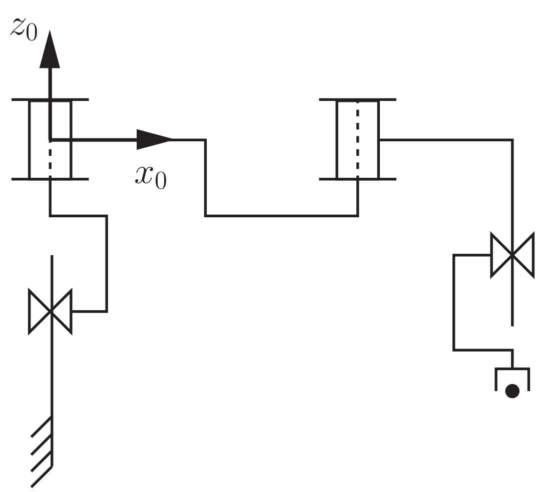
Advanced Robotic Manipulation (CS 327A) Homework #1

(Spring 2022/2023) Due: **Friday, April 21 11:59pm**

In this homework we will examine how the Jacobian is used to connect the joint space and task space of a robot manipulator, and how the null space of the Jacobian in a redundant robot can be used. We will also start familiarizing ourselves with the SAI simulation environment, which will facilitate our investigation of future topics in this class.

Problem 1 - Basic vs Task Specific Jacobian

Consider the PRRP manipulator in the figure below.



The manipulator has the following forward kinematics:

$$T_{04} = \begin{bmatrix} c_{23} & -s_{23} & 0 & l_2c_2 + l_3c_{23} \\ s_{23} & c_{23} & 0 & l_2s_2 + l_3s_{23} \\ 0 & 0 & 1 & d_1 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

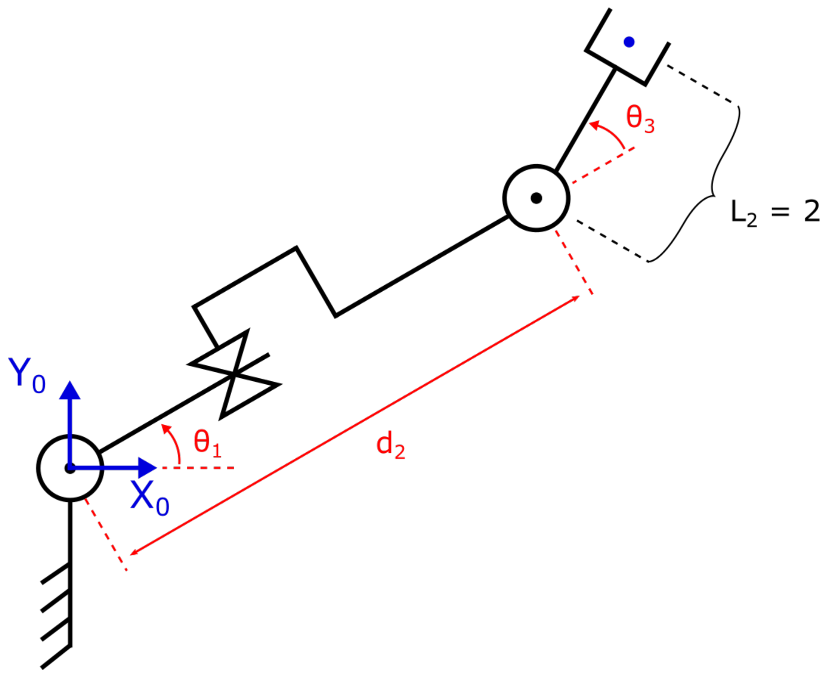
(a) Compute the basic Jacobian J_0 for this manipulator

(b) Suppose the manipulator has the task of positioning the end effector in space. Is this manipulator redundant for that task? If so, what is the degree of redundancy?

(c) For the positioning task, we would like to specify the target position using cylindrical coordinates (ρ, θ, z). Find the corresponding task-specific Jacobian for this representation. You may leave your answer as a product of matrices.

Problem 2 - Redundancy and the Null Space of the Jacobian

Consider the RPR manipulator in the figure below.



This robot's task is to **position** an object in the $x_0 - y_0$ plane. The associated task Jacobian J is as follows:

$$J = \begin{bmatrix} -d_2 s_1 - 2s_{13} & c_1 & -2s_{13} \\ d_2 c_1 + 2c_{13} & s_1 & 2c_{13} \end{bmatrix}$$

(a) What is the degree of redundancy with respect to this task?

(b) Sketch the manipulator in a few different configurations which all have the same end effector position.

(c) Since this manipulator is redundant, we can achieve a secondary objective at the same time as the primary end effector positioning task. Illustrate the joint-configuration that achieves the end-effector position $x_e = [5 \;\; 5]^T$ and minimizes d_2.

(d) Compute the Jacobian J and its right pseudo-inverse J^+ at the configuration in Part (c).

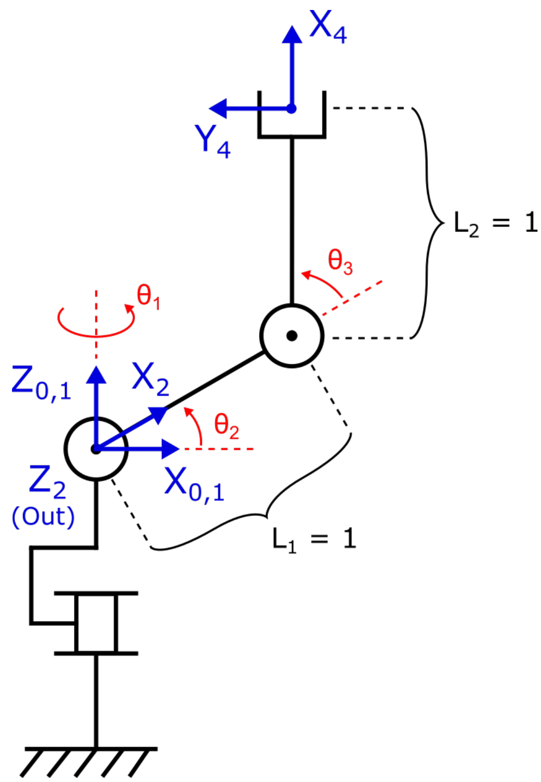
(e) Compute the null space projection matrix N_{proj}.

(f) What do the columns of N_{proj} represent in terms of how the projection matrix affects an arbitrary joint displacement δq_0? How do they relate to the two link-lengths (d_2 and $l_2 = 2$)?

Problem 3 - Null Space Projection at Singularities

Consider the RRR-spatial manipulator in the figure below.

The task of this manipulator is to position the end effector in XYZ Cartesian space



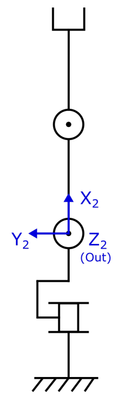
The forward kinematics of the manipulator are as follows:

$$T_{04} = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & c_1(c_2 + c_{23}) \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & s_1(c_2 + c_{23}) \\ s_{23} & c_{23} & 0 & s_2 + s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The manipulator has the following task Jacobian, expressed in both frame 0 and in frame 2 for your convenience

$$^0J = \begin{bmatrix} -s_1(c_2 + c_{23}) & -c_1(s_2 + s_{23}) & -c_1 s_{23} \\ c_1(c_2 + c_{23}) & -s_1(s_2 + s_{23}) & -s_1 s_{23} \\ 0 & c_2 + c_{23} & c_{23} \end{bmatrix} \; ^2J = \begin{bmatrix} 0 & -s_3 & -s_3 \\ 0 & 1 + c_3 & c_3 \\ -c_2 - c_{23} & 0 & 0 \end{bmatrix}$$

(a) Identify and sketch the singular configurations of the manipulator. For each configuration, clearly indicate which direction or directions are singular



Now, let's consider the configuration when $\theta_2 = 90°$ and $\theta_3 = 0°$ as shown above. For reference, the position of the end effector with respect to frame 0 is ${}^0x_{ee} = [0, 0, 2]^T$. In this configuration the arm is vertically stretched out in a compound singular configuration, and it loses 2 d.o.f. in Cartesian space (cannot move in the $\hat{x}_2$ direction and in the $\hat{z}_2$ direction). Here, the manipulator can be treated as a redundant manipulator in the subspace orthogonal to the singular directions, i.e. it is redundant along the non-singular direction $\hat{y}_2$.

(b) Find the Jacobian matrix for the task of positioning the end effector along the $\hat{y}_2$ direction, expressed in frame $\{2\}$. This matrix should be 1×3 in size.

(c) Find the right pseudo-inverse of the Jacobian matrix from Part (b).

(d) Find a matrix N_{proj} that will project a given vector of elementary joint displacements δq into the null space associated with the Jacobian from Part (b).

(e) Suppose now you want to apply the following vector of small joint value adjustments (in degrees) to the current configuration: $\delta q_0 = [5°, 5°, 5°]^T$.

Using the forward kinematics of the manipulator provided, what will the end effector position be for the new joint configuration $q_{new} = q_{current} + \delta q_0$?

Assume $\theta_1 = 0°$.

(f) Now project that same vector of small joint value adjustments into the null space using the matrix you found in Part (d): $\delta q_n = N_{proj} \delta q_0$.

What will the end effector position be if you instead apply the projected vector of joint value adjustments to the original configuration i.e. $q_{new} = q_{current} + \delta q_n$?

(g) Comment on the difference in effect of the two vectors. What do you notice with respect to the new position of the end effector in Part (f) along the $\hat{y}_2$ direction? In general, what motions of the end effector are possible when we project an arbitrary vector δq_0 into the null space of the Jacobian at this configuration?

Problem 4 - Introduction to SAI

In this problem you will be getting familiar with the basics of SAI.

(a) Problem Setup

- Open terminal and clone the `cs327a` directory into your `apps` folder: `https://github.com/manips-sai-org/cs327a`
- Download and unzip the Robot Mesh Files from Canvas. Create a folder inside the cs327a directory called `resources` and place the `rpr_robot`, `kukaiiwa`, `puma`, `panda` folders inside this `resources` folders. The reason why you need to do this manually is that robot mesh files can be quite large, so it is bad practice to commit them to the repo.
- Build the repo as you did for each SAI2.0 module (see SAI2.0 installation instructions if you have questions). This build will compile the starter code under `cs327a/hw1/` and create two executables in the folder `cs327a/bin/hw1/`. Note that these executable must be run from within the `cs327a/bin/hw1` directory for the graphic models to be loaded correctly.
- Run the `hw1_viz` executable to verify that the program starts and a static 3 DoF RPR manipulator is visible.

  ```
  $ cd bin/hw1
  $ ./hw1_viz
  ```

  Press ESC to close the window.

(b) Obtaining Manipulator Properties from Simulation

- In SAI we typically divide a simulation into two different executable - the simulation and visualization environment (simviz) and the controller (control).
- For this problem you will be editing the control executable in order to change the manipulator's configuration.
- Open up the `hw1_control.cpp` file inside of the `cs327a/hw1/`. Closely examine the example commands and the comments documenting them. Feel free to examine the header files in the sources of each sai2 module for more information.
- When you are ready, adjust the code as necessary to change the the manipulator's position for the following questions. **Remember that you have to rebuild the repository every time you change the `hw1_control.cpp` file in order for the changes to take effect.**

(c) Response Questions

(i) Find the end effector position in frame $\{0\}$, ${}^{0}P_4$, for the two configurations below.

(1) $\theta_1 = -30^\circ$, $d_2 = 0.2\,\text{m}$, $\theta_3 = 0^\circ$

(2) $\theta_1 = 0^\circ$, $d_2 = 0.1\,\text{m}$, $\theta_3 = -90^\circ$

(ii) Write out the linear Jacobian J_v for Configuration 2. What do you notice about the rank of Jacobian? What does this tell you about this configuration for the manipulator?

(iii) Write out the mass matrix for both Configuration 1 and Configuration 2. Compare the off-diagonal terms in both cases. Explain what this means for the inertial coupling of the manipulator in each configuration.

(iv) Now We want to examine how manipulator's gravity vector G is affected by changes in θ_3.

Set $\theta_1 = -45^\circ$ and $d_2 = 0.15\,\text{m}$. Vary the value of θ_3 over the range of 0° to 180°.

Record the gravity vector from the simulation at regular intervals. Use whatever external software you prefer (Matlab, Python, Excel, etc.) to generate a plot of the vector's value from the recorded data. Explain why your plot is shaped the way it is using the physical structure of the manipulator.