CS327A HW3

1\. (a) $\dot{x}_{ee} = J\dot{q} = \dot{d}_1 + \dot{d}_2 \Rightarrow J = [1, 1]$

$$\dot{q} = \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \end{bmatrix}$$

(b) $J^{+} = J^{T}(JJ^{T})^{-1} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

$$N: \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}[1, 1] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}$$

$$\delta q = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \delta x_{ee} + \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix} \delta q_0$$

(c) $W^{-1} = \begin{bmatrix} \frac{1}{w_1} & 0 \\ 0 & \frac{1}{w_2} \end{bmatrix}$ $\quad J^{\#} = \begin{bmatrix} \frac{1}{w_1} & 0 \\ 0 & \frac{1}{w_2} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\left([1 \ \ 1] \times \begin{bmatrix} \frac{1}{w_1} & 0 \\ 0 & \frac{1}{w_2} \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)^{-1}$

$$= \begin{bmatrix} \frac{1}{w_1} \\ \frac{1}{w_2} \end{bmatrix} \cdot \frac{w_1 w_2}{w_1 + w_2} = \begin{bmatrix} \frac{w_2}{w_1+w_2} \\ \frac{w_1}{w_1+w_2} \end{bmatrix}$$

$$I - J^{\#}J = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{w_2}{w_1+w_2} \\ \frac{w_1}{w_1+w_2} \end{bmatrix}[1, 1] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{w_2}{w_1+w_2} & \frac{w_2}{w_1+w_2} \\ \frac{w_1}{w_1+w_2} & \frac{w_1}{w_1+w_2} \end{bmatrix} = \begin{bmatrix} \frac{w_1}{w_1+w_2} & \frac{-w_2}{w_1+w_2} \\ -\frac{w_1}{w_1+w_2} & \frac{w_2}{w_1+w_2} \end{bmatrix}$$

$$\delta q = \begin{bmatrix} \frac{w_2}{w_1+w_2} \\ \frac{w_1}{w_1+w_2} \end{bmatrix} \delta x_{ee} + \frac{1}{w_1+w_2}\begin{bmatrix} w_1 & -w_2 \\ -w_1 & w_2 \end{bmatrix} \delta q_0$$

(d) $W = A = \begin{bmatrix} 9 & 3 \\ 3 & 3 \end{bmatrix}$ $\qquad$ let $\delta q_0 = 0$

with pseudo-inverse J^{+} $\quad \boxed{\delta q = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \delta x_{ee}}$

with inertia-weighted inverse $\bar{J}$. $\quad J^{\#} = \begin{bmatrix} 9 & 3 \\ 3 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\left([1 \ \ 1] \times \begin{bmatrix} 9 & 3 \\ 3 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)^{-1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\boxed{\delta q = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \delta x_{ee}}$$

The solution using pseudo-inverse distribute motions to both joints, where as the solution using inertia-weight inverse tends to minimize the energy required to move the manipulator. In this case Joint 2 has less weight than Joint 1, thus inertia-weight inverse will assign motion to the joint with less weight.

2 b. The first three joints are fairly stable, but the end effector is very unstable

This is because the pseudo inverse doesn't filter out the accelerations of the joint accelerations that are projected to the nullspace.

c. The manipulator is stable with the ee rotates at a fixed position. The dynamically consistent Jacobian inverse filters out both the velocity of the end effector and and accelerations of joint that are projected to the nullspace.

3. a $\Omega = \begin{pmatrix} R_F^T \Sigma_F R_F & 0 \\ 0 & R_m^T \Sigma_m R_m \end{pmatrix}$

$R_F^T \Sigma_F R_F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_m^T \Sigma_m R_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow \Omega = \left(\begin{array}{ccc|ccc} 0 & 0 & 0 & & & \\ 0 & 1 & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ \hline & & & 1 & 0 & 0 \\ & 0 & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \end{array}\right)$

Similarly $\bar{\Omega} = \begin{pmatrix} R_F^T \Sigma_F R_F & 0 \\ 0 & R_m^T \Sigma_m R_m \end{pmatrix} = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ 0 & 0 & 0 & & 0 & \\ 0 & 0 & 0 & & & \\ \hline & & & 0 & 0 & 0 \\ & 0 & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \end{array}\right)$

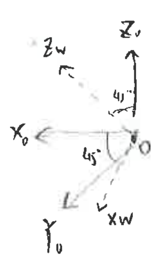
b $F_0 = F_{motion} + F_{active\text{-}force} + \hat{\Lambda} \times \text{select forces} \times (-K_{vx} \cdot V_{ee})$

$F_{motion} = \hat{\Lambda}_0(x)\, \Omega\, F^*_{motion} + \hat{p}_0(x)$

$F_{active\text{-}force} = \bar{\Omega} F^*_{active\text{-}force}$

$F^*_{motion} = \begin{bmatrix} F^* \\ m^* \end{bmatrix} = \begin{bmatrix} \ddot{x}_{pd} - K_p(x_p - x_{pd}) - k_v(\dot{x}_p - \dot{x}_{pd}) \\ \dot{\omega}_d - K_p \delta\varphi - k_v(\omega - \omega_d) \end{bmatrix}$

$F^*_{active\text{-}force} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

c 

If the window was tilted for 45°, we can obtain task coordinate frame from global frame by apply $R_F = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$

d. No, the forces fluctuates around 10N The performance of force control could be improved with a feedback control using force sensor to correct input force

Plot

Time

cs327a::hw3::robot::Kuka-IIWA::sensors::ee_force ✖

Rate: 0.1 Stop