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| --- |
| == Example 1: f(x1,x2) = 6x1^2 + 4x1x2 + 3x2^2, h(x1,x2) = x1 + x2 - 5 ==  Iter 1: x1 = 0.100000, x2 = 0.100000, h = -4.800000  Iter 2: x1 = 0.276000, x2 = 0.282000, h = -4.442000  Iter 3: x1 = 0.505280, x2 = 0.527720, h = -3.967000  Iter 4: x1 = 0.767058, x2 = 0.819366, h = -3.413577  Iter 5: x1 = 1.042959, x2 = 1.140244, h = -2.816796  Iter 6: x1 = 1.317318, x2 = 1.475235, h = -2.207448  Iter 7: x1 = 1.577316, x2 = 1.811113, h = -1.611571  Iter 8: x1 = 1.812992, x2 = 2.136753, h = -1.050254  Iter 9: x1 = 2.017141, x2 = 2.443206, h = -0.539652  Iter 10: x1 = 2.185115, x2 = 2.723687, h = -0.091198  Iter 11: x1 = 2.314568, x2 = 2.973475, h = 0.288043  Iter 12: x1 = 2.405149, x2 = 3.189752, h = 0.594901  Iter 13: x1 = 2.458174, x2 = 3.371394, h = 0.829568  Iter 14: x1 = 2.476285, x2 = 3.518732, h = 0.995017  Iter 15: x1 = 2.463121, x2 = 3.633296, h = 1.096417  Iter 16: x1 = 2.422997, x2 = 3.717556, h = 1.140553  Iter 17: x1 = 2.360624, x2 = 3.774671, h = 1.135296  Iter 18: x1 = 2.280851, x2 = 3.808254, h = 1.089105  Iter 19: x1 = 2.188448, x2 = 3.822155, h = 1.010603  Iter 20: x1 = 2.087936, x2 = 3.820275, h = 0.908211  Iter 21: x1 = 1.983444, x2 = 3.806413, h = 0.789857  Iter 22: x1 = 1.878616, x2 = 3.784132, h = 0.662747  Iter 23: x1 = 1.776545, x2 = 3.756668, h = 0.533213  Iter 24: x1 = 1.679748, x2 = 3.726861, h = 0.406609  Iter 25: x1 = 1.590159, x2 = 3.697114, h = 0.287273  Iter 26: x1 = 1.509151, x2 = 3.669377, h = 0.178529  Iter 27: x1 = 1.437579, x2 = 3.645149, h = 0.082728  Iter 28: x1 = 1.375825, x2 = 3.625499, h = 0.001324  Iter 29: x1 = 1.323870, x2 = 3.611100, h = -0.065031  Iter 30: x1 = 1.281353, x2 = 3.602270, h = -0.116377  Iter 31: x1 = 1.247645, x2 = 3.599026, h = -0.153329  Iter 32: x1 = 1.221918, x2 = 3.601130, h = -0.176952  Iter 33: x1 = 1.203206, x2 = 3.608148, h = -0.188646  Iter 34: x1 = 1.190465, x2 = 3.619501, h = -0.190035  Iter 35: x1 = 1.182627, x2 = 3.634510, h = -0.182863  Iter 36: x1 = 1.178643, x2 = 3.652446, h = -0.168910  Iter 37: x1 = 1.177519, x2 = 3.672565, h = -0.149916  Iter 38: x1 = 1.178344, x2 = 3.694140, h = -0.127516  Iter 39: x1 = 1.180309, x2 = 3.716489, h = -0.103201  Iter 40: x1 = 1.182722, x2 = 3.738997, h = -0.078281  Iter 41: x1 = 1.185012, x2 = 3.761125, h = -0.053862  Iter 42: x1 = 1.186731, x2 = 3.782423, h = -0.030845  Iter 43: x1 = 1.187549, x2 = 3.802531, h = -0.009920  Iter 44: x1 = 1.187244, x2 = 3.821179, h = 0.008423  Iter 45: x1 = 1.185694, x2 = 3.838185, h = 0.023880  Iter 46: x1 = 1.182864, x2 = 3.853447, h = 0.036310  Iter 47: x1 = 1.178788, x2 = 3.866930, h = 0.045718  Iter 48: x1 = 1.173559, x2 = 3.878666, h = 0.052224  Iter 49: x1 = 1.167313, x2 = 3.888732, h = 0.056045  Iter 50: x1 = 1.160217, x2 = 3.897246, h = 0.057463  Iter 51: x1 = 1.152454, x2 = 3.904356, h = 0.056810  Iter 52: x1 = 1.144216, x2 = 3.910226, h = 0.054442  Iter 53: x1 = 1.135689, x2 = 3.915033, h = 0.050722  Iter 54: x1 = 1.127054, x2 = 3.918952, h = 0.046006  Iter 55: x1 = 1.118472, x2 = 3.922155, h = 0.040627  Iter 56: x1 = 1.110087, x2 = 3.924805, h = 0.034892  Iter 57: x1 = 1.102019, x2 = 3.927048, h = 0.029067  Iter 58: x1 = 1.094365, x2 = 3.929014, h = 0.023379  Iter 59: x1 = 1.087196, x2 = 3.930815, h = 0.018011  Iter 60: x1 = 1.080563, x2 = 3.932541, h = 0.013104  Iter 61: x1 = 1.074493, x2 = 3.934265, h = 0.008758  Iter 62: x1 = 1.068994, x2 = 3.936040, h = 0.005035  Iter 63: x1 = 1.064058, x2 = 3.937903, h = 0.001961  Iter 64: x1 = 1.059662, x2 = 3.939873, h = -0.000465  Iter 65: x1 = 1.055772, x2 = 3.941959, h = -0.002268  Iter 66: x1 = 1.052348, x2 = 3.944157, h = -0.003495  Iter 67: x1 = 1.049340, x2 = 3.946454, h = -0.004205  Iter 68: x1 = 1.046700, x2 = 3.948832, h = -0.004467  Iter 69: x1 = 1.044376, x2 = 3.951268, h = -0.004355  Iter 70: x1 = 1.042319, x2 = 3.953735, h = -0.003946  Iter 71: x1 = 1.040480, x2 = 3.956208, h = -0.003312  Iter 72: x1 = 1.038817, x2 = 3.958659, h = -0.002524  Iter 73: x1 = 1.037290, x2 = 3.961064, h = -0.001646  Iter 74: x1 = 1.035866, x2 = 3.963401, h = -0.000733  Iter 75: x1 = 1.034515, x2 = 3.965652, h = 0.000167  Iter 76: x1 = 1.033215, x2 = 3.967800, h = 0.001015  Iter 77: x1 = 1.031948, x2 = 3.969834, h = 0.001782  Iter 78: x1 = 1.030700, x2 = 3.971746, h = 0.002446  Iter 79: x1 = 1.029464, x2 = 3.973530, h = 0.002994  Iter 80: x1 = 1.028234, x2 = 3.975187, h = 0.003420  Iter 81: x1 = 1.027008, x2 = 3.976716, h = 0.003723  Iter 82: x1 = 1.025787, x2 = 3.978122, h = 0.003909  Iter 83: x1 = 1.024575, x2 = 3.979410, h = 0.003985  Iter 84: x1 = 1.023376, x2 = 3.980588, h = 0.003964  Iter 85: x1 = 1.022194, x2 = 3.981665, h = 0.003860  Iter 86: x1 = 1.021036, x2 = 3.982650, h = 0.003686  Iter 87: x1 = 1.019908, x2 = 3.983551, h = 0.003459  Iter 88: x1 = 1.018814, x2 = 3.984379, h = 0.003193  Iter 89: x1 = 1.017760, x2 = 3.985142, h = 0.002902  Iter 90: x1 = 1.016749, x2 = 3.985850, h = 0.002599  Iter 91: x1 = 1.015786, x2 = 3.986509, h = 0.002295  Iter 92: x1 = 1.014872, x2 = 3.987128, h = 0.002000  Iter 93: x1 = 1.014009, x2 = 3.987712, h = 0.001721  Iter 94: x1 = 1.013197, x2 = 3.988267, h = 0.001464  Iter 95: x1 = 1.012436, x2 = 3.988796, h = 0.001233  Iter 96: x1 = 1.011726, x2 = 3.989305, h = 0.001030  Iter 97: x1 = 1.011063, x2 = 3.989794, h = 0.000858  Iter 98: x1 = 1.010447, x2 = 3.990267, h = 0.000715  Iter 99: x1 = 1.009875, x2 = 3.990725, h = 0.000600  Iter 100: x1 = 1.009343, x2 = 3.991169, h = 0.000512  Final solution: f(x) = 70.014766, constraint = 0.000512  == Example 2: g(y1,y2) = 5y1^2 + 2y1y2 + 7y2^2, c(y1,y2) = y1 - y2 - 3 ==  Iter 1: x1 = 0.060000, x2 = -0.060000, h = -2.880000  Iter 2: x1 = 0.170400, x2 = -0.168000, h = -2.661600  Iter 3: x1 = 0.320784, x2 = -0.311952, h = -2.367264  Iter 4: x1 = 0.500467, x2 = -0.480217, h = -2.019316  Iter 5: x1 = 0.698975, x2 = -0.661946, h = -1.639079  Iter 6: x1 = 0.906443, x2 = -0.847380, h = -1.246177  Iter 7: x1 = 1.113939, x2 = -1.028068, h = -0.857994  Iter 8: x1 = 1.313695, x2 = -1.197005, h = -0.489300  Iter 9: x1 = 1.499266, x2 = -1.348699, h = -0.152035  Iter 10: x1 = 1.665609, x2 = -1.479163, h = 0.144772  Iter 11: x1 = 1.809096, x2 = -1.585856, h = 0.394952  Iter 12: x1 = 1.927465, x2 = -1.667580, h = 0.595046  Iter 13: x1 = 2.019729, x2 = -1.724327, h = 0.744057  Iter 14: x1 = 2.086041, x2 = -1.757114, h = 0.843154  Iter 15: x1 = 2.127531, x2 = -1.767791, h = 0.895323  Iter 16: x1 = 2.146137, x2 = -1.758854, h = 0.904991  Iter 17: x1 = 2.144410, x2 = -1.733247, h = 0.877657  Iter 18: x1 = 2.125337, x2 = -1.694184, h = 0.819521  Iter 19: x1 = 2.092162, x2 = -1.644980, h = 0.737142  Iter 20: x1 = 2.048226, x2 = -1.588906, h = 0.637132  Iter 21: x1 = 1.996819, x2 = -1.529062, h = 0.525881  Iter 22: x1 = 1.941064, x2 = -1.468275, h = 0.409338  Iter 23: x1 = 1.883812, x2 = -1.409027, h = 0.292839  Iter 24: x1 = 1.827574, x2 = -1.353402, h = 0.180975  Iter 25: x1 = 1.774465, x2 = -1.303057, h = 0.077522  Iter 26: x1 = 1.726178, x2 = -1.259217, h = -0.014605  Iter 27: x1 = 1.683978, x2 = -1.222684, h = -0.093338  Iter 28: x1 = 1.648709, x2 = -1.193863, h = -0.157428  Iter 29: x1 = 1.620821, x2 = -1.172801, h = -0.206378  Iter 30: x1 = 1.600406, x2 = -1.159237, h = -0.240356  Iter 31: x1 = 1.587249, x2 = -1.152651, h = -0.260100  Iter 32: x1 = 1.580873, x2 = -1.152320, h = -0.266807  Iter 33: x1 = 1.580597, x2 = -1.157378, h = -0.262024  Iter 34: x1 = 1.585596, x2 = -1.166868, h = -0.247536  Iter 35: x1 = 1.594945, x2 = -1.179790, h = -0.225265  Iter 36: x1 = 1.607678, x2 = -1.195149, h = -0.197173  Iter 37: x1 = 1.622826, x2 = -1.211995, h = -0.165179  Iter 38: x1 = 1.639460, x2 = -1.229449, h = -0.131091  Iter 39: x1 = 1.656720, x2 = -1.246732, h = -0.096548  Iter 40: x1 = 1.673839, x2 = -1.263181, h = -0.062980  Iter 41: x1 = 1.690164, x2 = -1.278257, h = -0.031579  Iter 42: x1 = 1.705161, x2 = -1.291553, h = -0.003285  Iter 43: x1 = 1.718425, x2 = -1.302788, h = 0.021212  Iter 44: x1 = 1.729672, x2 = -1.311800, h = 0.041472  Iter 45: x1 = 1.738741, x2 = -1.318541, h = 0.057282  Iter 46: x1 = 1.745575, x2 = -1.323058, h = 0.068633  Iter 47: x1 = 1.750217, x2 = -1.325479, h = 0.075697  Iter 48: x1 = 1.752788, x2 = -1.326000, h = 0.078787  Iter 49: x1 = 1.753474, x2 = -1.324861, h = 0.078335  Iter 50: x1 = 1.752512, x2 = -1.322337, h = 0.074849  Iter 51: x1 = 1.750168, x2 = -1.318721, h = 0.068889  Iter 52: x1 = 1.746727, x2 = -1.314305, h = 0.061033  Iter 53: x1 = 1.742479, x2 = -1.309375, h = 0.051854  Iter 54: x1 = 1.737703, x2 = -1.304197, h = 0.041901  Iter 55: x1 = 1.732663, x2 = -1.299010, h = 0.031672  Iter 56: x1 = 1.727594, x2 = -1.294018, h = 0.021612  Iter 57: x1 = 1.722701, x2 = -1.289394, h = 0.012094  Iter 58: x1 = 1.718153, x2 = -1.285267, h = 0.003420  Iter 59: x1 = 1.714083, x2 = -1.281732, h = -0.004185  Iter 60: x1 = 1.710584, x2 = -1.278847, h = -0.010569  Iter 61: x1 = 1.707717, x2 = -1.276634, h = -0.015649  Iter 62: x1 = 1.705507, x2 = -1.275089, h = -0.019404  Iter 63: x1 = 1.703950, x2 = -1.274179, h = -0.021871  Iter 64: x1 = 1.703018, x2 = -1.273851, h = -0.023131  Iter 65: x1 = 1.702660, x2 = -1.274039, h = -0.023301  Iter 66: x1 = 1.702811, x2 = -1.274663, h = -0.022526  Iter 67: x1 = 1.703394, x2 = -1.275638, h = -0.020968  Iter 68: x1 = 1.704327, x2 = -1.276876, h = -0.018798  Iter 69: x1 = 1.705523, x2 = -1.278292, h = -0.016185  Iter 70: x1 = 1.706900, x2 = -1.279805, h = -0.013295  Iter 71: x1 = 1.708378, x2 = -1.281341, h = -0.010281  Iter 72: x1 = 1.709884, x2 = -1.282838, h = -0.007278  Iter 73: x1 = 1.711354, x2 = -1.284241, h = -0.004405  Iter 74: x1 = 1.712737, x2 = -1.285507, h = -0.001756  Iter 75: x1 = 1.713988, x2 = -1.286606, h = 0.000594  Iter 76: x1 = 1.715078, x2 = -1.287517, h = 0.002595  Iter 77: x1 = 1.715985, x2 = -1.288230, h = 0.004215  Iter 78: x1 = 1.716698, x2 = -1.288745, h = 0.005444  Iter 71: x1 = 1.708378, x2 = -1.281341, h = -0.010281  Iter 72: x1 = 1.709884, x2 = -1.282838, h = -0.007278  Iter 73: x1 = 1.711354, x2 = -1.284241, h = -0.004405  Iter 74: x1 = 1.712737, x2 = -1.285507, h = -0.001756  Iter 75: x1 = 1.713988, x2 = -1.286606, h = 0.000594  Iter 76: x1 = 1.715078, x2 = -1.287517, h = 0.002595  Iter 77: x1 = 1.715985, x2 = -1.288230, h = 0.004215  Iter 78: x1 = 1.716698, x2 = -1.288745, h = 0.005444  Iter 75: x1 = 1.713988, x2 = -1.286606, h = 0.000594  Iter 76: x1 = 1.715078, x2 = -1.287517, h = 0.002595  Iter 77: x1 = 1.715985, x2 = -1.288230, h = 0.004215  Iter 78: x1 = 1.716698, x2 = -1.288745, h = 0.005444  Iter 76: x1 = 1.715078, x2 = -1.287517, h = 0.002595  Iter 77: x1 = 1.715985, x2 = -1.288230, h = 0.004215  Iter 78: x1 = 1.716698, x2 = -1.288745, h = 0.005444  Iter 79: x1 = 1.717218, x2 = -1.289069, h = 0.006287  Iter 77: x1 = 1.715985, x2 = -1.288230, h = 0.004215  Iter 78: x1 = 1.716698, x2 = -1.288745, h = 0.005444  Iter 79: x1 = 1.717218, x2 = -1.289069, h = 0.006287  Iter 78: x1 = 1.716698, x2 = -1.288745, h = 0.005444  Iter 79: x1 = 1.717218, x2 = -1.289069, h = 0.006287  Iter 79: x1 = 1.717218, x2 = -1.289069, h = 0.006287  Iter 80: x1 = 1.717549, x2 = -1.289215, h = 0.006764  Iter 80: x1 = 1.717549, x2 = -1.289215, h = 0.006764  Iter 81: x1 = 1.717705, x2 = -1.289203, h = 0.006908  Iter 82: x1 = 1.717704, x2 = -1.289054, h = 0.006758  Iter 81: x1 = 1.717705, x2 = -1.289203, h = 0.006908  Iter 82: x1 = 1.717704, x2 = -1.289054, h = 0.006758  Iter 83: x1 = 1.717568, x2 = -1.288794, h = 0.006363  Iter 82: x1 = 1.717704, x2 = -1.289054, h = 0.006758  Iter 83: x1 = 1.717568, x2 = -1.288794, h = 0.006363  Iter 84: x1 = 1.717322, x2 = -1.288449, h = 0.005770  Iter 83: x1 = 1.717568, x2 = -1.288794, h = 0.006363  Iter 84: x1 = 1.717322, x2 = -1.288449, h = 0.005770  Iter 84: x1 = 1.717322, x2 = -1.288449, h = 0.005770  Iter 85: x1 = 1.716989, x2 = -1.288043, h = 0.005032  Iter 86: x1 = 1.716596, x2 = -1.287601, h = 0.004197  Iter 87: x1 = 1.716166, x2 = -1.287147, h = 0.003312  Iter 88: x1 = 1.715721, x2 = -1.286698, h = 0.002419  Iter 89: x1 = 1.715281, x2 = -1.286273, h = 0.001555  Iter 90: x1 = 1.714863, x2 = -1.285885, h = 0.000749  Iter 91: x1 = 1.714480, x2 = -1.285544, h = 0.000025  Iter 92: x1 = 1.714143, x2 = -1.285258, h = -0.000599  Iter 93: x1 = 1.713858, x2 = -1.285029, h = -0.001113  Iter 94: x1 = 1.713630, x2 = -1.284859, h = -0.001512  Iter 95: x1 = 1.713459, x2 = -1.284746, h = -0.001795  Iter 96: x1 = 1.713344, x2 = -1.284687, h = -0.001968  Iter 97: x1 = 1.713283, x2 = -1.284677, h = -0.002040  Iter 98: x1 = 1.713270, x2 = -1.284710, h = -0.002020  Iter 99: x1 = 1.713299, x2 = -1.284777, h = -0.001924  Iter 100: x1 = 1.713363, x2 = -1.284873, h = -0.001765  Final solution: f(x) = 21.831436, constraint = -0.001765 |

Code

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| --- |
| #include <iostream>  #include <cmath>  #include <iomanip>  #include <functional>  using namespace std;  const double EPSILON = 1e-6;  const int MAX\_ITER = 100;  const double ALPHA = 0.01;  void solveALM(      function<double(double, double)> f,      function<double(double, double)> h,      function<double(double, double, double)> df\_dx1,      function<double(double, double, double)> df\_dx2,      double x1\_init, double x2\_init,      string title  ) {      double x1 = x1\_init, x2 = x2\_init;      double lambda = 0.0;      double rk = 1.0;      cout << "== " << title << " ==" << endl;      cout << fixed << setprecision(6);      for (int k = 0; k < MAX\_ITER; ++k) {          double constraint = h(x1, x2);            double g1 = df\_dx1(x1, x2, lambda + 2 \* rk \* constraint);          double g2 = df\_dx2(x1, x2, lambda + 2 \* rk \* constraint);          x1 -= ALPHA \* g1;          x2 -= ALPHA \* g2;          constraint = h(x1, x2);          lambda += 2 \* rk \* constraint;          cout << "Iter " << k + 1 << ": x1 = " << x1 << ", x2 = " << x2 << ", h = " << constraint << endl;          if (fabs(constraint) < EPSILON) break;      }      cout << "Final solution: f(x) = " << f(x1, x2) << ", constraint = " << h(x1, x2) << "\n\n";  }  int main() {      //  1: f(x) = 6x1^2 + 4x1x2 + 3x2^2, h(x) = x1 + x2 - 5      solveALM(          [](double x1, double x2) {              return 6 \* x1 \* x1 + 4 \* x1 \* x2 + 3 \* x2 \* x2;          },          [](double x1, double x2) {              return x1 + x2 - 5;          },          [](double x1, double x2, double lag\_term) {              return 12 \* x1 + 4 \* x2 + lag\_term;          },          [](double x1, double x2, double lag\_term) {              return 4 \* x1 + 6 \* x2 + lag\_term;          },          0.0, 0.0,          "Example 1: f(x1,x2) = 6x1^2 + 4x1x2 + 3x2^2, h(x1,x2) = x1 + x2 - 5"      );      //  2: g(y) = 5y1^2 + 2y1y2 + 7y2^2, c(y) = y1 - y2 - 3      solveALM(          [](double y1, double y2) {              return 5 \* y1 \* y1 + 2 \* y1 \* y2 + 7 \* y2 \* y2;          },          [](double y1, double y2) {              return y1 - y2 - 3;          },          [](double y1, double y2, double lag\_term) {              return 10 \* y1 + 2 \* y2 + lag\_term;          },          [](double y1, double y2, double lag\_term) {              return 2 \* y1 + 14 \* y2 - lag\_term;          },          0.0, 0.0,          "Example 2: g(y1,y2) = 5y1^2 + 2y1y2 + 7y2^2, c(y1,y2) = y1 - y2 - 3"      );      return 0;  } |