

Chi-square for each group:

1	70.978
2	54.542

Model test baseline model:

<i>Minimum Function Test Statistic</i>	967.228
<i>Degrees of freedom</i>	72
<i>P-value</i>	0.000

User model versus baseline model:

<i>Comparative Fit Index (CFI)</i>	0.920
<i>Tucker-Lewis Index (TLI)</i>	0.893

Loglikelihood and Information Criteria:

<i>Loglikelihood user model (H0)</i>	-3708.679
<i>Loglikelihood unrestricted model (H1)</i>	-3645.919
<i>Number of free parameters</i>	54
<i>Akaike (AIC)</i>	7525.357
<i>Bayesian (BIC)</i>	7725.541
<i>Sample-size adjusted Bayesian (BIC)</i>	7554.284

Root Mean Square Error of Approximation:

RMSEA		0.094
90 Percent Confidence Interval	0.072	0.115
P-value RMSEA <= 0.05		0.001

Standardized Root Mean Square Residual:

SRMR	0.069
------	-------

Parameter Estimates:

Information	Expected
Standard Errors	Standard

Group 1 [1]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
visual =~				
x1	1.000			
x2	0.539	0.181	2.979	0.003
x3	0.908	0.230	3.941	0.000
textual =~				
x4	1.000			
x5	1.167	0.105	11.117	0.000
x6	0.935	0.087	10.707	0.000
speed =~				
x7	1.000			
x8	1.489	0.308	4.825	0.000
x9	0.689	0.141	4.881	0.000
Std.lv	Std.all			
0.789	0.708			
0.425	0.338			
0.717	0.607			
0.924	0.831			
1.077	0.859			
0.863	0.816			
0.642	0.638			
0.955	0.859			
0.442	0.494			

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				
textual	0.316	0.092	3.424	0.001
speed	0.155	0.067	2.308	0.021
textual ~~				
speed	0.099	0.062	1.598	0.110
Std.lv	Std.all			
0.434	0.434			
0.306	0.306			
0.166	0.166			

Intercepts:

		Estimate	Std.Err	z-value	P(> z)
.x1	(.25.)	5.071	0.089	57.290	0.000
.x2	(.26.)	6.159	0.078	79.316	0.000
.x3	(.27.)	2.327	0.079	29.545	0.000
.x4	(.28.)	2.963	0.088	33.687	0.000
.x5	(.29.)	4.232	0.097	43.655	0.000
.x6	(.30.)	2.091	0.082	25.545	0.000
.x7	(.31.)	4.106	0.067	61.515	0.000
.x8	(.32.)	5.480	0.067	81.668	0.000
.x9	(.33.)	5.306	0.073	72.415	0.000
visual		0.000			
textual		0.000			
speed		0.000			
Std.lv	Std.all				
5.071	4.550				
6.159	4.897				
2.327	1.971				
2.963	2.665				
4.232	3.374				
2.091	1.976				
4.106	4.083				
5.480	4.930				
5.306	5.920				
0.000	0.000				
0.000	0.000				
0.000	0.000				

Variances:

		Estimate	Std.Err	z-value	P(> z)
.x1		0.620	0.164	3.775	0.000
.x2		1.401	0.176	7.968	0.000
.x3		0.880	0.161	5.465	0.000
.x4		0.383	0.068	5.602	0.000
.x5		0.413	0.085	4.859	0.000
.x6		0.375	0.063	5.947	0.000
.x7		0.600	0.104	5.781	0.000
.x8		0.324	0.172	1.886	0.059
.x9		0.608	0.081	7.543	0.000
visual		0.623	0.194	3.213	0.001
textual		0.853	0.147	5.807	0.000
speed		0.412	0.122	3.370	0.001
Std.lv	Std.all				
0.620	0.499				
1.401	0.886				
0.880	0.631				
0.383	0.310				
0.413	0.262				
0.375	0.335				
0.600	0.593				
0.324	0.262				
0.608	0.756				
1.000	1.000				
1.000	1.000				
1.000	1.000				

Group 2 [2]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
visual =~				
x1	1.000			
x2	0.610	0.112	5.439	0.000
x3	0.670	0.113	5.927	0.000
textual =~				
x4	1.000			
x5	1.063	0.082	12.988	0.000
x6	0.907	0.070	12.966	0.000
speed =~				
x7	1.000			
x8	1.026	0.215	4.781	0.000
x9	1.664	0.344	4.833	0.000
Std.lv Std.all				

0.935 0.784
0.571 0.527
0.627 0.588

1.051 0.874
1.118 0.850
0.953 0.849

0.551 0.473
0.565 0.624
0.916 0.833

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				
textual	0.496	0.112	4.447	0.000
speed	0.316	0.085	3.723	0.000
textual ~~				
speed	0.209	0.070	2.990	0.003
Std.lv Std.all				

0.504 0.504
0.613 0.613

0.362 0.362

Intercepts:

		Estimate	Std.Err	z-value	P(> z)
.x1	(.25.)	5.071	0.089	57.290	0.000
.x2	(.26.)	6.159	0.078	79.316	0.000
.x3	(.27.)	2.327	0.079	29.545	0.000
.x4	(.28.)	2.963	0.088	33.687	0.000
.x5	(.29.)	4.232	0.097	43.655	0.000
.x6	(.30.)	2.091	0.082	25.545	0.000
.x7	(.31.)	4.106	0.067	61.515	0.000
.x8	(.32.)	5.480	0.067	81.668	0.000
.x9	(.33.)	5.306	0.073	72.415	0.000
visual		-0.283	0.125	-2.272	0.023
textual		0.178	0.123	1.442	0.149
speed		0.068	0.069	0.984	0.325
Std.lv Std.all					

5.071	4.251
6.159	5.685
2.327	2.181
2.963	2.464
4.232	3.220
2.091	1.864
4.106	3.530
5.480	6.052
5.306	4.822
-0.303	-0.303
0.169	0.169
0.123	0.123

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.548	0.133	4.134	0.000
.x2	0.847	0.110	7.673	0.000
.x3	0.745	0.103	7.212	0.000
.x4	0.341	0.065	5.218	0.000
.x5	0.479	0.081	5.916	0.000
.x6	0.351	0.059	5.939	0.000
.x7	1.049	0.131	8.001	0.000
.x8	0.501	0.074	6.734	0.000
.x9	0.372	0.126	2.942	0.003
visual	0.875	0.188	4.659	0.000
textual	1.105	0.168	6.589	0.000
speed	0.303	0.111	2.737	0.006
Std.lv	Std.all			
0.548	0.385			
0.847	0.722			
0.745	0.655			
0.341	0.236			
0.479	0.277			
0.351	0.279			
1.049	0.776			
0.501	0.611			
0.372	0.307			
1.000	1.000			
1.000	1.000			
1.000	1.000			

Checking if this constraint reduces fit:

```
> anova(fit.1, fit.4)
```

Chi Square Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff	Pr(>Chisq)
fit.1	48	7517.6	7740.1	105.80			
fit.4	54	7525.4	7725.5	125.52	19.724	6	0.0031

fit.1

fit.4 **

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Similarly, we see that constraining the intercepts also produces a difference, suggesting that men and women differ on some aspect.

4.0.4 Constraining Some Parameters, Not All

```
> groupmodel.5 <- ' visual =~ x1 + x2 + x3
+               textual =~ x4 + x5 + x6
+               speed  =~ x7 + x8 + x9 '
> fit.5 <- cfa(groupmodel.5,
+             data = HolzingerSwineford1939,
+             group = "sex", group.equal = c("loadings", "intercepts"),
+             group.partial = c("visual =~ x1 + x2 + x3",
+                               "textual =~ x4 + x5 + x6"))
> summary(fit.5, standardized=TRUE, fit.measures=TRUE)
```

lavaan (0.5-23.1097) converged normally after 63 iterations

<i>Number of observations per group</i>	
1	146
2	155

<i>Estimator</i>	<i>ML</i>
<i>Minimum Function Test Statistic</i>	144.998
<i>Degrees of freedom</i>	60
<i>P-value (Chi-square)</i>	0.000

Chi-square for each group:

1	86.018
2	58.979

Model test baseline model:

<i>Minimum Function Test Statistic</i>	967.228
<i>Degrees of freedom</i>	72
<i>P-value</i>	0.000

User model versus baseline model:

<i>Comparative Fit Index (CFI)</i>	0.905
<i>Tucker-Lewis Index (TLI)</i>	0.886

Loglikelihood and Information Criteria:

<i>Loglikelihood user model (H0)</i>	-3718.418
<i>Loglikelihood unrestricted model (H1)</i>	-3645.919
<i>Number of free parameters</i>	48
<i>Akaike (AIC)</i>	7532.835
<i>Bayesian (BIC)</i>	7710.777
<i>Sample-size adjusted Bayesian (BIC)</i>	7558.548

Root Mean Square Error of Approximation:

<i>RMSEA</i>	0.097
<i>90 Percent Confidence Interval</i>	0.077 0.117
<i>P-value RMSEA <= 0.05</i>	0.000

Standardized Root Mean Square Residual:

SRMR 0.075

Parameter Estimates:

Information Expected
Standard Errors Standard

Group 1 [1]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
visual =~				
x1	1.000			
x2 (.p2.)	0.610	0.099	6.138	0.000
x3 (.p3.)	0.783	0.107	7.296	0.000
textual =~				
x4	1.000			
x5 (.p5.)	1.106	0.065	17.082	0.000
x6 (.p6.)	0.920	0.055	16.780	0.000
speed =~				
x7	1.000			
x8 (.p8.)	1.129	0.165	6.842	0.000
x9 (.p9.)	1.209	0.178	6.809	0.000
Std.lv Std.all				
	0.810	0.720		
	0.494	0.386		
	0.634	0.552		
	0.945	0.840		
	1.046	0.846		
	0.869	0.819		
	0.556	0.552		
	0.628	0.590		
	0.672	0.724		

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				
textual	0.345	0.092	3.739	0.000
speed	0.208	0.064	3.237	0.001
textual ~~				
speed	0.104	0.058	1.782	0.075
Std.lv Std.all				
	0.450	0.450		
	0.462	0.462		
	0.198	0.198		

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1 (.25.)	5.097	0.089	57.448	0.000

.x2	(.26.)	6.177	0.078	78.759	0.000
.x3	(.27.)	2.367	0.081	29.178	0.000
.x4	(.28.)	2.984	0.088	33.774	0.000
.x5	(.29.)	4.254	0.098	43.621	0.000
.x6	(.30.)	2.111	0.082	25.632	0.000
.x7	(.31.)	4.142	0.072	57.211	0.000
.x8	(.32.)	5.471	0.075	73.290	0.000
.x9	(.33.)	5.337	0.073	73.609	0.000
visual		0.000			
textual		0.000			
speed		0.000			
Std.lv	Std.all				
5.097	4.533				
6.177	4.831				
2.367	2.060				
2.984	2.652				
4.254	3.441				
2.111	1.988				
4.142	4.111				
5.471	5.139				
5.337	5.752				
0.000	0.000				
0.000	0.000				
0.000	0.000				

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.609	0.135	4.509	0.000
.x2	1.391	0.175	7.965	0.000
.x3	0.918	0.132	6.972	0.000
.x4	0.372	0.066	5.635	0.000
.x5	0.434	0.079	5.490	0.000
.x6	0.372	0.061	6.102	0.000
.x7	0.706	0.100	7.035	0.000
.x8	0.740	0.110	6.710	0.000
.x9	0.409	0.087	4.708	0.000
visual	0.655	0.154	4.261	0.000
textual	0.894	0.134	6.662	0.000
speed	0.309	0.083	3.744	0.000
Std.lv	Std.all			
0.609	0.482			
1.391	0.851			
0.918	0.696			
0.372	0.294			
0.434	0.284			
0.372	0.330			
0.706	0.696			
0.740	0.652			
0.409	0.475			
1.000	1.000			
1.000	1.000			
1.000	1.000			

Group 2 [2]:

Latent Variables:

Estimate	Std.Err	z-value	P(> z)
----------	---------	---------	---------

```

visual =~
  x1          1.000
  x2      (.p2.) 0.610  0.099  6.138  0.000
  x3      (.p3.) 0.783  0.107  7.296  0.000
textual =~
  x4          1.000
  x5      (.p5.) 1.106  0.065 17.082  0.000
  x6      (.p6.) 0.920  0.055 16.780  0.000
speed =~
  x7          1.000
  x8      (.p8.) 1.129  0.165  6.842  0.000
  x9      (.p9.) 1.209  0.178  6.809  0.000
Std.lv Std.all

0.884  0.747
0.539  0.504
0.692  0.635

1.033  0.867
1.143  0.858
0.950  0.849

0.615  0.531
0.695  0.744
0.744  0.698

```

Covariances:

	Estimate	Std.Err	z-value	P(> z)
visual ~~				
textual	0.452	0.104	4.350	0.000
speed	0.307	0.076	4.014	0.000
textual ~~				
speed	0.218	0.071	3.087	0.002
Std.lv Std.all				
	0.495	0.495		
	0.565	0.565		
	0.343	0.343		

Intercepts:

	Estimate	Std.Err	z-value	P(> z)
.x1 (.25.)	5.097	0.089	57.448	0.000
.x2 (.26.)	6.177	0.078	78.759	0.000
.x3 (.27.)	2.367	0.081	29.178	0.000
.x4 (.28.)	2.984	0.088	33.774	0.000
.x5 (.29.)	4.254	0.098	43.621	0.000
.x6 (.30.)	2.111	0.082	25.632	0.000
.x7 (.31.)	4.142	0.072	57.211	0.000
.x8 (.32.)	5.471	0.075	73.290	0.000
.x9 (.33.)	5.337	0.073	73.609	0.000
visual	-0.315	0.121	-2.598	0.009
textual	0.153	0.121	1.258	0.208
speed	0.056	0.082	0.692	0.489
Std.lv Std.all				
	5.097	4.311		
	6.177	5.779		
	2.367	2.175		

2.984	2.504
4.254	3.195
2.111	1.887
4.142	3.575
5.471	5.857
5.337	5.009
-0.356	-0.356
0.148	0.148
0.092	0.092

Variances:

	Estimate	Std.Err	z-value	P(> z)
.x1	0.617	0.126	4.903	0.000
.x2	0.852	0.109	7.796	0.000
.x3	0.706	0.105	6.710	0.000
.x4	0.353	0.063	5.602	0.000
.x5	0.467	0.080	5.845	0.000
.x6	0.350	0.057	6.085	0.000
.x7	0.964	0.125	7.701	0.000
.x8	0.390	0.078	5.016	0.000
.x9	0.582	0.100	5.833	0.000
visual	0.781	0.165	4.722	0.000
textual	1.067	0.153	6.976	0.000
speed	0.378	0.102	3.715	0.000
Std.lv	Std.all			
0.617	0.441			
0.852	0.746			
0.706	0.596			
0.353	0.248			
0.467	0.263			
0.350	0.279			
0.964	0.718			
0.390	0.447			
0.582	0.513			
1.000	1.000			
1.000	1.000			
1.000	1.000			

Checking if this constraint reduces fit:

```
> anova(fit.1, fit.5)
```

Chi Square Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff	Pr(>Chisq)
fit.1	48	7517.6	7740.1	105.8			
fit.5	60	7532.8	7710.8	145.0	39.202	12	9.745e-05

fit.1

fit.5 ***

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Again, constraining the loadings and intercepts makes the fit worse.