Conditional Models

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Packages

```r
library(psych)
library(sjPlot)
library(broom)
library(lme4)
library(MuMIn)
library(merTools)
library(reghelper)
library(stargazer)
library(lsmeans)
library(multcompView)
library(plyr)
library(tidyverse)
```

Basic Syntex

From last week:

- **Level 1:** $Y_{ij} = \beta_{0j} + \varepsilon ij$
- **Level 2:** $\beta_{0j} = \gamma_{00} + U_{0j}$

Sample Data

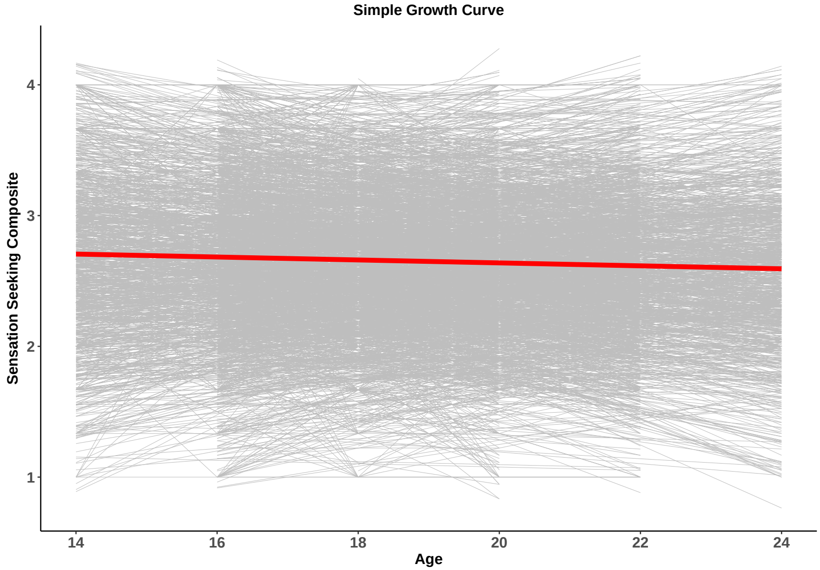
The National Longitudinal Study of Youths 1979 Child and Young Adult Sample (NLSYCYA) is a longitudinal study conducted by the National Bureau of Labor Statistics. The sample includes the children of the original 1979 sample. Here, we are going to use a subset of the more than 11,000 variables available that include the following.

Item Name	Description	Time-Varying?
PROC_CID	Participant ID	No
Dem_DOB	Year of Date of Birth	No
groups	Jail, Community Service, None	No
DemPWeight	Weight Percentile at age 10	No
age	Age of participant	Yes
Year	Year of Survey	Yes
age0	Age of participant (centered)	Yes
SensSeek	Sensation-Seeking Composite	Yes
CESD	CESD Depression Composite	Yes

Simple Growth Curve Model

- **Level 1:** $Y_{ij} = \beta_{0j} + \beta_{1j} * time_{ij} + \varepsilon ij$
- **Level 2:**
  - $\beta_{0j} = \gamma_{00} + U_{0j}$
  - $\beta_{1j} = \gamma_{10} + U_{1j}$

Simple Growth Curve Model



In R

```r
mod0 <- lmer(SensSeek ~ age0 + (1|PROC_CID), data = sample_dat)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: SensSeek ~ age0 + (1 | age0)
##    Data: sample_dat
##
## REML criterion at convergence: 28493.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.99678 -0.61465  0.01054  0.59619  2.47175
##
## Random effects:
##  Groups   Name        Variance  Std.Dev.
##  age0     (Intercept) 3.053e-15 5.525e-08
##  Residual             3.240e-01 5.692e-01
## Number of obs: 16645, groups:  age0, 6
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  2.705676   0.007061   383.2
## age0        -0.011252   0.001388    -8.1
##
## Correlation of Fixed Effects:
##      (Intr)
## age0 -0.781
```

Conditional Models: Adding Predictors

Let's see if we can better predict participants' change in sensation seeking over time by adding covariates.

Predictor	Continuous	Categorical
Time Invariant	Weight for Age	Group
Time Varying	CESD Scores	Depression

Time Invariant Predictors

Time Invariant Predictors: Continuous

The basic equation, specifying a random intercept and slope:

- **Level 1:** $Y_{ij} = \beta_{0j} + \beta_{1j} * time_{1j} + \varepsilon ij$
- **Level 2:**
  - $\beta_{0j} = \gamma_{00} + \gamma_{01} * X_{2j} + U_{0j}$
  - $\beta_{1j} = \gamma_{10} + \gamma_{11} * X_{2j} + U_{1j}$

But we need to break this down to see that adding additional predictors results in interaction terms:

$$Y_{ij} = \gamma_{00} + \gamma_{01} * X_{2j} + U_{0j} + (\gamma_{10} + \gamma_{11} * X_{2j} + U_{1j}) * X_{1j} + \varepsilon ij$$
$$Y_{ij} = \gamma_{00} + \gamma_{01} * X_{2j} + \gamma_{10} * X_{1j} + \gamma_{11} * X_{2j} * X_{1j} + U_{0j} + U_{1j} * X_{1j} + \varepsilon ij$$

We can also fit this with intercepts depending on weight, but without the change (slope) dependent on weight:

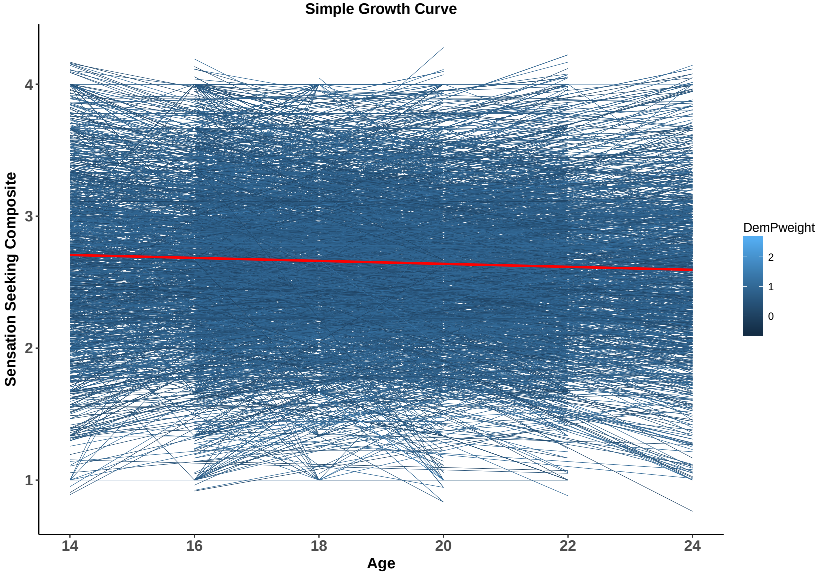
$$Y_{ij} = \gamma_{00} + \gamma_{01} * X_{2j} + U_{0j} + (\gamma_{10} + U_{1j}) * X_{1j} + \varepsilon ij$$
$$Y_{ij} = \gamma_{00} + \gamma_{01} * X_{2j} + \gamma_{10} * X_{1j} + U_{0j} + U_{1j} * X_{1j} + \varepsilon ij$$

Time Invariant Predictors: Continuous Example - Weight for Age Percentile

```r
describe(sample_dat$DemPweight)
```

```
##    vars     n mean   sd median trimmed  mad   min max range skew kur
## X1    1 16645 0.66 0.32   0.71    0.68 0.36 -0.64 2.7  3.35 -0.3
##    se
## X1  0
```

Time Invariant Predictors: Continuous Example - Weight for Age Percentile



Time Invariant Predictors: Continuous Example - Weight for Age Percentile

```r
# time invariant covariate with random intecept (with weight as covariate)
# and slope (without weight as a covariate)
mod1a <- lmer(SensSeek ~ age0 + DemPweight + (age0|PROC_CID),
              data = sample_dat)

summary(mod1a)

# time invariant predictor with random slope and intercept
mod1b <- lmer(SensSeek ~ age0 + DemPweight + age0*DemPweight +
                (age0|PROC_CID), data = sample_dat)

summary(mod1b)
```

Time Invariant Predictors: Categorical Example - 2 level group

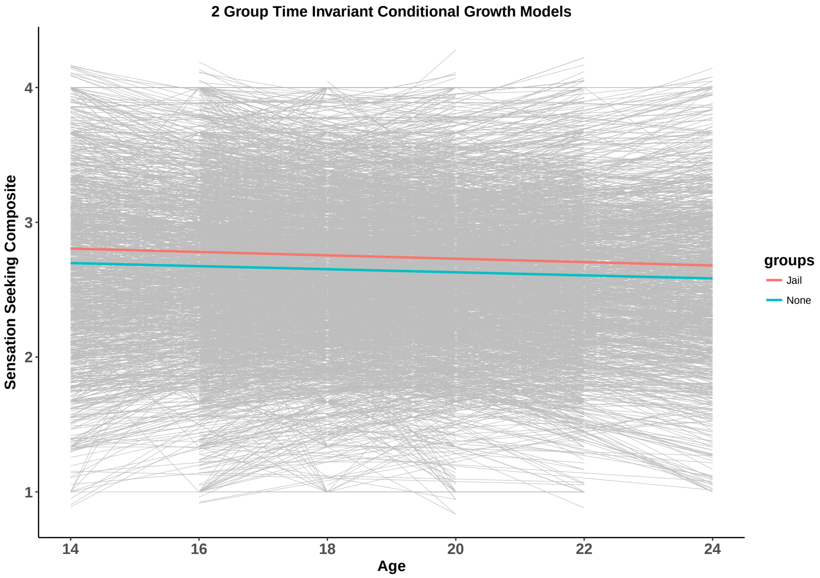
Lets's start with 2 groups: Jail v. None

- **Level 1:** $Y_{ij} = \beta_{0j} + \beta_{1j} * time_{1j} + \varepsilon ij$
- **Level 2:**
  - $\beta_{0j} = \gamma_{00} + \gamma_{01} * X_{2j} + U_{0j}$
  - $\beta_{1j} = \gamma_{10} + \gamma_{11} * X_{2j} + U_{1j}$

Time Invariant Predictors: Example - 2 level group

- **Level 1:** $Y_{ij} = \beta_{0j} + \beta_{1j} * age0_{ij} + \varepsilon ij$
- **Level 2:**
  - $\beta_{0j} = \gamma_{00} + \gamma_{01} * groupsNone + U_{0j}$
  - $\beta_{1j} = \gamma_{10} + \gamma_{11} * groupsNone + U_{1j}$

Variable	D1
Jail	0
None	1



2 Group Time Invariant Conditional Growth Models
Sensation Seeking Composite
4
3
2
1
14
16
18
20
22
24
Age
groups
Jail
None

Time Invariant Predictors: Example - 2 level group

```r
mod2g <- lmer(SensSeek ~ age0 + groups + age0*groups + (age0|PROC_CID),
              data = sample_dat %>% filter(groups != "CommServ"))
summary(mod2g)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: SensSeek ~ age0 + groups + age0 * groups + (age0 | PROC_CID)
##    Data: sample_dat %>% filter(groups != "CommServ")
##
## REML criterion at convergence: 25439.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8386 -0.4713  0.0097  0.4741  3.5087
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  PROC_CID (Intercept) 0.1601936 0.40024
##           age0        0.0009029 0.03005  -0.40
##  Residual             0.1757529 0.41923
## Number of obs: 16134, groups:  PROC_CID, 7188
##
## Fixed effects:
##                  Estimate Std. Error t value
## (Intercept)      2.811511   0.030945   90.86
## age0            -0.011601   0.005078   -2.28
## groupsNone      -0.115815   0.031847   -3.64
## age0:groupsNone  0.002546   0.005231    0.49
##
## Correlation of Fixed Effects:
##             (Intr) age0   grpsNn
## age0        -0.633
## groupsNone  -0.972  0.615
## age0:grpsNn  0.615 -0.971 -0.634
```

Side Note: `lme4` helper functions

```
vcov(mod2g)
VarCorr(mod2g)
fixef(mod2g)
ranef(mod2g)
coef(mod2g)
confint.merMod(mod2g, method = "boot")
reghelper::ICC(mod2g)
MuMIn::r.squaredGLMM(mod2g)
```

```r
vcov(mod2g)
```

```
## 4 x 4 Matrix of class "dpoMatrix"
##                    (Intercept)          age0    groupsNone age
## (Intercept)      9.575941e-04 -9.950566e-05 -9.575941e-04
## age0            -9.950566e-05  2.578270e-05  9.950566e-05   -
## groupsNone      -9.575941e-04  9.950566e-05  1.014221e-03   -
## age0:groupsNone  9.950566e-05 -2.578270e-05 -1.055740e-04
```

```r
VarCorr(mod2g)
```

```
##  Groups   Name        Std.Dev. Corr
##  PROC_CID (Intercept) 0.400242
##           age0        0.030048 -0.397
##  Residual             0.419229
```

```r
fixef(mod2g)
```

```
##     (Intercept)            age0      groupsNone age0:groupsNo
##     2.811510889    -0.011600946    -0.115815331     0.0025459
```

```r
ranef(mod2g)
```

```
## $PROC_CID
##           (Intercept)          age0
## 301      0.2386857200 -2.795195e-03
## 302      0.0209145813  7.910316e-04
## 303     -0.1976874625  1.107953e-03
## 401      0.4385500788 -9.212628e-03
## 801      0.0521633381 -1.724546e-02
## 802      0.5713034994 -1.458977e-02
## 803      0.0540967467 -8.653343e-03
## 1001     0.3045440068 -2.515398e-03
## 1602     0.1451053206 -4.326995e-03
## 1603     0.1515629720 -2.989324e-03
## 2001    -0.1018502691 -5.327357e-05
## 2501    -0.2280747057  6.426858e-03
## 2502     0.1451053206 -4.326995e-03
## 2701     0.1223335127 -3.781162e-03
## 2702     0.1515629720 -2.989324e-03
## 3002    -0.0178411027 -1.821252e-02
## 3003    -0.2294312321  5.748447e-03
## 4301     0.3124375351  2.608497e-03
```

```r
confint.merMod(mod2g, method = "boot", nsim = 10)
```

```
##                       2.5 %       97.5 %
## .sig01            0.386164710  0.405363134
## .sig02           -0.421473723 -0.280943625
## .sig03            0.026226070  0.033223113
## .sigma            0.411804912  0.422225484
## (Intercept)       2.777500804  2.896510060
## age0             -0.020983416 -0.002090931
## groupsNone       -0.193656044 -0.073856220
## age0:groupsNone  -0.006936207  0.011103280
```

All units of the random effects are in standard deviation units (which means you need to square them to get the variance!!)

- .sig01 = sd of random intercept = $\sqrt{\tau_{00}}$
- .sig02 = correlation between slope and intercept = $\sqrt{\tau_{10}}$
- .sig03 = sd of random slope = $\sqrt{\tau_{11}}$
- .sigma = residual variance = $\hat{\sigma}$

```r
reghelper::ICC(mod2g)
```

```
## [1] 0.4782449
```

Conditional R^2**:** How much variance fixed + random effects explain
Marginal R^2**:** how much variance the fixed effects explain

explained here

```
MuMIn::r.squaredGLMM(mod2g)
```

```
##          R2m         R2c
## 0.004487822 0.455541793
```

Side Note: Creating MLM Tables

There are lots of helpful packages for this, including `stargazer` and `sjPlot`, which are demonstrated below.

```
stargazer::stargazer(mod2g)
sjPlot::sjt.lmer(mod2g)
```

The problem is that `stargazer()` doesn't include all the terms we want, and `sjt.lmer()` only renders html. Embedded in the `.Rmd` version of these slides is some code that should help you to extract the terms you need and create a table using `dplyr` and `tidyr` that you can render in LaTeXusing `stargazer`.

Side Note: Creating MLM Tables

But let's understand where those variables came from. To do so, we'll use the `broom` package in R to grab the terms we need.

Description	Math Notation
Fixed Effect Intercept	γ_{00}
Fixed Effect Group Intercept	γ_{01}
Fixed Effect Age Slope	γ_{10}
Fixed Effect Group Slope	γ_{11}
Individual Random Intercepts	U_{0j}
Variance of Random Intercepts	τ_{00}
Random Age Slopes	U_{10}
Variance of Random Age Slopes	τ_{11}
Correlation b/w Random Slopes and Intercepts	τ_{10}
Residual Variance	$\hat{\sigma}^2$
Intraclass Correlation	ICC
Conditional R^2	R^2_c
Marginal R^2	R^2_m

Side Note: Creating MLM Tables

```r
broom::tidy(mod2g)
broom::glance(mod2g)
```

```
##                           term     estimate   std.error  statistic
## 1                  (Intercept)  2.811510889 0.030945018 90.8550409
## 2                         age0 -0.011600946 0.005077667 -2.2846999
## 3                   groupsNone -0.115815331 0.031846832 -3.6366359
## 4              age0:groupsNone  0.002545993 0.005231305  0.4866841
## 5       sd_(Intercept).PROC_CID  0.400241919          NA         NA
## 6              sd_age0.PROC_CID  0.030048359          NA         NA
## 7 cor_(Intercept).age0.PROC_CID -0.397195984          NA         NA
## 8       sd_Observation.Residual  0.419228923          NA         NA
##      group
## 1    fixed
## 2    fixed
## 3    fixed
## 4    fixed
## 5 PROC_CID
## 6 PROC_CID
## 7 PROC_CID
## 8 Residual
```

```
##       sigma    logLik      AIC      BIC deviance df.residual
## 1 0.4192289 -12719.82 25455.64 25517.15  25405.3       16126
```

Side Note: Creating MLM Tables

```r
options(knitr.kable.NA = '')
knitr::kable(tab, caption = "Ugly MLM Table Example")
```

Table 5: Ugly MLM Table Example

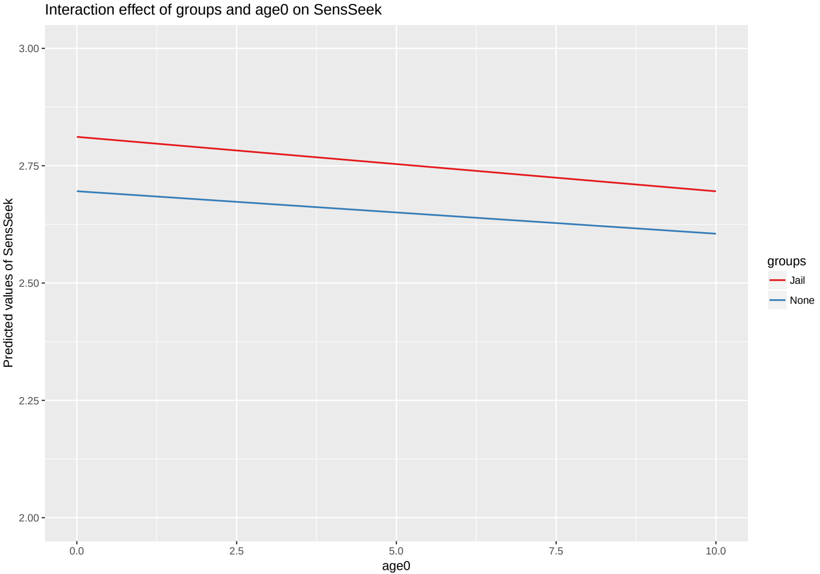
type	term	estimate	CI
Fixed Parts	(Intercept)	2.81	(2.77, 2.86)
Fixed Parts	age0	-0.01	(-0.02, -0.00)
Fixed Parts	groupsNone	-0.12	(-0.15, -0.07)
Fixed Parts	age0:groupsNone	0.00	(-0.01, 0.01)
Random Parts	τ00	0.16	(0.16, 0.17)
Random Parts	τ_{11}	0.00	(0.00, 0.00)
Random Parts	τ_{10}	0.16	(0.21, 0.09)
Random Parts	$\hat{\sigma}^2$	0.18	(0.17, 0.18)
mod_terms	ICC	0.48	
mod_terms	R_m^2	0.00	
mod_terms	R_c^2	0.46	

Side Note: Plotting Simple Effects

```r
# categorical
sjp.int(mod2g, type = "eff", p.kr = F, swap.pred = T)
# continuous
sjp.int(mod1b, type = "eff", p.kr = F, swap.pred = T,
        mdrt.values = "meansd")
```

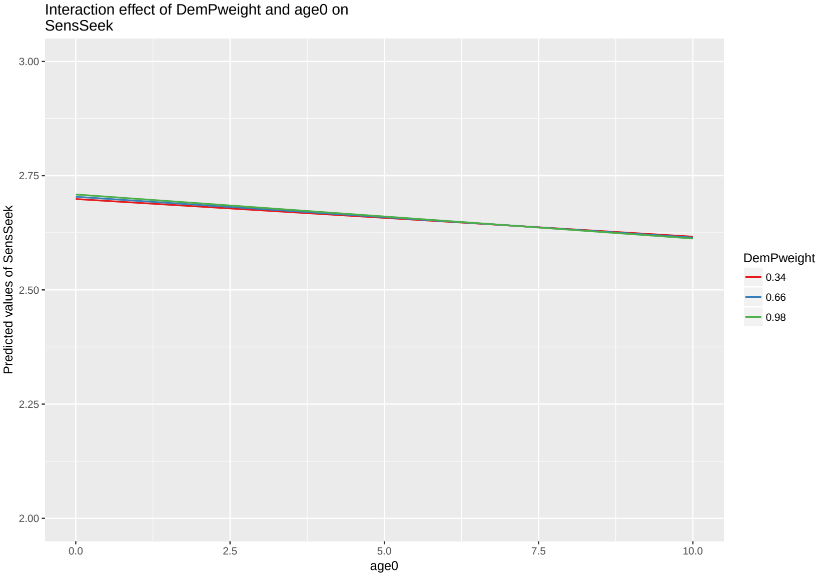
Side Note: Plotting Simple Effects (Categorical)

```
sjp.int(mod2g, type = "eff", p.kr = F, swap.pred = T)
```



Side Note: Plotting Simple Effects (Continuous)

```
sjp.int(mod1b, type = "eff", p.kr = F, swap.pred = T, mdrt.value
```



Side Note: Comparisons with `lsmeans`

```r
# create a reference grid
ref.grid2g <- ref.grid(mod2g)
# create the lsmeans object
lsgroups   <- lsmeans(ref.grid2g, "groups")
# compact letter display
cld(lsgroups, alpha = .10)
# plot
plot(lsgroups)
# contrasts of the ref.grid object
contrast(ref.grid2g, method = "eff")
# comparisons
groups.sum <- summary(lsgroups, infer = c(TRUE,TRUE),
                      level = .90, adjust = "bon", by = "groups")
```

```r
# create a reference grid
(ref.grid2g <- ref.grid(mod2g))
```

```
## 'ref.grid' object with variables:
##     age0 = 3.9838
##     groups = Jail, None
```

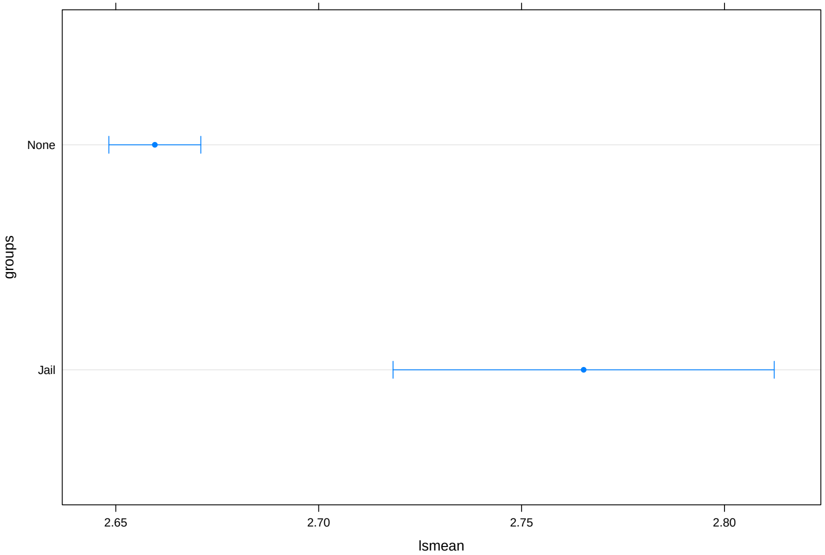
```r
# create the lsmeans object
(lsgroups   <- lsmeans(ref.grid2g, "groups"))
```

```
##  groups   lsmean          SE      df lower.CL upper.CL
##  Jail   2.765295 0.023957487 6923.38 2.718331 2.812260
##  None   2.659623 0.005780429 7125.64 2.648291 2.670954
##
## Degrees-of-freedom method: satterthwaite
## Confidence level used: 0.95
```

```r
# compact letter display
cld(lsgroups, alpha = .10)
```

```
##  groups   lsmean          SE      df lower.CL upper.CL
##  None   2.659623 0.005780429 7125.64 2.648291 2.670954
##  Jail   2.765295 0.023957487 6923.38 2.718331 2.812260
##
## Degrees-of-freedom method: satterthwaite
## Confidence level used: 0.95
## significance level used: alpha = 0.1
```

```r
# plot
plot(lsgroups)
```



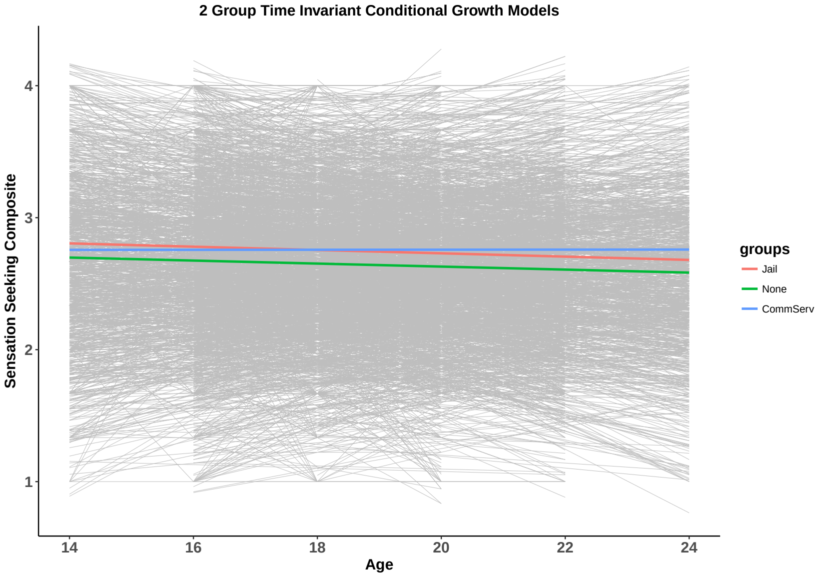
```r
# contrasts of the ref.grid object
contrast(ref.grid2g, method = "eff")
```

```
##  contrast                              estimate         SE
##  3.9837610016115,Jail effect  0.05283635 0.01232249 6934
##  3.9837610016115,None effect -0.05283635 0.01232249 6934
##  p.value
##   <.0001
##   <.0001
##
## P value adjustment: fdr method for 2 tests
```

```r
# comparisons
(groups.sum <- summary(lsgroups, infer = c(TRUE,TRUE),
          level = .90, adjust = "bon", by = "groups"))
```

```
## groups = Jail:
##    lsmean          SE      df lower.CL upper.CL t.ratio
##  2.765295 0.023957487 6923.38 2.725884 2.804707 115.425
##
## groups = None:
##    lsmean          SE      df lower.CL upper.CL t.ratio
##  2.659623 0.005780429 7125.64 2.650114 2.669132 460.108
##
## Degrees-of-freedom method: satterthwaite
## Confidence level used: 0.9
```

Time Invariant Predictors: Example - 3 level group



Time Invariant Predictors: Example - 3 level group

- **Level 1:** $Y_{ij} = \beta_{0j} + \beta_{1j} * age0_{ij} + \varepsilon ij$
- **Level 2:**
  - $\beta_{0j} = \gamma_{00} + \gamma_{01} * D1 + \gamma_{02} * D2 + U_{0j}$
  - $\beta_{1j} = \gamma_{10} + \gamma_{11} * D1 + \gamma_{12} * D2 + U_{1j}$

Variable	D1	D2
Jail	0	0
None	1	0
CommServ } 0	1	

Time Invariant Predictors: Example - 3 level group

```r
mod3g <- lmer(SensSeek ~ age0 + groups + age0*groups +
                  (age0|PROC_CID), data = sample_dat)
summary(mod3g)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: SensSeek ~ age0 + groups + age0 * groups + (age0 | PROC_CID)
##    Data: sample_dat
##
## REML criterion at convergence: 26251.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8387 -0.4727  0.0097  0.4728  3.5010
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  PROC_CID (Intercept) 0.1579466 0.39742
##           age0        0.0008989 0.02998  -0.39
##  Residual             0.1761279 0.41968
## Number of obs: 16645, groups:  PROC_CID, 7423
##
## Fixed effects:
##                        Estimate Std. Error t value
## (Intercept)            2.811413   0.030865   91.09
## age0                  -0.011577   0.005082   -2.28
## groupsNone            -0.115728   0.031765   -3.64
## groupsCommServ        -0.058406   0.050348   -1.16
## age0:groupsNone        0.002527   0.005236    0.48
## age0:groupsCommServ    0.013322   0.008875    1.50
##
## Correlation of Fixed Effects:
##             (Intr) age0   grpsNn grpsCS ag0:gN
```

Time Varying Predictors

Time Varying Predictors: Continuous

Next, we'll add in a time-varying predictor. Maybe it's not that our participants sensation seeking is moderated by early life experiences of jail or court-ordered community service. Instead, their sensation seeking is moderated by depression.
How does this look?

- **Level 1:** $Y_{ij} = \beta_{0j} + \beta_{1j} * time + \beta_{2j} * CESD + \varepsilon ij$
- **Level 2:**
  - $\beta_{0j} = \gamma_{00} + \gamma_{01} + U_{0j}$
  - $\beta_{1j} = \gamma_{10} + U_{1j}$
  - $\beta_{2j} = \gamma_{20}$

Time Varying Predictors: Continuous

To Interaction or Not - That Is the Question

- **Level 1:** $Y_{ij} = \beta_{0j} + \beta_{1j} * age0 + \beta_{2j} * CESD + \varepsilon ij$
- **Level 2:**
  - $\beta_{0j} = \gamma_{00} + \gamma_{01} + U_{0j}$
  - $\beta_{1j} = \gamma_{10} + U_{1j}$
  - $\beta_{2j} = \gamma_{20}$

$$Y_{ij} = \gamma_{00} + \gamma_{01} + U_{0j} + (\gamma_{10} + U_{1j}) * age0 + \gamma_{20} * CESD$$

Time Varying Predictors: Continuous

```r
modTV1 <- lmer(SensSeek ~ age0 + CESD + (age0|PROC_CID), data =
summary(modTV1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: SensSeek ~ age0 + CESD + (age0 | PROC_CID)
##    Data: sample_dat
##
## REML criterion at convergence: 26222.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8913 -0.4785  0.0140  0.4705  3.5504
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  PROC_CID (Intercept) 0.1572108 0.39650
##           age0        0.0008911 0.02985  -0.38
##  Residual             0.1759002 0.41940
## Number of obs: 16645, groups:  PROC_CID, 7423
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  2.672365   0.008797  303.78
## age0        -0.009076   0.001205   -7.53
## CESD         0.049854   0.008088    6.16
##
## Correlation of Fixed Effects:
##      (Intr) age0
## age0 -0.507
## CESD -0.580 -0.020
```

Time Varying Predictors: Categorical

Next, we'll add in a time-varying predictor. Maybe it's not that our participants sensation seeking is moderated by early life experiences of jail or court-ordered community service. Instead, their sensation seeking is moderated by depression.
How does this look?

- **Level 1:** $Y_{ij} = \beta_{0j} + \beta_{1j} * time + \beta_{2j} * depressed + \varepsilon ij$
- **Level 2:**
  - $\beta_{0j} = \gamma_{00} + \gamma_{01} + U_{0j}$
  - $\beta_{1j} = \gamma_{10} + U_{1j}$
  - $\beta_{2j} = \gamma_{20}$

Time Varying Predictors: Categorical

```r
# creating a dummy variable for time varying categorical depress
sample_dat <- sample_dat %>%
  mutate(depressed =
           factor(ifelse(CESD <= 1.5, 0, 1), levels = c(0,1),
                  labels = c("Depressed", "Not Depressed")))
modTV2 <- lmer(SensSeek ~ age0 + depressed + (age0|PROC_CID),
               data = sample_dat)
summary(modTV2)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: SensSeek ~ age0 + depressed + (age0 | PROC_CID)
##    Data: sample_dat
##
## REML criterion at convergence: 26257.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8439 -0.4702  0.0091  0.4721  3.4959
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  PROC_CID (Intercept) 0.1583541 0.39794
##           age0        0.0008978 0.02996  -0.38
##  Residual             0.1761783 0.41974
## Number of obs: 16645, groups:  PROC_CID, 7423
##
## Fixed effects:
##                          Estimate Std. Error t value
```

confidence intervals and effect size

fitted / predicted values

Other Things

autoregressive models, autoregressive errors

cohen's d - changing intercept with 0 at last time point