

计算机视觉与模式识别



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Hyun Soon Park, University of Minnesota

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第十一章 摄像机参数

摄像机是计算机视觉系统的眼睛。

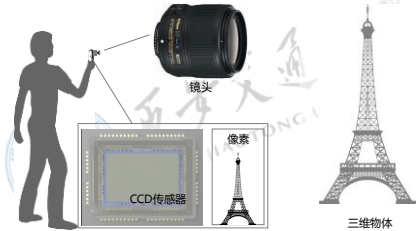


- 一、内参
- 二、外参
- 三、畸变
- 四、摄像机标定

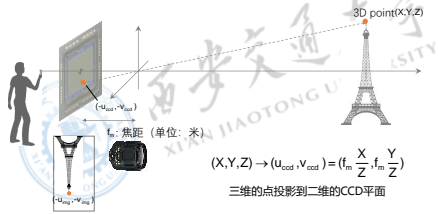
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = L \begin{bmatrix} K & R & t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

传感器的配置 (内部参数)
传感器和小孔之间的空间位置关系 (内部参数)
摄像机的空间配置 (外部参数)

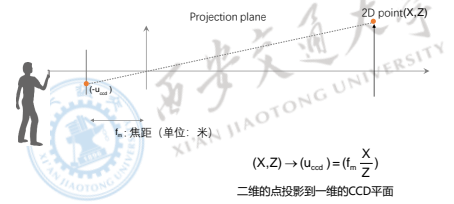
摄像机内参矩阵K



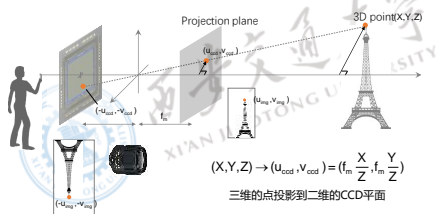
三维点的投影(度量空间)



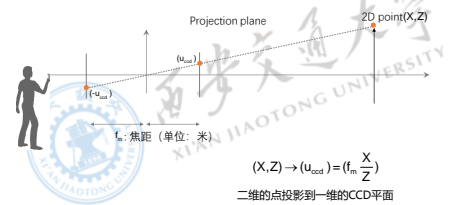
二维点的投影(一个简化的相机)



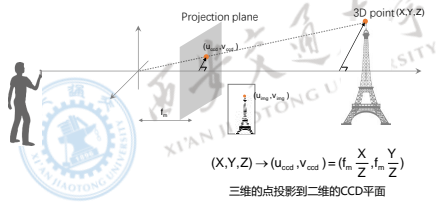
三维点的投影(度量空间)



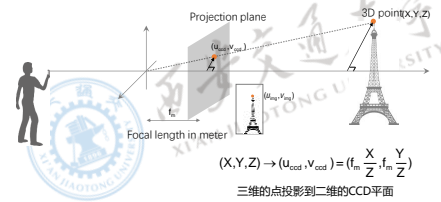
二维点的投影(一个简化的相机)



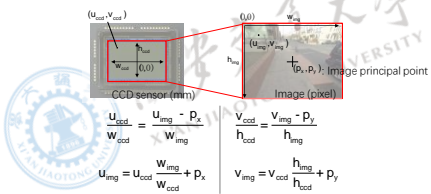
三维点的投影(度量空间)



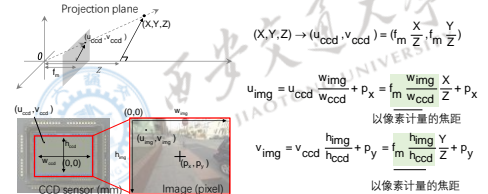
三维点的投影(度量空间)



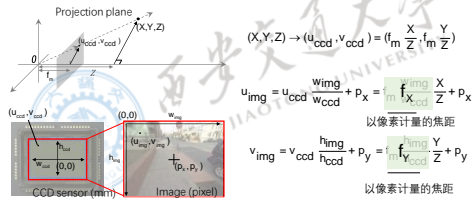
三维点的投影(像素空间)



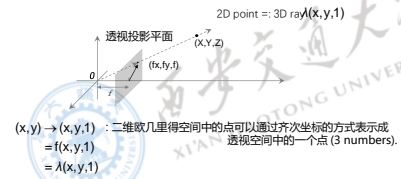
三维点的投影(像素空间)



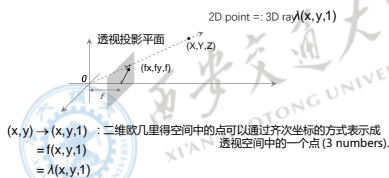
三维点的投影(像素空间)



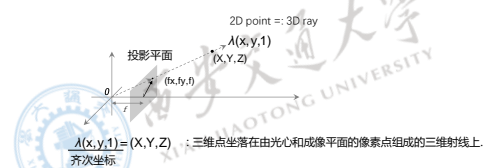
齐次坐标



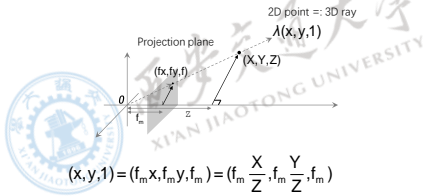
齐次与非齐次坐标之间的转换



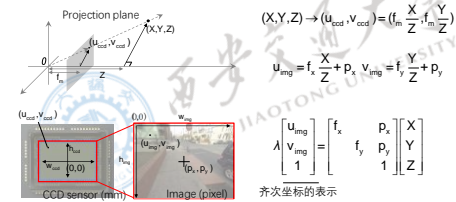
齐次坐标的物理意义



三维透视投影变换



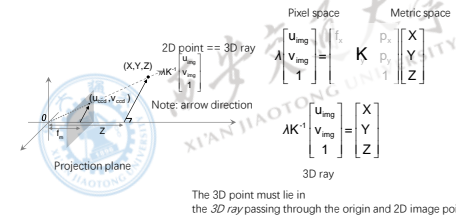
三维透视投影变换（像素空间）



摄像机内部参数



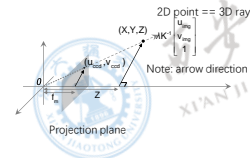
逆向投影



摄像机的一些基本概念



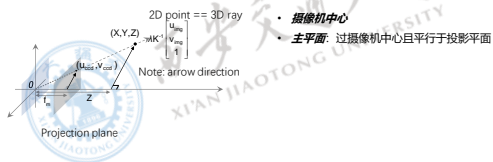
摄像机中心



基本概念

• 摄像机中心

主平面

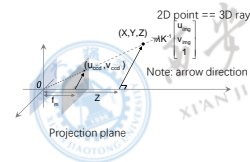


基本概念

• 摄像机中心

• 主平面: 过摄像机中心且平行于投影平面

主方向



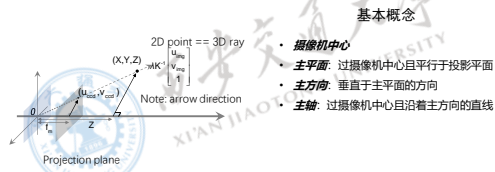
基本概念

• 摄像机中心

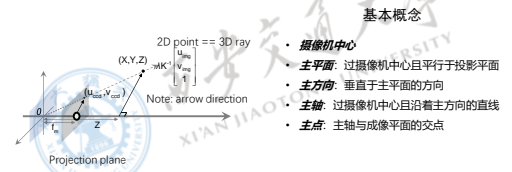
• 主平面: 过摄像机中心且平行于投影平面

• 主方向: 垂直于主平面的方向

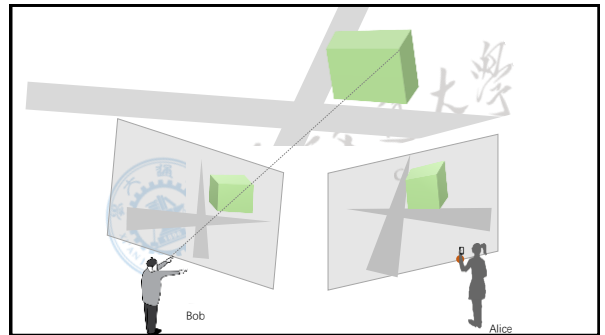
主轴



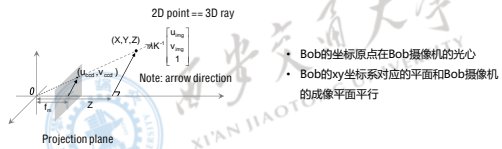
主点(Principle Point)



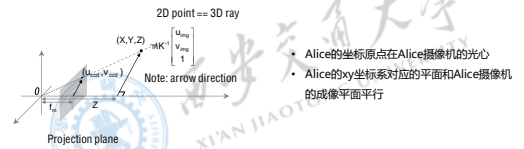
摄像机外参矩阵[R, t]



Bob's的投影

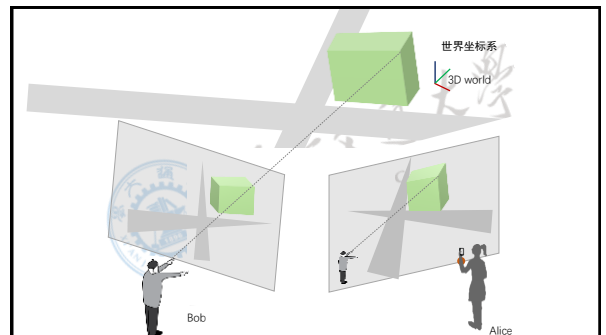


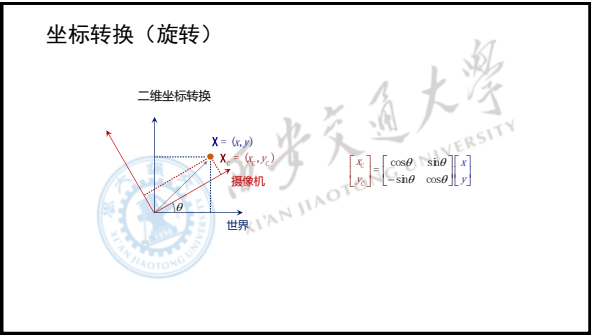
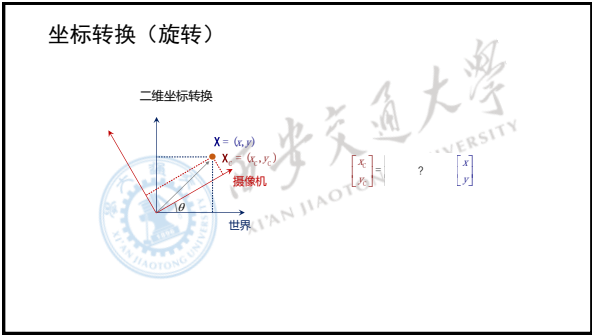
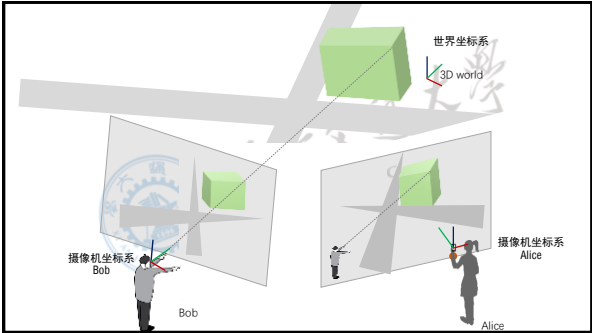
Alice's的投影

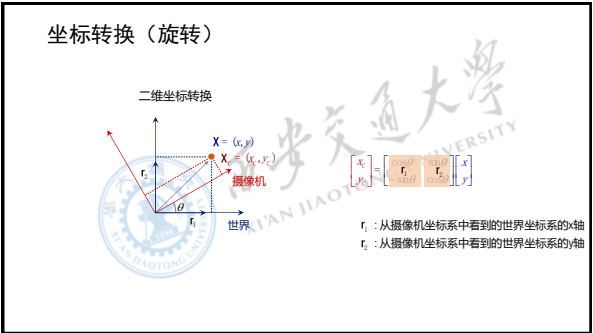
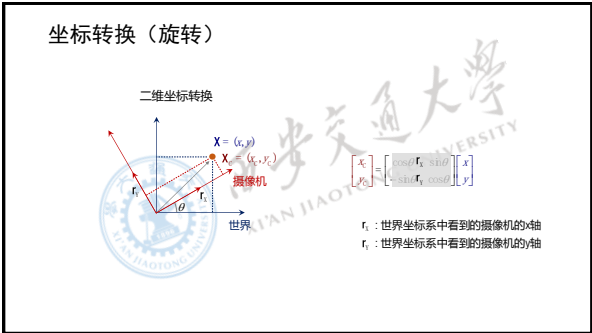
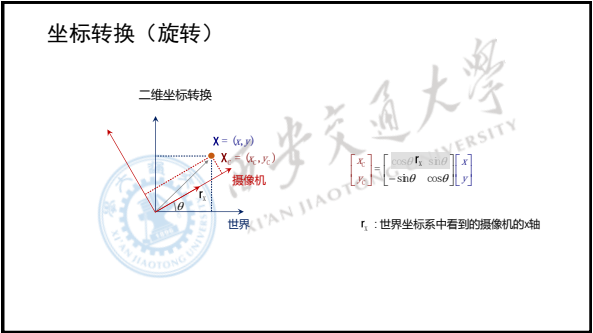
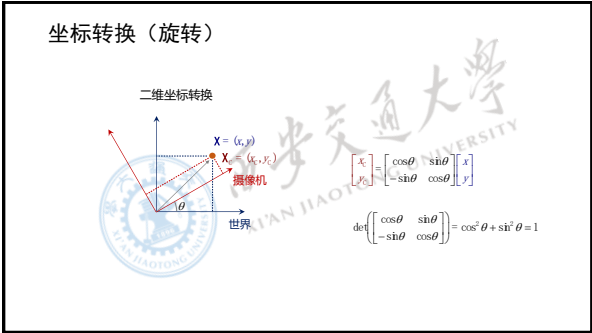


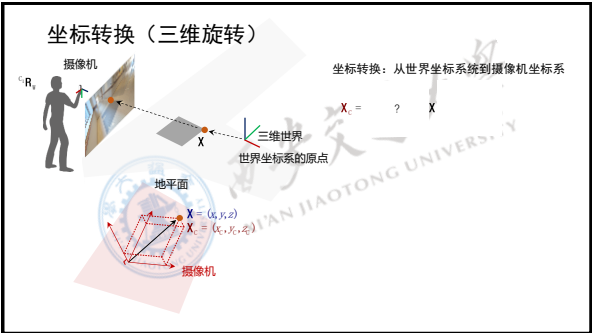
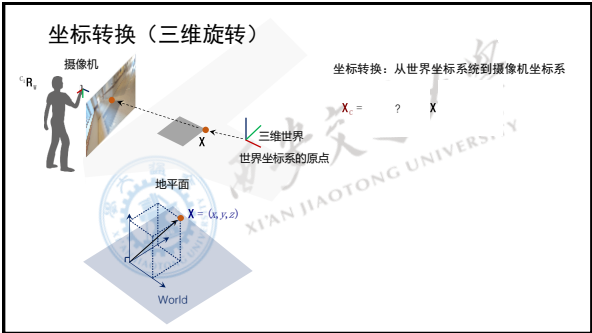
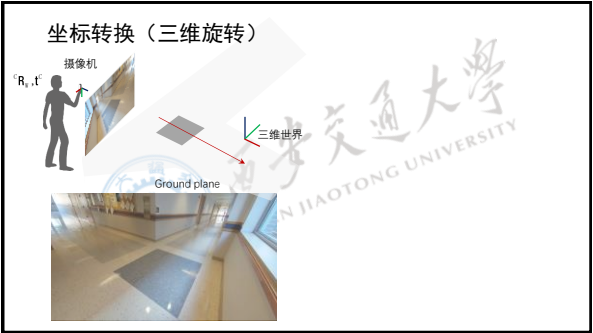
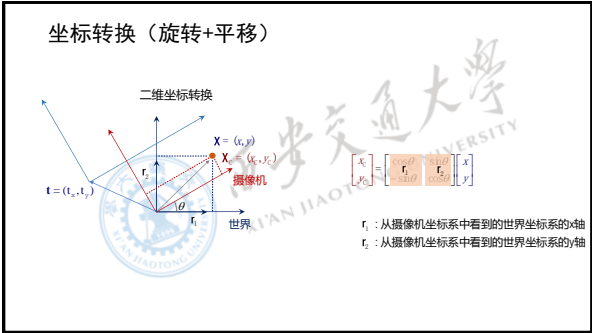
Bob's and Alice's的投影

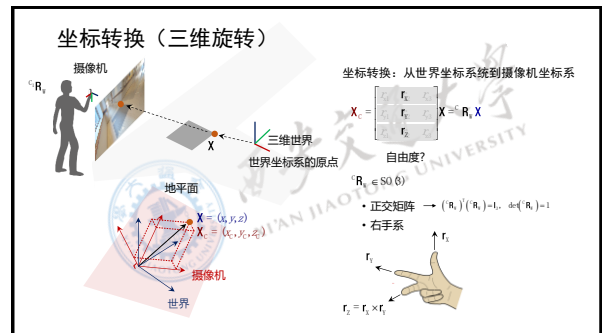
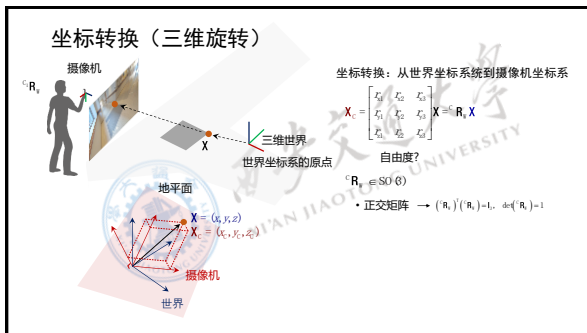
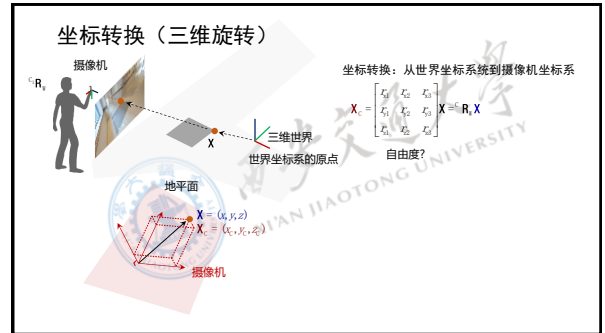
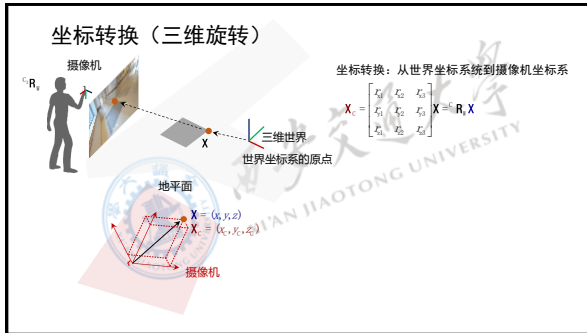
- Bob和Alice似乎都活在自己的世界中
- 我们怎么样子把他们联系起来呢?

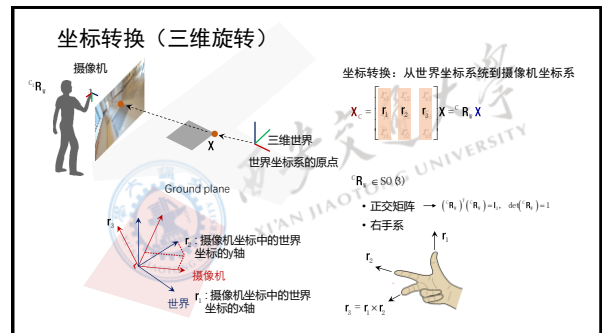
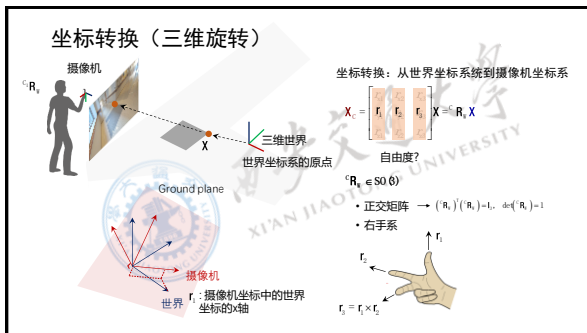
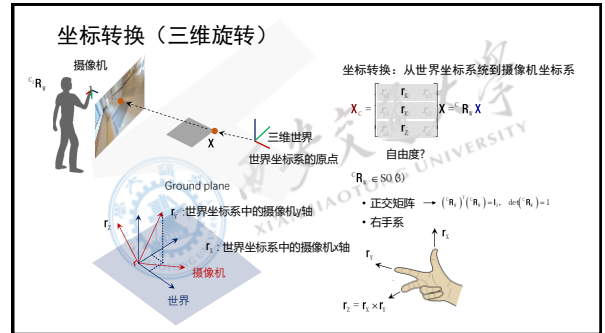
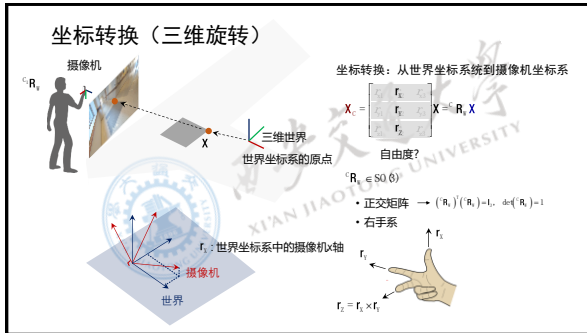


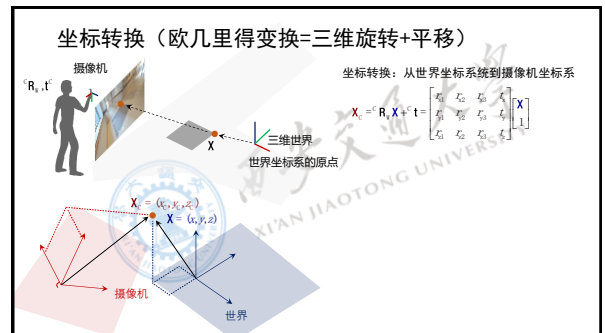
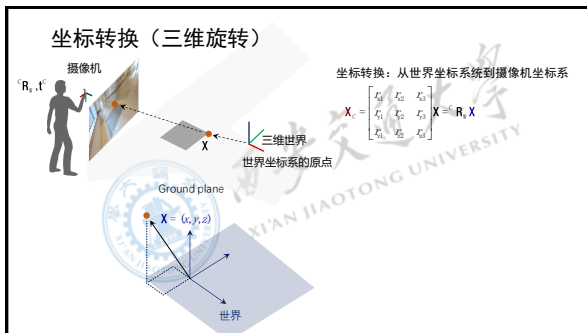
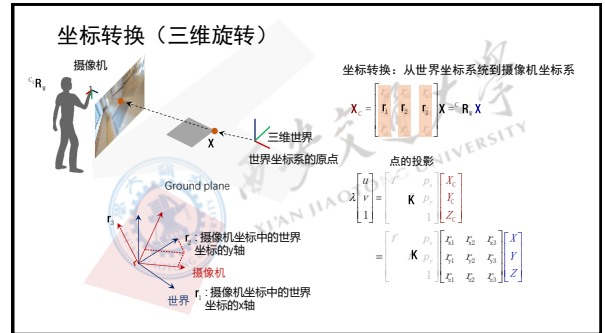
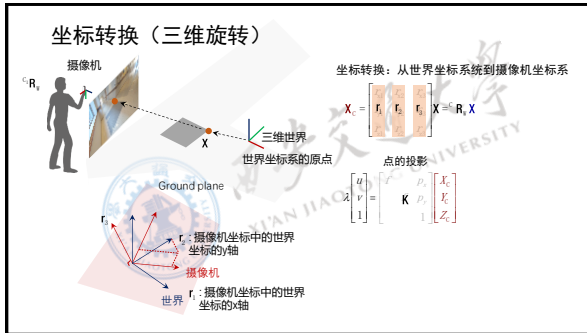


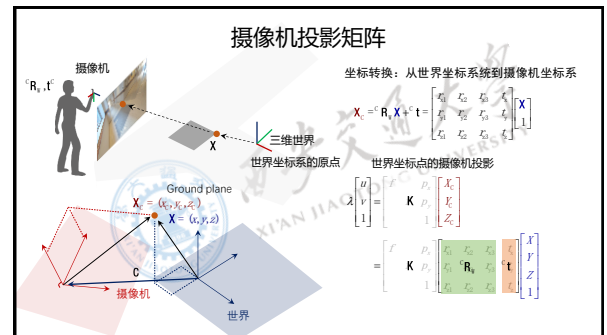
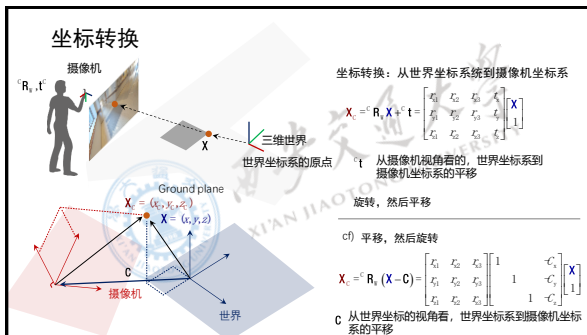
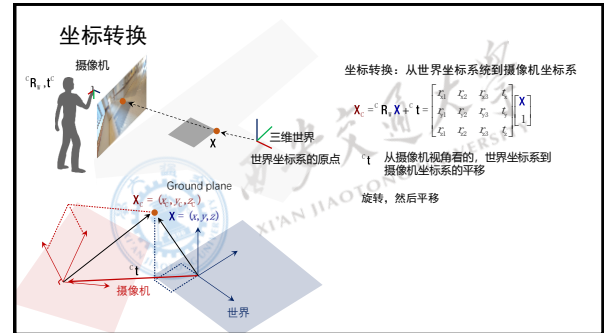
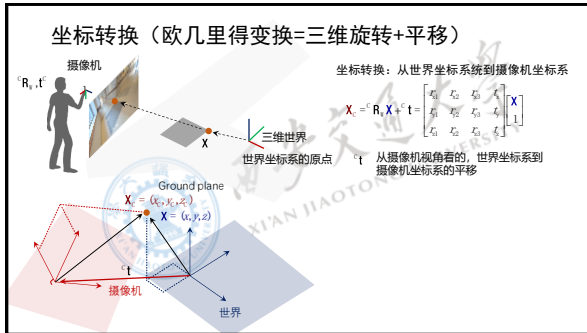












一个示例

图像的投影：示例

$$C = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}$$

图像的投影：示例

$$C = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

图像的投影：示例

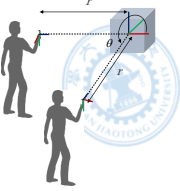
$$C = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

图像的投影：示例

$$C = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} \quad R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

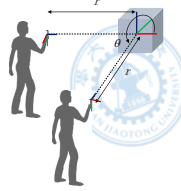
$$C = \begin{bmatrix} r \cos \theta \\ r \sin \theta \\ 0 \end{bmatrix} \quad R = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



图像的投影：示例

$$C = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} \quad R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} r \cos \theta \\ r \sin \theta \\ 0 \end{bmatrix} \quad R = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



图像的投影：示例

$$C = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} \quad R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

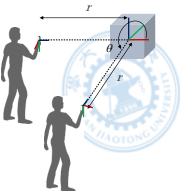
$$C = \begin{bmatrix} r \cos \theta \\ r \sin \theta \\ 0 \end{bmatrix} \quad R = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```

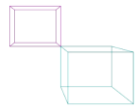
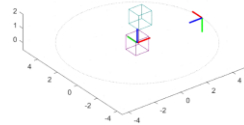
RotateCamera.m
for i = 1: length(theta)
    camera_offset = (radius*cos(theta(i)), radius*sin(theta(i)), 0);
    camera_center = camera_offset + center_of_mass;

    rz = [-cos(theta(i)), -sin(theta(i)), 0];
    ry = [0 0 -1];
    rx = [-sin(theta(i)), cos(theta(i)), 0];
    R = [rx; ry; rz];
    C = camera_center;
    P = R' * [1; 0; 0] * R;
    proj(i) = P;
end
end

```



图像的投影：示例



透视投影变换的性质



西安交通大学
XI'AN JIAOTONG UNIVERSITY

摄像机模型



镜头的配置 (内部参数)

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = L \left(\begin{bmatrix} K & R & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \right)$$

传感器和小孔之间的空间位置关系
(内部参数)

摄像机的空间配置
(外部参数)

畸变模型L



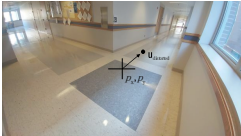
西安交通大学
XI'AN JIAOTONG UNIVERSITY



Lens Radial Distortion

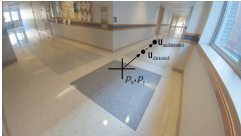
径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



$$\bar{u}_{distorted} = L(\rho) \bar{u}_{undistorted}$$

where $\rho = \|\bar{u}_{undistorted}\|$

$$L(\rho) = 1 + k_1 \rho^2 + k_2 \rho^4 + \dots$$

径向畸变模型

$$\bar{u}_{distorted} = L(\rho) \bar{u}_{undistorted}$$
$$L(\rho) = 1 + k_1 \rho^2 + k_2 \rho^4 + \dots$$



径向畸变模型

$$\bar{u}_{distorted} = L(\rho) \bar{u}_{undistorted}$$
$$L(\rho) = 1 + k_1 \rho^2 + k_2 \rho^4 + \dots$$

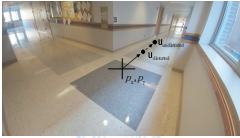


$k_1 > 0$

$k_1 < 0$

径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



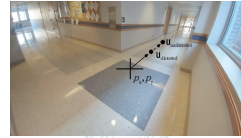
$$\bar{u}_{\text{distorted}} = L(\rho) \bar{u}_{\text{undistorted}}$$

$$\text{where } \rho = \|\bar{u}_{\text{undistorted}}\|$$

$$L(\rho) = 1 + k_1 \rho^2 + k_2 \rho^4 + \dots$$

径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



Normalized point:

$$\bar{u}_{\text{distorted}} = K \bar{u}_{\text{undistorted}}, \quad \bar{u}_{\text{undistorted}} = K^{-1} \bar{u}_{\text{distorted}}$$

$$\bar{u}_{\text{distorted}} = L(\rho) \bar{u}_{\text{undistorted}}$$

$$\text{where } \rho = \|\bar{u}_{\text{undistorted}}\|$$

$$L(\rho) = 1 + k_1 \rho^2 + k_2 \rho^4 + \dots$$

径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



Normalized point:

$$\bar{u}_{\text{distorted}} = K \bar{u}_{\text{undistorted}}, \quad \bar{u}_{\text{undistorted}} = K^{-1} \bar{u}_{\text{distorted}}$$

$$\bar{u}_{\text{distorted}} = L(\rho) \bar{u}_{\text{undistorted}}$$

$$\text{where } \rho = \|\bar{u}_{\text{undistorted}}\|$$

$$L(\rho) = 1 + k_1 \rho^2 + k_2 \rho^4 + \dots$$

$$r = f \frac{\rho_{aa}}{f_{aa}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{pk}$$

$$P_1 = \frac{f_{aa}}{2} = \frac{3840}{2} = 1920 \text{pk} \quad P_1 = \frac{f_{aa}}{2} = \frac{2160}{2} = 1080 \text{pk}$$

径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



```
im = imread('image.jpg');
f = 1224;
k1 = 0.08;
px = sum(im,2)/2;
py = size(im,1)/2;
```

$$r = f \frac{\rho_{aa}}{f_{aa}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{pk}$$

$$P_1 = \frac{f_{aa}}{2} = \frac{3840}{2} = 1920 \text{pk} \quad P_1 = \frac{f_{aa}}{2} = \frac{2160}{2} = 1080 \text{pk}$$

径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



$$r = \frac{f}{f_{\text{std}}} = 0.00153 \frac{2840}{0.0048} = 1224 \text{pk}$$

$$P_1 = \frac{f_{\text{std}}}{2} = \frac{3840}{2} = 1920 \text{pk} \quad P_2 = \frac{f_{\text{std}}}{2} = \frac{2160}{2} = 1080 \text{pk}$$

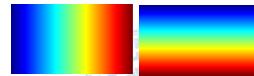
```

im = imread('image.jpg');
f = 1224;
% k = 0.0k;
px = size(im,2)/2;
py = size(im,1)/2;
im_new = zeros(size(im)); % create a new image
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        dx = [0,1] * (px-pv)/f;
        y = expm(jdx);
        l = 1 + k*r*r;
        x = l*(dx+px);
        y = l*(py+py);
        K(u) = u;
    end
    if floor(x(1)) <= 0 || floor(x(1)) > size(im,2) || floor(x(2)) <= 0 || floor(x(2)) > size(im,1)
        continue;
    end
    im_new(j,:) = im(floor(x(2)), floor(x(1)),:);
end
end

```

径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



X

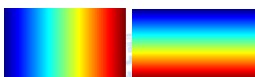
Y

$$r = \frac{f}{f_{\text{std}}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{pk}$$

$$P_1 = \frac{f_{\text{std}}}{2} = \frac{3840}{2} = 1920 \text{pk} \quad P_2 = \frac{f_{\text{std}}}{2} = \frac{2160}{2} = 1080 \text{pk}$$

径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



X

Y

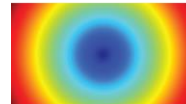
$$r = \frac{f}{f_{\text{std}}} = 0.00153 \frac{2840}{0.0048} = 1224 \text{pk}$$

$$P_1 = \frac{f_{\text{std}}}{2} = \frac{3840}{2} = 1920 \text{pk} \quad P_2 = \frac{f_{\text{std}}}{2} = \frac{2160}{2} = 1080 \text{pk}$$

[X,Y] = meshgrid(1:size(m,2), 1:size(m,1)); % XY coordinate
h = size(X,1); w = size(X,2);
Xn = (X-pv)/f;
Yn = (Y-pv)/f;
r = sqrt(Xn.^2 + Yn.^2);
u_distorted = K * u_undistorted

径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



$$\rho = \|u_{\text{undistorted}}\|$$

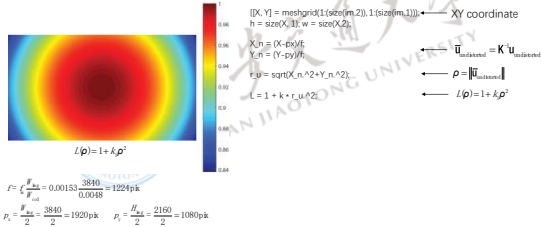
$$r = \frac{f}{f_{\text{std}}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{pk}$$

$$P_1 = \frac{f_{\text{std}}}{2} = \frac{3840}{2} = 1920 \text{pk} \quad P_2 = \frac{f_{\text{std}}}{2} = \frac{2160}{2} = 1080 \text{pk}$$

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Yn = (Y-pv)/f;
r = sqrt(Xn.^2 + Yn.^2);
u_distorted = K * u_undistorted
rho = \|u_undistorted\|

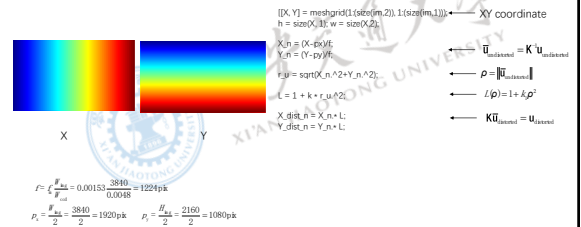
径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



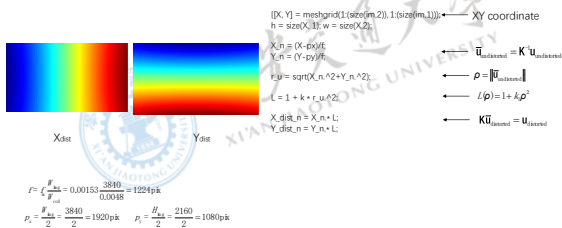
径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



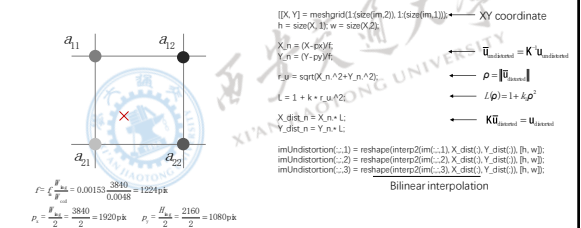
径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



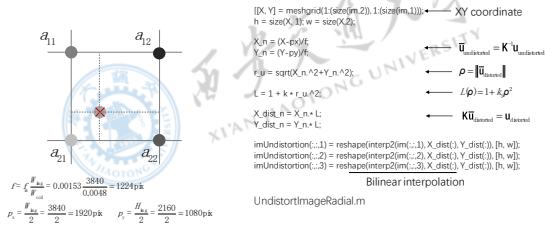
径向畸变模型

假设：镜头的畸变是距离Principle Point距离的一个函数



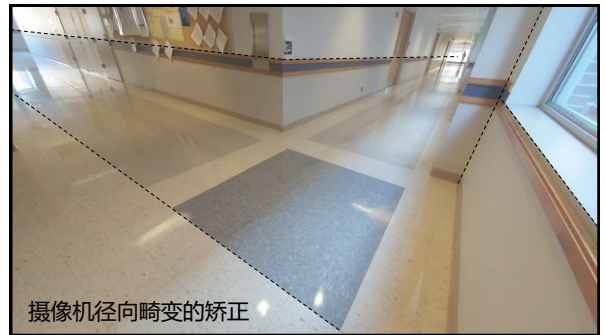
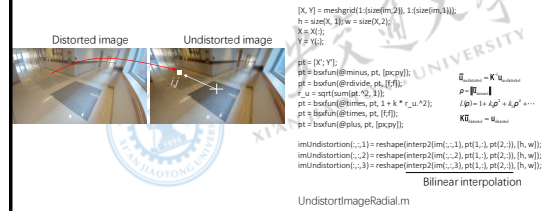
径向畸变模型

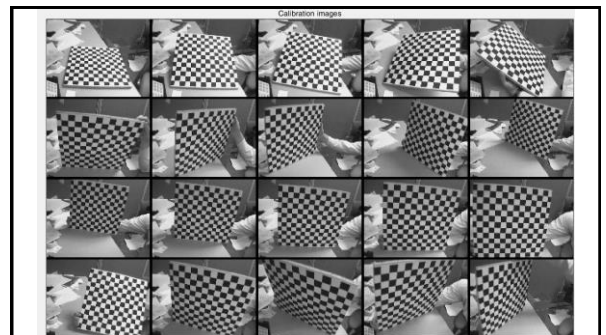
假设：镜头的畸变是距离Principle Point距离的一个函数

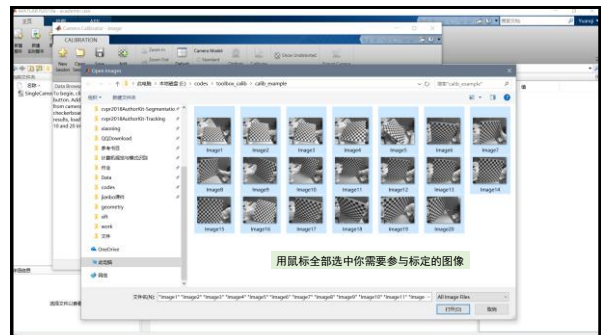
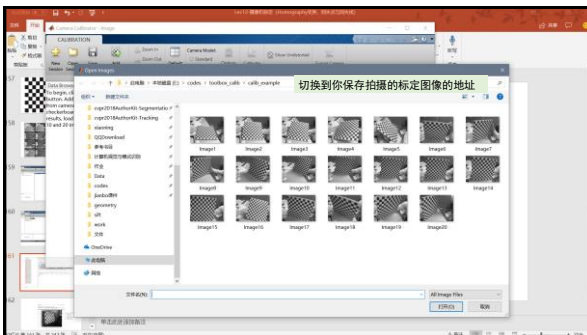
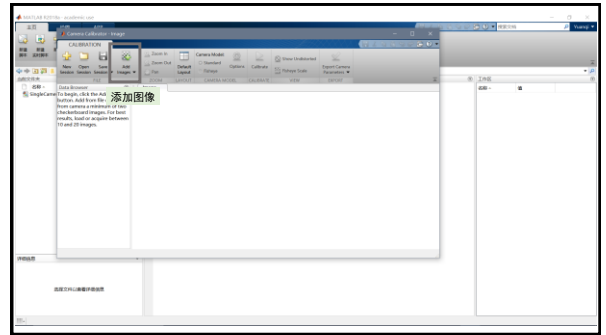
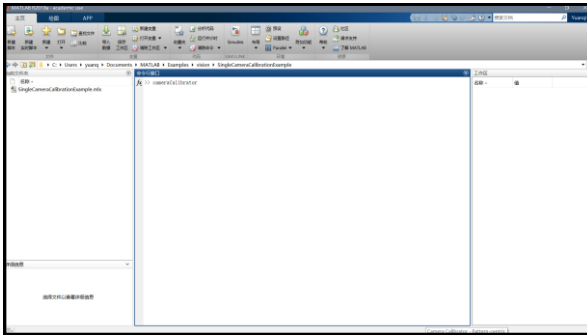


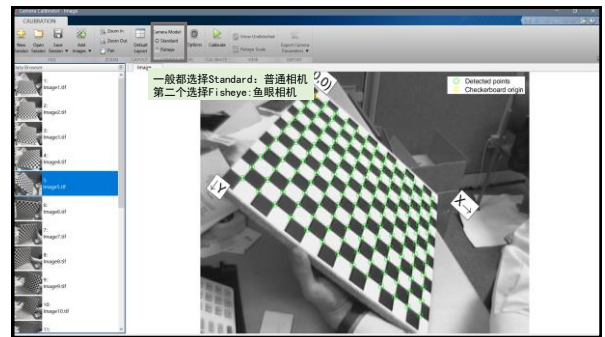
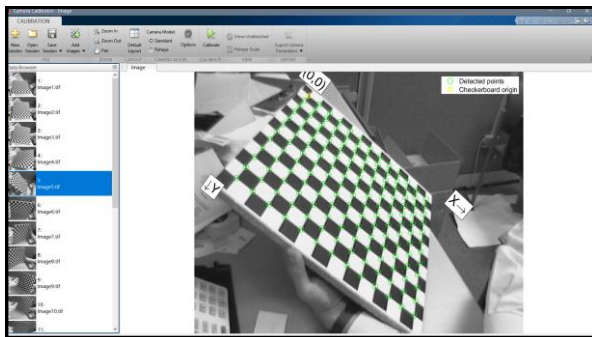
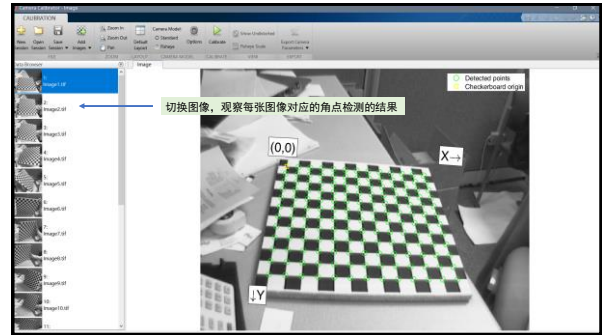
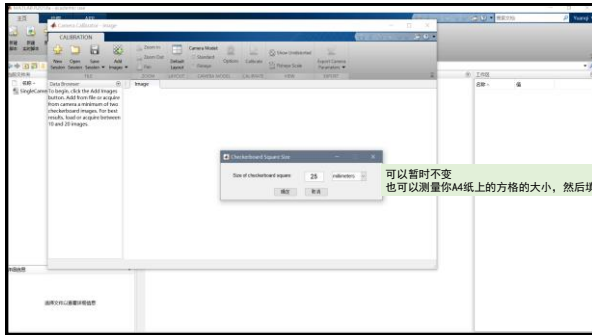
径向畸变模型

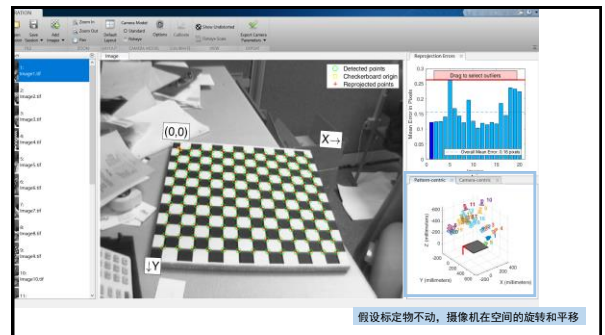
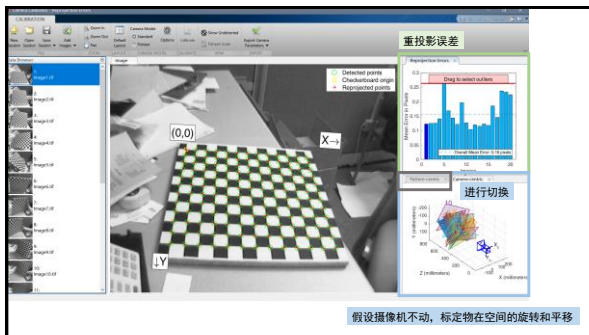
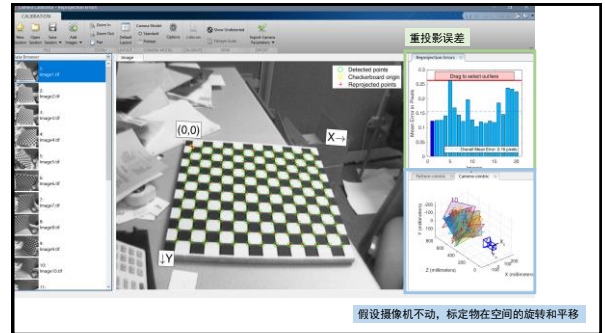
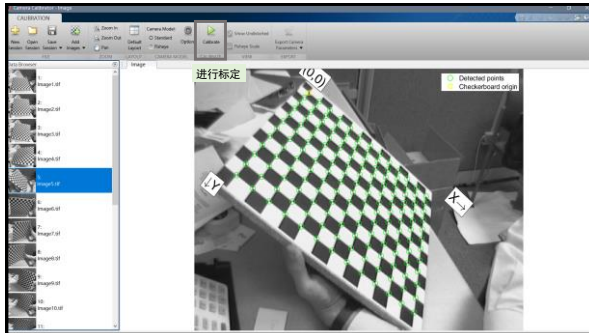
假设：镜头的畸变是距离Principle Point距离的一个函数

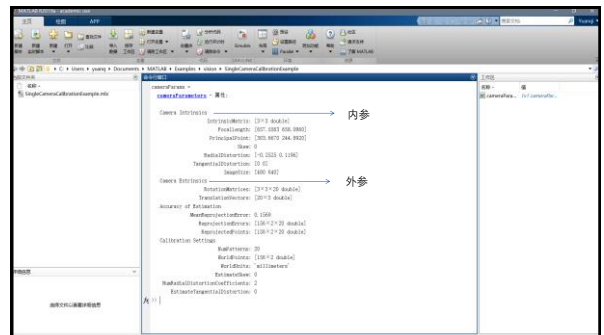
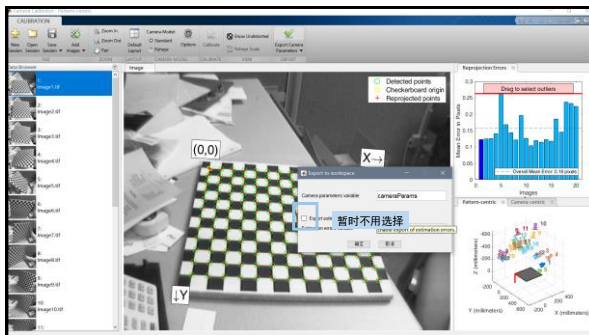
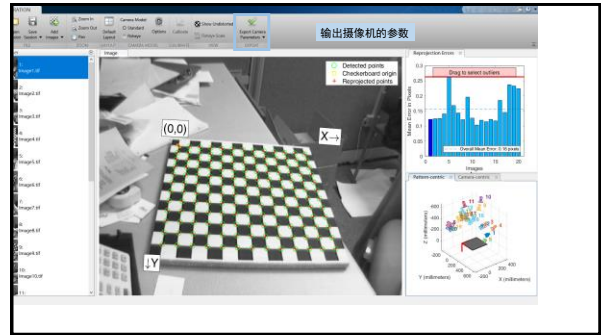
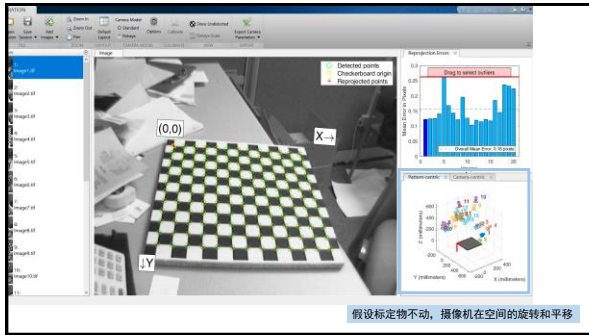


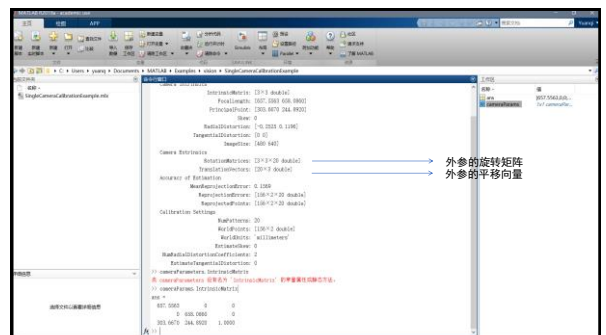
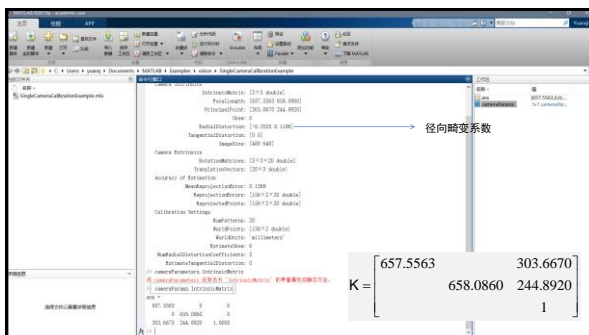
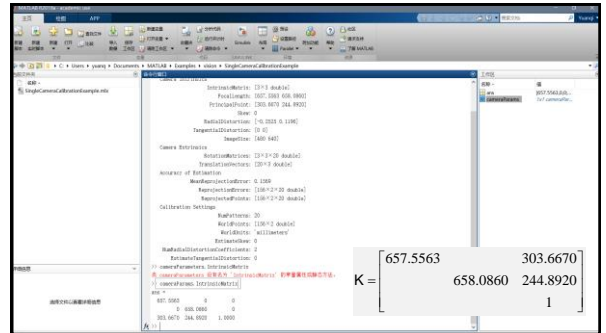
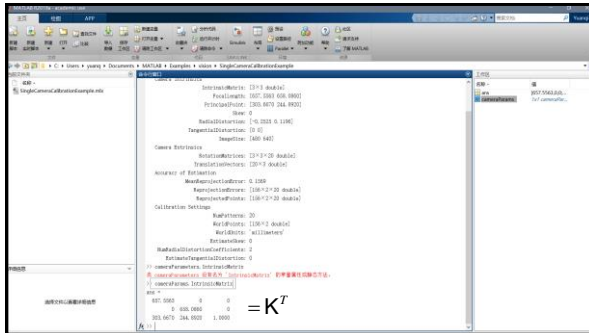












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Radial Distortion Parameter Estimation (2nd order)

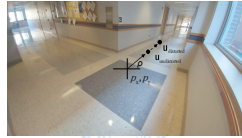


Normalized point:

$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1} \mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1} \mathbf{u}_{\text{undistorted}}$$

$$\bar{\mathbf{u}}_{\text{distorted}} = (1 + k_1 \rho^2 + k_2 \rho^4) \bar{\mathbf{u}}_{\text{undistorted}}$$

Radial Distortion Parameter Estimation (2nd order)



Normalized point:

$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1} \mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1} \mathbf{u}_{\text{undistorted}}$$

$$\bar{\mathbf{u}}_{\text{distorted}} = (1 + k_1 \rho^2 + k_2 \rho^4) \bar{\mathbf{u}}_{\text{undistorted}}$$

$$\begin{bmatrix} \rho^2 \bar{\mathbf{u}}_{\text{undistorted}} & \rho^2 \bar{\mathbf{u}}_{\text{undistorted}} \\ \vdots & \vdots \\ \rho^2 \bar{\mathbf{u}}_{\text{undistorted}} & \rho^2 \bar{\mathbf{u}}_{\text{undistorted}} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{u}}_{\text{undistorted}} - \bar{\mathbf{u}}_{\text{distorted}} \\ \vdots \\ \bar{\mathbf{u}}_{\text{undistorted}} - \bar{\mathbf{u}}_{\text{distorted}} \end{bmatrix} \quad m: \# \text{ of points}$$

Radial Distortion Parameter Estimation (2nd order)



Normalized point:

$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1} \mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1} \mathbf{u}_{\text{undistorted}}$$

$$\bar{\mathbf{u}}_{\text{distorted}} = (1 + k_1 \rho^2 + k_2 \rho^4) \bar{\mathbf{u}}_{\text{undistorted}}$$

$$\begin{bmatrix} \rho^2 \bar{\mathbf{u}}_{\text{undistorted}} & \rho^2 \bar{\mathbf{u}}_{\text{undistorted}} \\ \vdots & \vdots \\ \rho^2 \bar{\mathbf{u}}_{\text{undistorted}} & \rho^2 \bar{\mathbf{u}}_{\text{undistorted}} \end{bmatrix} \mathbf{x} = \mathbf{b} \quad m: \# \text{ of points}$$