CS 327A HW1

1. a

$$ {}^0J = \begin{bmatrix} \frac{\partial}{\partial q_1}{}^0x_p, & \frac{\partial}{\partial q_2}{}^0x_p, & \frac{\partial}{\partial q_3}{}^0x_p & \frac{\partial}{\partial q_4}{}^0x_p \\ \bar{\epsilon}_1({}^0_1R_7) & \bar{\epsilon}({}^0_2R_7) & \bar{\epsilon}({}^0_3R_7) & \bar{E}({}^0_4R_7) \end{bmatrix} $$

$$ \frac{\partial}{\partial d_1}x_p = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \frac{\partial}{\partial \theta_2} = \begin{bmatrix} -l_2S_2 - l_3S_{23} \\ l_2C_2 + l_3C_{23} \\ 0 \end{bmatrix} $$

$$ \frac{\partial}{\partial \theta_3} = \begin{bmatrix} -l_3S_{23} \\ l_3C_{23} \\ 0 \end{bmatrix} \quad \frac{\partial}{\partial d_4} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} $$

$$ = \begin{bmatrix} 0 & -l_2S_2 - l_3S_{23} & -l_3S_{23} & 0 \\ 0 & l_2C_2 + l_3C_{23} & l_3C_{23} & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} $$

b. Yes. $n = 4 \quad m_{k(v)} = 3 \quad 4 - 3 = 1$

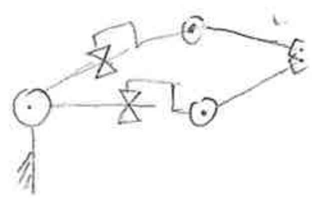
c $J(q) = E(x)J_0(q)$

$$ E(x) = \begin{bmatrix} E_p(x_p) & 0 \\ 0 & E_r(x_r) \end{bmatrix}, \quad E_p(x_p) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta/\rho & \cos\theta/\rho & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_r(x_r) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} $$

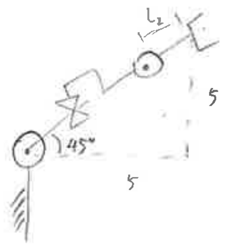
Thus $J(q) = \begin{bmatrix} E_p(x_p) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = E_p(x_p)J_v = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta/\rho & \cos\theta/\rho & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{3\times3} \begin{bmatrix} 0 & -l_2S_2 - l_3S_{23} & -l_2S_{23} & 0 \\ 0 & l_2C_2 + l_3C_{23} & l_3C_{23} & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

2. (a) $N = 3$ $m_K = 2$ Degree of redundancy = 1

(b)



(c)



$d_{2\,min} = 5\sqrt{2} - 2$

(d) $$J = \begin{bmatrix} -(5\sqrt{2}-2)\frac{\sqrt{2}}{2} - 2\cdot\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -2\cdot\frac{\sqrt{2}}{2} \\ (5\sqrt{2}-2)\frac{\sqrt{2}}{2} + 2\cdot\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2\cdot\frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} -5 & \frac{\sqrt{2}}{2} & -\sqrt{2} \\ 5 & \frac{\sqrt{2}}{2} & \sqrt{2} \end{bmatrix}$$

right-inverse

$$J^+ = J^T(JJ^T)^{-1} = \begin{bmatrix} -0.093 & 0.093 \\ 0.707 & 0.707 \\ -0.026 & 0.026 \end{bmatrix}$$

(e) $$N_{proj} = I_3 - J^+J = \begin{bmatrix} -0.074 & 0 & -0.262 \\ 0 & 0 & 0 \\ -0.262 & 0 & -0.926 \end{bmatrix}$$

(f) Columns of N_{proj} represent how we can move each Joint without moving End effector. In our case 2^{nd} column is 0, which means $\delta d_2 \triangleq 0$ in the nullspace. In otherwords, To maintain the position of EE, the prismatic Joint 2 should not move and d_2 should remain fixed in the nullspace.

3 (a) $\det(J_0) = -S_1(C_2+C_{23}) \cdot \left(-S_1(S_2+S_{23})C_{23} + S_1 S_{23}(C_2+C_{23})\right)$

$+ C_1(S_2+S_{23})\left(C_1(C_2+C_{23})C_{23} - C_1 S_{23}\left(C_1(C_2+C_{23})(C_2+C_{23})\right)\right)$

$= -S_1^2(C_2+C_{23})\left(-(S_2+S_{22})C_{23} + S_{23}(C_2+C_{23})\right)$

$+ C_1^2(S_2+S_{23})\left((C_2+C_{23})C_{23} - C_1^2 S_{23}(C_2+C_{23})^2\right.$

$= -S^2 S_{23}(C_2+C_{23})^2 + S_1^2 C_{23}(S_2+S_{23})(C_2+C_{23}) + C_1^2 C_{23}(S_2+S_{23})(C_2+C_{23}) - C_1^2 S_{23}(C_2+C_{23})^2$

$= -S_{23}(C_2+C_{23})^2 + C_{23}(S_2+S_{23})(C_2+C_{23}) = 0$

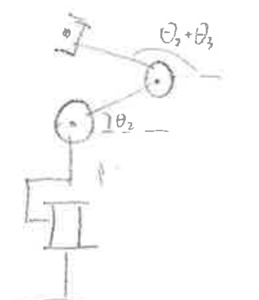
$= (C_2+C_{23})\left(-S_{23}C_2 - \cancel{S_{23}C_{23}} + C_{23}S_2 + \cancel{C_{23}S_{23}}\right) = 0$

$= (C_2+C_{23})(C_{23}S_2 - S_{23}C_2) = 0$

Singularity 1: $C_2 = -C_{23}$

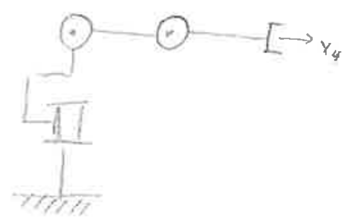
Singularity 2: $C_{23}S_2 = S_{23}C_2 \Rightarrow \tan 23 = \tan 2$

Sing 1

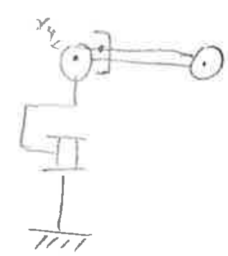


Sing 2

fully extended



fully flexed.



Z_4 is the singular direction

X_4 is the singular direction

(b) $J_{y_2} = [0 \quad 1+C_3 \quad C_3] = [0,\ 2,\ 1]$

(c) $J^+ = J_{y_2}^T (J_{y_2} J_{y_2}^T)^{-1} = \frac{1}{5}\begin{bmatrix}0\\2\\1\end{bmatrix}$

(d) $N_{proj} = I_3 - J^+J = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} - [0,\ 2,\ 1]\cdot\frac{1}{5}\begin{bmatrix}0\\2\\1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&0.2&-0.4\\0&-0.4&0.8\end{bmatrix}$

(e) ${}^0P_{EE} = \begin{bmatrix}C_1(C_2+C_{23})\\S_1(C_2+C_{23})\\S_2+S_{23}\end{bmatrix} = \begin{bmatrix}\cos 5^\circ(c95^\circ + c100^\circ)\\ s5\ (c95^\circ + c100^\circ)\\ \sin 95^\circ + \sin 100^\circ\end{bmatrix} = \begin{bmatrix}-0.259\\-0.022\\1.98\end{bmatrix}$

(f) $\delta q_n = N_{proj}\begin{bmatrix}5^\circ\\5^\circ\\5^\circ\end{bmatrix} = \begin{bmatrix}5^\circ\\-1^\circ\\2^\circ\end{bmatrix}$ $\quad q_{new} = [5^\circ, 59^\circ, 2^\circ]$ $\quad {}^0P_{EE} = \begin{bmatrix}\cos 5^\circ\ (c89^\circ + c91^\circ)\\ S5^\circ\ (c89^\circ + c91^\circ)\\ \sin 89^\circ + \sin 91^\circ\end{bmatrix} = \begin{bmatrix}0\\0\\1.9997\end{bmatrix}$

(g) The new position ${}^0P_{EE}$ along y_2 is 0. when an arbitrary vector δq_0 is projected to the nullspace of this config, The EE could move in x_2, z_2 directions, but no in y_2 direction

4. (i) (1) $^{0}P_4 = \begin{bmatrix} 0.303 \\ -0.175 \\ 0.244 \end{bmatrix}$ (2). $^{0}P_4 = \begin{bmatrix} 0.1 \\ 0 \\ 0.394 \end{bmatrix}$

(ii) $J_v = \begin{bmatrix} 0 & 1 & 0.15 \\ 0.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ rank $=2 < 3 \Rightarrow$ The manipulator is singular in this configuration

(iii) Config 1

$$M_1 = \begin{bmatrix} 0.317 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.102 \end{bmatrix}$$

No off diagonal terms, no inertial coupling

Config 2

$$M_2 = \begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.2 & 0.015 \\ 0 & 0.015 & 0.10225 \end{bmatrix}$$

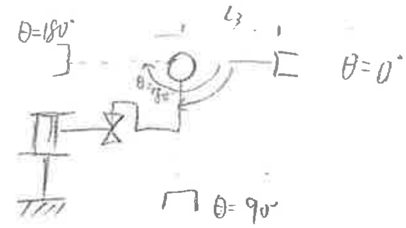
non-zero off diagonal terms at M_{23} and M_{32}, which indicate inertial coupling of joint 2 and 3.

(iv)

θ_3	G_1
$\theta_3 = 0$	$[0,0,-0.14715]$
30	$[0,0,-0.127]$
60	$[0,0,-0.0735]$
90	$[0,0,0]$
120	$[0,0,0.0735]$
150	$[0,0,0.127]$
180	$[0,0,0.14715]$

when $\theta_3 = 0$ or $180°$

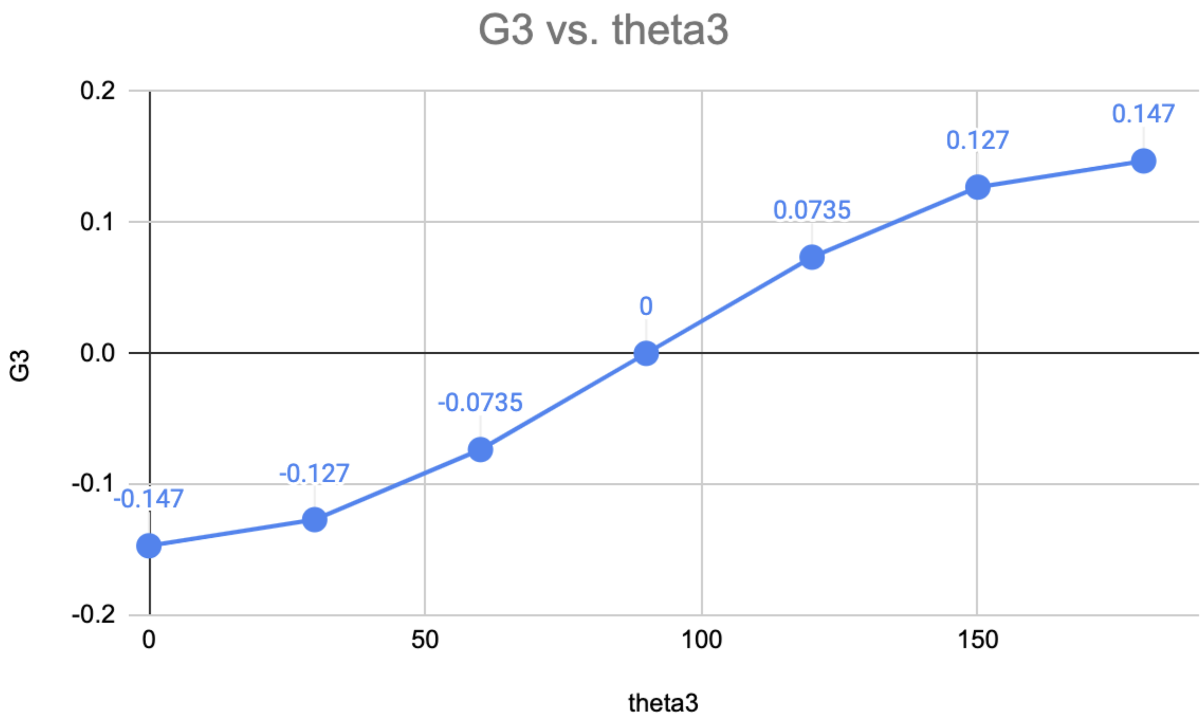
τ_3 reaches max value, because moment arm of weight at EE is at its max ($\tau_3 = W_{EE} \cdot L_3$ at $0°$ and $180°$)



when $\theta_3 = 90$, the moment arm of the weight at EE is 0

thus $\tau_3 = W_{EE} \cdot L_3 \cdot \cos 90° = 0$

τ_1 τ_2 remain 0 as θ_3 changes from 0 to $180°$ because the weight on EE do not induce movement on Joint 1 and Joint 2, and therefore no extra torques needed to compensate for gravity.



G3 vs. theta3
0.2
0.1
0.0
-0.1
-0.2
G3
0
50
100
150
theta3
-0.147
-0.127
-0.0735
0
0.0735
0.127
0.147