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# 1 Modeling

## 1.1 Splines

### 1.1.1 Cubic Hermite Interpolation

Each point is defined by its position  $h_n$  and slope  $h_{m+n}$ ,  $m$  being the number of control points. To simplify calculations, it is assumed that  $t_0 = 0$  and  $t_1 = 1$ .

The goal is to convert from a monomial basis

$$\begin{aligned}\phi_0(t) &= 1 \\ \phi_1(t) &= t \\ \phi_2(t) &= t^2 \\ \phi_3(t) &= t^3\end{aligned}$$

to a hermite basis

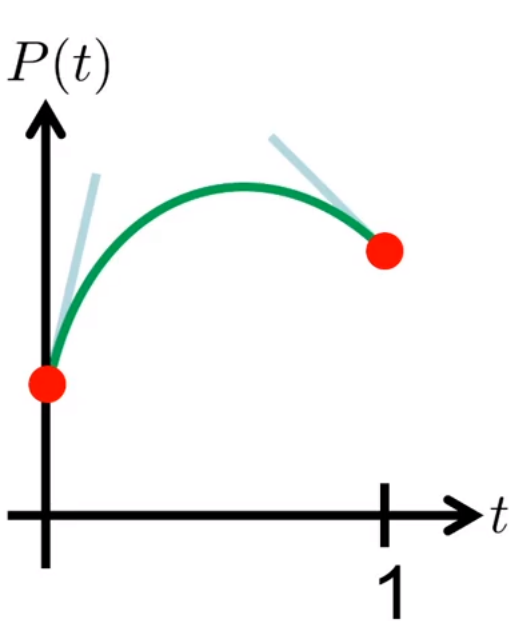
$$\begin{aligned}H_0(t) &= 2t^3 - 3t^2 + 1 \\ H_1(t) &= -2t^3 + 3t^2 \\ H_2(t) &= t^3 - 2t^2 + t \\ H_3(t) &= t^3 - t^2\end{aligned}$$

so that instead of having to manipulate polynomial coefficients

$$f(t) = a\phi_3(t) + b\phi_2(t) + c\phi_1(t) + d\phi_0(t)$$

an easier point slope method can be used:

$$f(t) = h_0H_0(t) + h_1H_1(t) + h_2H_2(t) + h_3H_3(t)$$



$$\begin{aligned}P(t) &= at^3 + bt^2 + ct + d \\ P'(t) &= 3at^2 + 2bt + c\end{aligned}$$

$$\begin{aligned}h_0 &= P(0) = d \\ h_1 &= P(1) = a + b + c + d \\ h_2 &= P'(0) = c \\ h_3 &= P'(1) = 3a + 2b + c\end{aligned}$$

Unknowns in this equation are  $a$ ,  $b$ ,  $c$ , and  $d$ , so a matrix can be used to solve the systems of equations:

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

$a$ ,  $b$ ,  $c$ , and  $d$  can be obtained from  $h$  values by inverting the matrix:

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

From these solved  $h$  values,  $P(t)$  can now be converted to a form that is easier for a user to manipulate, in terms of  $h$  values:

$$\begin{aligned}
 P(t) &= at^3 + bt^2 + ct + d \\
 &= (2h_0 - 2h_1 + h_2 + h_3)t^3 \\
 &\quad + (-3h_0 + 3h_1 - 2h_2 - h_3)t^2 \\
 &\quad + h_2t + h_0 \\
 &= h_0(2t^3 - 3t^2 + 1) + h_1(-2t^3 + 3t^2) + \\
 &\quad h_2(t^3 - 2t^2 + t) + h_3(t^3 - t^2)
 \end{aligned}$$

Each equation in  $P(t) = h_0(2t^3 - 3t^2 + 1) + h_1(-2t^3 + 3t^2) + h_2(t^3 - 2t^2 + t) + h_3(t^3 - t^2)$  that is multiplied by an  $h$  value is called a cubic hermite.

### 1.1.2 More than 1D

A parametric curve described by  $\vec{\gamma}(t) = (\gamma_0(t), \gamma_1(t))$  can be converted into hermite basis like this:

$$\underbrace{\vec{\gamma}'(t)}_{\text{Tangent}} = \underbrace{(\gamma_0'(t), \gamma_1'(t))}_{\text{Slopes}}$$

where cubic hermite interpolation can be done for both dimensions.

### 1.1.3 Cubic blossom

The cubic blossom of a function  $f(t)$  is  $F(t_1, t_2, t_3)$ .

Cubic blossoms have three properties:

1. Symmetric
  - $F(t_1, t_2, t_3) = F(t_1, t_3, t_2) = F(t_3, t_1, t_2) \dots$
2. Affine
  - $F(\alpha u + (1 - \alpha)v, t_2, t_3) = \alpha F(u, t_2, t_3) + (1 - \alpha)F(v, t_2, t_3)$
3. Diagonal
  - $f(t) = F(t, t, t)$

Blossoming example: Given function  $f(t) = t^3$ , one way to write  $F(t_1, t_2, t_3)$  is  $F(t_1, t_2, t_3) = t_1 t_2 t_3$ .