

# Week3\_\_MLM

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### Problem 1:

Run a series of models using a time-invariant nominal covariate. a) where the covariate only predicts the intercept b) predicts both intercept and slope c) is rescaled eg centering. For all models, how does your model change from model to model. What is your final model?

#### *Part A: Fixed effect estimates*

Intercept = 40.57; mean of Controls at mean age (age.centered = 0)

Age.centered = 2.05; increase in words correct/year controlling for Group

Group = -1.4; difference in words correct between Control and PKU controlling for mean age

Pseudo-R2 = .38 (marginal; fixed) and .69 (conditional; fixed + random)

#### *Part B: Fixed effect estimates*

Intercept = 40.93; mean of Controls at mean age (age.centered = 0)

Age.centered = 2.33; increase in words correct/year for Controls

Group = -2.25; difference in words correct between Control and PKU age mean age (age.centered = 0)

Age.centered:GROUP = -.77; difference in slope between Control and PKU

Pseudo-R2 = .38 (marginal; fixed) and .70 (conditional; fixed + random)

#### *Part C: Fixed effect estimates*

Intercept = 38.69; mean of PKU at mean age (age.centered = 0)

Age.centered = 1.56; increase in words correct/year for PKU

Group = 2.25; difference in words correct between PKU and Controls at mean age (age.centered = 0)

Age.centered:GROUP = .77; difference in slope between PKU and Control

Pseudo-R2 = .38 (marginal; fixed) and .70 (conditional; fixed + random)

**Likelihood ratio test suggests that simpler model, where covariate only predicts intercept, is preferred.**

```
#Model from HW #2
```

```
library(lme4)
```

```
## Loading required package: Matrix
```

```
##
```

```
## Attaching package: 'Matrix'
```

```
## The following object is masked from 'package:tidyr':
```

```
##
```

```
## expand
```

```
library(MuMIn)
wide_to_long_merged$Timepoint <- as.factor(wide_to_long_merged$Timepoint)
lin.mlm <- lmer(Sem_TotalCorrect ~ age.centered + (age.centered | ID2), data = wide_to_long_merged)
summary(lin.mlm)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered + (age.centered | ID2)
## Data: wide_to_long_merged
##
## REML criterion at convergence: 1381.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.9380 -0.5849 -0.0708  0.4632  3.2995
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## ID2 (Intercept) 30.9260 5.5611
## age.centered 0.7509 0.8665 0.45
## Residual 37.3791 6.1138
## Number of obs: 200, groups: ID2, 67
##
## Fixed effects:
## Estimate Std. Error t value
## (Intercept) 40.0957 0.8712 46.02
## age.centered 2.0512 0.2504 8.19
##
## Correlation of Fixed Effects:
## (Intr)
## age.centerd 0.360
```

```
anova(lin.mlm)
```

```
## Analysis of Variance Table
## Df Sum Sq Mean Sq F value
## age.centered 1 2507.2 2507.2 67.074
```

```
r.squaredGLMM(lin.mlm)
```

```
## R2m R2c
## 0.3786386 0.6876268
```

*#Part a: adding group, a time-invariant nominal covariate that only predicts the intercept*

```
wide_to_long_merged$GROUP <- relevel(wide_to_long_merged$GROUP, ref = "Control")
lin.nom1 <- lmer(Sem_TotalCorrect ~ age.centered + GROUP +
  (age.centered | ID2), data = wide_to_long_merged)
summary(lin.nom1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered + GROUP + (age.centered | ID2)
## Data: wide_to_long_merged
##
## REML criterion at convergence: 1378.1
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
```

```
## -1.9198 -0.6026 -0.0533 0.4658 3.2676
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## ID2 (Intercept) 30.7729 5.5473
## age.centered 0.7385 0.8594 0.41
## Residual 37.3936 6.1150
## Number of obs: 200, groups: ID2, 67
##
## Fixed effects:
## Estimate Std. Error t value
## (Intercept) 40.5682 1.0526 38.54
## age.centered 2.0531 0.2501 8.21
## GROUPPKU -1.4062 1.7262 -0.81
##
## Correlation of Fixed Effects:
## (Intr) ag.cnt
## age.centerd 0.299
## GROUPPKU -0.564 -0.030
```

```
anova(lin.nom1)
```

```
## Analysis of Variance Table
## Df Sum Sq Mean Sq F value
## age.centered 1 2506.32 2506.32 67.0254
## GROUP 1 24.82 24.82 0.6636
```

```
r.squaredGLMM(lin.nom1)
```

```
## R2m R2c
## 0.3787432 0.6872902
```

```
#Part b: adding group, a time-invariant nominal covariate the predicts intercept and slope
wide_to_long_merged$GROUP <- relevel(wide_to_long_merged$GROUP, ref = "Control")
lin.nom2 <- lmer(Sem_TotalCorrect ~ age.centered + GROUP + age.centered:GROUP +
  (age.centered | ID2), data = wide_to_long_merged)
summary(lin.nom2)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered + GROUP + age.centered:GROUP +
## (age.centered | ID2)
## Data: wide_to_long_merged
##
## REML criterion at convergence: 1375.5
##
## Scaled residuals:
## Min 1Q Median 3Q Max
## -1.9346 -0.6113 -0.0769 0.4429 3.2882
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## ID2 (Intercept) 30.6481 5.5361
## age.centered 0.8873 0.9419 0.34
## Residual 36.4284 6.0356
## Number of obs: 200, groups: ID2, 67
##
```

```
## Fixed effects:
##               Estimate Std. Error t value
## (Intercept)      40.9334      1.0788  37.94
## age.centered       2.3289      0.3124   7.46
## GROUPPKU          -2.2454      1.8333  -1.22
## age.centered:GROUPPKU -0.7721      0.5411  -1.43
##
## Correlation of Fixed Effects:
##      (Intr) ag.cnt GROUPP
## age.centerd  0.334
## GROUPPKU    -0.588 -0.197
## a.:GROUPPKU -0.193 -0.577  0.305
```

```
anova(lin.nom2)
```

```
## Analysis of Variance Table
##               Df Sum Sq Mean Sq F value
## age.centered    1 2391.59 2391.59 65.6519
## GROUP           1  25.01  25.01  0.6866
## age.centered:GROUP 1  74.16  74.16  2.0357
```

```
r.squaredGLMM(lin.nom2)
```

```
##      R2m      R2c
## 0.3824455 0.7000320
```

```
#Part c: rescaling nominal variable (i.e., changing dummy coding such that refernce group becomes PKU)
wide_to_long_merged$GROUP <- relevel(wide_to_long_merged$GROUP, ref = "PKU")
lin.nom3 <- lmer(Sem_TotalCorrect ~ age.centered + GROUP + age.centered:GROUP +
                 (age.centered | ID2), data = wide_to_long_merged)
summary(lin.nom3)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered + GROUP + age.centered:GROUP +
##      (age.centered | ID2)
##      Data: wide_to_long_merged
##
## REML criterion at convergence: 1375.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.9346 -0.6113 -0.0769  0.4429  3.2882
##
## Random effects:
##      Groups      Name      Variance Std.Dev. Corr
##      ID2      (Intercept) 30.6481  5.5361
##      age.centered 0.8873  0.9419  0.34
##      Residual    36.4284  6.0356
## Number of obs: 200, groups: ID2, 67
##
## Fixed effects:
##               Estimate Std. Error t value
## (Intercept)      38.6880      1.4823  26.100
## age.centered       1.5568      0.4419   3.523
## GROUPControl       2.2454      1.8333   1.225
## age.centered:GROUPControl 0.7721      0.5411   1.427
```

```
##
## Correlation of Fixed Effects:
##          (Intr) ag.cnt GROUPl
## age.centerd  0.291
## GROUPlContrl -0.809 -0.235
## ag.c:GROUPl -0.237 -0.817  0.305

anova(lin.nom3)

## Analysis of Variance Table
##              Df  Sum Sq Mean Sq F value
## age.centered    1 2391.59 2391.59 65.6519
## GROUPl          1   25.01   25.01  0.6866
## age.centered:GROUPl 1   74.16   74.16  2.0357

r.squaredGLMM(lin.nom3)

##          R2m          R2c
## 0.3824455 0.7000318

#Likelihood ratio test
anova(lin.nom1, lin.nom2)

## refitting model(s) with ML (instead of REML)

## Data: wide_to_long_merged
## Models:
## lin.nom1: Sem_TotalCorrect ~ age.centered + GROUPl + (age.centered | ID2)
## lin.nom2: Sem_TotalCorrect ~ age.centered + GROUPl + age.centered:GROUPl +
## lin.nom2:          (age.centered | ID2)
##              Df      AIC      BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## lin.nom1    7 1395.5 1418.5 -690.73  1381.5
## lin.nom2    8 1395.5 1421.9 -689.77  1379.5 1.9047      1      0.1675
```

## Problem 2:

### Part A: Fixed effect estimates

Intercept = 37.46; mean across all participants at means levels of age (i.e., age.centered = 0) and when baselinePho = 0

Age.centered = 1.91; slope (increase in words correct/year) controlling for baselinePho

baselinePho = .13; slope (increase in words correct/unit of baselinePho) controlling for Age.centered

Pseudo-R<sup>2</sup> = .39 (marginal; fixed) and .68 (conditional; fixed + random)

### Part B: Fixed effect estimates

Intercept = 37.46; mean across all participants at mean age (age.centered = 0) and when baselinePho = 0

Age.centered = 1.80; slope (increase in words correct/year) when baselinePho = 0

baselinePho = .13; slope (increase in words correct/unit of baselinePho) at mean age (age.centered = 0)

Age.centered:baselinePho = .006; extent to which relationship between age.centered and Sem\_Total\_Correct changes at different levels of baselinePho (change is minimal)

Pseudo-R<sup>2</sup> = .39 (marginal; fixed) and .68 (conditional; fixed + random)

### Part C: Fixed effect estimates

Intercept = 39.94; mean across all participants at means levels of age and baselinePho (i.e., age.centered and baselinePho.centered = 0)

Age.centered = 1.90; slope (increase in words correct/year) at mean levels of baselinePho (baselinePho.centered = 0)

baselinePho = .13; slope (increase in words correct/unit of baselinePho) at mean age (age.centered = 0)

Age.centered:baselinePho = .006; extent to which relationship between age.centered and Sem\_Total\_Correct changes at different levels of baselinePho (change is minimal; this does not change as a function of centering)  
Pseudo-R<sup>2</sup> = .39 (marginal; fixed) and .68 (conditional; fixed + random)

Likelihood ratio test suggests that simpler model, where covariate only predicts intercept, is preferred.

*#Part a: adding baselinePho, a time-invariant continuous covariate that only predicts the intercept*

```
lin.cont1 <- lmer(Sem_TotalCorrect ~ age.centered + baselinePho +
                  (age.centered | ID2), data = wide_to_long_merged)
summary(lin.cont1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered + baselinePho + (age.centered |
##      ID2)
##      Data: wide_to_long_merged
##
## REML criterion at convergence: 1382.9
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.9522 -0.5507 -0.0656  0.4575  3.2804
##
## Random effects:
##      Groups      Name      Variance Std.Dev. Corr
##      ID2      (Intercept) 29.8950  5.4676
##              age.centered  0.5985  0.7736  0.46
##      Residual              38.2696  6.1862
## Number of obs: 200, groups:  ID2, 67
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)   37.4659    2.5437  14.729
## age.centered    1.9052    0.2668   7.140
## baselinePho     0.1335    0.1250   1.068
##
## Correlation of Fixed Effects:
##              (Intr) ag.cnt
## age.centerd  0.492
## baselinePho -0.942 -0.407
```

```
anova(lin.cont1)
```

```
## Analysis of Variance Table
##              Df Sum Sq Mean Sq F value
## age.centered  1 2632.65 2632.65 68.7923
## baselinePho   1   43.62   43.62  1.1399
```

```
r.squaredGLMM(lin.cont1)
```

```
##      R2m      R2c
## 0.3891391 0.6786925
```

*#Part b: adding group, a time-invariant nominal covariate the predicts intercept and slope*

```
lin.cont2 <- lmer(Sem_TotalCorrect ~ age.centered + baselinePho + age.centered:baselinePho +
                  (age.centered | ID2), data = wide_to_long_merged)
summary(lin.cont2)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula:
## Sem_TotalCorrect ~ age.centered + baselinePho + age.centered:baselinePho +
##   (age.centered | ID2)
##   Data: wide_to_long_merged
##
## REML criterion at convergence: 1387.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.9473 -0.5522 -0.0711  0.4504  3.2828
##
## Random effects:
##   Groups   Name                Variance Std.Dev. Corr
##   ID2      (Intercept)    30.2017   5.496
##           age.centered    0.6741   0.821   0.44
##   Residual                38.1032   6.173
## Number of obs: 200, groups: ID2, 67
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)    37.455111   2.650150  14.133
## age.centered     1.797015   0.723892   2.482
## baselinePho      0.132251   0.126930   1.042
## age.centered:baselinePho 0.006416   0.034494   0.186
##
## Correlation of Fixed Effects:
##           (Intr) ag.cnt bslnPh
## age.centerd  0.416
## baselinePho -0.936 -0.274
## ag.cntrd:bP -0.259 -0.928  0.135
```

```
anova(lin.cont2)
```

```
## Analysis of Variance Table
##              Df Sum Sq Mean Sq F value
## age.centered    1 2576.52 2576.52 67.6194
## baselinePho      1  40.13   40.13  1.0531
## age.centered:baselinePho 1    1.32    1.32  0.0346
```

```
r.squaredGLMM(lin.cont2)
```

```
##           R2m       R2c
## 0.3905637 0.6847858
```

```
#Part c: rescaling nominal variable (i.e., centering baselinePho)
```

```
wide_to_long_merged$baselinePho.centered <- wide_to_long_merged$baselinePho -
  mean(wide_to_long_merged$baselinePho, na.rm = T)
```

```
lin.cont3 <- lmer(Sem_TotalCorrect ~ age.centered + baselinePho.centered + age.centered:baselinePho.cen
  (age.centered | ID2), data = wide_to_long_merged)
```

```
summary(lin.cont3)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula:
## Sem_TotalCorrect ~ age.centered + baselinePho.centered + age.centered:baselinePho.centered +
##   (age.centered | ID2)
```

```
## Data: wide_to_long_merged
##
## REML criterion at convergence: 1387.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.9473 -0.5522 -0.0711  0.4504  3.2828
##
## Random effects:
##   Groups   Name                Variance Std.Dev. Corr
##   ID2      (Intercept)    30.2017   5.496
##           age.centered    0.6741   0.821   0.44
##   Residual                    38.1032   6.173
## Number of obs: 200, groups: ID2, 67
##
## Fixed effects:
##
##              Estimate Std. Error t value
## (Intercept)    39.942761    0.937405   42.61
## age.centered    1.917692    0.271151    7.07
## baselinePho.centered 0.132251    0.126930    1.04
## age.centered:baselinePho.centered 0.006416    0.034494    0.19
##
## Correlation of Fixed Effects:
##      (Intr) ag.cnt bslnP.
## age.centerd  0.347
## bslnPh.cntr -0.099 -0.409
## ag.cntrd:P. -0.388 -0.084  0.135
```

```
anova(lin.cont3)
```

```
## Analysis of Variance Table
##
##              Df   Sum Sq Mean Sq F value
## age.centered    1 2576.52 2576.52 67.6194
## baselinePho.centered 1   40.13   40.13  1.0531
## age.centered:baselinePho.centered 1    1.32    1.32  0.0346
```

```
r.squaredGLMM(lin.cont3)
```

```
##      R2m      R2c
## 0.3905637 0.6847858
```

```
#Likelihood ratio test
anova(lin.cont1, lin.cont2)
```

```
## refitting model(s) with ML (instead of REML)
## Data: wide_to_long_merged
## Models:
## lin.cont1: Sem_TotalCorrect ~ age.centered + baselinePho + (age.centered |
## lin.cont1:      ID2)
## lin.cont2: Sem_TotalCorrect ~ age.centered + baselinePho + age.centered:baselinePho +
## lin.cont2:      (age.centered | ID2)
##
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## lin.cont1  7 1395.0 1418.0 -690.48  1381.0
## lin.cont2  8 1396.9 1423.3 -690.47  1380.9 0.0184    1    0.8921
```

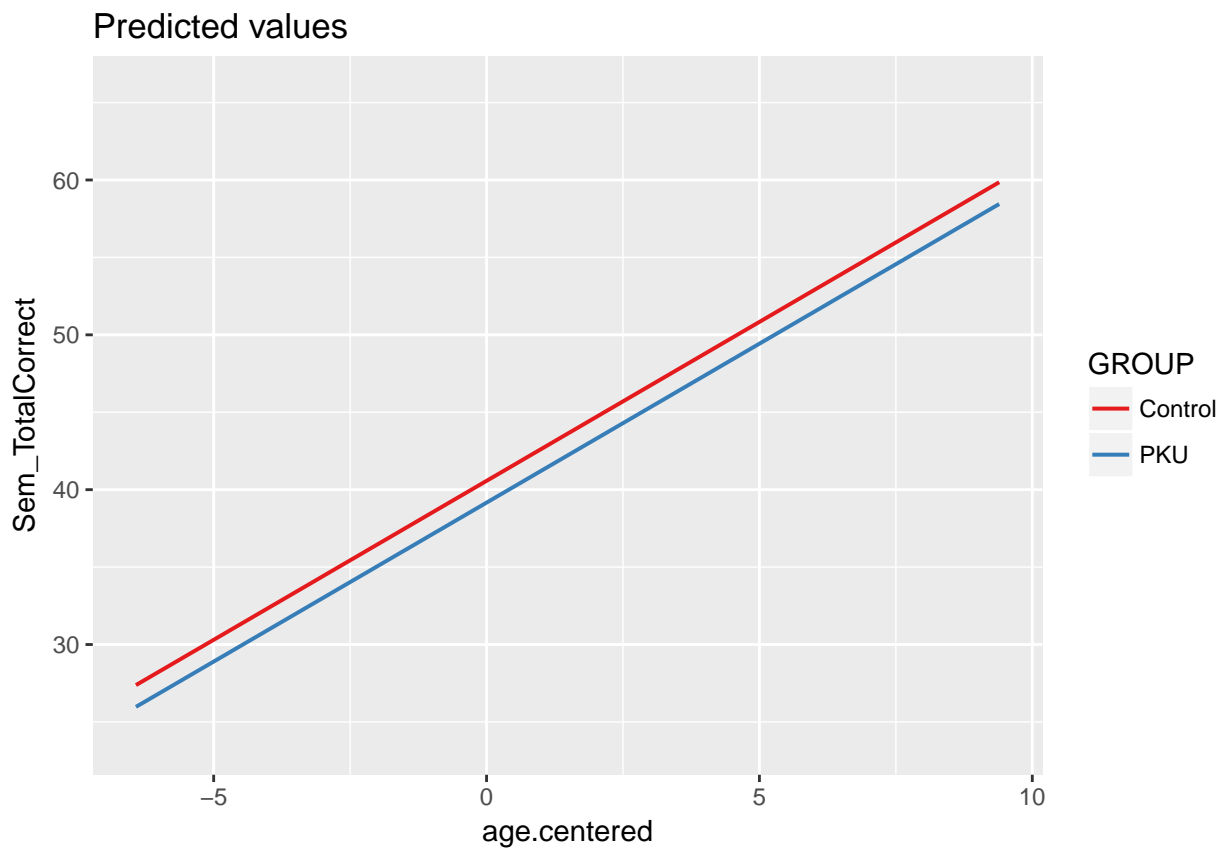


### Problem 3:

Graph both of your final models for the continuous and nominal models above.

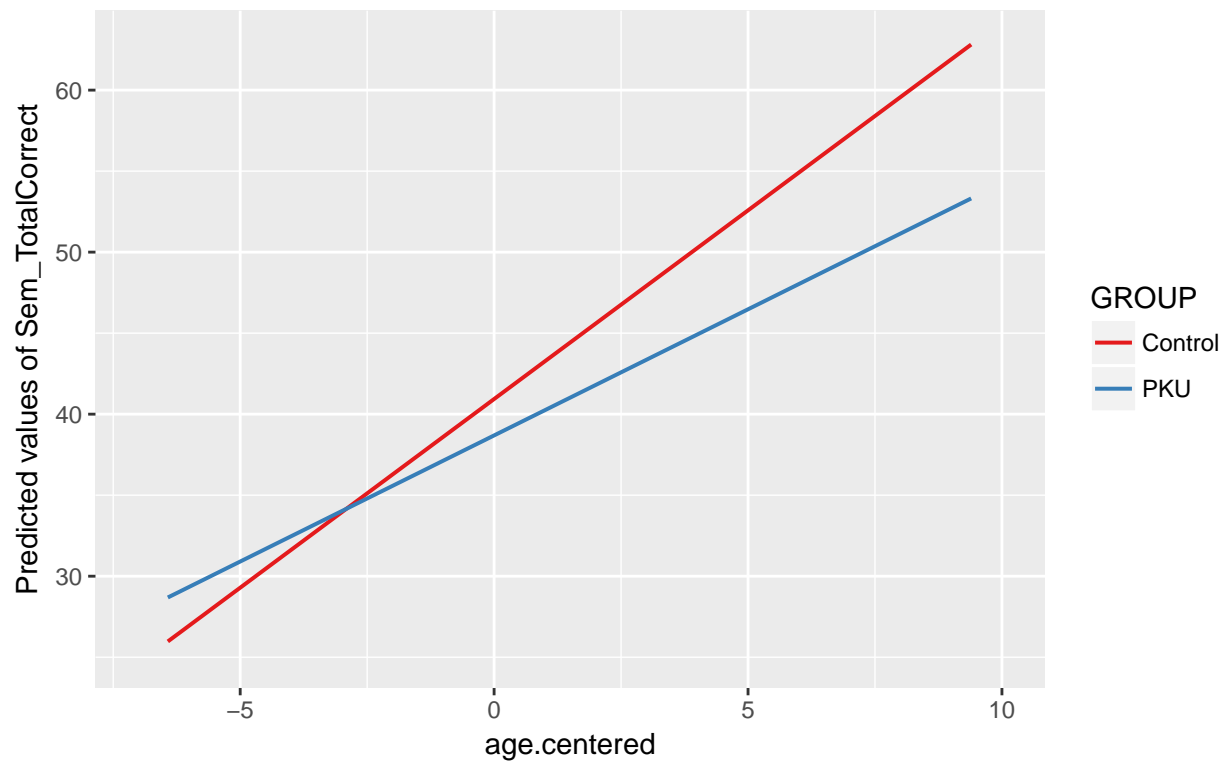
```
#Graphing nominal  
library(sjPlot)
```

```
## Install package "strengexjacke" from GitHub (`devtools::install_github("strengexjacke/strengexjacke")`)  
sjp.lmer(lin.nom1, type = "pred.fe", var = c("age.centered", "GROUP"), facet = FALSE, show.scatter = FALSE)
```

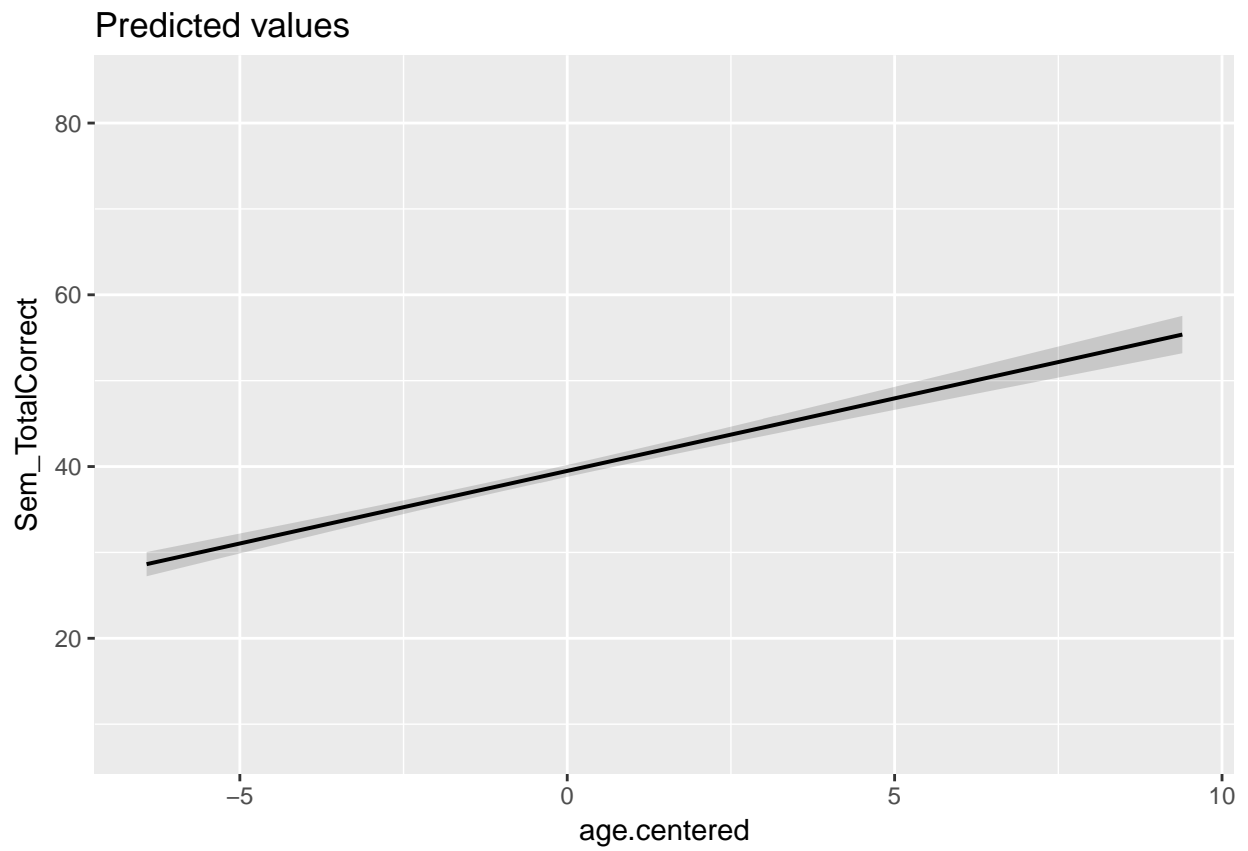


```
sjp.int(lin.nom2, swap.pred = T)
```

Interaction effect of GROUP and age.centered on Sem\_TotalCorrect

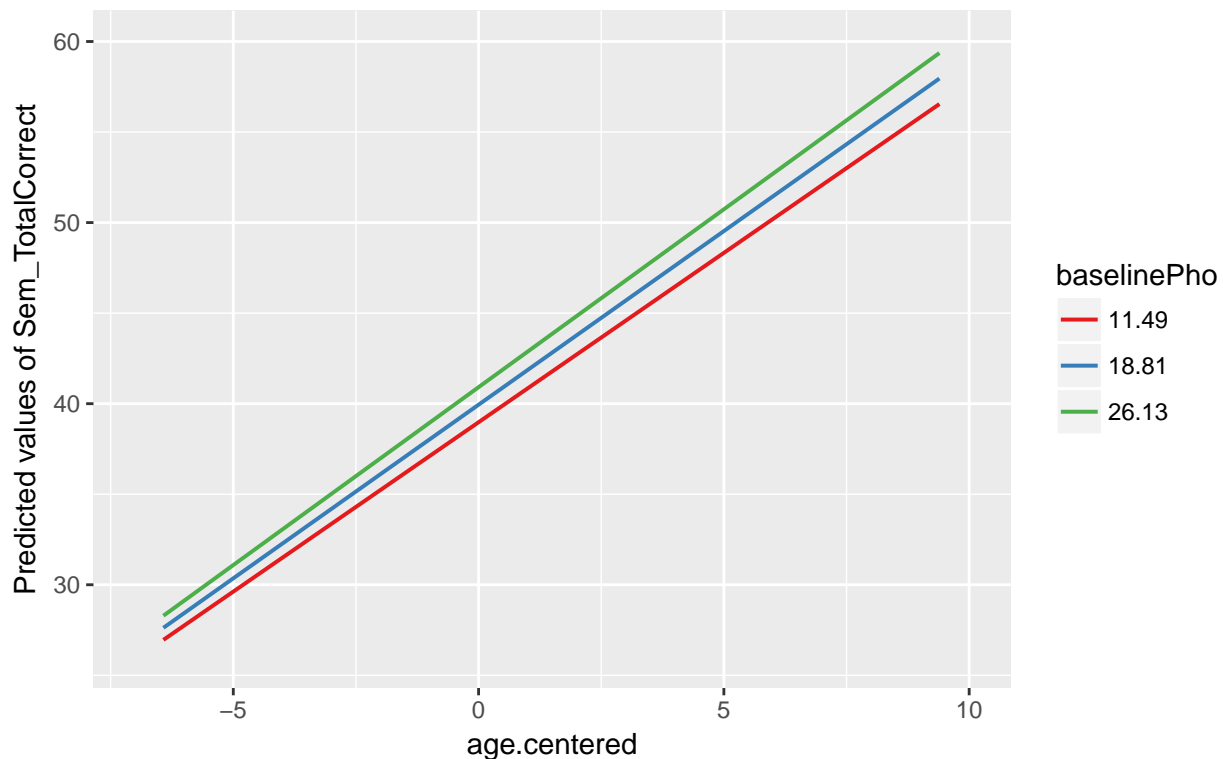


```
#Graphing continuous  
sjp.lmer(lin.cont1, type = "pred", var = c("age.centered"), show.scatter = FALSE, show.ci = TRUE)
```



```
sjp.int(lin.cont2, swap.pred = T, mdrt.values = "meansd")
```

### Interaction effect of baselinePho and age.centered on Sem\_TotalCorrect



### Problem 4:

Calculate confidence intervals around your estimates for your final models

```
#Confidence intervals around nominal model
lin.nom.ci <- confint(lin.nom1, level = .95, oldNames = F, method = "boot", nsim = 100)

## Computing bootstrap confidence intervals ...
broom::tidy(lin.nom.ci)

##           .rownames      X2.5..   X97.5..
## 1      sd_(Intercept)|ID2  3.727115452  6.933463
## 2 cor_age.centered.(Intercept)|ID2 -0.531284335  1.000000
## 3      sd_age.centered|ID2  0.008625606  1.562232
## 4              sigma  5.338576188  7.081255
## 5      (Intercept) 38.654186279 42.312612
## 6      age.centered  1.467206393  2.520117
## 7      GROUPPKU -4.361017521  2.020003

#Confidence intervals around continuous model
lin.cont.ci <- confint.merMod(lin.cont1, level = .95, oldNames = F, method = "boot", nsim = 100)

## Computing bootstrap confidence intervals ...
broom::tidy(lin.cont.ci)

##           .rownames      X2.5..   X97.5..
## 1      sd_(Intercept)|ID2  3.47857343  7.3439076
```

```
## 2 cor_age.centered.(Intercept)|ID2 -1.00000000 1.00000000
## 3          sd_age.centered|ID2  0.03364713 1.5253887
## 4          sigma  5.33728031 6.9600579
## 5          (Intercept) 32.00964743 42.9467729
## 6          age.centered  1.30143830 2.4664208
## 7          baselinePho -0.15729827 0.4003399
```

## Problem 5:

Include both types of covariates in a single model. How does your interpretation of parameters change?

*Fixed effect estimates:*

**Intercept** = 40.94; mean of Controls at mean levels of age and baselinePho (i.e., age.centered and baselinePho.centered = 0)

**Age.centered** = 2.25; slope of Controls (increase in words correct/year) at mean levels of baselinePho (i.e., baselinePho.centered = 0)

**baselinePho.centered** = .10; slope of Controls (increase in words correct/unit of baselinePho) at mean levels of age (i.e., age.centered = 0)

**GROUP** = -2.73; difference in words correct between Control and PKU at mean levels of age and baselinePho (i.e., when age.centered and baselinePho.centered = 0)

**age.centered:baselinePho.centered** = -.02; extent to which relationship between age.centered and Sem\_TotalCorrect changes at different levels of baselinePho.centered, when group = Controls

**age.centered:GROUP** = -.63; extent to which relationship between age.centered and Sem\_TotalCorrect changes at different levels of GROUP, when baselinePho.centered = 0

**baselinePho.centered:GROUP** = -.05; extent to which relationship between baselinePho.centered and Sem\_TotalCorrect changes at different levels of GROUP, when age.centered = 0

**age.centered:baselinePho.centered:GROUP** = .09; extent to which the interaction between age.centered and baselinePho.centered changes at different levels of GROUP

Pseudo-R<sup>2</sup> = .39 (marginal; fixed) and .70 (conditional; fixed + random)

```
wide_to_long_merged$GROUP <- relevel(wide_to_long_merged$GROUP, ref = "Control")
lin.comb <- lmer(Sem_TotalCorrect ~ age.centered*baselinePho.centered*GROUP +
  (age.centered | ID2), data = wide_to_long_merged)
summary(lin.comb)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Sem_TotalCorrect ~ age.centered * baselinePho.centered * GROUP +
##      (age.centered | ID2)
##      Data: wide_to_long_merged
##
## REML criterion at convergence: 1384.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.9767 -0.5934 -0.0664  0.4473  3.2374
##
## Random effects:
##      Groups   Name                Variance Std.Dev. Corr
##      ID2      (Intercept)  31.896     5.6476
##              age.centered   0.767     0.8758  0.31
##      Residual                36.961     6.0796
## Number of obs: 200, groups:  ID2, 67
##
## Fixed effects:
```

```
##                                Estimate Std. Error t value
## (Intercept)                   40.94211    1.22024   33.55
## age.centered                   2.24740    0.34845    6.45
## baselinePho.centered           0.09661    0.15203    0.64
## GROUPPKU                       -2.72955    2.10022   -1.30
## age.centered:baselinePho.centered -0.02067    0.04099   -0.50
## age.centered:GROUPPKU          -0.63273    0.59736   -1.06
## baselinePho.centered:GROUPPKU   -0.04609    0.32795   -0.14
## age.centered:baselinePho.centered:GROUPPKU 0.08539    0.08517    1.00
##
## Correlation of Fixed Effects:
##      (Intr) ag.cnt bslnP. GROUPP ag.:P. a.:GRO bP.:GR
## age.centerd  0.417
## bslnPh.cntr -0.254 -0.431
## GROUPPKU    -0.581 -0.242  0.148
## ag.cntrd:P. -0.402 -0.204  0.104  0.234
## a.:GROUPPKU -0.243 -0.583  0.251  0.144  0.119
## bP.:GROUPPK  0.118  0.200 -0.464  0.167 -0.048 -0.382
## a.:P.:GROUP  0.194  0.098 -0.050 -0.341 -0.481  0.073  0.119
```

```
anova(lin.comb)
```

```
## Analysis of Variance Table
```

```
##                                Df  Sum Sq Mean Sq F value
## age.centered                   1 2440.47  2440.47  66.0274
## baselinePho.centered           1   32.64    32.64   0.8830
## GROUP                           1   14.13    14.13   0.3823
## age.centered:baselinePho.centered 1    2.26     2.26   0.0610
## age.centered:GROUP              1   67.13    67.13   1.8163
## baselinePho.centered:GROUP       1    2.52     2.52   0.0683
## age.centered:baselinePho.centered:GROUP 1   37.16    37.16   1.0052
```

```
r.squaredGLMM(lin.comb)
```

```
##      R2m      R2c
## 0.3900541 0.7022353
```