CS 327A HW2

(i) $\dot{x}_d = 0$

$$\dot{y}_d = -\frac{\pi}{25}\sin\left(\frac{2\pi t}{5}\right)$$

$$\dot{z}_d = \frac{\pi}{25}\sin\left(\frac{4\pi t}{5}\right)$$

$$\dot{\lambda}_{0d} = \dot{\lambda}_{2d} = \frac{-\pi^2}{10\sqrt{2}}\cos\left(\frac{\pi}{4}\cos\left(\frac{2\pi t}{5}\right)\right)\sin\left(\frac{2\pi t}{5}\right)$$

$$\dot{\lambda}_{1d} = \dot{\lambda}_{3d} = \frac{\pi^2}{10\sqrt{2}}\sin\left(\frac{\pi}{4}\cos\left(\frac{2\pi t}{5}\right)\right)\sin\left(\frac{2\pi t}{5}\right)$$

$$\dot{X}_d = [\dot{x}_d, \dot{y}_d, \dot{z}_d, \dot{\lambda}_{0d}\ \dot{\lambda}_{1d}\ \dot{\lambda}_{2d}\ \dot{\lambda}_{3d}]^T$$

(ii) $\ddot{x}_d = 0$

$$\ddot{y}_d = -\frac{2\pi^2}{125}\cos\left(\frac{2\pi t}{5}\right)$$

$$\ddot{z}_d = \frac{4\pi^2}{125}\cos\left(\frac{4\pi t}{5}\right)$$

$$\ddot{\lambda}_{0d} = \ddot{\lambda}_{2d} = -\frac{\pi^2}{10\cdot\sqrt{2}}\left(\frac{2\pi}{5}\cos\left(\frac{2\pi t}{5}\right)\cos\left(\frac{\pi}{4}\cos\left(\frac{2\pi t}{5}\right)\right) + \frac{\pi^2\sin^2\left(\frac{2\pi t}{5}\right)\sin\left(\frac{\pi}{4}\cos\left(\frac{2\pi t}{5}\right)\right)}{10}\right)$$

$$\ddot{\lambda}_{1d} = \ddot{\lambda}_{3d} = \frac{\pi^2}{10\sqrt{2}}\left(\frac{2\pi}{5}\cos\left(\frac{2\pi t}{5}\right)\sin\left(\frac{\pi}{4}\cos\left(\frac{2\pi t}{5}\right)\right) - \frac{\pi^2\sin^2\left(\frac{2\pi t}{5}\right)\cos\left(\frac{\pi}{4}\cos\left(\frac{2\pi t}{5}\right)\right)}{10}\right)$$

$$\ddot{X}_d = [\ddot{x}_d\ \ddot{y}_d\ \ddot{z}_d\ \ddot{\lambda}_{0d}\ \ddot{\lambda}_{1d}\ \ddot{\lambda}_{2d}\ \ddot{\lambda}_{3d}]^T$$

(iii) $E(X) = \begin{pmatrix} E_p(X_p) & 0 \\ 0 & E_r(X_r) \end{pmatrix}$

where $E_p(X_p) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for cartesian coordinates

$$E_r(X_r) = \frac{1}{2}\begin{bmatrix} -\lambda_1 & -\lambda_2 & -\lambda_3 \\ \lambda_0 & \lambda_3 & -\lambda_2 \\ -\lambda_3 & \lambda_0 & \lambda_1 \\ \lambda_2 & -\lambda_1 & \lambda_0 \end{bmatrix}$$

$E^+(X) = \begin{pmatrix} E_p^+(X_p) & 0 \\ 0 & E_r^+(X_r) \end{pmatrix}$

where $E_p^+(X_p) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$E_r^+(X_r) = 2\begin{bmatrix} -\lambda_1 & \lambda_0 & -\lambda_3 & \lambda_2 \\ -\lambda_2 & \lambda_3 & \lambda_0 & -\lambda_1 \\ -\lambda_3 & -\lambda_2 & \lambda_1 & \lambda_0 \end{bmatrix}$$

(iv) From eqn 4.61 $\delta\varphi = -2\check{\lambda}^T\lambda_d$, where $2\check{\lambda}^T = E_r^+(X_r)$

$$= -E_r^+(X_r)\cdot\lambda_d = -E_r^+(X_r)\cdot\begin{bmatrix} \lambda_{0d} \\ \lambda_{1d} \\ \lambda_{2d} \\ \lambda_{3d} \end{bmatrix}$$

b i

$$F^* = \ddot{x}_{pd} - k_p(x_p - x_{pd}) - k_v(\dot{x}_p - \dot{x}_{pd})$$

$$M^* = \dot{\omega}_d - k_p\delta\varphi - k_v(\omega - \omega_d)$$

where $\dot{\omega}_d = 2\check{\lambda}'\ddot{\lambda}_d$

$$\Rightarrow \begin{bmatrix} F^* \\ M^* \end{bmatrix} = F_0^*$$

ii From 4.21 $F_0 = \hat{\Lambda}_0\begin{bmatrix} F^* \\ M^* \end{bmatrix} + \hat{P}_0$

$$\hat{\Lambda}_0 = \hat{\Lambda}_0 = J_0^{-T}(q)A(q)J_0^{-1}(q)$$

$$\hat{P}_0 = P_0 = J_0^{-T}(q)g(q)$$

(c) i $\underset{6\times1}{X} = \underset{6\times7}{J(q)}\,\underset{7\times1}{q}$ $J(q)$ dimension is 6×7

ii Task Jacobian's null space dimensionality is $7-6=1$

iii Null space matrix $N_{proj} = (I - J_0^{+} J_0)$, where I is a 7×7 identity matrix

iv No for a 6DOF Puma, the degree of redundancy is $6-6=0$. There is no extra DOF for obstacle avoidance.

v 3DOF is required to follow a trajectory. Null space dimensionality would be $7-3=4$ DOF

2\. part b The end effector is fairly stable with the need rotate between 0-180° degree. However the other joints tend to rotate in a fixed range, which cause the robot arm "spin" in a circle due to lack of joint damping

part c The end effector moves in at a much stable state with much less movements at other joints. The addition of joint damping terms has reduced the oscillation of joints and thus resut in a much stabler robot

part d. The motion of robot in part d has slightly greater movement that part c. The overshoot of joint motion is due to the lack of operational space dynamic compensation in the identity matrix