

计算机视觉与模式识别

苏远岐，新型计算机研究所

第九章 角点检测与特征描述-1

最小二乘法与SVD分解是我们在计算机视觉中最常用的方法。

一、几何变换的参数估计：最佳仿射变换的估计

二、最小二乘法与SVD：梯度

三、Harris角点检测：角点的判定依据

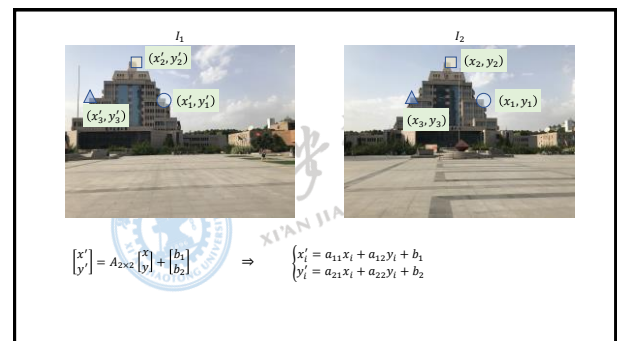
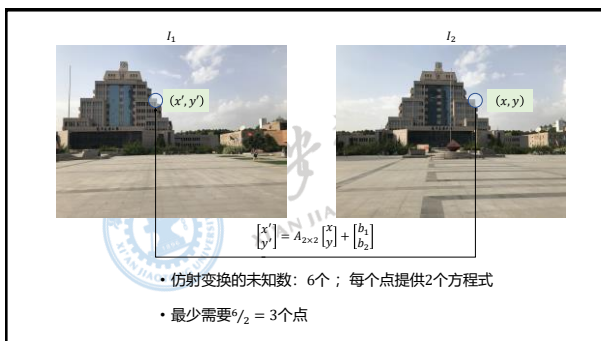
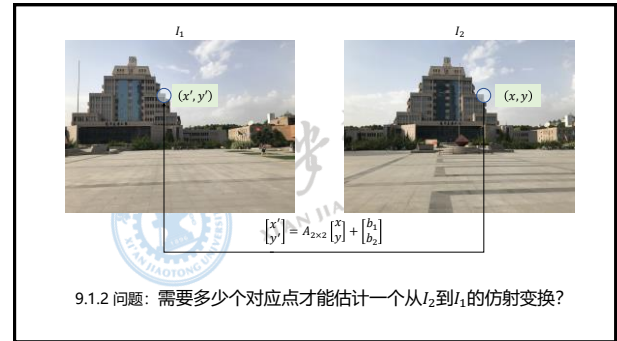
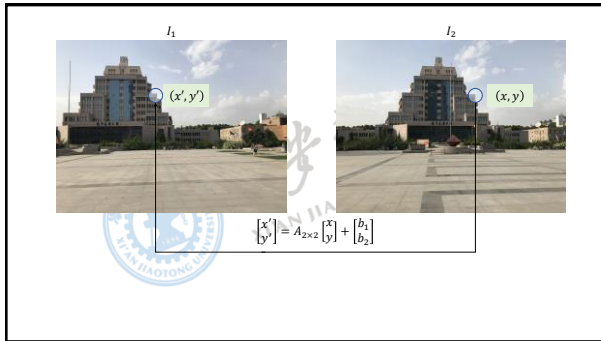
四、模板匹配：SAD, MSE, NCC

五、拉普拉斯算子与SIFT特征点

一、几何变换的参数估计



9.1.1 问题：如何估计一个从 I_2 到 I_1 的几何变换？



9.1.3 方程组的求解

$$\begin{cases} x'_1 = a_{11}x_1 + a_{12}y_1 + b_1 \\ y'_1 = a_{21}x_1 + a_{22}y_1 + b_2 \end{cases}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}$$

9.1.3 方程组的求解

$$\begin{cases} x'_1 = a_{11}x_1 + a_{12}y_1 + b_1 \\ y'_1 = a_{21}x_1 + a_{22}y_1 + b_2 \end{cases}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}$$

9.1.3 方程组的求解

$$\begin{cases} x'_1 = a_{11}x_1 + a_{12}y_1 + b_1 \\ y'_1 = a_{21}x_1 + a_{22}y_1 + b_2 \end{cases}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}$$

9.1.3 方程组的求解

$$\begin{cases} x'_1 = a_{11}x_1 + a_{12}y_1 + b_1 \\ y'_1 = a_{21}x_1 + a_{22}y_1 + b_2 \end{cases}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \mathbf{A} \mathbf{x} = \mathbf{b}$$

二、最小二乘法与SVD

9.2.1 方程组的求解：最小二乘法

$$\min_{\mathbf{x}} L = \left\| \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \mathbf{x} - \begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \\ a_{31} \\ a_{32} \end{bmatrix} \right\|_2^2$$

9.2.1 方程组的求解：最小二乘法

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix}^T \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \mathbf{x} - \begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \\ a_{31} \\ a_{32} \end{bmatrix}$$

$$L = \mathbf{x}^T \mathbf{A}^T \mathbf{A} \mathbf{x} - \mathbf{b}^T \mathbf{A} \mathbf{x} - \mathbf{x}^T \mathbf{A}^T \mathbf{b} + \mathbf{b}^T \mathbf{b}$$

9.2.1 方程组的求解：最小二乘法

$$L = \mathbf{x}^T \mathbf{A}^T \mathbf{A} \mathbf{x} - \mathbf{b}^T \mathbf{A} \mathbf{x} - \mathbf{x}^T \mathbf{A}^T \mathbf{b} + \mathbf{b}^T \mathbf{b}$$

$$\frac{\partial L}{\partial \mathbf{x}} = \mathbf{A}^T \mathbf{A} \mathbf{x} + (\mathbf{x}^T \mathbf{A}^T \mathbf{A})^T - (\mathbf{b}^T \mathbf{A})^T - \mathbf{A}^T \mathbf{b}$$

$$\frac{\partial L}{\partial \mathbf{x}} = 2\mathbf{A}^T \mathbf{A} \mathbf{x} - 2\mathbf{A}^T \mathbf{b} = 0$$

9.2.1 方程组的求解：最小二乘法

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A}^+)^{-1} \mathbf{A}^T \mathbf{b} \quad \text{伪逆}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}^{-1} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}^T \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

9.2.1 方程组的求解：最小二乘法

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A}^+)^{-1} \mathbf{A}^T \mathbf{b} \quad \text{伪逆}$$

Matlab:

>> x = pinv(A) * b

>> x = A\b

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}^{-1} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}^T \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

9.2.2 方程组的求解：SVD分解

$$\mathbf{A} = \mathbf{U} \mathbf{S} \mathbf{V}^T$$

Matlab:

>> [U, S, V] = svd(A)

正交矩阵

对角矩阵

正交矩阵

9.2.2 方程组的求解：SVD分解

$$\mathbf{A} = \mathbf{U} \mathbf{S} \mathbf{V}^T$$

Matlab:

>> [U, S, V] = svd(A)

正交矩阵

对角矩阵

正交矩阵

9.2.2 方程组的求解: SVD分解

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Matlab:

>> [U, S, V] = svd(A)

$$A = USV^T$$

正交矩阵

对角矩阵

正交矩阵

U =	S =	V =
~0.1125 ~0.8229 ~0.3919 ~0.3900	14.2091 0	~0.0111 0.7072
~0.3409 ~0.4714 ~0.3420 ~0.0207	9 0.6208	~0.7872 ~0.5814
~0.5474 ~0.0201 0.6979 ~0.4514	0 0	0 0
~0.7448 0.3812 ~0.3462 0.0407	0 0	0 0

9.2.2 方程组的求解: SVD分解

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Matlab:

>> [U, S, V] = svd(A)

$$A = USV^T$$

正交矩阵

对角矩阵

正交矩阵

U =	S =	V =
~0.1125 ~0.8229 ~0.3919 ~0.3900	14.2091 0	~0.0111 0.7072
~0.3409 ~0.4714 ~0.3420 ~0.0207	9 0.6208	~0.7872 ~0.5814
~0.5474 ~0.0201 0.6979 ~0.4514	0 0	0 0
~0.7448 0.3812 ~0.3462 0.0407	0 0	0 0

9.2.2 方程组的求解: SVD分解

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Matlab:

>> [U, S, V] = svd(A)

$$A^T = VS^T U^T$$

正交矩阵

对角矩阵

正交矩阵

U =	S =	V =
~0.1125 ~0.8229 ~0.3919 ~0.3900	14.2091 0	~0.0111 0.7072
~0.3409 ~0.4714 ~0.3420 ~0.0207	9 0.6208	~0.7872 ~0.5814
~0.5474 ~0.0201 0.6979 ~0.4514	0 0	0 0
~0.7448 0.3812 ~0.3462 0.0407	0 0	0 0

9.2.3 方程组的求解: 最小二乘法

$$2(VS^T U^T USV^T x - VS^T U^T b) = 0$$

$$A = USV^T$$

$$\frac{\partial L}{\partial x} = 2A^T Ax - 2A^T b = 0$$

9.2.3 方程组的求解：最小二乘法

$$VS^T SV^T x = VS^T U^T b$$

U 是正交矩阵

$$2(VS^T U^T U SV^T x - VS^T U^T b) = 0$$

$$A = USV^T$$

$$\frac{\partial L}{\partial x} = 2A^T Ax - 2A^T b = 0$$

9.2.3 方程组的求解：最小二乘法

$$VS^T SV^T x = VS^T U^T b$$

$S^T SV^T x = S^T U^T b$
 V 是正交矩阵

$$2(VS^T U^T U SV^T x - VS^T U^T b) = 0$$

$$A = USV^T$$

$$\frac{\partial L}{\partial x} = 2A^T Ax - 2A^T b = 0$$

9.2.4 特例1：如果A是满秩方阵

$$VS^T SV^T x = VS^T U^T b$$

V 是正交矩阵

$$S^T SV^T x = S^T U^T b$$

S 对角矩阵，非奇异

$$x = VS^{-1}U^T b$$

9.2.4 特例1：如果A是满秩方阵

$$VS^T SV^T x = VS^T U^T b$$

V 是正交矩阵

$$S^T SV^T x = S^T U^T b$$

S 对角矩阵，非奇异

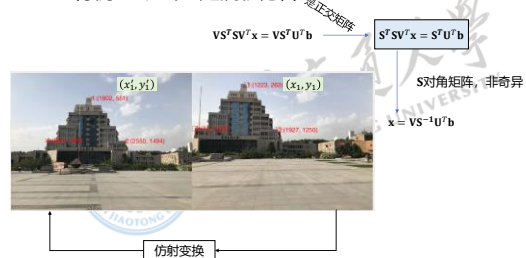
$$x = VS^{-1}U^T b$$



9.2.4 特例1: 如果A是满秩方阵



9.2.4 特例1: 如果A是满秩方阵



9.2.5 方程组的求解

$$\begin{cases} x'_1 = a_{11}x_1 + a_{12}y_1 + b_1 \\ y'_1 = a_{21}x_1 + a_{22}y_1 + b_2 \end{cases}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}$$

$$(x_1 = 1223, y_1 = 263)$$



9.2.5 方程组的求解

$$\begin{cases} x'_1 = a_{11}x_1 + a_{12}y_1 + b_1 \\ y'_1 = a_{21}x_1 + a_{22}y_1 + b_2 \end{cases}$$

$$\begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}$$

$$(x_1 = 1223, y_1 = 263)$$



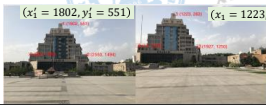
9.2.5 方程组的求解

$$\begin{cases} x'_i = a_{11}x_i + a_{12}y_i + b_1 \\ y'_i = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} 1802 \\ 551 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}$$

$$(x'_1 = 1802, y'_1 = 551)$$

$$(x_1 = 1223, y_1 = 263)$$



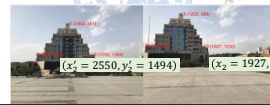
9.2.5 方程组的求解

$$\begin{cases} x'_i = a_{11}x_i + a_{12}y_i + b_1 \\ y'_i = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ 1927 & 1250 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1927 & 1250 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} 1802 \\ 551 \\ 2550 \\ 1494 \\ x'_4 \\ y'_4 \end{bmatrix}$$

$$(x'_1 = 1802, y'_1 = 551)$$

$$(x_1 = 1223, y_1 = 263)$$



9.2.5 方程组的求解

$$\begin{cases} x'_i = a_{11}x_i + a_{12}y_i + b_1 \\ y'_i = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ 1927 & 1250 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1927 & 1250 & 1 \\ 74 & 1223 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 74 & 1223 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} 1802 \\ 551 \\ 2550 \\ 1494 \\ 800 \\ 1488 \end{bmatrix}$$

$$(x'_1 = 1802, y'_1 = 551)$$

$$(x_1 = 1223, y_1 = 263)$$



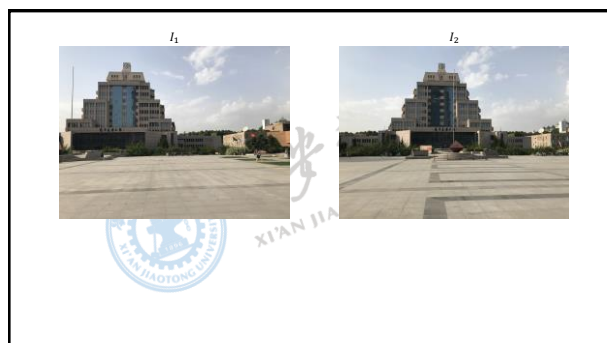
9.2.5 方程组的求解

$$\begin{cases} x'_i = a_{11}x_i + a_{12}y_i + b_1 \\ y'_i = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

$$\text{SVD} \quad \begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ 1927 & 1250 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1927 & 1250 & 1 \\ 74 & 1223 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 74 & 1223 & 1 \end{bmatrix} \mathbf{A}$$

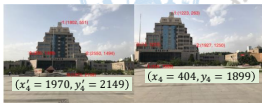
Matlab:

>> [U, S, V] = svd(A)



9.2.6 一般情况:

$$\begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ 1927 & 1250 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1927 & 1250 & 1 \\ 74 & 1223 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 74 & 1223 & 1 \\ 404 & 1899 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 404 & 1899 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} 1802 \\ 551 \\ 2550 \\ 1494 \\ 800 \\ 1488 \\ 1970 \\ 2149 \end{bmatrix}$$



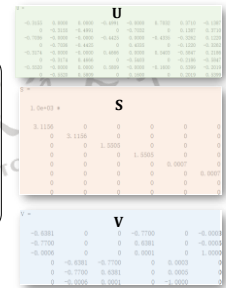
$$\begin{cases} x'_i = a_{11}x_i + a_{12}y_i + b_1 \\ y'_i = a_{21}x_i + a_{22}y_i + b_2 \end{cases}$$

9.2.6 方程组的求解 $A = USV^T$

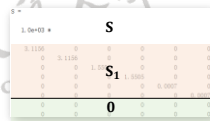
$$\text{SVD} \quad \begin{bmatrix} 1223 & 263 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1223 & 263 & 1 \\ 1927 & 1250 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1927 & 1250 & 1 \\ 74 & 1223 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 74 & 1223 & 1 \\ 404 & 1899 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 404 & 1899 & 1 \end{bmatrix} A \begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \\ a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} 1802 \\ 551 \\ 2550 \\ 1494 \\ 800 \\ 1488 \\ 1970 \\ 2149 \end{bmatrix}$$

Matlab:

>> [U, S, V] = svd(A)

9.2.6 方程组的求解 $A = USV^T$

$$S = \begin{bmatrix} s_1 \\ 0 \end{bmatrix}$$



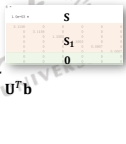
Matlab:

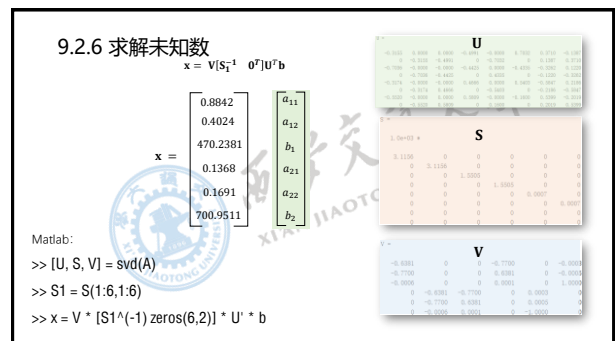
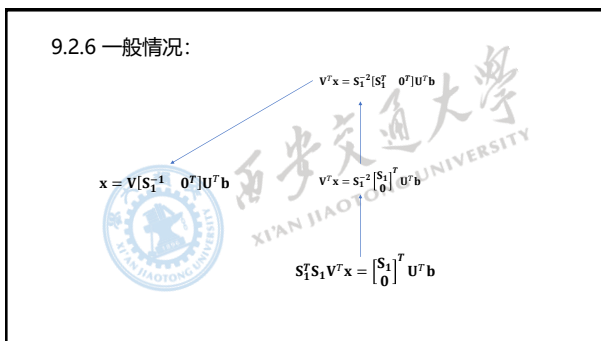
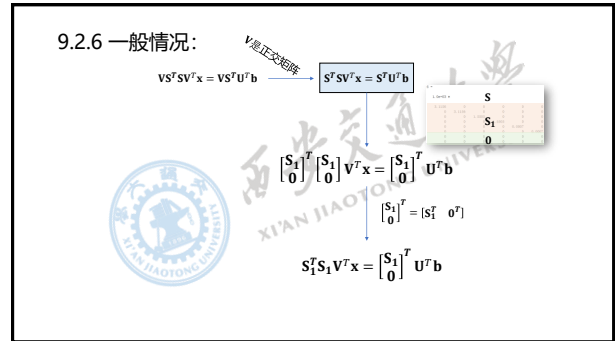
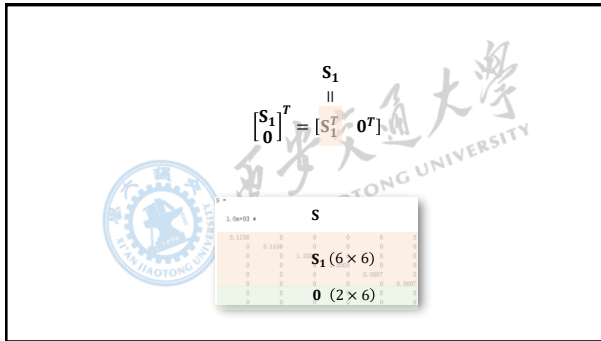
>> [U, S, V] = svd(A)

9.2.6 一般情况:

$$VS^T SV^T x = VS^T U^T b \xrightarrow{V \text{ 左正交矩阵}} S^T SV^T x = S^T U^T b$$

$$\begin{bmatrix} s_1 & 0 \\ 0 & 0 \end{bmatrix}^T \begin{bmatrix} s_1 \\ 0 \end{bmatrix} V^T x = \begin{bmatrix} s_1 \\ 0 \end{bmatrix}^T U^T b$$







如何让匹配的过程自动化呢？



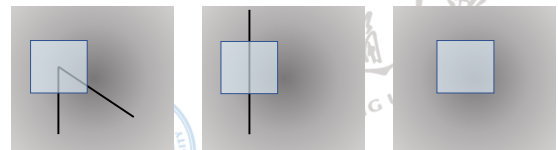
西安交通大学
XI'AN JIAOTONG UNIVERSITY

三、Harris角点检测



西安交通大学
XI'AN JIAOTONG UNIVERSITY

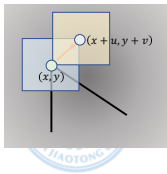
9.3.1 Harris角点检测的基本原理



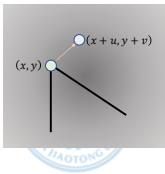
利用当前矩形框与周边矩形框的差异来度量当前点的独特性。

- 差异越大，越具有独特性
- 差异越小，越不具备独特性

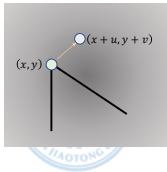
9.3.2 差异度量



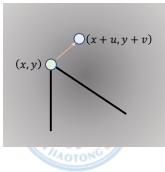
9.3.2 差异度量



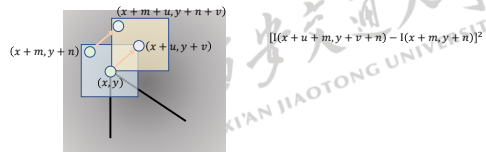
9.3.2 差异度量



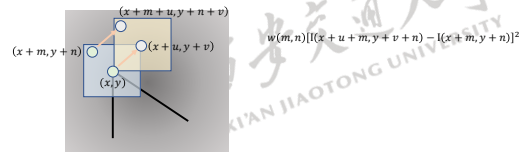
9.3.2 差异度量



9.3.2 差异度量

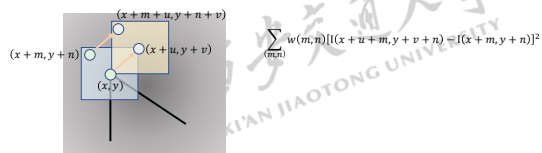


9.3.2 差异度量



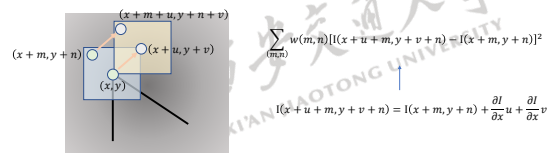
距离当前点越近, 差异的影响越大; 否则越小。

9.3.2 差异度量



距离当前点越近, 差异的影响越大; 否则越小。

9.3.3 一阶Taylor展开



9.3.3 一阶Taylor展开

$$l(x+m+u, y+n+v) = l(x+m, y+n) + \frac{\partial l}{\partial x} u + \frac{\partial l}{\partial y} v + \sum_{(m,n)} w(m,n) \left[\frac{\partial l(x+m, y+n)}{\partial x} u + \frac{\partial l(x+m, y+n)}{\partial y} v \right]^2$$

9.3.3 一阶Taylor展开

$$l(x+m+u, y+n+v) = l(x+m, y+n) + \frac{\partial l}{\partial x} u + \frac{\partial l}{\partial y} v + \sum_{(m,n)} w(m,n) \left[\frac{\partial l(x+m, y+n)}{\partial x} u + \frac{\partial l(x+m, y+n)}{\partial y} v \right]^2$$

9.3.3 一阶Taylor展开

$$l(x+m+u, y+n+v) = l(x+m, y+n) + \frac{\partial l}{\partial x} u + \frac{\partial l}{\partial y} v + \sum_{(m,n)} w(m,n) \left[l_x(x+m, y+n) u + l_y(x+m, y+n) v \right]^2$$

9.3.3 一阶Taylor展开

$$l(x+m+u, y+n+v) = l(x+m, y+n) + \frac{\partial l}{\partial x} u + \frac{\partial l}{\partial y} v + \sum_{(m,n)} w(m,n) \left[l_x(x+m, y+n) u + l_y(x+m, y+n) v \right]^2$$

9.3.3 一阶Taylor展开

$$I(x+m, y+n) + I_x(x+m, y+n)u + I_y(x+m, y+n)v + \frac{1}{2} [I_{xx}(x+m, y+n)u^2 + 2I_{xy}(x+m, y+n)uv + I_{yy}(x+m, y+n)v^2] + \dots$$

9.3.4 矩阵 $H(x, y)$

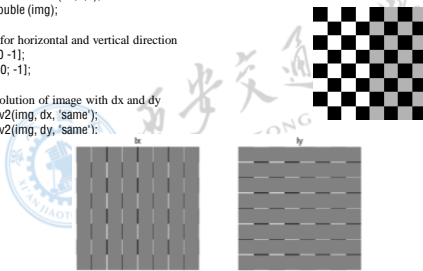
$$H(x, y) = \begin{bmatrix} \sum_{(m,n)} w(m,n) I_{xx}^2(x+m, y+n) & \sum_{(m,n)} w(m,n) I_{xy}(x+m, y+n) I_{yx}(x+m, y+n) \\ \sum_{(m,n)} w(m,n) I_{xy}(x+m, y+n) I_{yx}(x+m, y+n) & \sum_{(m,n)} w(m,n) I_{yy}^2(x+m, y+n) \end{bmatrix}$$

9.3.5 如何计算

```
img = checkerboard(50,4,4);
img = double(img);
```

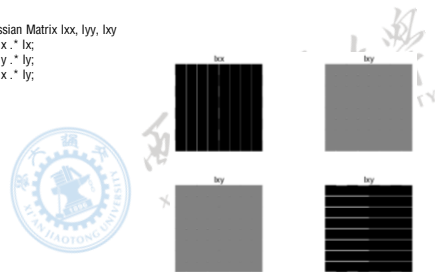
```
%Filter for horizontal and vertical direction
dx = [1 0 -1];
dy = [1; 0; -1];
```

```
% Convolution of image with dx and dy
lx = conv2(img, dx, 'same');
ly = conv2(img, dy, 'same');
```



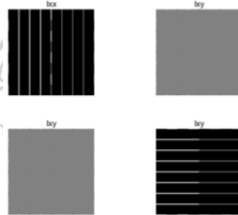
```
% Hessian Matrix lxx, lyy, lxy
```

```
lxx = lx .* lx;
lyy = ly .* ly;
lxy = lx .* ly;
```

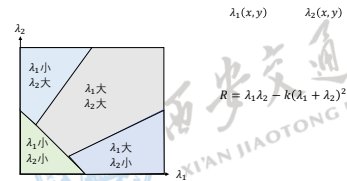


```
%Gaussian filter definition (https://en.wikipedia.org/wiki/Canny_edge_detector)
G = [2, 4, 5, 4, 2; 4, 9, 12, 9, 4, 5, 12, 15, 12, 5, 4, 9, 12, 9, 4, 2, 4, 5, 4, 2];
G = 1/159.* G;
```

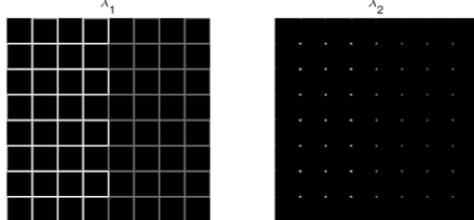
```
% Convolution with Gaussian filter
hox = conv2(hox, G, 'same');
hyy = conv2(hyy, G, 'same');
hxy = conv2(hxy, G, 'same');
```



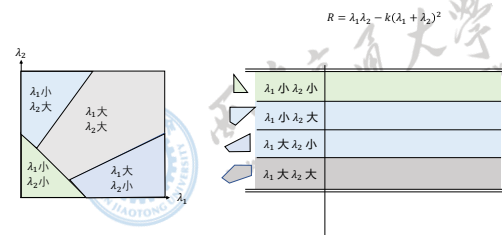
9.3.6 矩阵 $H(x, y)$



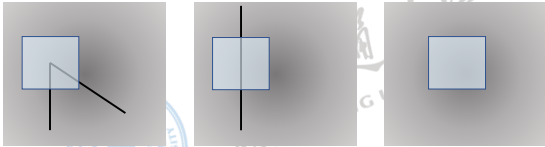
9.3.6 矩阵 $H(x, y)$



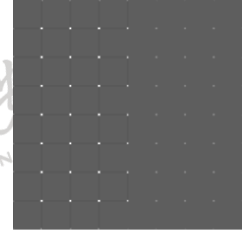
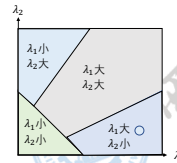
9.3.6 矩阵 $H(x, y)$



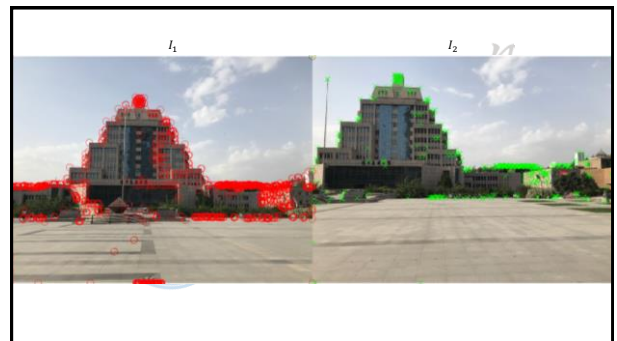
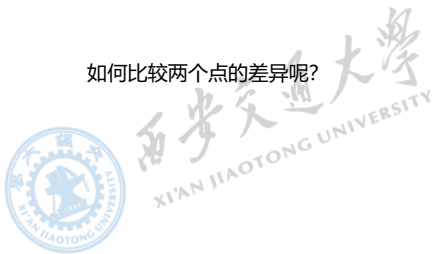
9.3.6 四种特征值的组合分别对应那种情况

9.3.6 矩阵 $H(x, y)$

$$R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$$



如何比较两个点的差异呢?



如何比较两个点的差异呢?

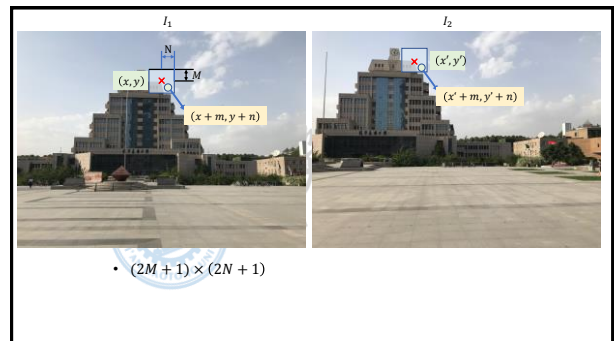
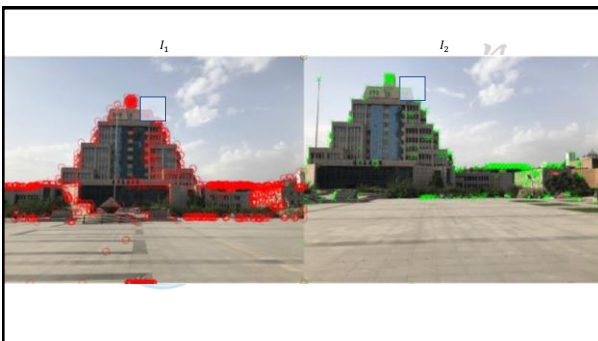


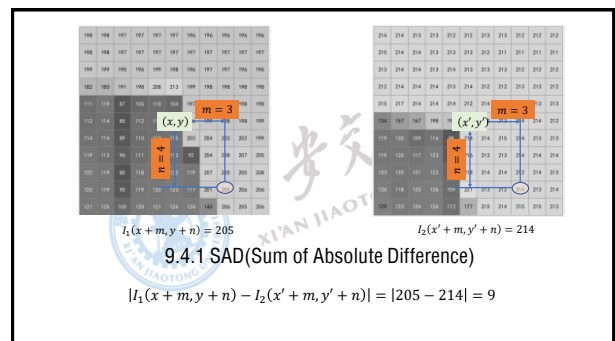
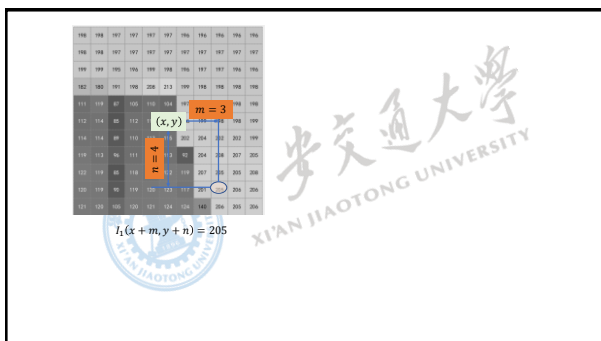
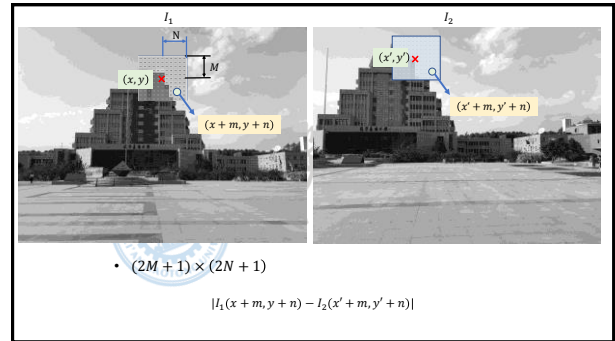
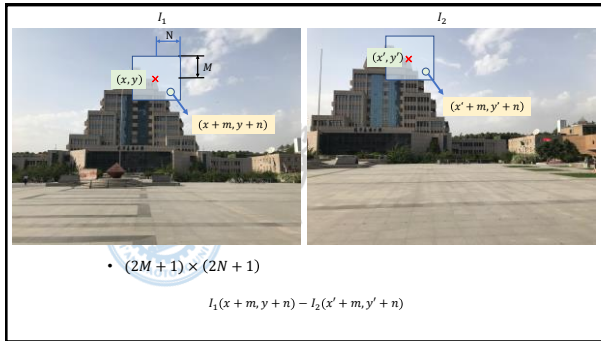
西安交通大学
XI'AN JIAOTONG UNIVERSITY

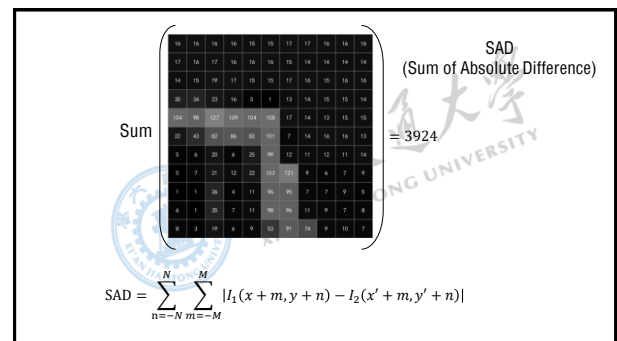
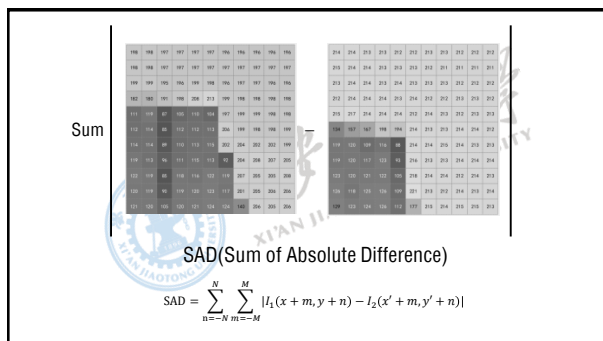
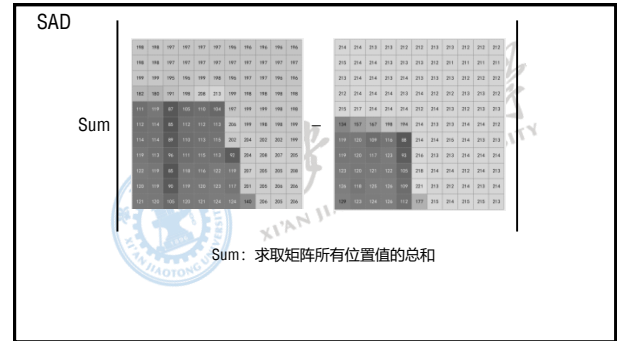
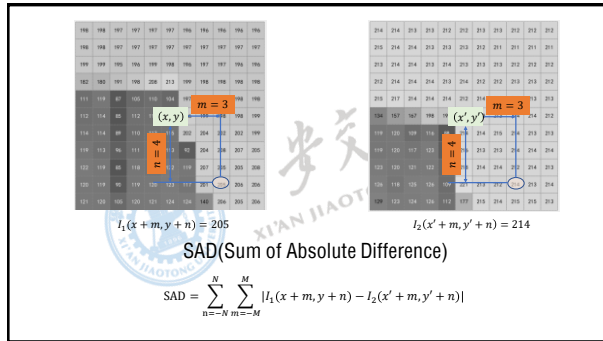
四、模板匹配



西安交通大学
XI'AN JIAOTONG UNIVERSITY







9.4.2 MSE
Mean Squared Error

Mean

168	168	167	167	167	166	166	166	166	166
168	168	167	167	167	166	166	166	166	166
168	168	167	167	167	166	166	166	166	166
168	168	167	167	167	166	166	166	166	166
168	168	167	167	167	166	166	166	166	166
168	168	167	167	167	166	166	166	166	166
168	168	167	167	167	166	166	166	166	166
168	168	167	167	167	166	166	166	166	166
168	168	167	167	167	166	166	166	166	166
168	168	167	167	167	166	166	166	166	166

254	254	253	253	252	252	253	253	253	252
253	254	254	253	253	252	252	251	251	251
253	254	254	253	253	252	252	251	251	251
253	254	254	253	253	252	252	251	251	251
253	254	254	253	253	252	252	251	251	251
253	254	254	253	253	252	252	251	251	251
253	254	254	253	253	252	252	251	251	251
253	254	254	253	253	252	252	251	251	251
253	254	254	253	253	252	252	251	251	251
253	254	254	253	253	252	252	251	251	251

$$MSE = \frac{\sum_{n=-N}^N \sum_{m=-M}^M -(I_1(x+m, y+n) - I_2(x'+m, y'+n))^2}{(2N+1)(2M+1)}$$

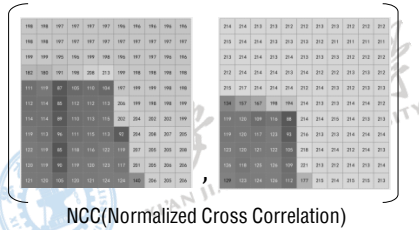
Mean

260	280	290	300	310	320	330	340	350	360	370	380	390	400	410	420	430	440	450	460	470	480	490	500	510	520	530	540	550	560	570	580	590	600	610	620	630	640	650	660	670	680	690	700	710	720	730	740	750	760	770	780	790	800	810	820	830	840	850	860	870	880	890	900	910	920	930	940	950	960	970	980	990
261	281	291	301	311	321	331	341	351	361	371	381	391	401	411	421	431	441	451	461	471	481	491	501	511	521	531	541	551	561	571	581	591	601	611	621	631	641	651	661	671	681	691	701	711	721	731	741	751	761	771	781	791	801	811	821	831	841	851	861	871	881	891	901	911	921	931	941	951	961	971	981	991
262	282	292	302	312	322	332	342	352	362	372	382	392	402	412	422	432	442	452	462	472	482	492	502	512	522	532	542	552	562	572	582	592	602	612	622	632	642	652	662	672	682	692	702	712	722	732	742	752	762	772	782	792	802	812	822	832	842	852	862	872	882	892	902	912	922	932	942	952	962	972	982	992
263	283	293	303	313	323	333	343	353	363	373	383	393	403	413	423	433	443	453	463	473	483	493	503	513	523	533	543	553	563	573	583	593	603	613	623	633	643	653	663	673	683	693	703	713	723	733	743	753	763	773	783	793	803	813	823	833	843	853	863	873	883	893	903	913	923	933	943	953	963	973	983	993
264	284	294	304	314	324	334	344	354	364	374	384	394	404	414	424	434	444	454	464	474	484	494	504	514	524	534	544	554	564	574	584	594	604	614	624	634	644	654	664	674	684	694	704	714	724	734	744	754	764	774	784	794	804	814	824	834	844	854	864	874	884	894	904	914	924	934	944	954	964	974	984	994
265	285	295	305	315	325	335	345	355	365	375	385	395	405	415	425	435	445	455	465	475	485	495	505	515	525	535	545	555	565	575	585	595	605	615	625	635	645	655	665	675	685	695	705	715	725	735	745	755	765	775	785	795	805	815	825	835	845	855	865	875	885	895	905	915	925	935	945	955	965	975	985	995
266	286	296	306	316	326	336	346	356	366	376	386	396	406	416	426	436	446	456	466	476	486	496	506	516	526	536	546	556	566	576	586	596	606	616	626	636	646	656	666	676	686	696	706	716	726	736	746	756	766	776	786	796	806	816	826	836	846	856	866	876	886	896	906	916	926	936	94					

9.4.3 NCC

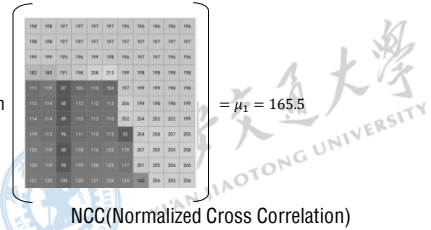
Normalized Cross Correlation

NCC

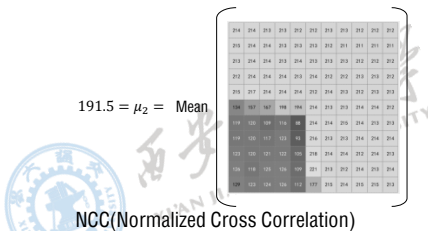


$$NCC = \frac{\sum_{n=-N}^N \sum_{m=-M}^M (I_1(x+m, y+n) - \mu_1)(I_2(x'+m, y'+n) - \mu_2)}{[(2N+1)(2M+1)-1] \times \text{std}_1 \times \text{std}_2}$$

Mean

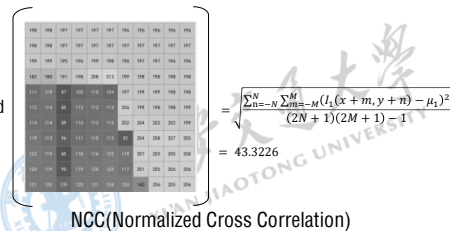


$$NCC = \frac{\sum_{n=-N}^N \sum_{m=-M}^M (I_1(x+m, y+n) - \mu_1)(I_2(x'+m, y'+n) - \mu_2)}{[(2N+1)(2M+1)-1] \times \text{std}_1 \times \text{std}_2}$$

191.5 = μ_2 = Mean

$$NCC = \frac{\sum_{n=-N}^N \sum_{m=-M}^M (I_1(x+m, y+n) - \mu_1)(I_2(x'+m, y'+n) - \mu_2)}{[(2N+1)(2M+1)-1] \times \text{std}_1 \times \text{std}_2}$$

Std



$$NCC = \frac{\sum_{n=-N}^N \sum_{m=-M}^M (I_1(x+m, y+n) - \mu_1)(I_2(x'+m, y'+n) - \mu_2)}{[(2N+1)(2M+1)-1] \times \text{std}_1 \times \text{std}_2}$$

$$\sqrt{\frac{\sum_{n=-N}^N \sum_{m=-M}^M (I_2(x+m, y+n) - \mu_2)^2}{(2N+1)(2M+1) - 1}} = \text{Std}$$

39.6518 =

NCC(Normalized Cross Correlation)

$$\text{NCC} = \frac{\sum_{n=-N}^N \sum_{m=-M}^M (I_1(x+m, y+n) - \mu_1)(I_2(x'+m, y'+n) - \mu_2)}{[(2N+1)(2M+1) - 1] \times \text{std}_1 \times \text{std}_2}$$

Sum

$-\mu_1(165.5)$ $-\mu_2(191.5)$

NCC(Normalized Cross Correlation)

$$\text{NCC} = \frac{\sum_{n=-N}^N \sum_{m=-M}^M (I_1(x+m, y+n) - \mu_1)(I_2(x'+m, y'+n) - \mu_2)}{[(2N+1)(2M+1) - 1] \times \text{std}_1 \times \text{std}_2} = 1.4034\text{e}+05$$

Sum

$-\mu_1(165.5)$ $-\mu_2(191.5)$

NCC(Normalized Cross Correlation)

$$\text{NCC} = \frac{\sum_{n=-N}^N \sum_{m=-M}^M (I_1(x+m, y+n) - \mu_1)(I_2(x'+m, y'+n) - \mu_2)}{[(2N+1)(2M+1) - 1] \times \text{std}_1 \times \text{std}_2} = \frac{1.4034\text{e}+05}{120 \times 43.3226 \times 39.6518}$$

Sum

$-\mu_1(165.5)$ $-\mu_2(191.5)$

NCC(Normalized Cross Correlation)

$$\text{NCC} = \frac{\sum_{n=-N}^N \sum_{m=-M}^M (I_1(x+m, y+n) - \mu_1)(I_2(x'+m, y'+n) - \mu_2)}{[(2N+1)(2M+1) - 1] \times \text{std}_1 \times \text{std}_2} = 0.6808$$

Normalized Cross Correlation

NCC

198	198	197	197	197	196	196	196	214	214	213	213	213	213	212	212	212
198	198	197	197	197	197	197	197	213	214	214	213	213	213	211	211	211
199	199	195	196	199	196	196	197	212	214	214	214	214	213	212	212	212
192	192	191	196	198	213	199	198	212	214	214	214	214	213	212	212	212
111	119	87	165	116	166	197	199	210	217	214	214	214	213	213	213	213
112	114	85	112	112	113	204	199	198	197	197	198	194	214	213	214	212
114	114	89	113	113	116	202	204	202	202	199	198	197	197	198	194	212
117	113	96	111	112	113	16	204	204	207	205	114	116	117	121	11	214
117	119	85	116	114	127	119	207	205	205	206	113	116	117	121	119	214
120	119	91	119	120	127	117	201	205	204	204	116	118	119	121	119	214
117	118	100	119	117	120	120	204	205	205	206	119	121	124	121	117	213

$$-1 \leq \text{NCC} \leq 1$$

9.4.4 匹配与筛选

