



# **Chapter 6: Functions**

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Introduction to Computer Programming  
(C language)

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# Course Content

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- ❑ C.1. Introduction to Computers and Programming
- ❑ C.2. C Program Structure and its Components
- ❑ C.3. Variables and Basic Data Types
- ❑ C.4. Selection Statements
- ❑ C.5. Repetition Statements
- ❑ **C.6. Functions**
- ❑ C.7. Arrays
- ❑ C.8. Pointers
- ❑ C.9. File Processing

# References

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- ▣ [1] "*C: How to Program*", 7<sup>th</sup> Ed. – Paul Deitel and Harvey Deitel, Prentice Hall, 2012.
- ▣ [2] "*The C Programming Language*", 2<sup>nd</sup> Ed. – Brian W. Kernighan and Dennis M. Ritchie, Prentice Hall, 1988
- ▣ and others, especially those on the Internet

# Content

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- ▣ Introduction
- ▣ Functions in the standard library
- ▣ An example of a function
- ▣ Components of a function
- ▣ Function call
- ▣ Recursion
- ▣ Summary

# Introduction

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□ In the previous chapters, we have used several so-called functions in the library:

- printf in stdio.h
- scanf in stdio.h
- fflush in stdio.h
- sqrt in math.h
- pow in math.h
- system in stdlib.h
- strcmp in string.h
- strcpy in string.h

Those functions are modular processing units that are:

→ Responsible for a certain task

→ Reusable in many various programs

# Functions in the standard library

---

- |              |              |              |
|--------------|--------------|--------------|
| □ <assert.h> | □ <locale.h> | □ <stddef.h> |
| □ <ctype.h>  | □ <math.h>   | □ <stdio.h>  |
| □ <errno.h>  | □ <setjmp.h> | □ <stdlib.h> |
| □ <float.h>  | □ <signal.h> | □ <string.h> |
| □ <limits.h> | □ <stdarg.h> | □ <time.h>   |

# Functions in the standard library

---

## □ Some functions in `<stdio.h>`

### ■ `int printf(const char *format, ...)`

- Sends formatted output to stdout

### ■ `int scanf(const char *format, ...)`

- Reads formatted input from stdin

### ■ `int getchar(void)`

- Gets a character (an unsigned char) from stdin

### ■ `char *gets(char *str)`

- Reads a line from stdin and stores it into the string pointed to, by str. It stops when either the newline character ('\n') is read or when the end-of-file (EOF) is reached, whichever comes first.

# Functions in the standard library

---

## □ Some functions in `<math.h>`

### ■ `double cos(double x)`

- Returns the cosine of a radian angle **x**

### ■ `double pow(double x, double y)`

- Returns **x** raised to the power of **y**

### ■ `double sqrt(double x)`

- Returns the square root of **x**

### ■ `double ceil(double x)`

- Returns the smallest integer value greater than or equal to **x**

### ■ `double floor(double x)`

- Returns the largest integer value less than or equal to **x**



# Functions in the standard library

---

## □ Some functions in `<stdlib.h>`

### ■ `void *malloc(size_t size)`

- Allocates the requested memory and returns a pointer to it

### ■ `void free(void *ptr)`

- Deallocates the memory previously allocated by a call to *calloc*, *malloc*, or *realloc*

### ■ `int rand(void)`

- Returns a pseudo-random number in the range of 0 to *RAND\_MAX* (at least 32767, up to implementation)

### ■ `int system(const char *string)`

- The command specified by *string* is passed to the host environment to be executed by the command processor
  - E.g. "pause", "cls", "date"

```
//Chapter 5 - while.. and for.. statements
//Squared numbers smaller than N which is input by a user
```

```
#include <stdio.h>
```

```
void main() {
```

```
    int N = 0, i;
```

```
    do {
```

```
        printf("\n\nEnter a natural number greater than 0: N = ");
        scanf("%d", &N);
        fflush(stdin);
```

```
    } while (N<=0);
```

```
    printf("\n\nAll the squared numbers smaller than N are:");
```

```
    for (i=1; i*i<N; i++)
```

```
}
```

```
//Chapter 5 - repetition statements
//Two opposite triangles
```

```
#include <stdio.h>
```

```
void main() {
```

```
    int N; //height
```

```
    do {
```

```
        printf("\n\nEnter a natural number greater than 0: N = ");
        scanf("%d", &N);
```

```
        fflush(stdin);
```

```
    }
```

```
    while (N<=0);
```

```
    int i; //row index
```

```
    for (i=1; i<=N; i++) {
```

```
        int j; //column index for the 1st triangle
```

```
        for (j=1; j<=i; j++) printf("*");
```

```
        int k; //column index for the 2nd triangle
```

```
        for (k=1; k<=2*N-2*i-1; k++) printf(" ");
```

```
        for (k=1; k<=i; k++)
```

```
            if (k<N) printf("*");
```

```
        printf("\n");
```

```
    }
```

```
}
```

Repeated code!!!

Can we just code them once and then make use of them over the time just like those in the standard library?

# Introductions

---

- Let's define a function: `getNaturalNumber()`

```
#include <stdio.h>

unsigned int getNaturalNumber() {

    int N;

    do {
        printf("\n\nEnter a natural number greater than 0: N = ");
        scanf("%d", &N);

        fflush(stdin);
    }
    while (N<=0);

    return N;
}
```

Declared in a header file

`C6_function_getNaturalNumber_1.h`

for multiple uses

`unsigned int getNaturalNumber();`

Source code file: `C6_function_getNaturalNumber.c`

Purpose: to ask users to input a natural number  
until a valid number is input

```
//Chapter 5 - while.. and for.. statements
//Squared numbers smaller than N which is input by a user
```

```
#include <stdio.h>
```

```
#include "C6_function_getNaturalNumber_1.h"
```

```
void main() {
```

```
    int N = 0, i;
```

```
    N = getNaturalNumber();
```

```
    printf("\n\nAll the squared number
```

```
    for (i=1; i*i<N; i++) printf("%d \
```

```
}
```

Use of our previously defined function, which is declared in a header file:

"C6\_function\_getNaturalNumber\_1.h"

```
//Chapter 5 - repetition statements
```

```
//Two opposite triangles
```

```
#include "stdio.h"
```

```
#include "C6_function_getNaturalNumber_1.h"
```

```
void main() {
```

```
    int N; //height
```

```
    N = getNaturalNumber();
```

```
    int i; //row index
```

```
    for (i=1; i<=N; i++) {
```

```
        int j; //column index for the 1st triangle
```

```
        for (j=1; j<=i; j++) printf("*");
```

```
        int k; //column index for the 2nd triangle
```

```
        for (k=1; k<=2*N-2*i-1; k++) printf(" ");
```

```
        for (k=1; k<=i; k++)
```

```
            if (k<N) printf("*");
```

Compile:

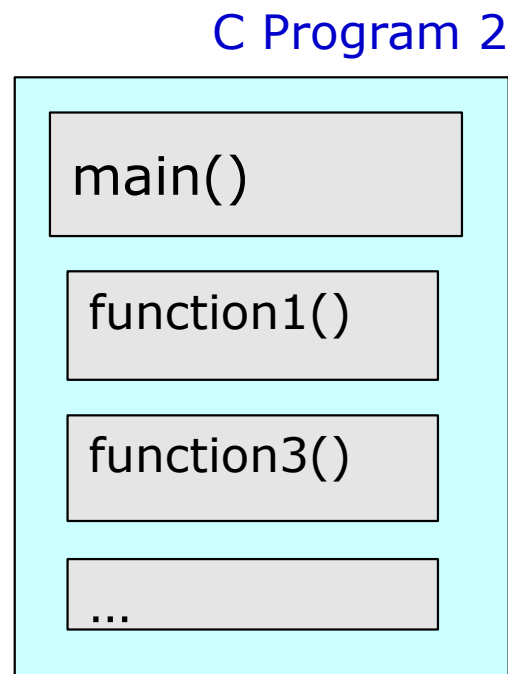
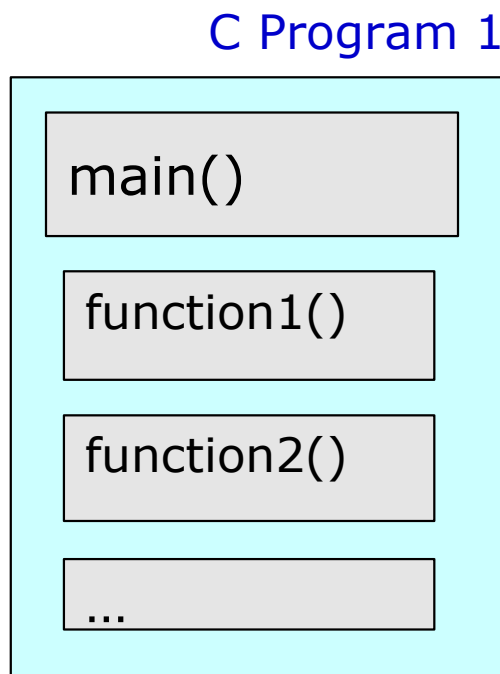
```
gcc C6_starTriangle_function_1.c C6_function_getNaturalNumber.c
-o C6_starTriangle_function_1.exe
```

# Introduction

---

## □ A function

- A processing unit to perform a specific task
- A means to modularize a program for manageable program development



Divide-and-conquer  
Reusable  
Information hiding  
Abstraction  
Easy for debugging

# Introduction

---

- Issues related to functions
  - Function definition
  - Function declaration
  - Function call

# An example of a function

---

- ▣ Prepare your own library for numbers
  - Compute the sum of N first natural numbers
    - ▣  $\text{sum} = 1 + 2 + 3 + \dots + (N-1) + N$
  - Compute the factorial of N, a natural number
    - ▣  $\text{factorial} = 1 * 2 * 3 * \dots * (N-1) * N$
  - Compute the n-th power of x, a floating-point number
    - ▣  $x^n = x * x * x * \dots * x$
  - Count the number of digits in N, a natural number
    - ▣  $N = 123456 \quad \Rightarrow \text{Number of digits} = 6$
  - Round x, a floating-point number, with two digits after the decimal point
    - ▣  $x = 1.23456 \quad \Rightarrow x = 1.23$
    - ▣  $x = 9.87654321 \quad \Rightarrow x = 9.88$

# An example of a function

---

- Prepare your own library for numbers
  - Check if a natural number is a prime number
    - $7 \Rightarrow \text{true} (1)$
    - $8 \Rightarrow \text{false} (0)$
  - Check if a natural number is a squared number
    - $4 \Rightarrow \text{true} (1)$
    - $8 \Rightarrow \text{false} (0)$
  - Toggle non-zero digits to '9' digits in an integer number to generate its 9-based mask
    - $113789 \Rightarrow 999999$
    - $-10789 \Rightarrow -90999$
  - Count the number of occurrences of each digit in an integer number
    - $113789 \Rightarrow 0: 0; 1: 2; 2: 0; 3: 1; 4: 0; 5: 0; 6: 0; 7: 1; 8: 1; 9: 1$
    - $-20054 \Rightarrow 0: 2; 1: 1; 2: 1; 3: 0; 4: 1; 5: 1; 6: 0; 7: 0; 8: 0; 9: 0$



# An example of a function

---

- Prepare your own library for numbers
  - Estimate a value of e, the natural number

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} \approx 2.71828$$

- Estimate a value of  $e^x$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

- Estimate a value of PI

$$\pi = \sum_{k=0}^{\infty} \frac{(-1)^k 4}{2k+1}$$

- ...

# Components of a function

---

- Given  $N$ , a natural number, calculate the *factorial of  $N$* :  $N! = 1*2*..*N = (N-1)!*N$

```
//input: a natural number n
//output: -1: invalid input; >=1: the factorial of n
//precondition: n>=0
double factorial(int n=1) {

    if (n<0) return -1;

    if (n==0) return 1;

    double aFact = 1;
    int i;
    for (i=1; i<=n; i++) aFact *= i;

    return aFact;
}
```

# Components of a function

- Given  $N$ , a natural number, calculate the *factorial of  $N$* :  $N! = 1 * 2 * \dots * N = (N-1)! * N$

```
//input: a natural number n
//output: -1: invalid input; >=1: the factorial of n
//precondition: n>=0
double factorial(int n=1) {
    if (n<0) return -1;
    if (n==0) return 1;
    double aFact = 1;
    int i;
    for (i=1; i<=n; i++) aFact *= i;
    return aFact;
}
```

Return type which is a data type of the value returned

Function name

Parameter list with comma separation. No default value for each parameter in C functions.

**return** statement to return a value of a return type to the caller

Function body that includes declarations and statements performed for a specific task

# Components of a function

---

```
[static] return-type function-name (argument-declarations)
{
    declarations and statements
}
```

- *function-name*: a valid identifier

Part of the input {  
- *argument-declarations*: a list of formal parameters in communication  
+ Each parameter is regarded as a local variable with a data type.  
+ Each parameter can be specified with "**const**" for unchanged intention.  
+ Each parameter can be passed by value or by reference if it is a pointer.

Processing in its body {  
- *declarations*: a list of local variables  
+ Each variable can be **auto** or **static** with one single initialization.  
- *statements*: zero, one, or many statements of any kind

Part of the output {  
- *return-type*: a valid data type or **void**  
+ Statement **return** [<expression>]; in the body is used to end the called function and return [a value] to its caller. If not, close brace of the body ends the called function and program control is switched to its caller.

Characteristic {  
- [**static**]: optional specification to make the function available only in the file where it is defined.

# Concepts related to functions

---

## □ *Function definition:*

```
[static] return-type function-name (argument-declarations)
{
    declarations and statements
}
```

## □ *Function prototype:*

```
return-type function-name (argument-declarations);
```

## □ *Function signature:*

```
function-name (argument-declarations)
```

## □ No concept of “nested functions”!

- Implementation-dependent

# Where is a function defined and declared?

---

- A function definition can be placed in:
  - the same file where the main() function is
    - Before the main() function
    - After the main() function
  - a separated file where the main() function is not
- Regardless of where a function is defined, its declaration is required before any call to it.
  - Function prototype in the global declaration section
  - Function prototype in a header file for common use

```
#define e 2.718281
```

```
//input: a natural number n  
//output: -1: invalid input; >=1: the factorial of n  
//precondition: n>=0  
double factorial(int n) {
```

```
    if (n<0) return -1;
```

```
    if (n==0) return 1;
```

```
    int i;
```

```
    double aFact = 1;
```

```
    for (i=1; i<=n; i++) aFact *= i;
```

```
    return aFact;
```

```
}
```

```
double power(double x, int n);
```

Function declaration

```
void main() {
```

```
    int i, n;
```

```
    double x, e_x = 0;
```

```
    printf("Enter a natural number: ");
```

```
    scanf("%d", &n);
```

```
    printf("\nEnter a floating point number: ");
```

```
    scanf("%lf", &x);
```

```
    for (i=0; i<=n; i++) e_x += power(x, i)/factorial(i);
```

```
    printf("\ne^x is real as: %g\n\n", pow(e, x));
```

```
    printf("\ne^x is approximated as: %g\n\n", e_x);
```

```
}
```

```
//input: a floating point number x and a natural number n
```

```
//output: a floating point number which is the n-th power of x
```

```
double power(double x, int n) {
```

```
    if (n==0) return 1;
```

```
    int i;
```

```
    double aPower = 1;
```

```
    for (i=1; i<=n; i++) aPower *= x;
```

```
    return aPower;
```

```
}
```

A program for an approximation  
of the **x**-th power of **e**

Function definition

= Function declaration

(as it is placed before its call)

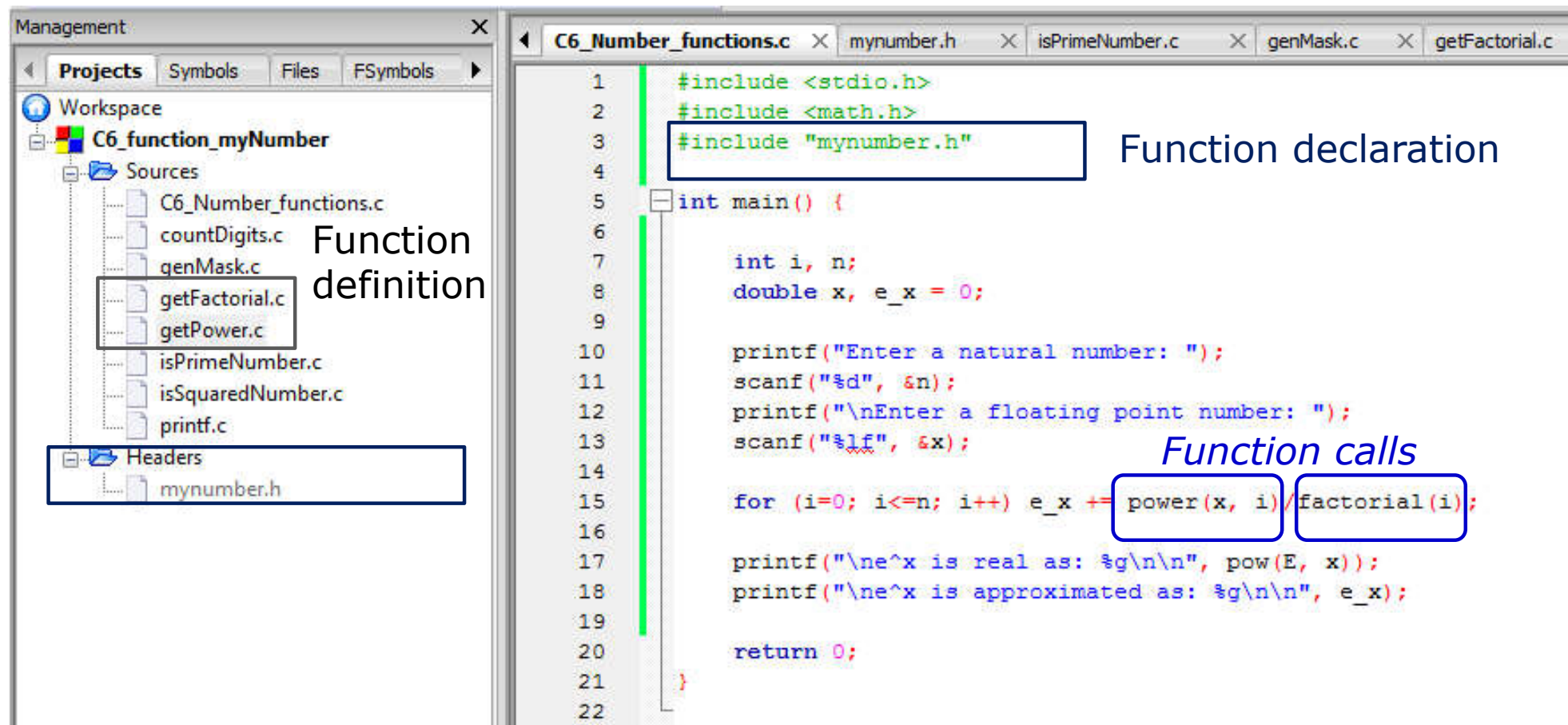
*a call to a function*

Function definition

(a function declaration is required  
as it is placed after its call.)

# Where is a function defined and declared?

## SEPARATED FILES



The image shows a C project workspace and a source code editor. The workspace on the left displays a project named 'C6\_function\_myNumber' with a 'Sources' folder containing several .c files and a 'Headers' folder containing 'mynumber.h'. The source code editor on the right shows the contents of 'C6\_Number\_functions.c'.

**Function declaration:** The code includes 'mynumber.h' (line 3), which contains the function declarations for 'power' and 'factorial'.

**Function definition:** The 'Sources' folder in the workspace contains the implementation files for 'countDigits.c', 'genMask.c', 'getFactorial.c', 'getPower.c', 'isPrimeNumber.c', 'isSquaredNumber.c', and 'printf.c'.

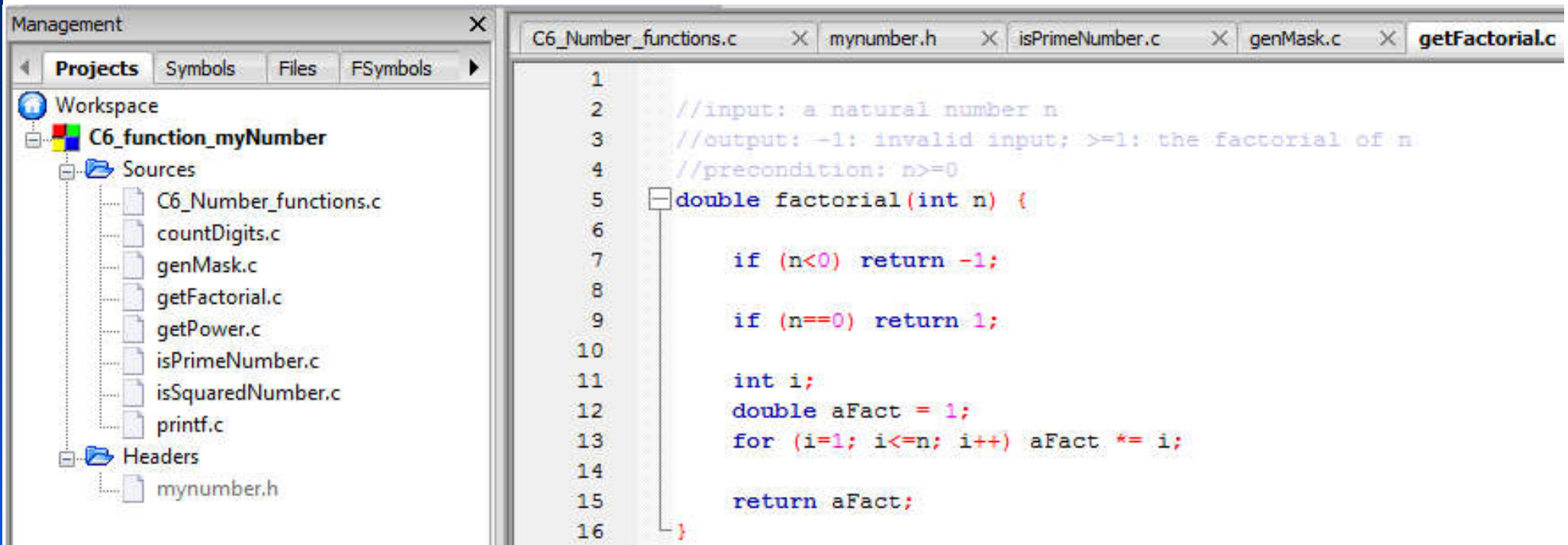
**Function calls:** The 'main' function (lines 5-21) calls 'power(x, i)' and 'factorial(i)' in the loop (line 15), and 'pow(E, x)' in the printf statement (line 17).

```
1  #include <stdio.h>
2  #include <math.h>
3  #include "mynumber.h"
4
5  int main() {
6
7      int i, n;
8      double x, e_x = 0;
9
10     printf("Enter a natural number: ");
11     scanf("%d", &n);
12     printf("\nEnter a floating point number: ");
13     scanf("%lf", &x);
14
15     for (i=0; i<=n; i++) e_x += power(x, i)/factorial(i);
16
17     printf("\ne^x is real as: %g\n\n", pow(E, x));
18     printf("\ne^x is approximated as: %g\n\n", e_x);
19
20     return 0;
21 }
```



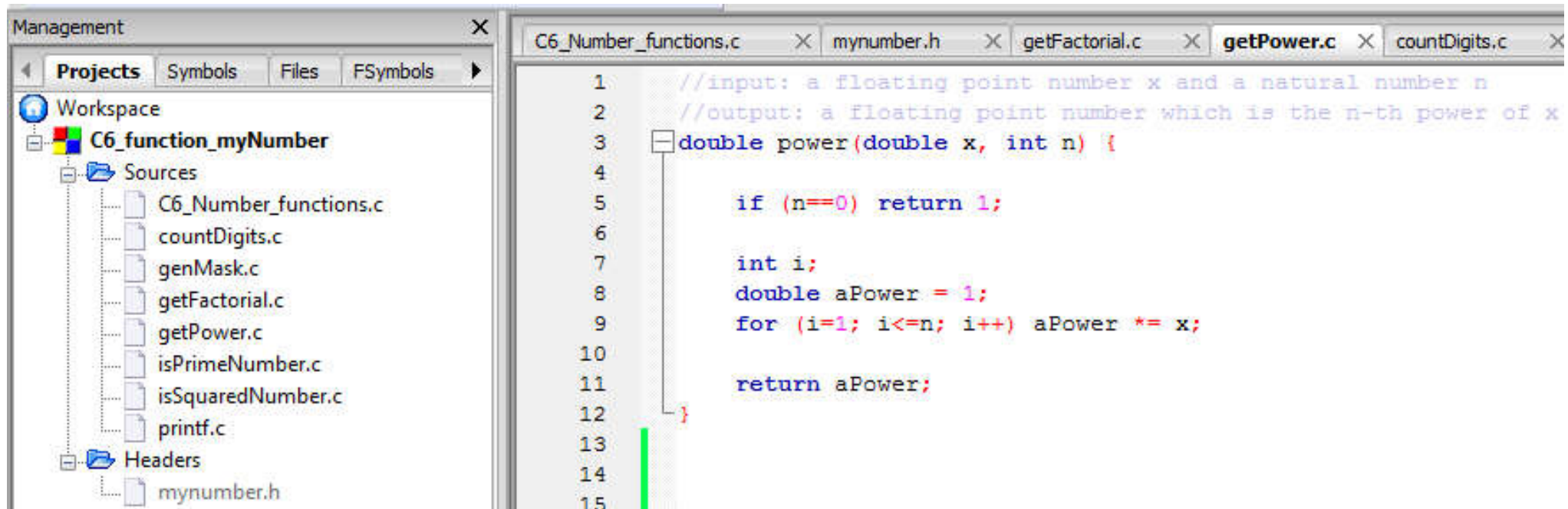
# Where is a function defined and declared?

Source file  
getFactorial.c



# Where is a function defined and declared?

Source file  
getPower.c

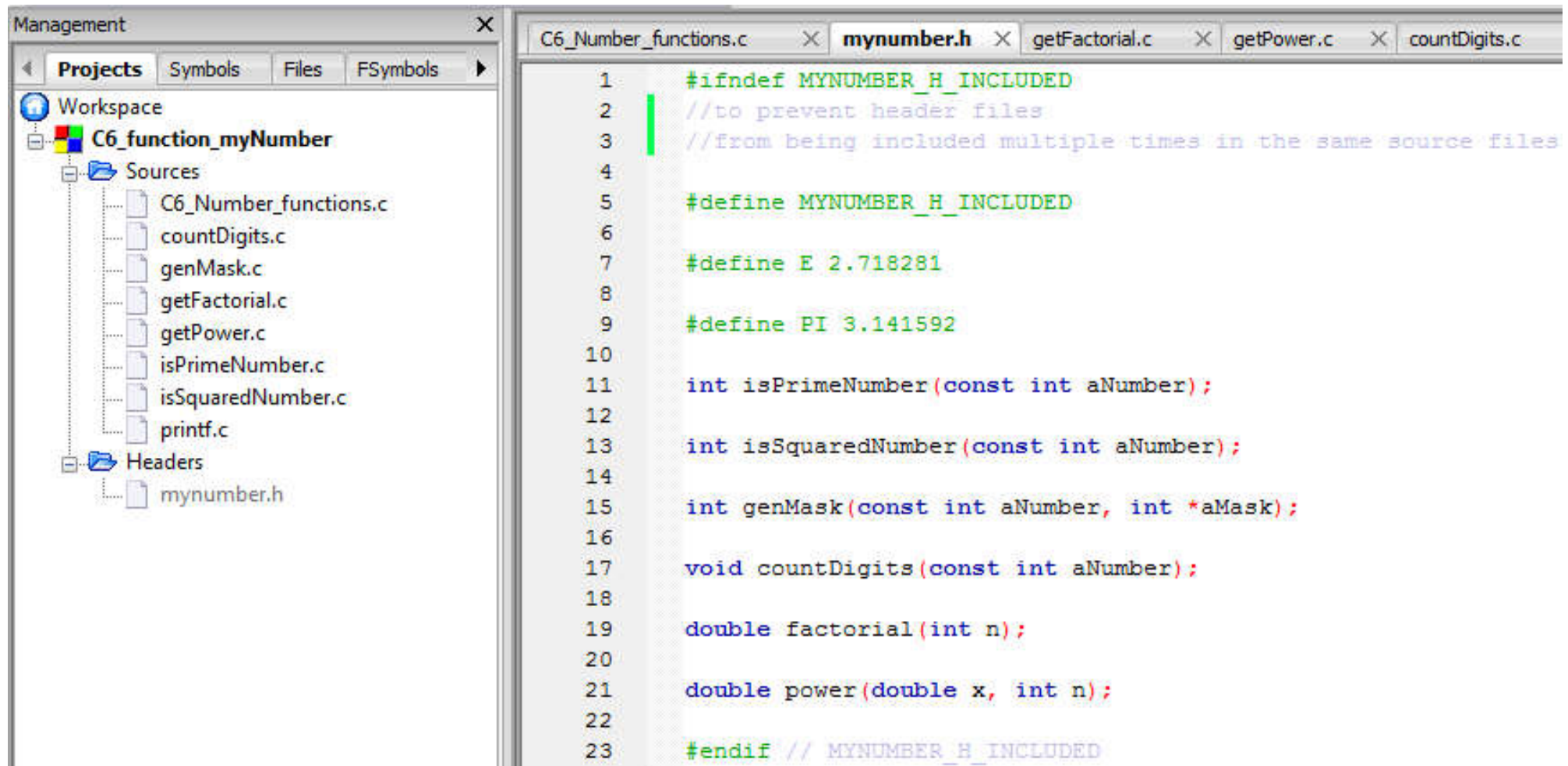


The screenshot shows a C64 IDE interface. On the left, the 'Management' pane displays a project named 'C6\_function\_myNumber' with a 'Sources' folder containing several .c files and a 'Headers' folder containing 'mynumber.h'. The 'getPower.c' file is highlighted. On the right, the 'getPower.c' file is open, showing the following code:

```
1 //input: a floating point number x and a natural number n
2 //output: a floating point number which is the n-th power of x
3 double power(double x, int n) {
4
5     if (n==0) return 1;
6
7     int i;
8     double aPower = 1;
9     for (i=1; i<=n; i++) aPower *= x;
10
11     return aPower;
12 }
```

# Where is a function defined and declared?

Header file for common use  
mynumber.h



The screenshot shows a code editor with a project structure on the left and a header file on the right. The project structure, titled 'C6\_function\_myNumber', includes a 'Sources' folder with files like C6\_Number\_functions.c, countDigits.c, genMask.c, getFactorial.c, getPower.c, isPrimeNumber.c, isSquaredNumber.c, and printf.c. It also has a 'Headers' folder containing mynumber.h. The main editor window displays the content of mynumber.h, which includes preprocessor directives for header inclusion, macro definitions for E and PI, and function declarations for isPrimeNumber, isSquaredNumber, genMask, countDigits, factorial, and power.

```
1  #ifndef MYNUMBER_H_INCLUDED
2  //to prevent header files
3  //from being included multiple times in the same source files
4
5  #define MYNUMBER_H_INCLUDED
6
7  #define E 2.718281
8
9  #define PI 3.141592
10
11 int isPrimeNumber(const int aNumber);
12
13 int isSquaredNumber(const int aNumber);
14
15 int genMask(const int aNumber, int *aMask);
16
17 void countDigits(const int aNumber);
18
19 double factorial(int n);
20
21 double power(double x, int n);
22
23 #endif // MYNUMBER_H_INCLUDED
```

# Function call

---

- ❑ Function call is a mention of a function to another function or itself.
  - The function whose function body contains a a mention is called a calling function or a caller.
  - The function whose name is mentioned in the caller's function body is called a called function or a callee.
  - The caller is the same as or different from the callee.
- ❑ The program in C6\_function\_myNumber
  - The main() function calls the printf(), scanf(), pow(), factorial(), and power() functions.
    - ❑ Caller = main
    - ❑ Callees = printf, scanf, pow, factorial, power

# Function call

---

- A function call is mentioned in the function body of the caller as:

*function-name (argument-list)*

- *argument-list*

- Optional, i.e. empty if the callee has no argument
  - Each argument is called actual parameter which is an expression corresponding to a formal parameter by order.
  - Each argument has a type compatible with the type of its corresponding formal parameter. Otherwise, type conversion with promotion or truncation is performed.
  - Assignment of each expression value to the corresponding formal parameter's memory is performed.
- A value of a return type (if it is not **void**) is returned via this function call.

```
#include <stdio.h>
#include <math.h>
#include "mynumber.h"
```

```
int main() {

    int i, n;
    double x, e_x = 0;

    printf("Enter a natural number: ");
    scanf("%d", &n);
    printf("\nEnter a floating point number: ");
    scanf("%lf", &x);

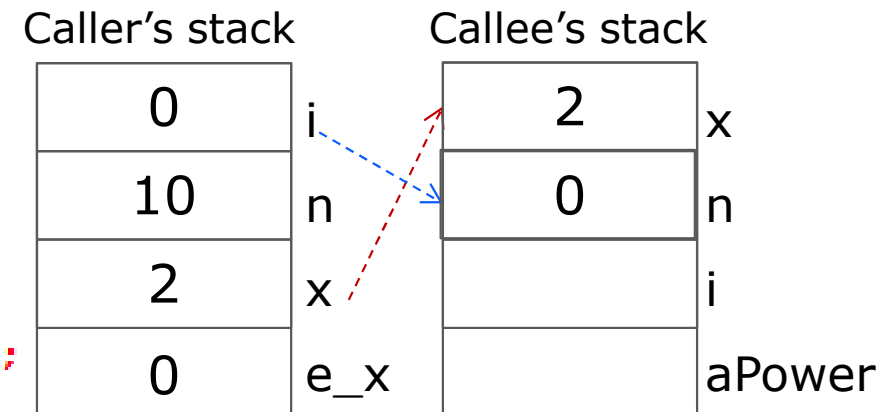
    for (i=0; i<=n; i++) e_x += power(x, i)/factorial(i);

    printf("\ne^x is real as: %g\n\n", pow(E, x));
    printf("\ne^x is approximated as: %g\n\n", e_x);

    return 0;
}
```

The caller

Stack's values when i=0 in the main() function



```
double power(double x, int n) {

    if (n==0) return 1;

    int i;
    double aPower = 1;
    for (i=1; i<=n; i++) aPower *= x;

    return aPower;
}
```

The callee

# Function call

---

## □ Function call by value

- Parameters are passed by value.
  - The actual parameter values are copied into local storage of the formal parameters of the callee.
- The caller and callee do not share any memory.

## □ Function call by reference

- C has **no explicit reference parameters** but implements them via pointers, i.e. address passing.
- Pointers are passed to the arguments.
  - There is only one copy of the value at any time.
- The caller and callee have access to the value in their shared memory through pointers.

# Function call

---

A function to swap two integer numbers

```
void swap(int a, int b){  
    int temp;  
    temp = a;  
    a = b;  
    b = temp;  
}
```

**a** and **b** will be passed by values of int type.

```
void swap(int *a, int *b){  
    int temp;  
    temp = *a;  
    *a = *b;  
    *b = temp;  
}
```

**a** and **b** will be passed by pointers to int values, i.e. addresses of the memory that contains int values.

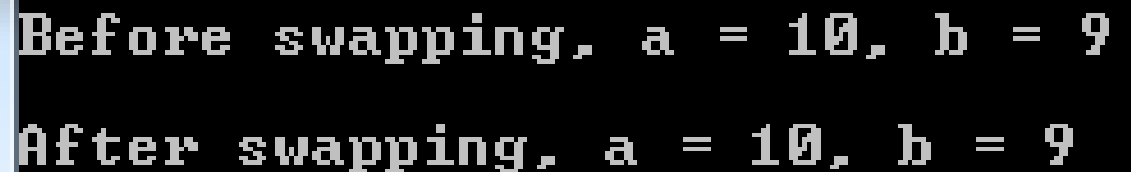


# Function call by value

```
#include <stdio.h>
```

```
void swap (int a, int b) {  
    int temp;  
  
    temp = a;  
    a = b;  
    b = temp;  
}
```

```
void main() {  
    int a = 10, b = 9;  
  
    printf("\nBefore swapping, a = %d, b = %d \n", a, b);  
  
    swap(a, b);  
  
    printf("\nAfter swapping, a = %d, b = %d \n", a, b);  
}
```



```
Before swapping, a = 10, b = 9  
After swapping, a = 10, b = 9
```

- ❑ Change on formal parameters in the callee has no impact on actual parameters in the caller.

# Function call by value

```
#include <stdio.h>
```

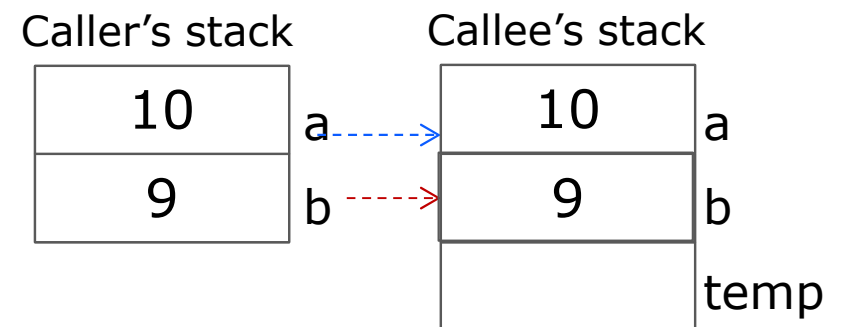
```
void swap (int a, int b) {  
    int temp;  
  
    temp = a;  
    a = b;  
    b = temp;  
}
```

```
void main() {  
    int a = 10, b = 9;  
  
    printf("\nBefore swapping, a = %d, b = %d \n", a, b);  
  
    swap(a, b);  
  
    printf("\nAfter swapping, a = %d, b = %d \n", a, b);  
}
```

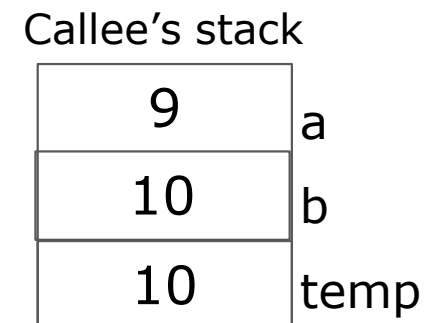
```
Before swapping, a = 10, b = 9
```

```
After swapping, a = 10, b = 9
```

Stack's values when a=10 and b=9 in the main() function



Stack's values before the callee ends



# Function call by reference

```
#include <stdio.h>
```

```
void swap (int *a, int *b) {  
    int temp;  
  
    temp = *a;  
    *a = *b;  
    *b = temp;  
}
```

```
void main() {  
    int a = 10, b = 9;  
  
    printf("\nBefore swapping, a = %d, b = %d \n", a, b);  
  
    swap(&a, &b);  
  
    printf("\nAfter swapping, a = %d, b = %d \n", a, b);  
}
```

Before swapping, a = 10, b = 9  
After swapping, a = 9, b = 10

- Change on the shared memory can be made by both callee and caller. Pointers are used for reference.

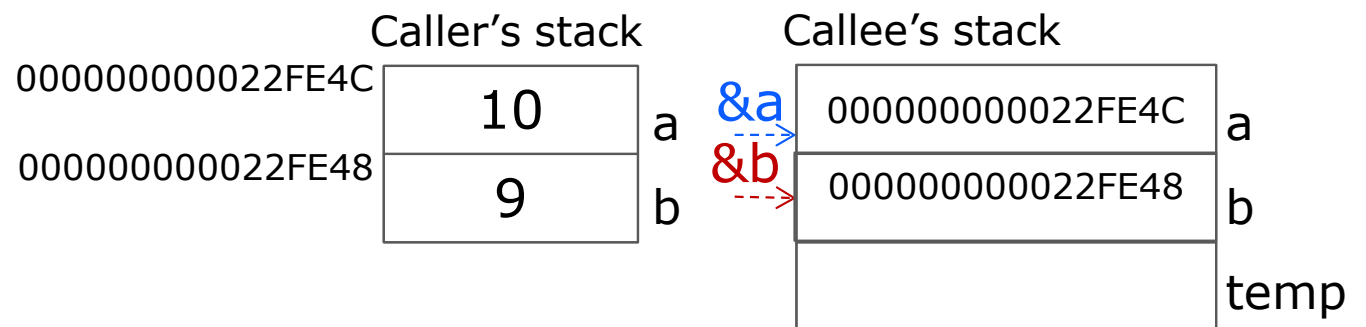
# Function call by reference

```
#include <stdio.h>
```

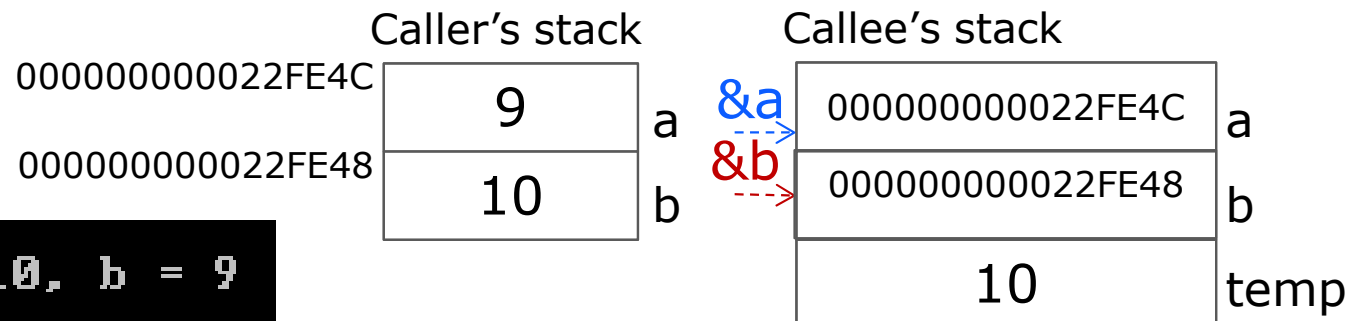
```
void swap (int *a, int *b) {  
    int temp;  
  
    temp = *a;  
    *a = *b;  
    *b = temp;  
}
```

```
void main() {  
    int a = 10, b = 9;  
  
    printf("\nBefore swapping, a = %d, b = %d \n", a, b);  
  
    swap(&a, &b);  
  
    printf("\nAfter swap  
}
```

Stack's values when a=10 and b=9 in the main() function



Stack's values before the callee ends



```
Before swapping, a = 10, b = 9  
After swapping, a = 9, b = 10
```

# Recursion

---

- ❑ A recursive function is a function that calls itself either directly or indirectly.
- ❑ When a function calls itself recursively, each invocation gets a fresh set of all the automatic variables, independent of the previous set.

```
double factorial(int n) {  
  
    if (n<0) return -1;  
  
    if (n==0) return 1;  
  
    int i;  
    double aFact = 1;  
    for (i=1; i<=n; i++) aFact *= i;  
  
    return aFact;  
}
```

```
double rFactorial(int n) {  
  
    if (n<0) return -1;  
  
    if (n==0) return 1;  
  
    return n*rFactorial(n-1);  
}
```

Function to compute the factorial of n:

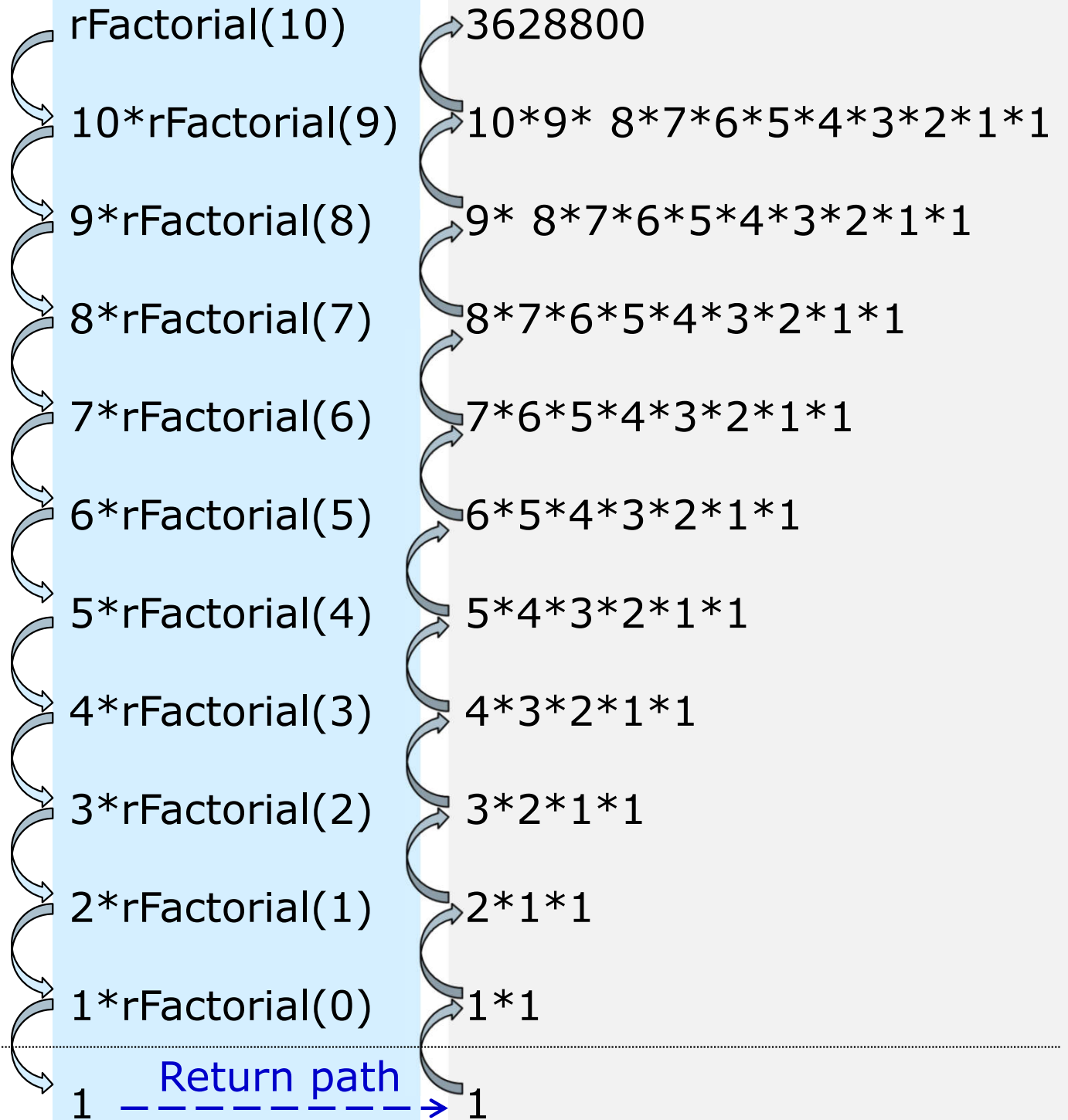
non-recursive vs. recursive versions 37

# rFactorial(10)

Recursion path

Recursive cases  
with smaller sizes

Base case



# Recursion

---

- Writing a recursive function
  - Determine and write the **base cases** and their solutions
    - No recursive call is specified for those base cases.
  - Determine and write the **recursive (inductive) cases** and their solutions
    - Establish a connection between the larger problem and the smaller problems using recursive calls
  - Determine the other cases that are neither base nor recursive cases
    - Check for other constraints with no recursive call

# Recursion

---

- Compute the sum of  $N$  first natural numbers
  - $\text{sum} = 1 + 2 + 3 + \dots + (N-1) + N$ 
    - Base case:  $\text{sum}(1) = 1$
    - Recursive case:  $\text{sum}(N) = \text{sum}(N-1) + N$
- Compute the factorial of  $N$ , a natural number
  - $\text{factorial} = 1 * 2 * 3 * \dots * (N-1) * N$ 
    - Base case:  $\text{factorial}(0) = \text{factorial}(1) = 1$
    - Recursive case:  $\text{factorial}(N) = \text{factorial}(N-1) * N$
- Compute the  $n$ -th power of  $x$ , a floating-point number
  - $x^n = x * x * x * \dots * x$ 
    - Base case:  $\text{power}(x, 0) = 1$
    - Recursive case:  $\text{power}(x, n) = \text{power}(x, n-1) * x$



# Recursion

---

- Estimate a value of e, the natural number

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} \approx 2.71828$$

- Base case:  $e(0) = 1$
- Recursive case:  $e(n) = e(n-1) + 1/\text{factorial}(n)$

# Recursion

---

- Estimate a value of  $e^x$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

- Base case:  $e\_x(0) = 1$
- Recursive case:

$$e\_x(n) = e\_x(n-1) + \text{power}(x, n)/\text{factorial}(n)$$

# Recursion

---

## □ Estimate a value of PI

$$\pi = \sum_{k=0}^{\infty} \frac{(-1)^k 4}{2k+1}$$

- Base case:  $\text{pi}(0) = 4$

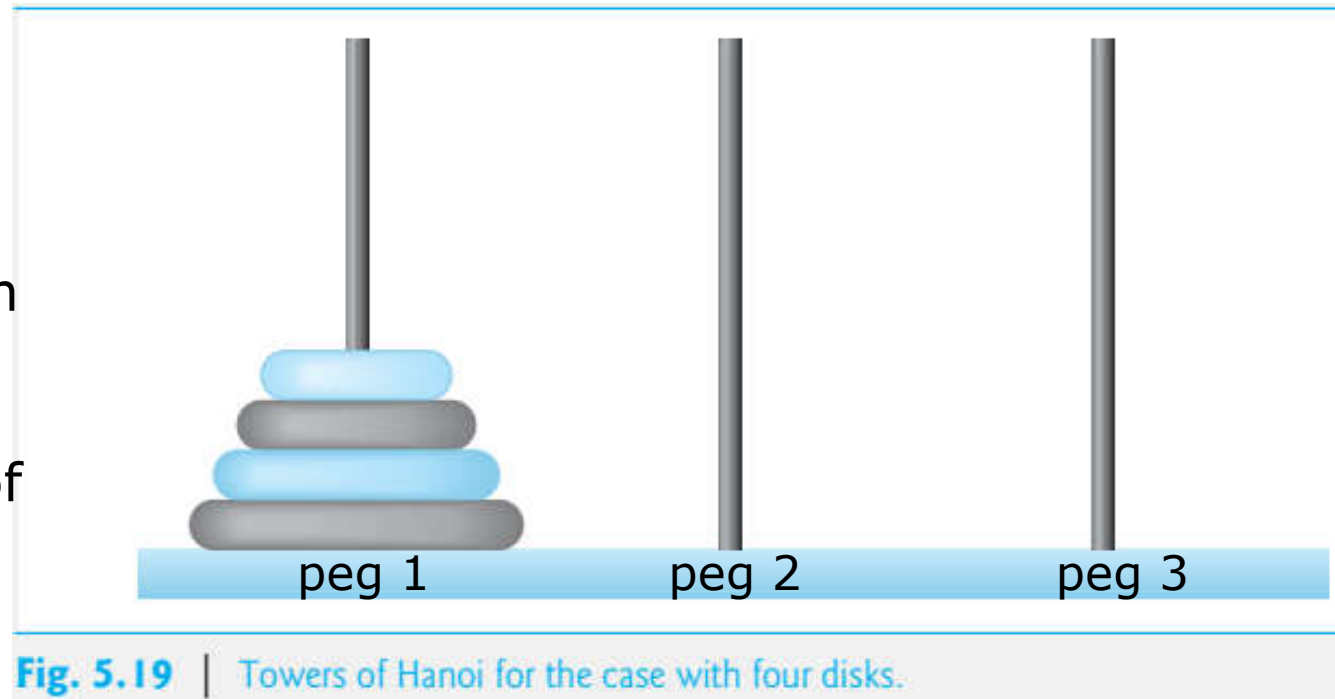
- Recursive case:

$$\text{pi}(k) = \text{pi}(k-1) + \text{power}(-1, k) * 4 / (2 * k + 1)$$

# Recursion

## □ Hanoi Tower

Move disks from peg 1 to peg 3 using peg 2 as a temporary holding area in such a way that smaller disks are always on top of larger disks and one disk is moved at a time



A recursive function with 4 parameters:

- a). The number of disks to be moved
- b). The peg on which these disks are initially threaded
- c). The peg to which this stack of disks to be moved
- d). The peg to be used as a temporary holding area

# Recursion

## □ Hanoi Tower

```
#include <stdio.h>

void towerHanoi(int n, int a, int b, int t) {

    if (n==1) printf("\nMove %d->%d\n", a, b);
    else {
        towerHanoi(n-1, a, t, b);
        towerHanoi(1, a, b, t);
        towerHanoi(n-1, t, b, a);
    }
}

void main() {

    int n = 4;

    towerHanoi(n, 1, 3, 2);
}
```

```
Move 1->2
Move 1->3
Move 2->3
Move 1->2
Move 3->1
Move 3->2
Move 1->2
Move 1->3
Move 2->3
Move 2->1
Move 3->1
Move 2->3
Move 1->2
Move 1->3
Move 2->3
```

# Hanoi Tower

Move 1→2  
Move 1→3  
Move 2→3  
Move 1→2  
Move 3→1  
Move 3→2  
Move 1→2  
Move 1→3  
Move 2→3  
Move 2→1  
Move 3→1  
Move 2→3  
Move 1→2  
Move 1→3  
Move 2→3

```
towerHanoi(4, 1, 3, 2)
  towerHanoi(3, 1, 2, 3)
    towerHanoi(2, 1, 3, 2)
      towerHanoi(1, 1, 2, 3)
      towerHanoi(1, 1, 3, 2)
      towerHanoi(1, 2, 3, 1)
    towerHanoi(1, 1, 2, 3)
    towerHanoi(2, 3, 2, 1)
      towerHanoi(1, 3, 1, 2)
      towerHanoi(1, 3, 2, 1)
      towerHanoi(1, 1, 2, 3)
  towerHanoi(1, 1, 3, 2)
  towerHanoi(3, 2, 3, 1)
    towerHanoi(2, 2, 1, 3)
      towerHanoi(1, 2, 3, 1)
      towerHanoi(1, 2, 1, 3)
      towerHanoi(1, 3, 1, 2)
    towerHanoi(1, 2, 3, 1)
    towerHanoi(2, 1, 3, 2)
      towerHanoi(1, 1, 2, 3)
      towerHanoi(1, 1, 3, 2)
      towerHanoi(1, 2, 3, 1)
```

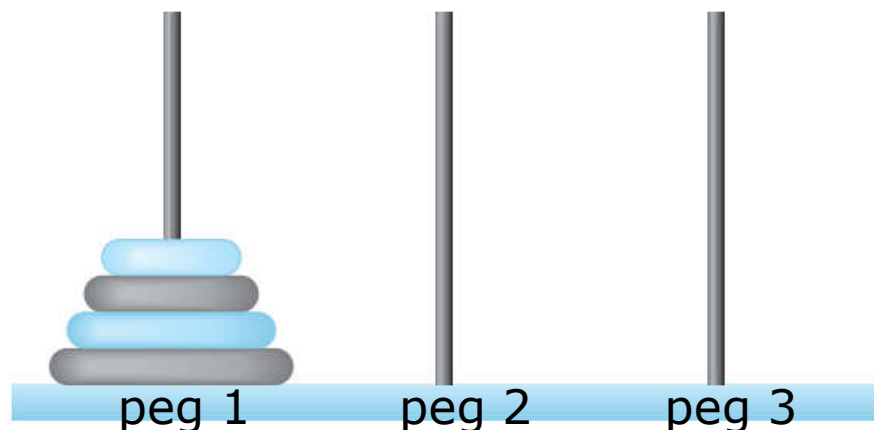


Fig. 5.19 | Towers of Hanoi for the case with four disks.

# Recursion

---

- ❑ Recursive function's concern
  - No saving in storage
  - Not faster
  - Infinite recursion
    - ❑ No base case is defined or base case can not be reached.
- ❑ Recursive function's advantages
  - Compact recursive code
  - Easy to write and understand
  - Convenient for recursively defined data structures and problems

# Summary

---

- A function is a processing unit for a specific task.
  - Divide-and-conquer approach
  - Reusability
  - Information hiding
  - Abstraction
  - Support for debugging
- Manageable program development



# Summary

---

- Function definition
- Function declaration
- Function call
  - By value
  - By reference with pointer implementation
- Recursion
  - Recursive problem
  - Recursive data structure

# Chapter 6: Functions

---

