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Module 2: Problem Set

Section 3.13(4)

$$5x + 2 \equiv 3x - 7 \pmod{31}$$

- Subtract $3x$ from both sides:

$$(5x - 3x) + 2 \equiv -7 \pmod{31}$$

$$2x + 2 \equiv -7 \pmod{31}$$

- Subtract 2 from both sides:

$$2x \equiv -9 \pmod{31}$$

- Since $-9 \mod 31 = 22$, we rewrite:

$$2x \equiv 22 \pmod{31}$$

- Multiply both sides by the modular inverse of 2 modulo 31.
  The inverse of 2 modulo 31 is 16, because:

$$2 \times 16 = 32 \equiv 1 \pmod{31}$$

  So:

$$x \equiv 16 \times 22 = 352 \equiv 11 \pmod{31}$$

Answer:

$$x \equiv 11 \pmod{31}$$

Section 3.13(16)

Solve the system of congruences:

$$\begin{cases} x \equiv 3 \pmod{5} \\ x \equiv 9 \pmod{11} \end{cases}$$

5 and 11 are coprime. This satisfies the condition for applying the Chinese Remiander Theorem, which guarantes a unique solution modulo the product of the moduli.

Compute the product of the moduli.

$$N = 5 \times 11 = 55$$

Compute the individual terms.
For modulus 5:

$$N_1 = \frac{N}{5} = \frac{55}{5} = 11$$

Find the inverse of N_1 modulo 5, i.e., find M_1 such that:

$$11 \times M_1 \equiv 1 \pmod{5}$$

Since $11 \mod 5 = 1$, we have:

$$M_1 = 1$$

For modulus 11:

$$N_2 = \frac{N}{11} = \frac{55}{11} = 5$$

Find the inverse of N_2 modulo 11, i.e., find M_2 such that:

$$5 \times M_2 \equiv 1 \pmod{11}$$

Since $5 \times 9 = 45 \equiv 1 \pmod{11}$, we have:

$$M_2 = 9$$

Compute the solution.

$$x = (3 \times N_1 \times M_1) + (9 \times N_2 \times M_2) \mod N$$

$$x = (3 \times 11 \times 1) + (9 \times 5 \times 9) \mod 55$$
$$x = 33 + 405 \mod 55$$
$$x = 438 \mod 55$$

Reduce modulo 55.

$$438 \div 55 = 7 \text{ with a remainder of } 53$$
$$x \equiv 53 \pmod{55}$$

Section 3.13(24)

$$3, 9, 8, 7, 6, 5, 4, 3, 2, 1$$

Compute the alternating sum:

$$3 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1 = -2$$

Now reduce $-2 \mod 11$:

$$-2 \mod 11 = 9$$