

Pl. a $W = \begin{bmatrix} I_3 & I_3 & I_3 & 0 & 0 & 0 \\ \hat{r}_1 & \hat{r}_2 & \hat{r}_3 & I_3 & I_3 & I_3 \end{bmatrix}$ where $r_1 = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$ $\hat{r}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$

$r_2 = \begin{bmatrix} 1.5 \\ 1.5 \\ 0 \end{bmatrix}$ $\hat{r}_2 = \begin{bmatrix} 0 & 0 & 1.5 \\ 0 & 0 & -1.5 \\ -1.5 & 1.5 & 0 \end{bmatrix}$

$r_3 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ $\hat{r}_3 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -2 \\ 1 & 2 & 0 \end{bmatrix}$

b. $e_{12} = \begin{bmatrix} \frac{3.5}{\sqrt{1.5^2 + 3.5^2}} \\ \frac{1.5}{\sqrt{1.5^2 + 3.5^2}} \\ 0 \end{bmatrix} = \begin{bmatrix} 0.919 \\ 0.399 \\ 0 \end{bmatrix}$ $e_{23} = \begin{bmatrix} \frac{0.5}{\sqrt{0.5^2 + 2.5^2}} \\ \frac{2.5}{\sqrt{0.5^2 + 2.5^2}} \\ 0 \end{bmatrix} = \begin{bmatrix} 0.196 \\ 0.981 \\ 0 \end{bmatrix}$ $e_{31} = \begin{bmatrix} \frac{4}{\sqrt{1^2 + 4^2}} \\ \frac{1}{\sqrt{1^2 + 4^2}} \\ 0 \end{bmatrix} = \begin{bmatrix} 0.970 \\ 0.242 \\ 0 \end{bmatrix}$

$$E = \begin{bmatrix} -e_{12} & 0 & e_{31} \\ e_{12} & -e_{23} & 0 \\ 0 & e_{23} & -e_{31} \end{bmatrix}$$

c. (i) $G = \begin{bmatrix} W_f & W_m \\ \bar{E} & 0_{9 \times 9} \\ 0_{9 \times 9} & I_9 \end{bmatrix}$ (ii) $W = [W_f \ W_m] = 6 \times 18$ (iii) $\bar{E} = (E^T E)^{-1} E^T$

$\bar{E} = 3 \times 9$
 $I_9 = 9 \times 9$

d (i) $6 \times (3-1) = 12$

(ii) $21 - 12 - 6 = 3$