Week3 MLM

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Problem 1:

Run a series of models using a time-invariant nominal covariate. a) where the covariate only predicts the intercept b) predicts both intercept and slope c) is rescaled eg centering. For all models, how does your model change from model to model. What is your final model?

```
Part A: Fixed effect estimates
Intercept = 40.57; mean of Controls at mean age (age.centered = 0)
Age.centered = 2.05; increase in words correct/year for all participants
Group = -1.4; difference in words correct between Control and PKU at mean age (age.centered = 0)
Pseudo-R2 = .38 (marginal; fixed) and .69 (conditional; fixed + random)
Part B: Fixed effect estimates
Intercept = 40.93; mean of Controls at mean age (age.centered = 0)
Age.centered = 2.33; increase in words correct/year for Controls
Group = -2.25; difference in words correct between Control and PKU age mean age (age.centered = 0)
Age.centered:GROUP = -.77; difference in slope between Control and PKU
Pseudo-R2 = .38 (marginal; fixed) and .70 (conditional; fixed + random)
Part C: Fixed effect estimates
Intercept = 38.69; mean of PKU at mean age (age.centered = 0)
Age.centered = 1.56; increase in words correct/year for PKU
Group = 2.25; difference in words correct between PKU and Controls at mean age (age.centered = 0)
Age.centered:GROUP = .77; difference in slope between PKU and Control
Pseudo-R2 = .38 (marginal; fixed) and .70 (conditional; fixed + random)
```

Likelihood ratio test suggests that simpler model, where covariate only predicts intercept, is preferred.

```
#Model from HW #2
library(lme4)

## Loading required package: Matrix

##
## Attaching package: 'Matrix'

## The following object is masked from 'package:tidyr':
##
## expand
```

```
library(MuMIn)
wide_to_long_merged$Timepoint <- as.factor(wide_to_long_merged$Timepoint)</pre>
lin.mlm <- lmer(Sem_TotalCorrect ~ age.centered + (age.centered | ID2), data = wide_to_long_merged)</pre>
summary(lin.mlm)
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered + (age.centered | ID2)
      Data: wide_to_long_merged
##
## REML criterion at convergence: 1381.7
##
## Scaled residuals:
       Min
                1Q Median
##
                                3Q
                                       Max
## -1.9380 -0.5849 -0.0708 0.4632 3.2995
##
## Random effects:
## Groups
             Name
                          Variance Std.Dev. Corr
             (Intercept) 30.9260 5.5611
## ID2
##
             age.centered 0.7509 0.8665
                                            0.45
## Residual
                          37.3791 6.1138
## Number of obs: 200, groups: ID2, 67
## Fixed effects:
                Estimate Std. Error t value
                40.0957
                            0.8712 46.02
## (Intercept)
## age.centered 2.0512
                             0.2504
                                       8.19
## Correlation of Fixed Effects:
##
               (Intr)
## age.centerd 0.360
anova(lin.mlm)
## Analysis of Variance Table
                Df Sum Sq Mean Sq F value
## age.centered 1 2507.2 2507.2 67.074
r.squaredGLMM(lin.mlm)
##
         R2m
                   R2c
## 0.3786386 0.6876268
#Part a: adding group, a time-invariant nominal covariate that only predicts the intercept
wide_to_long_merged$GROUP <- relevel(wide_to_long_merged$GROUP, ref = "Control")</pre>
lin.nom1 <- lmer(Sem_TotalCorrect ~ age.centered + GROUP +</pre>
                   (age.centered | ID2), data = wide_to_long_merged)
summary(lin.nom1)
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered + GROUP + (age.centered | ID2)
      Data: wide_to_long_merged
##
## REML criterion at convergence: 1378.1
## Scaled residuals:
##
       Min
               1Q Median
                                3Q
                                       Max
```

```
## -1.9198 -0.6026 -0.0533 0.4658 3.2676
##
## Random effects:
                          Variance Std.Dev. Corr
## Groups
           Name
##
             (Intercept) 30.7729 5.5473
##
             age.centered 0.7385 0.8594
                                            0.41
                          37.3936 6.1150
## Residual
## Number of obs: 200, groups: ID2, 67
##
## Fixed effects:
                Estimate Std. Error t value
## (Intercept)
                                    38.54
                40.5682
                            1.0526
                                       8.21
## age.centered 2.0531
                             0.2501
## GROUPPKU
                -1.4062
                             1.7262
                                    -0.81
##
## Correlation of Fixed Effects:
##
               (Intr) ag.cnt
## age.centerd 0.299
## GROUPPKU
               -0.564 - 0.030
anova(lin.nom1)
## Analysis of Variance Table
##
                Df Sum Sq Mean Sq F value
## age.centered 1 2506.32 2506.32 67.0254
## GROUP
                 1
                     24.82
                             24.82 0.6636
r.squaredGLMM(lin.nom1)
##
         R2m
                   R2c
## 0.3787432 0.6872902
#Part b: adding group, a time-invariant nominal covariate the predicts intercept and slope
wide_to_long_merged$GROUP <- relevel(wide_to_long_merged$GROUP, ref = "Control")</pre>
lin.nom2 <- lmer(Sem_TotalCorrect ~ age.centered + GROUP + age.centered:GROUP +</pre>
                   (age.centered | ID2), data = wide_to_long_merged)
summary(lin.nom2)
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered + GROUP + age.centered:GROUP +
##
       (age.centered | ID2)
##
      Data: wide_to_long_merged
## REML criterion at convergence: 1375.5
##
## Scaled residuals:
      Min
              1Q Median
                                30
                                       Max
## -1.9346 -0.6113 -0.0769 0.4429 3.2882
##
## Random effects:
## Groups
                          Variance Std.Dev. Corr
## ID2
             (Intercept) 30.6481 5.5361
##
             age.centered 0.8873 0.9419
                                            0.34
## Residual
                          36.4284 6.0356
## Number of obs: 200, groups: ID2, 67
```

```
## Fixed effects:
##
                        Estimate Std. Error t value
## (Intercept)
                         40.9334
                                     1.0788
                                               37.94
## age.centered
                           2.3289
                                      0.3124
                                               7.46
## GROUPPKU
                          -2.2454
                                      1.8333
                                               -1.22
## age.centered:GROUPPKU -0.7721
                                      0.5411
                                             -1.43
## Correlation of Fixed Effects:
##
               (Intr) ag.cnt GROUPP
## age.centerd 0.334
## GROUPPKU
              -0.588 -0.197
## a.:GROUPPKU -0.193 -0.577 0.305
anova(lin.nom2)
## Analysis of Variance Table
##
                      Df Sum Sq Mean Sq F value
## age.centered
                       1 2391.59 2391.59 65.6519
## GROUP
                           25.01 25.01 0.6866
## age.centered:GROUP 1
                           74.16 74.16 2.0357
r.squaredGLMM(lin.nom2)
##
         R<sub>2</sub>m
                   R2c
## 0.3824455 0.7000320
#Part c: rescaling nominal variable (i.e., changing dummy coding such that reference group becomes PKU)
wide_to_long_merged$GROUP <- relevel(wide_to_long_merged$GROUP, ref = "PKU")</pre>
lin.nom3 <- lmer(Sem_TotalCorrect ~ age.centered + GROUP + age.centered:GROUP +
                   (age.centered | ID2), data = wide_to_long_merged)
summary(lin.nom3)
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered + GROUP + age.centered:GROUP +
##
       (age.centered | ID2)
##
     Data: wide_to_long_merged
##
## REML criterion at convergence: 1375.5
##
## Scaled residuals:
            1Q Median
      Min
                                ЗQ
                                       Max
## -1.9346 -0.6113 -0.0769 0.4429 3.2882
##
## Random effects:
## Groups
            Name
                          Variance Std.Dev. Corr
             (Intercept) 30.6481 5.5361
             age.centered 0.8873 0.9419
##
                                            0.34
## Residual
                          36.4284 6.0356
## Number of obs: 200, groups: ID2, 67
## Fixed effects:
                            Estimate Std. Error t value
##
## (Intercept)
                                          1.4823 26.100
                            38.6880
## age.centered
                              1.5568
                                          0.4419 3.523
## GROUPControl
                               2.2454
                                          1.8333
                                                   1.225
## age.centered:GROUPControl 0.7721
                                          0.5411 1.427
```

```
##
## Correlation of Fixed Effects:
                (Intr) ag.cnt GROUPC
##
## age.centerd 0.291
## GROUPContrl -0.809 -0.235
## ag.c:GROUPC -0.237 -0.817 0.305
anova(lin.nom3)
## Analysis of Variance Table
                       Df Sum Sq Mean Sq F value
## age.centered
                         1 2391.59 2391.59 65.6519
## GROUP
                             25.01
                                      25.01 0.6866
## age.centered:GROUP
                             74.16
                                      74.16 2.0357
r.squaredGLMM(lin.nom3)
##
         R.2m
                    R2c
## 0.3824455 0.7000318
#Likelihood ratio test
anova(lin.nom1, lin.nom2)
## refitting model(s) with ML (instead of REML)
## Data: wide_to_long_merged
## Models:
## lin.nom1: Sem_TotalCorrect ~ age.centered + GROUP + (age.centered | ID2)
## lin.nom2: Sem_TotalCorrect ~ age.centered + GROUP + age.centered:GROUP +
                  (age.centered | ID2)
## lin.nom2:
            Df
                   AIC
                           BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## lin.nom1 7 1395.5 1418.5 -690.73
                                          1381.5
                                                                      0.1675
## lin.nom2 8 1395.5 1421.9 -689.77
                                          1379.5 1.9047
                                                               1
Problem 2:
Part A: Fixed effect estimates
Intercept = 37.46; mean across all participants at means levels of age (i.e., age.centered = 0) and when
baselinePho = 0
Age.centered = 1.91; slope (increase in words correct/year) at when baselinePho = 0
baselinePho = .13; slope (increase in words correct/unit of baselinePho) at mean age (age.centered = 0)
Pseudo-R2 = .39 (marginal; fixed) and .68 (conditional; fixed + random)
Part B: Fixed effect estimates
Intercept = 37.46; mean across all participants at mean age (age.centered = 0) and when baselinePho = 0
Age.centered = 1.80; slope (increase in words correct/year) when baselinePho = 0
baselinePho = .13; slope (increase in words correct/unit of baselinePho) at mean age (age.centered = 0)
Age.centered:baselinePho = .006; extent to which relationship between age.centered and Sem_Total_Correct
changes at different levels of baselinePho (change is minimal)
Pseudo-R2 = .39 (marginal; fixed) and .68 (conditional; fixed + random)
Part C: Fixed effect estimates
Intercept = 39.94; mean across all participants at means levels of age and baselinePho (i.e., age.centered
and baselinePho.centered = 0)
Age.centered = 1.90; slope (increase in words correct/year) at mean levels of baselinePho (base-
linePho.centered = 0
```

baselinePho = .13; slope (increase in words correct/unit of baselinePho) at mean age (age.centered = 0)

Age.centered:baselinePho = .006; extent to which relationship between age.centered and Sem_Total_Correct changes at different levels of baselinePho (change is minimal; this does not change as a function of centering)

Pseudo-R2 = .39 (marginal; fixed) and .68 (conditional; fixed + random)

Likelihood ratio test suggests that simpler model, where covariate only predicts intercept, is preferred.

```
#Part a: adding baselinePho, a time-invariant continuous covariate that only predicts the intercept
lin.cont1 <- lmer(Sem_TotalCorrect ~ age.centered + baselinePho +</pre>
                   (age.centered | ID2), data = wide_to_long_merged)
summary(lin.cont1)
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered + baselinePho + (age.centered |
##
       ID2)
##
      Data: wide_to_long_merged
##
## REML criterion at convergence: 1382.9
##
## Scaled residuals:
       Min
##
                1Q Median
                                3Q
                                       Max
  -1.9522 -0.5507 -0.0656 0.4575 3.2804
##
## Random effects:
##
   Groups
                          Variance Std.Dev. Corr
##
   ID2
             (Intercept)
                          29.8950 5.4676
##
             age.centered 0.5985 0.7736
                                             0.46
##
  Residual
                          38.2696 6.1862
## Number of obs: 200, groups: ID2, 67
##
## Fixed effects:
##
                Estimate Std. Error t value
## (Intercept)
                 37.4659
                             2.5437 14.729
                                     7.140
## age.centered
                  1.9052
                             0.2668
## baselinePho
                  0.1335
                             0.1250
                                      1.068
##
## Correlation of Fixed Effects:
##
               (Intr) ag.cnt
## age.centerd 0.492
## baselinePho -0.942 -0.407
anova(lin.cont1)
## Analysis of Variance Table
                Df Sum Sq Mean Sq F value
## age.centered 1 2632.65 2632.65 68.7923
## baselinePho
                 1
                     43.62
                             43.62 1.1399
r.squaredGLMM(lin.cont1)
         R2m
                   R2c
## 0.3891391 0.6786925
#Part b: adding group, a time-invariant nominal covariate the predicts intercept and slope
lin.cont2 <- lmer(Sem_TotalCorrect ~ age.centered + baselinePho + age.centered:baselinePho +
                   (age.centered | ID2), data = wide_to_long_merged)
summary(lin.cont2)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula:
## Sem_TotalCorrect ~ age.centered + baselinePho + age.centered:baselinePho +
       (age.centered | ID2)
##
##
      Data: wide_to_long_merged
##
## REML criterion at convergence: 1387.8
##
## Scaled residuals:
      Min
##
               1Q Median
                                3Q
                                       Max
## -1.9473 -0.5522 -0.0711 0.4504
                                   3.2828
##
## Random effects:
## Groups
            Name
                          Variance Std.Dev. Corr
             (Intercept) 30.2017 5.496
## ID2
             age.centered 0.6741 0.821
##
                                            0.44
                          38.1032 6.173
## Residual
## Number of obs: 200, groups: ID2, 67
##
## Fixed effects:
##
                             Estimate Std. Error t value
## (Intercept)
                            37.455111
                                      2.650150 14.133
## age.centered
                                                   2.482
                             1.797015
                                        0.723892
## baselinePho
                                                   1.042
                             0.132251
                                        0.126930
## age.centered:baselinePho 0.006416
                                       0.034494
                                                   0.186
## Correlation of Fixed Effects:
##
               (Intr) ag.cnt bslnPh
## age.centerd 0.416
## baselinePho -0.936 -0.274
## ag.cntrd:bP -0.259 -0.928 0.135
anova(lin.cont2)
## Analysis of Variance Table
##
                            Df Sum Sq Mean Sq F value
## age.centered
                             1 2576.52 2576.52 67.6194
                                         40.13 1.0531
## baselinePho
                                 40.13
                             1
## age.centered:baselinePho 1
                                  1.32
                                          1.32 0.0346
r.squaredGLMM(lin.cont2)
##
         R2m
                   R2c
## 0.3905637 0.6847858
#Part c: rescaling nominal variable (i.e., centering baselinePho)
wide_to_long_merged$baselinePho.centered <- wide_to_long_merged$baselinePho -
  mean(wide_to_long_merged$baselinePho, na.rm = T)
lin.cont3 <- lmer(Sem_TotalCorrect ~ age.centered + baselinePho.centered + age.centered:baselinePho.cen
                   (age.centered | ID2), data = wide_to_long_merged)
summary(lin.cont3)
## Linear mixed model fit by REML ['lmerMod']
## Formula:
## Sem_TotalCorrect ~ age.centered + baselinePho.centered + age.centered:baselinePho.centered +
       (age.centered | ID2)
```

```
##
     Data: wide_to_long_merged
##
## REML criterion at convergence: 1387.8
##
## Scaled residuals:
            1Q Median
##
      Min
                                3Q
                                       Max
## -1.9473 -0.5522 -0.0711 0.4504 3.2828
##
## Random effects:
## Groups
            Name
                          Variance Std.Dev. Corr
             (Intercept) 30.2017 5.496
             age.centered 0.6741 0.821
##
                                            0.44
## Residual
                          38.1032 6.173
## Number of obs: 200, groups: ID2, 67
##
## Fixed effects:
##
                                      Estimate Std. Error t value
## (Intercept)
                                     39.942761
                                                0.937405
                                                          42.61
## age.centered
                                      1.917692 0.271151
                                                             7.07
## baselinePho.centered
                                      0.132251
                                                 0.126930
                                                             1.04
## age.centered:baselinePho.centered 0.006416 0.034494
                                                             0.19
## Correlation of Fixed Effects:
               (Intr) ag.cnt bslnP.
## age.centerd 0.347
## bslnPh.cntr -0.099 -0.409
## ag.cntrd:P. -0.388 -0.084 0.135
anova(lin.cont3)
## Analysis of Variance Table
##
                                     Df Sum Sq Mean Sq F value
## age.centered
                                      1 2576.52 2576.52 67.6194
                                                 40.13 1.0531
## baselinePho.centered
                                          40.13
## age.centered:baselinePho.centered 1
                                           1.32
                                                   1.32 0.0346
r.squaredGLMM(lin.cont3)
                   R<sub>2</sub>c
##
         R<sub>2</sub>m
## 0.3905637 0.6847858
#Likelihood ratio test
anova(lin.cont1, lin.cont2)
## refitting model(s) with ML (instead of REML)
## Data: wide_to_long_merged
## Models:
## lin.cont1: Sem_TotalCorrect ~ age.centered + baselinePho + (age.centered |
## lin.cont1:
                 ID2)
## lin.cont2: Sem_TotalCorrect ~ age.centered + baselinePho + age.centered:baselinePho +
## lin.cont2:
                  (age.centered | ID2)
                         BIC logLik deviance Chisq Chi Df Pr(>Chisq)
            Df
                  AIC
## lin.cont1 7 1395.0 1418.0 -690.48
                                       1381.0
## lin.cont2 8 1396.9 1423.3 -690.47
                                       1380.9 0.0184
```

Problem 3:

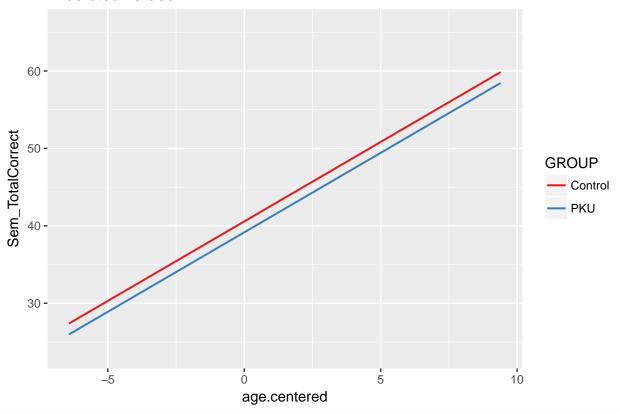
Graph both of your final models for the continuous and nominal models above.

```
#Graphing nominal
library(sjPlot)
```

Learn more about sjPlot with 'browseVignettes("sjPlot")'.

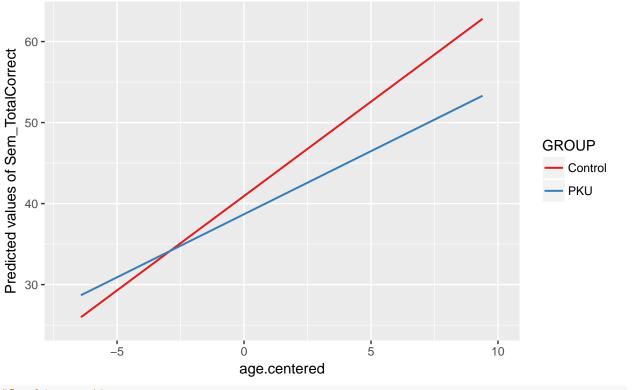
sjp.lmer(lin.nom1, type = "pred.fe", var = c("age.centered", "GROUP"), facet = FALSE, show.scatter = FA

Predicted values



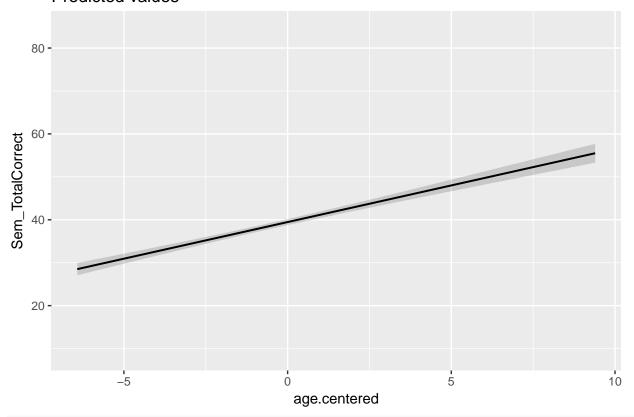
sjp.int(lin.nom2, swap.pred = T)

Interaction effect of GROUP and age.centered on Sem_TotalCorrect



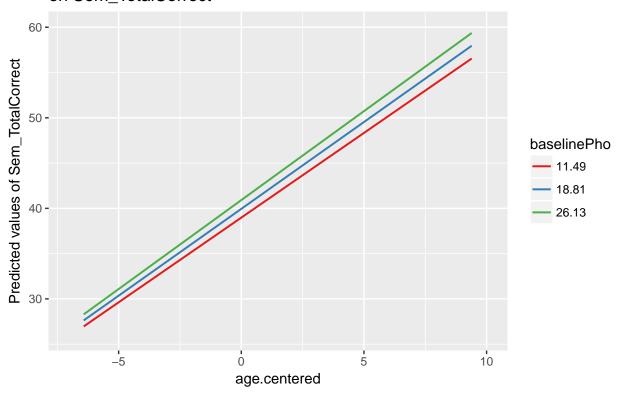
#Graphing continuous sjp.lmer(lin.cont1, type = "pred", var = c("age.centered"), show.scatter = FALSE, show.ci = TRUE)

Predicted values



sjp.int(lin.cont2, swap.pred = T, mdrt.values = "meansd")

Interaction effect of baselinePho and age.centered on Sem TotalCorrect



Problem 4:

##

1

```
Calculate confidence intervals around your estimates for your final models
#Confidence intervals around nominal model
lin.nom.ci <- confint(lin.nom1, level = .95, oldNames = F, method = "boot", nsim = 100)
## Computing bootstrap confidence intervals ...
broom::tidy(lin.nom.ci)
##
                                            X2.5..
                                                     X97.5..
                             .rownames
                   sd (Intercept) | ID2 3.53824533 6.865300
## 2 cor_age.centered.(Intercept)|ID2 -1.00000000
                                                    1.000000
## 3
                  sd_age.centered|ID2 0.07626682
                                                    1.461375
## 4
                                sigma 5.27803170 6.968268
## 5
                          (Intercept) 38.81178998 43.139264
## 6
                         age.centered 1.54485037
                                                    2.625827
                             GROUPPKU -4.58047174 1.371266
## 7
#Confidence intervals around continuous model
lin.cont.ci <- confint.merMod(lin.cont1, level = .95, oldNames = F, method = "boot", nsim = 100)
## Computing bootstrap confidence intervals ...
broom::tidy(lin.cont.ci)
```

sd_(Intercept)|ID2 3.60094141 7.2187336

X2.5..

.rownames

X97.5..

Problem 5:

Include both types of covariates in a single model. How does your interpretation of parameters change?

Fixed effect estimates:

Intercept = 40.94; mean of Controls at mean levels of age and baselinePho (i.e., age.centered and baselinePho.centered = 0)

Age.centered = 2.25; slope of Controls (increase in words correct/year) at mean levels of baselinePho (i.e., baselinePho.centered = 0)

baselinePho.centered = .10; slope of Controls (increase in words correct/unit of baselinePho) at mean levels of age (i.e., age.centered = 0)

GROUP = -2.73; difference in words correct between Control and PKU at mean levels of age and baselinePho (i.e., when age.centered and baselinePho.centered = 0)

age.centered:baselinePho.centered = -.02; extent to which relationship between age.centered and Sem_TotalCorrect changes at different levels of baselinePho.centered, when group = Controls

age.centered:GROUP = -.63; extent to which relationship between age.centered and Sem_TotalCorrect changes at different levels of GROUP, when baselinePho.centered = 0

baselinePho.centered: GROUP = -.05; extent to which relationship between baselinePho.centered and Sem TotalCorrect changes at different levels of GROUP, when age.centered = 0

age.centered:baselinePho.centered:GROUP = .09; extent to which the interaction between age.centered and baselinePho.centered changes at different levels of GROUP

Pseudo-R2 = .39 (marginal; fixed) and .70 (conditional; fixed + random)

```
## Linear mixed model fit by REML ['lmerMod']
  Formula: Sem_TotalCorrect ~ age.centered * baselinePho.centered * GROUP +
##
       (age.centered | ID2)
##
      Data: wide_to_long_merged
##
## REML criterion at convergence: 1384.6
##
## Scaled residuals:
##
                1Q Median
                                 3Q
                                        Max
  -1.9767 -0.5934 -0.0664
##
                            0.4473
                                     3.2374
##
## Random effects:
    Groups
                           Variance Std.Dev. Corr
    ID2
                                    5.6476
##
             (Intercept)
                           31.896
##
             age.centered 0.767
                                    0.8758
                                             0.31
                           36.961
                                    6.0796
##
    Residual
## Number of obs: 200, groups: ID2, 67
##
## Fixed effects:
```

```
Estimate Std. Error t value
##
## (Intercept)
                                           40.94211 1.22024 33.55
                                                      0.34845 6.45
## age.centered
                                           2.24740
## baselinePho.centered
                                            0.09661 0.15203 0.64
## GROUPPKU
                                           -2.72955
                                                      2.10022
                                                               -1.30
## age.centered:baselinePho.centered
                                           -0.02067 0.04099 -0.50
## age.centered:GROUPPKU
                                           -0.63273
                                                      0.59736 - 1.06
## baselinePho.centered:GROUPPKU
                                           -0.04609
                                                      0.32795 -0.14
## age.centered:baselinePho.centered:GROUPPKU 0.08539
                                                      0.08517
                                                               1.00
##
## Correlation of Fixed Effects:
##
              (Intr) ag.cnt bslnP. GROUPP ag.:P. a.:GRO bP.:GR
## age.centerd 0.417
## bslnPh.cntr -0.254 -0.431
## GROUPPKU
              -0.581 -0.242 0.148
## ag.cntrd:P. -0.402 -0.204 0.104 0.234
## a.:GROUPPKU -0.243 -0.583 0.251 0.144 0.119
## bP.:GROUPPK 0.118 0.200 -0.464 0.167 -0.048 -0.382
## a.:P.:GROUP 0.194 0.098 -0.050 -0.341 -0.481 0.073 0.119
anova(lin.comb)
## Analysis of Variance Table
                                        Df Sum Sq Mean Sq F value
##
## age.centered
                                         1 2440.47 2440.47 66.0274
## baselinePho.centered
                                             32.64 32.64 0.8830
## GROUP
                                             14.13 14.13 0.3823
                                              2.26
## age.centered:baselinePho.centered
                                         1
                                                    2.26 0.0610
## age.centered:GROUP
                                        1 67.13 67.13 1.8163
## baselinePho.centered:GROUP
                                         1
                                             2.52
                                                   2.52 0.0683
## age.centered:baselinePho.centered:GROUP 1
                                             37.16 37.16 1.0052
r.squaredGLMM(lin.comb)
##
        R2m
                  R2c
## 0.3900541 0.7022353
```