In this homework you will examine how both manipulator configuration and manipulator structure can affect the effective inertial properties of the manipulator's end effector.

For this homework there are no SAI implementation problems.

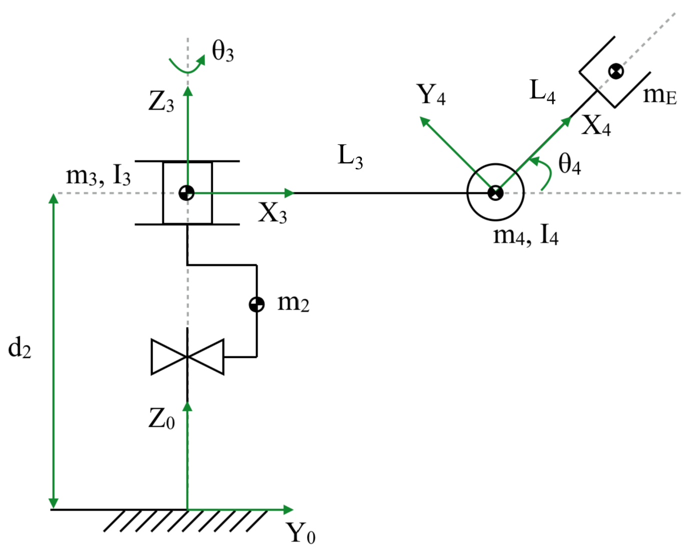
Problem 1: Inertial properties at different configurations

Consider the PRR manipulator shown below. (Note the non-standard joint mass and link numbering - this will be used in the next problem).

The task of this manipulator is to **position** the end-effector in the 3-D workspace. Use position of the end-effector in Cartesian coordinates x_E, y_E, and z_E as the operational space coordinates for this task.

L_3 is the link length of Link 3 (measured from the Frame 3 to Frame 4), and L_4 is the link length of Link 4 (measured from Frame 4 to mass m_E). The configuration shown below is when θ_3 = 0. The inertia tensors of links 3 and 4 in their respective link frames are:

$$^3I_3 = \begin{bmatrix} I_{xx,3} & 0 & 0 \\ 0 & I_{yy,3} & 0 \\ 0 & 0 & I_{zz,3} \end{bmatrix} \quad ^4I_4 = \begin{bmatrix} I_{xx,4} & 0 & 0 \\ 0 & I_{yy,4} & 0 \\ 0 & 0 & I_{zz,4} \end{bmatrix}$$



To simplify our analysis, we will assume that m_E is a point mass and that $m_2 = 0$. (m_2 will be left out of the matrices related to inertial properties)

The task Jacobian for this manipulator is as follows:

$$
{}^0J = {}^0J_v = \begin{bmatrix} 0 & -c_3(L_3 + L_4c_4) & L_4s_3s_4 \\ 0 & -s_3(L_3 + L_4c_4) & -L_4c_3s_4 \\ 1 & 0 & L_4c_4 \end{bmatrix}
$$

(a) Complete the joint space kinetic energy matrix A given below.

**** Hint **** You can fill in the missing values below using the properties of the joint space kinetic energy matrix and your intuition. An explicit calculation is not necessary.

$$
A = \begin{bmatrix} m_3 + m_4 + m_E & 0 & m_EL_4c_4 \\ 0 & m_4L_3^2 + m_E(L_3 + L_4c_4)^2 + I_{zz,3} + I_{xx,4}s_4^2 + I_{yy,4}c_4^2 & 0 \\ \_ & \_ & \_ \end{bmatrix}
$$

(b) **Draw the configuration** when $d_2 = 1m$, $\theta_3 = 0°$, and $\theta_4 = 90°$.

Then, **fill in the missing term** in the upper left-hand corner of the operational space inertia matrix at the end-effector ${}^4\Lambda^{-1}$ given below, calculated for the above configuration.

Note that this matrix is expressed in the end effector frame $\{4\}$

**** Hint **** Again, you do not have to calculate it explicitly. The missing term can be identified by considering the effective mass in the appropriate direction.

$$
{}^4\Lambda^{-1} = \begin{bmatrix} \_ & 0 & 0 \\ 0 & \frac{L_4^2}{I_{zz,4}+m_EL_4^2} & 0 \\ 0 & 0 & \frac{L_3^2}{I_{zz,3}+m_4L_3^2+I_{xx,4}+m_EL_3^2} \end{bmatrix}
$$

(c) Now for your subsequent calculations, use the following:

- The masses are $m_2 = 0kg$ and $m_3 = m_4 = m_E = 1kg$
- the inertias are $I_{xx,3} = I_{xx,4} = 2kg.m^2$, $I_{yy,3} = I_{yy,4} = 2kg.m^2$ and $I_{zz,3} = I_{zz,4} = 1kg.m^2$
- The link lengths are $L_3 = 2m$ and $L_4 = 1m$.

We will now examine the effective mass of this manipulator over the following 4 configurations:

(1) $d_2 = 1m$, $\theta_3 = 0°$, $\theta_4 = 90°$.
(2) $d_2 = 1m$, $\theta_3 = 0°$, $\theta_4 = 45°$.
(3) $d_2 = 1m$, $\theta_3 = 0°$, $\theta_4 = 30°$.
(4) $d_2 = 1m$, $\theta_3 = 0°$, $\theta_4 = 15°$.

Plot the 2D slice of the belted ellipsoid of effective mass in the $X_0 - Y_0$ plane for each configuration.

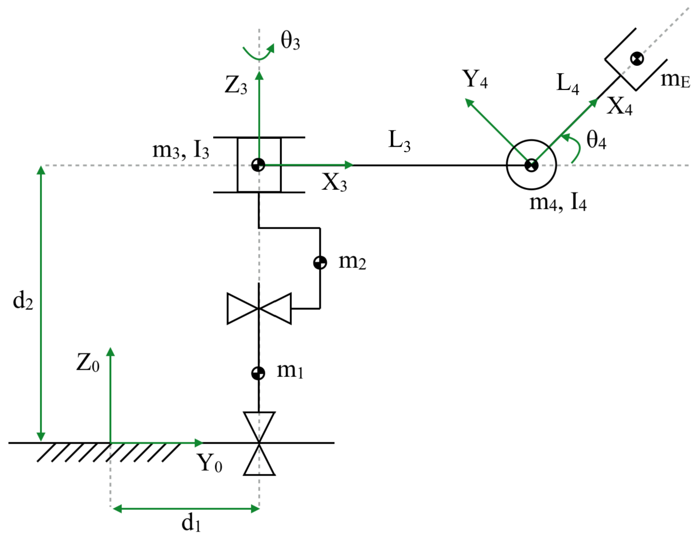
*** Note *** You can use the MATLAB, python, or any other language you like. Include your code together with the figure.

(d) Using your plots from part (c), describe how the shape of the belted ellipsoid changes as θ_4 approaches $0°$, particularly in the Y_0 direction. What does this mean for the effective mass of the end effector in the Y_0 direction as the robot approaches the $\theta_4 = 0°$ configuration? Explain your reasoning.

(e) When the robot reaches the $\theta_4 = 0°$ configuration (with $d_2 = 1m$, $\theta_3 = 0°$), in which direction can the robot no longer move? Which direction does the robot now have a redundancy in? Is this a Type 1 or Type 2 singularity? Explain your reasoning.

Problem 2: Macro-Mini Inertial Properties

Now consider the following PPRR manipulator. This manipulator is similar to the one from Problem 1, just with an extra prismatic joint added before the rest of the manipulator. We can think of this robot as a macro-mini system, where the macro part is the new prismatic joint, and the mini-part is the original PRR manipulator.



The task of this macro-mini manipulator is still to **position** the end-effector Again, assume that m_E is a point mass and $m_2 = 0$.

(a) Find the new task Jacobian matrix, ${}^0J$.

(b) Complete the joint space kinetic energy matrix A.

** **Hint** ** The lower 3x3 block matrix can be filled in by considering the mass properties of the macro-mini manipulator structure.

$$A = \begin{bmatrix} m_1 + m_3 + m_4 + m_E & 0 & -m_4 L_3 s_3 - m_E(L_3 s_3 + L_4 s_3 c_4) & -m_E L_4 c_3 s_4 \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

(c) For $d_1 = 0m$, $d_2 = 1m$, $\theta_3 = 0°$ and $\theta_4 = 90°$, the inverse of the operational space inertia matrix at the end-effector ${}^4\Lambda^{-1}$ is given by

$$
{}^4\Lambda^{-1} = \begin{bmatrix} \frac{1}{m_3+m_4+m_E} & 0 & 0 \\ 0 & \frac{L_4^2}{I_{zz,4}+m_E L_4^2}(1+\epsilon) & 0 \\ 0 & 0 & \frac{L_3^2}{I_{zz,3}+m_4 L_3^2+I_{xx,4}+m_E L_3^2} \end{bmatrix}
$$

where

$$
\epsilon = \frac{I_{zz,4}^2}{\left((m_1+m_3+m_4+m_E)I_{zz,4}+(m_1+m_3+m_4)m_E L_4^2\right) L_4^2}
$$

Explain how this shows that, in the given configuration, the effective mass in the Y_4-direction for this macro-mini manipulator is smaller than the effective mass in the Y_4-direction for the PRR mini manipulator alone

(d) For the same manipulator configurations from Part 1 (c), plot the 2D slice of the belted ellipsoid of effective mass in the $X_0 - Y_0$ plane of this new manipulator. Overlay your plots onto those from Part 1 (c) in a different color.

Use the same manipulator properties with the addition of the following:

- $m_1 = 1kg$
- $d_1 = 0m$

(e) Comment on the difference between the belted ellipsoids of the PPRR macro-mini manipulator as compared to the original PRR mini manipulator, particularly in the Y_0 direction. Explain why this difference occurs. What will happen to the effective mass of the macro-mini robot's end effector in the Y_0 direction as θ_4 approaches $0°$?