计算机视觉与模式识别

苏远歧，新型计算机研究所

第九章 角点检测与特征描述-1

最小二乘法与SVD分解是我们在计算机视觉中最常用的方法。

一、几何变换的参数估计：最佳仿射变换的估计
二、最小二乘法与SVD：梯度
三、Harris角点检测：角点的判定依据
四、模板匹配：SAD，MSE，NCC
五、拉普拉斯算子与SIFT特征点

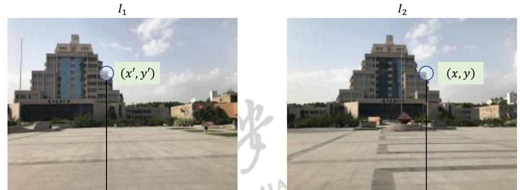
一、几何变换的参数估计

9.1.1 问题：如何估计一个从I_2到I_1的几何变换？

I_1 I_2

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = A_{2\times 2} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

I_1 I_2



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = A_{2\times 2} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

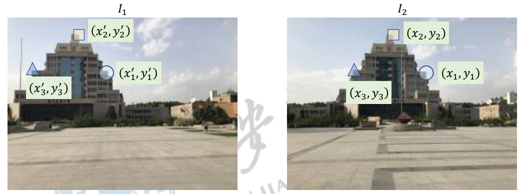
9.1.2 问题：需要多少个对应点才能估计一个从I_2到I_1的仿射变换?

I_1 I_2

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = A_{2\times 2} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

- 仿射变换的未知数：6个； 每个点提供2个方程式
- 最少需要$^6/_2 = 3$个点

I_1 I_2



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = A_{2\times 2} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \Rightarrow \begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

9.1.3 方程组的求解

$$\begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \end{bmatrix}$$

9.1.3 方程组的求解

$$\begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \end{bmatrix}$$

9.1.3 方程组的求解

$$\begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \end{bmatrix}$$

9.1.3 方程组的求解

$$\begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$A \quad x = b$$

二、最小二乘法与SVD

9.2.1 方程组的求解：最小二乘法

$$\min_{\mathbf{x}} L = \left\| \mathrm{A}\,\mathrm{x} - \mathrm{b} \right\|^2$$

9.2.1 方程组的求解：最小二乘法

$$\left[\mathrm{A}\,\mathrm{x} - \mathrm{b}\right]^{\mathrm{T}}\left[\mathrm{A}\,\mathrm{x} - \mathrm{b}\right]$$

$$L = \mathbf{x}^T\mathbf{A}^T\mathbf{A}\mathbf{x} - \mathbf{b}^T\mathbf{A}\mathbf{x} - \mathbf{x}^T\mathbf{A}^T\mathbf{b} + \mathbf{b}^T\mathbf{b}$$

9.2.1 方程组的求解：最小二乘法

$$L = \mathbf{x}^T\mathbf{A}^T\mathbf{A}\mathbf{x} - \mathbf{b}^T\mathbf{A}\mathbf{x} - \mathbf{x}^T\mathbf{A}^T\mathbf{b} + \mathbf{b}^T\mathbf{b}$$

$$\frac{\partial L}{\partial \mathbf{x}} = \mathbf{A}^T\mathbf{A}\mathbf{x} + (\mathbf{x}^T\mathbf{A}^T\mathbf{A})^T - (\mathbf{b}^T\mathbf{A})^T - \mathbf{A}^T\mathbf{b}$$

$$\frac{\partial L}{\partial \mathbf{x}} = 2\mathbf{A}^T\mathbf{A}\mathbf{x} - 2\mathbf{A}^T\mathbf{b} = \mathbf{0}$$

9.2.1 方程组的求解：最小二乘法

$\mathbf{x} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b}$

$\mathbf{x} = \mathbf{A}^{+}\mathbf{b}$ 伪逆

$$\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix}}_{\mathrm{A}} \underbrace{\begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix}}_{\mathrm{x}} = \underbrace{\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}}_{\mathrm{b}}$$

9.2.1 方程组的求解：最小二乘法

$\mathbf{x} = \mathbf{A}^{+}\mathbf{b}$ 伪逆

Matlab：

```
>> x = pinv(A) * b
>> x = A\b
```

$$\mathrm{A}\,\mathrm{x} = \mathrm{b}$$

9.2.2 方程组的求解：SVD分解

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

U：正交矩阵　S：对角矩阵　V：正交矩阵

Matlab：

```
>> [U, S, V] = svd(A)
```

9.2.2 方程组的求解：SVD分解

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

U：正交矩阵　S：对角矩阵　V：正交矩阵

Matlab：

```
>> [U, S, V] = svd(A)
```

9.2.2 方程组的求解：SVD分解

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\mathbf{A} = \mathbf{USV}^T$$

Matlab:

```
>> [U, S, V] = svd(A)
```

正交矩阵　　对角矩阵　　正交矩阵

```
U =

   -0.1525   -0.8226   -0.3945   -0.3800
   -0.3499   -0.4214    0.2428    0.8007
   -0.5474   -0.0201    0.6979   -0.4514
   -0.7448    0.3812   -0.5462    0.0407

S =

   14.2691         0
         0    0.6268
         0         0
         0         0

V =

   -0.6414    0.7672
   -0.7672   -0.6414
```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$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\mathbf{A} = \mathbf{USV}^T$$

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   14.2691         0
         0    0.6268
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V =

   -0.6414    0.7672
   -0.7672   -0.6414
```


9.2.2 方程组的求解：SVD分解

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\mathbf{A}^T = \mathbf{VS}^T\mathbf{U}^T$$

Matlab:

```
>> [U, S, V] = svd(A)
```

正交矩阵　　对角矩阵　　正交矩阵

```
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   -0.1525   -0.8226   -0.3945   -0.3800
   -0.3499   -0.4214    0.2428    0.8007
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   14.2691         0
         0    0.6268
         0         0
         0         0

V =

   -0.6414    0.7672
   -0.7672   -0.6414
```

9.2.3 方程组的求解：最小二乘法

$$2\left(\mathbf{VS}^T\mathbf{U}^T\mathbf{USV}^T\mathbf{x} - \mathbf{VS}^T\mathbf{U}^T\mathbf{b}\right) = \mathbf{0}$$

$$\mathbf{A} = \mathbf{USV}^T$$

$$\frac{\partial L}{\partial \mathbf{x}} = 2\mathbf{A}^T\mathbf{A}\mathbf{x} - 2\mathbf{A}^T\mathbf{b} = \mathbf{0}$$

9.2.3 方程组的求解：最小二乘法

$$\frac{\partial L}{\partial \mathbf{x}} = 2\mathbf{A}^T\mathbf{A}\mathbf{x} - 2\mathbf{A}^T\mathbf{b} = \mathbf{0}$$

$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^T$

$$2\left(\mathbf{V}\mathbf{S}^T\mathbf{U}^T\mathbf{U}\mathbf{S}\mathbf{V}^T\mathbf{x} - \mathbf{V}\mathbf{S}^T\mathbf{U}^T\mathbf{b}\right) = \mathbf{0}$$

U是正交矩阵

$$\mathbf{V}\mathbf{S}^T\mathbf{S}\mathbf{V}^T\mathbf{x} = \mathbf{V}\mathbf{S}^T\mathbf{U}^T\mathbf{b}$$

9.2.3 方程组的求解：最小二乘法

$$\frac{\partial L}{\partial \mathbf{x}} = 2\mathbf{A}^T\mathbf{A}\mathbf{x} - 2\mathbf{A}^T\mathbf{b} = \mathbf{0}$$

$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^T$

$$2\left(\mathbf{V}\mathbf{S}^T\mathbf{U}^T\mathbf{U}\mathbf{S}\mathbf{V}^T\mathbf{x} - \mathbf{V}\mathbf{S}^T\mathbf{U}^T\mathbf{b}\right) = \mathbf{0}$$

$$\mathbf{V}\mathbf{S}^T\mathbf{S}\mathbf{V}^T\mathbf{x} = \mathbf{V}\mathbf{S}^T\mathbf{U}^T\mathbf{b} \xrightarrow{\text{V是正交矩阵}} \boxed{\mathbf{S}^T\mathbf{S}\mathbf{V}^T\mathbf{x} = \mathbf{S}^T\mathbf{U}^T\mathbf{b}}$$

9.2.4 特例1：如果A是满秩方阵

$$\mathbf{V}\mathbf{S}^T\mathbf{S}\mathbf{V}^T\mathbf{x} = \mathbf{V}\mathbf{S}^T\mathbf{U}^T\mathbf{b} \xrightarrow{V\text{是正交矩阵}} \boxed{\mathbf{S}^T\mathbf{S}\mathbf{V}^T\mathbf{x} = \mathbf{S}^T\mathbf{U}^T\mathbf{b}}$$

S对角矩阵，非奇异

$$\mathbf{x} = \mathbf{V}\mathbf{S}^{-1}\mathbf{U}^T\mathbf{b}$$

9.2.4 特例1：如果A是满秩方阵

$$\mathbf{V}\mathbf{S}^T\mathbf{S}\mathbf{V}^T\mathbf{x} = \mathbf{V}\mathbf{S}^T\mathbf{U}^T\mathbf{b} \xrightarrow{V\text{是正交矩阵}} \boxed{\mathbf{S}^T\mathbf{S}\mathbf{V}^T\mathbf{x} = \mathbf{S}^T\mathbf{U}^T\mathbf{b}}$$

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$$\mathbf{VS}^T\mathbf{SV}^T\mathbf{x} = \mathbf{VS}^T\mathbf{U}^T\mathbf{b} \xrightarrow{V\text{是正交矩阵}} \boxed{\mathbf{S}^T\mathbf{SV}^T\mathbf{x} = \mathbf{S}^T\mathbf{U}^T\mathbf{b}}$$

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仿射变换

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$$\mathbf{VS}^T\mathbf{SV}^T\mathbf{x} = \mathbf{VS}^T\mathbf{U}^T\mathbf{b} \xrightarrow{V\text{是正交矩阵}} \boxed{\mathbf{S}^T\mathbf{SV}^T\mathbf{x} = \mathbf{S}^T\mathbf{U}^T\mathbf{b}}$$

S对角矩阵，非奇异

$$\mathbf{x} = \mathbf{VS}^{-1}\mathbf{U}^T\mathbf{b}$$

仿射变换

9.2.5 方程组的求解

$$\begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \end{bmatrix}$$

$(x_1 = 1223, y_1 = 263)$

9.2.5 方程组的求解

$$\begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \end{bmatrix}$$

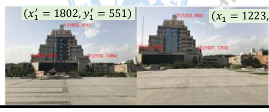
$(x_1 = 1223, y_1 = 263)$

9.2.5 方程组的求解

$$\begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} 1802 \\ 551 \\ x_2' \\ y_2' \\ x_3' \\ y_3' \end{bmatrix}$$

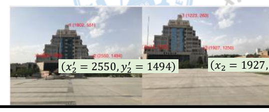
$(x_1' = 1802, y_1' = 551)$ $(x_1 = 1223, y_1 = 263)$



9.2.5 方程组的求解

$$\begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ 1927 & 1250 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1927 & 1250 & 1 \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} 1802 \\ 551 \\ 2550 \\ 1494 \\ x_3' \\ y_3' \end{bmatrix}$$

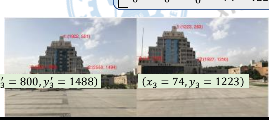


$(x_2' = 2550, y_2' = 1494)$ $(x_2 = 1927, y_2 = 1250)$

9.2.5 方程组的求解

$$\begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ 1927 & 1250 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1927 & 1250 & 1 \\ 74 & 1223 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 74 & 1223 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} 1802 \\ 551 \\ 2550 \\ 1494 \\ 800 \\ 1488 \end{bmatrix}$$



$(x_3' = 800, y_3' = 1488)$ $(x_3 = 74, y_3 = 1223)$

9.2.5 方程组的求解

$$\begin{cases} x_i' = a_{11}x_i + a_{12}y_i + b_1 \\ y_i' = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\mathrm{SVD}\left(\underset{A}{\begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ 1927 & 1250 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1927 & 1250 & 1 \\ 74 & 1223 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 74 & 1223 & 1 \end{bmatrix}} \right)$$

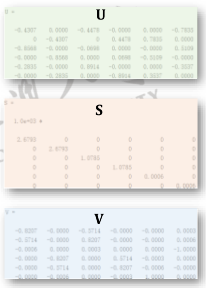
Matlab:

```
>> [U, S, V] = svd(A)
```

9.2.5 方程组的求解

$\mathbf{A} = \mathbf{USV}^T$

$$\text{SVD}\left(\mathbf{A} = \begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ 1927 & 1250 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1927 & 1250 & 1 \\ 74 & 1223 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 74 & 1223 & 1 \end{bmatrix}\right)$$



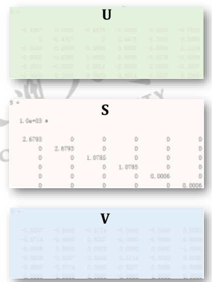
Matlab:

>> [U, S, V] = svd(A)

9.2.5 聚焦对角阵S

$\mathbf{A} = \mathbf{USV}^T$

$$\text{SVD}\left(\mathbf{A} = \begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ 1927 & 1250 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1927 & 1250 & 1 \\ 74 & 1223 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 74 & 1223 & 1 \end{bmatrix}\right)$$



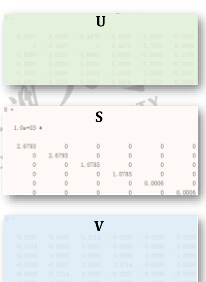
Matlab:

>> [U, S, V] = svd(A)

9.2.5 求解未知数

$\mathbf{x} = \mathbf{VS}^{-1}\mathbf{U}^T\mathbf{b}$

$$\mathbf{x} = \begin{bmatrix} 0.9432 \\ 0.0851 \\ 626.1134 \\ -0.0108 \\ 0.9631 \\ 310.9023 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix}$$



Matlab:

>> [U, S, V] = svd(A)

>> x = V * S^(-1) * U' * b

9.2.5 从未知数到仿射变换矩阵

$$\mathbf{x} = \begin{bmatrix} 0.9432 \\ 0.0851 \\ 626.1134 \\ -0.0108 \\ 0.9631 \\ 310.9023 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0.9432 & 0.0851 \\ -0.0108 & 0.9631 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 626.1134 \\ 310.9023 \end{bmatrix}$$

Matlab:

>> [U, S, V] = svd(A)

>> x = V * S^(-1) * U' * b

I_1

I_2

从未

134]
023]

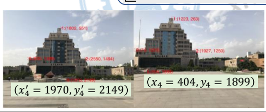
Matlab:

```
>> [U, S, V
>> x = V *
```

如果匹配点的个数大于3呢?

9.2.6 一般情况：

$$
\begin{bmatrix}
1223 & 263 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1223 & 263 & 1 \\
1927 & 1250 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1927 & 1250 & 1 \\
74 & 1223 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 74 & 1223 & 1 \\
404 & 1899 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 404 & 1899 & 1
\end{bmatrix}
\begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix}
=
\begin{bmatrix} 1802 \\ 551 \\ 2550 \\ 1494 \\ 800 \\ 1488 \\ 1970 \\ 2149 \end{bmatrix}
$$



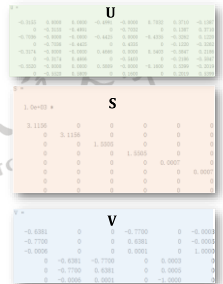
$$
\begin{cases}
x'_i = a_{11}x_i + a_{12}y_i + b_1 \\
y'_i = a_{21}x_i + a_{22}y_i + b_2
\end{cases}
$$

9.2.6 方程组的求解

$\mathbf{A} = \mathbf{USV}^T$

$$
\text{SVD}\left(
\begin{bmatrix}
1223 & 263 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1223 & 263 & 1 \\
1927 & 1250 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1927 & 1250 & 1 \\
74 & 1223 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 74 & 1223 & 1 \\
404 & 1899 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 404 & 1899 & 1
\end{bmatrix}
\right)
$$

A



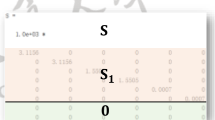
Matlab：

>> [U, S, V] = svd(A)

9.2.6 方程组的求解

$\mathbf{A} = \mathbf{USV}^T$

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{0} \end{bmatrix}$$

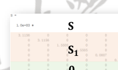


Matlab：

>> [U, S, V] = svd(A)

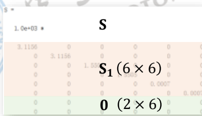
9.2.6 一般情况：

$\mathbf{VS}^T\mathbf{SV}^T\mathbf{x} = \mathbf{VS}^T\mathbf{U}^T\mathbf{b} \xrightarrow{V\text{是正交矩阵}} \boxed{\mathbf{S}^T\mathbf{SV}^T\mathbf{x} = \mathbf{S}^T\mathbf{U}^T\mathbf{b}}$



$$\begin{bmatrix} \mathbf{S}_1 \\ \mathbf{0} \end{bmatrix}^T \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{0} \end{bmatrix} \mathbf{V}^T\mathbf{x} = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{0} \end{bmatrix}^T \mathbf{U}^T\mathbf{b}$$

$$\mathbf{S_1}$$
$$\begin{bmatrix}\mathbf{S_1}\\ \mathbf{0}\end{bmatrix}^T = [\underset{\overset{\|}{\mathbf{S_1}}}{\mathbf{S}_1^T} \quad \mathbf{0}^T]$$



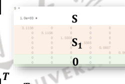
9.2.6 一般情况：

$$\mathbf{V}\mathbf{S}^T\mathbf{S}\mathbf{V}^T\mathbf{x} = \mathbf{V}\mathbf{S}^T\mathbf{U}^T\mathbf{b} \xrightarrow{V是正交矩阵} \boxed{\mathbf{S}^T\mathbf{S}\mathbf{V}^T\mathbf{x} = \mathbf{S}^T\mathbf{U}^T\mathbf{b}}$$

$$\begin{bmatrix}\mathbf{S_1}\\ \mathbf{0}\end{bmatrix}^T \begin{bmatrix}\mathbf{S_1}\\ \mathbf{0}\end{bmatrix}\mathbf{V}^T\mathbf{x} = \begin{bmatrix}\mathbf{S_1}\\ \mathbf{0}\end{bmatrix}^T\mathbf{U}^T\mathbf{b}$$

$$\begin{bmatrix}\mathbf{S_1}\\ \mathbf{0}\end{bmatrix}^T = [\mathbf{S}_1^T \quad \mathbf{0}^T]$$

$$\mathbf{S}_1^T\mathbf{S}_1\mathbf{V}^T\mathbf{x} = \begin{bmatrix}\mathbf{S_1}\\ \mathbf{0}\end{bmatrix}^T\mathbf{U}^T\mathbf{b}$$



9.2.6 一般情况：

$$\mathbf{S}_1^T\mathbf{S}_1\mathbf{V}^T\mathbf{x} = \begin{bmatrix}\mathbf{S_1}\\ \mathbf{0}\end{bmatrix}^T\mathbf{U}^T\mathbf{b}$$

$$\mathbf{V}^T\mathbf{x} = \mathbf{S}_1^{-2}\begin{bmatrix}\mathbf{S_1}\\ \mathbf{0}\end{bmatrix}^T\mathbf{U}^T\mathbf{b}$$

$$\mathbf{V}^T\mathbf{x} = \mathbf{S}_1^{-2}[\mathbf{S}_1^T \quad \mathbf{0}^T]\mathbf{U}^T\mathbf{b}$$

$$\mathbf{x} = \mathbf{V}[\mathbf{S}_1^{-1} \quad \mathbf{0}^T]\mathbf{U}^T\mathbf{b}$$

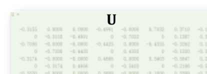
9.2.6 求解未知数

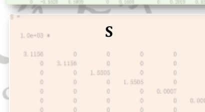
$$\mathbf{x} = \mathbf{V}[\mathbf{S}_1^{-1} \quad \mathbf{0}^T]\mathbf{U}^T\mathbf{b}$$

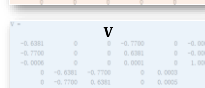
$$\mathbf{x} = \begin{bmatrix}0.8842\\0.4024\\470.2381\\0.1368\\0.1691\\700.9511\end{bmatrix} \quad \begin{bmatrix}a_{11}\\a_{12}\\b_1\\a_{21}\\a_{22}\\b_2\end{bmatrix}$$

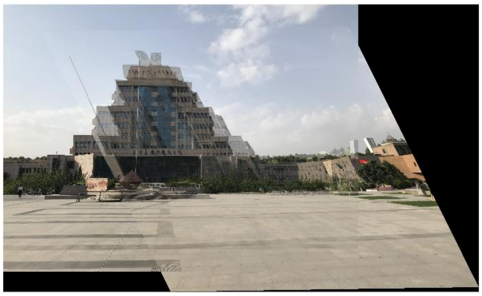
Matlab：

```
>> [U, S, V] = svd(A)
>> S1 = S(1:6,1:6)
>> x = V * [S1^(-1) zeros(6,2)] * U' * b
```





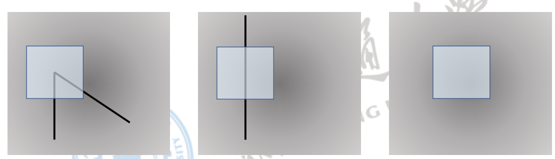




如何让匹配的过程自动化呢?

三、Harris角点检测

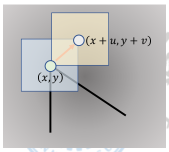
9.3.1 Harris角点检测的基本原理



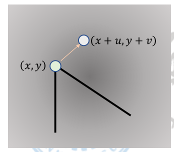
利用当前矩形框与周边矩形框的差异来度量当前点的独特性。

- 差异越大，越具有独特性
- 差异越小，越不具备独特性

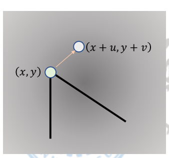
9.3.2 差异度量



9.3.2 差异度量

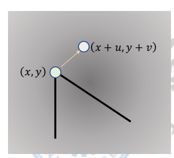


9.3.2 差异度量



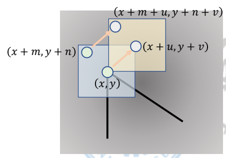
$\mathrm{I}(x+u, y+v) - \mathrm{I}(x, y)$

9.3.2 差异度量



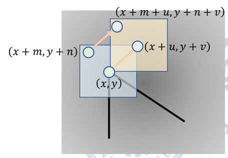
$[\mathrm{I}(x+u, y+v) - \mathrm{I}(x, y)]^2$

9.3.2 差异度量



$$[\mathrm{I}(x+u+m, y+v+n) - \mathrm{I}(x+m, y+n)]^2$$

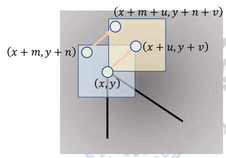
9.3.2 差异度量



$$w(m,n)[\mathrm{I}(x+u+m, y+v+n) - \mathrm{I}(x+m, y+n)]^2$$

距离当前点越近，差异的影响越大；否则越小。

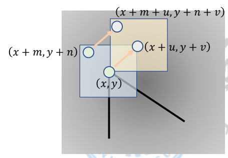
9.3.2 差异度量



$$\sum_{(m,n)} w(m,n)[\mathrm{I}(x+u+m, y+v+n) - \mathrm{I}(x+m, y+n)]^2$$

距离当前点越近，差异的影响越大；否则越小。

9.3.3 一阶Taylor展开

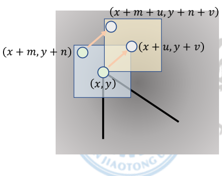


$$\sum_{(m,n)} w(m,n)[\mathrm{I}(x+u+m, y+v+n) - \mathrm{I}(x+m, y+n)]^2$$

$$\uparrow$$

$$\mathrm{I}(x+u+m, y+v+n) = \mathrm{I}(x+m, y+n) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial x}v$$

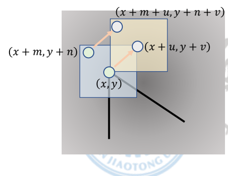
9.3.3 一阶Taylor展开



$$\sum_{(m,n)} w(m,n)\left[\frac{\partial I(x+m,y+n)}{\partial x}u+\frac{\partial I(x+m,y+n)}{\partial x}v\right]^2$$

$$\mathrm{I}(x+u+m, y+v+n)=\mathrm{I}(x+m,y+n)+\frac{\partial I}{\partial x}u+\frac{\partial I}{\partial x}v$$

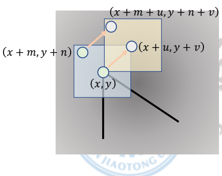
9.3.3 一阶Taylor展开



$$I_x=\frac{\partial I}{\partial x} \qquad I_y=\frac{\partial I}{\partial y}$$

$$\sum_{(m,n)} w(m,n)\left[\frac{\partial I(x+m,y+n)}{\partial x}u+\frac{\partial I(x+m,y+n)}{\partial x}v\right]^2$$

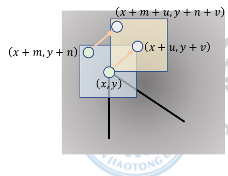
9.3.3 一阶Taylor展开



$$I_x=\frac{\partial I}{\partial x} \qquad I_y=\frac{\partial I}{\partial y}$$

$$\sum_{(m,n)} w(m,n)[I_x(x+m,y+n)u+I_y(x+m,y+n)v]^2$$

9.3.3 一阶Taylor展开



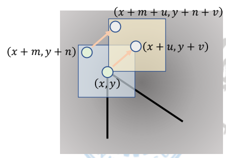
$$I_x=\frac{\partial I}{\partial x} \qquad I_y=\frac{\partial I}{\partial y}$$

$$\sum_{(m,n)} w(m,n)[I_x(x+m,y+n)u+I_y(x+m,y+n)v]^2$$

$$\Downarrow$$

$$[u \quad v]\begin{bmatrix} I_x(x+m,y+n) \\ I_y(x+m,y+n) \end{bmatrix}[I_x(x+m,y+n) \quad I_y(x+m,y+n)]\begin{bmatrix} u \\ v \end{bmatrix}$$

9.3.3 一阶Taylor展开



$$I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y}$$

$$\sum_{(m,n)} w(m,n)\left[I_x(x+m,y+n)u + I_y(x+m,y+n)v\right]^2$$

$$\Downarrow$$

$$[u \quad v]\begin{bmatrix} I_x^2(x+m,y+n) & \\ & I_y^2(x+m,y+n) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

$$I_x(x+m,y+n)I_y(x+m,y+n)$$

9.3.4 矩阵$H(x,y)$

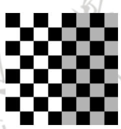
$$\begin{bmatrix} \sum_{(m,n)} w(m,n) I_x^2(x+m,y+n) & \sum_{(m,n)} w(m,n) I_x(x+m,y+n) I_y(x+m,y+n) \\ \sum_{(m,n)} w(m,n) I_x(x+m,y+n) I_y(x+m,y+n) & \sum_{(m,n)} w(m,n) I_y^2(x+m,y+n) \end{bmatrix}$$

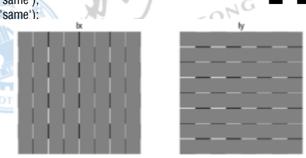
9.3.5 如何计算

```
img = checkerboard(50,4,4);
img = double (img);

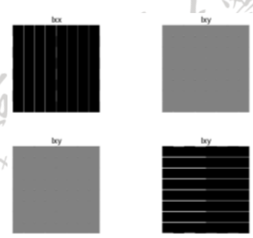
%Filter for horizontal and vertical direction
dx = [1 0 -1];
dy = [1; 0; -1];

% Convolution of image with dx and dy
Ix = conv2(img, dx, 'same');
Iy = conv2(img, dy, 'same');
```



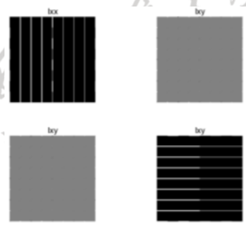



```
% Hessian Matrix Ixx, Iyy, Ixy
Ixx = Ix .* Ix;
Iyy = Iy .* Iy;
Ixy = Ix .* Iy;
```



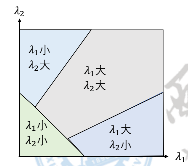
```
%Gaussian filter definition (https://en.wikipedia.org/wiki/Canny_edge_detector)
G = [2, 4, 5, 4, 2; 4, 9, 12, 9, 4;5, 12, 15, 12, 5;4, 9, 12, 9, 4;2, 4, 5, 4, 2];
G = 1/159.* G;

% Convolution with Gaussian filter
Ixx = conv2(Ixx, G, 'same');
Iyy = conv2(Iyy, G, 'same');
Ixy = conv2(Ixy, G, 'same');
```



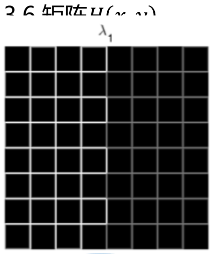
9.3.6 矩阵$H(x,y)$

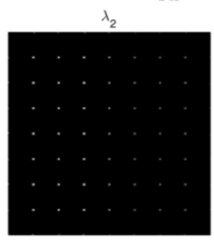
$\lambda_1(x,y)$ $\lambda_2(x,y)$



$R = \lambda_1\lambda_2 - k(\lambda_1 + \lambda_2)^2$

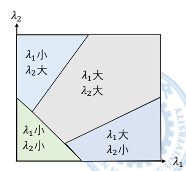
9.3.6 矩阵$H(x,y)$





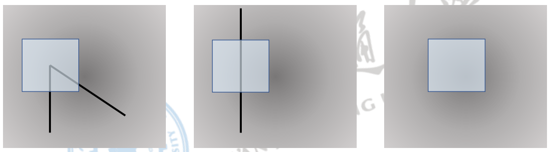
9.3.6 矩阵$H(x,y)$

$R = \lambda_1\lambda_2 - k(\lambda_1 + \lambda_2)^2$



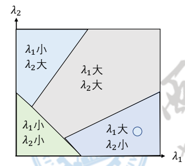
λ_1 小 λ_2 小	
λ_1 小 λ_2 大	
λ_1 大 λ_2 小	
λ_1 大 λ_2 大	

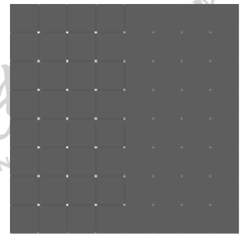
9.3.6 四种特征值的组合分别对应那种情况



9.3.6 矩阵$H(x,y)$

$R=\lambda_1\lambda_2-k(\lambda_1+\lambda_2)^2$

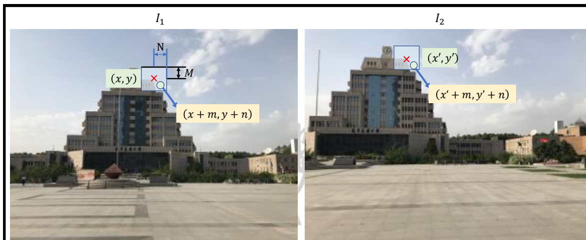




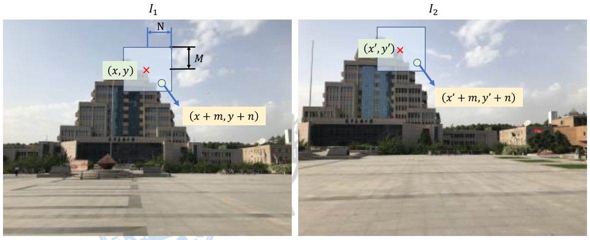
如何比较两个点的差异呢?

如何比较两个点的差异呢?

四、模板匹配

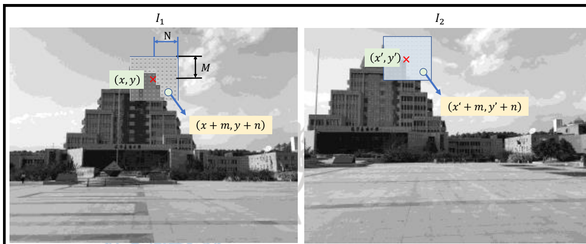


- $(2M+1) \times (2N+1)$



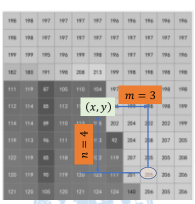
- $(2M+1) \times (2N+1)$

$$I_1(x+m, y+n) - I_2(x'+m, y'+n)$$

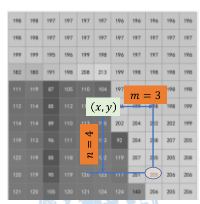


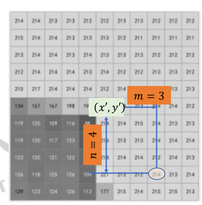
- $(2M+1) \times (2N+1)$

$$|I_1(x+m, y+n) - I_2(x'+m, y'+n)|$$



$I_1(x+m, y+n) = 205$





$I_1(x+m, y+n) = 205$ $I_2(x'+m, y'+n) = 214$

9.4.1 SAD(Sum of Absolute Difference)

$$|I_1(x+m, y+n) - I_2(x'+m, y'+n)| = |205 - 214| = 9$$

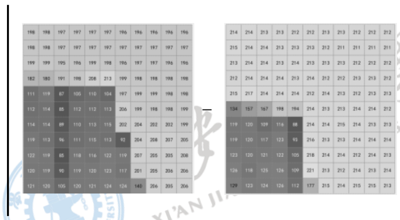
$I_1(x+m, y+n) = 205$ $I_2(x'+m, y'+n) = 214$

SAD(Sum of Absolute Difference)

$$\text{SAD} = \sum_{n=-N}^{N} \sum_{m=-M}^{M} |I_1(x+m, y+n) - I_2(x'+m, y'+n)|$$

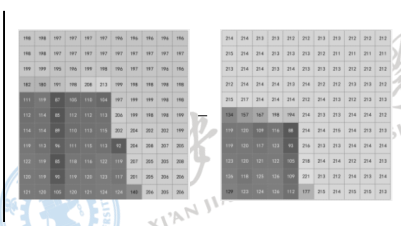
SAD

Sum



Sum：求取矩阵所有位置值的总和

Sum

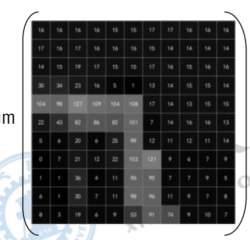


SAD(Sum of Absolute Difference)

$$\text{SAD} = \sum_{n=-N}^{N} \sum_{m=-M}^{M} |I_1(x+m, y+n) - I_2(x'+m, y'+n)|$$

SAD
(Sum of Absolute Difference)

Sum

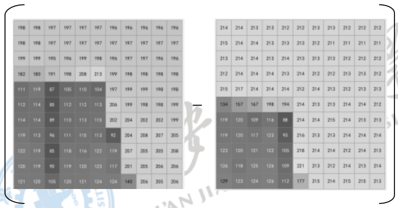


= 3924

$$\text{SAD} = \sum_{n=-N}^{N} \sum_{m=-M}^{M} |I_1(x+m, y+n) - I_2(x'+m, y'+n)|$$

9.4.2 MSE
Mean Squared Error

Mean

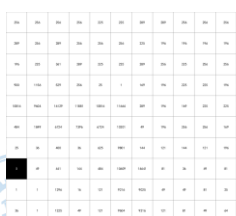


2

MSE(Mean Squared Error)

$$\mathrm{MSE}=\frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}\left(I_1(x+m,y+n)-I_2(x'+m,y'+n)\right)^2}{(2N+1)(2M+1)}$$

Mean

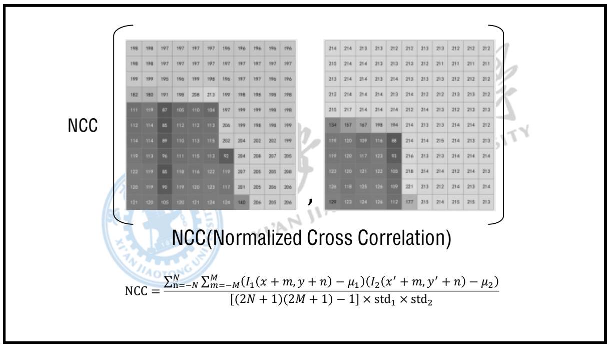


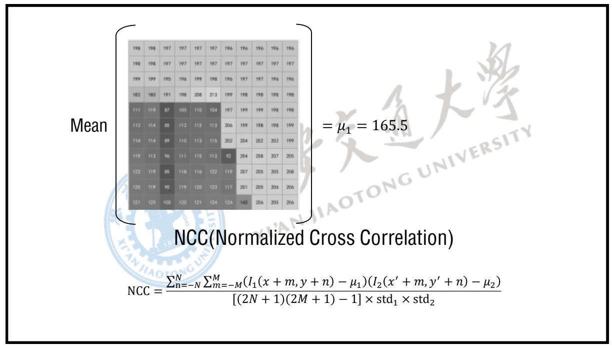
$= 1772.6$

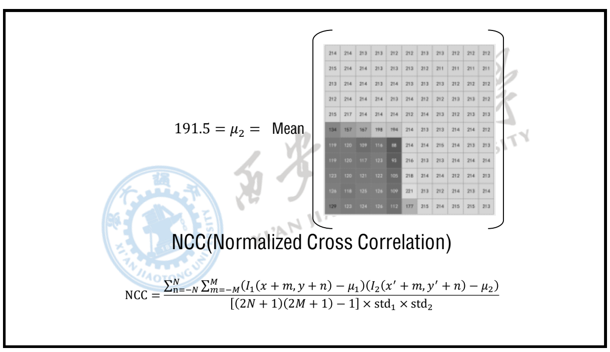
MSE(Mean Squared Error)

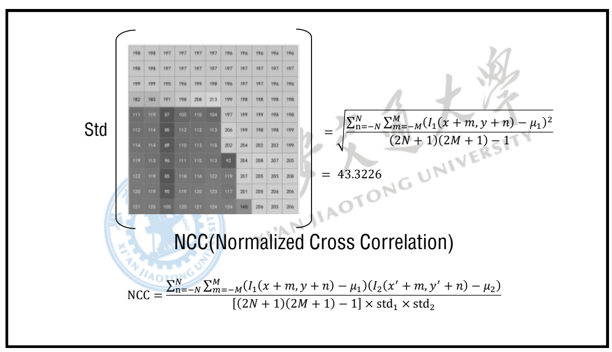
$$\mathrm{MSE}=\frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}\left(I_1(x+m,y+n)-I_2(x'+m,y'+n)\right)^2}{(2N+1)(2M+1)}$$

9.4.3 NCC
Normalized Cross Correlation


NCC
NCC(Normalized Cross Correlation)
$\mathrm{NCC} = \frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}(I_1(x+m,y+n)-\mu_1)(I_2(x'+m,y'+n)-\mu_2)}{[(2N+1)(2M+1)-1]\times \mathrm{std}_1 \times \mathrm{std}_2}$

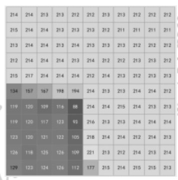

Mean
$= \mu_1 = 165.5$
NCC(Normalized Cross Correlation)
$\mathrm{NCC} = \frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}(I_1(x+m,y+n)-\mu_1)(I_2(x'+m,y'+n)-\mu_2)}{[(2N+1)(2M+1)-1]\times \mathrm{std}_1 \times \mathrm{std}_2}$


$191.5 = \mu_2 =$ Mean
NCC(Normalized Cross Correlation)
$\mathrm{NCC} = \frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}(I_1(x+m,y+n)-\mu_1)(I_2(x'+m,y'+n)-\mu_2)}{[(2N+1)(2M+1)-1]\times \mathrm{std}_1 \times \mathrm{std}_2}$


Std
$= \sqrt{\frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}(I_1(x+m,y+n)-\mu_1)^2}{(2N+1)(2M+1)-1}}$
$= 43.3226$
NCC(Normalized Cross Correlation)
$\mathrm{NCC} = \frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}(I_1(x+m,y+n)-\mu_1)(I_2(x'+m,y'+n)-\mu_2)}{[(2N+1)(2M+1)-1]\times \mathrm{std}_1 \times \mathrm{std}_2}$

$$\sqrt{\frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}(I_2(x+m,y+n)-\mu_2)^2}{(2N+1)(2M+1)-1}} = \quad \text{Std}$$

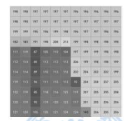
$39.6518 =$



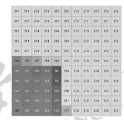
NCC(Normalized Cross Correlation)

$$\text{NCC} = \frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}(I_1(x+m,y+n)-\mu_1)(I_2(x'+m,y'+n)-\mu_2)}{[(2N+1)(2M+1)-1]\times \text{std}_1 \times \text{std}_2}$$

Sum



$-\mu_1(165.5)$,

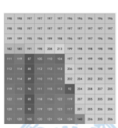


$-\mu_2(191.5)$

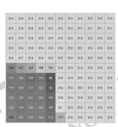
NCC(Normalized Cross Correlation)

$$\text{NCC} = \frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}(I_1(x+m,y+n)-\mu_1)(I_2(x'+m,y'+n)-\mu_2)}{[(2N+1)(2M+1)-1]\times \text{std}_1 \times \text{std}_2} \quad 1.4034e+05$$

Sum



$-\mu_1(165.5)$,

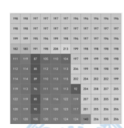


$-\mu_2(191.5)$

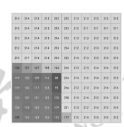
NCC(Normalized Cross Correlation)

$$\text{NCC} = \frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}(I_1(x+m,y+n)-\mu_1)(I_2(x'+m,y'+n)-\mu_2)}{[(2N+1)(2M+1)-1]\times \text{std}_1 \times \text{std}_2} \quad \frac{1.4034e+05}{120\times 43.3226\times 39.6518}$$

Sum



$-\mu_1(165.5)$,



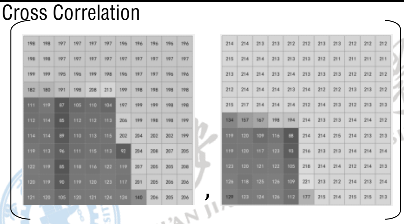
$-\mu_2(191.5)$

NCC(Normalized Cross Correlation)

$$\text{NCC} = \frac{\sum_{n=-N}^{N}\sum_{m=-M}^{M}(I_1(x+m,y+n)-\mu_1)(I_2(x'+m,y'+n)-\mu_2)}{[(2N+1)(2M+1)-1]\times \text{std}_1 \times \text{std}_2} = 0.6808$$

Normalized Cross Correlation

NCC



$$-1 \leq \text{NCC} \leq 1$$

9.4.4 匹配与筛选