# Assignment 6: SEM

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# 1 Reading the Data

I am going to use the class dataset for Questions 1 and 2, and the lavaan dataset, HolzingerSwineford1939 for the multiple group analyses.

- > library(lavaan)
- > head(HolzingerSwineford1939)

```
school grade
  id sex ageyr agemo
                                          x1
   1
            13
                   1 Pasteur
                                  7 3.333333 7.75 0.375
2
   2
       2
            13
                   7 Pasteur
                                  7 5.333333 5.25 2.125
3
  3
       2
            13
                   1 Pasteur
                                  7 4.500000 5.25 1.875
  4
       1
            13
                   2 Pasteur
                                  7 5.333333 7.75 3.000
5
  5
       2
            12
                   2 Pasteur
                                  7 4.833333 4.75 0.875
       2
            14
                    1 Pasteur
                                  7 5.333333 5.00 2.250
                                 x7
                       x6
                                      x8
                                               x9
        x4
             х5
1 2.333333 5.75 1.2857143 3.391304 5.75 6.361111
2 1.666667 3.00 1.2857143 3.782609 6.25 7.916667
3 1.000000 1.75 0.4285714 3.260870 3.90 4.416667
4 2.666667 4.50 2.4285714 3.000000 5.30 4.861111
5 2.666667 4.00 2.5714286 3.695652 6.30 5.916667
6 1.000000 3.00 0.8571429 4.347826 6.65 7.500000
```

```
> sem_long = read.csv("SEM_long.csv", header = TRUE, sep = ",")
```

> head(sem\_long)

```
PosAFF11 PosAFF21 PosAFF31
                                 NegAFF11
1 3.407195916 3.161366 2.691232 2.4207020 2.3531140
2 2.594690943 2.682974 2.676173 1.6652403 0.8228416
3 3.168698274 2.854090 3.277873 1.5646086 1.6909398
4 4.600159845 4.698997 4.751315 0.9352668 0.7944903
5 1.911294244 2.184561 1.661057 1.6076395 1.5046016
6 3.472353729 4.016982 4.021543 1.9754108 1.1348223
  NegAFF31 PosAFF12 PosAFF22 PosAFF32 NegAFF12 NegAFF22
1 2.338767 3.608672 2.385396 3.029975 2.283468 2.3173734
2 1.083938 3.103346 2.666753 3.251786 1.651941 1.7019345
3 1.502032 2.327999 3.061529 3.272173 1.749022 1.6237575
4 0.205178 4.254750 3.792440 3.934392 1.279030 0.8741891
5 1.209850 3.971308 4.674580 4.093508 1.019751 1.0376248
6 1.729405 3.845904 4.186825 4.316398 1.189548 1.1866801
   NegAFF32 PosAFF13 PosAFF23 PosAFF33 NegAFF13 NegAFF23
1 2.3932485 3.561401 3.506353 3.862975 2.090484 1.3276333
2 1.3462416 2.893546 2.804488 2.637762 1.257376 1.6585273
```

```
3 1.5763265 2.705572 2.310391 2.841122 1.699849 1.7373582 4 1.1822674 3.212826 2.775791 3.239004 1.824778 1.3016450 5 0.7776716 3.487932 2.673107 3.093565 1.316975 0.7120284 6 1.3158520 3.236481 3.256453 4.023383 1.053895 1.4449087 NegAFF33 1 1.6680488 2 1.4298172 3 1.8542867 4 1.5356285 5 0.7083461 6 0.9600753
```

# 2 Testing Measurement Invariance

## Configural

```
> config <- '
+ ## define latent variables
+ PosAffect1 =~ PosAFF11 + PosAFF21 + PosAFF31
+ PosAffect2 =~ PosAFF12 + PosAFF22 + PosAFF32
+ PosAffect3 =~ PosAFF13 + PosAFF23 + PosAFF33
+
+ ## correlated residuals across time
+ PosAFF11 ~~ PosAFF12 + PosAFF13
+ PosAFF12 ~~ PosAFF13
+ PosAFF12 ~~ PosAFF23 + PosAFF23
+ PosAFF22 ~~ PosAFF23
+ PosAFF31 ~~ PosAFF32 + PosAFF33
+ PosAFF32 ~~ PosAFF33
+ '
> config <- cfa(config, data=sem_long, meanstructure=TRUE, std.lv=TRUE)
> summary(config, standardized=TRUE, fit.measures=TRUE)
```

lavaan (0.5-23.1097) converged normally after 106 iterations

Number of observations	Used 368	Total 369
Estimator Minimum Function Test Statistic Degrees of freedom P-value (Chi-square)	ML 8.366 15 0.908	
Model test baseline model:		
Minimum Function Test Statistic Degrees of freedom P-value	2617.825 36 0.000	
User model versus baseline model:		
Comparative Fit Index (CFI) Tucker-Lewis Index (TLI)	1.000 1.006	

Loglikelihood and Information Criteria:

_		er model ( restricted		(1)	NA NA
Number of	f frag n				39
Number of	_	arameters			NA
Akaike (A Bayesian					NA NA
bayesian	(B10)				IVA
Root Mean S	Square E	rror of Ap	proximati	on:	
RMSEA					0.000
90 Percer	nt Confid	dence Inte	erval	0.00	0.020
P-value F	RMSEA <=	0.05			0.998
Standardize	ed Root l	Mean Squar	e Residua	1:	
SRMR					0.011
					****
Parameter E	Estimate	5:			
Informati	ion				Expected
Standard	Errors				Standard
Latent Vari	lables:				
		Estimate	Std.Err	z-value	P(> z )
PosAffect		00 057	4 500	00 570	0 000
PosAFF1		92.657	4.503		0.000
PosAFF2		0.652	0.031		0.000
PosAFF3		0.681	0.031	21.954	0.000
PosAffect PosAFF1		0.554	0.026	21.131	0.000
PosAFF2		0.643	0.020	21.131	0.000
PosAFF3		0.642	0.030	23.351	0.000
PosAffect		0.042	0.021	20.001	0.000
PosAFF1	-	0.508	0.024	21.023	0.000
PosAFF2		0.546	0.027	20.364	
PosAFF3		0.538	0.026	20.777	0.000
Std.lv	Std.all				
92.657	0.872				
0.652	0.876				
0.681	0.907				
0.554	0.881				
0.643	0.894				
0.642	0.936				
0.508	0.887				
0.546	0.868				
0.538	0.879				
Covariances	s:		Q. 1 F	7	D(x L L)
Dog A FF1 1	~~	Estimate	Std.Err	z-value	Y(> Z )
.PosAFF11 .PosAFF1	12	0.235	1.092	0.215	0.830
.Posaff1		0.235	1.092	0.215	0.633
.PosAFF12	~~	0.501	1.040	0.410	0.033
.1 OBHFF 12					

0.003

0.006

.PosAFF13

0.528

.PosAFF21	~~				
.PosAFF		0.006	0.008	0.653	0.514
.PosAFF		0.005	0.008	0.618	0.537
.PosAFF22	~~				
.PosAFF	23	0.011	0.007	1.490	0.136
.PosAFF31	~~				
.PosAFF	32	0.008	0.007	1.059	0.290
.PosAFF	33	0.016	0.008	2.098	0.036
.PosAFF32	~~				
.PosAFF	33	0.005	0.006	0.777	0.437
PosAffec	t1 ~~				
PosAff	ect2	0.479	0.044	10.783	0.000
PosAff	ect3	0.408	0.048	8.451	0.000
PosAffec	t.2 ~~				
PosAff		0.451	0.046	9.831	0.000
	Std.all	0.101	0.010	0.001	0.000
Sta.iv	btu.all				
0.235	0.015				
0.501	0.036				
0.001	0.000				
0.003	0.039				
0.003	0.039				
0.006	0.040				
0.006	0.048				
0.005	0.045				
0.044	0 100				
0.011	0.108				
0.008	0.100				
0.016	0.173				
0.005	0.068				
0.479	0.479				
0.408	0.408				
0.451	0.451				
Intercepts	:				
_		Estimate	Std.Err	z-value	P(> z )
.PosAFF	11	185.500	5.539	33.488	0.000
.PosAFF	21	3.105	0.039	80.038	0.000
.PosAFF	31	3.248	0.039	82.950	0.000
.PosAFF	12	3.253	0.033	99.150	0.000
.PosAFF		3.126	0.038	83.334	0.000
.PosAFF		3.274	0.036	91.558	0.000
.PosAFF		3.302	0.030	110.547	0.000
.Posaff.			0.030	94.373	0.000
		3.094			
.PosAFF		3.280	0.032	102.804	0.000
PosAff		0.000			
PosAff		0.000			
PosAff		0.000			
Std.lv	Std.all				
185.500	1.746				
3.105	4.172				
3.248	4.324				
3.253	5.169				
3.126	4.344				
3.274	4.773				
0.217	1.770				

```
      3.302
      5.763

      3.094
      4.920

      3.280
      5.359

      0.000
      0.000

      0.000
      0.000

      0.000
      0.000
```

#### Variances:

		Estimate	Std.Err	z-value	P(> z )
.PosAFF	'11	2705.921	288.582	9.377	0.000
.PosAFF	21	0.129	0.014	9.222	0.000
.PosAFF	31	0.100	0.013	7.514	0.000
.PosAFF	12	0.089	0.009	9.956	0.000
.PosAFF	22	0.104	0.011	9.351	0.000
.PosAFF	32	0.058	0.009	6.464	0.000
.PosAFF	13	0.070	0.008	8.256	0.000
.PosAFF	23	0.098	0.011	9.207	0.000
.PosAFF	33	0.085	0.010	8.700	0.000
PosAff	ect1	1.000			
PosAff	ect2	1.000			
PosAff	ect3	1.000			
Std.lv	Std.all				
2705.921	0.240				
0.129	0.233				
0.100	0.178				
0.089	0.224				
0.104	0.201				
0.058	0.124				
0.070	0.213				
0.098	0.247				
0.085	0.228				
1.000	1.000				
1.000	1.000				
1.000	1.000				

# Weak: Constrain Loadings

```
> weak <- '
+ ## define latent variables
+ PosAffect1 =~ L1*PosAFF11 + L2*PosAFF21 + L3*PosAFF31
+ PosAffect2 =~ L1*PosAFF12 + L2*PosAFF22 + L3*PosAFF32
+ PosAffect3 =~ L1*PosAFF13 + L2*PosAFF23 + L3*PosAFF33
+
+ ## free latent variances at later times (only set the scale once)
+ PosAffect2 ~~ NA*PosAffect2
+ PosAffect3 ~~ NA*PosAffect3
+
+ ## correlated residuals across time
+ PosAFF11 ~~ PosAFF12 + PosAFF13
+ PosAFF12 ~~ PosAFF13
+ PosAFF21 ~~ PosAFF23
+ PosAFF21 ~~ PosAFF23
+ PosAFF31 ~~ PosAFF32 + PosAFF33
+ PosAFF32 ~~ PosAFF33
+ PosAFF32 ~~ PosAFF33
+ PosAFF32 ~~ PosAFF33
+ PosAFF32 ~~ PosAFF33
+</pre>
```

> weak <- cfa(weak, data=sem\_long, meanstructure=TRUE, std.lv=TRUE)
> summary(weak, standardized=TRUE, fit.measures=TRUE)

laman (0 F 02 1007)			20		
lavaan (0.5-23.1097) conve	rged normall	ly after 8	sy iteratio	ns	
			Used	Total	
Number of observations			368	369	
Estimator			ML		
Minimum Function Test St	atistic	4	425.559		
Degrees of freedom			19		
P-value (Chi-square)			0.000		
Model test baseline model:					
Minimum Function Test St	atistic	20	617.825		
Degrees of freedom			36		
P-value			0.000		
User model versus baseline	model:				
Comparative Fit Index (C	FI)		0.843		
Tucker-Lewis Index (TLI)	·		0.702		
Loglikelihood and Informat	ion Criteria	ì:			
Loglikelihood user model	(HO)		NA		
Loglikelihood unrestrict		1)	NA		
Number of free parameter	a		35		
Number of free parameter Akaike (AIC)	5		NA		
Bayesian (BIC)			NA NA		
D . W . G					
Root Mean Square Error of	Approximatio	on:			
RMSEA			0.241		
90 Percent Confidence In	terval	0.222	0.261		
P-value RMSEA <= 0.05			0.000		
Standardized Root Mean Squ	are Residual	1:			
SRMR			0.186		
Parameter Estimates:					
Information		E	xpected		
Standard Errors			tandard		
Latent Variables:					
Estimat	e Std.Err	z-value	P(> z )		
PosAffect1 =~					
PosAFF11 (L1) 0.5		20.353	0.000		
PosAFF21 (L2) 0.6			0.000		
PosAFF31 (L3) 0.6	66 0.030	22.490	0.000		
PosAffect2 =~	0.4	00 055	0.000		
PosAFF12 (L1) 0.5	91 0.029	20.353	0.000		

PosAFF22	(L2)	0.667	0.029	22.740	0.000
PosAFF32		0.666	0.029	22.740	0.000
PosAffect3		0.000	0.030	22.430	0.000
PosAFF13		0.591	0.029	20.353	0.000
PosAFF23		0.667		22.740	0.000
			0.029		
PosAFF33		0.666	0.030	22.490	0.000
Std.lv	Std.all				
0.591	0.006	<b>:</b>			
0.667	0.891				
0.666	0.890				
0.666	0.890	1			
0.564	0.886	}			
0.636	0.892	•			
0.636	0.932				
0.487	0.872				
0.549	0.870	)			
0.549	0.886	;			
Covariances:					
		Estimate	Std.Err	z-value	P(> z )
.PosAFF11 ~	~				
.PosAFF12	?	1.235	1.862	0.663	0.507
.PosAFF13	}	1.533	1.781	0.861	0.389
.PosAFF12 ~	~				
.PosAFF13	}	0.003	0.006	0.525	0.600
.PosAFF21 ~	~				
.PosAFF22	•	-0.001	0.010	-0.157	0.876
.PosAFF23	}	0.004	0.009	0.393	0.694
.PosAFF22 ~	~				
.PosAFF23	}	0.011	0.007	1.451	0.147
.PosAFF31 ~	~~				
.PosAFF32	)	0.013	0.008	1.535	0.125
.PosAFF33		0.020	0.009	2.241	0.025
.PosAFF32 ~	~	0.020			0.020
.PosAFF33	}	0.005	0.006	0.818	0.414
PosAffect1		0.000	0.000	0.010	0.111
PosAffec		0.463	0.050	9.345	0.000
PosAffec		0.328	0.045	7.296	0.000
PosAffect2		0.020	0.010	7.200	0.000
PosAffec		0.352	0.051	6.875	0.000
	Std.all		0.001	0.070	0.000
202121	2047411				
1.235	0.040	)			
1.533	0.053				
1.000	0.000				
0.003	0.038	}			
-0.001	-0.014				
0.004	0.034				
0.001	0.001				
0.011	0.107	•			
0.011	3.107				
0.013	0.154				
0.020	0.134				
0.020	0.202				
0.005	0.072	)			
0.003	0.072	•			

0.485 0.485 0.398 0.398

0.448 0.448

## Intercepts:

		Estimate	Std.Err	z-value	P(> z )
.PosAFF1	1	185.500	5.514	33.642	0.000
.PosAFF2	?1	3.105	0.039	79.606	0.000
.PosAFF3	31	3.248	0.039	83.222	0.000
.PosAFF1	2	3.253	0.033	97.976	0.000
.PosAFF2	22	3.126	0.037	84.067	0.000
.PosAFF3	32	3.274	0.036	92.051	0.000
.PosAFF1	3	3.302	0.029	113.457	0.000
.PosAFF2	23	3.094	0.033	94.047	0.000
.PosAFF3	33	3.280	0.032	101.557	0.000
PosAffe	ct1	0.000			
PosAffe	ct2	0.000			
PosAffe	ct3	0.000			
Std.lv	Std.all				
185.500	1.754	<u>!</u>			
3.105	4.150	)			
3.248	4.338	3			
3.253	5.107	•			
3.126	4.382	?			
3.274	4.799	)			
3.302	5.914	!			
3.094	4.903	3			
3.280	5.294	!			
0.000	0.000	)			
0.000	0.000	)			
0.000	0.000	)			

#### Variances:

0.061

0.075

0.131

		Estimate	Std.Err	z-value	P(> z )
PosAffe	ct2	0.912	0.095	9.612	0.000
PosAffe	ct3	0.679	0.074	9.113	0.000
.PosAFF1	1	11188.266	824.813	13.565	0.000
.PosAFF2	?1	0.115	0.020	5.647	0.000
.PosAFF3	31	0.117	0.020	5.696	0.000
.PosAFF1	2	0.087	0.009	9.870	0.000
.PosAFF2	22	0.104	0.011	9.527	0.000
.PosAFF3	32	0.061	0.009	6.935	0.000
.PosAFF1	3	0.075	0.008	9.201	0.000
.PosAFF2	23	0.097	0.010	9.292	0.000
.PosAFF3	3	0.082	0.010	8.568	0.000
PosAffe	ct1	1.000			
Std.lv	Std.all	<u>!</u>			
1.000	1.000	)			
1.000	1.000	)			
11188.266	1.000	)			
0.115	0.206	3			
0.117	0.208	3			
0.087	0.215	5			
0.104	0.204	1			

#### Strong: Constrain Intercepts, Slopes

0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

0.097

0.082

0.243

```
> strong <- '
+ ## define latent variables
+ Pos1 =~ L1*PosAFF11 + L2*PosAFF21 + L3*PosAFF31
+ Pos2 =~ L1*PosAFF12 + L2*PosAFF22 + L3*PosAFF32
+ Pos3 =~ L1*PosAFF13 + L2*PosAFF23 + L3*PosAFF33
+ ## free latent variances at later times (only set the scale once)
+ Pos2 ~~ NA*Pos2
+ Pos3 ~~ NA*Pos3
+ ## correlated residuals across time
+ PosAFF11 ~~ PosAFF12 + PosAFF13
+ PosAFF12 ~~ PosAFF13
+ PosAFF21 ~~ PosAFF22 + PosAFF23
+ PosAFF22 ~~ PosAFF23
+ PosAFF31 ~~ PosAFF32 + PosAFF33
+ PosAFF32 ~~ PosAFF33
+ ## constrain intercepts across time
+ PosAFF11 ~ t1*1
+ PosAFF21 ~ t2*1
+ PosAFF31 ~ t3*1
+ PosAFF12 ~ t1*1
+ PosAFF22 ~ t2*1
+ PosAFF32 ~ t3*1
+ PosAFF13 ~ t1*1
```

```
+ PosAFF23 ~ t2*1
```

+ PosAFF33 ~ t3\*1

į

+ ## free latent means at later times (only set the scale once)

- + Pos2 ~ NA\*1
- + Pos3 ~ NA\*1'
- $\verb| > strong <- cfa(strong, data=sem\_long, meanstructure=TRUE, std.lv=TRUE)| \\$
- > summary(strong, standardized=TRUE, fit.measures=TRUE)

lavaan (0.5-23.1097) converged normally after 165 iterations

	Used	Total
Number of observations	368	369
Estimatos	MT	

Estimator ML
Minimum Function Test Statistic 936.404
Degrees of freedom 23
P-value (Chi-square) 0.000

Model test baseline model:

Minimum	Function Test Statistic	2617.825
Degrees	of freedom	36
P-value		0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.646
Tucker-Lewis Index (TLI)	0.446

Loglikelihood and Information Criteria:

Loglikelihood	user model (HO)		NA
Loglikelihood	unrestricted model	(H1)	NA

Number of free parameters 31
Akaike (AIC) NA
Bayesian (BIC) NA

Root Mean Square Error of Approximation:

RMSEA		0.329
90 Percent Confidence Interval	0.311	0.347
P-value RMSEA <= 0.05		0.000

Standardized Root Mean Square Residual:

SRMR 0.498

Parameter Estimates:

Information Expected Standard Errors Standard

Latent Variables:

Estimate Std.Err z-value P(>|z|)

Pos1 =~					
PosAFF11		0.591	0.029	20.342	0.000
PosAFF21		0.666	0.029	22.725	0.000
PosAFF31	(L3)	0.666	0.030	22.484	0.000
Pos2 =~					
PosAFF12		0.591	0.029	20.342	0.000
PosAFF22	(L2)	0.666	0.029	22.725	0.000
PosAFF32	(L3)	0.666	0.030	22.484	0.000
Pos3 =~					
PosAFF13	(L1)	0.591	0.029	20.342	0.000
PosAFF23	(L2)	0.666	0.029	22.725	0.000
PosAFF33	(L3)	0.666	0.030	22.484	0.000
${\it Std.lv}$	Std.all				
0.591	0.003	1			
0.666	0.890	)			
0.666	0.890	)			
0.566	0.887	•			
0.638	0.892	?			
0.637	0.933	1			
0.486	0.869				
0.548	0.869				
0.547	0.885				
~ .					
Covariances:		<b>.</b>	a	-	D(: 1 1)
D4EE11 ^	~~	Estimate	Std.Err	z-value	P(> z )
.PosAFF11 ^					
D = = A E E 1 C		0 407	2 700	0 115	0 000
.PosAFF12		-0.427	3.709	-0.115	0.908
.PosAFF13		-0.427 10.469	3.709 3.625	-0.115 2.888	0.908 0.004
.PosAFF13 .PosAFF12 ^	}	10.469	3.625	2.888	0.004
.PosAFF13 .PosAFF12 ^ .PosAFF13	}				
.PosAFF13 .PosAFF12 .PosAFF13 .PosAFF21	} -~ } -~	0.003	3.625 0.006	2.888 0.452	0.004 0.651
.PosAFF13 .PosAFF12 .PosAFF13 .PosAFF21 .PosAFF22	3 3 -~ ?	10.469 0.003 -0.001	3.625 0.006 0.010	2.888 0.452 -0.149	0.004 0.651 0.881
.PosAFF13 .PosAFF12 .PosAFF13 .PosAFF22 .PosAFF22	3 3 -~ ?	0.003	3.625 0.006	2.888 0.452	0.004 0.651
.PosAFF13 .PosAFF12 .PosAFF21 .PosAFF23 .PosAFF23	3 3 3 2 3	10.469 0.003 -0.001 0.004	3.625 0.006 0.010 0.009	2.888 0.452 -0.149 0.398	0.004 0.651 0.881 0.691
.PosAFF13 .PosAFF12 .PosAFF21 .PosAFF23 .PosAFF23 .PosAFF22	3 3 3 2 3	10.469 0.003 -0.001	3.625 0.006 0.010	2.888 0.452 -0.149	0.004 0.651 0.881
.PosAFF13 .PosAFF13 .PosAFF21 .PosAFF22 .PosAFF22 .PosAFF22 .PosAFF23	3 3 3 2 3 3 3	0.003 -0.001 0.004 0.010	3.625 0.006 0.010 0.009 0.007	2.888 0.452 -0.149 0.398 1.396	0.004 0.651 0.881 0.691 0.163
.PosAFF13 .PosAFF13 .PosAFF21 .PosAFF22 .PosAFF23 .PosAFF23 .PosAFF31 .PosAFF31	3 3 2 3 3 3	10.469 0.003 -0.001 0.004 0.010	3.625 0.006 0.010 0.009 0.007	2.888 0.452 -0.149 0.398 1.396 1.438	0.004 0.651 0.881 0.691 0.163
.PosAFF13 .PosAFF13 .PosAFF21 .PosAFF22 .PosAFF23 .PosAFF23 .PosAFF31 .PosAFF32	3 3 2 3 3 3	0.003 -0.001 0.004 0.010	3.625 0.006 0.010 0.009 0.007	2.888 0.452 -0.149 0.398 1.396	0.004 0.651 0.881 0.691 0.163
.PosAFF13 .PosAFF12 .PosAFF21 .PosAFF23 .PosAFF23 .PosAFF23 .PosAFF31 .PosAFF33 .PosAFF33	3 3 2 3 3 3 3 2 3	10.469 0.003 -0.001 0.004 0.010 0.012 0.020	3.625 0.006 0.010 0.009 0.007 0.008 0.009	2.888 0.452 -0.149 0.398 1.396 1.438 2.299	0.004 0.651 0.881 0.691 0.163 0.150 0.021
.PosAFF13 .PosAFF12 .PosAFF21 .PosAFF23 .PosAFF23 .PosAFF23 .PosAFF31 .PosAFF33 .PosAFF32 .PosAFF32 .PosAFF32	3 3 2 3 3 3 3 2 3	10.469 0.003 -0.001 0.004 0.010	3.625 0.006 0.010 0.009 0.007	2.888 0.452 -0.149 0.398 1.396 1.438	0.004 0.651 0.881 0.691 0.163
.PosAFF13 .PosAFF12 .PosAFF21 .PosAFF22 .PosAFF23 .PosAFF23 .PosAFF31 .PosAFF32 .PosAFF32 .PosAFF33 .PosAFF33	3 3 2 3 3 3 3 2 3	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005	3.625 0.006 0.010 0.009 0.007 0.008 0.009	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817	0.004 0.651 0.881 0.691 0.163 0.150 0.021
.PosAFF13 .PosAFF21 .PosAFF21 .PosAFF22 .PosAFF22 .PosAFF23 .PosAFF31 .PosAFF32 .PosAFF32 .PosAFF33 .PosAFF33 .PosAFF33	3 3 2 3 3 3 3 2 3	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000
POSAFF13 POSAFF21 POSAFF23 POSAFF23 POSAFF23 POSAFF31 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33	3 3 2 3 3 3 3 2 3	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005	3.625 0.006 0.010 0.009 0.007 0.008 0.009	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817	0.004 0.651 0.881 0.691 0.163 0.150 0.021
POSAFF13 POSAFF13 POSAFF21 POSAFF23 POSAFF23 POSAFF31 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33	3 3 2 3 3 3 3 2 3	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469 0.323	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050 0.045	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444 7.183	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000 0.000
POSAFF13 POSAFF13 POSAFF21 POSAFF23 POSAFF23 POSAFF23 POSAFF31 POSAFF33	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469 0.323 0.352	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000
Posaff13 Posaff13 Posaff21 Posaff23 Posaff23 Posaff23 Posaff31 Posaff33	3 3 2 3 3 3 3 2 3 3	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469 0.323 0.352	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050 0.045	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444 7.183	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000 0.000
POSAFF13 POSAFF13 POSAFF21 POSAFF22 POSAFF23 POSAFF23 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POS1 POS2 POS3 POS2 POS3 POS3 Std.1v	Std.all	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469 0.323 0.352	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050 0.045	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444 7.183	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000 0.000
POSAFF13 POSAFF12 POSAFF21 POSAFF22 POSAFF23 POSAFF23 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POS1 POS2 POS3 POS2 POS3 POS2 POS3 Std.1v	Std.all	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469 0.323 0.352	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050 0.045	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444 7.183	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000 0.000
POSAFF13 POSAFF13 POSAFF21 POSAFF22 POSAFF23 POSAFF23 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POS1 POS2 POS3 POS2 POS3 POS3 Std.1v	Std.all	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469 0.323 0.352	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050 0.045	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444 7.183	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000 0.000
POSAFF13 POSAFF21 POSAFF23 POSAFF23 POSAFF23 POSAFF31 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POS1 POS2 POS3 POS2 POS3 Std.1v -0.427 10.469	Std.all -0.007 0.179	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469 0.323 0.352	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050 0.045	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444 7.183	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000 0.000
POSAFF13 POSAFF12 POSAFF21 POSAFF22 POSAFF23 POSAFF23 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POS1 POS2 POS3 POS2 POS3 POS2 POS3 Std.1v	Std.all	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469 0.323 0.352	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050 0.045	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444 7.183	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000 0.000
POSAFF13 POSAFF22 POSAFF23 POSAFF23 POSAFF31 POSAFF31 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POS1 POS2 POS3 POS2 POS3 Std.1v -0.427 10.469 0.003	Std.all -0.007 0.179	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469 0.323 0.352	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050 0.045	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444 7.183	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000 0.000
POSAFF13 POSAFF21 POSAFF23 POSAFF23 POSAFF23 POSAFF31 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POSAFF33 POS1 POS2 POS3 POS2 POS3 Std.1v -0.427 10.469	Std.all -0.007 0.179	10.469 0.003 -0.001 0.004 0.010 0.012 0.020 0.005 0.469 0.323 0.352	3.625 0.006 0.010 0.009 0.007 0.008 0.009 0.006 0.050 0.045	2.888 0.452 -0.149 0.398 1.396 1.438 2.299 0.817 9.444 7.183	0.004 0.651 0.881 0.691 0.163 0.150 0.021 0.414 0.000 0.000

0.010	0.103
0.012 0.020	0.145 0.207
0.005	0.072
0.490 0.393	0.490 0.393
0.447	0.447

### Intercepts:

		Estimate	${\it Std.Err}$	z-value	P(> z )
.PosAFF11	(t1)	3.242	0.035	92.993	0.000
.PosAFF21	(t2)	3.096	0.037	82.836	0.000
.PosAFF31	(t3)	3.256	0.038	86.197	0.000
.PosAFF12	(t1)	3.242	0.035	92.993	0.000
.PosAFF22	(t2)	3.096	0.037	82.836	0.000
.PosAFF32	(t3)	3.256	0.038	86.197	0.000
.PosAFF13	(t1)	3.242	0.035	92.993	0.000
.PosAFF23	(t2)	3.096	0.037	82.836	0.000
.PosAFF33	(t3)	3.256	0.038	86.197	0.000
Pos2		0.031	0.056	0.551	0.581
Pos3		0.022	0.058	0.382	0.702
Pos1		0.000			
Std.lv S	td.all	1			

3.242 0.015 3.096 4.140 4.351 3.256 3.242 5.077 3.096 4.335 3.256 4.765 3.242 5.793 4.912 3.096 5.266 3.256 0.032 0.032 0.027 0.027 0.000 0.000

# Variances:

vailances.				
	Estimate	Std.Err	z-value	P(> z )
Pos2	0.917	0.095	9.628	0.000
Pos3	0.676	0.074	9.088	0.000
.PosAFF11	44406.486	3273.692	13.565	0.000
.PosAFF21	0.116	0.020	5.677	0.000
.PosAFF31	0.117	0.020	5.700	0.000
.PosAFF12	0.087	0.009	9.861	0.000
.PosAFF22	0.104	0.011	9.540	0.000
.PosAFF32	0.061	0.009	6.925	0.000
.PosAFF13	0.077	0.008	9.274	0.000
.PosAFF23	0.098	0.010	9.327	0.000
.PosAFF33	0.083	0.010	8.571	0.000
Pos1	1.000			
0+17- 0+1-7	7			

 Std.lv
 Std.all

 1.000
 1.000

 1.000
 1.000

 44406.486
 1.000

```
0.116
        0.207
0.117
         0.208
0.087
        0.214
0.104
        0.203
        0.130
0.061
0.077
        0.245
0.098
        0.246
0.083
         0.216
1.000
         1.000
```

> anova(weak, strong)

```
Chi Square Difference Test

Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)

weak 19 425.56

strong 23 936.40 510.85 4 < 2.2e-16 ***

---

Signif. codes:
0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Yes, there is strong measurement invariance and we can run growth models.

### 3 Second Order Growth Model

```
> sec.order <- '
+ ## define latent variables
+ Pos1 = NA*PosAFF11 + L1*PosAFF11 + L2*PosAFF21 + L3*PosAFF31
+ Pos2 = NA*PosAFF12 + L1*PosAFF12 + L2*PosAFF22 + L3*PosAFF32
+ Pos3 = NA*PosAFF13 + L1*PosAFF13 + L2*PosAFF23 + L3*PosAFF33
+ ## intercepts
+ PosAFF11 ~ t1*1
+ PosAFF21 ~ t2*1
+ PosAFF31 ~ t3*1
+ PosAFF12 ~ t1*1
+ PosAFF22 ~ t2*1
+ PosAFF32 ~ t3*1
+ PosAFF13 ~ t1*1
+ PosAFF23 ~ t2*1
+ PosAFF33 ~ t3*1
+ ## correlated residuals across time
+ PosAFF11 ~~ PosAFF12 + PosAFF13
+ PosAFF12 ~~ PosAFF13
+ PosAFF21 ~~ PosAFF22 + PosAFF23
+ PosAFF22 ~~ PosAFF23
+ PosAFF31 ~~ PosAFF32 + PosAFF33
+ PosAFF32 ~~ PosAFF33
+ ## latent variable intercepts
```

```
+ Pos1 ~ 0*1
+ Pos2 ~ 0*1
+ Pos3 ~ 0*1
+ #model constraints for effect coding
+ ## loadings must average to 1
+ L1 == 3 - L2 - L3
+ ## means must average to 0
+ t1 == 0 - t2 - t3
+ i =~ 1*Pos1 + 1*Pos2 + 1*Pos3
+ s =~ 0*Pos1 + 1*Pos2 + 2*Pos3 '
> fit.sec.order <- growth(sec.order, data=sem_long, missing = "ML")</pre>
> summary(fit.sec.order, fit.measures=TRUE)
lavaan (0.5-23.1097) converged normally after 135 iterations
 Number of observations
                                                    369
 Number of missing patterns
                                                      2
 Estimator
                                                    ML
 Minimum Function Test Statistic
                                               934.954
 Degrees of freedom
                                                     24
 P-value (Chi-square)
                                                 0.000
Model test baseline model:
 Minimum Function Test Statistic
                                              2617.825
 Degrees of freedom
                                                     36
 P-value
                                                 0.000
User model versus baseline model:
  Comparative Fit Index (CFI)
                                                 0.647
  Tucker-Lewis Index (TLI)
                                                 0.471
Loglikelihood and Information Criteria:
 Loglikelihood user model (HO)
                                                    NA
 Loglikelihood unrestricted model (H1)
                                                    NA
 Number of free parameters
                                                     30
  Akaike (AIC)
                                                    NA
 Bayesian (BIC)
                                                    NA
Root Mean Square Error of Approximation:
                                                 0.321
 RMSEA
                                          0.303 0.338
 90 Percent Confidence Interval
 P-value RMSEA <= 0.05
                                                  0.000
Standardized Root Mean Square Residual:
  SRMR
                                                  0.494
```

Parameter Estimates:

Observed Standard

#### Information Standard Errors

Latell Valiables.	Latent	Variables.
-------------------	--------	------------

Education variable	100.	Estimate	Std.Err	z-value	P(> z )
Pos1 =~					
PosAFF11	(L1)	0.922	0.018	50.272	0.000
PosAFF21	(L2)	1.039	0.018	58.235	0.000
PosAFF31	(L3)	1.039	0.017	61.108	0.000
Pos2 =~	,				
PosAFF12	(L1)	0.922	0.018	50.272	0.000
PosAFF22	(L2)	1.039	0.018	58.235	0.000
PosAFF32	(L3)	1.039	0.017	61.108	0.000
Pos3 =~	(20)	1.000	0.017	01.100	0.000
PosAFF13	(L1)	0.922	0.018	50.272	0.000
PosAFF23	(L2)	1.039	0.018	58.235	0.000
PosAFF33	(L3)	1.039	0.017	61.108	0.000
i =~	(LU)	1.003	0.017	01.100	0.000
r - Pos1		1.000			
Pos2		1.000			
Pos3		1.000			
s =~		0 000			
Pos1		0.000			
Pos2		1.000			
Pos3		2.000			
Covariances:					
covariances.		Estimate	Std Err	z-value	P(> z )
.PosAFF11 ~~		Estimate	Sta.EII	z varue	I (>121)
.PosAFF12		-0.242	6.436	-0.038	0.970
.PosAFF13		10.274	6.282	1.635	0.370
.PosAFF12 ~~		10.274	0.202	1.035	0.102
.PosAFF13		0.003	0.006	0.450	0.653
.PosAFF21 ~~		0.003	0.000	0.430	0.000
		0 001	0.000	0 140	0.000
.PosAFF22		-0.001	0.009	-0.148	0.883
.PosAFF23		0.004	0.009	0.396	0.692
.PosAFF22 ~~		0.010	0 007	4 000	0.407
.PosAFF23		0.010	0.007	1.382	0.167
.PosAFF31 ~~		2 212	0.000	4 404	0.450
.PosAFF32		0.012	0.009	1.431	0.153
.PosAFF33		0.020	0.009	2.271	0.023
.PosAFF32 ~~					
.PosAFF33		0.005	0.006	0.822	0.411
i ~~					
S		-0.060	0.023	-2.549	0.011
Intercepts:					
intercepts.		Estimate	Std.Err	z-value	P(> z )
.PosAFF11	(t1)	0.292	0.055	5.276	0.000
.PosAFF21	(t2)	-0.226	0.057	-3.976	0.000
.PosAFF31	(t2)	-0.226	0.054	-1.215	0.224
.PosAFF12	(t3)	0.292	0.054	5.276	0.224
.PosAFF22					0.000
	(t2)	-0.226	0.057	-3.976	
.PosAFF32	(t3)	-0.066	0.054	-1.215	0.224
.PosAFF13	(t1)	0.292	0.055	5.276	0.000
.PosAFF23	(t2)	-0.226	0.057	-3.976	0.000
.PosAFF33	(t3)	-0.066	0.054	-1.215	0.224

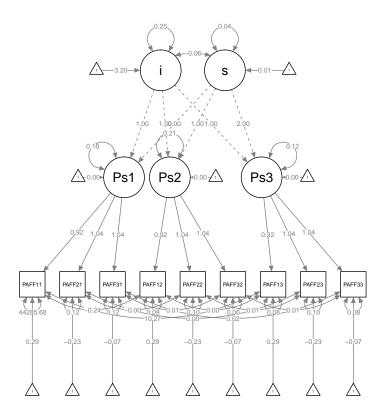
Pos1	0.000			
Pos2	0.000			
Pos3	0.000			
i	3.202	0.035	90.708	0.000
s	0.007	0.019	0.352	0.724
Variances:				
	Estimate	Std.Err	z-value	P(> z )
.PosAFF11	44285.676	0.001	29682467.681	0.000
.PosAFF21	0.116	0.020	5.673	0.000
.PosAFF31	0.117	0.021	5.685	0.000
.PosAFF12	0.087	0.009	9.901	0.000
.PosAFF22	0.104	0.011	9.536	0.000
.PosAFF32	0.061	0.009	6.917	0.000
.PosAFF13	0.077	0.009	8.473	0.000
.PosAFF23	0.098	0.011	9.278	0.000
.PosAFF33	0.083	0.010	8.575	0.000
Pos1	0.159	0.043	3.729	0.000
Pos2	0.208	0.023	9.233	0.000
Pos3	0.121	0.034	3.583	0.000
i	0.252	0.044	5.777	0.000
s	0.036	0.018	1.951	0.051
Constraints:				
			Sla	ck
L1 - (3-L2-L3)			0.	000
t1 - (0-t2-t3)			0.	000

# Plotting

<sup>&</sup>gt; library(semPlot)

<sup>&</sup>gt; library(semTools)

<sup>&</sup>gt; semPaths(fit.sec.order, whatLabels = "est", panelGroups = TRUE)



## Comparing with Normal Growth Model

```
> simple.growth <- 'i_1 = ~ 1*PosAFF11 + 1*PosAFF21 + 1*PosAFF31

+ s_1 = ~ 0*PosAFF11 + 1*PosAFF21 + 2*PosAFF31

+ i_2 = ~ 1*PosAFF12 + 1*PosAFF22 + 1*PosAFF32

+ s_2 = ~ 0*PosAFF12 + 1*PosAFF22 + 2*PosAFF32

+ i_3 = ~ 1*PosAFF13 + 1*PosAFF23 + 1*PosAFF33

+ s_3 = ~ 0*PosAFF13 + 1*PosAFF23 + 2*PosAFF33

+ '
> fit.growth <- growth(simple.growth, data=sem_long)

> summary(fit.growth, fit.measures = TRUE)
```

lavaan (0.5-23.1097) converged normally after 169 iterations

Number of observations	<i>Used</i> 368	Total 369
Estimator	ML	
Minimum Function Test Statistic	734.084	
Degrees of freedom	18	
P-value (Chi-square)	0.000	

Model test baseline model:

Minimum Function Test Statistic 2617.825

Degrees of free	dom			36
P-value				0.000
User model versus	baseline mo	del:		
Comparative Fit	Index (CFI)			0.723
Tucker-Lewis Inc				0.445
Loglikelihood and	Information	Criteria	ı <b>:</b>	
Loglikelihood u	ser model (H	r()		NA
Loglikelihood u			)	NA NA
.0		•	•	
Number of free p	parameters			36
Akaike (AIC)				NA
Bayesian (BIC)				NA
Doot Many Courses	E			
Root Mean Square	Error of App	Proximatic	т:	
RMSEA				0.329
90 Percent Conf.	idence Inter	val	0.309	
P-value RMSEA <				0.000
Standardized Root	Mean Square	Residual	:	
GDMD.				0.750
SRMR				0.753
Parameter Estimate	og :			
ratameter cstimat				
ralametel Estimato	ED.			
Information	55.		E:	xpected
	-s.			xpected tandard
Information				_
Information			S	tandard
Information Standard Errors Latent Variables:	Estimate	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 =~	Estimate	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 =~ PosAFF11	Estimate	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21	Estimate 1.000 1.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31	Estimate	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~	Estimate	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11	Estimate	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF11 PosAFF21	Estimate	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11	Estimate	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF21 PosAFF21 PosAFF31	Estimate	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF21 PosAFF31 i_2 = ~	Estimate  1.000 1.000 1.000 0.000 1.000 2.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF21 PosAFF31 i_2 = ~ PosAFF12	Estimate  1.000 1.000 1.000 0.000 1.000 2.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF21 PosAFF31 i_2 = ~ PosAFF12 PosAFF22	Estimate  1.000 1.000 1.000 0.000 1.000 2.000 1.000 1.000 1.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF21 PosAFF31 i_2 = ~ PosAFF12 PosAFF22 PosAFF32	Estimate  1.000 1.000 1.000 0.000 1.000 2.000 1.000 1.000 1.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF21 PosAFF31 i_2 = ~ PosAFF12 PosAFF22 PosAFF32 s_2 = ~ PosAFF12 PosAFF12 PosAFF32	Estimate  1.000 1.000 1.000 0.000 1.000 2.000 1.000 1.000 0.000 1.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31  s_1 = ~ PosAFF11 PosAFF21 PosAFF31  i_2 = ~ PosAFF12 PosAFF22 PosAFF32  s_2 = ~ PosAFF12 PosAFF32  s_1 = ~ PosAFF32  s_2 = ~ PosAFF32  PosAFF32	Estimate  1.000 1.000 1.000 0.000 2.000 1.000 1.000 1.000 0.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31  s_1 = ~ PosAFF11 PosAFF31  i_2 = ~ PosAFF12 PosAFF22 PosAFF32  s_2 = ~ PosAFF12 PosAFF32  i_3 = ~	Estimate  1.000 1.000 1.000 0.000 1.000 2.000 1.000 1.000 0.000 1.000 2.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF31 i_2 = ~ PosAFF12 PosAFF22 PosAFF32 s_2 = ~ PosAFF32 s_2 = ~ PosAFF32 i_3 = ~ PosAFF32 i_3 = ~ PosAFF13	Estimate  1.000 1.000 1.000 0.000 1.000 2.000 1.000 1.000 1.000 2.000 1.000 1.000 1.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF21 PosAFF31 i_2 = ~ PosAFF12 PosAFF22 PosAFF22 PosAFF32 s_2 = ~ PosAFF12 PosAFF32 i_3 = ~ PosAFF13 PosAFF13 PosAFF13	Estimate  1.000 1.000 1.000 0.000 1.000 2.000 1.000 1.000 1.000 2.000 1.000 1.000 1.000 1.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF21 PosAFF31 i_2 = ~ PosAFF12 PosAFF22 PosAFF32 s_2 = ~ PosAFF12 PosAFF32 s_1 = ~ PosAFF13 PosAFF32 i_3 = ~ PosAFF33 PosAFF33	Estimate  1.000 1.000 1.000 0.000 1.000 2.000 1.000 1.000 1.000 2.000 1.000 1.000 1.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31  s_1 = ~ PosAFF11 PosAFF21 PosAFF31  i_2 = ~ PosAFF12 PosAFF22 PosAFF32  s_2 = ~ PosAFF12 PosAFF32  i_3 = ~ PosAFF33 PosAFF33 S_3 = ~	Estimate  1.000 1.000 1.000 0.000 1.000 2.000 1.000 1.000 1.000 1.000 2.000 1.000 1.000 1.000 1.000 1.000	Std.Err	S	tandard
Information Standard Errors  Latent Variables:  i_1 = ~ PosAFF11 PosAFF21 PosAFF31 s_1 = ~ PosAFF11 PosAFF21 PosAFF31 i_2 = ~ PosAFF12 PosAFF22 PosAFF32 s_2 = ~ PosAFF12 PosAFF32 s_1 = ~ PosAFF13 PosAFF32 i_3 = ~ PosAFF33 PosAFF33	Estimate  1.000 1.000 1.000 0.000 1.000 2.000 1.000 1.000 1.000 2.000 1.000 1.000 1.000 1.000	Std.Err	S	tandard

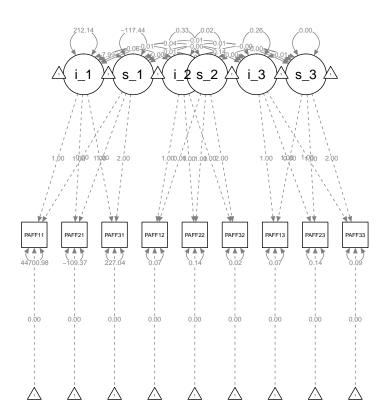
2.000

PosAFF33

Covariances:				
	Estimate	Std.Err	z-value	P(> z )
i_1 ~~				
s_1	7.985	5.307	1.505	0.132
i_2	0.061	0.028	2.168	0.030
s_2	-0.005		-0.627	0.531
i_3	0.042		1.643	0.100
s_3	-0.008	0.009	-0.912	0.362
s_1 ~~	0 005	0.046	0.000	0.770
i_2	0.005		0.292	0.770
s_2 i_3	0.007		1.353 -0.319	0.176 0.749
s_3	-0.005 0.010	0.014	1.964	0.749
i_2 ~~	0.010	0.005	1.304	0.000
s_2	0.014	0.013	1.065	0.287
i_3	0.141		7.035	0.000
s_3	-0.002	0.006	-0.383	0.702
s_2 ~~	0.002	0.000	0.000	0.702
i_3	0.005	0.006	0.798	0.425
s_3	0.002	0.002	1.145	0.252
i_3 ~~				
s_3	0.007	0.013	0.545	0.586
Intercepts:				
	Estimate	Std.Err	z-value	P(> z )
.PosAFF11	0.000			
.PosAFF21	0.000			
.PosAFF31	0.000			
.PosAFF12	0.000			
.PosAFF22	0.000			
.PosAFF32	0.000			
.PosAFF13	0.000			
.PosAFF23	0.000			
.PosAFF33	0.000		45 440	
i_1	2.098	0.044		0.000
s_1	0.109			0.000
i_2	3.223	0.033	98.490	0.000
s_2	0.020	0.010	1.969	0.049 0.000
i_3 s_3	3.263 -0.015		109.830 -1.496	0.000
5_0	0.013	0.010	1.430	0.133
Variances:				
, 42 242000	Estimate	Std.Err	z-value	P(> z )
.PosAFF11	44700.976			0.000
.PosAFF21	-109.366	8.471	-12.911	0.000
.PosAFF31	227.036		13.029	0.000
.PosAFF12	0.069			0.004
.PosAFF22	0.139	0.015	8.956	0.000
.PosAFF32	0.024	0.026	0.950	0.342
.PosAFF13	0.070	0.023	2.986	0.003
.PosAFF23	0.137	0.015	9.196	0.000
.PosAFF33	0.085		3.386	0.001
i_1	212.144		11.675	0.000
s_1	-117.437		-12.113	0.000
i_2	0.332		9.560	0.000
s_2	0.015		1.262	0.207
i_3	0.262	0.030	8.607	0.000

s\_3 0.000 0.012 0.026 0.979

> semPaths(fit.growth, whatLabels = "est", panelGroups = TRUE)



#### > anova(fit.sec.order, fit.growth)

#### Chi Square Difference Test

Df AIC BIC Chisq Chisq diff Df diff
fit.growth 18 734.08
fit.sec.order 24 934.95 200.87 6
Pr(>Chisq)
fit.growth
fit.sec.order < 2.2e-16 \*\*\*
--Signif. codes:
0 '\*\*\*' 0.001 '\*\*' 0.05 '.' 0.1 ' ' 1

# 4 Multiple Groups

```
> groupmodel.1 <- ' visual = ~ x1 + x2 + x3
+ textual = ~ x4 + x5 + x6
```

```
> summary(fit.1, standardized=TRUE, fit.measures=TRUE)
```

Number of observations per group       1       146         2       155         Estimator       ML         Minimum Function Test Statistic       105.795         Degrees of freedom       48         P-value (Chi-square)       0.000         Chi-square for each group:       61.395         1       61.395         2       44.401         Model test baseline model:       967.228         Degrees of freedom       72         P-value       0.000         User model versus baseline model:       Comparative Fit Index (CFI)       0.935         Tucker-Lewis Index (TLI)       0.903         Loglikelihood and Information Criteria:       Loglikelihood unrestricted model (H1)       -3698.817         Loglikelihood unrestricted model (H1)       -3645.919         Number of free parameters       60         Akaike (AIC)       7517.633         Bayesian (BIC)       7549.774	
Estimator ML Minimum Function Test Statistic 105.795 Degrees of freedom 48 P-value (Chi-square) 0.000  Chi-square for each group:  1 61.395 2 44.401  Model test baseline model:  Minimum Function Test Statistic 967.228 Degrees of freedom 72 P-value 0.000  User model versus baseline model:  Comparative Fit Index (CFI) 0.935 Tucker-Lewis Index (TLI) 0.903  Loglikelihood and Information Criteria:  Loglikelihood user model (H0) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7740.060 Sample-size adjusted Bayesian (BIC) 7549.774	
Estimator ML Minimum Function Test Statistic 105.795 Degrees of freedom 48 P-value (Chi-square) 0.000  Chi-square for each group:  1 61.395 2 44.401  Model test baseline model:  Minimum Function Test Statistic 967.228 Degrees of freedom 72 P-value 0.000  User model versus baseline model:  Comparative Fit Index (CFI) 0.935 Tucker-Lewis Index (TLI) 0.903  Loglikelihood and Information Criteria:  Loglikelihood user model (H0) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7549.774	
Minimum Function Test Statistic       105.795         Degrees of freedom       48         P-value (Chi-square)       0.000         Chi-square for each group:       61.395         1       61.395         2       44.401         Model test baseline model:       967.228         Minimum Function Test Statistic       967.228         Degrees of freedom       72         P-value       0.000         User model versus baseline model:         Comparative Fit Index (CFI)       0.935         Tucker-Lewis Index (TLI)       0.903         Loglikelihood and Information Criteria:         Loglikelihood user model (H0)       -3698.817         Loglikelihood unrestricted model (H1)       -3645.919         Number of free parameters       60         Akaike (AIC)       7517.633         Bayesian (BIC)       7740.060         Sample-size adjusted Bayesian (BIC)       7549.774	
Degrees of freedom         48           P-value (Chi-square)         0.000           Chi-square for each group:         61.395           1         61.395           2         44.401           Model test baseline model:         967.228           Minimum Function Test Statistic         967.228           Degrees of freedom         72           P-value         0.000           User model versus baseline model:         Comparative Fit Index (CFI)         0.935           Tucker-Lewis Index (TLI)         0.903           Loglikelihood and Information Criteria:         Loglikelihood unrestricted model (H0)         -3698.817           Loglikelihood unrestricted model (H1)         -3645.919           Number of free parameters         60           Akaike (AIC)         7517.633           Bayesian (BIC)         7740.060           Sample-size adjusted Bayesian (BIC)         7549.774	
P-value (Chi-square)         0.000           Chi-square for each group:         61.395           1         61.395           2         44.401           Model test baseline model:         967.228           Degrees of freedom         72           P-value         0.000           User model versus baseline model:         0.935           Tucker-Lewis Index (CFI)         0.935           Tucker-Lewis Index (TLI)         0.903           Loglikelihood and Information Criteria:         0.903           Loglikelihood user model (H0)         -3698.817           Loglikelihood unrestricted model (H1)         -3645.919           Number of free parameters         60           Akaike (AIC)         7517.633           Bayesian (BIC)         7549.774	
Chi-square for each group:  1 61.395 2 44.401  Model test baseline model:  Minimum Function Test Statistic 967.228 Degrees of freedom 72 P-value 0.000  User model versus baseline model:  Comparative Fit Index (CFI) 0.935 Tucker-Lewis Index (TLI) 0.903  Loglikelihood and Information Criteria:  Loglikelihood user model (HO) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7740.060 Sample-size adjusted Bayesian (BIC) 7549.774	
1 61.395 2 44.401  odel test baseline model:  Minimum Function Test Statistic 967.228 Degrees of freedom 72 P-value 0.000  ser model versus baseline model:  Comparative Fit Index (CFI) 0.935 Tucker-Lewis Index (TLI) 0.903  oglikelihood and Information Criteria:  Loglikelihood user model (H0) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7549.774	
2 44.401  odel test baseline model:  Minimum Function Test Statistic 967.228  Degrees of freedom 72  P-value 0.000  ser model versus baseline model:  Comparative Fit Index (CFI) 0.935  Tucker-Lewis Index (TLI) 0.903  oglikelihood and Information Criteria:  Loglikelihood user model (H0) -3698.817  Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60  Akaike (AIC) 7517.633  Bayesian (BIC) 7740.060  Sample-size adjusted Bayesian (BIC) 7549.774	
Minimum Function Test Statistic 967.228 Degrees of freedom 72 P-value 0.000  Ser model versus baseline model:  Comparative Fit Index (CFI) 0.935 Tucker-Lewis Index (TLI) 0.903  coglikelihood and Information Criteria:  Loglikelihood user model (HO) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7549.774	
Minimum Function Test Statistic  Degrees of freedom  P-value  Comparative Fit Index (CFI)  Tucker-Lewis Index (TLI)  Oglikelihood and Information Criteria:  Loglikelihood user model (HO)  Loglikelihood unrestricted model (H1)  Number of free parameters  Akaike (AIC)  Bayesian (BIC)  Sample-size adjusted Bayesian (BIC)  72  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.00000  0.00000  0.000000	
Degrees of freedom 72 P-value 0.000  er model versus baseline model:  Comparative Fit Index (CFI) 0.935 Tucker-Lewis Index (TLI) 0.903  eglikelihood and Information Criteria:  Loglikelihood user model (H0) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7549.774	
P-value 0.000  ser model versus baseline model:  Comparative Fit Index (CFI) 0.935 Tucker-Lewis Index (TLI) 0.903  oglikelihood and Information Criteria:  Loglikelihood user model (H0) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7740.060 Sample-size adjusted Bayesian (BIC) 7549.774	
Ser model versus baseline model:  Comparative Fit Index (CFI) 0.935 Tucker-Lewis Index (TLI) 0.903  oglikelihood and Information Criteria:  Loglikelihood user model (HO) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7740.060 Sample-size adjusted Bayesian (BIC) 7549.774	
Comparative Fit Index (CFI) 0.935 Tucker-Lewis Index (TLI) 0.903  oglikelihood and Information Criteria:  Loglikelihood user model (H0) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7740.060 Sample-size adjusted Bayesian (BIC) 7549.774	
Tucker-Lewis Index (TLI)  O.903  Oglikelihood and Information Criteria:  Loglikelihood user model (H0)  Loglikelihood unrestricted model (H1)  Number of free parameters  Akaike (AIC)  Bayesian (BIC)  Sample-size adjusted Bayesian (BIC)  7.549.774	
Loglikelihood user model (HO) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7740.060 Sample-size adjusted Bayesian (BIC) 7549.774	
Loglikelihood user model (HO) -3698.817 Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7740.060 Sample-size adjusted Bayesian (BIC) 7549.774	
Loglikelihood unrestricted model (H1) -3645.919  Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7740.060 Sample-size adjusted Bayesian (BIC) 7549.774	
Number of free parameters 60 Akaike (AIC) 7517.633 Bayesian (BIC) 7740.060 Sample-size adjusted Bayesian (BIC) 7549.774	
Akaike (AIC)       7517.633         Bayesian (BIC)       7740.060         Sample-size adjusted Bayesian (BIC)       7549.774	
Bayesian (BIC) 7740.060 Sample-size adjusted Bayesian (BIC) 7549.774	
Sample-size adjusted Bayesian (BIC) 7549.774	
oot Mean Square Error of Approximation:	
RMSEA 0.089	
90 Percent Confidence Interval 0.066 0.113	
P-value RMSEA <= 0.05 0.004	
tandardized Root Mean Square Residual:	
SRMR 0.063	
Parameter Estimates:	
Information Expected	
Standard Errors Standard	

Group 1 [1]:

T	77 . 7 7
Latent	Variables:

		Estimate	Std.Err	z-value	P(> z )
visual =	~				
<i>x</i> 1		1.000			
<i>x</i> 2		0.511	0.175	2.924	0.003
хЗ		0.873	0.221	3.950	0.000
textual	=~				
x4		1.000			
x5		1.159	0.103	11.204	0.000
x6		0.930	0.086	10.812	0.000
speed =~					
<i>x</i> 7		1.000			
<i>x8</i>		1.480	0.304	4.863	0.000
x9		0.679	0.139	4.895	0.000
Std.lv	Std.all				
0.808	0.725				
0.413	0.329				
0.706	0.602				
0.927	0.835				
1.075	0.857				
0.863	0.818				
0.646	0.644				
0.957	0.864				
0.439	0.490				

## Covariances:

		Estimate	Std.Err	z-value	P(> z )	
visual ~	~					
textua	1	0.327	0.094	3.491	0.000	
speed		0.157	0.068	2.304	0.021	
textual	~~					
speed		0.098	0.062	1.584	0.113	
Std.lv	Std.all					
0.436	0.436					
0.301	0.301					
0.164	0.164					

# Intercepts:

	Estimate	${\it Std.Err}$	z-value	P(> z )
.x1	5.033	0.092	54.589	0.000
.x2	6.231	0.104	59.969	0.000
.x3	2.462	0.097	25.368	0.000
.x4	2.918	0.092	31.741	0.000
.x5	4.264	0.104	41.082	0.000
.x6	2.173	0.087	24.887	0.000
.x7	4.056	0.083	48.796	0.000
.x8	5.571	0.092	60.757	0.000
.x9	5.326	0.074	71.828	0.000
visual	0.000			

textua	1	0.000
speed		0.000
Std.lv	${\it Std.all}$	
5.033	4.518	
6.231	4.963	
2.462	2.099	
2.918	2.627	
4.264	3.400	
2.173	2.060	
4.056	4.038	
5.571	5.028	
5.326	5.945	
0.000	0.000	
0.000	0.000	
0.000	0.000	

#### Variances:

	Estimate	Std.Err	z-value	P(> z )
.x1	0.589	0.168	3.504	0.000
.x2	1.406	0.175	8.022	0.000
.x3	0.878	0.157	5.573	0.000
.x4	0.374	0.067	5.535	0.000
.x5	0.417	0.084	4.948	0.000
.x6	0.369	0.062	5.938	0.000
.x7	0.591	0.103	5.713	0.000
.x8	0.312	0.171	1.828	0.068
.x9	0.610	0.080	7.588	0.000
visual	0.653	0.200	3.270	0.001
textual	0.860	0.147	5.855	0.000
speed	0.418	0.123	3.404	0.001
Std ly Std all				

Std.lv Std.all 0.474 0.589 1.406 0.892 0.878 0.638 0.303 0.374 0.417 0.265 0.369 0.331 0.591 0.586 0.312 0.254 0.760 0.610 1.000 1.000 1.000 1.000 1.000 1.000

# Group 2 [2]:

#### Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
visual =~				
x1	1.000			
x2	0.557	0.108	5.153	0.000
х3	0.586	0.107	5.470	0.000
textual =~				
x4	1.000			
x5	1.075	0.083	12.898	0.000
x6	0.928	0.071	12.996	0.000
speed =~				

<i>x</i> 7		1.000			
x8		1.076	0.227	4.746	0.000
х9		1.691	0.354	4.773	0.000
Std.lv	Std.all				
0.988	0.820				
0.550	0.511				
0.579	0.553				
1.041	0.872				
1.118	0.850				
0.966	0.856				
0.538	0.469				
0.579	0.638				
0.910	0.827				
Covariance	s:				
	· ~	Estimate	Std.Err	z-value	P(> z )
visual ~	7	0 510	0 444	A	0 000
textua	.1	0.519	0.114		
speed	~~	0.319	0.086	3.718	0.000
textual		2 222	0 000	0.070	0 000
speed	a	0.202	0.068	2.973	0.003
Std.1v	Std.all				
0.504	0.504				
0.599	0.599				
0.361	0.361				
Intercepts	:				
-		Estimate	Std.Err	z-value	P(> z )
.x1		4.844	0.097	50.058	0.000
.x2		5.953	0.087	68.783	0.000
. x3		2.051	0.084	24.367	0.000
. x4		3.196	0.096	33.349	0.000
.x5		4.413	0.106	41.757	0.000
.x6		2.197	0.091	24.233	0.000
.x7		4.308	0.092	46.747	0.000
.x8		5.486	0.073	75.197	0.000
.x9		5.419	0.088	61.316	0.000
visual		0.000			
textua		0.000			
speed		0.000			
Std.lv	Std.all				
4.844	4.021				
5.953	5.525				
2.051	1.957				
3.196	2.679				
4.413	3.354				
2.197	1.946				
4.308	3.755				
5.486	6.040				
5.419	4.925				
0.000	0.000				
0.000	0.000				
0.000	0.000				

#### Variances: Estimate Std.Err z-value P(>|z|)0.474 0.144 3.284 0.001 .x1 .x2 0.858 0.110 7.800 0.000 0.762 0.101 7.511 0.000 .x3 0.340 0.064 5.311 0.000 .x4 0.081 5.956 0.000 .x5 0.481 .x6 0.341 0.059 5.803 0.000 1.027 0.128 8.018 0.000 .x7 0.489 0.075 6.547 0.000 .x8 0.002 .x9 0.382 0.125 3.053 visual 0.977 0.206 4.754 0.000 0.000 textual 1.083 0.165 6.560 0.290 2.695 0.007 speed 0.108 Std.lv Std.all 0.474 0.327 0.858 0.739 0.762 0.694 0.340 0.239 0.481 0.278 0.341 0.268 1.027 0.780 0.489 0.593 0.382 0.316 1.000 1.000 1.000 1.000 1.000 1.000

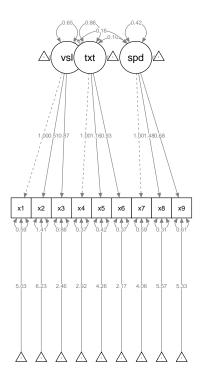
## **Plotting**

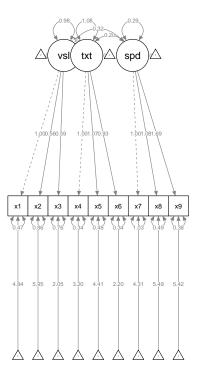
```
> library(semPlot)
```

<sup>&</sup>gt; library(semTools)

<sup>&</sup>gt; semPaths(fit.1, whatLabels = "est", panelGroups = TRUE)

2





### **Constraining Parameters**

## 4.0.1 Constraining Means

#### Constraining Visual Means

### lavaan (0.5-23.1097) converged normally after 53 iterations

Number of observations per group	
1	146
2	155
Estimator	ML
Minimum Function Test Statistic	108.223
Degrees of freedom	50
P-value (Chi-square)	0.000

Chi-square for each group:

1				62.988
2				45.235
Model test baselin	e model:			
Minimum Function	Test Stat	istic		967.228
Degrees of freed	om			72
P-value				0.000
User model versus	baseline m	odel:		
Comparative Fit	Index (CFI	.)		0.935
Tucker-Lewis Ind	ex (TLI)			0.906
Loglikelihood and	Informatio	n Criteri	a:	
Loglikelihood us	er model (	(HO)	_	3700.031
Loglikelihood un				3645.919
Number of force				FO
Number of free parts (AIC)	arameters			58 7516.061
Bayesian (BIC)				7731.074
Sample-size adju	sted Bayes	ian (BIC)		7547.131
Root Mean Square E	rror of Ap	proximati	on:	
RMSEA				0.088
90 Percent Confi	dence Inte	rval	0.06	
P-value RMSEA <=	0.05			0.004
Standardized Root	Mean Squar	e Residua	1:	
SRMR				0.065
Parameter Estimate	s:			
Information				Expected
Standard Errors				Standard
Group 1 [1]:				
Latent Variables:				
	Estimate	Std.Err	z-value	P(> z )
visual =~	1 000			
x1 (L1) x2 (L2)	1.000 0.532	0.092	5.782	0.000
x3 (L3)	0.532	0.092	6.839	0.000
textual =~	0.070	0.000	0.000	0.000
x4	1.000			
x5	1.160	0.104	11.207	0.000
x6	0.932	0.086	10.820	0.000

1.000

1.507

0.677

0.313 0.139

speed =~ x7

х8

*x9* 

Std.lv Std.all

4.815

4.872

0.000

```
0.866
      0.772
0.461
     0.361
0.581
     0.508
0.927
      0.834
1.075
      0.857
        0.818
0.864
0.641
       0.639
0.966
        0.872
0.434
        0.484
```

### ${\it Covariances:}$

		Estimate	Std.Err	z-value	P(> z )
visual ~	~				
textua	.1	0.359	0.095	3.793	0.000
speed		0.166	0.070	2.366	0.018
textual	~~				
speed		0.098	0.061	1.591	0.112
Std.lv	Std.all				
0.448	0.448				
0.298	0.298				
0.164	0.164				

### Intercepts:

		Estimate	Std.Err	z-value	P(> z )
.x1		5.033	0.093	54.232	0.000
.x2		6.231	0.106	58.899	0.000
.x3		2.462	0.095	26.046	0.000
.x4		2.918	0.092	31.741	0.000
.x5		4.264	0.104	41.082	0.000
.x6		2.173	0.087	24.887	0.000
.x7		4.056	0.083	48.796	0.000
.x8		5.571	0.092	60.757	0.000
.x9		5.326	0.074	71.828	0.000
visual		0.000			
textua.	1	0.000			
speed		0.000			
Std.lv	Std.all				
5.033	4.488				
6.231	4.874				
2.462	2.156				
2.918	2.627				
4.264	3.400				
2.173	2.060				
4.056	4.038				
5.571	5.028				
5.326	5.945				
0.000	0.000				
0.000	0.000				
0.000	0.000				

#### Variances:

Estimate Std.Err z-value P(>|z|)0.508 0.149 3.402 0.001 .x1

.x2		1.422	0.176	8.072	0.000
.x3		0.968	0.132	7.322	0.000
.x4		0.375	0.067	5.563	0.000
.x5		0.417	0.084	4.956	0.000
.x6		0.368	0.062	5.928	0.000
.x7		0.597	0.104	5.770	0.000
.x8		0.294	0.175	1.677	0.093
.x9		0.614	0.081	7.620	0.000
visual		0.750	0.175	4.288	0.000
textua	1	0.858	0.147	5.849	0.000
speed		0.411	0.122	3.371	0.001
Std.lv	${\it Std.all}$				
0.508	0.404				
1.422	0.870				
0.968	0.742				
0.375	0.304				
0.417	0.265				
0.368	0.330				
0.597	0.592				
0.294	0.239				
0.614	0.765				
1.000	1.000				
1.000	1.000				
1.000	1.000				

# Group 2 [2]:

### Latent Variables:

		Estimate	Std.Err	z-value	P(> z )
visual =	~				
x1	(L1)	1.000			
x2	(L2)	0.532	0.092	5.782	0.000
xЗ	(L3)	0.670	0.098	6.839	0.000
textual	=~				
x4		1.000			
x5		1.075	0.083	12.887	0.000
x6		0.929	0.072	12.993	0.000
speed =^	•				
<i>x</i> 7		1.000			
x8		1.078	0.227	4.751	0.000
x9		1.688	0.353	4.785	0.000
Std.lv	Std.all				
0.958	0.800				
0.510	0.480				
0.642	0.599				
1.040	0.872				
1.118	0.850				
0.967	0.856				
0.539	0.469				
0.580	0.639				
0.909	0.826				

## ${\it Covariances:}$

Estimate Std.Err z-value P(>|z|)

visual ~	~				
textua	7	0.500	0.111	4.494	0.000
speed	.1	0.315	0.084	3.737	0.000
textual	~~	0.010	0.004	0.707	0.000
speed		0.202	0.068	2.975	0.003
Std.lv	Std.all	0.202	0.000	2.010	0.000
Btu.iv	Dua.aii				
0.502	0.502				
0.611	0.611				
0.361	0.361				
Intercepts	:				
		Estimate	Std.Err	z-value	P(> z )
.x1		4.844	0.096	50.332	0.000
.x2		5.953	0.085	69.789	0.000
.x3		2.051	0.086	23.824	0.000
.x4		3.196	0.096	33.348	0.000
.x5		4.413	0.106	41.757	0.000
.x6		2.197	0.091	24.233	0.000
.x7		4.308	0.092	46.747	0.000
.x8		5.486	0.073	75.197	0.000
.x9		5.419	0.088	61.316	0.000
visual		0.000			
textua	.1	0.000			
speed		0.000			
Std.lv	Std.all				
4.844	4.043				
5.953	5.606				
2.051	1.914				
3.196	2.679				
4.413	3.354				
2.197	1.946				
4.308	3.755				
5.486	6.040				
5.419	4.925				
0.000	0.000				
0.000	0.000				
0.000	0.000				
Variances:		Fatimata	C+ -1 F	7	D(>1-1)
1		Estimate 0.518	Std.Err	z-value	P(> z ) 0.000
.x1 .x2		0.868	0.132	3.923	0.000
.x2 .x3			0.108	8.005 7.218	0.000
		0.736 0.342	0.102 0.064		0.000
. x4 . x5		0.342	0.084	5.328 5.950	0.000
.x6		0.340	0.059	5.782	0.000
.xo .x7		1.027	0.039	8.020	0.000
.x8		0.488	0.128	6.548	0.000
.x9		0.384	0.124	3.091	0.002
visual		0.918	0.124	4.930	0.002
visuai textua		1.081	0.165	4.930 6.552	0.000
speed		0.290	0.105	2.698	0.000
speed Std.lv	Std.all	0.290	0.107	∠.090	0.007
0.518	0.361				
0.868	0.769				
0.736	0.769				
0.730	0.041				

```
0.342
         0.240
0.480
         0.277
0.340
         0.267
1.027
         0.780
         0.592
0.488
0.384
         0.317
         1.000
1.000
1.000
         1.000
1.000
         1.000
```

Now we compare the two fits:

```
> anova(fit.1,fit.2.1)
```

```
Chi Square Difference Test
```

```
Df AIC BIC Chisq Chisq diff Df diff

fit.1 48 7517.6 7740.1 105.80

fit.2.1 50 7516.1 7731.1 108.22 2.4282 2

Pr(>Chisq)

fit.1

fit.2.1 0.297
```

Thus, contraining the means did not make the model fit worse i.e. the groups are not reliably different from each other. If they were, constraining the means to be the same would have produced a worse fit. Thus, men and women don't differ on visual ability in these data.

#### Constraining Textual Means

Minimum Function Test Statistic

```
lavaan (0.5-23.1097) converged normally after 59 iterations
 Number of observations per group
                                                    146
                                                    155
 Estimator
                                                     ML
 Minimum Function Test Statistic
                                                106.329
 Degrees of freedom
                                                     50
 P-value (Chi-square)
                                                  0.000
Chi-square for each group:
  1
                                                 61.700
  2
                                                 44.629
Model test baseline model:
```

Degrees of	freedom	1			72
P-value					0.000
User model v	ersus ba	seline m	odel:		
Comparativ	e Fit In	dex (CFT	.)		0.937
Tucker-Lew			,		0.909
Loglikelihoo	d and In	formatio	n Criteri	a:	
Loglikelih Loglikelih					3699.083 3645.919
Number of	free par	ameters			58
Akaike (AI					7514.166
Bayesian (	BIC)				7729.179
Sample-siz	e adjust	ed Bayes	ian (BIC)		7545.236
Root Mean Sq	uare Err	or of Ap	proximati	on:	
DMCEA					0 007
RMSEA 90 Percent	Confide	nco Into	ruo I	0.06	0.087 4 0.109
P-value RM			Ival	0.00	0.006
1 /0100 101					0.000
Standardized	Root Me	an Squar	e Residua	1:	
SRMR					0.064
Parameter Es	timates:				
Informatio	n				Expected
Standard E	rrors				Standard
Group 1 [1]:					
Latent Varia	hles:				
Lacent varia		Stimate	Std.Err	z-value	P(> z )
visual =~					
x1		1.000			
x2		0.509	0.174	2.926	0.003
х3		0.872	0.220	3.970	0.000
textual =~					
x4	(L1)	1.000			
x5	(L2)	1.110	0.065	17.106	0.000
x6	(L3)	0.930	0.055	16.885	0.000
speed =~		4 000			
x7 x8		1.000 1.480	0.204	1 060	0.000
хо х9		0.679	0.304 0.139	4.863 4.895	0.000 0.000
	td.all	0.079	0.139	4.090	0.000
0.809	0.726				
0.412	0.328				
0.705	0.601				

0.941

1.046

0.841

0.875 0.8230.647 0.644

0.957 0.863 0.439 0.490

#### Covariances:

		Estimate	Std.Err	z-value	P(> z )
visual ~	~				
textua	1	0.336	0.094	3.566	0.000
speed		0.157	0.068	2.306	0.021
textual	~~				
speed		0.100	0.063	1.587	0.113
Std.lv	Std.all				
0.441	0.441				
0.301	0.301				
0.164	0.164				

### Intercepts:

		Estimate	Std.Err	z-value	P(> z )
.x1		5.033	0.092	54.589	0.000
.x2		6.231	0.104	59.969	0.000
.x3		2.462	0.097	25.368	0.000
.x4		2.918	0.093	31.494	0.000
.x5		4.264	0.102	41.652	0.000
.x6		2.173	0.088	24.706	0.000
.x7		4.056	0.083	48.796	0.000
.x8		5.571	0.092	60.757	0.000
.x9		5.326	0.074	71.828	0.000
visual		0.000			
textua	1	0.000			
speed		0.000			
Std.lv	Std.all				
5.033	4.518				
6.231	4.963				
2.462	2.099				
2.918	2.606				
4.264	3.447				
2.173	2.045				
4.056	4.038				
5.571	5.028				
5.326	5.945				
0.000	0.000				
0.000	0.000				
0.000	0.000				

## Variances:

	Estimate	Std.Err	z-value	P(> z )
.x1	0.587	0.167	3.511	0.000
.x2	1.407	0.175	8.027	0.000
.x3	0.879	0.157	5.600	0.000
.x4	0.367	0.065	5.645	0.000
.x5	0.437	0.079	5.539	0.000
.x6	0.364	0.060	6.035	0.000
.x7	0.591	0.103	5.711	0.000
.x8	0.312	0.171	1.830	0.067

.x9		0.610	0.080	7.587	0.000
visual		0.654	0.199	3.284	0.001
textua	1	0.886	0.133	6.671	0.000
speed		0.418	0.123	3.404	0.001
Std.lv	${\it Std.all}$				
0.587	0.473				
1.407	0.892				
0.879	0.639				
0.367	0.293				
0.437	0.285				
0.364	0.322				
0.591	0.586				
0.312	0.254				
0.610	0.760				
1.000	1.000				
1.000	1.000				
1.000	1.000				

# Group 2 [2]:

### Latent Variables:

		Estimate	Std.Err	z-value	P(> z )
visual =	~				
x1		1.000			
x2		0.557	0.108	5.151	0.000
xЗ		0.586	0.107	5.467	0.000
textual	=~				
x4	(L1)	1.000			
x5	(L2)	1.110	0.065	17.106	0.000
x6	(L3)	0.930	0.055	16.885	0.000
speed =~	•				
x7		1.000			
x8		1.076	0.227	4.744	0.000
х9		1.692	0.355	4.772	0.000
Std.lv	Std.all				
0.988	0.820				
0.550	0.511				
0.580	0.553				
1.029	0.868				
1.143	0.858				
0.957	0.852				
0.538	0.469				
0.579	0.638				
0.911	0.828				

### Covariances:

	Estimate	Std.Err	z-value	P(> z )
visual ~~				
textual	0.511	0.112	4.568	0.000
speed	0.319	0.086	3.717	0.000
textual ~~				
speed	0.200	0.067	2.984	0.003
Std.lv Std.all	-			

```
0.503 0.503
0.599 0.599
```

0.362 0.362

## Intercepts:

		Estimate	${\it Std.Err}$	z-value	P(> z )
.x1		4.844	0.097	50.058	0.000
.x2		5.953	0.087	68.783	0.000
.x3		2.051	0.084	24.367	0.000
.x4		3.196	0.095	33.553	0.000
.x5		4.413	0.107	41.246	0.000
.x6		2.197	0.090	24.372	0.000
.x7		4.308	0.092	46.747	0.000
.x8		5.486	0.073	75.197	0.000
.x9		5.419	0.088	61.316	0.000
visual		0.000			
textua	1	0.000			
speed		0.000			
Std.lv	Std.all				
4.844	4.021				
5.953	5.525				
2.051	1.957				
3.196	2.695				
4.413	3.313				
2.197	1.958				
4.308	3.755				
5.486	6.040				
5.419	4.925				
0.000	0.000				
0.000	0.000				
0.000	0.000				

#### Variances:

	Estimate	Std.Err	z-value	P(> z )
.x1	0.475	0.145	3.283	0.001
.x2	0.858	0.110	7.798	0.000
.x3	0.762	0.102	7.508	0.000
.x4	0.347	0.062	5.616	0.000
.x5	0.468	0.079	5.896	0.000
.x6	0.345	0.057	6.040	0.000
.x7	1.027	0.128	8.020	0.000
.x8	0.490	0.075	6.552	0.000
.x9	0.382	0.125	3.046	0.002
visual	0.977	0.206	4.751	0.000
textual	1.059	0.152	6.987	0.000
speed	0.289	0.107	2.694	0.007
0.17 0.1 77				

Std.lv Std.all 0.475 0.327 0.739 0.858 0.694 0.762 0.247 0.347 0.468 0.264 0.345 0.274 0.780 1.027 0.490 0.593 0.382 0.315 1.000 1.000

```
1.000
         1.000
1.000
         1.000
```

Now we compare the two fits:

#### > anova(fit.1,fit.2.2)

#### Chi Square Difference Test

Df AICBIC Chisq Chisq diff Df diff 48 7517.6 7740.1 105.80 fit.1 fit.2.2 50 7514.2 7729.2 106.33 0.53332 Pr(>Chisq)

fit.1

fit.2.2 0.7659

Similar to visual, men and women do not differ on textual ability.

#### Constraining Speed Means

```
> groupmodel.2.3 <- ' visual = x1 + x2 + x3
                    textual = ^{\sim} x4 + x5 + x6
speed = ^{\sim} c(L1,L1)*x7 + c(L2, L2)*x8 + c(L3,L3)*x9 '
> fit.2.3 <- cfa(groupmodel.2.3,
              data = HolzingerSwineford1939,
               group = "sex")
> summary(fit.2.3, standardized=TRUE, fit.measures=TRUE)
```

#### lavaan (0.5-23.1097) converged normally after 57 iterations

Number of observations per group  1  2	146 155
Estimator Minimum Function Test Statistic Degrees of freedom P-value (Chi-square)	ML 122.650 50 0.000

#### Chi-square for each group:

1	71.011
2	51.639

#### Model test baseline model:

${\it Minimum}$	Function Test Statistic	967.228
Degrees	of freedom	72
P-value		0.000

#### User model versus baseline model:

Comparative Fit Index (CFI)	0.919
Tucker-Lewis Index (TLI)	0.883

#### Loglikelihood and Information Criteria:

Loglikelihood user model (HO)	-3707.244
Loglikelihood unrestricted model (H1)	-3645.919
Number of free parameters	58
Akaike (AIC)	7530.488
Bayesian (BIC)	7745.500
Sample-size adjusted Bayesian (BIC)	7561.557

Root Mean Square Error of Approximation:

 RMSEA
 0.098

 90 Percent Confidence Interval
 0.076
 0.120

 P-value RMSEA <= 0.05</td>
 0.000

Standardized Root Mean Square Residual:

SRMR 0.072

Parameter Estimates:

Information Expected Standard Errors Standard

Estimate Std.Err z-value P(>|z|)

Group 1 [1]:

Latent Variables:

		<b>DD</b> 0 I ma 0 C	Dog. HII	z varac	1 ( 121)
visual =	~				
x1		1.000			
x2		0.557	0.180	3.085	0.002
<i>x3</i>		0.951	0.227	4.192	0.000
textual	=~				
x4		1.000			
x5		1.158	0.103	11.204	0.000
x6		0.929	0.086	10.805	0.000
speed =~					
<i>x</i> 7	(L1)	1.000			
<i>x8</i>	(L2)	1.157	0.162	7.133	0.000
x9	(L3)	1.059	0.149	7.122	0.000
Std.lv	Std.all				
0.776	0.696				
0.432	0.344				
0.737	0.629				
0.928	0.836				
1.075	0.857				
0.862	0.817				
0.606	0.606				
0.701	0.661				
0.642	0.683				

 ${\it Covariances:}$ 

Estimate Std.Err z-value P(>|z|)

visual ~~

textual speed textual ~~	~	0.307 0.192	0.091 0.066	3.372 2.894	0.001 0.004
speed Std.lv S	Std.all	0.104	0.061	1.688	0.091
0.427	0.427				
0.409	0.409				
0.184	0.184				
Intercepts:					
.x1		Estimate 5.033	Std.Err 0.092	z-value 54.589	P(> z ) 0.000
.x2		6.231	0.092	59.969	0.000
.x3		2.462	0.097	25.368	0.000
.x4		2.918	0.092	31.741	0.000
.x5		4.264	0.104	41.082	0.000
.x6		2.173	0.087	24.887	0.000
.x7		4.056	0.083	49.022	0.000
.x8		5.571	0.088	63.400	0.000
.x9		5.326	0.078	68.442	0.000
visual		0.000			
textual		0.000			
speed	~	0.000			
	Std.all				
5.033 6.231	4.518 4.963				
2.462	2.099				
2.918	2.627				
4.264	3.400				
2.173	2.060				
4.056	4.057				
5.571	5.247				
5.326	5.664				
0.000	0.000				
0.000	0.000				
0.000	0.000				
Variances:			a	_	56.1.1
1		Estimate 0.640	Std.Err	z-value	P(> z )
.x1 .x2		1.390	0.153 0.174	4.188 7.969	0.000
.x3		0.832	0.174	5.327	0.000
.x4		0.372	0.068	5.513	0.000
.x5		0.417	0.084	4.937	0.000
.x6		0.370	0.062	5.950	0.000
.x7		0.632	0.096	6.619	0.000
.x8		0.635	0.107	5.944	0.000
.x9		0.472	0.084	5.605	0.000
visual		0.601	0.182	3.300	0.001
textual		0.861	0.147	5.860	0.000
speed	a. 1 ==	0.367	0.091	4.016	0.000
	Std.all				
0.640 1.390	0.515 0.882				
0.832	0.882				
0.372	0.302				
0.012	0.002				

0.417	0.265
0.370	0.333
0.632	0.633
0.635	0.564
0.472	0.534
1.000	1.000
1.000	1.000
1.000	1.000

## Latent Variables:

		Estimate	Std.Err	z-value	P(> z )
visual =	~				
x1		1.000			
x2		0.563	0.112	5.013	0.000
xЗ		0.586	0.112	5.248	0.000
textual	=~				
x4		1.000			
x5		1.072	0.083	12.892	0.000
x6		0.927	0.071	13.006	0.000
speed =~					
<i>x</i> 7	(L1)	1.000			
x8	(L2)	1.157	0.162	7.133	0.000
x9	(L3)	1.059	0.149	7.122	0.000
Std.lv	Std.all				
0.986	0.818				
0.555	0.515				
0.578	0.552				
1.042	0.873				
1.117	0.849				
0.966	0.855				
0.632	0.548				
0.731	0.781				
0.669	0.637				

## Covariances:

		Estimate	Std.Err	z-value	P(> z )
visual ~	~				
textua	1	0.518	0.114	4.536	0.000
speed		0.312	0.081	3.844	0.000
textual	~~				
speed		0.212	0.072	2.953	0.003
Std.lv	Std.all				
0.505	0.505				
0.500	0.500				
0.322	0.322				

## Intercepts:

	Estimate	${\it Std.Err}$	z-value	P(> z )
.x1	4.844	0.097	50.058	0.000
.x2	5.953	0.087	68.783	0.000

```
.x3
                   2.051
                             0.084
                                     24.367
                                               0.000
                                               0.000
.x4
                   3.196
                             0.096
                                     33.348
                                     41.757
                                               0.000
.x5
                   4.413
                             0.106
                   2.197
                             0.091
                                     24.233
                                               0.000
.x6
.x7
                   4.308
                             0.093
                                     46.520
                                               0.000
                             0.075
                                     72.988
                                               0.000
.x8
                   5.486
                   5.419
                             0.084
                                     64.207
                                               0.000
.x9
visual
                   0.000
textual
                   0.000
speed
                   0.000
Std.lv Std.all
 4.844
          4.021
 5.953
          5.525
 2.051
          1.957
 3.196
          2.679
 4.413
          3.354
 2.197
          1.946
 4.308
          3.737
 5.486
          5.863
 5.419
          5.157
 0.000
          0.000
 0.000
          0.000
 0.000
          0.000
```

#### Variances:

		Estimate	Std.Err	z-value	P(> z )
.x1		0.480	0.153	3.126	0.002
.x2		0.853	0.111	7.686	0.000
.x3		0.764	0.103	7.406	0.000
.x4		0.338	0.064	5.276	0.000
.x5		0.484	0.081	5.980	0.000
.x6		0.342	0.059	5.804	0.000
.x7		0.930	0.123	7.574	0.000
.x8		0.341	0.081	4.193	0.000
.x9		0.656	0.098	6.681	0.000
visual		0.972	0.212	4.591	0.000
textua	1	1.086	0.165	6.571	0.000
speed		0.399	0.102	3.900	0.000
Std.lv	Std.all				
0.480	0.331				
0.853	0.735				
0.764	0.696				
0.338	0.237				
0.484	0.279				
0.342	0.268				
0.930	0.700				
0.341	0.390				
0.656	0.594				
1.000	1.000				
1.000	1.000				
1.000	1.000				

Now we compare the two fits:

## > anova(fit.1,fit.2.3)

Chi Square Difference Test

```
Df
             AIC BIC Chisq Chisq diff Df diff
fit.1 48 7517.6 7740.1 105.80
fit.2.3 50 7530.5 7745.5 122.65
                                  16.855 2
       Pr(>Chisq)
fit.1
fit.2.3 0.0002188 ***
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Here, we see a difference in fits, such that constraining the means for speed in men and women makes the model fir worse. Thus, men and women do in fact differ in their speeds.

## 4.0.2 Constraining Factor Loadings

Akaike (AIC)

```
> groupmodel.3 <- ' visual = ^{\sim} x1 + x2 + x3
                   textual = ^{\sim} x4 + x5 + x6
                   speed = x7 + x8 + x9'
```

<pre>&gt; fit.3 &lt;- cfa(groupmodel.3,</pre>		
+ data = HolzingerSwineford1939		
<pre># group = "sex", group.equal =</pre>		
> summary(fit.3, standardized=TRUE, fit.me	easures=TRUE)	
lavaan (0.5-23.1097) converged normally af	tor 17 iterations	
lavaan (0.5-25.1097) converged normally an	ter 47 Iterations	
Number of observations per group		
1	146	
2	155	
Estimator	ML	
Minimum Function Test Statistic	126.226	
Degrees of freedom	54	
P-value (Chi-square)	0.000	
Chi-square for each group:		
	70.504	
1	72.534	
2	53.692	
Model test baseline model:		
Minimum Function Test Statistic	967.228	
Degrees of freedom	72	
P-value	0.000	
User model versus baseline model:		
Comparative Fit Index (CFI)	0.919	
Tucker-Lewis Index (TLI)	0.892	
Loglikelihood and Information Criteria:		
Loglikelihood user model (HO)	-3709.032	
Loglikelihood unrestricted model (H1)	-3645.919	
Number of free parameters	54	

7526.064

Povogion	(PTC)				7706 040
Bayesian Sample-s		sted Bayes	sian (BIC)		7726.248 7554.990
Root Mean	Square E	rror of Ap	proximati	on:	
RMSEA					0.094
	nt Confi	dence Inte	rval	0.07	
	RMSEA <=		i vai	0.07	0.001
Standardiz	ed Root	Mean Squar	e Residua	1:	
SRMR					0.074
Parameter	Estimate	s:			
Informat	ion				Expected
Standard					Standard
Group 1 [1	]:				
Latent Var	riables:				
		Estimate	Std.Err	z-value	P(> z )
visual =	·~				
x1		1.000			
<i>x</i> 2	(.p2.)		0.096		0.000
xЗ	(.p3.)	0.694	0.103	6.747	0.000
textual	=~				
x4		1.000			
x5	(.p5.)		0.065		0.000
x6	(.p6.)	0.930	0.055	16.895	0.000
speed =^	•				
<i>x</i> 7		1.000			
x8	(.p8.)	1.168	0.163	7.170	0.000
х9	(.p9.)	1.037	0.145	7.157	0.000
Std.lv	Std.all				
0.851	0.757				
0.470	0.368				
0.591	0.519				
0.941	0.841				
1.045	0.845				
0.876	0.824				
0.614	0.614				
0.717	0.675				
0.636	0.674				
Covariance	es:				
		Estimate	Std.Err	z-value	P(> z )
visual ^	~~				
textua	1	0.364	0.095	3.844	0.000
speed		0.203	0.069	2.923	0.003

textual ~~

speed Std.lv Std.all 0.105

0.063

1.680

0.093

```
0.454 0.454
0.388 0.388
```

0.183 0.183

## Intercepts:

_		Estimate	Std.Err	z-value	P(> z )
.x1		5.033	0.093	54.090	0.000
.x2		6.231	0.106	58.928	0.000
.x3		2.462	0.094	26.122	0.000
.x4		2.918	0.093	31.489	0.000
.x5		4.264	0.102	41.664	0.000
.x6		2.173	0.088	24.704	0.000
.x7		4.056	0.083	49.034	0.000
.x8		5.571	0.088	63.420	0.000
.x9		5.326	0.078	68.204	0.000
visual		0.000			
textua	1	0.000			
speed		0.000			
Std.lv	Std.all				
5.033	4.477				
6.231	4.877				
2.462	2.162				
2.918	2.606				
4.264	3.448				
2.173	2.044				
4.056	4.058				
5.571	5.249				
5.326	5.645				
0.000	0.000				
0.000	0.000				
0.000	0.000				

## Variances:

	Estimate	${\it Std.Err}$	z-value	P(> z )
.x1	0.540	0.145	3.710	0.000
.x2	1.412	0.176	8.043	0.000
.x3	0.948	0.131	7.240	0.000
.x4	0.367	0.065	5.654	0.000
.x5	0.437	0.079	5.551	0.000
.x6	0.363	0.060	6.028	0.000
.x7	0.622	0.095	6.558	0.000
.x8	0.613	0.107	5.755	0.000
.x9	0.486	0.084	5.768	0.000
visual	0.724	0.170	4.251	0.000
textual	0.886	0.133	6.670	0.000
speed	0.377	0.093	4.056	0.000

Std.lv Std.all 0.540 0.427 0.865 1.412 0.731 0.948 0.293 0.367 0.437 0.286 0.321 0.363 0.622 0.623 0.613 0.544 0.545 0.486 1.000 1.000

1.000	1.000
1.000	1.000

I at ant	Variables:	۰
Басень	variautes.	

Latent Var	iables:				
		Estimate	Std.Err	z-value	P(> z )
visual =	·~				
x1		1.000			
x2	(.p2.)	0.552	0.096	5.766	0.000
xЗ	(.p3.)	0.694	0.103	6.747	0.000
textual	=~ -				
x4		1.000			
x5	(.p5.)	1.110	0.065	17.092	0.000
x6	(.p6.)	0.930	0.055	16.895	0.000
speed =~					
x7		1.000			
х8	(.p8.)	1.168	0.163	7.170	0.000
x9	(.p9.)	1.037	0.145	7.157	0.000
Std.lv	Std.all				
0.941	0.787				
0.520	0.489				
0.653	0.608				
0.000	0.000				
1.029	0.868				
1.142	0.857				
0.957	0.853				
0.007	0.000				
0.632	0.548				
0.738	0.788				
0.655	0.625				
0.000	0.020				
Covariance	s:				
00.0110100		Estimate	Std.Err	z-value	P(> z )
visual ^	~~	2501ma 00	Dourer	2 varao	1 (* 121)
textua	7	0.484	0.108	4.474	0.000
speed	· <b>-</b>	0.303	0.079	3.854	0.000
textual	~~	0.000	0.075	0.001	0.000
speed		0.207	0.071	2.932	0.003
Std.lv	Std.all	0.207	0.071	2.002	0.000
Bou.iv	Dou.uii				
0.500	0.500				
0.510	0.510				
0.010	0.010				
0.318	0.318				
0.010	0.010				

## Intercepts:

	Estimate	${\it Std.Err}$	z-value	P(> z )
.x1	4.844	0.096	50.439	0.000
.x2	5.953	0.085	69.757	0.000
.x3	2.051	0.086	23.763	0.000
.x4	3.196	0.095	33.557	0.000
.x5	4.413	0.107	41.234	0.000
.x6	2.197	0.090	24.374	0.000
.x7	4.308	0.093	46.504	0.000
.x8	5.486	0.075	72.953	0.000

```
.x9
                       5.419
                                0.084
                                         64.410
                                                   0.000
                       0.000
    visual
    textual
                       0.000
                       0.000
    speed
   Std.lv
          Std.all
    4.844
             4.051
    5.953
             5.603
    2.051
             1.909
             2.695
    3.196
             3.312
    4.413
             1.958
    2.197
    4.308
             3.735
             5.860
    5.486
    5.419
             5.174
             0.000
    0.000
    0.000
             0.000
    0.000
             0.000
Variances:
                   Estimate
                              Std.Err z-value
                                                 P(>|z|)
   .x1
                       0.544
                                0.136
                                          4.007
                                                   0.000
                       0.859
                                0.109
                                          7.897
                                                   0.000
   .x2
                       0.728
                                0.104
                                          6.999
   .x3
                                                   0.000
   .x4
                       0.347
                                0.062
                                          5.607
                                                   0.000
   .x5
                       0.471
                                0.080
                                         5.911
                                                   0.000
                                0.057
                                          6.015
                                                   0.000
   .x6
                       0.343
   .x7
                       0.931
                                0.123
                                          7.578
                                                   0.000
                       0.332
                                0.082
                                          4.051
                                                   0.000
   .x8
                       0.668
                                0.098
                                          6.826
                                                   0.000
   .x9
                       0.886
                                0.185
                                          4.793
                                                   0.000
    visual
    textual
                       1.059
                                0.152
                                          6.987
                                                   0.000
                       0.399
                                0.102
                                          3.918
                                                   0.000
```

speed Std.lv Std.all0.544 0.381 0.859 0.761 0.728 0.631 0.347 0.247 0.471 0.265 0.343 0.272 0.700 0.931 0.379 0.332 0.668 0.609 1.000 1.000

1.000

1.000

Checking if this constraint reduces fit:

1.000

1.000

### > anova(fit.1, fit.3)

#### Chi Square Difference Test

```
Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq) fit.1 48 7517.6 7740.1 105.80 fit.3 54 7526.1 7726.2 126.23 20.431 6 0.00232
```

fit.1

```
fit.3 **
---
Signif. codes:
0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

And indeed it does – this means that men and women do in fact differ on some construct, as constraining the factor loadings to be the same produces a worse fit.

## 4.0.3 Constraining Intercepts

```
> groupmodel.4 <- ' visual = x1 + x2 + x3
+ textual = x4 + x5 + x6
+ speed = x7 + x8 + x9 '
> fit.4 <- cfa(groupmodel.4,
+ data = HolzingerSwineford1939,
+ group = "sex", group.equal = "intercepts")
> summary(fit.4, standardized=TRUE, fit.measures=TRUE)
lavaan (0.5-23.1097) converged normally after 68 iterations
```

Number of observations per group	
1	146
2	155
Estimator	ML
Minimum Function Test Statistic	125.520
Degrees of freedom	54
P-value (Chi-square)	0.000

#### Chi-square for each group:

1	70.978
2	54.542

#### Model test baseline model:

${\tt Minimum}$	Function Test Statistic	967.228
Degrees	of freedom	72
P-value		0.000

#### User model versus baseline model:

Comparative Fit Index (CFI)	0.920
Tucker-Lewis Index (TLI)	0.893

#### Loglikelihood and Information Criteria:

Loglikelihood user model (HO)	-3708.679
Loglikelihood unrestricted model (H1)	-3645.919
Number of free parameters	54
Akaike (AIC)	7525.357
Bayesian (BIC)	7725.541
Sample-size adjusted Bayesian (BIC)	7554.284

Root Mean Square Error of Approximation:

RMSEA					0.094
90 Percei	nt Confid	dence Inte	rval	0.07	2 0.115
P-value l	RMSEA <=	0.05			0.001
Standardize	ed Root N	Mean Squar	e Residua	1:	
SRMR					0.069
Parameter 1	Estimates	s:			
Informati	ion				Expected
Standard	Errors				Standard
Group 1 [1]	7:				
Latent Var	iahles:				
Laveno Val.	TANTED.	Estimate	Std.Err	z-value	P(> z )
visual =	~		204.111	_ ,	- (- (21)
x1		1.000			
x2		0.539	0.181	2.979	0.003
xЗ		0.908	0.230	3.941	0.000
textual =	=~				
x4		1.000			
<i>x</i> 5		1.167	0.105	11.117	0.000
x6		0.935	0.087	10.707	0.000
speed =~					
<i>x</i> 7		1.000			
х8		1.489	0.308	4.825	0.000
x9		0.689	0.141	4.881	0.000
Std.lv	Std.all				
0.789	0.708				
0.425	0.338				
0.717	0.607				
0.924	0.831				
1.077	0.859				
0.863	0.816				
0.642	0.638				
0.955	0.859				
0.442	0.494				
Covariances	s:				
		Estimate	Std.Err	z-value	P(> z )
visual ~	~				
textual	1	0.316	0.092	3.424	0.001
speed		0.155	0.067	2.308	0.021
textual	~ ~				
speed	a	0.099	0.062	1.598	0.110
Std.1v	Std.all				

0.434 0.306

0.166

0.434 0.306

0.166

```
Intercepts:
                   Estimate Std.Err z-value
                                                P(>|z|)
   .x1
            (.25.)
                      5.071
                                0.089
                                        57.290
                                                   0.000
                      6.159
                                0.078
                                        79.316
                                                   0.000
   .x2
            (.26.)
   .x3
            (.27.)
                      2.327
                                0.079
                                        29.545
                                                   0.000
   .x4
            (.28.)
                      2.963
                                0.088
                                        33.687
                                                   0.000
   .x5
            (.29.)
                      4.232
                                0.097
                                        43.655
                                                   0.000
   .x6
            (.30.)
                      2.091
                                0.082
                                        25.545
                                                   0.000
                      4.106
                                0.067
                                        61.515
                                                   0.000
   .x7
            (.31.)
   .x8
                                0.067
                                                   0.000
            (.32.)
                      5.480
                                        81.668
            (.33.)
                      5.306
                                0.073
                                        72.415
                                                   0.000
   .x9
    visual
                      0.000
                      0.000
    textual
                      0.000
    speed
   Std.lv
           Std.all
    5.071
             4.550
    6.159
             4.897
    2.327
             1.971
    2.963
             2.665
    4.232
             3.374
    2.091
             1.976
    4.106
             4.083
             4.930
    5.480
    5.306
             5.920
    0.000
             0.000
             0.000
    0.000
    0.000
             0.000
Variances:
                   Estimate
                             Std.Err z-value P(>|z|)
                                                   0.000
   .x1
                      0.620
                                0.164
                                         3.775
   .x2
                       1.401
                                0.176
                                         7.968
                                                   0.000
                      0.880
                                0.161
                                         5.465
                                                   0.000
   .x3
                                                   0.000
   .x4
                      0.383
                                0.068
                                         5.602
   .x5
                      0.413
                                0.085
                                         4.859
                                                   0.000
                      0.375
                                0.063
                                         5.947
                                                   0.000
   .x6
                                         5.781
                                                   0.000
   .x7
                      0.600
                                0.104
                                0.172
                                         1.886
                                                   0.059
   .x8
                      0.324
   .x9
                      0.608
                                0.081
                                         7.543
                                                   0.000
                                                   0.001
    visual
                      0.623
                                0.194
                                         3.213
                                         5.807
                      0.853
                                0.147
                                                   0.000
    textual
                                                   0.001
    speed
                      0.412
                                0.122
                                         3.370
   Std.lv Std.all
             0.499
    0.620
    1.401
             0.886
    0.880
             0.631
    0.383
             0.310
    0.413
             0.262
    0.375
             0.335
             0.593
    0.600
    0.324
             0.262
    0.608
             0.756
    1.000
             1.000
    1.000
             1.000
    1.000
             1.000
```

Latent Var	iables:				
Latent var	Tabics.	Estimate	Std.Err	z-value	P(> z )
visual =	~				,
x1		1.000			
x2		0.610	0.112	5.439	0.000
хЗ		0.670	0.113	5.927	0.000
textual	=~				
x4		1.000			
x5		1.063	0.082	12.988	0.000
x6		0.907	0.070	12.966	0.000
speed =~					
<i>x</i> 7		1.000			
x8		1.026	0.215	4.781	0.000
х9		1.664	0.344	4.833	0.000
Std.lv	Std.all				
0.935	0.784				
0.935	0.784				
0.627	0.527				
0.627	0.588				
1.051	0.874				
1.118	0.850				
0.953	0.849				
0.551	0.473				
0.565	0.624				
0.916	0.833				
Covariance	s:				
	s:	Estimate	Std.Err	z-value	P(> z )
visual ~	~				
visual ~ textua	~	0.496	0.112	4.447	0.000
visual ~ textua speed	~				
visual ~ textua speed textual	~	0.496 0.316	0.112 0.085	4.447 3.723	0.000
visual ~ textua speed textual speed	~ 1 ~~	0.496	0.112	4.447	0.000
visual ~ textua speed textual	~ 1 ~~	0.496 0.316	0.112 0.085	4.447 3.723	0.000
visual ~ textua speed textual speed Std.lv	l 22 Std.all	0.496 0.316	0.112 0.085	4.447 3.723	0.000
visual ~ textua speed textual speed Std.lv	~ 1 ~~ Std.all 0.504	0.496 0.316	0.112 0.085	4.447 3.723	0.000
visual ~ textua speed textual speed Std.lv	l 22 Std.all	0.496 0.316	0.112 0.085	4.447 3.723	0.000
visual ~ textua speed textual speed Std.lv 0.504 0.613	2 1 22 Std.all 0.504 0.613	0.496 0.316	0.112 0.085	4.447 3.723	0.000
visual ~ textua speed textual speed Std.lv	~ 1 ~~ Std.all 0.504	0.496 0.316	0.112 0.085	4.447 3.723	0.000
visual ~ textua speed textual speed Std.lv 0.504 0.613	Std.all 0.504 0.613 0.362	0.496 0.316	0.112 0.085	4.447 3.723	0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613	Std.all 0.504 0.613 0.362	0.496 0.316	0.112 0.085	4.447 3.723	0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613	Std.all 0.504 0.613 0.362	0.496 0.316 0.209	0.112 0.085 0.070	4.447 3.723 2.990	0.000 0.000 0.003
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362	Std.all 0.504 0.613 0.362	0.496 0.316 0.209	0.112 0.085 0.070 Std.Err	4.447 3.723 2.990 z-value	0.000 0.000 0.003
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362 Intercepts .x1	Std.all 0.504 0.613 0.362 : (.25.)	0.496 0.316 0.209 Estimate 5.071	0.112 0.085 0.070 Std.Err 0.089	4.447 3.723 2.990 z-value 57.290	0.000 0.000 0.003 P(> z ) 0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362 Intercepts .x1 .x2	Std.all 0.504 0.613 0.362 : (.25.) (.26.)	0.496 0.316 0.209 Estimate 5.071 6.159	0.112 0.085 0.070 Std.Err 0.089 0.078	4.447 3.723 2.990 z-value 57.290 79.316	0.000 0.000 0.003 P(> z ) 0.000 0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362  Intercepts .x1 .x2 .x3	Std.all 0.504 0.613 0.362 : (.25.) (.26.) (.27.)	0.496 0.316 0.209 Estimate 5.071 6.159 2.327	0.112 0.085 0.070 Std.Err 0.089 0.078 0.079	4.447 3.723 2.990 z-value 57.290 79.316 29.545	0.000 0.000 0.003 P(> z ) 0.000 0.000 0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362  Intercepts .x1 .x2 .x3 .x4	Std.all 0.504 0.613 0.362 : (.25.) (.26.) (.27.) (.28.)	0.496 0.316 0.209 Estimate 5.071 6.159 2.327 2.963	0.112 0.085 0.070 Std.Err 0.089 0.078 0.079 0.088	2.990 z-value 57.290 79.316 29.545 33.687	0.000 0.000 0.003 0.003 P(> z ) 0.000 0.000 0.000 0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362  Intercepts .x1 .x2 .x3 .x4 .x5	Std.all 0.504 0.613 0.362 : (.25.) (.26.) (.27.) (.28.) (.29.)	0.496 0.316 0.209 Estimate 5.071 6.159 2.327 2.963 4.232	0.112 0.085 0.070 Std.Err 0.089 0.078 0.079 0.088 0.097	2-value 57.290 79.316 29.545 33.687 43.655	0.000 0.000 0.003 0.003 P(> z ) 0.000 0.000 0.000 0.000 0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362  Intercepts .x1 .x2 .x3 .x4 .x5 .x6	Std.all  0.504 0.613  0.362  :  (.25.) (.26.) (.27.) (.28.) (.29.) (.30.)	0.496 0.316 0.209 Estimate 5.071 6.159 2.327 2.963 4.232 2.091	0.112 0.085 0.070 Std.Err 0.089 0.078 0.079 0.088 0.097 0.082	2-value 57.290 79.316 29.545 33.687 43.655 25.545	0.000 0.000 0.003 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362 Intercepts .x1 .x2 .x3 .x4 .x5 .x6 .x7	Std.all 0.504 0.613 0.362 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.)	0.496 0.316 0.209 Estimate 5.071 6.159 2.327 2.963 4.232 2.091 4.106	0.112 0.085 0.070 Std.Err 0.089 0.078 0.079 0.088 0.097 0.082 0.067	z-value 57.290 79.316 29.545 33.687 43.655 25.545 61.515	0.000 0.000 0.003 0.003 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000 0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362 Intercepts .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8	Std.all  0.504 0.613  0.362  :  (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)	0.496 0.316 0.209 0.209 Estimate 5.071 6.159 2.327 2.963 4.232 2.091 4.106 5.480	0.112 0.085 0.070 0.070 Std.Err 0.089 0.078 0.079 0.088 0.097 0.082 0.067 0.067	2-value 57.290 79.316 29.545 33.687 43.655 25.545 61.515 81.668	0.000 0.000 0.003 0.003 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362 Intercepts .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8 .x9	Std.all  0.504 0.613  0.362  :  (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)	0.496 0.316 0.209 Estimate 5.071 6.159 2.327 2.963 4.232 2.091 4.106 5.480 5.306	0.112 0.085 0.070 0.070 Std.Err 0.089 0.078 0.079 0.088 0.097 0.082 0.067 0.067 0.073	2-value 57.290 79.316 29.545 33.687 43.655 25.545 61.515 81.668 72.415	0.000 0.000 0.003 0.003 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362 Intercepts .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8 .x9 visual	Std.all  0.504 0.613  0.362  :  (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)	0.496 0.316 0.209 0.209 Estimate 5.071 6.159 2.327 2.963 4.232 2.091 4.106 5.480 5.306 -0.283	0.112 0.085 0.070 0.070 Std.Err 0.089 0.078 0.079 0.088 0.097 0.082 0.067 0.067 0.073 0.125	2-value 57.290 79.316 29.545 33.687 43.655 25.545 61.515 81.668 72.415 -2.272	0.000 0.000 0.003 0.003 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
visual ~ textual speed textual speed Std.lv 0.504 0.613 0.362 Intercepts .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8 .x9 visual textual	Std.all  0.504 0.613  0.362  :  (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)	0.496 0.316 0.209 0.209 Estimate 5.071 6.159 2.327 2.963 4.232 2.091 4.106 5.480 5.306 -0.283 0.178	0.112 0.085 0.070 0.070 Std.Err 0.089 0.078 0.079 0.088 0.097 0.082 0.067 0.067 0.073 0.125 0.123	2-value 57.290 79.316 29.545 33.687 43.655 25.545 61.515 81.668 72.415 -2.272 1.442	0.000 0.000 0.000 0.003 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.023 0.149

```
5.071
       4.251
6.159
       5.685
2.327
       2.181
       2.464
2.963
4.232
      3.220
2.091
       1.864
4.106
       3.530
        6.052
5.480
5.306
        4.822
-0.303
       -0.303
0.169
       0.169
        0.123
 0.123
```

#### Variances:

		Estimate	Std.Err	z-value	P(> z )
.x1		0.548	0.133	4.134	0.000
.x2		0.847	0.110	7.673	0.000
. x3		0.745	0.103	7.212	0.000
.x4		0.341	0.065	5.218	0.000
. x5		0.479	0.081	5.916	0.000
.x6		0.351	0.059	5.939	0.000
.x7		1.049	0.131	8.001	0.000
.x8		0.501	0.074	6.734	0.000
. x9		0.372	0.126	2.942	0.003
visual		0.875	0.188	4.659	0.000
textua	1	1.105	0.168	6.589	0.000
speed		0.303	0.111	2.737	0.006
Std.lv	Std.all				
0.548	0.385				
0.847	0.722				
0.745	0.655				
0.341	0.236				
0.479	0.277				
0.351	0.279				
1.049	0.776				
0.501	0.611				
0.372	0.307				
1.000	1.000				
1.000	1.000				
1.000	1.000				

Checking if this constraint reduces fit:

## > anova(fit.1, fit.4)

```
Chi Square Difference Test

Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)
fit.1 48 7517.6 7740.1 105.80
fit.4 54 7525.4 7725.5 125.52 19.724 6 0.0031
fit.1
fit.4 **
---
Signif. codes:
```

0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Similarly, we see that constraining the intercepts also produces a difference, suggesting that men and women differ on some aspect.

## 4.0.4 Constraining Some Parameters, Not All

```
> groupmodel.5 <- ' visual = x1 + x2 + x3
                  textual = ^{\sim} x4 + x5 + x6
                  speed = x7 + x8 + x9
> fit.5 <- cfa(groupmodel.5,</pre>
             data = HolzingerSwineford1939,
             group = "sex", group.equal = c("loadings", "intercepts"),
             group.partial = c("visual = x1 + x2 + x3",
                               "textual = x4 + x5 + x6"))
> summary(fit.5, standardized=TRUE, fit.measures=TRUE)
lavaan (0.5-23.1097) converged normally after 63 iterations
 Number of observations per group
                                                    146
 2
                                                    155
 Estimator
                                                    ML
                                                144.998
 Minimum Function Test Statistic
 Degrees of freedom
                                                     60
 P-value (Chi-square)
                                                  0.000
Chi-square for each group:
  1
                                                 86.018
                                                 58.979
Model test baseline model:
                                                967.228
 Minimum Function Test Statistic
 Degrees of freedom
                                                     72
 P-value
                                                  0.000
User model versus baseline model:
                                                  0.905
  Comparative Fit Index (CFI)
  Tucker-Lewis Index (TLI)
                                                  0.886
Loglikelihood and Information Criteria:
                                                      8
```

Loglikelihood user model (HO)	-3718.418
Loglikelihood unrestricted model (H1)	-3645.919
Number of free parameters	48
Akaike (AIC)	7532.835
Bayesian (BIC)	7710.777
Sample-size adjusted Bayesian (BIC)	7558.548

## Root Mean Square Error of Approximation:

RMSEA		0.097
90 Percent Confidence Interval	0.077	0.117
P-value RMSEA <= 0.05		0.000

## ${\it Standardized \ Root \ Mean \ Square \ Residual:}$

SRMR 0.075

Parameter Estimates:

Information Expected Standard Errors Standard

## Group 1 [1]:

## Latent Variables:

Latent	Varı	ables:				
			Estimate	Std.Err	z-value	P(> z )
visua	al =~					
x1			1.000			
x2		(.p2.)	0.610	0.099	6.138	0.000
хЗ		(.p3.)	0.783	0.107	7.296	0.000
texti	ıal =	·~				
x4			1.000			
х5		(.p5.)	1.106	0.065	17.082	0.000
x6		(.p6.)	0.920	0.055	16.780	0.000
speed	d =~					
<i>x</i> 7			1.000			
x8		(.p8.)	1.129	0.165	6.842	0.000
х9		(.p9.)	1.209	0.178	6.809	0.000
Std.	.lv	Std.all				
0.8	310	0.720				
0.4	194	0.386				
0.6	634	0.552				
0.9	945	0.840				
1.0	046	0.846				
0.8	369	0.819				
0.5	556	0.552				
0.6	528	0.590				
0.6	372	0.724				

#### Covariances:

		Estimate	${\it Std.Err}$	z-value	P(> z )
visual ~	~				
textua	.1	0.345	0.092	3.739	0.000
speed		0.208	0.064	3.237	0.001
textual	~~				
speed		0.104	0.058	1.782	0.075
Std.lv	Std.all				
0.450	0.450				
0.462	0.462				
0.198	0.198				

## Intercepts:

Estimate Std.Err z-value P(>|z|)
.x1 (.25.) 5.097 0.089 57.448 0.000

```
.x2
            (.26.)
                       6.177
                                0.078
                                        78.759
                                                   0.000
                                0.081
                                                   0.000
   .x3
            (.27.)
                       2.367
                                         29.178
   .x4
            (.28.)
                       2.984
                                0.088
                                         33.774
                                                   0.000
   .x5
            (.29.)
                       4.254
                                0.098
                                         43.621
                                                   0.000
            (.30.)
                                0.082
                                         25.632
                                                   0.000
   .x6
                       2.111
   .x7
            (.31.)
                       4.142
                                0.072
                                        57.211
                                                   0.000
            (.32.)
                       5.471
                                0.075
                                         73.290
                                                   0.000
   .x8
   .x9
            (.33.)
                       5.337
                                0.073
                                         73.609
                                                   0.000
    visual
                       0.000
   textual
                       0.000
                       0.000
    speed
   Std.lv
          Std.all
    5.097
             4.533
    6.177
             4.831
             2.060
    2.367
    2.984
             2.652
    4.254
             3.441
             1.988
    2.111
    4.142
             4.111
    5.471
             5.139
    5.337
             5.752
             0.000
    0.000
             0.000
    0.000
    0.000
             0.000
Variances:
                   Estimate Std.Err z-value P(>|z|)
                       0.609
                                0.135
                                         4.509
                                                   0.000
   .x1
                       1.391
                                0.175
                                         7.965
                                                   0.000
   .x2
                       0.918
                                0.132
                                          6.972
                                                   0.000
   .хЗ
   .x4
                       0.372
                                0.066
                                         5.635
                                                   0.000
   .x5
                       0.434
                                0.079
                                         5.490
                                                   0.000
   .x6
                                0.061
                                         6.102
                                                   0.000
                       0.372
                                0.100
                                          7.035
                                                   0.000
   .x7
                       0.706
   .x8
                       0.740
                                0.110
                                          6.710
                                                   0.000
                                0.087
   .x9
                       0.409
                                         4.708
                                                   0.000
    visual
                       0.655
                                0.154
                                          4.261
                                                   0.000
                                          6.662
    textual
                       0.894
                                0.134
                                                   0.000
                       0.309
                                0.083
                                         3.744
                                                   0.000
    speed
   Std.lv Std.all
             0.482
    0.609
             0.851
    1.391
    0.918
             0.696
    0.372
             0.294
             0.284
    0.434
    0.372
             0.330
    0.706
             0.696
    0.740
             0.652
    0.409
             0.475
```

1.000

1.000

1.000

1.000

1.000

1.000

#### Latent Variables:

Estimate Std.Err z-value P(>|z|)

visual =	.~				
x1		1.000			
x2	(.p2.)	0.610	0.099	6.138	0.000
<i>x</i> 3	(.p3.)	0.783	0.107	7.296	0.000
textual	=~				
x4		1.000			
x5	(.p5.)	1.106	0.065	17.082	0.000
x6	(.p6.)	0.920	0.055	16.780	0.000
speed =~					
x7		1.000			
x8	(.p8.)	1.129	0.165	6.842	0.000
x9	(.p9.)	1.209	0.178	6.809	0.000
Std.lv	Std.all				
0.884	0.747				
0.539	0.504				
0.692	0.635				
1.033	0.867				
1.143	0.858				
0.950	0.849				
0.500	0.045				
0.615	0.531				
0.695	0.744				
0.744	0.698				
0.744	0.050				
Covariance	s.				
oovar rance	υ.	Estimate	Std.Err	z-value	P(> z )
visual ~	~	LBUIMAUC	Dou.LII	Z varuc	1 ( > 121 )
vibuai					
tortus	7	0 152	0 104	1 350	0 000
textua	1	0.452	0.104	4.350	0.000
speed	1 ~~	0.452 0.307	0.104 0.076	4.350 4.014	0.000 0.000
speed textual	1 ~~	0.307	0.076	4.014	0.000
speed textual speed	~~				
speed textual	1 ~~ Std.all	0.307	0.076	4.014	0.000
speed textual speed Std.lv	~~ Std.all	0.307	0.076	4.014	0.000
speed textual speed Std.lv 0.495	~~ Std.all 0.495	0.307	0.076	4.014	0.000
speed textual speed Std.lv	~~ Std.all	0.307	0.076	4.014	0.000
speed textual speed Std.lv 0.495 0.565	Std.all 0.495 0.565	0.307	0.076	4.014	0.000
speed textual speed Std.lv 0.495	~~ Std.all 0.495	0.307	0.076	4.014	0.000
speed textual speed Std.lv 0.495 0.565	Std.all 0.495 0.565 0.343	0.307	0.076	4.014	0.000
speed textual speed Std.lv 0.495 0.565	Std.all 0.495 0.565 0.343	0.307 0.218	0.076 0.071	4.014 3.087	0.000
speed textual speed Std.lv 0.495 0.565 0.343 Intercepts	Std.all 0.495 0.565 0.343	0.307 0.218 Estimate	0.076 0.071 Std.Err	4.014 3.087 z-value	0.000 0.002 P(> z )
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1	Std.all 0.495 0.565 0.343 : (.25.)	0.307 0.218 Estimate 5.097	0.076 0.071 Std.Err 0.089	2-value 57.448	0.000 0.002 P(> z ) 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1 .x2	Std.all 0.495 0.565 0.343 : (.25.) (.26.)	0.307 0.218 Estimate 5.097 6.177	0.076 0.071 Std.Err 0.089 0.078	2-value 57.448 78.759	0.000 0.002 P(> z ) 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1 .x2 .x3	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.)	0.307 0.218 Estimate 5.097 6.177 2.367	0.076 0.071 Std.Err 0.089 0.078 0.081	2-value 57.448 78.759 29.178	0.000 0.002 P(> z ) 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1 .x2 .x3 .x4	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.)	0.307 0.218 Estimate 5.097 6.177 2.367 2.984	0.076 0.071 Std.Err 0.089 0.078 0.081 0.088	z-value 57.448 78.759 29.178 33.774	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1 .x2 .x3 .x4 .x5	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.)	0.307 0.218 Estimate 5.097 6.177 2.367 2.984 4.254	0.076 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098	2-value 57.448 78.759 29.178 33.774 43.621	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1 .x2 .x3 .x4 .x5 .x6	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.)	0.307 0.218 Estimate 5.097 6.177 2.367 2.984 4.254 2.111	0.076 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098 0.098	z-value 57.448 78.759 29.178 33.774 43.621 25.632	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1 .x2 .x3 .x4 .x5 .x6 .x7	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.)	0.307 0.218 Estimate 5.097 6.177 2.367 2.984 4.254 2.111 4.142	0.076 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098 0.098 0.082 0.072	z-value 57.448 78.759 29.178 33.774 43.621 25.632 57.211	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000 0.000
speed textual speed Std.1v  0.495 0.565  0.343  Intercepts  .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.)	0.307 0.218 Estimate 5.097 6.177 2.367 2.984 4.254 2.111 4.142 5.471	0.076 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098 0.098 0.072 0.075	z-value 57.448 78.759 29.178 33.774 43.621 25.632 57.211 73.290	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts  .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8 .x9	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)	0.307 0.218 Estimate 5.097 6.177 2.367 2.984 4.254 2.111 4.142 5.471 5.337	0.076 0.071 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098 0.098 0.072 0.075 0.073	z-value 57.448 78.759 29.178 33.774 43.621 25.632 57.211 73.290 73.609	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts  .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8 .x9 visual	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)	0.307 0.218 0.218 Estimate 5.097 6.177 2.367 2.984 4.254 2.111 4.142 5.471 5.337 -0.315	0.076 0.071 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098 0.098 0.072 0.075 0.073 0.121	z-value 57.448 78.759 29.178 33.774 43.621 25.632 57.211 73.290 73.609 -2.598	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts  .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8 .x9 visual textua	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)	0.307 0.218 0.218 Estimate 5.097 6.177 2.367 2.984 4.254 2.111 4.142 5.471 5.337 -0.315 0.153	0.076 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098 0.098 0.072 0.075 0.073 0.121 0.121	z-value 57.448 78.759 29.178 33.774 43.621 25.632 57.211 73.290 73.609 -2.598 1.258	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8 .x9 visual textua speed	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)	0.307 0.218 0.218 Estimate 5.097 6.177 2.367 2.984 4.254 2.111 4.142 5.471 5.337 -0.315	0.076 0.071 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098 0.098 0.072 0.075 0.073 0.121	z-value 57.448 78.759 29.178 33.774 43.621 25.632 57.211 73.290 73.609 -2.598	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8 .x9 visual textual speed Std.lv	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)  Std.all	0.307 0.218 0.218 Estimate 5.097 6.177 2.367 2.984 4.254 2.111 4.142 5.471 5.337 -0.315 0.153	0.076 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098 0.098 0.072 0.075 0.073 0.121 0.121	z-value 57.448 78.759 29.178 33.774 43.621 25.632 57.211 73.290 73.609 -2.598 1.258	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8 .x9 visual textual speed Std.lv 5.097	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)  Std.all 4.311	0.307 0.218 0.218 Estimate 5.097 6.177 2.367 2.984 4.254 2.111 4.142 5.471 5.337 -0.315 0.153	0.076 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098 0.098 0.072 0.075 0.073 0.121 0.121	z-value 57.448 78.759 29.178 33.774 43.621 25.632 57.211 73.290 73.609 -2.598 1.258	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
speed textual speed Std.lv  0.495 0.565  0.343  Intercepts .x1 .x2 .x3 .x4 .x5 .x6 .x7 .x8 .x9 visual textual speed Std.lv	Std.all 0.495 0.565 0.343 : (.25.) (.26.) (.27.) (.28.) (.29.) (.30.) (.31.) (.32.) (.33.)  Std.all	0.307 0.218 0.218 Estimate 5.097 6.177 2.367 2.984 4.254 2.111 4.142 5.471 5.337 -0.315 0.153	0.076 0.071 Std.Err 0.089 0.078 0.081 0.088 0.098 0.098 0.072 0.075 0.073 0.121 0.121	z-value 57.448 78.759 29.178 33.774 43.621 25.632 57.211 73.290 73.609 -2.598 1.258	0.000 0.002 P(> z ) 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000

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2.984
         2.504
 4.254
         3.195
         1.887
2.111
4.142
         3.575
5.471
         5.857
5.337
        5.009
         -0.356
-0.356
 0.148
         0.148
 0.092
         0.092
```

#### Variances:

		Estimate	${\it Std.Err}$	z-value	P(> z )
.x1		0.617	0.126	4.903	0.000
.x2		0.852	0.109	7.796	0.000
.x3		0.706	0.105	6.710	0.000
.x4		0.353	0.063	5.602	0.000
. x5		0.467	0.080	5.845	0.000
.x6		0.350	0.057	6.085	0.000
.x7		0.964	0.125	7.701	0.000
.x8		0.390	0.078	5.016	0.000
.x9		0.582	0.100	5.833	0.000
visual		0.781	0.165	4.722	0.000
textua	1	1.067	0.153	6.976	0.000
speed		0.378	0.102	3.715	0.000
Std.lv	Std.all				
0.617	0.441				
0.852	0.746				
0.706	0.596				
0.353	0.248				
0.467	0.263				
0.350	0.279				
0.964	0.718				
0.390	0.447				
0.582	0.513				
1.000	1.000				
1.000	1.000				
1.000	1.000				

Checking if this constraint reduces fit:

Again, constraining the loadings and intercepts makes the fit worse.