Unit 7
Predictive Parsing

Nguyen Thi Thu Huong

Hanoi University of Technology

Predictive Parsers

- Parser can “predict” which production to use
  - By looking at the next few tokens
  - No backtracking
- Predictive parsers accept LL(k) grammars
  - L means “left-to-right” scan of input
  - L means “leftmost derivation”
  - k means “predict based on k tokens of lookahead”
- In practice, LL(1) is used

- accomplished using a *predictive parsing table* M and a stack.

A stringent condition

The grammar must not be left recursive and no two right sides of a production have a common prefix.

Left Recursion

A grammar is ***left recursive*** if it has a non-terminal A such that there is a derivation.

$$A \Rightarrow A\alpha \text{ for some string } \alpha$$

Top-down parsing techniques **cannot** handle left-recursive grammars.
So, we have to convert our left-recursive grammar into an equivalent grammar which is not left-recursive.
The left-recursion may appear in a single step of the derivation (*immediate left-recursion*), or may appear in more than one step of the derivation.

Immediate Left-Recursion

$A \rightarrow A\,\alpha \mid \beta$ where β does not start with A

$\Downarrow$ eliminate immediate left recursion

$A \rightarrow \beta\,A'$

$A' \rightarrow \alpha\,A' \mid \varepsilon$ an equivalent grammar

In general,

$A \rightarrow A\,\alpha_1 \mid \ldots \mid A\,\alpha_m \mid \beta_1 \mid \ldots \mid \beta_n$ where $\beta_1 \ldots \beta_n$ do not start with A

$\Downarrow$ eliminate immediate left recursion

$A \rightarrow \beta_1\,A' \mid \ldots \mid \beta_n\,A'$

$A' \rightarrow \alpha_1\,A' \mid \ldots \mid \alpha_m\,A' \mid \varepsilon$ an equivalent grammar

Left-Recursion -- Problem

- A grammar cannot be immediately left-recursive, but it still can be left-recursive.
- By just eliminating the immediate left-recursion, we may not get a grammar which is not left-recursive.

$S \rightarrow Aa \mid b$
$A \rightarrow Sc \mid d$ This grammar is not immediately left-recursive, but it is still left-recursive.

$\underline{S} \Rightarrow Aa \Rightarrow \underline{S}ca$ or
$\underline{A} \Rightarrow Sc \Rightarrow \underline{A}ac$ causes to a left-recursion

- So, we have to eliminate all left-recursions from our grammar

Eliminate Left-Recursion -- Algorithm

- Arrange non-terminals in some order: $A_1 \ldots A_n$
- **for** i **from** 1 **to** n **do** {
  - **for** j **from** 1 **to** i-1 **do** {
    replace each production
    $A_i \rightarrow A_j \gamma$
    by
    $A_i \rightarrow \alpha_1 \gamma \mid \ldots \mid \alpha_k \gamma$
    where $A_j \rightarrow \alpha_1 \mid \ldots \mid \alpha_k$
  }
  - eliminate immediate left-recursions among A_i productions

}

Immediate Left-Recursion -- Example

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow id \mid (E)$

$\Downarrow$ eliminate immediate left recursion

$E \rightarrow T\ E'$

$E' \rightarrow +T\ E' \mid \varepsilon$

$T \rightarrow F\ T'$

$T' \rightarrow *F\ T' \mid \varepsilon$

$F \rightarrow id \mid (E)$

Predictive Parsing and Left Factoring

- In the grammar

  $E \rightarrow T + E \mid T$

  $T \rightarrow \text{int} \mid \text{int} * T \mid (E)$

- Hard to predict because
  - For T two productions start with int
  - For E it is not clear how to predict

- A grammar must be left-factored before use for predictive parsing

Left Factoring

- A predictive parser (a top-down parser without backtracking) insists that the grammar must be *left-factored*.

  grammar ➔ a new equivalent grammar suitable for predictive parsing

If_stmt → `if` expr `then` stmt `else` stmt |
`if` expr `then` stmt

- when we see `if`, we cannot now which production rule to choose to re-write *stmt* in the derivation.

Left Factoring (con'd)

In general,

$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2$ where α is non-empty and the first symbols of β_1 and β_2 (if they have one)are different.

when processing α we cannot know whether expand

A to $\alpha\beta_1$ or

A to $\alpha\beta_2$

But, if we re-write the grammar as follows

$A \rightarrow \alpha A'$

$A' \rightarrow \beta_1 \mid \beta_2$ so, we can immediately expand A to $\alpha A'$

Left factoring algorithm

- For each non-terminal A with two or more alternatives (production rules) with a common non-empty prefix, let say

$$A \rightarrow \alpha\beta_1 \mid \ldots \mid \alpha\beta_n \mid \gamma_1 \mid \ldots \mid \gamma_m$$

convert it into

$$A \rightarrow \alpha A' \mid \gamma_1 \mid \ldots \mid \gamma_m$$
$$A' \rightarrow \beta_1 \mid \ldots \mid \beta_n$$

Left Factoring example

S --> if E then S | if E then S else S

can be rewritten as

S --> if E then S S'

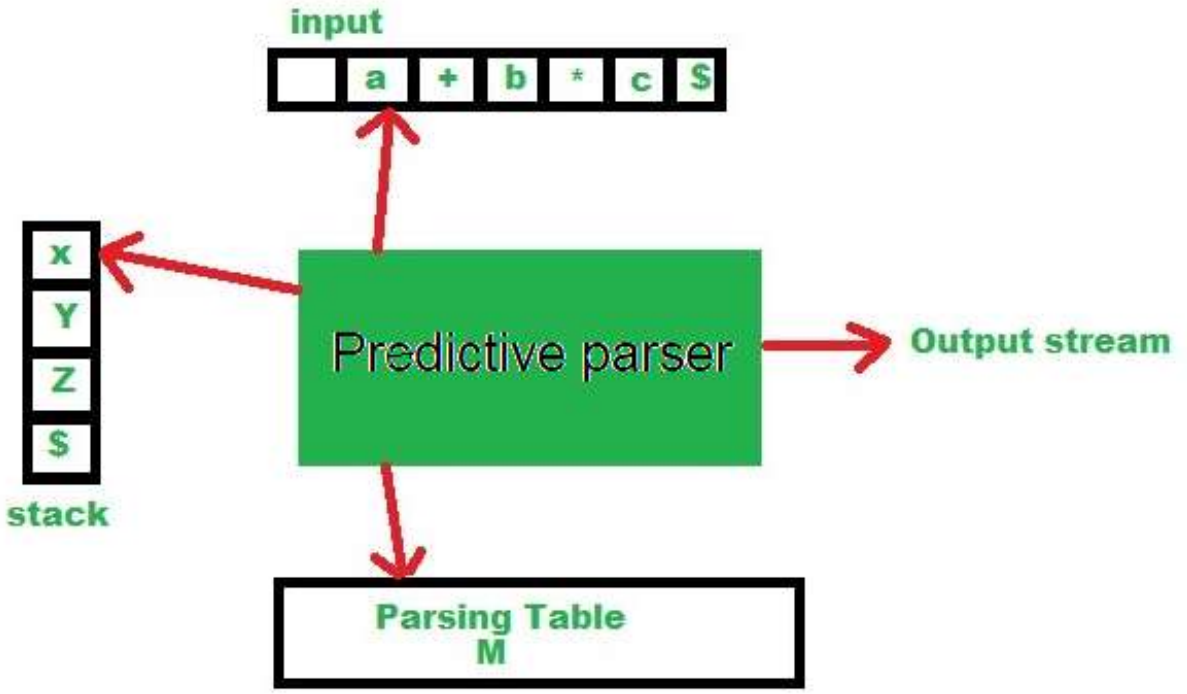
S' --> else S | ε

In KPL

```
IfSt ::= KW_IF Condition KW_THEN
  Statement ElseSt

ElseSt ::= KW_ELSE Statement
ElseSt ::= ε
```

(Non recursive) Predictive parser



Parsing table M

- *M*[*X*, *token*] indicates which production to use if the top of the stack is a nonterminal *X* and the current token is equal to *token*;
- in that case we pop *X* from the stack and we push all the rhs symbols of the production *M*[*X*, *token*] in reverse order.
- We use a special symbol $ to denote the end of file. Let *S* be the start symbol

Non recursive Predictive Parser

- The input contains the string to be parsed, followed by $ (EOF)
- The stack contains a sequence of grammar symbols, preceded by #, the bottom-of-stack marker.
- Initially the stack contains the start symbol of the grammar preceded by $.
- The parsing table is a two dimensional array M[A,a], where A is a nonterminal, and a is a terminal or the symbol $.

Parsing table for grammar S→ aSb|c

	a	b	c	$
S	S → aSb	Error	S → c	Error
a	Push	Error	Error	Error
b	Error	Push	Error	Error
c	Error	Error	Push	Error
#	Error	Error	Error	Accept

LL(1) Parsing Tables. Errors

- Yellow entries indicate error situations
  - Consider the [S,b] entry
  - “There is no way to derive a string starting with b from non-terminal S

Using Parsing Tables

- Method similar to recursive descent, except
  - For each non-terminal S
  - We look at the next token a
  - And chose the production shown at [S,a]
- We use a stack to keep track of pending non-terminals
- We reject when we encounter an error state
- We accept when we encounter end-of-input

LL(1) Parsing Algorithm

initialize stack = <S #>
repeat
  case stack of
    <X, rest> : if T[X,*next] = T $\rightarrow$ $Y_1 \ldots Y_n$
                 then stack $\leftarrow$ <$Y_1 \ldots$ Y_n rest>;
                 else error (); //X-nonterminal
    <t, rest> : if t == *next ++
                 then stack $\leftarrow$ <rest>;
                 else error (); //X-nonterminal
until stack == < >
//*next refers to the symbol to be checked

LL(1) Parsing Example for aacbb

Stack	Input	Action
S#	aacbb$	S $\rightarrow$ aSb
aSb#	aacbb$	push
Sb#	acbb$	S $\rightarrow$ aSb
aSbb#	acbb$	push
Sbb#	cbb$	S $\rightarrow$ c
cbb#	cbb$	push
bb#	bb$	push
b#	b$	push
#	$	ACCEPT

Constructing Parsing Tables

- LL(1) languages are those defined by a parsing table for the LL(1) algorithm
- No table entry can be multiply defined
- We want to generate parsing tables from CFG

Constructing Parsing Tables (Cont.)

- If $A \rightarrow \alpha$, where in the line of A we place α ?
- In the column of t (t is a terminal) where t can start a string derived from α
  - $\alpha \rightarrow^* t\ \beta$
  - We say that $t \in First(\alpha)$
- In the column of t if α is ε and t can follow an A
  - $S \rightarrow^* \beta\ A\ t\ \delta$
  - We say $t \in Follow(A)$

Computing First Sets

Definition: $First(X) = \{ t \mid X \rightarrow^* t\alpha \} \cup \{\varepsilon \mid X \rightarrow^* \varepsilon\}$

Algorithm sketch :

1. for all terminals t do $First(X) \leftarrow \{ t \}$ //if X is terminal t
2. for each production $X \rightarrow \varepsilon$ do $First(X) \leftarrow \{ \varepsilon \}$
3. if $X \rightarrow A_1 \ldots A_n \alpha$ and $\varepsilon \in First(A_i)$, $1 \leq i \leq n$ do
   - add $First(\alpha)$ to $First(X)$
4. for each $X \rightarrow A_1 \ldots A_n$ s.t. $\varepsilon \in First(A_i)$, $1 \leq i \leq n$ do
   - add ε to First(X)
5. repeat steps 4 & 5 until no First set can be grown

First Sets. Example

- Recall the grammar

$E \rightarrow T\ E'$ $\quad\quad$ $T \rightarrow FT'$

$E' \rightarrow +\ E \mid \varepsilon$ $\quad\quad$ $T' \rightarrow *F \mid \varepsilon$

$F \rightarrow (\ E\ ) \mid int$

First sets

First(( ) = { (}

First( )) = {) }

First(int) = { int }

First(+) = { + }

First(*) = { * }

First(T) = First (F) = {int, (}

First(E) = {int, (}

First(E') = {+, ε }

First(T') = {*, ε }

Computing Follow Sets

- Definition:

  $$\text{Follow}(X) = \{ t \mid S \rightarrow^* \beta\, X\, t\, \delta \}$$

- Intuition
  - If S is the start symbol then \$ $\in$ Follow(S)
  - If $X \rightarrow A\ B$ then First(B) $\subseteq$ Follow(A) and Follow(X) $\subseteq$ Follow(B)
  - Also if $B \rightarrow^* \varepsilon$ then Follow(X) $\subseteq$ Follow(A)

Computing Follow Sets (Cont.)

Algorithm sketch:

1. Follow(S) $\leftarrow$ { $ }
2. For each production A $\rightarrow \alpha$ X β
   - add First(β) - {ε} to Follow(X)
3. For each A $\rightarrow \alpha$ X β where $\varepsilon \in$ First(β)
   - add Follow(A) to Follow(X)

- repeat step(s) 2, 3 until no Follow set grows

Follow Sets. Example

- Recall the grammar

  $E \rightarrow T\ E'$ $\qquad$ $T \rightarrow FT'$

  $E' \rightarrow +\ E \mid \varepsilon$ $\qquad$ $T' \rightarrow *F \mid \varepsilon$

  $F \rightarrow (\ E\ ) \mid int$

- Follow sets

  Follow(+) = { int, (} $\qquad$ Follow(*) = { int, (}

  Follow(( ) = { int, (} $\qquad$ Follow(E) = {), $}

  Follow(E') = {$,) } $\qquad$ Follow(T) = {+,) , $}

  Follow( )) = {+,) , $} $\qquad$ Follow(T') = {+,) , $}

  Follow(int) = {*, +,) , $}

Constructing LL(1) Parsing Tables

- Construct a parsing table T for CFG G

- For each production $A \rightarrow \alpha$ in G do:
  - For each terminal $t \in First(\alpha)$ do
    - T[A, t] = α
  - If $\varepsilon \in First(\alpha)$, for each $t \in Follow(A)$ do
    - T[A, t] = ε
  - If $\varepsilon \in First(\alpha)$ and \$ $\in Follow(A)$ do
    - T[A, \$] = ε

- Grammar G:

$E \rightarrow TE'$
$E' \rightarrow +TE' \mid \varepsilon$
$T \rightarrow FT'$
$T' \rightarrow *FT' \mid \varepsilon$
$F \rightarrow (E) \mid \mathbf{int}$

It's possible to implement a predictive parser for *G*

Parsing table

	+	*	(	)	int	$
E			E→TE'		E→TE'	
E'	E'→+TE'			E'→ε		E'→ε
T			T→FT'		T→FT'	
T'	T'→ε	T'→*FT'		T'→ε		T'→ε
F			F→(E)		F→int	
+	Push					
*		Push				
(			Push			
)				Pushy		
int					Push	
#						Accept

Notes on LL(1) Parsing Tables

- If any entry is multiply defined then G is not LL(1)
  - If G is ambiguous
  - If G is left recursive
  - If G is not left-factored
- Most programming language grammars are not LL(1)
- There are tools that build LL(1) tables