derivatives2_assignment_2018

April 16, 2018

1 Instructions

1. Due: 23:59 23/04/2018.
2. Answers must be
   - named as assignment2.(pdf/ipynb)
   - uploaded directly to Google drive, under **students/YourName** folder.

2 Questions

Q1. (European Put-Call parity revisited) We consider portfolio A: long a call and short a put on the same underlying S, strike-price K and maturity T. Portfolio B consists of a long **prepaid** forward contract on S for the same maturity T, as well as borrowing the present value of the strike-price K to be repaid at T.

a. What is the initial cost, intermediate cash-flow and final payoff of A?

b. What is the initial cost, intermediate cash-flow and final payoff of B? (Let's denote $F_{0,T}(S)$ for the price of a prepaid forward contract on S for delivery at T, $P(0,T)$ is the price of a zero-coupon bond paying \$1 at T.)

c. Using the no-arbitrage principale to get a generic form of the Put-Call parity.

d. Suppose the risk-free interest rate is r, provide the Put-Call parity in three specific cases: i. S pays no dividend; ii. S pays n discrete dividends d_i at t_i for $i \in [1,n]$; iii. S pays a continuous dividends at the rate q.

Q2. Let's first recall that the Put-Call parity for a stock paying dividends

$$C(K,T) + Ke^{-rT} = P(K,T) + S_0 e^{-qT}\ ,\ q:\ \text{dividend yield over } (0,T).$$

The **Implied Dividend** yield is the value of q such that the Put-Call parity holds true.

$$\text{IDIV}(K,T) = -\frac{1}{T}\log\frac{C(K,T) - P(K,T) + Ke^{-rT}}{S}$$

a. Get prices of AAPL option on 01/06/2017 from the data file.

b. Get risk-free interst rate for the same date from this link.

c. Compute the implied dividend for different maturity T, use the Actual/360 day convention - see this page.

d. Compare the IDIV with the historical dividends from this page.

Q3. (Model risk in binomial tree framework) We re-consider the binomial tree model, in which the price at $t = 1$ has the following dynamics:

$$\begin{cases} S_u = uS_0 & \text{with probability } p \\ S_d = dS_0 & \text{with probability } 1-p \end{cases}$$

Let's suppose the risk-free rate is constant, thus from $B_0 = 1$ we have $B_1 = 1 + r$ for an investment in the bank account.

a. Verify that in order to exclude the opportunity arbitrage, one must have

$$d < 1 + r < u.$$

At $t = 0$ we sell a derivative with payoff $g(S_1)$ at the price g_0. We would like to hedge our position in setting a self-financing strategy. The strategy consists in holding Δ units of S and investing the rest in the bank account B.

b. Show that Δ and the price of this derivative are given by:

$$\Delta = \frac{g(S_u) - g(S_d)}{S_u - S_d}.$$

$$g_0 = \frac{qg_u + (1-q)g_d}{1+r} \text{ where } q = \frac{(1+r)-d}{u-d}.$$

Now instead of supposing S_1 takes only two values, we relax this assumption: without knowing the exact dynamics of S_1, we only know with probability 1:

$$S_1 \in [S_d, S_u].$$

We suppose further that the payoff function is **convex**, i.e., for $y_d \leq y \leq y_u$,

$$g(y_d) + \frac{g(y_u) - g(y_d)}{y_u - y_d}(y - y_d) \geq g(y).$$

c. We keep using the same hedging strategy as in b., what is the PnL of the hedging strategy in the new model?

d. Show that with the same price g_0 as in b., we have a positive PnL with probability 1.

e. (Optional) Generalize this problem in multi-period setting: give $0 = t_0 < t_1 < \cdots < t_N = T$, $t_n = nT/N = nh$, with probability 1

$$S_{(n+1)h} \in [dS_{nh}, uS_{nh}] \ , \ d < 1 + r = e^{\rho h} < u.$$

(Hint: Δ should be the first derivative of the price with respect to the underlying.)