CS 327A HW 4 Xuyi Long

P1.

a. $A = \sum_{i=1}^{n} (m_i J_{v_i}^T J_{v_i} + J_{w_i}^T I_{C_i} J_{w_i})$

$^0P_{C_3} = \begin{bmatrix} 0 \\ 0 \\ d_2 \end{bmatrix}$ $^0P_{C_4} = \begin{bmatrix} L_3 S_3 \\ L_3 C_3 \\ d_2 \end{bmatrix}$

$J_{v_3}(q) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $J_{v_4}(q) = \begin{bmatrix} 0 & L_3 C_3 & 0 \\ 0 & -L_3 S_3 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $J_v = \begin{bmatrix} 0 & -C_3(L_3+L_4C_4) & L_4 S_3 S_4 \\ 0 & -S_3(L_3+L_4C_4) & -L_4 C_3 S_4 \\ 1 & 0 & L_4 C_4 \end{bmatrix}$

$J_{w_3}(q) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $J_{w_4}(q) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $J_w = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$A = m_3 J_{v_3}^T J_{v_3} + m_4 J_{v_4}^T J_{v_4} + m_E J_v^T J_v + J_{w_3}^T I_3 J_{w_3} + J_{w_4}^T I_4 J_{w_4}$

$= m_3 \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + m_4 \begin{bmatrix} 0 & 0 & 1 \\ L_4 C_4 & -L_3 S_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & L_3 C_3 & 0 \\ 0 & -L_3 S_3 & 0 \\ 1 & 0 & 0 \end{bmatrix} + m_E \begin{bmatrix} 0 & 0 & 1 \\ -C_3(L_3+L_4C_4) & -S_3(L_3+L_4C_4) & 0 \\ L_4 S_3 S_4 & -L_4 C_3 S_4 & L_4 C_4 \end{bmatrix} \begin{bmatrix} 0 & -C_3(L_3+L_4C_4) & L_4 S_3 S_4 \\ 0 & -S_3(L_3+L_4C_4) & -L_4 C_3 S_4 \\ 1 & 0 & L_4 C_4 \end{bmatrix}$

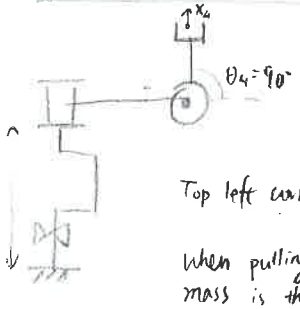
$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{xx3} & 0 & 0 \\ 0 & I_{yy3} & 0 \\ 0 & 0 & I_{zz3} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{xx4} & 0 & 0 \\ 0 & I_{yy4} & 0 \\ 0 & 0 & I_{zz4} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{zz3} \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & I_{zz4} \\ 0 & I_{zz4} & 0 \end{bmatrix}$

$A_{33} = m_E (L_4^2 S_3^2 S_4^2 + L_4^2 C_3^2 S_4^2 + L_4^2 C_4^2) + I_{zz4} = m_E L_4^2 + I_{zz4}$

$$A = \begin{bmatrix} \sim & \sim & \sim \\ \sim & \sim & \sim \\ m_E L_4 C_4 & 0 & m_E L_4^2 + I_{xx4} \end{bmatrix}$$

b)

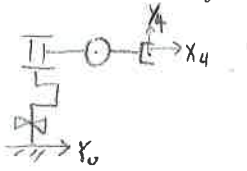


Top left corner: $\frac{1}{m_3 + m_4 + m_E}$

When pulling in X_4 direction at EE, the effective mass is the whole weight of manipulator.

c) See attached plot.

d) As θ_4 approaches 0, the shape of belted ellipsoid extends along Y_0 direction. This indicates that the effective mass of the end effector in Y_0 direction is increasing. This makes sense because when $\theta_4 = 0$, as shown in figure below, the ee can no longer move in Y_0 direction thus the effective mass is infinity



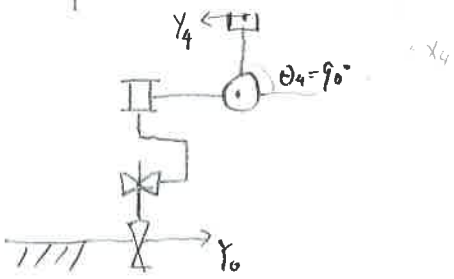
(e) When $\theta_4 = 0$, robot can't move in X_4 direction. The robot has redundancy in Y_4 direction, because the nullspace can move in X_4 direction, this is type I singularity

P2. (a) The jacobian for the 1st joint (prismatic) is $\begin{bmatrix}0\\1\\0\end{bmatrix}$. By combining it with task jacobian of the mini manipulator from P1, we have ${}^0J = \begin{bmatrix} 0 & 0 & -C_3(L_3+L_4C_4) & L_4S_3S_4 \\ 1 & 0 & -S_3(L_3+L_4C_4) & -L_4C_3S_4 \\ 0 & 1 & 0 & L_4C_4 \end{bmatrix}$

(b)

$$A = \begin{bmatrix} m_1+m_3+m_4+m_E & 0 & -m_4L_3S_3 - m_E(L_3S_3+L_4S_3C_4) & -m_EL_4C_3S_4 \\ 0 & m_4+m_3+m_E & 0 & m_EL_4C_4 \\ -m_4L_3S_3 - m_E(L_3S_3+L_4S_3C_4) & 0 & m_4L_3^2 + m_E(L_3+L_4C_4)^2 + I_{zz3} + I_{xx4}S_4^2 + I_{yy4}C_4^2 & 0 \\ -m_EL_4C_3S_4 & m_EL_4C_4 & 0 & m_EL_4^2 + I_{zz4} \end{bmatrix}$$

(c)



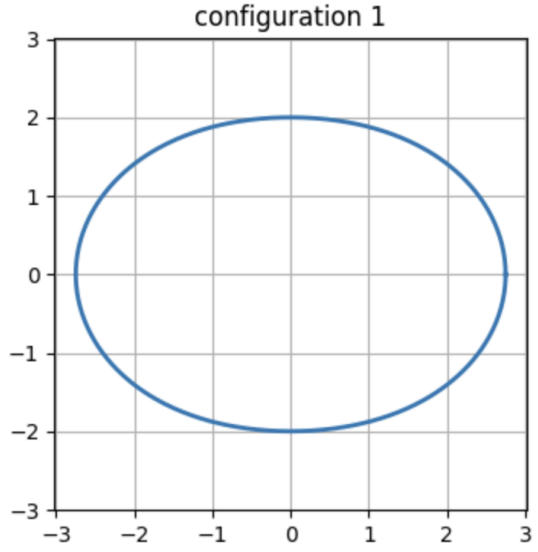
In this config, the effective mass in Y_4 direction is smaller than the effective mass in Y_4 direction for PRR mini manipulator, because the added joint is free to move in Y_4 direction. Intuively the mass felt at EE for macro-mini manipulator will be less than the mass felt in Y_4 direction for mini manipulator

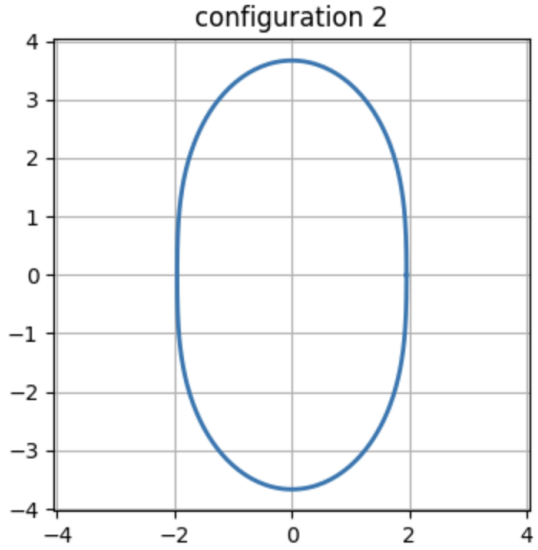
(d) See attached plot

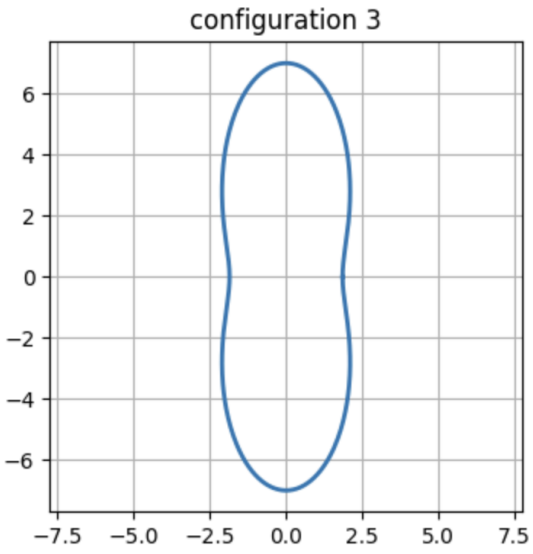
(e) As $\theta_4 \to 0°$, the ellipsoid of PRR in Y_0 goes to infinity and the ellipsoid of PPRR in Y_0 reaches the total mass of robot, which is $m_1+m_3+m_4+m_E = 4$. This difference is because of freedom of movement in Y_0 direction from the 1st prismatic joint. In PRR manipulator, when $\theta_4 = 0°$, the effective mass felt at EE when pulling in Y_0 direction will be infinite because the EE is not able to move. However, in PPRR, when pulling ee in Y_0 direction, the EE can move, and thus the effective mass felt at EE would be the entire mass of manipulator, which is $m_1+m_3+m_4+m_E = 4$.

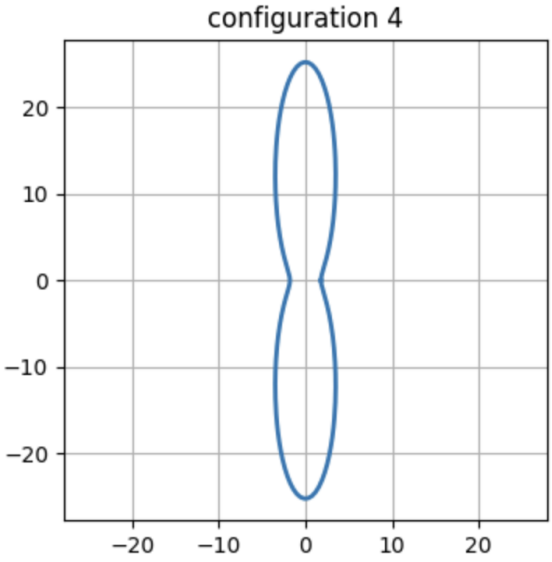
HW4 Colab code:
https://colab.research.google.com/drive/1bKS01cplxrW2yO8uk1_BQcJ8lgEGrG7I?usp=sharing

Plots for problem 1c









Plot for problem 2d