

计算机视觉与模式识别

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图像金字塔



第六章 高斯拉普拉斯金字塔

高斯拉普拉斯金字塔是我们进行图像多尺度分析的基础。

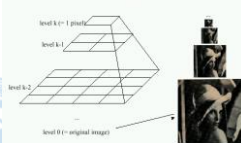
- 一、高斯金字塔与图像尺度：平滑与缩放
- 二、高斯金字塔的实现原理：平滑与降采样的配合使用
- 三、可分离的滤波器：可分离滤波
- 四、Reduce和Expand：快速的金字塔生成
- 五、基于金字塔的图像合成

一、高斯金字塔

Gaussian Pyramids

6.1.1 图像金字塔

Idea: Represent NxN image as a "pyramid" of $1 \times 1, 2 \times 2, 4 \times 4, \dots, 2^k \times 2^k$ images (assuming $N=2^k$)



Known as a Gaussian Pyramid [Burt and Adelson, 1983]

- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to wavelet transform

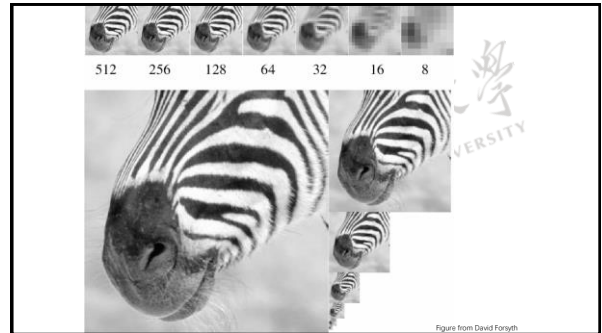


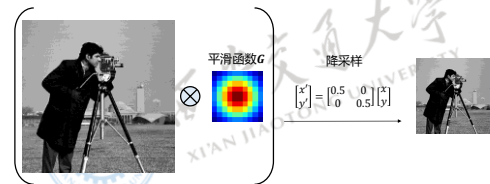
Figure from David Forsyth

6.1.2 高斯金字塔的实现



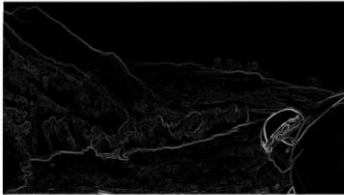
高斯平滑 → 降采样

Repeated Step: 平滑+降采样:





Different scale of image encodes different edge response.



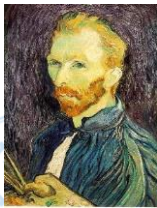
二、金字塔的采样

Sampling



西安交通大学
XI'AN JIAOTONG UNIVERSITY

6.2.1 降采样



1/4



1/8

Throw away every other row and column to create a 1/2 size image
- called *image sub-sampling*

6.2.1 降采样



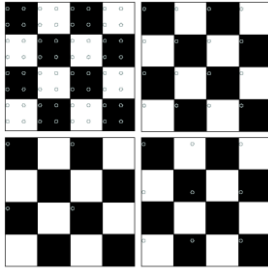
1/2

1/4 (2x zoom)

1/8 (4x zoom)

为什么看起来很糟糕?

6.2.2 奈奎斯特采样定律



好的采样:

- Sample often or,
- Sample wisely

差的采样:

- see aliasing in action!

6.2.3 平滑+采样



Gaussian 1/2



G 1/4



G 1/8

解决方案: 先平滑图像, 然后降采样

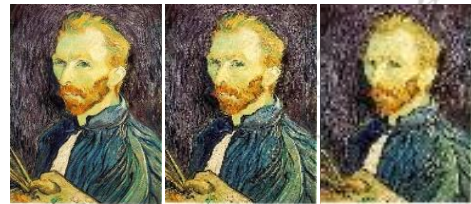
- Filter size should double for each $\frac{1}{2}$ size reduction. Why?

6.2.3 平滑+降采样

Solution: filter the image, *then* subsample

- Filter size should double for each $\frac{1}{2}$ size reduction. Why?
- How can we speed this up?

对比:

Solution: filter the image, *then* subsample

- Filter size should double for each $\frac{1}{2}$ size reduction. Why?
- How can we speed this up?

三、可分离滤波器 Separable Filters



6.3.1 模板运算

$$O(x, y) = \sum_{a=-m}^m \sum_{b=-n}^n I(x+a, y+b) f(a, b)$$

Diagram illustrating the template operation formula. The output image $O(x, y)$ is calculated as the sum over a and b of the input image $I(x+a, y+b)$ multiplied by the kernel image $f(a, b)$. Arrows point from the labels "Output Image", "Input Image", and "Kernel Image" to their respective terms in the formula.

6.3.1 模板运算

$$O(x, y) = \sum_{a=-m}^m \sum_{b=-n}^n I(x+a, y+b) f(a, b)$$

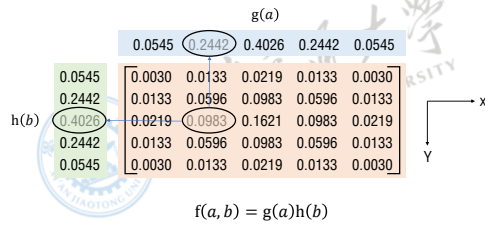
Diagram illustrating the template operation formula. The output image $O(x, y)$ is calculated as the sum over a and b of the input image $I(x+a, y+b)$ multiplied by the kernel image $f(a, b)$. An arrow points from the label $g(a)h(b)$ to the term $f(a, b)$ in the formula.

6.3.2 可分离滤波器：高斯

$$f(a, b) = \frac{1}{2\pi\sigma^2} e^{-\frac{a^2+b^2}{2\sigma^2}} = \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{a^2}{2\sigma^2}} \right) \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{b^2}{2\sigma^2}} \right)$$

Diagram illustrating the separable Gaussian filter formula. The 2D Gaussian function $f(a, b)$ is shown as the product of two 1D Gaussian functions, $g(a)$ and $h(b)$. Arrows point from the labels $g(a)$ and $h(b)$ to their respective terms in the product.

6.3.2 高斯滤波器的矩阵形式



6.3.3 可分离滤波

$$O(x,y) = \sum_{a=-m}^m \sum_{b=-n}^n I(x+a,y+b)g(a)h(b)$$

$$O(x,y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a,y+b)g(a) \right)$$

6.3.4 可分离滤波：一个示例

$$O(x,y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a,y+b)g(a) \right)$$



6.3.4 可分离滤波：一个示例

$$O(x,y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a,y+b)g(a) \right)$$



6.3.4 可分离滤波：一个示例

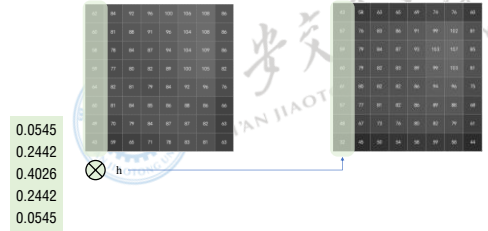
$$O(x,y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a,y+b)g(a) \right)$$

0.0545 0.2442 0.4026 0.2442 0.0545



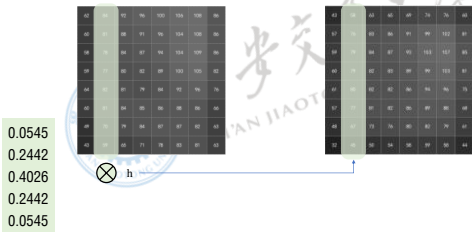
6.3.4 可分离滤波：一个示例

$$O(x,y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a,y+b)g(a) \right)$$



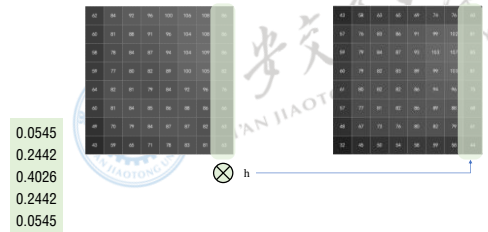
6.3.4 可分离滤波：一个示例

$$O(x,y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a,y+b)g(a) \right)$$



6.3.4 可分离滤波：一个示例

$$O(x,y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a,y+b)g(a) \right)$$

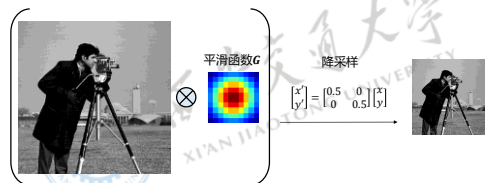


6.3.5 可分离滤波：复杂度比较

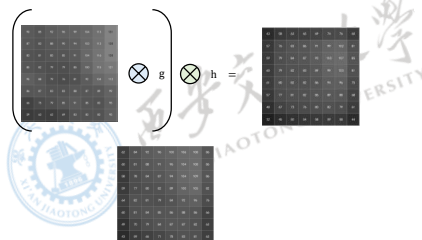
$$O(x, y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a, y+b) g(a) \right)$$

四、Expand and Reduce

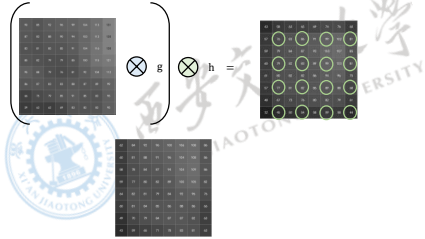
Repeated Step: 平滑+降采样:



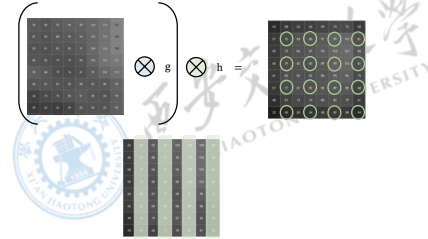
6.4.1 可分离滤波+降采样



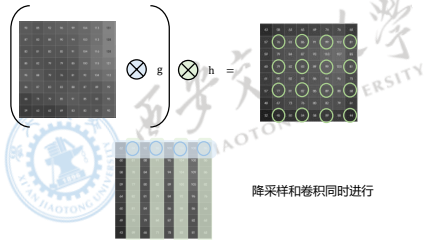
6.4.1 可分离滤波+降采样



6.4.1 可分离滤波+降采样

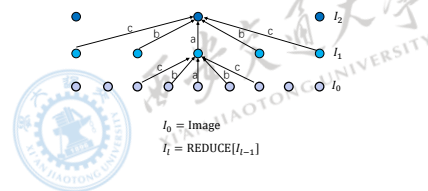


6.4.1 可分离滤波+降采样



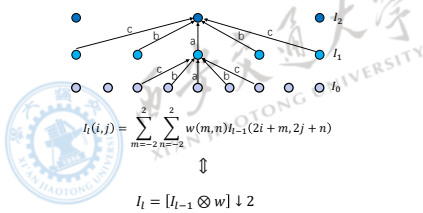
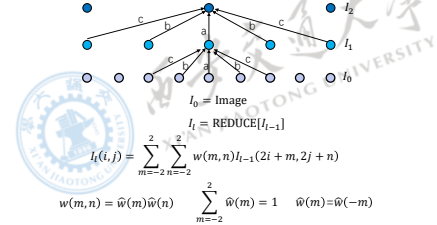
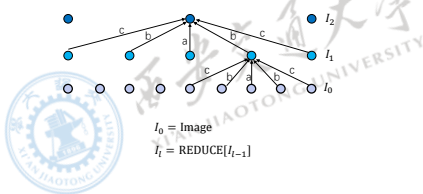
6.4.2 Reduce操作=滤波+降采样

[Burt & Adelson, 1983]



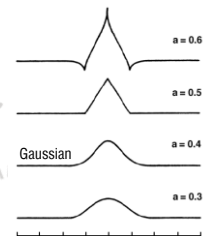
6.4.2 Reduce操作=滤波+降采样

[Burt & Adelson, 1983]

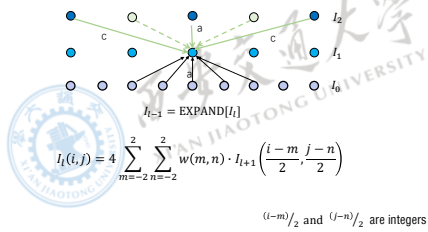


低通滤波器的选择

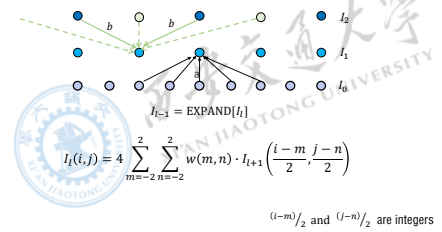
$$\begin{aligned} \hat{w}(0) &= a \\ \hat{w}(1) &= \hat{w}(-1) = 1/4 \\ \hat{w}(1) &= \hat{w}(-1) = 1/4 - a/2 \end{aligned}$$



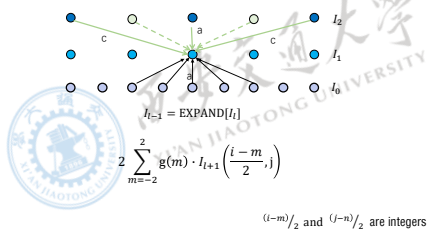
6.4.3 Expansion操作=滤波+增采样



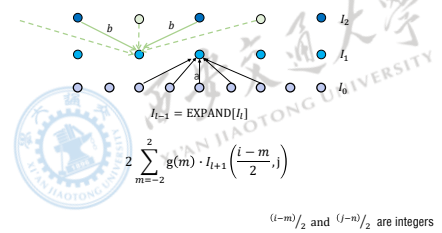
6.4.3 Expansion操作=滤波+增采样

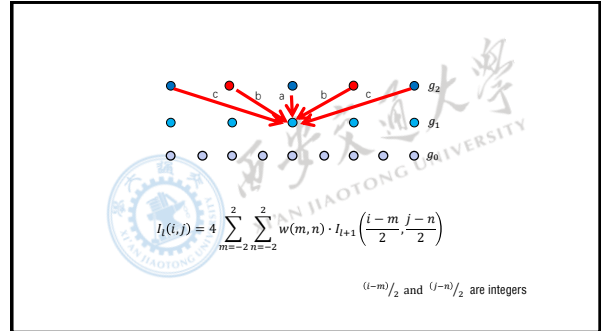
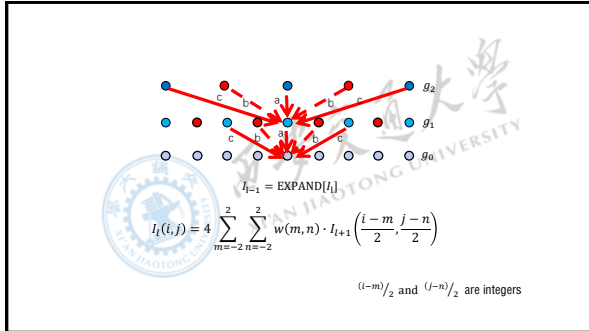


6.4.3 Expansion操作=滤波+增采样

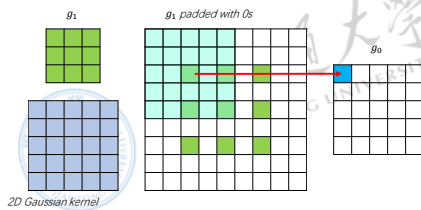


6.4.3 Expansion操作=滤波+增采样

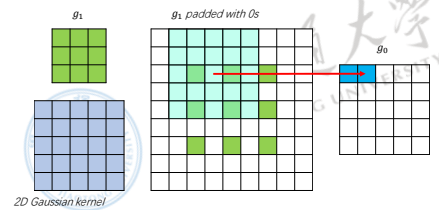




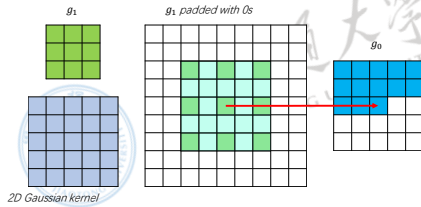
6.4.4 二维Expansion操作=滤波+增采样 (Part1)



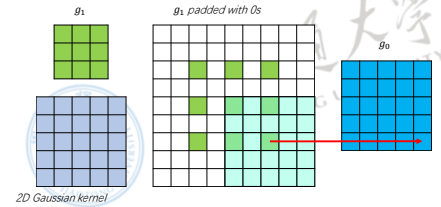
6.4.4 二维Expansion操作=滤波+增采样 (Part2)



6.4.4 二维Expansion操作=滤波+增采样 (Part3)

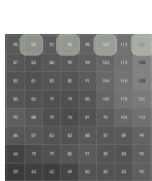


6.4.4 二维Expansion操作=滤波+增采样 (Part4)



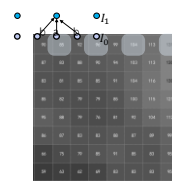
6.4.5 一个示例:Reduce

$$O(x, y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a, y+b) g(a) \right)$$



6.4.5 一个示例:Reduce

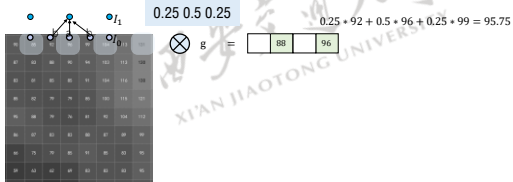
$$O(x, y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a, y+b) g(a) \right)$$



$$0.25 \cdot 90 + 0.5 \cdot 85 + 0.25 \cdot 92 = 88$$

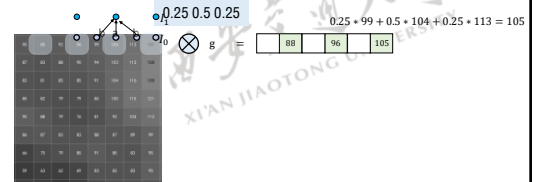
6.4.5 一个示例:Reduce

$$O(x, y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a, y+b) g(a) \right)$$



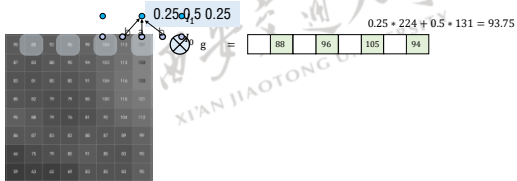
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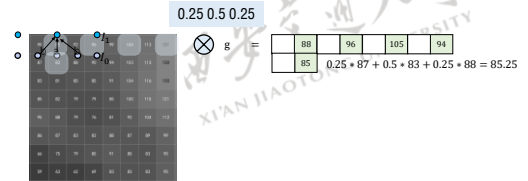
6.4.5 一个示例:Reduce

$$O(x, y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a, y+b) g(a) \right)$$



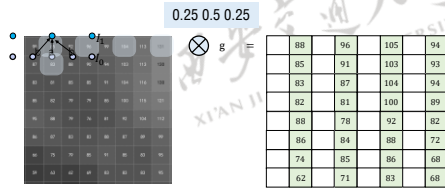
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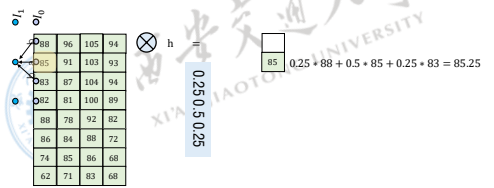
6.4.5 一个示例: Reduce

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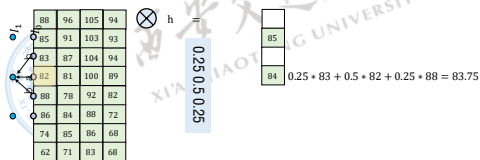
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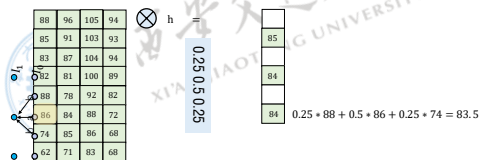
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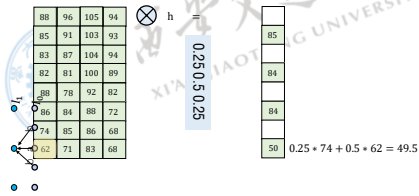
6.4.5 一个示例: Reduce

$$O(x, y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m I(x+a, y+b) g(a) \right)$$



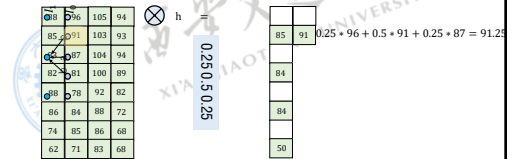
6.4.5 一个示例: Reduce

$$O(x, y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m l(x+a, y+b) g(a) \right)$$



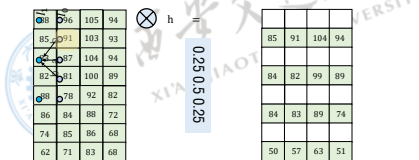
6.4.5 一个示例: Reduce

$$O(x, y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m l(x+a, y+b) g(a) \right)$$



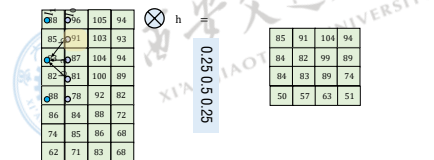
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6.4.5 一个示例: Reduce

$$O(x, y) = \sum_{b=-n}^n h(b) \left(\sum_{a=-m}^m l(x+a, y+b) g(a) \right)$$





6.4.5 一个示例:Expand

$$I_i(i, j) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) \cdot I_{i+1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

	85	91	104	94			
	84	82	99	89			
	84	83	89	74			
	50	57	63	51			

$$2 \cdot 0.5 \cdot 85 = 85$$

6.4.5 一个示例:Expand

$$I_i(i, j) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) \cdot I_{i+1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

	85	91	104	94			
	84	82	99	89			
	84	83	89	74			
	50	57	63	51			

$$2 \cdot (0.25 \cdot 85 + 0.25 \cdot 84) = 84.5$$

6.4.5 一个示例:Expand

$$I_i(i, j) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) \cdot I_{i+1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

	85	91	104	94			
	84	82	99	89			
	84	83	89	74			
	50	57	63	51			

$$2 \cdot (0.5 \cdot 84) = 84$$

6.4.5 一个示例:Expand

$$I_i(i, j) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) \cdot I_{i+1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

	85	91	104	94			
	84	82	99	89			
	84	83	89	74			
	50	57	63	51			

$$2 \cdot (0.25 \cdot 84 + 0.25 \cdot 84) = 84$$

6.4.5 一个示例:Expand

$$I_i(i, j) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) \cdot I_{i+1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

85	91	104	94
84	82	99	89
84	83	89	74
50	57	63	51

43
85
85
84
84

$$2 * (0.5 * 84) = 84$$

6.4.5 一个示例:Expand

$$I_i(i, j) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) \cdot I_{i+1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

85	91	104	94
84	82	99	89
84	83	89	74
50	57	63	51

43
85
85
84
84
84
67

$$2 * (0.25 * 84 + 0.25 * 50) = 67$$

6.4.5 一个示例:Expand

$$I_i(i, j) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) \cdot I_{i+1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

85	91	104	94
84	82	99	89
84	83	89	74
50	57	63	51

43
85
85
84
84
84
67
50

$$2 * (0.5 * 50) = 50$$

6.4.5 一个示例:Expand

$$I_i(i, j) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) \cdot I_{i+1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

85	91	104	94
84	82	99	89
84	83	89	74
50	57	63	51

43
46
85
85
84
84
84
67
50

$$2 * (0.25 * 91 + 0.25 * 50) = 45.5$$

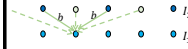
6.4.5 一个示例:Expand

$$I_1(i, j) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) \cdot I_{i+1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

85	87	99	89
84	83	89	74
50	57	63	51

43	46	52	47
85	91	104	94
85	87	102	92
84	82	99	89
84	83	94	82
84	83	89	74
67	70	76	63
50	57	63	51

6.4.5 一个示例:Expand



43	46	52	47
85	91	104	94
85	87	102	92
84	82	99	89
84	83	94	82
84	83	89	74
67	70	76	63
50	57	63	51

22
43
43
42
42
42
34
25

43
85
85
84
84
84
67
50

$$= 2 + (0.25)$$

6.4.5 一个示例:Expand

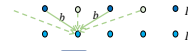


22	43	46	52	47
43	85	91	104	94
43	85	87	102	92
42	84	82	99	89
42	84	83	94	82
42	84	83	89	74
34	67	70	76	63
25	50	57	63	51

43	43
85	85
85	84
84	84
84	84
67	67
50	50

$$= 2 + (0.5)$$

6.4.5 一个示例:Expand



22	43	46	52	47
43	85	91	104	94
43	85	87	102	92
42	84	82	99	89
42	84	83	94	82
42	84	83	89	74
34	67	70	76	63
25	50	57	63	51

22
43
43
42
42
42
34
25

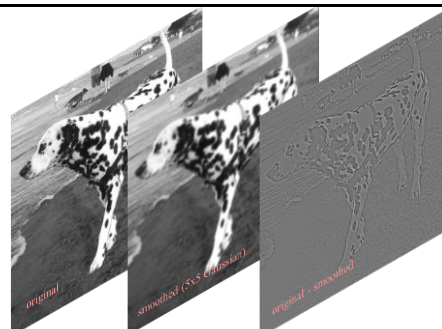
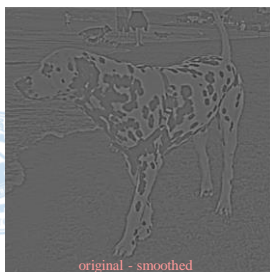
43	46
85	91
85	87
84	82
84	83
67	70
50	57

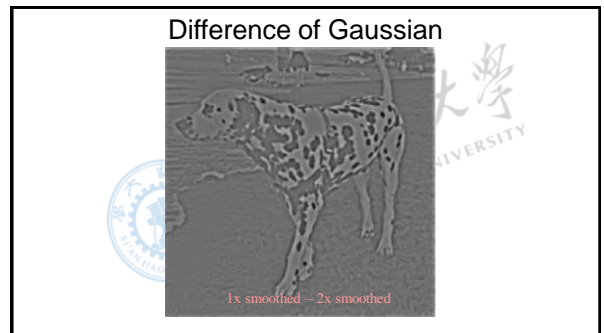
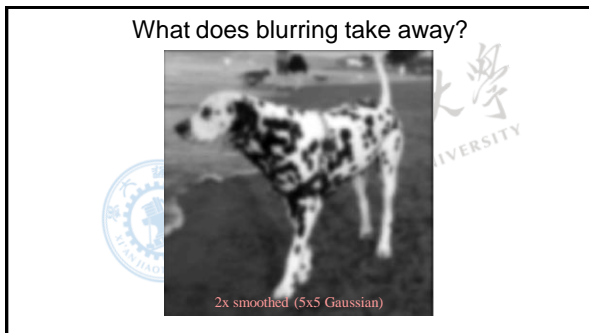
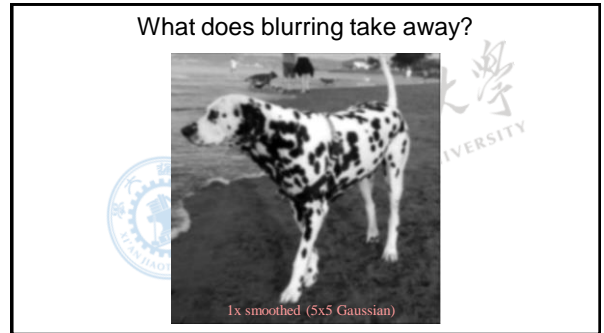
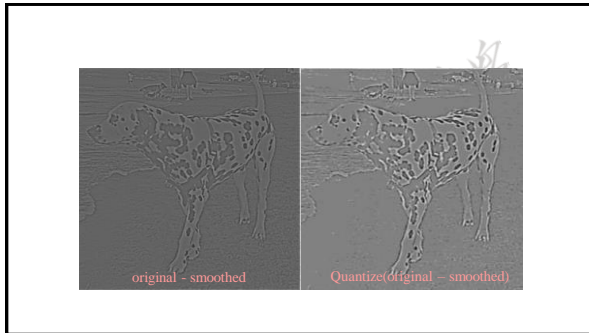
$$= 2 + (0.25) + 0.25$$

6.4.5 去除平滑后的图像是什么样子的呢?

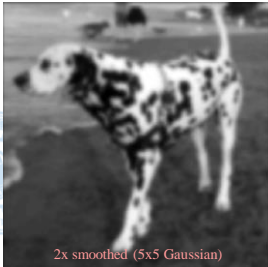


Difference as result of smoothing



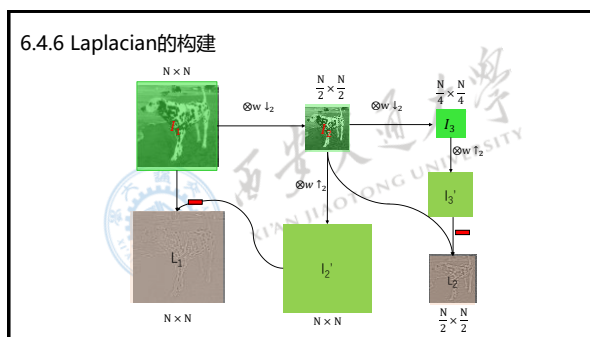
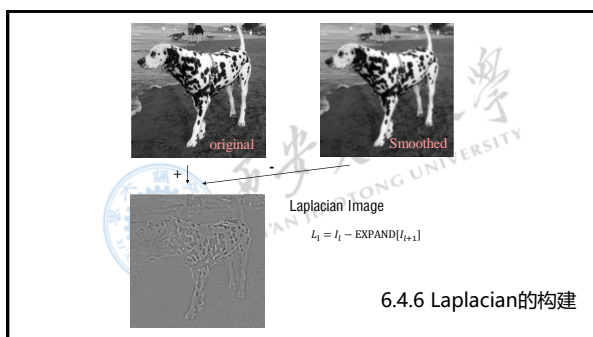
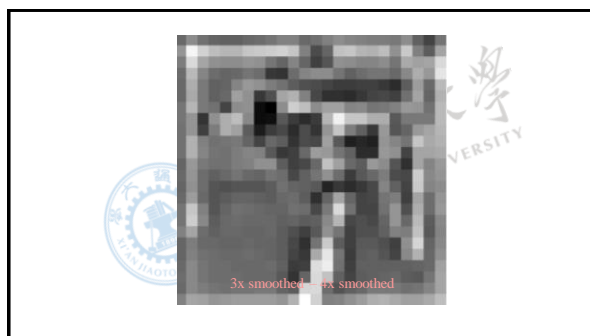
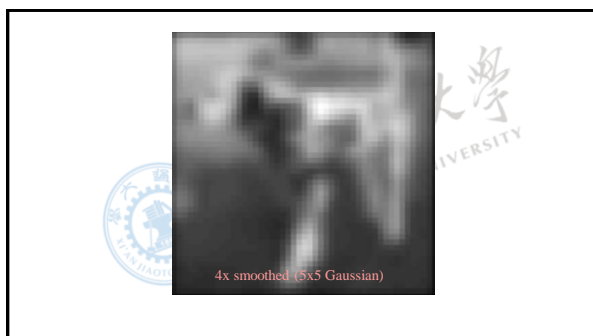


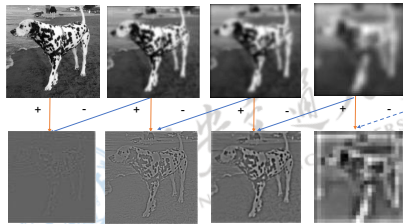
What does blurring take away?



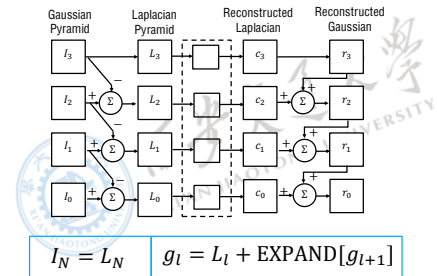
Difference of Gaussian







Gaussian pyramid is smooth=> can be subsampled
Laplacian pyramid has narrow band of frequency=> compressed

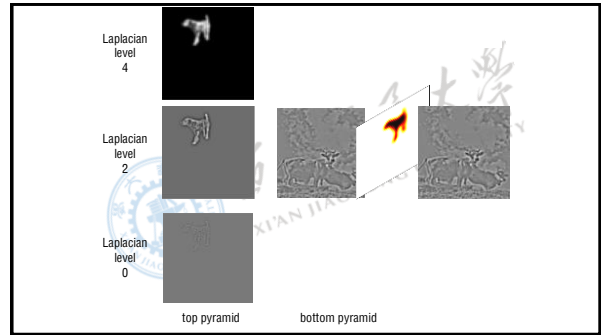
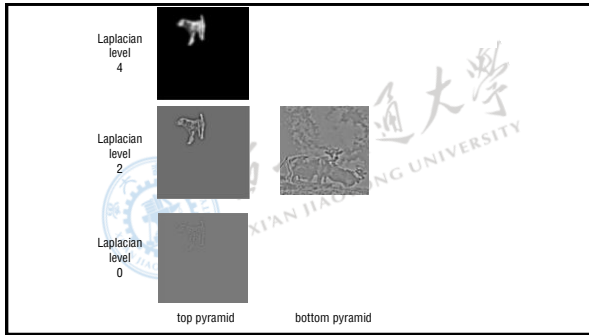


五、基于金字塔的图像合成 Pyramids Blending

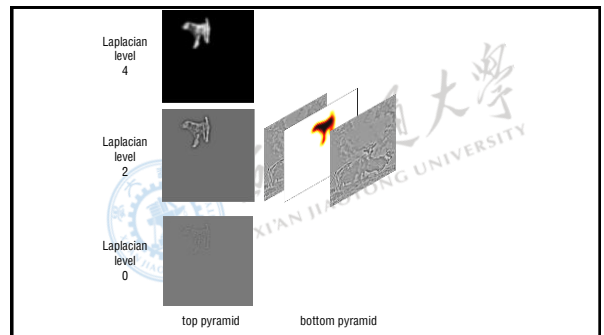
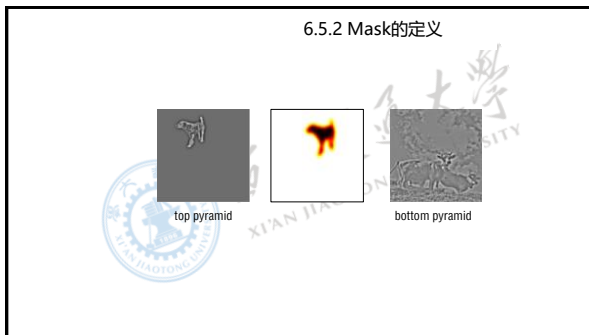


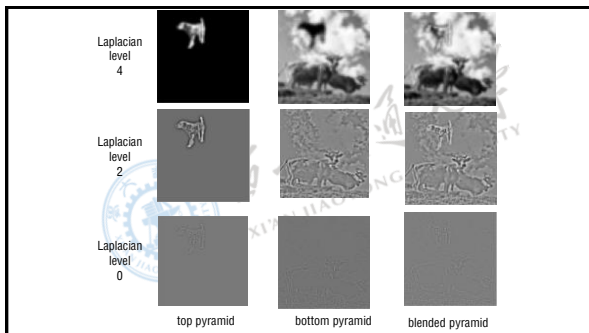
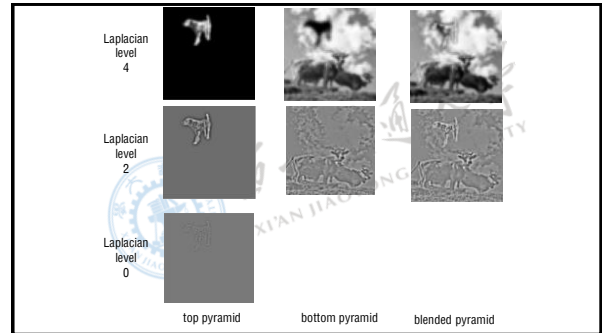
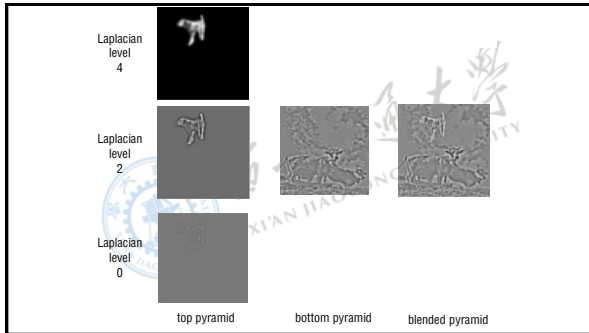
6.5.1 将左侧的照片合成到右侧的图像中





6.5.2 Mask的定义





Laplacian Pyramid: Blending

General Approach:

1. Build Laplacian pyramids LA and LB from images A and B
2. Build a Gaussian pyramid $MASK$ from selected region R
3. Form a combined pyramid LS from LA and LB using nodes of GR as weights:

$$LS(i,j) = MASK(i,j) \cdot LA(i,j) + (1 - MASK(i,j)) \cdot LB(i,j)$$

4. Collapse the LS pyramid to get the final blended image

