

# 计算机视觉与模式识别

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## 第九章 角点检测与特征描述-1

最小二乘法与SVD分解是我们在计算机视觉中最常用的方法。

一、几何变换的参数估计：最佳仿射变换的估计

二、最小二乘法与SVD：梯度

三、Harris角点检测：角点的判定依据

四、模板匹配：SAD, MSE, NCC

## 第九章 角点检测与特征描述-2

SIFT特征是一个里程碑意义的算法。

五、拉普拉斯算子与尺度稳定的特征点

六、LoG和DoG

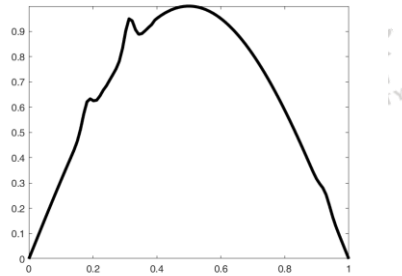
七、SIFT特征点提取

八、SIFT特征的方向估计

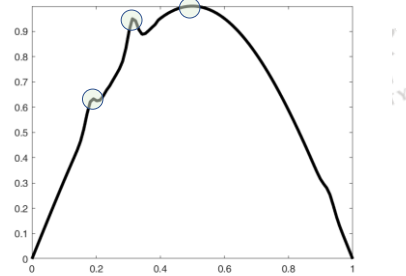
九、HOG和SIFT特征的描述子

五、拉普拉斯算子与尺度稳定的特征点

## 9.5.1 一维的信号



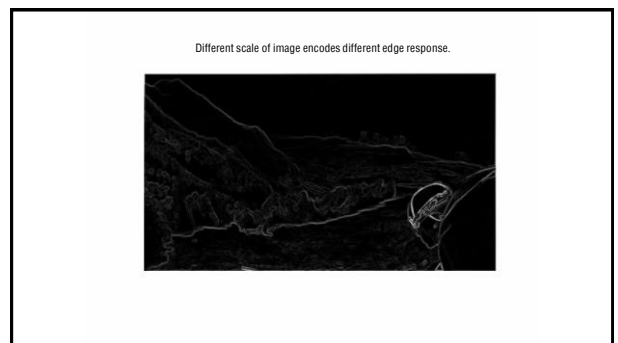
## 9.5.1 一维的信号



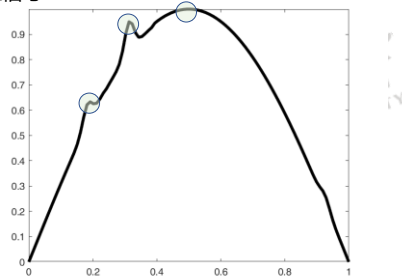
## 9.5.2 这些点稳定吗？

- 这些特征点和周边的点有不同的地方（空间的特殊性）
- 这些特征点会不会随着信号的平滑而丢失（在不同尺度上是否具有稳定性）

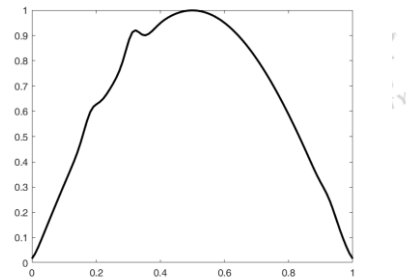




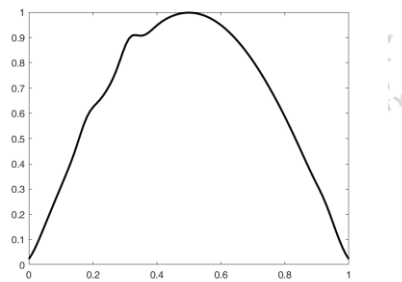
9.5.3 一维信号



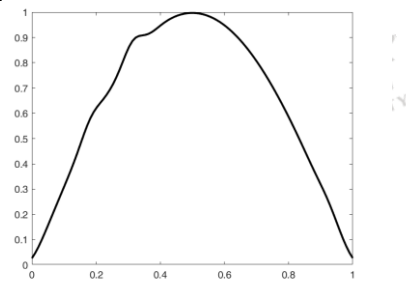
1x smooth



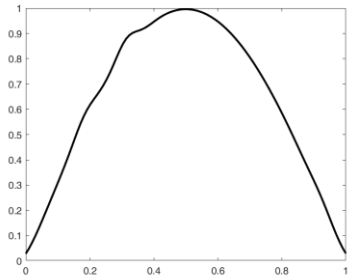
2x smooth



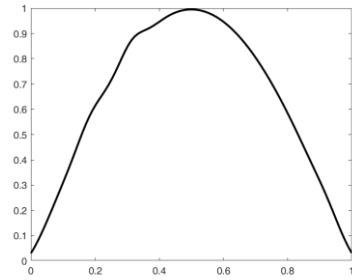
3x smooth



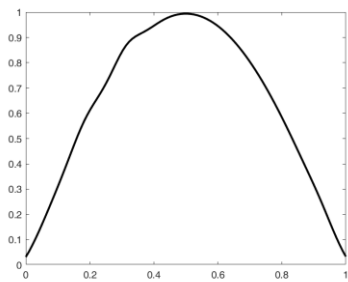
4x smooth



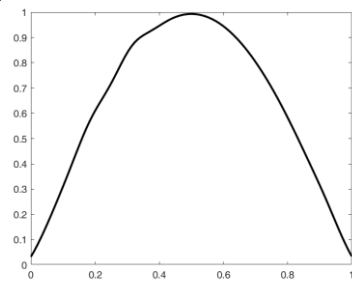
5x smooth



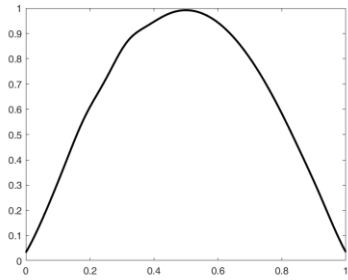
6x smooth



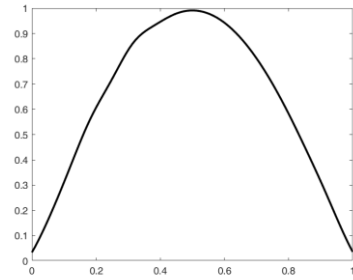
7x smooth



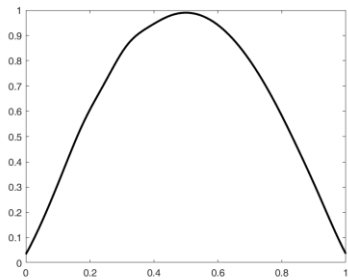
8x smooth



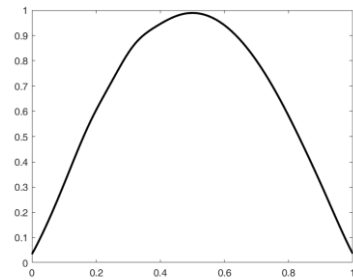
9x smooth



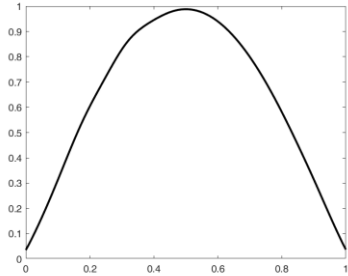
10x smooth



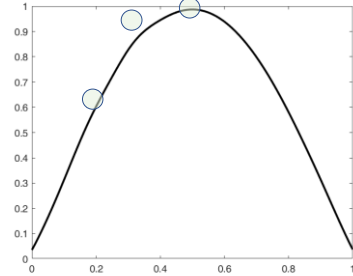
11x smooth



12x smooth



13x smooth

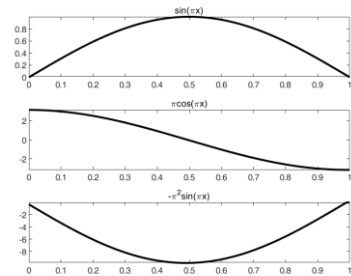


#### 9.5.4 一维信号二阶导数

- 这些特征点会不会随着信号的平滑而丢失 (在不同尺度上是否具有稳定性)
- 他们会不会随着信号的缩放而丢失



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## 9.5.5 二维函数的拉普拉斯算子



$$\Delta I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

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## 9.5.6 最简单的拉普拉斯算子



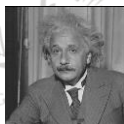
$$\begin{bmatrix} & -1 & \\ -1 & 4 & -1 \\ & -1 & \end{bmatrix}$$

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## 最简单的拉普拉斯算子

$$\frac{\partial^2 I}{\partial x^2} = \frac{\partial}{\partial x} \otimes \frac{\partial}{\partial x} \otimes I \Rightarrow$$

$$\begin{bmatrix} -1 & 1 \end{bmatrix} \otimes \begin{bmatrix} -1 & 1 \end{bmatrix} \otimes$$

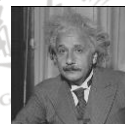


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## 最简单的拉普拉斯算子

$$\frac{\partial^2 I}{\partial x^2} = \frac{\partial}{\partial x} \otimes \frac{\partial}{\partial x} \otimes I \Rightarrow$$

$$\left[ \begin{bmatrix} -1 & 1 \end{bmatrix} \otimes \begin{bmatrix} -1 & 1 \end{bmatrix} \right] \otimes$$



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## 最简单的拉普拉斯算子

$$\frac{\partial^2 I}{\partial x^2} = \frac{\partial}{\partial x} \otimes \frac{\partial}{\partial x} \otimes I \Rightarrow \begin{bmatrix} \begin{bmatrix} -1 & 1 \end{bmatrix} \otimes \begin{bmatrix} -1 & 1 \end{bmatrix} \\ \begin{bmatrix} -1 & 1 \end{bmatrix} \otimes \begin{bmatrix} -1 & 1 \end{bmatrix} \\ \begin{bmatrix} -1 & 1 \end{bmatrix} \otimes \begin{bmatrix} -1 & 1 \end{bmatrix} \\ \begin{bmatrix} -1 & 2 & -1 \end{bmatrix} \end{bmatrix}$$

## 最简单的拉普拉斯算子

$$\frac{\partial^2 I}{\partial y^2} = \frac{\partial}{\partial y} \otimes \frac{\partial}{\partial y} \otimes I \Rightarrow \begin{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \end{bmatrix}$$

## 9.5.5 二维函数的拉普拉斯算子

$$\Delta I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

- 各向同性的最小阶线性算子!
- 最简单的拉普拉斯算子易受噪声影响

## 9.5.6 增加高斯平滑

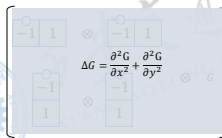
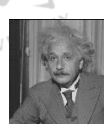
$$\frac{\partial^2 I}{\partial x^2} = \frac{\partial}{\partial x} \otimes \frac{\partial}{\partial x} \otimes G \otimes I$$

$$\frac{\partial^2 I}{\partial y^2} = \frac{\partial}{\partial y} \otimes \frac{\partial}{\partial y} \otimes G \otimes I$$


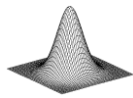
## 增加高斯平滑

$$\frac{\partial^2 I}{\partial x^2} = \frac{\partial}{\partial x} \otimes \frac{\partial}{\partial x} \otimes G \otimes I$$

$$\frac{\partial^2 I}{\partial y^2} = \frac{\partial}{\partial y} \otimes \frac{\partial}{\partial y} \otimes G \otimes I$$

$$\Delta G = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}$$



## 高斯函数的拉普拉斯算子

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial G}{\partial x} = \frac{1}{2\pi\sigma^2} \cdot \frac{-x}{\sigma^2} \cdot e^{-\frac{x^2+y^2}{2\sigma^2}}$$

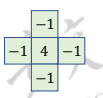
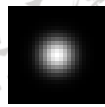
$$\frac{\partial^2 G}{\partial x^2} = \frac{-1}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{x^2}{2\pi\sigma^6} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial G}{\partial y} = \frac{1}{2\pi\sigma^2} \cdot \frac{-y}{\sigma^2} \cdot e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial^2 G}{\partial y^2} = \frac{-1}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{y^2}{2\pi\sigma^6} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\Delta G = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} = \frac{1}{\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}} \left( -1 + \frac{x^2+y^2}{2\sigma^2} \right)$$

## 9.5.7 构建高斯拉普拉斯算子

$$\Delta G = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} \Rightarrow$$



$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\Delta G = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} \Rightarrow$$



$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

控制分析的尺度

## 9.5.8 归一化的拉普拉斯算子

$$\Delta G = \sigma^2 \left( \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} \right) \Rightarrow \sigma^2 \begin{bmatrix} -1 & 4 & -1 \\ -1 & 4 & -1 \\ -1 & 4 & -1 \end{bmatrix} \otimes \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

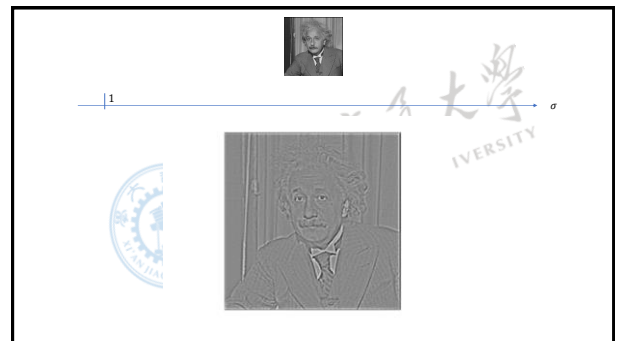
$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

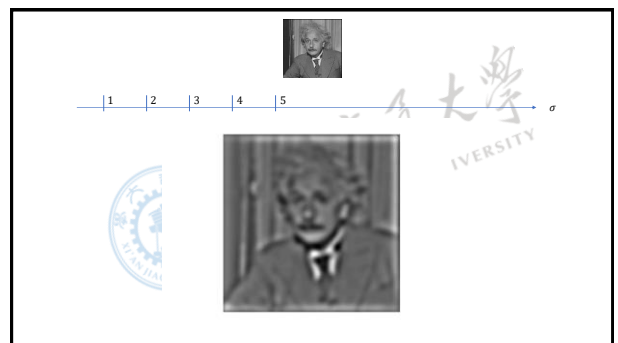
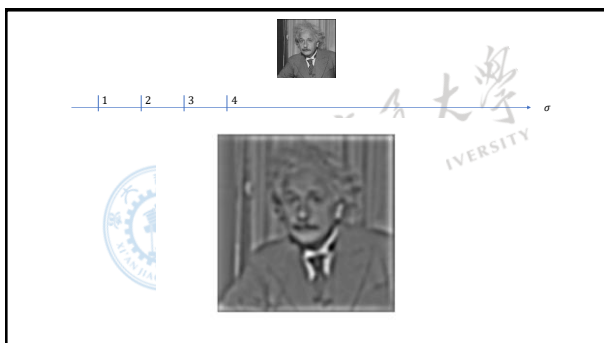
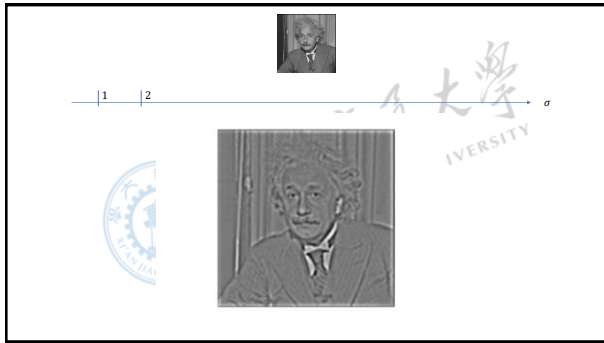
↑  
控制分析的尺度

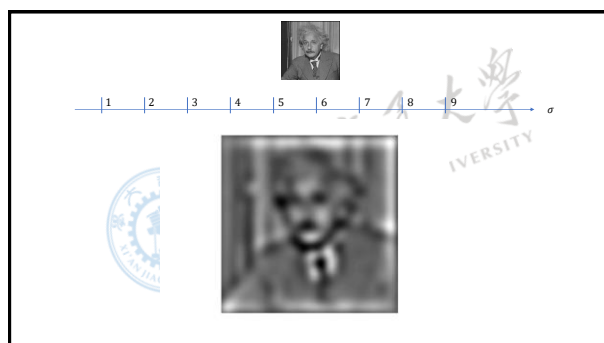
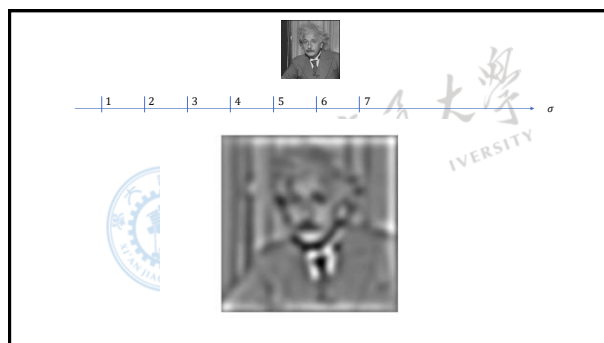
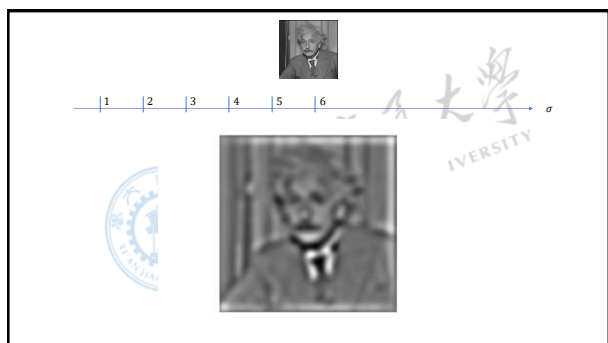
## 9.5.8 高斯函数的拉普拉斯算子

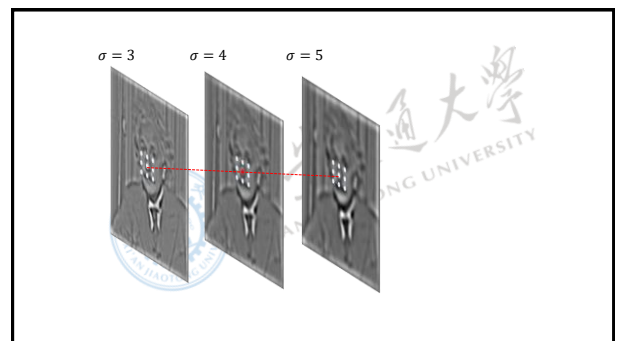
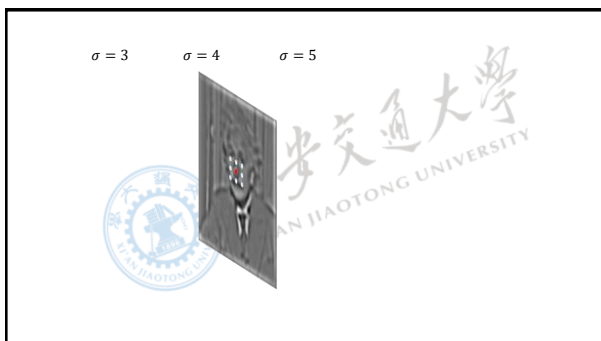
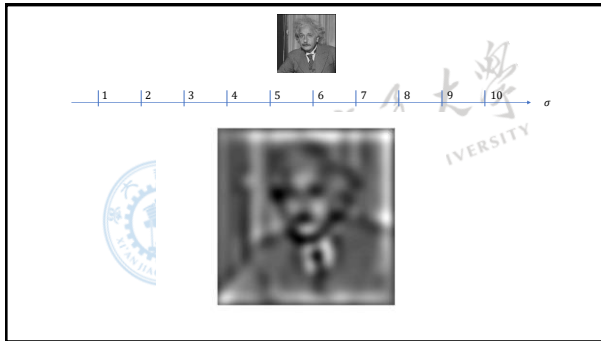
$$I(x, y) \otimes \Delta G(x, y, \sigma = 5) = LoG(x, y, \sigma)$$

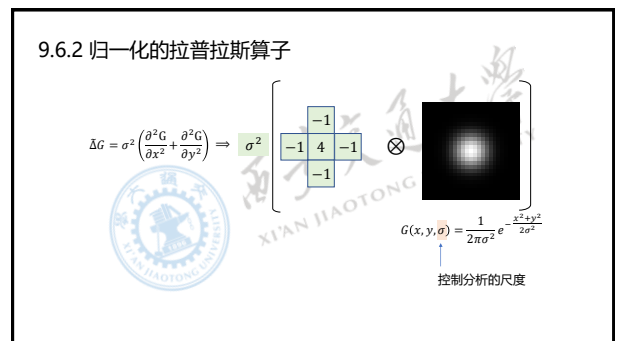
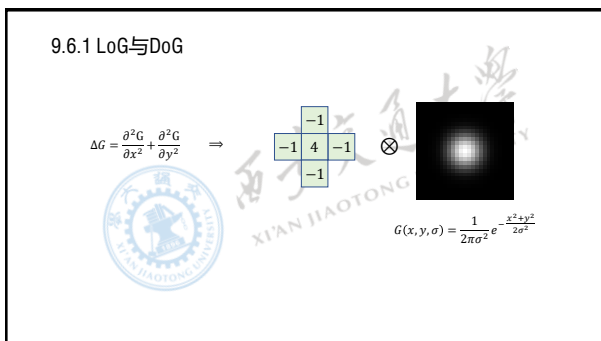
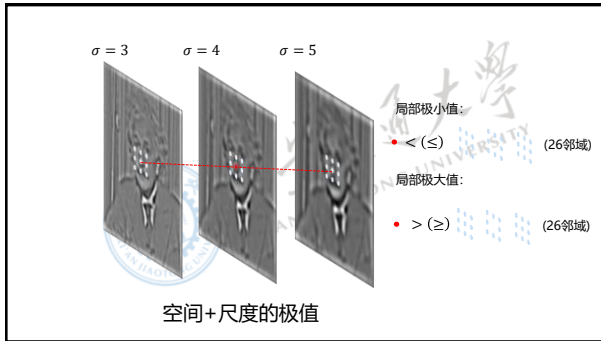
## 9.5.9 不同尺度的高斯拉普拉斯算子



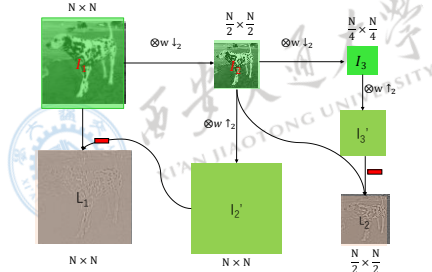








### 9.6.3 拉普拉斯金字塔的构建



### 9.6.4 高斯差分 (Difference of Gaussian)

$$DoG = \begin{cases} I \otimes G - I \\ I \otimes G \otimes G - I \otimes G \end{cases}$$

The diagram shows the Difference of Gaussian (DoG) operation. It illustrates the subtraction of two Gaussian-blurred versions of an image,  $L_1$  and  $L_2$ , to produce the DoG result.

### 9.6.5 不同尺度的高斯平滑滤波器

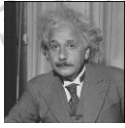


The diagram illustrates the Gaussian blur operation. It shows an input image  $I(x,y)$  being convolved with a Gaussian kernel  $G(x,y,\sigma)$  to produce a blurred image  $L(x,y,\sigma)$ . The Gaussian kernel is shown as a 2D Gaussian function.

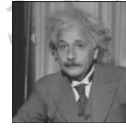
$$G(x,y,\sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



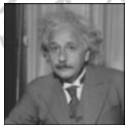
## 9.6.5 高斯滤波器的尺度



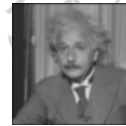
## 9.6.5 高斯滤波器的尺度



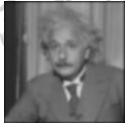
## 9.6.5 高斯滤波器的尺度



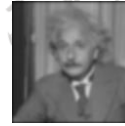
## 9.6.5 高斯滤波器的尺度



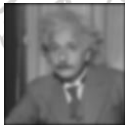
## 9.6.5 高斯滤波器的尺度



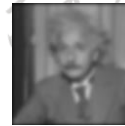
## 9.6.5高斯滤波器的尺度



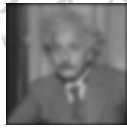
## 9.6.5 高斯滤波器的尺度



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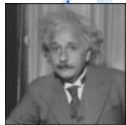
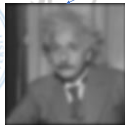
## 9.6.5 高斯滤波器的尺度



## 9.6.6 高斯差分图像



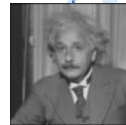
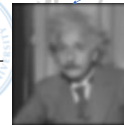
Difference of Gaussian

 $L(x, y, 1.5)$  $L(x, y, 4.0)$ 

## 9.6.6 高斯差分图像

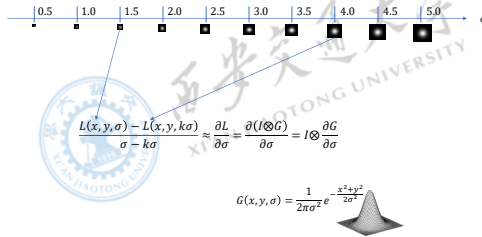


Difference of Gaussian (DOG)

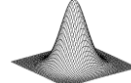
 $L(x, y, 1.5)$  $L(x, y, 4.0)$ 

(灰度偏移了128)

## 9.6.7 高斯差分图像是对尺度的有限差分



## 9.6.8 高斯函数相对于尺度的导数



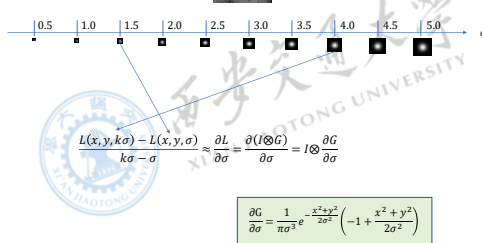
$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial G}{\partial x} = \frac{1}{2\pi\sigma^2} \cdot \frac{-x}{\sigma^2} \cdot e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial G}{\partial y} = \frac{1}{2\pi\sigma^2} \cdot \frac{-y}{\sigma^2} \cdot e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial G}{\partial \sigma} = \frac{1}{\pi\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} \left( -1 + \frac{x^2+y^2}{2\sigma^2} \right)$$

## 9.6.9 高斯差分图像是对尺度的有限差分



## 9.6.9 DoG: Difference of Gaussian

$$\sigma \Delta G = \frac{\partial G}{\partial \sigma} \approx \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k\sigma - \sigma}$$

DoG: Difference of Gaussian

$$G(x, y, k\sigma) - G(x, y, \sigma) = (k-1)\sigma^2 \Delta G$$

LoG: Laplacian of Gaussian

- LoG需要两个方向的高斯二阶微分卷积核，而DoG直接使用高斯卷积核，省去卷积核生成的运算量
- DoG可以保留各个高斯尺度空间的图像，这样子生成某一空间尺度的特征时，可以直接使用该尺度的图像，无需另行生成；
- LoG对斑点进行检测时，稳定性好，抗噪声干扰能力强，DoG是LoG的近似，也具备这一特征。

$$\sigma \Delta G = \frac{\partial G}{\partial \sigma} \approx \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k\sigma - \sigma}$$

DoG: Difference of Gaussian

$$G(x, y, k\sigma) - G(x, y, \sigma) = (k - 1)\sigma^2 \Delta G$$

LoG: Laplacian of Gaussian

$$\sigma \Delta G = \frac{\partial G}{\partial \sigma} \approx \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k\sigma - \sigma}$$

DoG: Difference of Gaussian

$$G(x, y, k\sigma) - G(x, y, \sigma) = (k - 1)\sigma^2 \Delta G(x, y, \sigma)$$

LoG (Normalized):  
Normalized Laplacian of Gaussian

$$\tilde{\Delta}G(x, y, \sigma)$$

## 七、SIFT特征点提取



## 9.7.2 SIFT的高斯金字塔生成



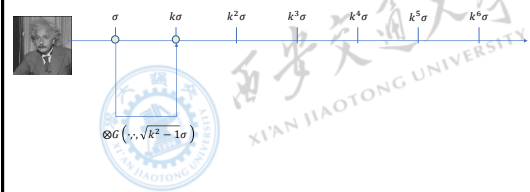
## 高斯卷积的性质

$$I \otimes G(\cdot, \sigma_1) \otimes G(\cdot, \sigma_2) \otimes \dots \otimes G(\cdot, \sigma_n) = I \otimes G\left(\cdot, \sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^2}\right)$$

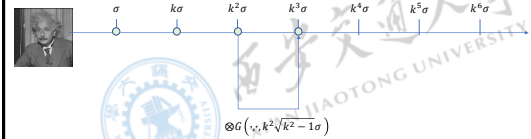
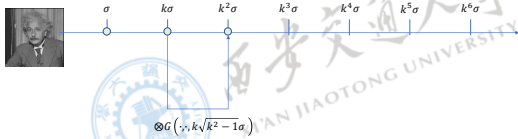
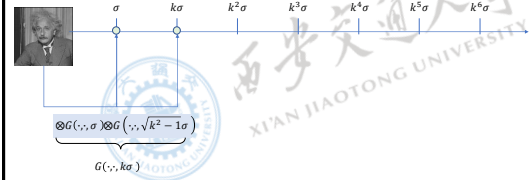
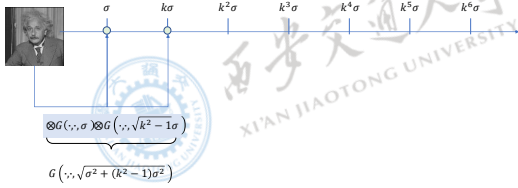
## 高斯卷积的性质

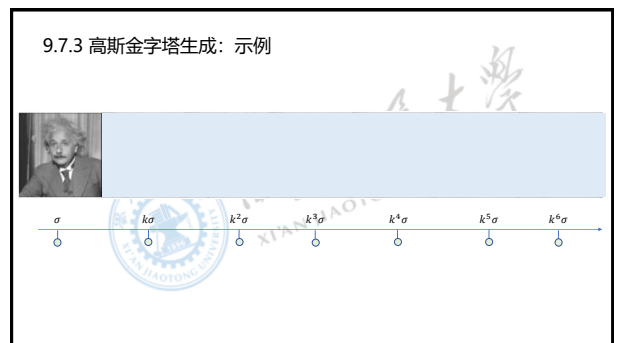
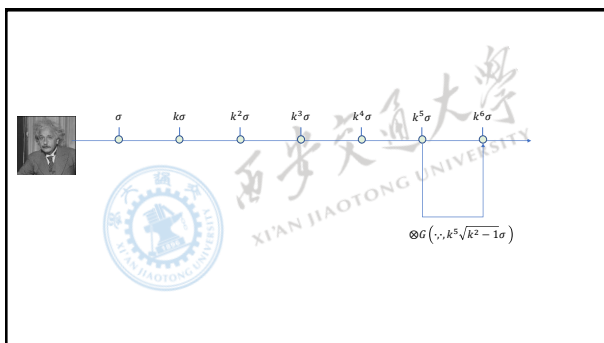
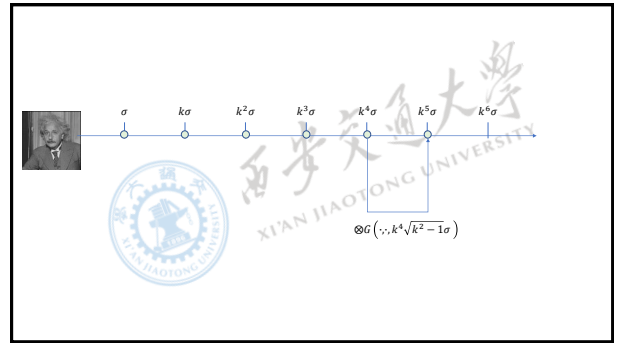
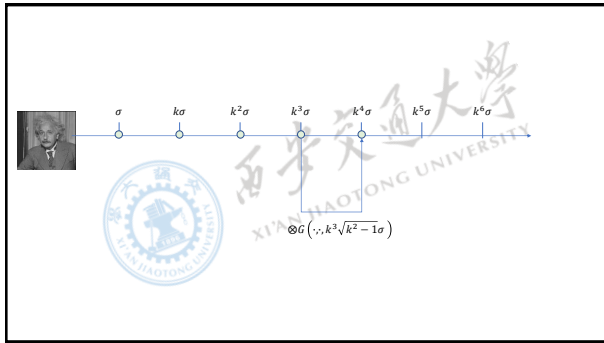
$$I \otimes G(\cdot, \sigma_1) \otimes G(\cdot, \sigma_2) \otimes \dots \otimes G(\cdot, \sigma_n) = I \otimes G\left(\cdot, \sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^2}\right)$$

## 9.7.2 SIFT的高斯金字塔生成

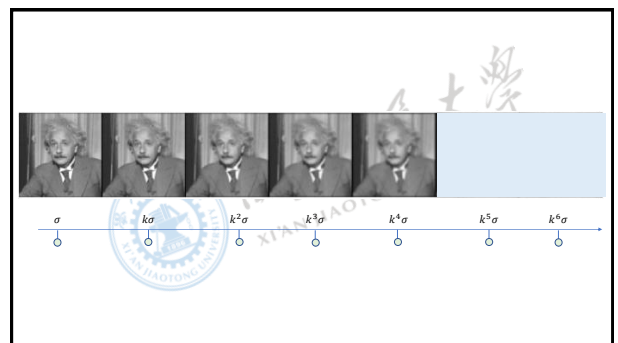
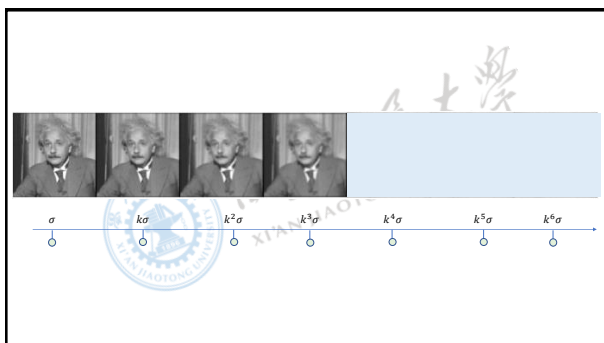
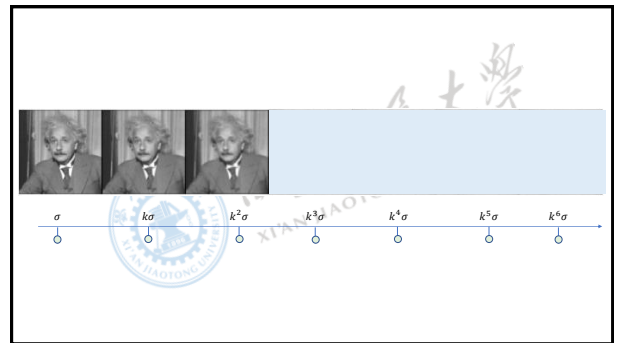
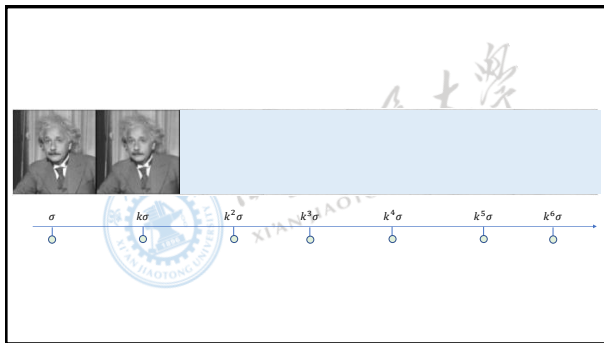


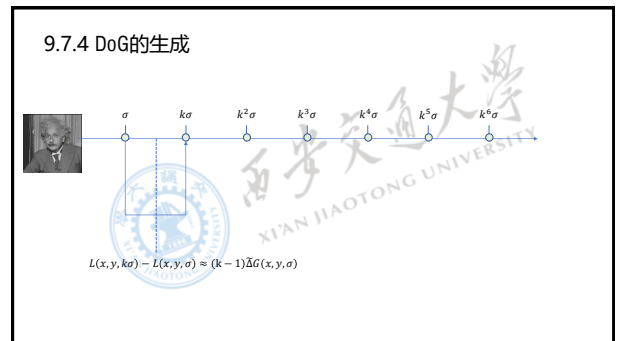
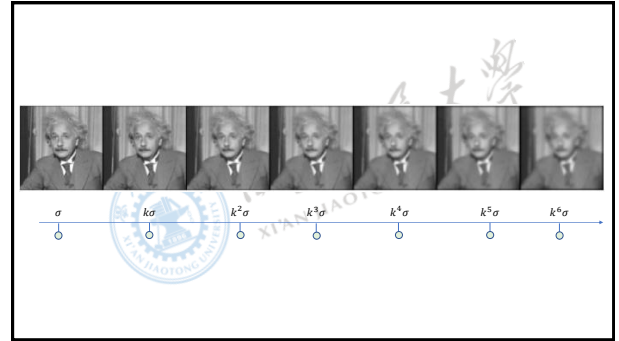
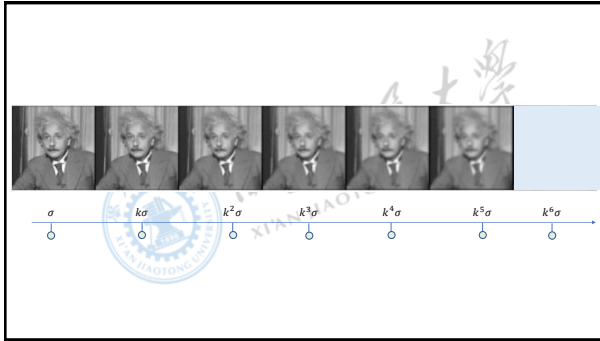
## 9.7.2 SIFT的高斯金字塔生成

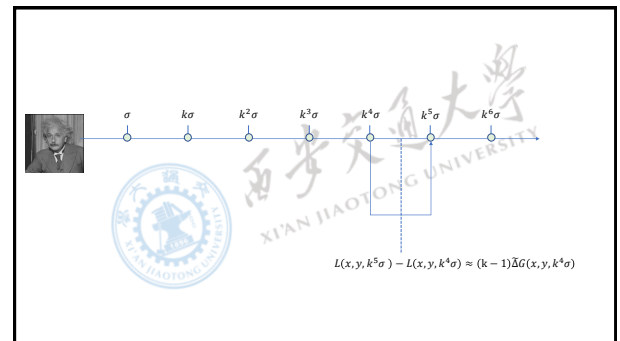
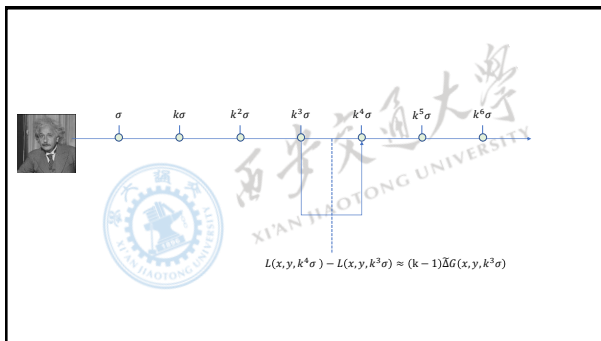
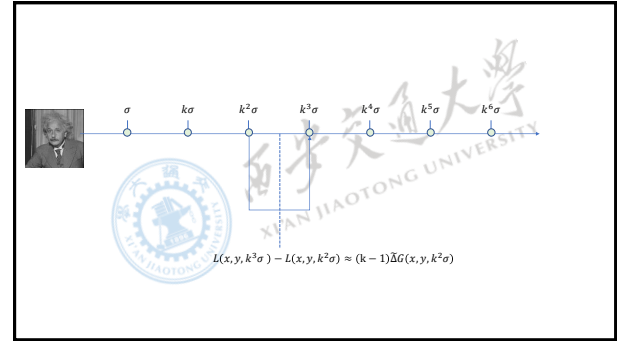
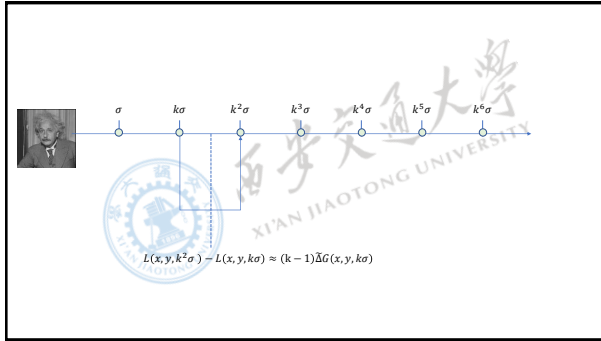


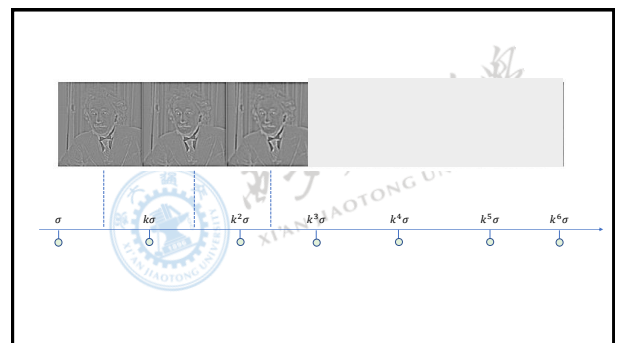
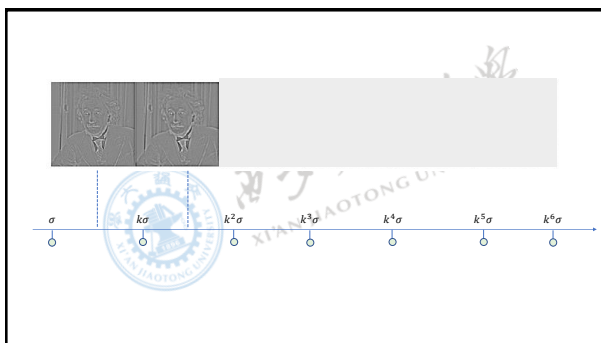
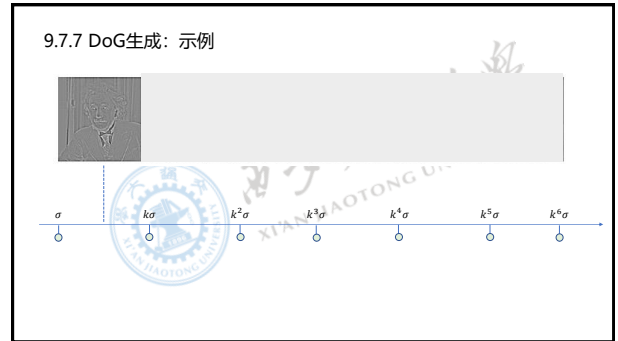
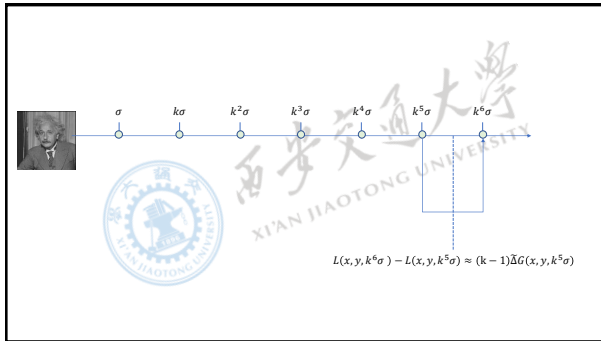


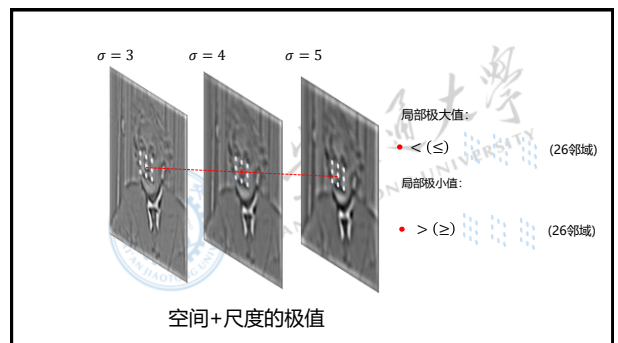
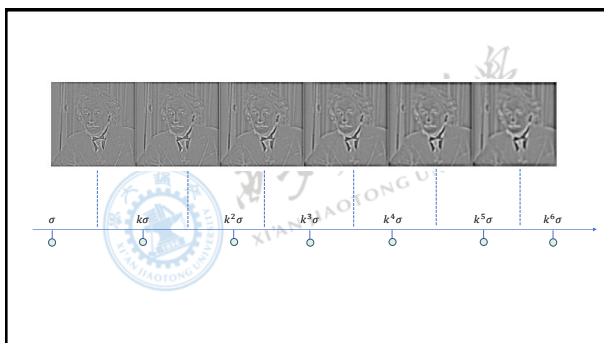
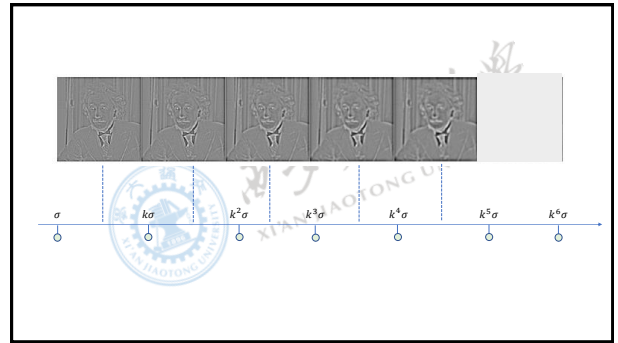
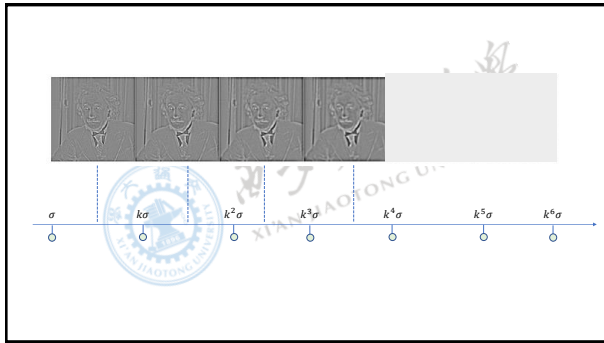












## 9.7.8 SIFT特征点的稳定性



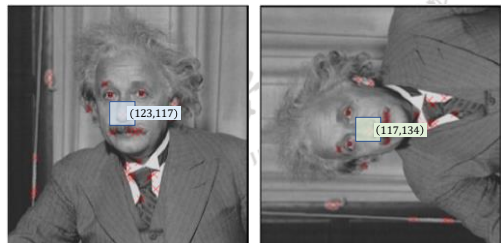
## SIFT特征点的稳定性



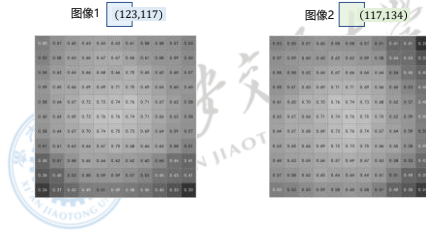
## 八、SIFT特征的方向估计



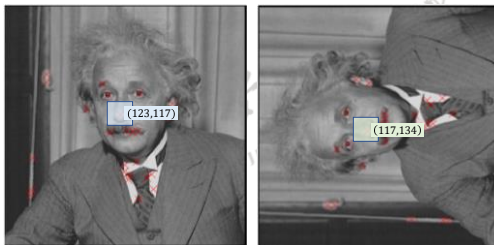
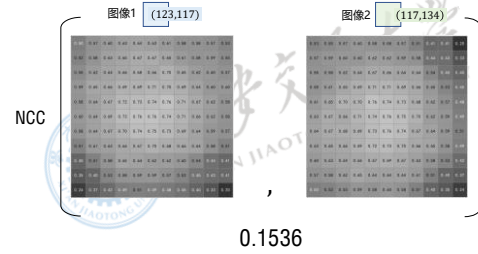
西安交通大学  
XI'AN JIAOTONG UNIVERSITY



## 9.8.1 特征匹配

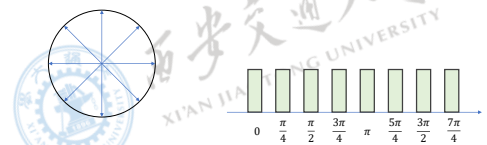


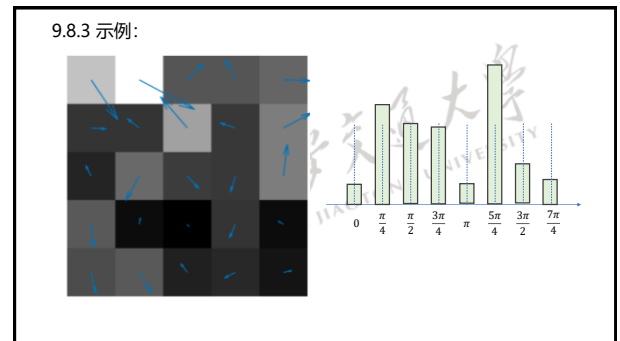
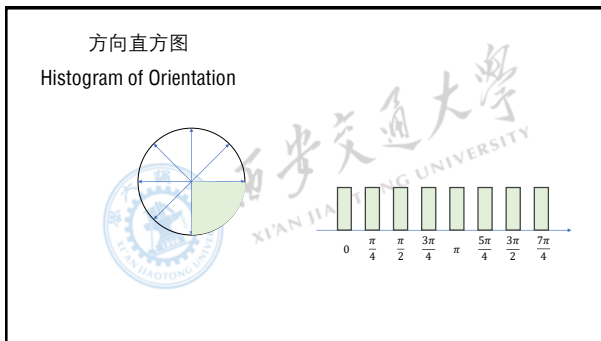
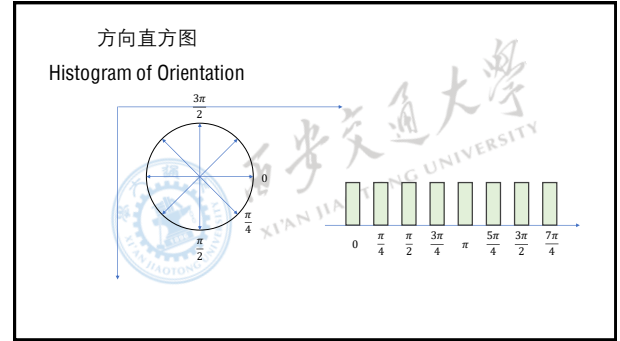
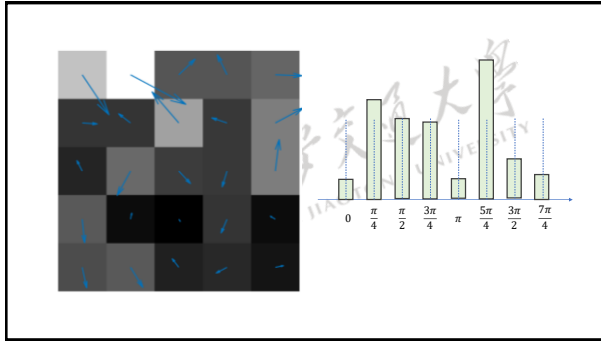
## 特征匹配



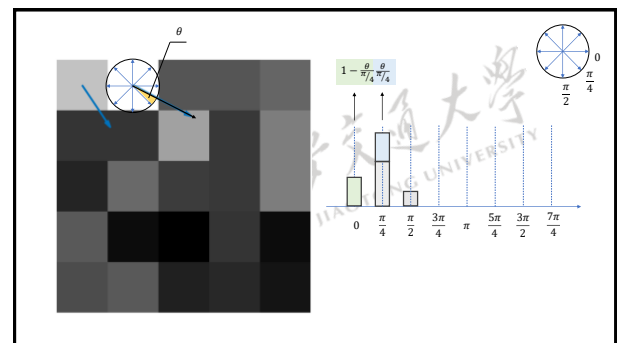
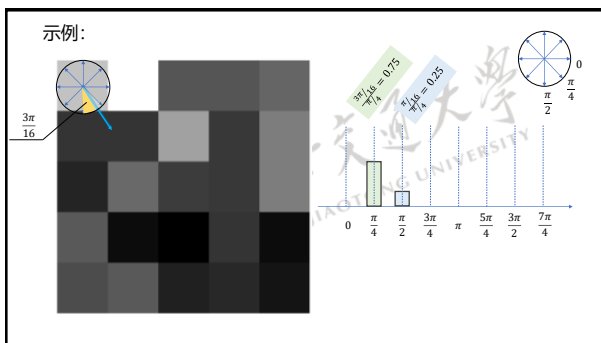
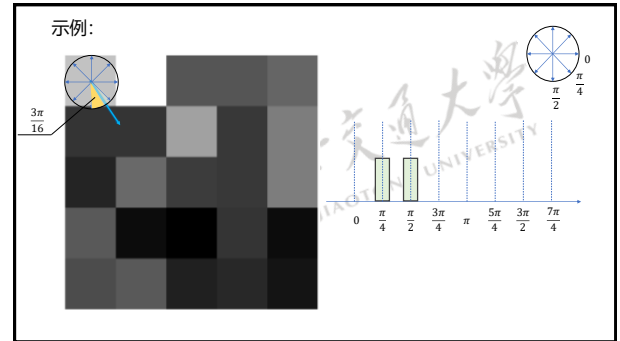
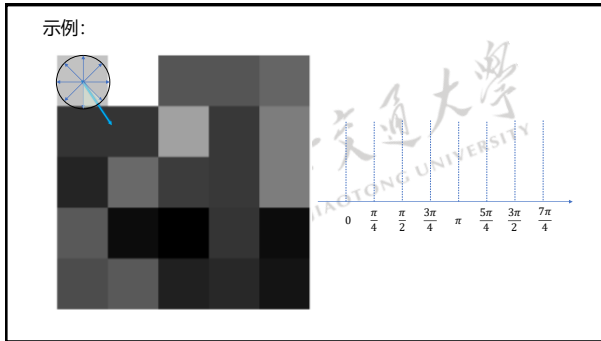
## 9.8.2 方向直方图

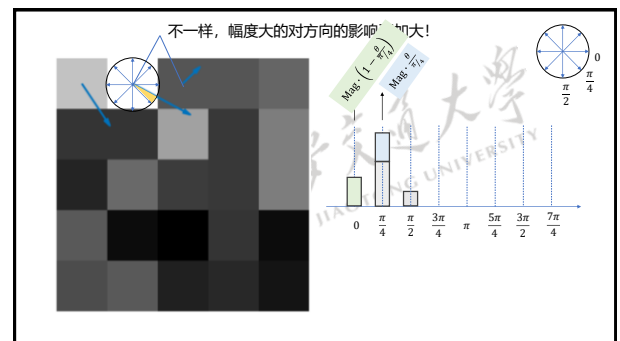
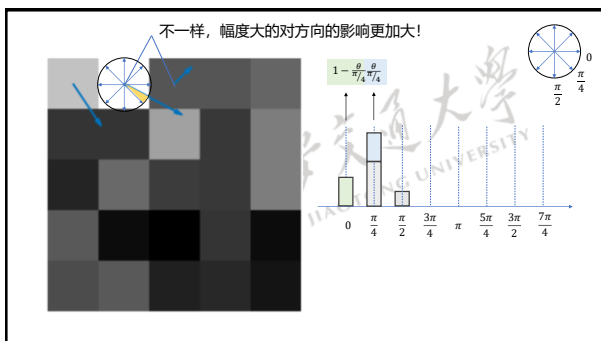
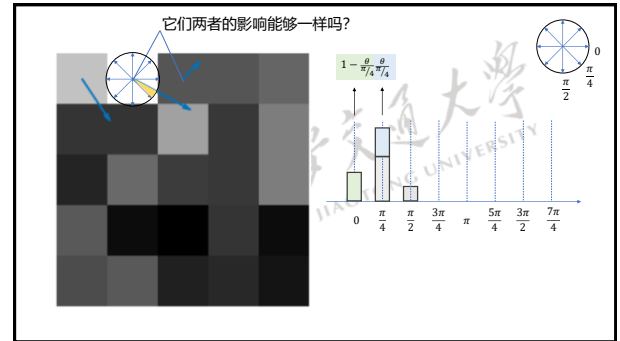
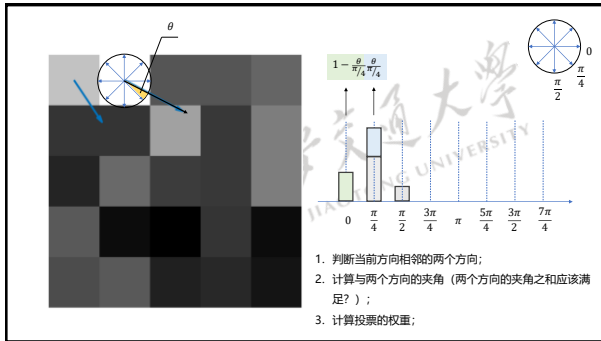
Histogram of Orientation

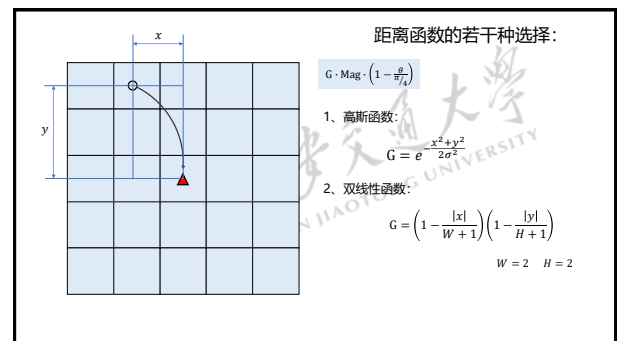
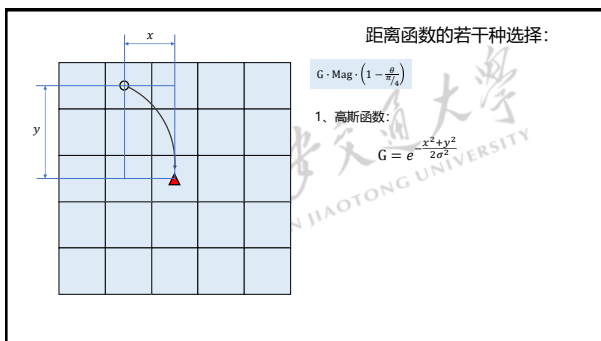
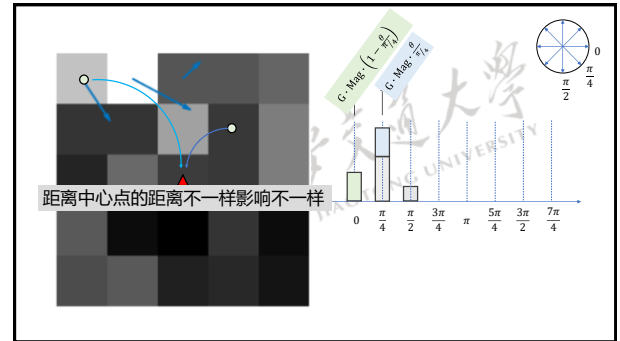
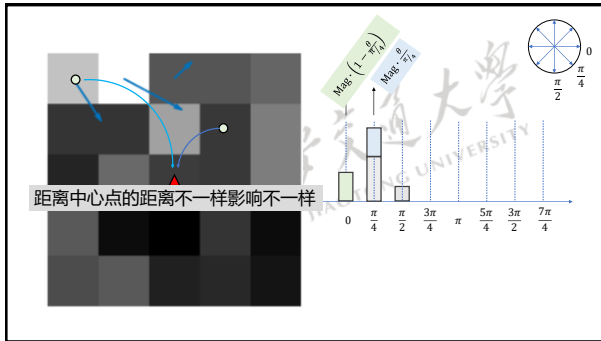












## 距离函数的若干种选择:

$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$
$\frac{2}{9}$	$\frac{4}{9}$	$\frac{6}{9}$	$\frac{4}{9}$	$\frac{2}{9}$
$\frac{3}{9}$	$\frac{6}{9}$	$\frac{9}{9}$	$\frac{6}{9}$	$\frac{3}{9}$
$\frac{2}{9}$	$\frac{4}{9}$	$\frac{6}{9}$	$\frac{4}{9}$	$\frac{2}{9}$
$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

$$G \cdot \text{Mag} \cdot \left(1 - \frac{\theta}{\pi/4}\right)$$

1、高斯函数:

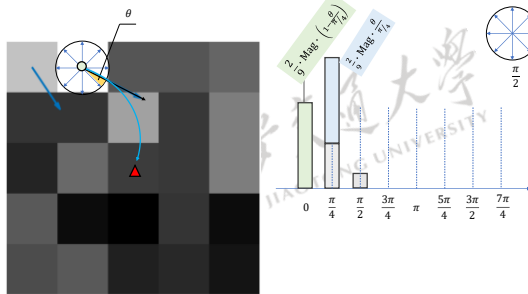
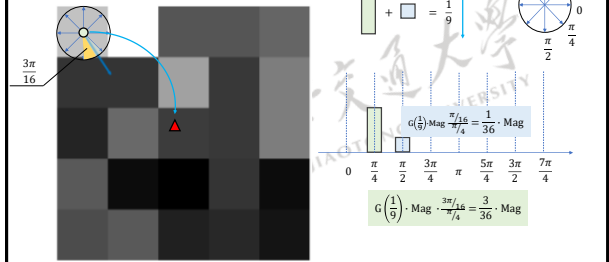
$$G = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

2、双线性函数:

$$G = \left(1 - \frac{|x|}{W+1}\right) \left(1 - \frac{|y|}{H+1}\right)$$

$W = 2 \quad H = 2$

## 示例:



## 距离函数的若干种选择:

$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$
$\frac{2}{9}$	$\frac{4}{9}$	$\frac{6}{9}$	$\frac{4}{9}$	$\frac{2}{9}$
$\frac{3}{9}$	$\frac{6}{9}$	$\frac{9}{9}$	$\frac{6}{9}$	$\frac{3}{9}$
$\frac{2}{9}$	$\frac{4}{9}$	$\frac{6}{9}$	$\frac{4}{9}$	$\frac{2}{9}$
$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

$$G \cdot \text{Mag} \cdot \left(1 - \frac{\theta}{\pi/4}\right)$$

1、高斯函数:

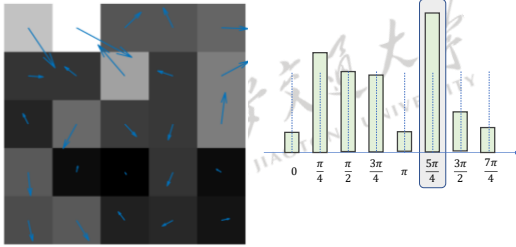
$$G = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

2、双线性函数:

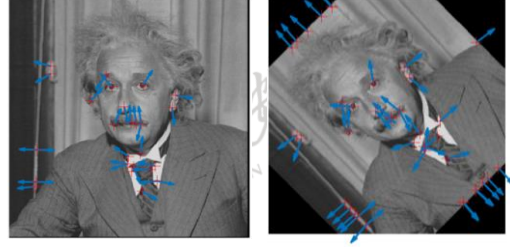
$$G = \left(1 - \frac{|x|}{W+1}\right) \left(1 - \frac{|y|}{H+1}\right)$$

$W = 2 \quad H = 2$

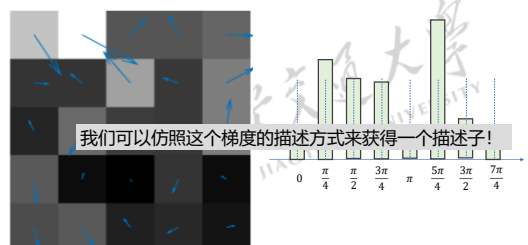
示例:



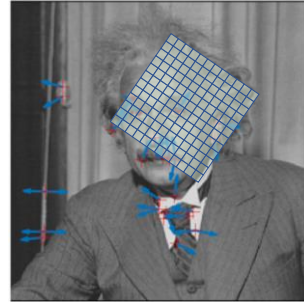
9.8.4 结果



我们可以仿照这个梯度的描述方式来获得一个描述子!

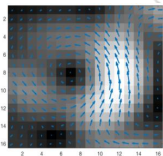
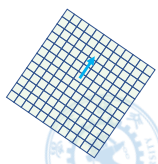


## 九、HOG和SIFT特征的描述子



9.9.1 SIFT特征的区域定义  
依据特征点的方向取一个小的图像块

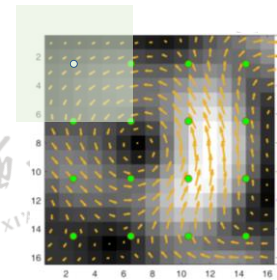
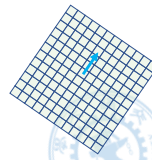
### 9.9.2 梯度直方图的优点



梯度而非原始的灰度

- 可以一定程度上不受光照的影响;
- 梯度包含了物体的局部特征, 细节, 明暗, 阴影等信息

### 9.9.3 局部 vs 整体

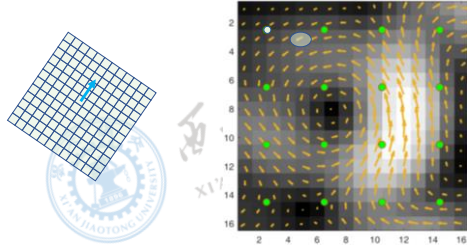


将图像划分成小块

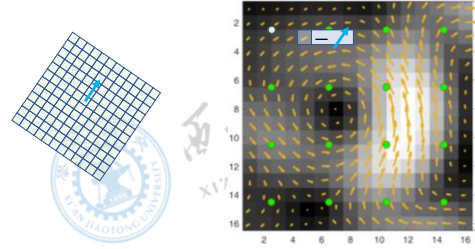
- 物体的局部信息;
- 局部信息的组合合成整体信息

## 9.9.4 梯度方向的补偿

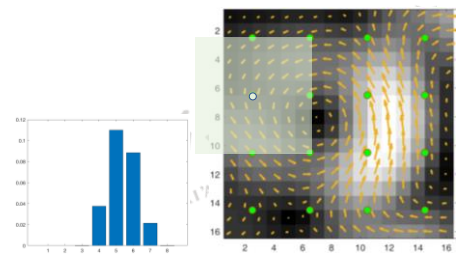
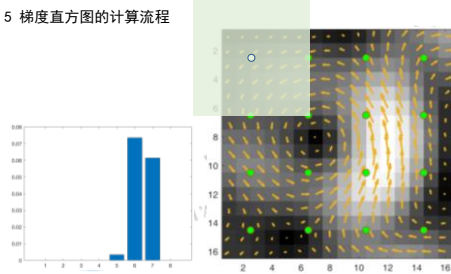
我们是直接使用这个方向吗？

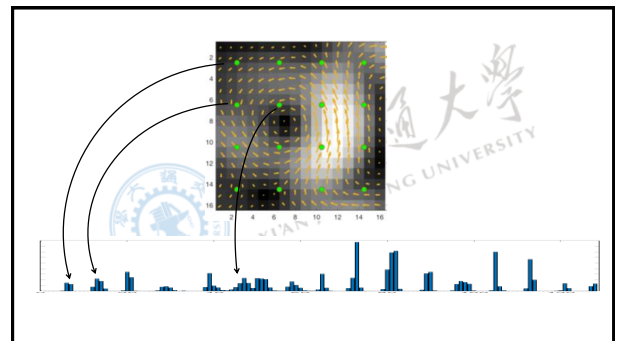
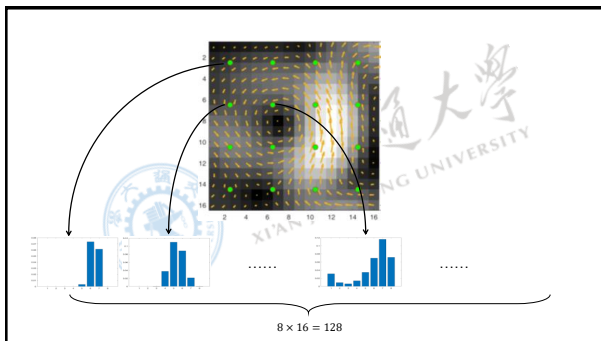
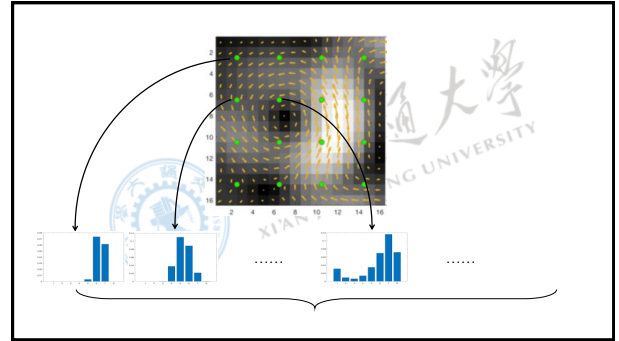
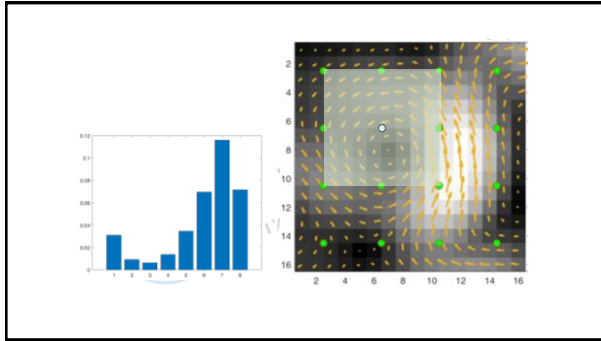


我们是直接使用这个方向吗？



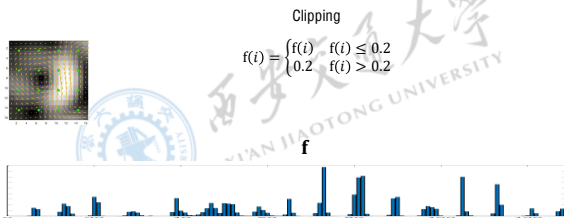
## 9.9.5 梯度直方图的计算流程



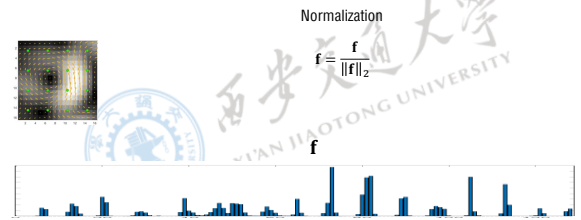




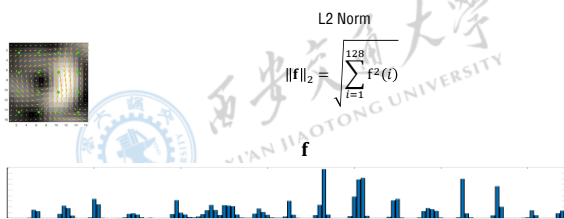
## 9.9.6 SIFT特征的归一化



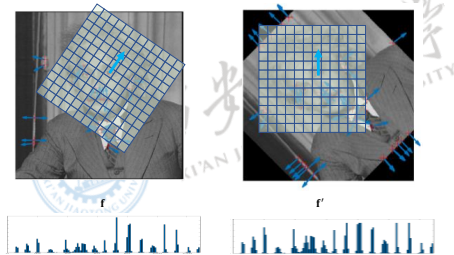
## 9.9.6 SIFT特征的归一化



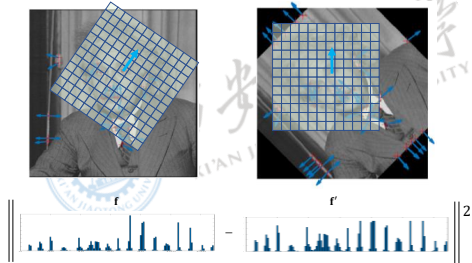
## 9.9.6 SIFT特征的归一化



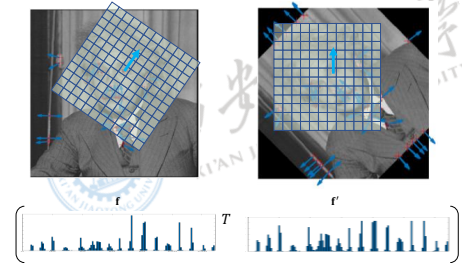
## 9.9.7 SIFT特征的匹配



9.9.7 SIFT特征的匹配: L2 Difference



9.9.7 SIFT特征的匹配: Cosine Distance

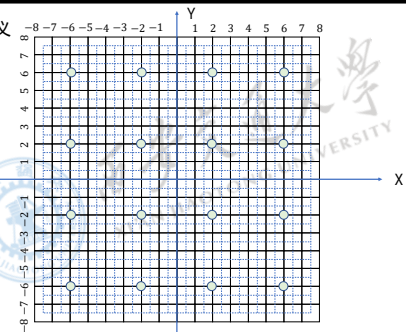


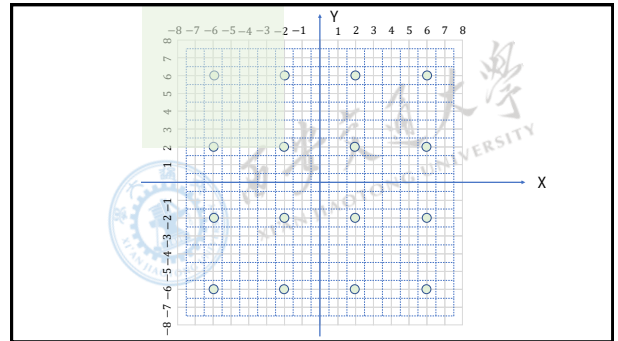
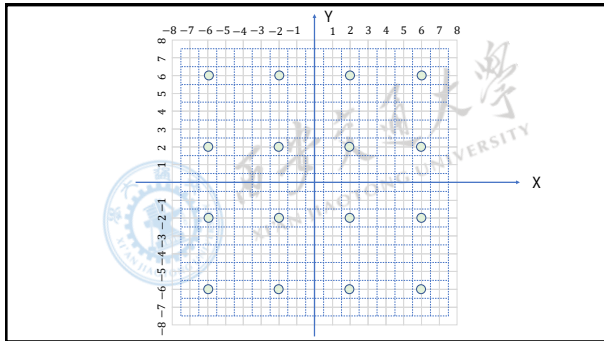
9.9.7 SIFT特征的匹配: 两者的等价性

$$\|f - f'\|^2 = (f - f')^T (f - f') = \|f\|^2 + \|f'\|^2 - 2f^T f'$$

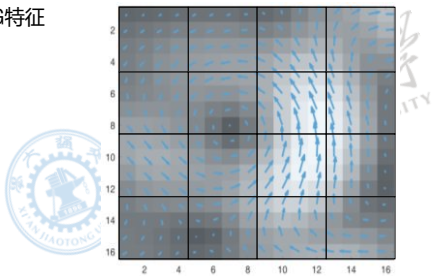
Diagram illustrating the equivalence between L2 Difference and Cosine Distance. The equation shows that the L2 difference can be expressed in terms of the L2 norms and the cosine distance:  $\|f - f'\|^2 = (f - f')^T (f - f') = \|f\|^2 + \|f'\|^2 - 2f^T f'$ .

9.9.8 网格定义

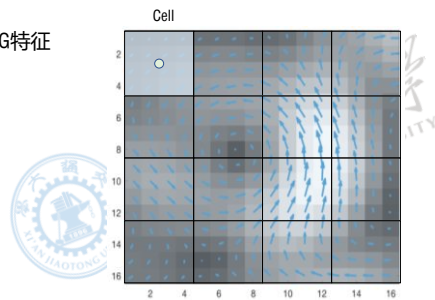




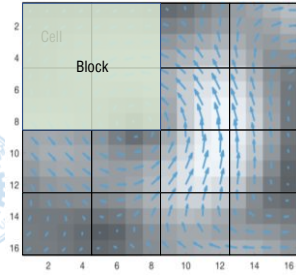
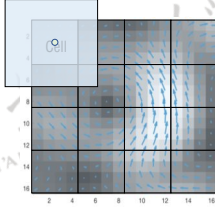
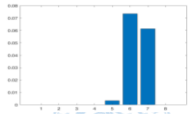
### 9.9.9 HoG特征



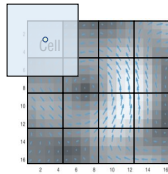
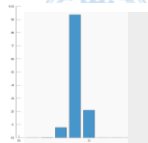
### 9.9.9 HoG特征



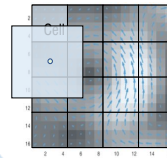
## 9.9.9 HoG特征



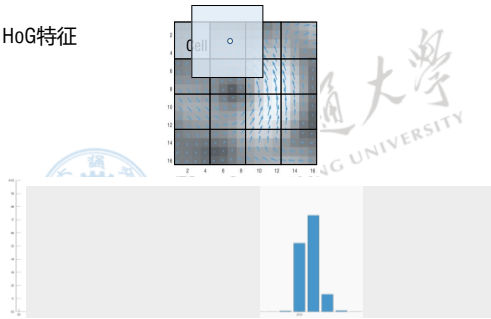
## 9.9.9 HoG特征



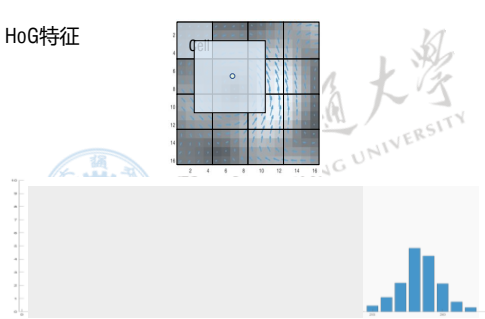
## 9.9.9 HoG特征



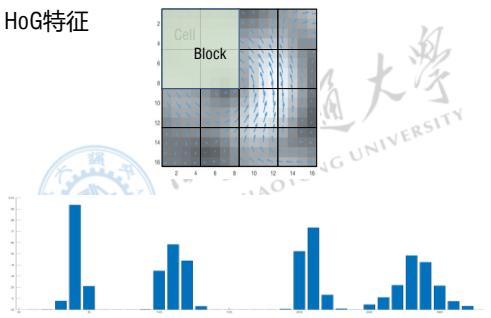
9.9.9 HoG特征



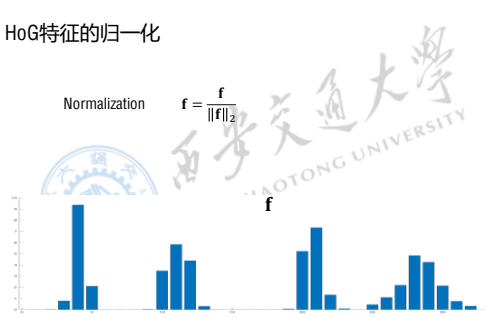
9.9.9 HoG特征



9.9.9 HoG特征



9.9.9 HoG特征的归一化



## 9.9.9 HoG特征的归一化

Normalization

$$\mathbf{f} = \frac{\mathbf{f}}{\|\mathbf{f}\|_1}$$

L1 Norm

$$\|\mathbf{f}\|_1 = \sum_{i=1}^{128} |f(i)|$$

