LongNGUYEN_DerivativePricing_Assignment1

April 2, 2018

```
In [10]: import math
         import numpy as np
         import pandas as pd
         import matplotlib.pyplot as plt
         import seaborn as sns
```

0.1 Question 1

(Payoff and prot)
For a given derivative, payoff is the value at the time of expiration, but for the prot one needs to take into account the initial investment, for instance, the derivative's cost. Payoff/prot at the expiration time in function of the underlying price are payoff/prot diagram.

a. Recall the payoff of a Call option with price at maturity S_T and strike price K.

Answer: > *Max(*S_T*-K,0)*

b. If the option cost at t = 0 is c, what would be the prot of the option buyer at t = T.

Answer: > - Consider "-c" is the cost to purchase option at time T=0 > - We got two cases:

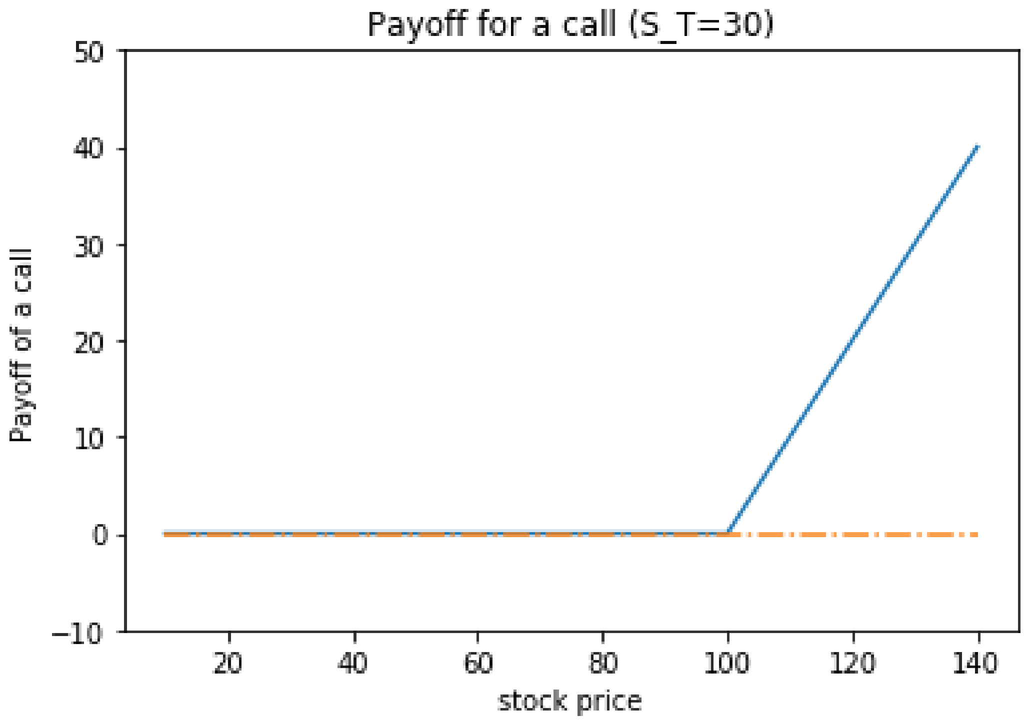
Time	T=0	T=t	Total Profit
Call Option	-c	Max(S_T-K,0)	-c*e^{rT}+Max(S_T-K,0)
Put Option	-c	Max(K-S_T,0)	-c*e^{rT}+Max(K-S_T,0)

c. Use Python to draw the payoff and prot diagrams for: K = 100, c = 1 (currency unit), T = 1 (year) and either r = 0 or r = 7 (% per annum).

Answer: > In this question, I assumed this is a Call Option and all the calculation from buyer perspectives

```
In [75]: import numpy as np
         import matplotlib.pyplot as plt
         s = np.arange(10,150,10)
         k=100
         payoff=(abs(s-k)+s-k)/2
         plt.ylim(-10,50)
```

```
plt.plot(s,payoff)
y2=sp.zeros(len(s))
plt.plot(s,y2,'-.')
plt.title("Payoff for a call (S_T=30)")
plt.xlabel("stock price")
plt.ylabel("Payoff of a call")
plt.show()
```




```
In [76]: import scipy as sp
         import matplotlib.pyplot as plt
         s = sp.arange(30,150,10)
         k=100;c=1*math.exp(0*1)
         y=(abs(s-k)+s-k)/2 -c
         y2=sp.zeros(len(s))
         plt.ylim(-2,40)
         plt.plot(s,y)
         plt.plot(s,y2,'-.')
         plt.title("Profit/Loss function with r=0%")
         plt.xlabel('Stock price')
         plt.ylabel('Profit (loss)')
         plt.show()
```


```
In [77]: import scipy as sp
         import matplotlib.pyplot as plt
         s = sp.arange(30,150,10)
         k=100;c=1*math.exp(0.07*1)
         y=(abs(s-k)+s-k)/2 -c
         y2=sp.zeros(len(s))
         plt.ylim(-2,40)
         plt.plot(s,y)
         plt.plot(s,y2,'-.')
         plt.title("Profit/Loss function with r=7%")
         plt.xlabel('Stock price')
         plt.ylabel('Profit (loss)')
         plt.show()
```

1 Question 2.

(Intrinsic and time value)

The price of a derivative can be decomposed into two components, intrinsic value (value of the derivative if exercised immediately), the rest is called the time value.

a. Use the option data sample (AAPL 01/06/2017), calculate the time value of each option.

b. Show that time values are always positive for a European Call/Put, i.e., C(S_t, t; K; T) max(S_t K, 0) and P(S_t, t; K; T) max(K S_t, 0). Verify numerically this fact in the data sample.

c. Verify if time value increases with time to maturity using the same data sample.

In [186]:
```
#import data from google doc
data = pd.read_csv('https://docs.google.com/spreadsheets/d/e/2PACX-1vSRmPANey5HKtRoW
```

In [187]:
```
data.head()
```

Out[187]:
```
  Symbol ExpirationDate  AskPrice  AskSize  BidPrice  BidSize  LastPrice  \
0   AAPL       06/02/17      0.02      NaN      0.00      NaN       0.04
1   AAPL       06/02/17      0.02      NaN      0.00      NaN       0.09
2   AAPL       06/02/17      0.23      NaN      0.19      NaN       0.23
3   AAPL       06/02/17      0.02      NaN      0.00      NaN       0.06
4   AAPL       06/02/17      4.55      NaN      4.10      NaN       4.50
```

```
   PutCall  StrikePrice  Volume  ImpliedVolatility   Delta    Gamma    Vega  \
0      put        122.0       0             1.7029 -0.0046   0.0983  0.1070
1      put        109.0       0             2.4536 -0.0032   0.0501  0.0786
2      put        152.5   18463             0.1566 -0.2914  27.4068  2.7423
3      put        118.0       0             1.9257 -0.0041   0.0780  0.0960
4      put        157.5     482             0.4620 -0.8731   5.6324  1.6626

   Rho  OpenInterest  UnderlyingPrice  DataDate
0  NaN            81           153.18  06/01/17
1  NaN            14           153.18  06/01/17
2  NaN         17502           153.18  06/01/17
3  NaN            50           153.18  06/01/17
4  NaN          1821           153.18  06/01/17
```

```
In [188]: data.dtypes
```

```
Out[188]: Symbol                object
          ExpirationDate        object
          AskPrice             float64
          AskSize              float64
          BidPrice             float64
          BidSize              float64
          LastPrice            float64
          PutCall               object
          StrikePrice          float64
          Volume                 int64
          ImpliedVolatility    float64
          Delta                float64
          Gamma                float64
          Vega                 float64
          Rho                  float64
          OpenInterest           int64
          UnderlyingPrice      float64
          DataDate              object
          dtype: object
```

```
In [171]: def poff (S,K,optiontype):
              if optiontype.lower() == 'call':
                  poff = (abs(S-K)+S-K)/2
              elif optiontype.lower() == 'put':
                  poff = (abs(K-S)+K-S)/2
              else:
                  poff = np.nan
              return payoff
```

a. Use the option data sample (AAPL 01/06/2017), calculate the time value of each option.

Answer > Instrinsic value

```
In [189]: data['intrinsic']=0
          data.loc[data.PutCall == "call",'intrinsic'] = np.maximum(data[data.PutCall == "call'
          data.loc[data.PutCall == "put",'intrinsic'] = np.maximum(data[data.PutCall == "put"]
```

Time values

```
In [190]: #timevalues
          data['timevalues']=data.AskPrice - data.intrinsic
```

b. Show that time values are always positive for a European Call/Put, i.e., C(S_t, t; K; T) max(S_t K, 0) and P(S_t, t; K; T) max(K S_t, 0). Verify numerically this fact in the data sample.

Answer > All of the time value is positive as they got 1860 positive values (True) - the whole dataset got 1860 variables.

```
In [230]: (data.timevalues>=0)

Out[230]: 0       True
          1       True
          2       True
          3       True
          4       True
          5       True
          6       True
          7       True
          8       True
          9       True
          10      True
          11      True
          12      True
          13      True
          14      True
          15      True
          16      True
          17      True
          18      True
          19      True
          20      True
          21      True
          22      True
          23      True
          24      True
          25      True
          26      True
          27      True
          28      True
          29      True
                  ...
          1830    True
```

```
1831    True
1832    True
1833    True
1834    True
1835    True
1836    True
1837    True
1838    True
1839    True
1840    True
1841    True
1842    True
1843    True
1844    True
1845    True
1846    True
1847    True
1848    True
1849    True
1850    True
1851    True
1852    True
1853    True
1854    True
1855    True
1856    True
1857    True
1858    True
1859    True
Name: timevalues, Length: 1860, dtype: bool
```

c. Verify if time value increases with time to maturity using the same data sample.

Answer > In short, it cannot be verify that Time Value increases with Time to Maturity within this sample data as we got some situations that Time to Maturity increased but Time Value reduced

In [260]:
```
#time to maturity in days
data['Timetomaturitydays'] = pd.to_datetime(data['ExpirationDate']) - pd.to_datetime
data['Timetomaturitydays'] = [d.days for d in data['Timetomaturitydays']]
```

In [262]:
```
data.set_index(['PutCall','StrikePrice','Timetomaturitydays'])[['timevalues']]
```

Out[262]:
```
                                          timevalues
PutCall StrikePrice Timetomaturitydays
put     122.0       1                           0.02
        109.0       1                           0.02
        152.5       1                           0.23
        118.0       1                           0.02
        157.5       1                           0.23
```

	117.0	1	0.02
	155.0	1	0.21
	165.0	1	0.43
	175.0	1	0.38
	160.0	1	0.28
	180.0	1	0.43
	162.5	1	0.43
	167.5	1	0.43
	190.0	1	0.28
	195.0	1	0.28
	200.0	1	0.38
	119.0	8	0.02
	124.0	8	0.02
	146.0	8	0.14
	149.0	8	0.31
	138.0	8	0.05
	172.5	8	0.43
	148.0	8	0.22
	155.0	8	0.75
	160.0	8	0.43
	165.0	8	0.43
	145.0	8	0.12
	117.0	8	0.02
	137.0	8	0.04
	141.0	8	0.07
...			...
call	177.5	15	0.01
	175.0	15	0.01
	205.0	15	0.02
	170.0	15	0.04
	225.0	15	0.02
	155.0	1	0.04
	165.0	8	0.03
	187.5	8	0.01
	182.5	15	0.01
	187.5	29	0.03
		36	0.04
	182.5	36	0.06
	215.0	78	0.04
	245.0	141	0.04
	167.5	15	0.05
	172.5	22	0.06
	170.0	22	0.07
	180.0	29	0.04
	190.0	29	0.03
	225.0	106	0.04
		141	0.06
	230.0	141	0.05

```
           245.0      169                   0.06
           250.0      169                   0.10
                      197                   0.07
           177.5      22                    0.04
           225.0      197                   0.16
           230.0      197                   0.14
           235.0      232                   0.16
           250.0      260                   0.15

[1860 rows x 1 columns]
```

2 Q3.

a/ What's the intrinsic and time value of ATM and OTM options? At ATM and OTM: the intrinsic is zero and the time value is the price of option. Because the sum of the intrinsic and time value is the price of option

Answer > As Table below

	Call Option	Call Option	Put Option
	ATM	OTM	ATM
	S_T=K	S_TK	
Intrinsic Value	0	less than 0	0
Time Value	option price	less than option price	option price

b/ Show that the time value of ATM/OTM European Call/Put options indeed increases with the option's time to maturity.

Answer > according to answer in question a, the final Option Price will be depend on Time Value as Intrinsic value will be equal to 0 or negative. Consequently, the longer time to maturity will bring higher time value due to our expectation

c/ Verify how time value evolves with respect to the log-moneyness (log St/K) using the data sample

```
In [198]: #logmoneyness
          data["logmoneyness"]=0
          data['logmoneyness']=np.log(data.UnderlyingPrice/data.StrikePrice)
```

```
In [200]: data.head(10)
```

```
Out[200]:   Symbol ExpirationDate  AskPrice  AskSize  BidPrice  BidSize  LastPrice  \
          0   AAPL       06/02/17      0.02      NaN      0.00      NaN       0.04
          1   AAPL       06/02/17      0.02      NaN      0.00      NaN       0.09
          2   AAPL       06/02/17      0.23      NaN      0.19      NaN       0.23
          3   AAPL       06/02/17      0.02      NaN      0.00      NaN       0.06
          4   AAPL       06/02/17      4.55      NaN      4.10      NaN       4.50
          5   AAPL       06/02/17      0.02      NaN      0.00      NaN       0.04
          6   AAPL       06/02/17      2.03      NaN      1.80      NaN       1.93
          7   AAPL       06/02/17     12.25      NaN     11.60      NaN      12.24
```

```
8   AAPL       06/02/17     22.20       NaN      21.60       NaN       21.20
9   AAPL       06/02/17      7.10       NaN       6.60       NaN        6.82

  PutCall  StrikePrice  Volume      ...         Gamma    Vega  Rho  \
0     put        122.0       0      ...        0.0983  0.1070  NaN
1     put        109.0       0      ...        0.0501  0.0786  NaN
2     put        152.5   18463      ...       27.4068  2.7423  NaN
3     put        118.0       0      ...        0.0780  0.0960  NaN
4     put        157.5     482      ...        5.6324  1.6626  NaN
5     put        117.0       0      ...        0.0742  0.0940  NaN
6     put        155.0    3528      ...       13.2819  2.1650  NaN
7     put        165.0       3      ...        2.0451  1.4476  NaN
8     put        175.0       0      ...        0.9848  1.0444  NaN
9     put        160.0       0      ...        3.5069  1.5038  NaN

   OpenInterest  UnderlyingPrice  DataDate  intrinsic timevalues  \
0            81           153.18  06/01/17       0.00       0.02
1            14           153.18  06/01/17       0.00       0.02
2         17502           153.18  06/01/17       0.00       0.23
3            50           153.18  06/01/17       0.00       0.02
4          1821           153.18  06/01/17       4.32       0.23
5            52           153.18  06/01/17       0.00       0.02
6          8697           153.18  06/01/17       1.82       0.21
7             4           153.18  06/01/17      11.82       0.43
8             0           153.18  06/01/17      21.82       0.38
9           186           153.18  06/01/17       6.82       0.28

   Timetomaturitydays  logmoneyness
0                   1      0.227593
1                   1      0.340266
2                   1      0.004449
3                   1      0.260929
4                   1     -0.027812
5                   1      0.269440
6                   1     -0.011811
7                   1     -0.074332
8                   1     -0.133172
9                   1     -0.043560

[10 rows x 22 columns]
```

In [216]:
```
fig, ax = plt.subplots(1,1,figsize=(15,15))
##data['cond'] = np.all((data['PutCall'].isin(['call']),data['ExpirationDate'].isin(

plt.scatter(data.loc[data.PutCall == "call",'logmoneyness'],data.loc[data.PutCall ==
plt.scatter(data.loc[data.PutCall == "put",'logmoneyness'],data.loc[data.PutCall == '

plt.legend()
```

```python
plt.grid(True)
plt.xlabel('Log Moneyness')
plt.ylabel('Time Value')
plt.show()
```