Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
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Dive into Music – The Universal Language of Mankind

The data of numerous artists and their compositions are gathered by the Integrative Collective Music (ICM) Society. And we hope, as H. W. Longfellow said,

"Music is the universal language of mankind."

to develop a general standard for measuring the influence of artists and their music, to explore those revolutionary artists and genres, and to appreciate the language of magnificent works throughout music history.

To carry out this challenging project, we've designed three main models:

1. **Influence Network** (**IN**) for modelling the influence of each artist and between pair of artists. The **PageRank** algorithm is applied to this task, and significant features within subnetworks are revealed by analyzing the (sub)INs. Moreover, the connectivity and centrality results of this model are mutually confirmed with the results derived in our next main model.
2. **Kernel Function** for classifying songs and learning their inner characteristics, and **Similarity Measurements** for quantifying the similarity between songs, artists, and even genres. We designed **MusicNet** and used tools in machine learning to learn a decent kernel function. Weaponed with kernel function and similarity measurements, we then capture a couple of contagious characteristics by **masking** certain characteristics, and reveal evolutions by **clustering** on feature space.
3. **Dynamic Influence Networks** (**DINs**) for identifying the dynamic levels of the influence of artists over time. By tracing the artists' influences, we can identify some outstanding artists and their influences upon artists or genres. Some might be found revolutionaries, some might be indentified as promoters of evolutions in music, while others may be considered insignificant according to our analysis.

In addition, we also briefly analyzed the potential impact of social and political factors on music.

Finally, we analyze the sensitivity, draw conclusions, and tell the strengths and weaknesses of our models.

Keywords: Influence Network; Pagerank; Kernel Function; MusicNet; Dynamic Influence Networks

Contents

1 Introduction

1.1 Background

Music is an essential component of cultural heritage, and it plays a significant role in our society. When people become free, it's enjoyable for them to appreciate some musical compositions. Many of us may have the experience that sometimes we can figure out the composer of a song without any searching. As an artist, one's music style is relatively stable, and the formation of one's music style is the result of comprehensive factors, such as other artists or genres, social enviroment, technological changes and so on.

Now, the Integrative Collective Music (ICM) Society wants to develop some mathematical models to do quantitative research on musical influence and to examine evolutionary and revolutionary trends of artists and genres. Also, four data sets about music and artists are provided by ICM Society for us to utilize.

1.2 Problem Restatement

In general, we should solve these problems:

(a) Create one or some musical influence network(s), and develop parameters to evaluate musical influence, then substract some subnetworks which contain influencers and their followers.

(b) Develop metrics to measure the similarity between music.

Using the models built to solve the problems above, answer these questions:

- Are artists within genre more similar than artists between genres?
- Do influencers found in problem (a) really influence their followers?
- Are there any musical characteristics which have more significance than others?
- What distinguishes genres and how do genres change over time?
- Are there any characteristics that capture revolutions? Who are revolutionaries?
- How to find those dynamic influencers who promote the evolution of his genre? How do they change?
- How to express external influence on music, artists and genres?

2 Outline of Our Work

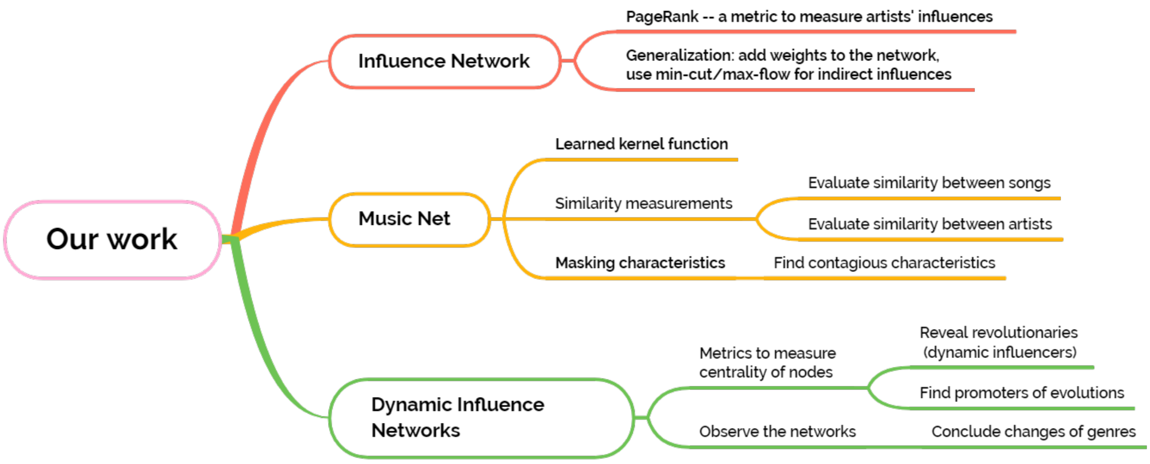


Figure 1: Mind map of our main work.

Firstly, using the data in *'influence_data.csv'*, we build an *Influence Network*, which is a directed graph. In this network, we use the metric *PageRank* to measure artists' influence and develop subnetworks.

Secondly, in order to study the similarity between music, we trained a deep neural network MusicNet to learn a good *Kernel Function*, which maps a song to a vector, then the *Similarity Measures* can be natrually defined.

Last but not least, we equip the influence network with timestamps and turn it into dynamic influence networks. In this network, genres' change can be observed, artists' influence over time can be measured and dynamic influencers can be signified.

3 General Assumptions

To simplify questions and models, we have the following general assumptions:

Assumption 1. *An artist only have one music genre.*

Assumption 2. *The songs created by the same artist are expected to have a more consistent style than the songs created by different artists.*

Assumption 3. *If an artist follows another artists, then he won't unfollow one or some artist(s) he once followed. This is because the data in 'influence_data.csv' didn't record it.*

Assumption 4. *If an artist quits the music scene or dies, he and his music will still influence other artists.*

4 Models

4.1 Influence Network (IN)

We develop a (static) **influence network** for Model I, which is a *directed* network and is *weighted* both on vertices and on edges.

1. First, formally define the influence network $G = (V, E)$, where $(x_1, x_2) \in E$ if and only if the follower x_1, as reported in 'influence_data.csv', is influenced by the influencer x_2.

2. Then, with the established network topology, we can weight each artist (i.e., the vertex $x \in V$) by the well-known **PageRanking** algorithm [1], thus deriving a function for measuring the **influcence** of an artist, $PR : V \longrightarrow [0, 1] \subset \mathbb{R}$.

   Our Algorithm 1 will return a *stable distribution* of PR based on the following equilibrium equations:

$$PR(x) = (1 - d) \cdot \frac{1}{N} + d \cdot \left(\sum_{y \text{ follows } x} \frac{PR(y)}{L(y)} + \sum_{z \ s.t. \ |L(z)|=0} \frac{PR(z)}{N} \right) \qquad \text{for all } x \in V$$

   where $N = |V|$, d is a factor within the interval $(0, 1)$, and $L(x)$ denotes x's influencers.

3. Finally, we weight each arc $(x_1, x_2) \in E$ using the PR values derived previously [1]:

$$w_{x_1 x_2} = \frac{PR_2}{out_1},$$

   where $PR_2 = PR(x_2)$ is the PR value for vertex x_2, and out_1 is the *out-degree* for vertex x_1. It's worth mentioning that the weights we define here measures only the **direct influence** between influencers and their followers, so if one artist is not following another, then current model will not measure a direct influence between them.

Moreover, this is a simple, static model, while a more sophisticated, dynamic model will be given in Section 4.3.

Algorithm 1 PageRank for influence network [2]

Require: The topology of influence network $G = (V, E)$.
Ensure: A PR function that maps each vertice to its influence value.

Initialize $d = 0.85$, $T = 750$, $N = |V|$, $x.F$ is the list of the artists followed by x, $x.I$ is the list of the artists following x.
for all $x \in V$ **do**
    $x.PR := \frac{1}{N}$.
end for
for t in $1, 2, \ldots, T$ **do**
    $dp := 0$
    for all $z \in V$ $s.t.$ $|z.F| = 0$ **do**
        $dp := dp + d * \frac{z.PR}{N}$
    end for
    for all $x \in V$ **do**
        $x.nPR := dp + \frac{1-d}{N}$
        for all $y \in x.I$ **do**
            $x.nPR := x.nPR + d * \frac{y.PR}{|y.F|}$
        end for
    end for
    for all $x \in V$ **do**
        $x.PR := x.nPR$
    end for
end for

4.2 Kernel Function and Similarity Measurements

In Model II, we explore the essential characteristics of a song, the similarity of two songs, and the similarity rate between two artists. The consturction of the model is also threefold:

1. Firstly, we derive a **kernel function** $\phi : S \longrightarrow B(\mathbf{0}; 1)(\subset \mathbb{R}^n)$, which maps a song $s \in S$ to a feature vector on the uni-sphere within high-dimensional feature space $\mathbb{R}^n$. With this carefully designed kernel function, we can define naturally the **distance** D between two songs $s_1, s_2 \in S$ as their distance in feature space:

$$D_\phi(s_1, s_2) = \|\phi(s_1) - \phi(s_2)\|.$$

   We now express the Assumption 2 more formally:

$$\mathbb{E}_{s_1, s_2 \in S_{[x]}} \left[D(s_1, s_2)^2 \right] < \mathbb{E}_{\substack{s_1 \in S_{[x_1]}, \\ s_2 \in S_{[x_2]}}} \left[D(s_1, s_2)^2 \right], \quad x \in V,\ x_1, x_2 \in V$$

   where $S_{[y]}$ denotes the songs created by artist y.

   The derivation of kernel function does not appeal to any musicology knowledge, and instead, it is built completely from the raw data in ‘full_music_data.csv’ based on Assumption 2.

Hence, as shown in Algorithm 2, we will apply **machine learning** techiniques to learn an apposite kernel function ϕ_θ, which is parameterized by θ.

Algorithm 2 Learning the kernel function ϕ.

Require: Songs' vectors: $S = \{s_i : 1 \le i \le |S|\ \}$, partitioned by their artists $S_{[1]}, \dots, S_{[|V|]}$, in which author i creates the songs in $S_{[i]}$.

Ensure: A kernel function $\phi_\theta : S \longrightarrow \mathbb{R}^n$, i.e., the function ϕ is parameterized by θ.

Initialize $t = 0$; initialize θ randomly; set the learning rate η to small constants; set the test interval $T = 100$; set the batch size $B = 32$; set the factor $\alpha \approx 1$ (e.g. $\alpha = 1.01$).

Split the songs into training set $S^{(\text{train})}$ and test set $S^{(\text{test})}$. (Then accordingly, we have $S^{(\text{train})}_{[i]}$ and $S^{(\text{test})}_{[i]}$ for each artist.)

repeat

    for b **in** $1, 2, \dots, B$ **do** (**in parallel**)

        Sample two random artists $i_b, j_b \in [1, |V|]$, and load their songs $S^{(train)}_{[i_b]}, S^{(train)}_{[j_b]}$.

        Derive the loss for artist i_b and j_b:

$$L^{(\text{train})}|_{i_b, j_b; \theta} = \sqrt{\frac{|S^{(train)}_{[i_b]}| \cdot |S^{(train)}_{[j_b]}|}{|S^{(train)}_{[i_b]}|^2 + |S^{(train)}_{[j_b]}|^2} \cdot \frac{\sum_{s_1, s_2 \in S^{(train)}_{[x]},\ x \in \{i_b, j_b\}} \|\phi_\theta(s_1) - \phi_\theta(s_2)\|^2}{\sum_{s_1 \in S^{(train)}_{[i_b]} \text{ and } s_2 \in S^{(train)}_{[j_b]}} \|\phi_\theta(s_1) - \phi_\theta(s_2)\|^2}}$$

        Update the function parameters:

$$\theta := \theta - \eta \frac{\partial L^{(\text{train})}}{\partial \theta}.$$

    end for

    if t is a multiple of T (i.e., $t = kT$) **then**

        Test the resulting parameters:

$$L^{(\text{test})}_k|_\theta = \frac{1}{|V|} \sum_{1 \le i \le |V|} \sqrt{\frac{|S^{(test)}_{[i]}| \cdot |S^{(test)}_{[i+1]}|}{|S^{(test)}_{[i]}|^2} \cdot \frac{\sum_{s_1, s_2 \in S^{(test)}_{[i]}} \|\phi_\theta(s_1) - \phi_\theta(s_2)\|^2}{\sum_{s_l \in S^{(test)}_{[i+l]},\ l=1,2} \|\phi_\theta(s_1) - \phi_\theta(s_2)\|^2}}$$

    end if

    $t := t + 1$

until the sequence $\{L^{(\text{test})}_k\}$ converges

2. After that, given the kernel function ϕ, the songs of the same artist or of similar style are mapped closer to one another. Now we are ready to, accordingly, derive **the similarity of two songs** $s_1, s_2 \in S$ as the inverse of their distance:

$$\begin{aligned} \mathcal{s}(s_1, s_2) &= \frac{1}{D_\phi(s_1, s_2) + \varepsilon} \\ &= \frac{1}{\|\phi(s_1) - \phi(s_2)\| + \varepsilon}, \end{aligned}$$

where $\varepsilon = 0.05$ is a small constant to ensure smoothness.

3. Finally, given all above, we can model **the similairty rate of the styles of two artists** (or in short, **the similarity rate of two artists**). Define the similarity of two artists $x_1, x_2 \in V$ as

$$\mathcal{S}(x_1, x_2) = \frac{1}{|S_{[x_1]}|} \sum_{s_1 \in S_{[x_1]}} \left(\max_{s_2 \in S_{[x_2]}} \mathcal{s}(s_1, s_2) \right).$$

We define so dedicately the above similarity rate as to ensure that it is nonsymmetric:

$$\mathcal{S}(x_1, x_2) \neq \mathcal{S}(x_2, x_1).$$

Our definition of $\mathcal{S}(x_1, x_2)$ actually reveals to what extent does x_1 differ from x_2's style. Figure 2 illustrates an example, in which $\mathcal{S}(x_1, x_2) \gg \mathcal{S}(x_2, x_1)$.

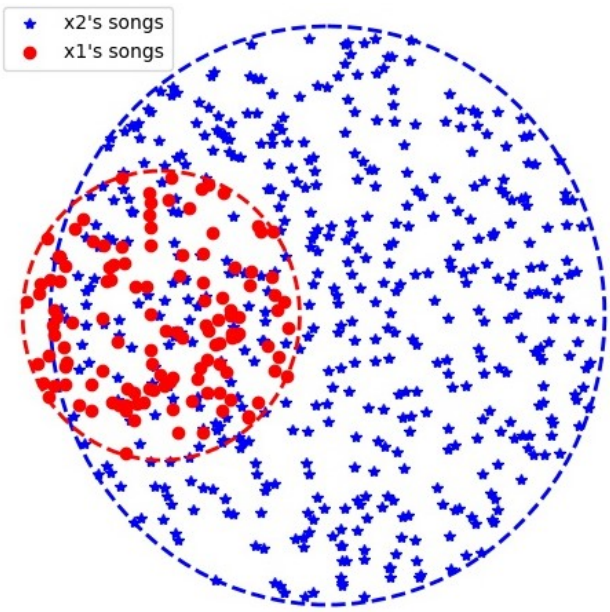


Figure 2: x_1 doesn't differ much from x_2, while x_2 differs a lot from x_1.

4.3 Dynamic Influence Networks (DINs)

In order to study the change of artists' influence over time and reveal the dynamic influencers, we extend our influence network to **dynamic influence networks** (**DINs**), which are networks that change over time, as shown in Figure 3.

Here we concentrate on networks in discrete times. Define the set of time $\mathcal{T}$ as

$$\mathcal{T} = \{t_1, t_2, \ldots, t_n\}, \quad \text{where } t_{i+1} - t_i = t_i - t_{i-1},$$

then the network at t_i is denoted by $G_i = (V_i, E_i)$. Similar to static networks, we can try to find one or some metric(s) to measure the artist's influence or centrality. So far, there are few researches in

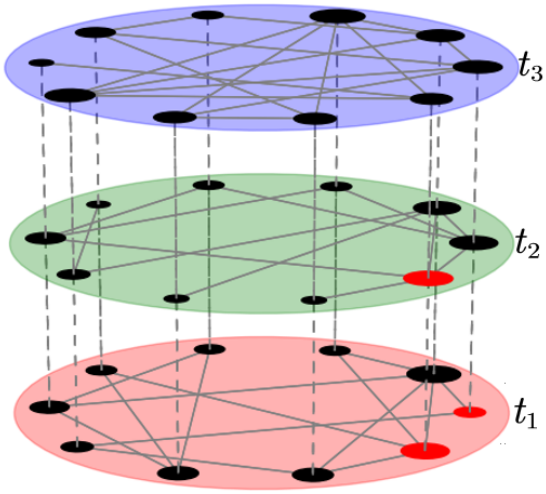


Figure 3: An example of dynamic networks. [3]

this field [4, 5] and their metrics are not suitable for our problem. Thanks to the inspiration from M. Ghanem et al. [6], we can define three relative metrics as follows.

For a node $v \in \cup_{i=1}^{n} V_i$, suppose $k(v) \in \{1, \ldots, n\}$ represents the duration time during which v exists in the network, which means

$$k(v) = \# \{i : v \in V_i, 1 \leq i \leq n\} .$$

And $R(v) = (r_i)_{i=1,\ldots,k(v)}$ is the sequence of ranks in a specific measure for v, then we can define three proportions to measure v's periods with high, low and middle influence:

$$Prop_{high}(v) = \frac{\#\left\{1 \leq i \leq k(v) - 1 : r_i < \lfloor 0.33|V_i|\rfloor\right\}}{k(v)}$$
$$Prop_{low}(v) = \frac{\#\left\{1 \leq i \leq k(v) - 1 : r_i > \lfloor 0.66|V_i|\rfloor\right\}}{k(v)}$$
$$Prop_{mid}(v) = \frac{\#\left\{1 \leq i \leq k(v) - 1 : \lfloor 0.33|V_i|\rfloor \leq r_i \leq \lfloor 0.66|V_i|\rfloor\right\}}{k(v)}$$

As to the sequence of ranks, it can be obtained from the ranks of *PageRank* in each network, which is introduced in the first model. We calculate each node's PageRank in graphs where it exsists, then rank these PageRank values and find which area the node belongs to, then three proportions which are introduced above of each node can be obtained.

In most cases, the value of $Prop_{high}$ has more importance. For a node (artist) v, higher value of $Prop_{high}(v)$ means more musical influence as of the time we calculate it. If one node has bigger value of $Prop_{high}(v)$ in his early yaers in the music scene but smaller value after his quit of the music scene, then he is just a *promoter of the evolution*. However, if a node's value of $Prop_{high}(v)$

is always big, he is identified as a *revolutionary*.

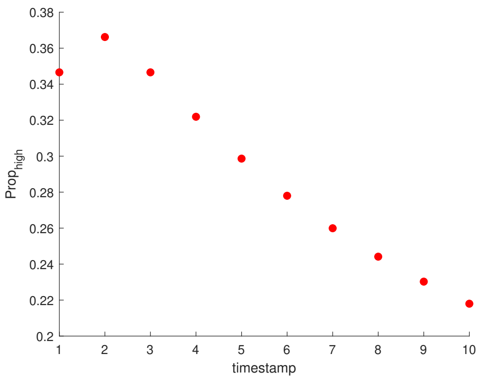


Figure 4: A simple illustration for evolution promoter's $Prop_{high}$.

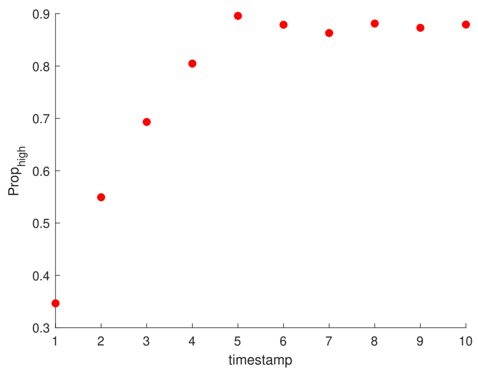


Figure 5: A simple illustration for revolutionary's $Prop_{high}$.

What's more, by plotting the figures of networks in different decades, we can observe the changes of genres. The figure and results refer to Section 5.3, 5.4.

5 Results of Our Model

5.1 IN and Its Subnetworks

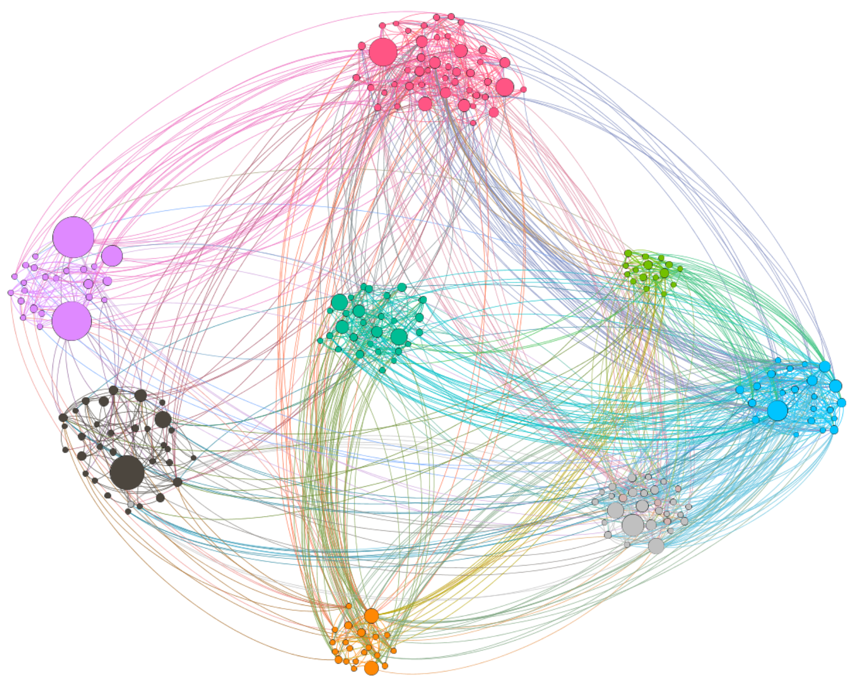


Figure 6: This is a figure of the influence network. In the figure, some insignificant nodes are omitted. The bigger the node is, the larger the PageRank value it has.

We calculate every artist's PageRank value and rank them in descending order, the most 20 influencing artists and their PageRank value are listed in Table 1, they are the biggest nodes in Figure 6.

Table 1: Top 20 in the rank of PR.

Name	Billie Holiday	Cab Calloway	Lester Young	Louis Jordan
$100 \times PR$	2.5091	2.4972	2.1418	1.5421
Rank	1	2	3	4
Name	Sister Rosetta Tharpe	T-Bone Walker	The Beatles	The Mills Brothers
$100 \times PR$	1.2193	1.1559	0.8989	0.8712
Rank	5	6	7	8
Name	M. F. McDowell	Mississippi Sheiks	Muddy Waters	Charlie Christian
$100 \times PR$	0.8249	0.7984	0.7967	0.7819
Rank	9	10	11	12
Name	Roy Acuff	Nat King Cole	Bob Dylan	Billy Eckstine
$100 \times PR$	0.7288	0.6130	0.5734	0.5695
Rank	13	14	15	16
Name	Woody Guthrie	Son House	Hank Williams	Chuck Berry
$100 \times PR$	0.5271	0.5216	0.4993	0.4967
Rank	17	18	19	20

Here are two of the subnetworks of the influence network in 1940.

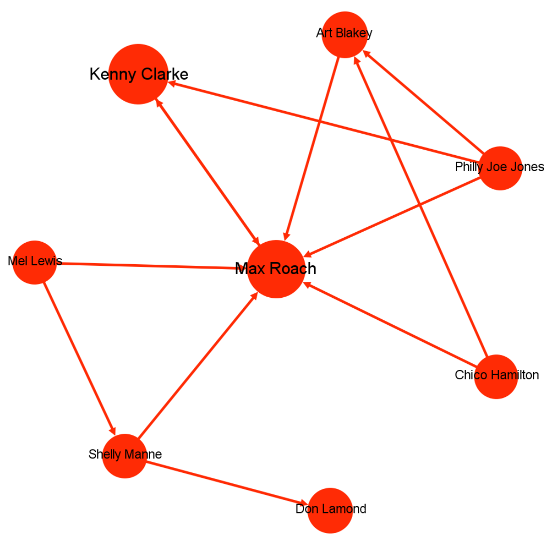


Figure 7: Max Roach and the artists he influenced in 1940.

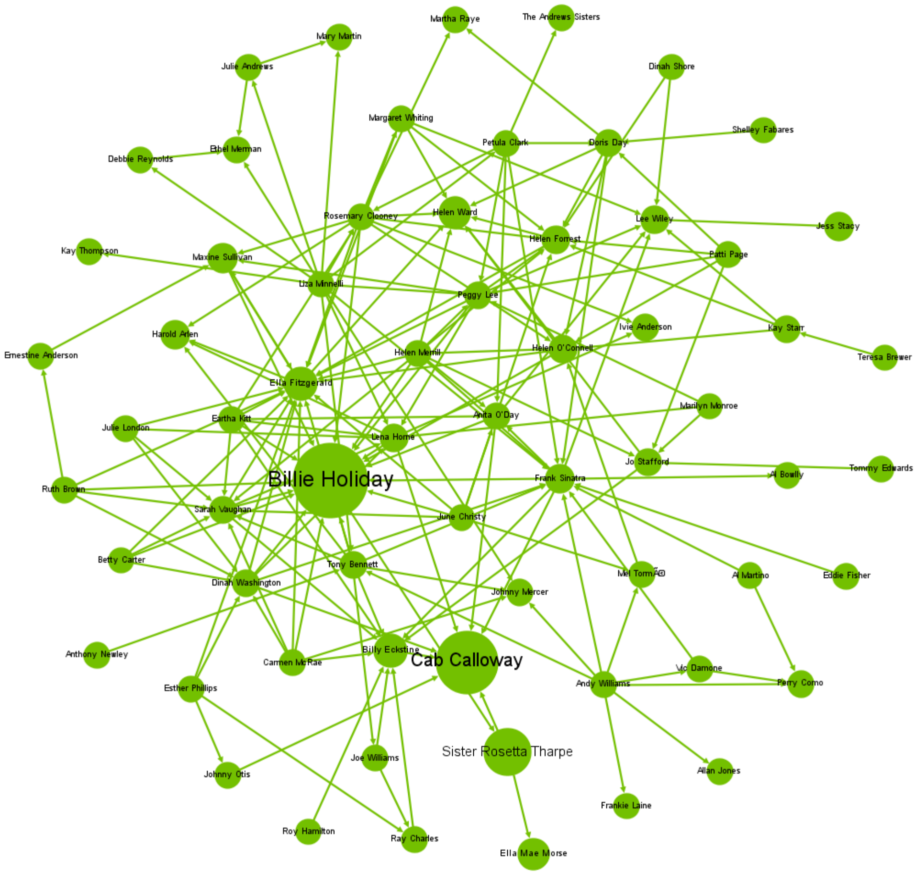


Figure 8: Billie Holiday and the artists she influenced in 1940.

The subnetwork shows the main influencer and the artists that he influences. In Figure 7, the PageRank value of Max Roach is small, so the size of this subnetwork is small. In Figure 8, the PageRank value of Billie Holiday is quite large, so the size of this subnetwork is larger.

According to the music history, Billie Holiday is a reprensentative artist in Jazz music, and her peak period was in the 1940s. As to Max Roach, he just entered the music world in the 1940s and didn't have much influence. That proves the effectiveness of the subnetwork from other angle.

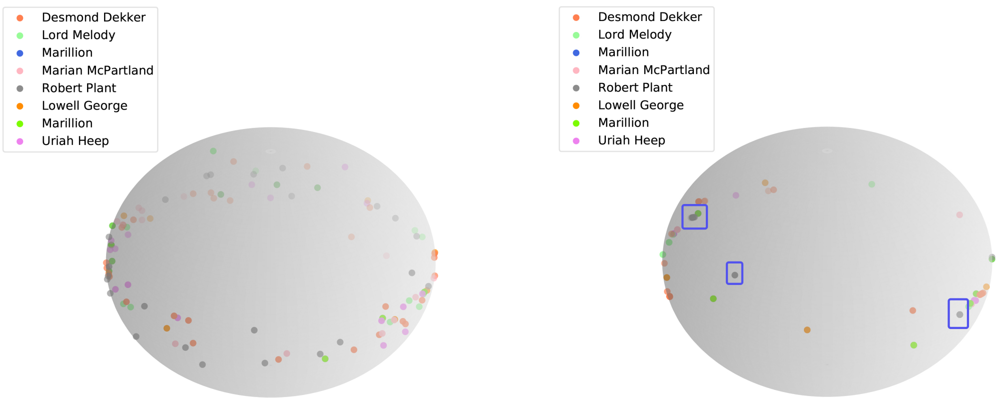
5.2 Well-Learned Kernel

We 've explored the given music dataset and trained MusicNet as a well-learned kernel function. Then, using the kernel, we measure the music similarity as well as the similarity among artists.

5.2.1 Illuastruction of Kernel Performence

We are to show our results of the kernel function that can grasp the features of music songs. In fact, it maps them to a high dimensional sphere, where the songs of a specific artist (or of similar styles) are close to each other and those of different artists (or of different styles) are at a relatively large distance.

Our trained kernel *MusicNet* has a nice performence as the Figure 9 showing below.



(a) without kernel mapping (b) after kernel mapping

Figure 9: Illustruction of high-dimensional kernel mapping, points with different colors denote songs from different artists, artists are sampled from the full-music dataset. It is projected and displayed on 3-dimension sphere by PCA feature selection.

Figure 9(a) is from the raw music characteristic data from the given data file and Figure 9(b) from the features after kernel mapping. It is clear that without kernal mapping, in Figure 9(a), the points spread out in the sphere surface, while, with mapping in Figure 9(b), the number of points seems to become *fewer*. This is just the result that the songs of similar styles are clustered together and largely overlapped.

For example, still in Figure 9, all Robert Plant (gray points) 's songs, which was previously scattered in the sphere in Figure 9(a), now has three gathering places on the sphere surface in Figure 9(b). It sugggests that the artist mainly tried three different styles of songs in different periods. Actually, Rober Plant is a lead singer of Led Zeppelin which is one of Britain's most famous rock band in 1970s. Their songs of Blue, Classic Rock, Jazz are famous all over the world.

Therefore, we're now ready to claim that:

> Our kernel function is WELL-LEARNED and has the ability to gather the songs of the same style together, and distinguish songs of different artists or different styles.

5.2.2 Measure the Similarity among Artists

As demonstrated in above section, we developed the *similarity rate* of styles of two artists $\mathcal{S}$ to measure the similarity between artists.

We choose three artists x_1, x_2, x_3 from genres Pop/Rock, Pop/Rock and Country respectively and calculate the similarity rate among them. The calculation results are showed in table 2.

Table 2: Calculation result of similarity rate among three artists.

Artist x_1	**Artist** x_2	**SimilarityRate** $\mathcal{S}(x_1, x_2)$
Marilyn Manson (Pop/Rock)	Hothouse Flowers (Pop/Rock)	3.469
Hothouse Flowers (Pop/Rock)	Marilyn Manson (Pop/Rock)	0.142
The Wilburn Brothers (Country)	Marilyn Manson (Pop/Rock)	0.075
Marilyn Manson (Pop/Rock)	The Wilburn Brothers (Country)	2.909
Hothouse Flowers (Pop/Rock)	The Wilburn Brothers (Country)	0.152
The Wilburn Brothers (Country)	Hothouse Flowers (Pop/Rock)	0.083

From results we would say it is more likely that artists within genre more similar than artists between genres, since value of $\mathcal{S}$ within Pop/Rock is significantly higher than that within Pop/Rock.

5.2.3 Contagious Music Characteristics

To find out which characteristic of music is more ůcontagious' , we use the trained kernel and perform a **mask process**, in which we mask the feature one by one and observe the changes of similarity rates among artists.

We carefully examined each characteristic in the full music dataset, mask one by one and compare the result with the previous result in table 2.

The distinguished music characterstics we found are *energy, tempo, loudness and acousticness*.

To be more specific, we find that when *energy* feature are masked out, the max similarity rate of two Pop/Rock artists (Marilyn Manson and Hothouse Flowers) leaped from 3.469 to 12.664, when *tempo, loudness* and *acousticness* are masked, that value changed to 34.071, 15.551 and 50.035 respectively. On the other hand, when other features are masked, their similarity rate doesn't changed a lot and still remain around 3.5. This result suggests that these four characteristics could distinguish artists within the same genre.

However, when we mask the other features, we find that most results remain unchanged for artists in different genres, they stay at around (3,0.075) and (0.15,0.08) as the last four rows in table

2. Only when *energy, tempo* and *loudness* are masked, similarity rate $\mathscr{S}$ changes significantly. For instance, when energy feature is set to 0, similarity rate between Hothouse Flowers (Pop/Rock) and The Wilburn Brothers (Country) becomes 0.2208 and 0.2246 respectively, which means their have a unsymmetric influence to each other becomes balanced. When tempo is masked, the same value rockets to 4.5904 and 3.0340 – it is a huge convesion,two artist of different genres become very 'same' when losing tempo feature.

Hence we claim that:

> The *tempo, loudness* and *acousticness* are the most distinguished characteristics, while the other features are considered somehow 'contagious'.

5.3 Observing DINs

The dynamic change of genres over time can be identified in our dynamic influence networks. We draw different networks in different decades, and arrange them into multi-layer networks, as shown in Figure 10.

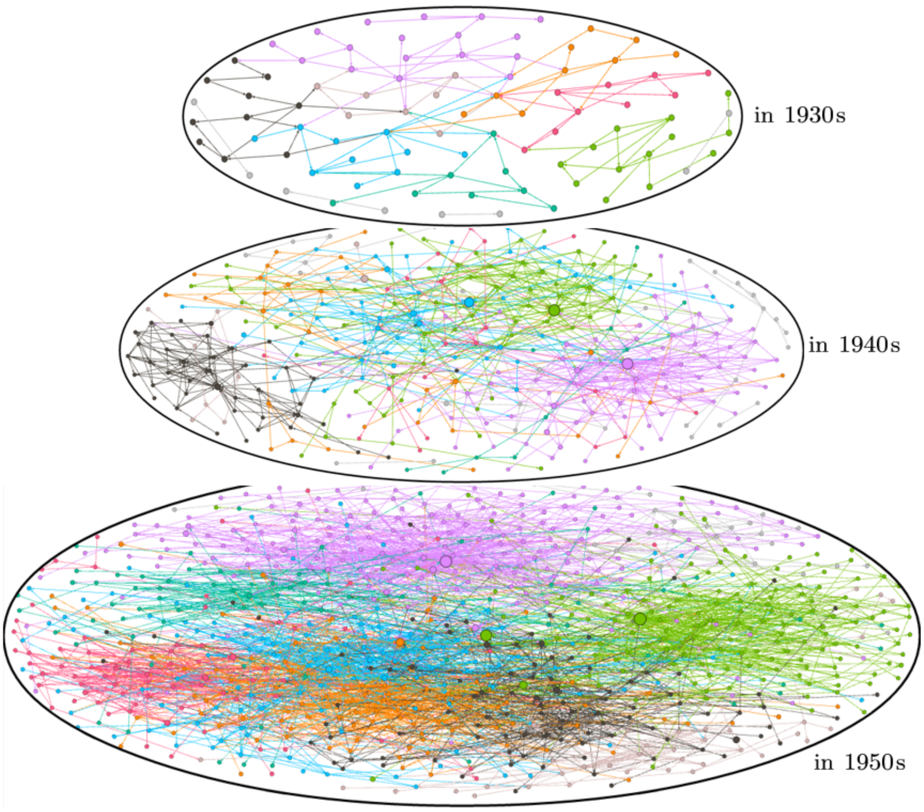


Figure 10: Influence networks from 1930s to 1950s.

In the figure, each node is colored according to their genres, different genres have different colors, and the size of a node depends on the PageRank value of the node. Some answers to our questions are expressed in the figure:

- As time moving on, representative artists(or dynamic influencers) appeared in some genres, while some genres didn't have some outstanding artists.
- Genres with more people grow faster than other genres.
- Different genres relate to each other in the form that artists follow other artists who have different genres from them, because we can find many edges between groups with different colors.
- Early in the music history, the media of music is underdeveloped, artists' influence was quite limited, as a result of which, there were many connected components in the network. With the development of music media, the network tends to be much more connected and have only one connected component.

5.4 Metrics in DINs

Totally, there are 295 artists who have $Prop_{high}$ value over 0.8000, their values range form 0.8000 to 0.8889, they are identified as revolutionaries.

Table 3: Statistics of artists' $Prop_{high} \in [0.8000, 0.8889]$

Number of artists	$Prop_{high}$	**Representative artists**
32	0.8889	Hank Williams, Billie Holiday, Howlin' Wolf, Bob Wills
33	0.8750	Miles Davis, John Coltrane, Ray Charles, Muddy Waters
69	0.8571	Marvin Gaye, Elvis Presley, Chuck Berry, James Brown
95	0.8333	The Beatles, Bob Dylan, The Rolling Stones, David Bowie
66	0.8000	Sex Pistols, The Clash, Brian Eno, Ramones

There are 261 artists who have $Prop_{high}$ value ranging from 0.6000 to 0.7778, they are identified as those who promote the evolution of music.

6 Sensitivity Analysis

In our first model 4.1, the algorithm used to calculate PR is just a process of iteration of vectors by left-multiplying a matrix, and iteration is a smooth process. Thus, it's a robust model.

In our second model 4.2, due to the complexity of machine learning, we actually learnt our kernel function by trials and errors. A small trick is shown in Appendix A. Another useful trick we

Table 4: Statistics of artists' $Prop_{high} \in [0.6000, 0.7778]$

Number of artists	$Prop_{high}$	**Representative artists**
19	0.7778	Gil Evans, Lightnin' Hopkins, Stan Kenton, Rufus Thomas
60	0.7500	Metallica, Nirvana, R.E.M., The Smiths
43	0.7143	Buddy Guy, Dusty Springfield, The Impressions, R. J. Elliott
82	0.6667	Radiohead, Eric Clapton, Jeff Beck, Pearl Jam
31	0.6250	Sonny Rollins, Peggy Lee, The Stanley Brothers
26	0.6000	Echo & The Bunnymen, Kate Bush, Teddy Pendergrass

take to avoid the unstability of similarity $\mathcal{J}$ (defined using the inverse of distanse D) is to apply a smoothing factor ε in the denominator.

In our third model 4.3, the range of $Prop_{high}$ is limited. To be more specific,

$$Prop_{high}(v) \in \left\{ \frac{p}{q} : p, q \in \mathbb{N}, 0 \leq p < q \leq n \right\},$$

where n is the number of timestamps. According to the data set, $n = 9$, but there are more than 5,000 artists, so there is not much difference between their value of $Prop_{high}(v)$, especially when $Prop_{high}(v)$ is very small. That's not what we hope to see. If we have more timestamps, for example, the data of *active_start* in *'influence_data.csv'* could be more accurate, then the value of n and the set of $\left\{ \frac{p}{q} : p, q \in \mathbb{N}, 0 \leq p < q \leq n \right\}$ will become bigger, then we can work out the metrics with higher degree of distinction.

7 Generalizations

7.1 Measure Indirect Influence

In our first model 4.1, though the algorithm of *PageRank* considers some of the indirect influence, its main component depends on the direct influence that influencers have on their followers (namely, directly linked nodes contribute most to one node's PR).

Hence, we consider to find some metrics that focus on indirect influence. The *indirect influence* between artists x_1 and x_2 (x_1 doesn't follow x_2) can be modeled by virtue of the same network. The *max-flow/min-cut* algorithm is suggested to do the job [7].

7.2 For Larger Data Set

If the data set provided becomes larger, in our first and second model 4.1, 4.2, our models still work well. As to the DINs model 4.3, its robustness will be strengthened, and the results of $Prop_{high}, Prop_{mid}, Prop_{low}$ will become more effective.

Besides, some other metrics for centrality can also be introduced into the DINs model. For example, **closeness centrality** [4, 8]. It involves a straightforward replacement of distance with **latency**. In a time-varying network, any time respecting path has a duration, namely the time it takes to follow that path. The fastest such path between two nodes is the latency, note that it is also dependent on the start time. The latency from node i to node j beginning at time t is denoted by $\lambda_{i,t}(j)$. Closeness centrality is defiened as:

$$C_c(i,t) = \frac{|V| - 1}{\sum\limits_{j \neq i} \lambda_{i,t}(j)}.$$

Using various metrics to measure artists' centrality, his music influence can be analysed more comprehensive.

8 Strengths and Weaknesses

Strengths of our models:

- The influence network model only relies on artists' relationship between following and being followed, which is very easy to construct.
- Compared with other simple metrics such as *In-degree*, *PageRank* is a comprehensive metric and it ensures the stability of our model.
- We define the similarity and feature for songs from raw data, without use of the profound music knowledge.
- Our models can still handle the problem when data set becomes larger.
- We have consistency and coherence between the first and the third model, because the PageRank can be used in both two models.

Weaknesses of our models:

- The measurement of indirect influence between artists (i.e., across one or more artists) may appeal to flow/cut algorithms, which requires high computational complexity. Due to limited computing power and limited time, we didn't estimate the indirect influence between all pairs of artists.

- When visualizing our results from the second model 4.2, the *Principal Component Analysis* method is used, during which process some information may be lost.
- So far, our DINs model hasn't been robust enough.

9 Document to ICM Society

Dear ICM fellows:

Given the data set of artists' influence relation and their songs' characteristics, we've developed several interesting models, which is absolutely inspring for understanding the beautiful languages of music!

Here are them:

1. The **Influence Network** (**IN**) model. It's really amazing that in such a simple, efficient and elegant way can we estimate the influence of every artist in the network. Especially as the scale of the music industry continues to grow and the distribution of music is increasing rapidly, our model will be greatly suitable for not only individual music entertainers but also the music industry.
2. The **Kernel Function** and **Similarity Measures**. What is the song's language in nature? Which style does the song follows? Is it a rare masterpiece, or a mediocre work? Did an artist, or a genre really shed light on the development of musics? All these are explained in this model.
3. The **Dynamic Influence Networks** (**DINs**). Tracing the evolutions of music history is a coooooool thing. We illustrate in this model which songs, or which artists had never faded away in decades. *"Emm, this team's work sounds very brilliant and nostalgic! I strongly recommend their works. (Oh, of course, I won't tell you that it's because they ranked me first.)"* – The Beatles.

For more details of our models and results, please refer to our paper!

Moreover, we're looking forward to a richer data set containg more diverse genres! Since all these established models are iterative method, they are still practical for larger input sizes and can perform robustly with large streams of input data.

Hope you can continue offering high-quality data, and they are indeed helpful for better analysis and wiser results!

Last but not least, it's subtle to take cultural or political factors into consideration. *"Music is the universal language of mankind."* Cultural backgrounds and political settings may leave an impact upon music works, and music works may, vice versa, indicate the evolution of cultural patterns and social patterns.

Sincerely,

Team 2123278

References

[1] Lawrence Page, Sergey Brin, Rajeev Motwani, and Terry Winograd. The pagerank citation ranking: Bringing order to the web. Technical report, Stanford InfoLab, 1999.

[2] Keshi Dai. Pagerank lecture note, 2009. `https://www.ccs.neu.edu/home/daikeshi/notes/PageRank.pdf`.

[3] Multilayer networks library for python (pymnet). `http://www.mkivela.com/pymnet/visualizing.html`.

[4] Peter Grindrod, Mark C. Parsons, Desmond J. Higham, and Ernesto Estrada. Communicability across evolving networks. *Phys. Rev. E*, 83:046120, Apr 2011.

[5] Petter Holme and Jari Saramäki. Temporal networks. *Physics Reports*, 519(3):97–125, 2012. Temporal Networks.

[6] M. Ghanem, C. Magnien, and F. Tarissan. Centrality metrics in dynamic networks: A comparison study. *IEEE Transactions on Network Science and Engineering*, 6(4):940–951, 2019.

[7] Béla Bollobás. *Modern graph theory*, volume 184. Springer Science & Business Media, 2013.

[8] Temporal network. `https://en.wikipedia.org/wiki/Temporal_network`.

Appendices

Appendix A A Shortcut for Estimating Songs' Similarity

It's not a walk in the park to calculate the distance/similarity between groups of songs. In fact, Algorithm 2, as well as the subsequent definitions of similarity, relies heavily on the computation of the distance between groups of songs, so an efficient method is needed to simplify and accelerate the calculation.

To state formally, i.e. for two sets of songs $S_{[i]}$ and $S_{[j]}$, we need to estimate

$$D = \sqrt{\frac{1}{|S_{[i]}| \cdot |S_{[j]}|} \sum_{\substack{s_1 \in S_{[i]},\\ s_2 \in S_{[j]}}} \|\phi(s_1) - \phi(s_2)\|^2}. \tag{1}$$

Since ϕ always maps a song onto a uni-sphere, for every $s \in S$, we will have

$$\|\phi(s)\| = r = 1.$$

Then, interestingly,

$$\begin{aligned} \|\phi(s_1) - \phi(s_2)\| &= \widehat{\phi(s_1)\ \phi(s_2)} \qquad (\widehat{AB} \text{ is arc } AB\text{'s length on uni-sphere}) \\ &= r \cdot \langle \phi(s_1), \phi(s_2) \rangle = \langle \phi(s_1), \phi(s_2) \rangle \\ &= \arccos\left(\phi(s_1)^T \phi(s_2)\right) \approx \frac{\pi}{2} - \phi(s_1)^T \phi(s_2), \end{aligned}$$

where the last approximation follows from Taylor expansion: $\arccos x = \frac{\pi}{2} - x + o(x^2)$ for $x \approx 0$.

Use the above approximation formula, we can rewrite the Formula 1 into

$$D = \sqrt{\frac{1}{|S_{[i]}| \cdot |S_{[j]}|} \sum_{\substack{s_1 \in S_{[i]},\\ s_2 \in S_{[j]}}} \langle \phi(s_1), \phi(s_2) \rangle^2} \approx \frac{1}{\sqrt{|S_{[i]}| \cdot |S_{[j]}|}} \cdot \|\frac{\pi}{2} - \phi(S_{[i]})^T \phi(S_{[j]})\|_F, \tag{2}$$

where $\phi(S_{[i]})$ is computed simply by feeding every song $s \in S_{[i]}$ into ϕ, and putting the outcoming feature vectors altogether into a matrix.

With Formula 2, the calculation of D is greatly simplified and many existing accelerating techiniques for matrix product now become available.

Appendix B Result of Music Feature Masking

We mask each feature of music characteristic and the results are as follows, we round the result in to three decimal.

Table 5: Similarity rate between Pop/Rock artists Marilyn Manson (x_1) and Hothouse Flowers (x_2).

masked feature	**similarity rate** $\mathcal{S}(x_1, x_2)$	**similarity rate** $\mathcal{S}(x_2, x_1)$
None	3.469	0.142
danceability	3.158	0.128
energy	12.665	0.116
valence	3.463	0.142
tempo	34.071	0.113
loudness	15.551	0.3345
mode	3.546	0.125
key	3.524	0.124
acousticness	50.035	0.157
instrumentalness	3.459	0.142
liveness	3.476	0.141
speechiness	3.474	0..131
explicit	3.710	0.129
duration(ms)	3.470	0.142

Table 6: Similarity rate between Pop/Rock artist Marilyn Manson (x_1) and Country artist The Wilburn Brothers (x_2)

masked feature	**similarity rate** $\mathcal{S}(x_1, x_2)$	**similarity rate** $\mathcal{S}(x_2, x_1)$
None	2.909	0.075
danceability	2.957	0.075
energy	3.128	0.075
valence	2.908	0.075
tempo	34.20	0.075
loudness	4.299	0.122
mode	2.979	0.075
key	3.048	0.075
acousticness	2.911	0.075
instrumentalness	2.908	0.075
liveness	2.911	0.075
speechiness	2.935	0.075
explicit	2.982	0.076
duration(ms)	2.909	0.075

Table 7: Similarity Rate between Pop/Rock artist Hothouse Flowers (x_1) and Country artist The Wilburn Brothers (x_2)

masked feature	**similarity rate** $\mathcal{S}(x_1, x_2)$	**similarity rate** $\mathcal{S}(x_2, x_1)$
None	0.152	0.083
danceability	0.152	0.083
energy	0.221	0.225
valence	0.152	0.083
tempo	4.590	3.034
loudness	0.208	0.156
mode	0.152	0.083
key	0.186	0.083
acousticness	0.117	0.086
instrumentalness	0.152	0.083
liveness	0.152	0.083
speechiness	0.152	0.083
explicit	0.152	0.083
duration(ms)	0.152	0.083