

## PROBLEM SET 1

*Information Theory, 2018*

**P1.** A bag contains fifteen balls distinguishable only by their colours; ten are blue and five are red. I reach into the bag with both hands and pull out two balls (one with each hand) and record their colours.

- (a) What is the random phenomenon?
- (b) What is the sample space?
- (c) Express the event that the ball in my left hand is red as a subset of the sample space.

**P2.** M&M sweets are of varying colours and the different colours occur in different proportions. The table below gives the probability that a randomly chosen M&M has each colour, but the value for tan candies is missing.

Colour	Brown	Red	Yellow	Green	Orange	Tan
Probability	0.3	0.2	0.2	0.1	0.1	?

- (a) What value must the missing probability be?
- (b) You draw an M&M at random from a packet. What is the probability of each of the following events
  - i. You get a brown one or a red one.
  - ii. You don't get a yellow one.
  - iii. You don't get either an orange one or a tan one.
  - iv. You get one that is brown or red or yellow or green or orange or tan.

**P3.** Not all dice are fair. In order to describe an unfair die properly, we must specify the probability for each of the six possible outcomes. The following table gives answers for each of 4 different dice.

Outcome	Probabilities			
	Die 1	Die 2	Die 3	Die 4
1	1/3	1/6	1/7	1/3
2	0	1/6	1/7	1/3
3	1/6	1/6	1/7	-1/6
4	0	1/6	1/7	-1/6
5	1/6	1/6	1/7	1/3
6	1/3	1/7	2/7	1/3

Which of the four dice have validly specified probabilities and which do not? In the case of an invalidly described die, explain why the probabilities are invalid.

**P4.** An examination consists of multiple-choice questions, each having five possible answers. Suppose you are a student taking the exam. and that you reckon you have probability 0.75 of knowing the answer to any question that may be asked and that, if you do not know, you intend to guess an answer with probability  $1/5$  of being correct. What is the probability you will give the correct answer to a question?

**P5.** Are  $X$  and  $Y$  independently distributed?

$x$	0	0	1	1
$y$	0	1	0	1
$p(x,y)$	0.32	0.08	0.48	0.12

**P6.** James is interested in the relationship between weather conditions and whether the downtown train runs on time. For a year, James records the weather each day as well as whether this train arrives on time or is delayed. Here are his results:

Weather condition	On-time	Delayed	Total
Sunny	167	3	170
Cloudy	115	5	120
Rainy	40	15	55
Snowy	8	12	20
Total	330	35	365

Find the marginal distribution of arrival status in percentages. Round to the nearest whole percent.