Non-Hermitian topological systems with eigenvalues that are always real

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Abstract The effect of non-Hermiticity in band topology has sparked many discussions on non-Hermitian topological physics. However, sufficiently strong non-Hermiticity will destroy the reality of energy spectra. Here, we show a systematic strategy to construct non-Hermitian topological systems exhibiting energy spectra that are always real, regardless of weak or strong non-Hermiticity.

Real spectra of non-Hermitian systems

Conservation of energy in quantum physics demands real eigenenergies for a closed system that is described by a Hermitian Hamiltonian. However, the non-Hermiticity is not a sufficient condition for the existence of complex spectra, e.g.,

Parity-time (*PT*) symmetry	$PT\, H\, PT^{-1} = H$
Pseudo-Hermiticity	$H^{\dagger} = \eta H \eta^{-1}$

However, these symmetries cannot guarantee the reality of energy spectra. For example, $H = i\sigma_x$, which is pseudo-Hermitian since $H^{\dagger} = \eta H \eta^{-1}$, where $\eta = \sigma_x$, but Hhas complex eigenvalues of $\pm i$.

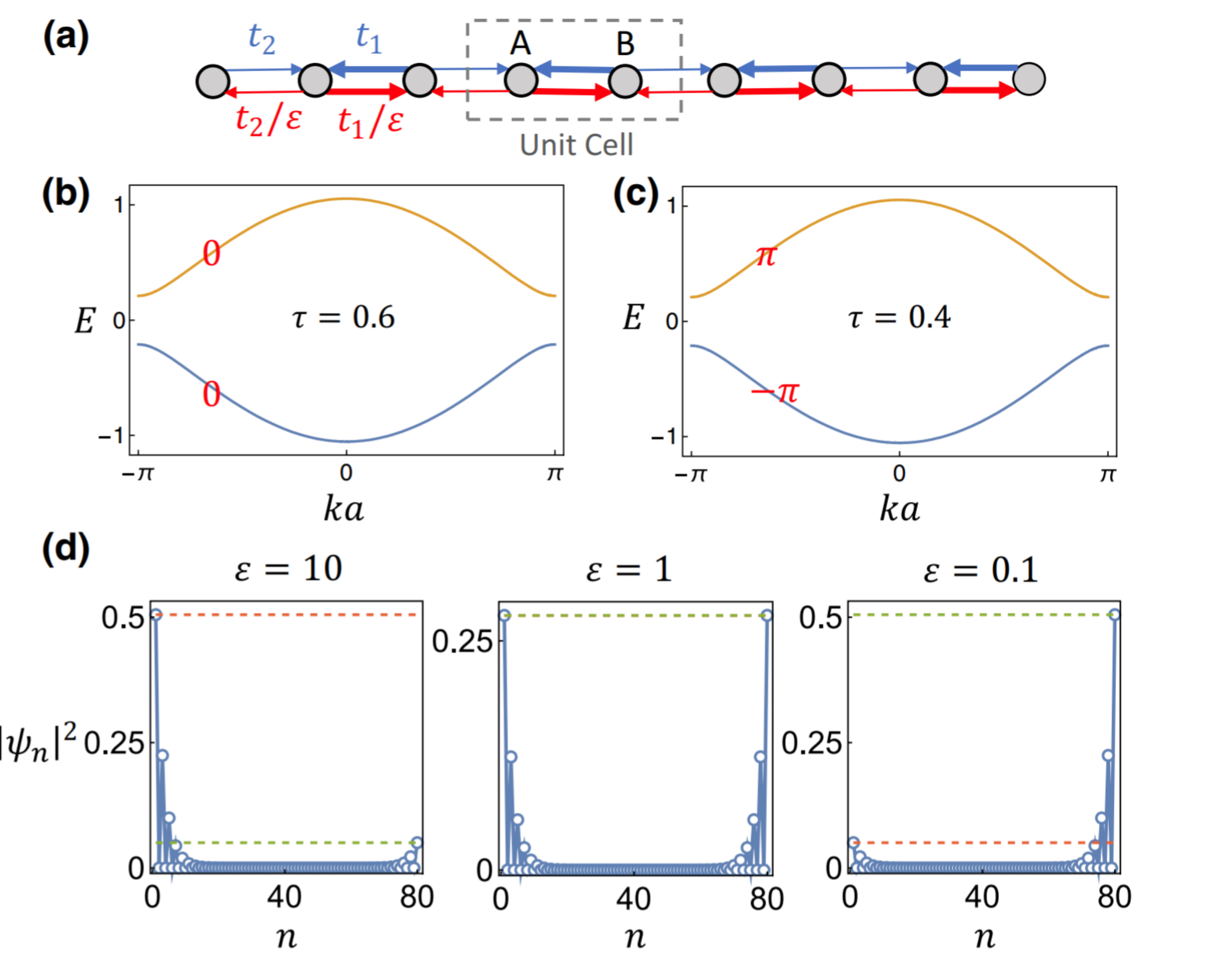
Sturm-Liouville theory

$$H_0 \Psi_n = E_n M\, \Psi_n \quad \Rightarrow \quad H = M^{-1} H_0$$

Where H_0 is a Hermitian Matrix, M is a real diagonal matrix with diagonal elements $M_{ii} > 0$. Importantly, $E_n \in \mathbb{R}$.

Although $H = M^{-1} H_0$ is non-Hermitian, it can have real eigenvalue sufficiently. Interestingly, the topology of H can be either inherited from a topologically nontrivial H_0 or induced by the M matrix with a trivial H_0.

1D non-Hermitian topological system with real spectra



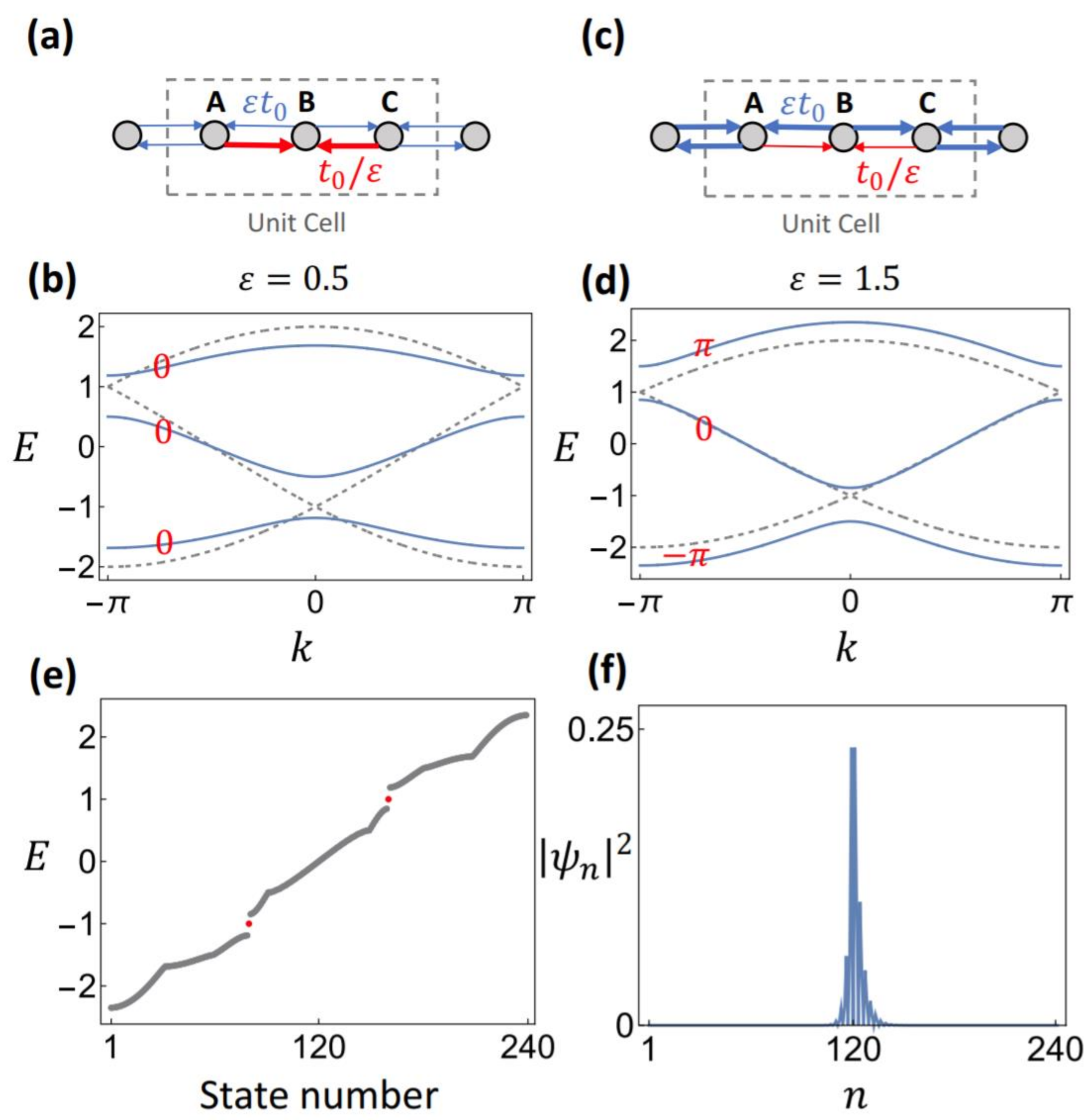
The Hamiltonian of Fig. (a) can be written as:

$$H_k = [t_1 + t_2\cos(ka)]\left(\frac{1+\varepsilon}{2\varepsilon}\sigma_x + i\frac{\varepsilon-1}{2\varepsilon}\sigma_y\right) + t_2\sin(ka)\left(\frac{1+\varepsilon}{2\varepsilon}\sigma_y - i\frac{\varepsilon-1}{2\varepsilon}\sigma_x\right)$$

The spectra are always real even for strong non-Hermiticity (i.e., $\varepsilon \ll 1$ or $\varepsilon \gg 1$). Compared with Hatano-Nelson model, the real spectra of our systems are insensitive to boundary conditions. This Hamiltonian satisfies the chiral symmetry: $H_k = -\sigma_z H_k \sigma_z$. The topology of our 1D non-Hermitian system can be characterized by the Zak phase (denoted in Fig.(b,c))

As shown in Fig. (d), the topological states can be affected by the non-Hermitian parameter ε. For a finite system with N sites, the ratio between the amplitudes of the left and right edge states is ε: $\frac{|\psi_1|^2}{|\psi_N|^2} = \varepsilon$, where $\psi_{1/N}$ denote the field on the left/right end site.

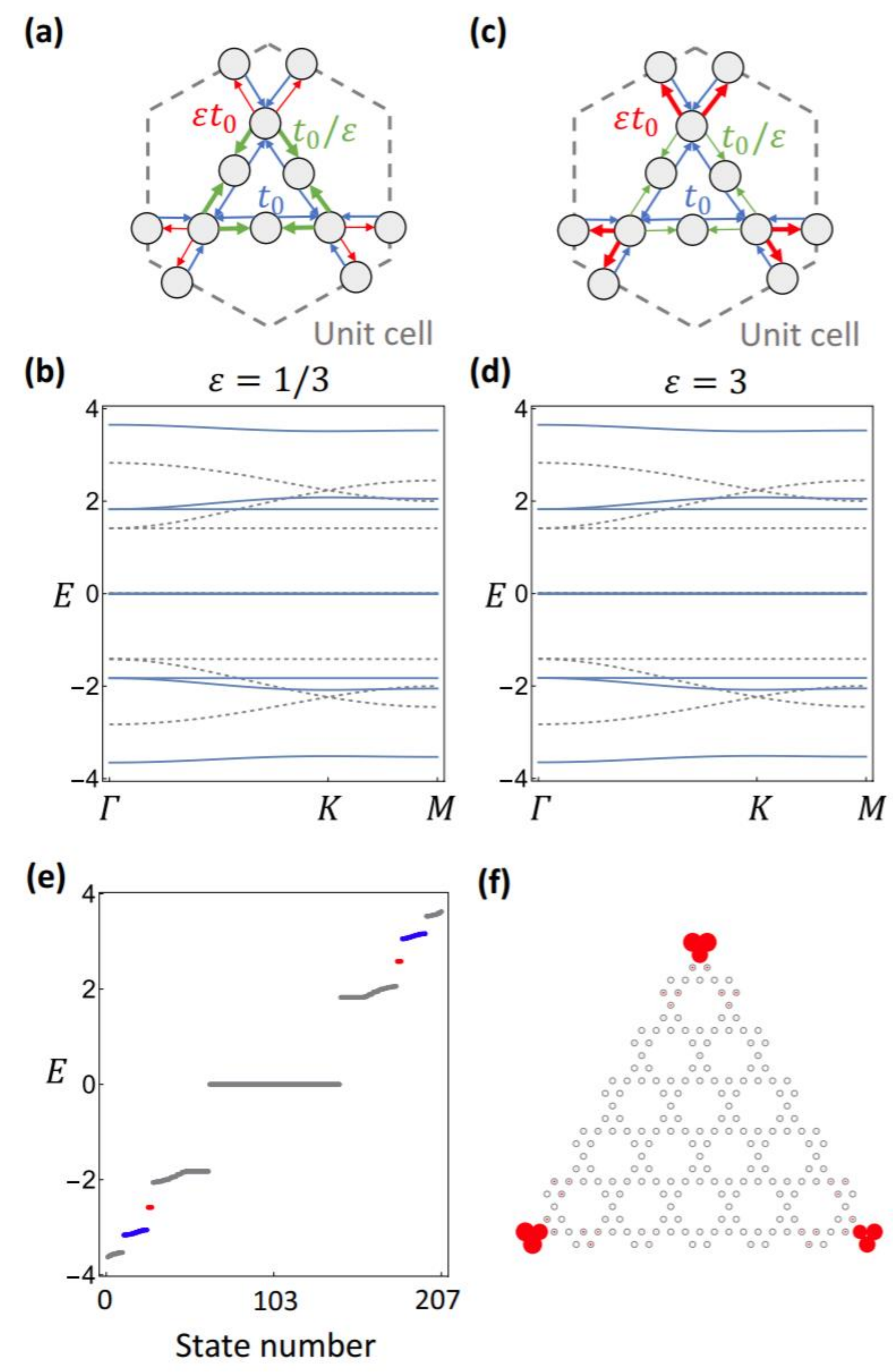
Non-Hermiticity-induced topological phase transition



Non-Hermiticity-induced topological phase transition in 1D system.

$$H_k = t_0\begin{pmatrix} 0 & \varepsilon & \varepsilon e^{-ika} \\ 1/\varepsilon & 0 & 1/\varepsilon \\ \varepsilon e^{ika} & \varepsilon & 0 \end{pmatrix}$$

When $\varepsilon < 1$ ($\varepsilon > 1$), the system becomes trivial (topological). After combining the trivial and topological systems into a chain, topological interface states emerge in the bulk gaps.



Non-Hermiticity-induced higher-order topological states

When $\varepsilon = 1$, the system corresponds to a trivial C_3-symmetric Hermitian system. The $\varepsilon \neq 1$ cannot only make the system non-Hermitian, but also open a gap at the valley point K.

The higher-order topological properties can be described by the topological bulk polarization $\boldsymbol{P}$ and fractional corner charge Q.

When $\varepsilon > 1$, $\boldsymbol{P} = \frac{2}{3}(\boldsymbol{a}_1 + \boldsymbol{a}_2)$ and $Q = 1/3$. we can see that there are corner states in the bulk gap for a finite system.

Conclusions

- We propose a systematic strategy to construct real-eigenvalued non-Hermitian topological systems
- The non-reciprocal-coupling-based non-Hermiticity can affect many properties of topological states. And it is able to determine the topology by inducing a topological phase transition

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