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BUFFER-STOCK SAVING AND THE LIFE CYCLE/PERMANENT INCOME HYPOTHESIS*

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This paper argues that the typical household's saving is better described by a "buffer-stock" version than by the traditional version of the Life Cycle/Permanent Income Hypothesis (LC/PIH) model. Buffer-stock behavior emerges if consumers with important income uncertainty are sufficiently impatient. In the traditional model, consumption growth is determined solely by tastes. In contrast, buffer-stock consumers set average consumption growth equal to average labor income growth, regardless of tastes. The model can explain three empirical puzzles: the "consumption/income parallel" documented by Carroll and Summers; the "consumption/income divergence" first documented in the 1930s; and the stability of the household age/wealth profile over time despite the unpredictability of idiosyncratic wealth changes.

I. INTRODUCTION

Of the consumers who participated in the Federal Reserve Board's 1983 *Survey of Consumer Finances,* 43 percent said that being prepared for emergencies was the most important reason for saving. Only 15 percent said that preparing for retirement was the most important saving motive.[1] These are not the answers that standard interpretations of the Life Cycle/Permanent Income Hypothesis (LC/PIH) model of saving would lead one to expect.

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1. Summarized in Avery and Kennickell [1989]. The other possible categories were "To buy something or for the family" (29 percent), and "Investment" (7 percent). Later SCF surveys produced similar results.

This paper will argue, however, that such responses, and a wide range of other evidence, are consistent with a version of the LC/PIH model in which consumers face important income uncertainty, but are also both "prudent," in Kimball's [1990b] sense that they have a precautionary saving motive, and "impatient" in the sense that if future income were known with certainty they would choose to consume more than their current income.[2] Under these conditions, consumers may engage in what I call "buffer-stock" saving behavior.[3] Buffer-stock savers have a target wealth-to-permanent-income ratio such that, if wealth is below the target, the precautionary saving motive will dominate impatience, and the consumer will save, while if wealth is above the target, impatience will dominate prudence, and the consumer will dissave.[4]

I first describe the main properties of the buffer-stock model in an infinite-horizon context where labor income growth is constant. The model's most surprising feature is its implication that, even with a fixed aggregate interest rate, if consumers are sufficiently impatient, average consumption growth will equal average labor income growth, either for individual households or for aggregate consumption.[5] This is true even though the consumers in the model behave according to the standard Euler equation which has been widely thought to imply that consumption growth depends only on tastes, and not on the growth rate of income. The problem in previous work has been in the common assumption that the second-order variance term in the log-linearized version of the Euler equation can safely be ignored. In fact, this variance term is an endogenous equilibrating variable: it will, on average,

2. This desire to borrow can be due either to a high time preference rate or to high expected income growth.

3. Prudence, in Kimball's sense of having a utility function with a positive third derivative, is not by itself sufficient to generate buffer-stock saving. The utility function must also exhibit Decreasing Absolute Prudence, as does the Constant Relative Risk Aversion (CRRA) utility function used in this paper. See Kimball [1990a, 1990b] for arguments that Decreasing Absolute Prudence is a natural condition to require of utility functions.

4. The proof that a target wealth-to-income ratio exists, and is stable, is contained, along with many other derivations and proofs, in a companion paper [Carroll 1996]. For a copy of that paper contact the author.

5. Standard general equilibrium models imply that the economy will converge to a steady state in which the growth rate of consumption equals the growth rate of income. However, the mechanism by which this is achieved is through the dependence of the interest rate on the capital stock. In the model considered here this channel is short-circuited by assuming a fixed aggregate interest rate.

take on whatever value is required to cause average consumption growth to equal average income growth.

I also present simulation evidence documenting other major differences between implications of the buffer-stock model and what I will refer to as the "standard" model (either the perfect-certainty model with Constant Relative Risk Aversion utility, or the Certainty Equivalent (CEQ) model in which utility is quadratic). In comparison with the standard model, the buffer-stock model predicts a much higher marginal propensity to consume out of transitory income, a much higher effective discount rate for future labor income, and a positive rather than negative sign for the correlation between saving and expected labor income growth.

Whether these results for the infinite-horizon model carry over to the finite-horizon context cannot be determined analytically. The next section of the paper therefore solves a finite-horizon version of the model under the same baseline parameter values used in the infinite-horizon model, but with age/income profiles roughly calibrated to U. S. household-level data. I show that this configuration of the model generates buffer-stock saving behavior over most of the working lifetime until roughly age 45 or 50, and behavior that resembles the standard LC/PIH model only for roughly the period between age 50 and retirement.

I then argue that the finite-horizon version of the model can explain three major stylized facts which, combined, cannot be explained by the principal alternative models: the standard LC/PIH model, a Keynesian alternative to the standard LC/PIH framework, or the Campbell and Mankiw [1989] combination of these two models. First of the three stylized facts is the "consumption/income parallel" documented by Carroll and Summers [1991] and Carroll [1994]: when consumption is aggregated by groups or by whole economies, it closely parallels growth in income over periods of more than a few years. The consumption/income parallel, inconsistent with the standard LC/PIH framework, is explained in the buffer-stock model as the result of consumers' impatience and their prudent unwillingness to borrow. The second fact is the "consumption/income divergence" that emerges from microeconomic consumer surveys: for individual households, consumption is often far from current income, implying that the aggregate consumption/income parallel does not arise from high frequency tracking of consumption to income at the household level. The consumption/income divergence, inconsistent with the Keynesian model, is explained with essentially the same logic Friedman used long ago: consumption does not respond one-for-one to transitory shocks to income because assets are used to buffer consumption against such shocks.

The final set of stylized facts is about the patterns of wealth accumulation over the lifetime. I show that the standard LC/PIH model implies that the productivity growth slowdown after 1973 should have resulted in massive increases in household wealth-income ratios (because slower expected growth implies lower current consumption). I then demonstrate that the modest observed changes in the actual age/median-wealth profile match the predictions of the buffer-stock model. Finally, I argue that the extraordinarily high volatility [Avery and Kennickell 1989] of household liquid wealth is difficult to explain with either the LC/PIH or the Keynesian models, but is a natural implication of a model in which the principal purpose of holding wealth is so that it can be used to absorb random shocks to income.

Although the implications of the buffer-stock version of the LC/PIH model differ in important respects from standard modern versions of the LC/PIH model, careful reading of Friedman [1957] suggests that the buffer-stock version of the model represents a close approximation to his original ideas. Direct quotations from Friedman will illustrate the similarities between his views and the implications of the buffer-stock model, and some of the empirical discussion will parallel arguments that Friedman used long ago to justify his conception of the Permanent Income Hypothesis.

The emphasis on Friedman is not meant to suggest that there has been no progress since his book. The buffer-stock model presented here owes much to the insights of Kimball [1990a, 1990b], Zeldes [1989a], and Deaton [1991], who have all emphasized the importance of precautionary motives for saving. Indeed, the model presented here is structurally similar to the model of Zeldes [1989a], with the important difference that I assume impatient consumers. Alternatively, the model is similar to Deaton's [1991] except that I do not directly impose liquidity constraints and my model has independent transitory and permanent shocks.

The rest of the paper is organized as follows. Section II sets forth the basic intertemporal optimization model, briefly describes the method of solution, discusses parametric assumptions, and, for reference purposes, presents what I will call the standard LC/PIH model. Section III describes the solution to the infinite-horizon version of the model. Section IV presents the finite-life solution to the model under age/income profiles calibrated from U. S. data and argues that the resulting behavior fits the stylized facts about household consumption, saving, and wealth better than the alternative models. Section V contains a brief discussion of the recent literature on models related to the buffer-stock model, and Section VI concludes.

II. The Basic Model

II.A. Solving the Model

I assume that the consumer solves the intertemporal optimization problem:

$$\max E_t \sum_{i=t}^{T} \beta^{i-t} u(C_i) \tag{1}$$

such that

$$\begin{aligned} W_{t+1} &= R[W_t + Y_t - C_t] \\ Y_t &= P_t V_t \\ P_t &= G_t P_{t-1} N_t, \end{aligned}$$

where Y is current labor income; P is "permanent labor income" defined as the labor income that would be received if the white noise multiplicative transitory shock to income, V, were equal to its mean value of one; N is a lognormally distributed white noise mean one multiplicative shock to permanent income; $G = (1 + g)$ is the growth factor for permanent labor income; W is the stock of physical net wealth; $R = (1 + r)$ is the (constant) gross interest rate; and $\beta = 1/(1 + \delta)$ is the discount factor where δ is the discount rate.

Optimal consumption in any period will depend on total current resources (or "gross wealth"), the sum of current assets and current income, which, following Deaton [1991], I will call X:

$$X_t = W_t + Y_t. \tag{2}$$

The evolution of gross wealth is given by

$$X_{t+1} = R[X_t - C_t] + Y_{t+1}. \tag{3}$$

Carroll [1996] demonstrates that this problem can be rewritten by dividing through all variables by the level of permanent labor income. Defining lowercase variables as the uppercase variable divided by the current level of permanent income (i.e., $c_t = C_t/P_t$), the general Euler equation for consumption then becomes

$$(4) \qquad 1 = R\beta E_{t-1}\left[\left\{c_t\left[R[x_{t-1} - c_{t-1}]/GN_t + V_t\right]GN_t/c_{t-1}\right\}^{-\rho}\right].$$

In the last period of life it is optimal to consume everything: $c_T[x_T] = x_T$. Thereafter, recursion on equation (4) implicitly defines a rule for the consumption ratio as a function of the gross wealth ratio in each period back to the beginning of life. Unfortunately, under general forms of income uncertainty the consumption rules do not have analytical formulas, so they must be approximated by numerical methods. The details of the method of numerical solution are contained in Appendix 1.

II.B. Parameter Values

I will solve the model and present most of my results for a single baseline set of parameter values, but will also present a summary of results for alternative choices of all parameter values so that readers who differ with any particular parametric choice can determine how sensitive my results are to changes in that parameter. The baseline values for characterizing the distribution of income shocks will be the same as in Carroll [1992]. Using data from the *Panel Study of Income Dynamics,* that paper found that household income uncertainty was well captured by a process that took the form,

$$V \sim \left|\begin{array}{ll} 0 & \text{with probability } p \\ Z & \text{with probability } (1-p) \end{array}\right|,$$

$$\ln Z \sim TN(\mu_Z, \sigma^2_{\ln Z}),$$

$$\ln N \sim TN\left(\frac{\sigma^2_{\ln N}}{2}, \sigma^2_{\ln Z}\right),$$

where TN signifies a truncated normal distribution (in practice, I truncate at three standard deviations above and below the mean), and $\mu_{\ln z}$ is chosen to make $E_t V_{t+1} = 1$. The choice of mean for ln N was similarly motivated by the wish to make $E_t N_{t+1} = 1$ regardless of the choice of $\sigma^2_{\ln N}$. This simplifies the analysis of the effect of changes in $\sigma^2_{\ln N}$ on wealth. The probability of zero income was estimated at about $p = 0.5$ percent per year, and the standard deviations of both transitory and permanent income shocks were estimated to be around 0.1 percent per year, after crudely accounting for the effects of measurement error.

The expected growth rate of labor income appropriate for calibrating this model is the growth rate in household labor income for working households. Carroll and Summers [1991] provide evidence that, over long periods, household income growth can be characterized as the sum of aggregate productivity growth and a household-specific component that reflects such factors as increasing job tenure, seniority, and experience. If we assume very conservatively that each of these factors contributes 1 percent annually to household income growth, the appropriate baseline assumption for the growth rate of household labor income is 2 percent per year. This will be the baseline assumption for the infinite-horizon version of the model. For the finite-horizon version of the model used later, the pattern of income growth over the lifetime will be calibrated explicitly using household data.

Carroll [1992] made the strong assumption that the time preference rate was 10 percent per year: this is a substantial departure from common assumptions in the economics literature. Much macroeconomic research assumes a discount rate of 1 percent per quarter, or about 4 percent per year.[6] In order to emphasize that the results obtained in this paper are not the result of extreme assumptions about the time preference rate, the baseline value of the discount rate assumed in this paper will be 4 percent per year.[7]

The baseline interest rate will be zero, also following Carroll [1992]. The asset in this model is perfectly riskless and perfectly liquid. The closest proxy is probably the three-month T-bill, whose after-tax rate of return over the postwar period has been roughly zero. Results for interest rates of 2 percent and 4 percent are qualitatively similar, and will be presented later.[8]

6. Examples include Kydland and Prescott [1982], Hansen [1985], and Benhabib, Rogerson, and Wright [1991].

7. I do not appeal here to empirical evidence on the discount rate because I will argue below that one of the implications of the theoretical results in this paper is that the empirical methods that have been used to estimate discount rates, as in, e.g., Lawrance [1991], are fundamentally flawed.

8. Readers accustomed to general equilibrium representative agent models may object to having such a large gap between the interest rate and the rate of time preference. My view is that this model is the right description of the behavior of the typical consumer, but probably not the right model for understanding where most of the aggregate capital stock comes from. See the conclusion for a more extended discussion of this point.

Estimates of the coefficient of relative risk aversion vary widely. Empirical estimates above 6 have often been obtained (see, e.g., Mankiw and Zeldes [1991]),[9] but many economists believe that values above about 5 imply greater risk aversion than is plausible. At the other extreme, log utility, which is the limit of the CRRA utility function as ρ approaches one, is a common assumption because under some circumstances it is analytically tractable. The baseline value of ρ for this paper will be $\rho = 2$, toward the low end of the usual range in order to avoid exaggerating the magnitude of precautionary saving effects.

II.C. Comparison to Previous Work

This model is similar in many respects to models considered by Deaton [1991] and Zeldes [1989a]. The finite-horizon version differs from Zeldes' model primarily in the parametric assumptions about income uncertainty and tastes. Zeldes did not calibrate his model explicitly using panel data on household income from the PSID, and, more important, assumed that consumers were substantially more patient than I assume here; Zeldes also did not examine an infinite-horizon version of his model. The infinite-horizon version of my model differs from Deaton's [1991] in simultaneously incorporating both transitory and permanent shocks to income; because of the presence of zero-income events; and because Deaton imposes explicit liquidity constraints. However, as Zeldes [1989a], and, earlier, Schechtman [1976] have pointed out, the combination of the assumption that income can go to zero in each period with the assumption that consumption must remain strictly positive is sufficient to guarantee that consumers will never borrow.[10]

Despite the formal modeling differences, I view the infinite-horizon version of this model and Deaton's as close substitutes because very similar household behavior emerges from the two models. One insight this similarity provides is that Deaton's quali-

9. However, see below for a critique of the method of estimating ρ in this and other similar papers.

10. They refuse to borrow essentially for precautionary reasons, fearing the consequences of borrowing and then earning zero income indefinitely, so that eventually consumption is driven to zero. The no-borrowing result is less special than it may appear, however; the qualitative characteristics of the model are unchanged if the lower bound on income is positive. In that case, consumers will sometimes borrow, but will never borrow more than the present discounted value of the minimum possible future income stream. In effect, this amounts only to a shift in the horizontal axis for the problem. For a more detailed discussion see Carroll [1992].

tative results are attributable mainly to his assumption that consumers are impatient rather than to the assumption of liquidity constraints. An analytical convenience of this formulation over Deaton's is that, here, consumption always obeys the standard Euler equation linking the marginal utility of consumption in one period to marginal utility in adjacent periods. In Deaton's model, the usual Euler equation is violated whenever the liquidity constraints are binding.

For purposes of comparison, a brief description is in order of the model I will refer to as the standard model. The specific model I will consider is the perfect certainty version of the CRRA model described above, i.e., the model that would apply if the transitory and permanent shocks were known in advance always to be equal to their expected values, $V_t = N_t = 1$ for all t. Results in most cases would be very similar for the version of the model with quadratic utility and uncertainty. Defining human wealth H as the present discounted value of the expected stream of future income, and denoting the marginal propensity to consume by k, the solution to this model in the finite and the infinite horizons is given by

Finite horizon	Infinite horizon
$C_t = k_t[X_t + H_t]$	$C_t = k_t[X_t + H_t]$
	$Y_{t+1} = GY_t$
$H_t = \sum_{i=t+1}^{T} R^{i-t} Y_i$	$H_t = \sum_{i=t+1}^{\infty} R^{i-t} Y_i \approx \dfrac{Y_t}{r - g}$
$k_t = \dfrac{\left(1 - [R^{-1}(\beta R)]^{1/\rho}\right)}{\left(1 - [R^{-1}(\beta R)^{1/\rho}]^{T-t+1}\right)}$	$k = \left(1 - [R^{-1}(\beta R)^{1/\rho}]\right).$

III. Characteristics of the Solution

III.A. The Optimal Consumption Rule and the Consumption Euler Equation

In the version of his model with only permanent shocks, Deaton shows that if the change in permanent income is distributed lognormally with variance $\sigma^2_{\ln N}$, then, making the usual approximations that $\ln[R] \approx r$, $\ln[\beta] \approx -\delta$, and $\ln[G] \approx g$, the successive consumption rules $c_t[x_t]$, $c_{t-1}[x_{t-1}]$, . . . , converge if[11]

$$\rho^{-1}(r - \delta) + (\rho/2)\sigma^2_{\ln N} < g - \sigma^2_{\ln N}/2. \tag{5}$$

Carroll [1996] proves that this same condition guarantees convergence of the consumption rules in the model in this paper.[12]

The intuition for this equation is easiest to grasp if we assume that $\sigma^2_{\ln N} = 0$ momentarily. In the standard model the growth rate of consumption is $\rho^{-1}(r - \delta)$.[13] Now consider a consumer with zero assets. Because the PDV of income must equal the PDV of consumption, if consumption growth will be slower than income growth over the remainder of the lifetime (i.e., if (5) holds), the *level* of consumption today must be higher than the *level* of income today. Thus, the condition boils down to whether the consumer is sufficiently impatient that he would wish to dissave (or borrow) today to finance current consumption, if future income were perfectly certain. The more general case, which the $\sigma^2_{\ln N}/2$ terms reflects, on the left-hand side of equation (5), the additional consumption growth induced by the permanent income shocks, and, on the right-hand side of the equation, the reduction in the mean growth of the log of income necessary to maintain $E_tN_{t+1} = 1$ (without this adjustment E_tN_{t+1} would increase with $\sigma^2_{\ln N}$). Equation (5) is the condition referred to informally in the introduction as the impatience assumption, although note that this equation can be satisfied by consumers who do not discount future utility at all ($\delta = 0$) but who face positive income growth.

Many of the important results from the buffer-stock model can be understood by considering the log-linearized consumption Euler equation, which takes the form,

$$E_t\Delta\ln C_{t+1} \approx \rho^{-1}(r - \delta) + (\rho/2)\text{var}_t(\Delta\ln C_{t+1}) + e_{t+1}, \tag{6}$$

11. This formula differs slightly from Deaton's, which lacks the $-\sigma^2 \ln N/2$ term on the right-hand side. The difference is merely notational: Deaton calls the mean of his lognormal permanent income shock g, while in my framework the mean of $\log(GN)$ is $g - \sigma^2_{\ln N}/2$. My definition was chosen because it implies that the expected value of the permanent shock is one regardless of the assumption about the variance of the permanent shocks.

12. The exact condition (without approximation) is $(R\beta)E_t[GN_{t+1}]^{-\rho} < 1$.

13. Again, this is an approximation. The exact result is that $(c_{t+1}/c_t) = (R\beta)^{-1/\rho}$. Henceforth, in the text and in figures, I will approximate log $(R\beta)$ with $(r - \delta)$ and log G with g without further comment, although in all the calculations the correct (not approximate) formulas are used.

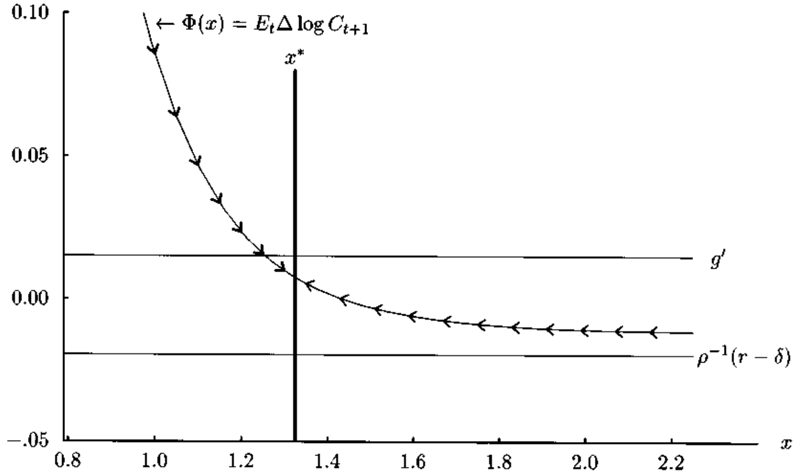

FIGURE Ia
Expected Consumption Growth as a Function of Cash on Hand

if shocks to consumption are lognormally distributed.[14] The bulk of previous work on consumption (see, e.g., Hansen and Singleton [1983], Hall [1988], Zeldes [1989b], and Lawrance [1991]) has essentially ignored the expected variance term in the consumption Euler equation, assuming it to be either a constant or zero.

Figure Ia summarizes many of the important features of the buffer-stock model. The curve labeled "$\Phi(x) = E_t\ \Delta \ln C_{t+1}$" corresponds to the expectation of consumption growth (calculated numerically using the converged consumption rule) as a function of the consumer's gross wealth-to-income ratio. The horizontal line drawn at $\rho^{-1}(r - \delta)$ indicates the growth rate of consumption that would prevail in a standard model with CRRA utility and baseline parameter values but with no labor income uncertainty.[15] The other horizontal line is the expected growth rate of permanent income, $g' = E_t\ \Delta \ln P_{t+1} = g - \sigma^2_{\ln N}/2$, which under these

14. See, e.g., Deaton [1992]. If shocks to consumption are not lognormally distributed, a similar equation can be derived using a Taylor expansion of the Euler equation, see Dynan [1993]. The formula in equation (6) was used because it is more intuitive than the expression from the Taylor expansion.

15. In fact, the CRRA model with certain income does not have a well-defined solution for the baseline income growth and interest rate parameter values, because with an interest rate less than the income growth rate, the present discounted value of future income is unbounded. However, for a finite-horizon version of the certainty model where the horizon is arbitrarily long, it remains true that the growth rate of consumption will be given by $\rho^{-1}(r - \delta)$.

parameter values is greater than $\rho^{-1}(r - \delta) - \rho\sigma^2_{\ln N}/2$, thus guaranteeing that these consumers are impatient in the required sense of equation (5). The vertical line labeled x^* represents the target value of the gross wealth ratio; i.e., x^* is the x_t such that $E_t x_{t+1} = x_t$.

The first point the figure illustrates is the inadequacy of the common assumption that the variance of consumption growth is constant or zero: the gap between the $E_t\, \Delta \ln C_{t+1}$ curve and the $\rho^{-1}(r - \delta)$ line is strongly declining in the level of the gross wealth ratio.[16] This happens for the intuitive reason that consumers with less wealth have less ability to buffer their consumption against shocks to income. More formally, the declining variance is a result of the fact that the optimal consumption rule is strictly concave. In other words, the marginal propensity to consume is a strictly decreasing function of the level of wealth. (Carroll and Kimball [1996] prove the strict concavity of the consumption function for a wide class of problems that includes this one.) The link between concavity and the variance term stems from the fact that at low levels of wealth, the marginal propensity to consume is high, so for a poor consumer a given amount of variation in income will induce a larger amount of variation in consumption than the same income variation would induce for a consumer with more wealth and thus a lower MPC.

Carroll [1996] formally proves a variety of propositions about this figure. First, as $x_t \to 0$, the expected rate of consumption growth goes to infinity (although for graphing purposes the expected consumption growth locus is truncated at 10 percent). This is essentially because as $x_t \to 0$, $C_t \to 0$, and therefore log $C_t \to -\infty$. Second, as $x_t \to \infty$, the expected growth rate of consumption approaches $\rho^{-1}(r - \delta)$, the growth rate that prevails in the perfect certainty model. This is because as wealth approaches infinity, the proportion of future consumption the consumer expects to finance out of his (uncertain) labor income becomes infinitesimal, so for all practical purposes labor income uncertainty becomes irrelevant.[17] Third, there exists a target wealth-to-income

16. Strictly speaking, the gap between $\Phi(x)$ and $\rho^{-1}(r - \delta)$ is not exactly proportional to the variance of expected consumption growth, because equation (6) is an approximation. Qualitative statements about the gap and the variance are interchangeable, however, so as a heuristic tool I will speak of the gap interchangably with the variance.

17. This proof requires an additional restriction on parameter values, $g < r$, which is not satisfied by my baseline parameter values but is satisfied by some of the alternative parameter values considered later.

ratio x^* such that if $x_t = x^*$, $E_t x_{t+1} = x^*$, and that target is "stable" in the sense that if $x_t > x^*$, $E_t x_{t+1} < x_t$ and vice versa. (This result justifies the directional arrows on the expected consumption growth locus.)

The fourth proposition is a correction of a proposition in Carroll [1992]. That paper argued that at the target gross wealth ratio x^*, expected consumption growth was "approximately" equal to expected permanent income growth,

$$E_t\left[\Delta \ln C_{t+1} | x_t = x^*\right] \approx E_t \Delta \ln P_{t+1}.$$

Carroll [1996] shows that a more appropriate approximation is

$$E_t\left[\Delta \ln C_{t+1} | x_t = x^*\right] \approx E_t \Delta \ln P_{t+1} + \eta''[x^*] E_t \frac{(x_{t+1} - x^*)^2}{2}, \tag{7}$$

where $\eta[x] = \log c[x]$.[18] Carroll [1996] uses the proof in Carroll and Kimball [1996] of the strict concavity of the consumption function to show that the $\eta''[x^*]$ term is strictly negative. Thus, expected consumption growth for consumers holding x^* is strictly less than the expected growth rate of permanent labor income. This is visible in the figure from the gap between the intersection of the x^* line with the $E_t[\Delta \ln C_{t+1}]$ locus and the intersection of the x^* line with the $E_t[\Delta \ln P_{t+1}]$ line.

Figure Ib provides an example of how to perform experiments with the figure. The solid lines represent a blowup of the middle portion of Figure Ia, while the dashed lines show how the figure changes if g, the expected growth rate of labor income, declines from the baseline value of $g = .02$ to a new value of $g = .005$ annually. (The dashed horizontal line indicates the new, lower expected income growth rate, $g_2' = g_2 - \sigma^2_{\ln N}/2 = 0.005 - 0.005 = 0$). Even though the expected income growth rate does not appear directly in the Euler equation for consumption growth (6), the locus depicting the expected growth rate of consumption does shift, from its original position $\Phi_1(x)$ to $\Phi_2(x)$, the downward-sloping dashed curve. The expected consumption growth curve shifts because the expected variance term in (6) at a given x changes; the new optimal consumption rule will differ from the old one, so at a given x the same amount of variation in income

18. The intuition here is Jensen's inequality: because the expected growth curve is convex, the average growth of consumption for consumers distributed around the target wealth will be greater than the expected growth rate at the target.

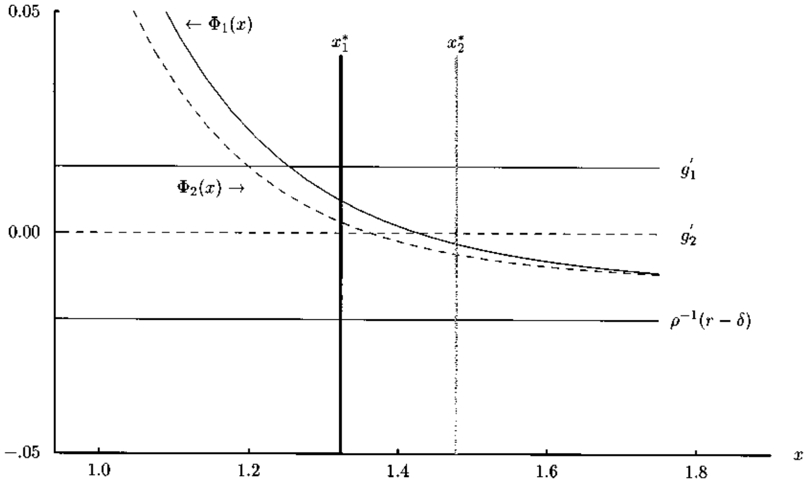

FIGURE Ib
The Effect of Reducing the Growth Rate of Income from g_1 to g_2

can produce a different amount of variation in consumption. The new target wealth ratio x_2^* is greater than the original one. Thus, as one would suspect, when consumers expect slower income growth, they hold more wealth. This is the manifestation of the "human wealth effect" in this model.

The new $\Phi_2(x)$ locus in Figure Ib was constructed by solving the whole model under the new growth rate assumption and again numerically calculating the expectation of consumption growth as a function of x. It would be convenient if there were a shortcut to this laborious process, but unfortunately there appears to be no other way to obtain accurate quantitative answers to questions about how target wealth changes when parameter values change. On the other hand, there is a simple procedure that appears always to give correct *qualitative* answers to such questions. Define $\gamma[x^*] = [E_t \, \Delta \ln C_{t+1} \mid x = x^*] - E_t \, \Delta \ln P_{t+1}$; that is, γ corresponds to the last term in equation (7), the gap between expected consumption growth and expected income growth for a consumer with wealth equal to target wealth. Now, to determine how a given parameter affects target wealth, shift only the curves that directly reflect the parameter in question, and find the value of x^* that leaves $\gamma[x]$ the same as under the original parameter values. Figure Ic illustrates how this procedure would apply to the experiment that is performed "correctly"

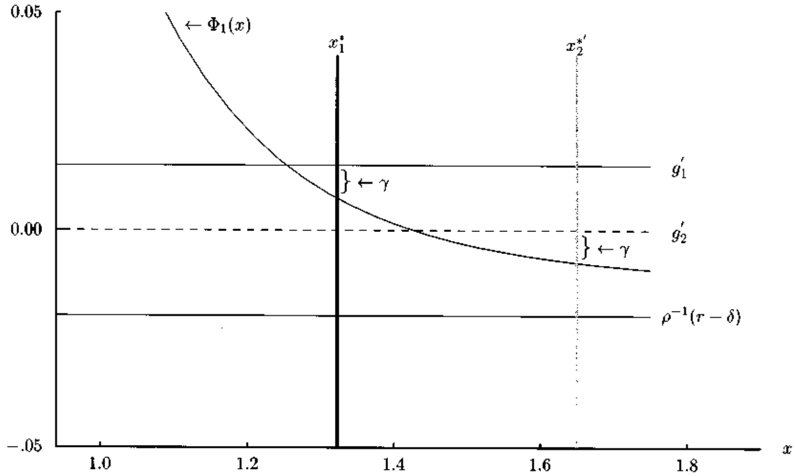

FIGURE Ic
Rough Method of Obtaining Qualitative Answer about the Effect of Reducing the Growth Rate of Income

in Figure Ib, a decline in the expected growth rate of income. Because the only locus that directly reflects the growth rate is the growth curve itself, that is the only curve in the figure that needs to be shifted. The new x^*, $x_2^{*\prime}$, is drawn at the point that leaves the gap γ between $E_t\,[\Delta \ln C_{t+1} \mid x = x^*]$ and $E_t\,\Delta \ln P_{t+1}$ unchanged. As with the correct procedure in Figure Ib, the qualitative answer this exercise yields is that a decline in the growth rate of income produces an increase in the target wealth ratio.

III.B. The Steady-State Distribution of Assets

The description of the implications of the model thus far has focused on features and implications of the optimal consumption rule. This is in keeping with most previous research, including Zeldes [1989a] and Kimball [1990a], who, for instance, examine the effect of uncertainty on the marginal propensity to consume at given levels of wealth or consumption, but do not examine what levels of wealth and consumption will prevail. As a result, the main implications they are able to draw about the differences between their models and standard models are qualitative—for their models, the marginal propensity to consume out of transitory income will be higher, consumers will hold more wealth, and the expected growth rate of consumption will be greater, than for the standard model. However, to compare the model with quantitative empirical results, it is necessary to be able to answer the question, *how much* greater will the MPC be, on average? How much higher is wealth, typically? How much faster is consumption growth, on average? And how do the answers to these questions depend on assumptions about the degree of income uncertainty, the value of taste parameters, and the expected growth rate of income?

To answer such questions, it is necessary to compute the distribution of wealth implied by the model. Presumably one reason Zeldes did not attempt this is that in a finite-horizon version of the model, the consumption rules and the distribution of wealth change in every period of life, so the answers to those questions would have been different for every period of life. It would be difficult, therefore, to summarize the results.

Things are potentially much simpler in the infinite-horizon context. Clarida [1987] showed that in a similar model with liquidity constraints, no permanent shocks, and no growth, the distribution of assets will be ergodic, converging toward a fixed steady-state distribution. If the permanent shocks in this model are removed (i.e., $N_{i,t} = 1$ for all t), this model satisfies the critical condition for ergodicity posed in Clarida [1987] (see Carroll [1996] for a proof). Unfortunately, I have been unable to construct a general proof of ergodicity for the model with permanent shocks. Nonetheless, for any particular converged consumption rule, if an ergodic distribution exists, it can be found by numerical methods. (See Appendix 2 for a description of the numerical procedure.)

Figure II shows the evolution of the distribution of the net wealth ratio over time in a simulated economy containing 20,000 households who all behave in every period according to the converged consumption rule derived under the baseline parameter values. Consumers in the simulation begin life with zero assets and permanent labor income in the first period of $P_{i,1} = 1$ for every household i. In each subsequent period of life, each household receives independent shocks drawn from the income distributions described above.[19] What is depicted in Figure II is the temporal evolution of the across-household distribution of the net wealth ratio, where net wealth is defined as the remaining wealth in a period after the household has drawn all shocks and

19. I assume that there are no aggregate shocks.

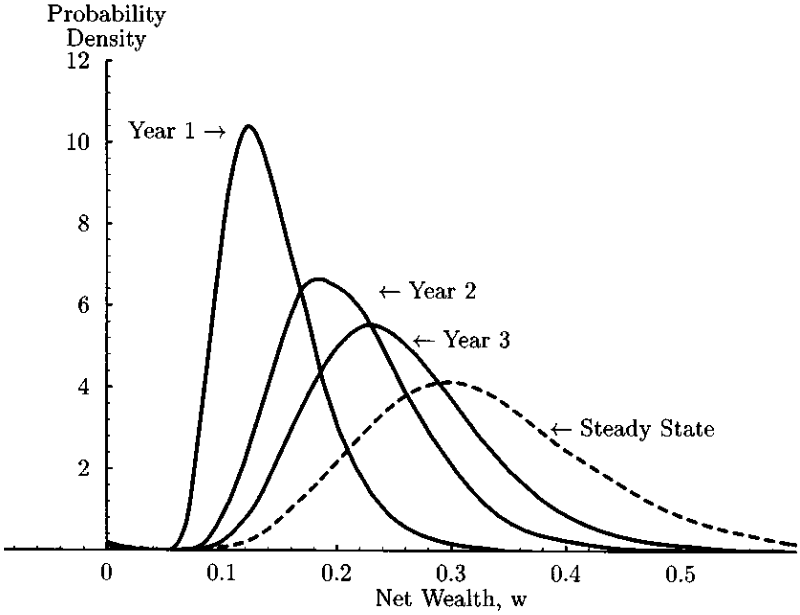

FIGURE II
Convergence of the Wealth Distribution

has consumed the desired amount. (This concept corresponds better to the data typically reported in wealth surveys than does the gross wealth ratio x used heretofore in this paper.)

Convergence of the distribution toward the steady-state distribution is rapid. From a starting value of zero, after only three periods of life the mean net wealth ratio is 0.27, compared with an estimated steady-state value of 0.34. The speed of convergence to the steady state is interesting because if convergence is rapid, it is more likely that any steady-state results derived will also apply, at least roughly, to collections of consumers out of steady state.

III.C. The Relationship between Consumption Growth and Income Growth

Table I presents some summary statistics about the steady-state behavior of collections of buffer-stock consumers who all have identical preferences. Each row presents results when all parameters are at their baseline values except the parameter designated in the first column, which takes the indicated value.

TABLE I
STEADY-STATE RESULTS FOR ALTERNATIVE PARAMETER VALUES

Parameter value	Growth rate of aggregate consumption	Average growth rate of household permanent income	Average growth rate of household consumption	Aggregate personal saving rate	Average MPC out of wealth	Average net wealth	Target net wealth
$g = .00$	0.00	−0.005	−0.005	0.002	0.16	0.66	0.62
$g = .02$†	0.02	0.015	0.015	0.007	0.33	0.35	0.32
$g = .04$	0.04	0.035	0.035	0.011	0.42	0.28	0.25
$\delta = .00$	0.02	0.015	0.015	0.015	0.15	0.66	0.61
$\delta = .04$†	0.02	0.015	0.015	0.007	0.33	0.35	0.32
$\delta = .10$	0.02	0.015	0.015	0.005	0.46	0.25	0.23
$r = .00$†	0.02	0.015	0.015	0.007	0.33	0.35	0.32
$r = .02$	0.02	0.015	0.015	0.009	0.26	0.45	0.42
$r = .04$	0.02	0.015	0.015	0.014	0.17	0.65	0.61
$\rho = 1$	0.02	0.015	0.015	0.003	0.49	0.14	0.11
$\rho = 2$†	0.02	0.015	0.015	0.007	0.33	0.35	0.32
$\rho = 5$	0.02	0.015	0.015	0.024	0.14	1.13	1.08
$\sigma_{\ln N} = .05$	0.02	0.019	0.019	0.006	0.38	0.30	0.28
$\sigma_{\ln N} = .10$†	0.02	0.015	0.015	0.007	0.33	0.35	0.32
$\sigma_{\ln N} = .15$	0.02	0.007	0.007	0.011	0.22	0.51	0.47
$\sigma_{\ln Z} = .05$	0.02	0.015	0.015	0.006	0.33	0.32	0.30
$\sigma_{\ln Z} = .10$†	0.02	0.015	0.015	0.007	0.33	0.35	0.32
$\sigma_{\ln Z} = .15$	0.02	0.015	0.015	0.008	0.32	0.39	0.35
$p = .001$	0.02	0.015	0.015	0.004	0.41	0.18	0.16
$p = .005$†	0.02	0.015	0.015	0.007	0.33	0.35	0.32
$p = .010$	0.02	0.015	0.015	0.010	0.30	0.46	0.43

† Designates the base value of the parameter.

The most striking result is shown in columns 2, 3, and 4: the expected growth rate of households' consumption always matches the expected growth rate of their permanent labor income, and the growth rate of their aggregate consumption always matches the growth rate of their aggregate labor income. This section explains how these results arise.

Consider a collection of ex ante identical buffer-stock consumers indexed by i who as of period t have achieved among them the steady-state distribution for $c_{i,t}$, the ratio of consumption to permanent income. Designating $E_{.,t}$ as the expectation taken across all households as of time t, the average expected growth rate of consumption for these consumers is given by

$$
\begin{aligned}
(8) \qquad E_{.,t}\Delta \ln C_{i,t+1} &= E_{.,t}\left[\ln C_{i,t+1} - \ln c_{i,t}\right] \\
&= E_{.,t}\left[\ln c_{i,t+1} G N_{i,t+1} P_{i,t} - \ln c_{i,t} P_{i,t}\right] \\
&= E_{.,t}\left[\ln G N_{i,t+1} + \ln c_{i,t+1} - \ln c_{i,t}\right] \\
&= E_{.,t}\left[\ln G N_{i,t+1}\right] + E_{.,t}\left[\ln c_{i,t+1}\right] - E_{.,t}\left[\ln c_{i,t}\right] \\
&= E_{.,t}\left[\ln G N_{i,t+1}\right] = g - \sigma^2_{\ln N}/2,
\end{aligned}
$$

where the last equality follows because in steady state the average value of the consumption ratio does not change from one period to the next, $E_{.,t}\,[\ln c_{i,t+1}] = E_{.,t}\,[\ln c_{i,t}]$.

In the same notation, equation (6) can be rewritten as

$$
(6') \qquad E_{.,t}\Delta \ln C_{i,t+1} \approx \rho^{-1}(r - \delta) + (\rho/2) E_{.,t}\left(\Delta \ln C_{i,t+1}\right).
$$

We now have two expressions, equations (8) and (6′), for the expected growth rate of consumption, taking expectations across consumers at a point in time. The two equations share a single endogenous variable, the expected variance of consumption growth. Obviously, this means we can solve for the endogenous value of the expected variance:

$$
(9) \qquad E_{.,t}\left[\mathrm{var}_{i,t}\left(\Delta \ln C_{i,t+1}\right)\right] \approx (\rho/2)\left[g - \sigma^2_{\ln N}/2 - \rho^{-1}(r - \delta)\right].
$$

Of course, as Figure I vividly illustrates, the variance of consumption growth is also a negative function of the level of wealth. Using that fact in conjunction with equation (9) yields intuitive predictions. For example, at higher interest rates the right-hand side of equation (9) will be smaller, corresponding to a smaller average consumption variance and thus necessarily a higher average level of wealth. In other words, the interest elasticity of average wealth is positive.[20] Similar logic can be used to show that wealth is higher for consumers who discount the future less (δ falls) or for consumers with lower expected permanent income growth (the human wealth effect). The coefficient of relative risk aversion has offsetting effects: a higher ρ represents a stronger precautionary saving motive (reflected in the $2/\rho$ term), and on its own would increase average wealth. But a higher ρ also corresponds to a lower intertemporal elasticity of substitution (reflected in the ρ^{-1} $(r - \delta)$ term), which should result in lower wealth if consumers are impatient. The overall effect of ρ on wealth is therefore ambiguous.

Because the average growth rate is not the same as the growth rate of the average, the above proof that $E_{.,t}\, \Delta \ln C_{i,t+1} = E_{.,t}\, \Delta \ln P_{i,t+1}$ does not prove that $\Delta \ln E_{.,t}\, C_{i,t+1} = \Delta \ln E_{.,t}\, P_{i,t+1}$. Yet Table I showed that in the simulations both propositions held true. It is surprisingly difficult to prove that the average growth rate of aggregate consumption is equal to the average growth rate of aggregate income in the buffer-stock model. Because the proof is difficult but not enlightening, it is not presented here; interested readers can find it in Carroll [1996].

A word is in order about the subtle but important difference between the kind of analysis contained in the previous section and embodied in Figure I, and the kind of analysis just presented that can be done using equations (6′) and (9). Figure I and its discussion reflected only statements about the optimal behavior of an individual consumer. Nowhere were implications about aggregates or averages across consumers derived or discussed. On the other hand, the logic used to obtain equation (9) relied critically on an assumption that there was a population of consumers across whom the distribution of consumption (and other variables) had reached its ergodic steady state.

20. The reasoning relating parameter values to mean wealth in this sentence and the rest of the paragraph cannot be formally justified, for several reasons. For example, the demonstration in Figure I that $\mathrm{var}_t(\Delta \ln C_{t+1})$ is a negative function of x_t does not absolutely guarantee that when the equilibrium variance falls mean wealth must rise. That would only follow with absolute necessity if the new steady-state distribution of x were identical to the original distribution except for a location parameter, and if the relationship between $\mathrm{var}_t(\Delta \ln C_{t+1})$ and x_t were linear, neither of which is true. Furthermore, even equation (9) itself relies on approximations. Equation (9), and the methods for reasoning about average wealth from it, should be viewed as a heuristic tool rather than as a rigorous analytical framework. That said, I have found no parameter values for which this kind of reasoning from equation (9) gives the wrong answer.

III.D. Implications for Empirical Research

The most important implication of equation (9) is that typical methods of Euler equation estimation, on either household or aggregate data, yield meaningless results if the consumers involved are buffer-stock savers, because typical methods assume that the variance term in the Euler equation is either zero or a constant. This subsection illuminates the potential pitfalls for Euler equation estimation using examples, first from the literature on Euler equation estimation with household data and then from the literature on aggregate Euler equation estimation.

Lawrance [1991] estimates consumption Euler equations across households of different educational levels, assuming a constant value of the variance term across households. Simplifying considerably, she finds that consumption growth is faster for households with greater education, and concludes that education must be correlated with the pure rate of time preference, δ. However, a large literature in labor economics has established that households with greater education have faster labor income growth. The dependence of consumption growth on income growth in the buffer-stock model strongly suggests that Lawrance's estimates of δ might simply be proxying for the effects on consumption growth of predictable differences in income growth.

Because the issues here are both subtle and important, it is vital to be perfectly clear about the nature of the problem. A simple example will illustrate how results like Lawrance's could arise in a buffer-stock framework despite identical time preference rates across households. Suppose that the population consists of two kinds of consumers, H and L, identical in every respect (including taste parameters) except that the mean growth rate of permanent labor income is higher for consumers in group H than for consumers in group L, $g_H > g_L$. Suppose further that the two groups of consumers have, respectively, converged to their steady-state wealth distributions. Now imagine estimating an equation of the form,

$$\Delta \ln C_{i,t+1} = \alpha_0 + \alpha_1 E_{i,t} r_{i,t+1} + \alpha_1 D_i + \varepsilon_{i,t+1},$$

across this whole population of consumers, with the dummy variable D equal to 1 for consumers of type H and 0 for consumers of type L. The estimated coefficient on D_i will be $(g_H - g_L)$, as can be see by plugging equation (9) with different growth rates into equation (6′). The Lawrance interpretation of this finding would be that consumers in group H have a lower discount rate than consumers in group L, even though by assumption the data were generated by consumers with identical time preference rates. The problem is in the omission from the estimating equation of the endogenous variance term, which buffer-stock theory indicates ought to be correlated with D_i. In econometric terms, this is an omitted variable problem, where the omitted variable is correlated with the included variable D.

Even papers that explicitly acknowledge the presence and potential nonconstancy of the variance term face substantial problems. One of the earliest and best of these papers is Dynan [1993]. She uses data from the Bureau of Labor Statistics' *Consumer Expenditure Surveys* to calculate consumption growth rates and variances of consumption growth rates for different groups of households and estimates an equation of the form,

$$\Delta \ln C_{i,t+1} = \alpha_0 + \alpha_1 E_{i,t} r_{i,t+1} + \alpha_2 E_{.,t} \operatorname{var}(\Delta \ln C_{i,t+1}) + \varepsilon_{i,t+1},$$

by instrumental variables, using as instruments the head of household's education, occupation, age, and a variety of other characteristics. She finds a coefficient close to zero on *both* the $E_{i,t} r_{i,t+1}$ term *and* the $E_{.,t}$ var(Δ ln $C_{i,t+1}$) term.[21] Dynan considers herself to be estimating equation (6) and thus expects the coefficient on $r_{,t+1}$ to equal ρ^{-1} and the coefficient on $E_{.,t}$ var(Δ ln $C_{i,t+1}$) to be ($\rho/2$). A coefficient estimate of zero for α_1 would therefore imply a coefficient of relative risk aversion of $\rho = \infty$, while a coefficient of zero on the variance term implies that $\rho = 0$. However, Dynan focuses on the coefficient estimate for $E_{.,t}$ var(Δ ln $C_{i,t+1}$), and concludes that her data provide no evidence for the existence of a precautionary saving motive (a precautionary motive requires a strictly positive ρ).

If Dynan's [1993] consumers were engaged in buffer-stock saving, however, coefficient estimates of zero on both these terms would be unsurprising. The simplest example of how such a result could arise is as follows. Consider a sample that contains households that fall into several groups identifiable to the econometrician via instruments like Dynan's (education groups, say). Suppose that the households in the groups are identical in every respect except for the interest rates they face and their rates of

21. An important subtlety: for simplicity, I will assume that Dynan's instruments are effectively isolating separate groups of consumers with different group characteristics. In this case the $E_{.,t}$ notation represents the instrumented value of the variable whose expectation is being taken.

time preference. In the buffer-stock framework, on average, the impatient groups of consumers will end up holding less wealth. Their smaller buffer-stocks reduce their ability to shield consumption against income shocks, resulting in a larger value for $(\rho/2)\ E_{.,t}\ \mathrm{var}(\Delta \ln C_{i,t+1})$. Under these circumstances, group membership would be highly statistically significant (as Dynan's instruments are) in a first-stage regression of the variance term and the interest rate term on group dummies, yet all consumers would end up with the same growth rate of consumption (their shared growth rate of income g) despite having predictably different values of both $E_{.,t}\ \mathrm{var}(\Delta \ln C_{i,t+1})$ and $E_{.,t}\ r_{i,t+1}$. The regression's intercept term α_0 would be estimated to equal g, and the coefficients on the interest rate and variance terms would both be estimated at zero, as Dynan found. Similar logic holds if the coefficient of relative risk aversion varies across groups; the coefficient estimates for α_1 and α_2 would again be zero. Taken as a whole, therefore, Dynan's findings are actually *supportive* of a buffer-stock model of precautionary saving, but only because, in contrast with the standard model, the buffer-stock model is capable of explaining how *both* of her coefficient estimates could be estimated at zero even if consumers in fact have a strong precautionary saving motive.

In principle, it is possible to estimate consumption Euler equations consistently across groups of buffer-stock consumers, but the conditions required are quite stringent. In particular, it is essential to have important and predictable differences across groups of consumers in both interest rates and income growth rates, and no differences at all in tastes across groups. Without variation in income growth rates, equation (9) indicates that the variance term and the interest rate term should be perfectly collinear, preventing accurate coefficient estimation on either term. Without variation in interest rates across groups, it is obviously not possible to estimate a coefficient on the interest rate term.

The implications of this discussion for the estimation of consumption Euler equations across households are thus rather grim, at least for groups of consumers who satisfy the impatience condition (5). One might hope that the traditional Euler equation estimation methods would at least be applicable to patient consumers who do not satisfy (5). Unfortunately, however, even for finite-horizon patient consumers, the variance term in the Euler equation will be a declining function of wealth, because the proof of the concavity of the consumption function in Carroll and Kimball [1996] holds regardless of taste parameters. Because the amount of wealth accumulation accomplished by a given age, and therefore the variance term in the Euler equation, depends on the time preference rate, the time preference rate cannot itself be estimated consistently using Euler equation methods across such groups of consumers. Similar logic applies to the coefficient of relative risk aversion. Of course, as the level of the gross wealth ratio gets very large, the variance of consumption growth approaches zero, and these problems disappear. In fact, the only consumers for whom it is absolutely clear that taste parameters can be consistently estimated via traditional consumption Euler equation methods are consumers who possess effectively infinite wealth.

The message of the buffer-stock model for the estimation of household taste parameters is not entirely nihilistic, however. Equation (9) itself can be estimated, most easily by regressing group variances of consumption growth on group income growth rates and group-specific interest rates: this yields a direct estimate of $(2/\rho)$ and therefore of ρ itself, assuming a ρ that is constant across groups. One advantage of estimating (9) over direct estimation of the consumption Euler equation is that consistent estimation of (9) need not require identical time preference rates across groups. Indeed, if good instruments can be found for the growth rate of income and for the interest rate, it should even be possible to use nonlinear methods to estimate group time preference rates from equation (9).

A second, cruder test of the foundations of the buffer-stock model, and indeed of the more fundamental property of the concavity of the consumption function, is suggested by Figure I: simply examine whether the variance of consumption growth is higher for consumers with lower wealth-to-permanent-income ratios. Perhaps the best empirical test along these lines would be to look for consumers who experienced a major recent drop in their wealth (possibly owing to a spell of unemployment), and to calculate the effects on the variance term and on subsequent consumption growth.

Equation (9) and Figure I do not exhaust the empirical implications of the model for household data. See Section V for a brief discussion of a variety of recent empirical work which matches the model to data in new ways that have little to do with Euler equation estimation.

Implications of the buffer-stock model for estimation of aggregate Euler equations are similar to those for estimation across groups of households. If the population is normalized at 1, the aggregate and average levels of consumption will be the same: designate both as $E_{,t+1}\ C_{i,t} = C_{.,t}$. In that case, the relationship between aggregate labor income growth g and aggregate consumption growth in the steady state can be written simply as

$$\Delta \ln C_{i,t+1} = g. \tag{10}$$

The mechanism by which the growth rate of aggregate consumption converges to the growth rate of aggregate labor income is essentially the same as what caused expected household consumption growth within groups of buffer-stock consumers to converge to expected household income growth: through the mediation of the endogenous variance term. Households living in a more impatient country will have a lower value of household wealth, on average, and therefore higher consumption variance, which will boost their consumption growth enough to make consumption growth match income growth.

The model's prediction that aggregate consumption growth should approximately equal aggregate permanent labor income growth, at least in steady state, may have the potential to help explain many empirical failures of the standard Euler equation framework estimated using aggregate data. The most obvious application is to the consumption/income parallel that Carroll and Summers [1991] documented for cross-country aggregate data over periods of three to five years and longer. Of course, standard growth models also predict that, in steady state, income growth and consumption growth will be "balanced." The distinction between this model and the standard model is that the transition to the steady state is much faster here: under baseline parameter values, the transition half-life in the buffer-stock model is generally about two years. In contrast, under standard parameter values the transition half-life in a Solow or Cass-Koopmans growth model is on the order of fifteen or twenty years, and some authors (Mankiw, Romer, and Weil [1992] in particular) favor parameter values that generate a half-life that is even longer. The slow rate of convergence in the standard model was a primary reason Carroll and Summers [1991] rejected the idea that the consumption/income parallel evident in aggregate data over periods of three to five years was a reflection of balanced growth steady states.

Another piece of evidence Carroll and Summers [1991] mustered against the Cass-Koopmans growth model as a description of aggregate consumption was that there is no apparent relationship across countries between the average growth rate of aggregate consumption and average country-specific interest rates. This result is easily explained in a buffer-stock framework as resulting from the endogeneity of the variance term, which is not observed in aggregate data and is hence inevitably omitted from aggregate Euler equation estimation.[22] As equation (9) indicates, the omitted variance term is in theory negatively correlated with the interest rate. In fact, equation (10) summarizes all the implications of the buffer-stock model for the steady-state relationship between consumption growth and interest rates, tastes, income uncertainty, and other variables. The coefficients on all such variables should be zero when the consumption variance term is omitted from the estimating equation.

The model may also help to shed some light on the findings of Campbell and Mankiw [1989], who estimated an equation of the form,

$$\Delta \ln C_{t+1} = \alpha_0 + \alpha_1 E_t r_{t+1} + \lambda E_t \Delta \ln Y_{t+1} + \varepsilon_{t+1},$$

where $E_t \Delta \ln Y_{t+1}$ was calculated using lagged variables that, according to traditional Euler equation analysis, should be uncorrelated with current consumption growth. They found a highly significant estimate of λ in the vicinity of 0.5. Their interpretation was that half of consumption is done by consumers who set consumption equal to income, while the rest is done by consumers who obey the standard Euler equation (although they did not find robust evidence of a positive coefficient on the interest rate as would be expected for the consumers who putatively obey the standard Euler equation).

Suppose that in the postwar period aggregate labor income grew according to

$$\Delta \ln Y_t = g_t + e_t,$$

where g represents the underlying rate of labor income growth and e represents transitory shocks to income growth. Furthermore, suppose that $g_t = g_1$ for t before 1973 and $g_t = g_2 < g_1$ for t after 1973, reflecting the productivity growth slowdown in the post-1973 period. Now suppose that some subset of Campbell and

22. It is important to recognize here that the right variable is the variance of consumption growth at the microeconomic level. The omitted variance term therefore cannot be recovered from any kind of ARCH or GARCH estimation using aggregate data. It must be calculated using household level data, if at all.

Mankiw's [1989] instruments also experienced a regime shift in the post-1973 period, and therefore instruments dated t will do a good job indicating whether the economy is in the slow-growth or fast-growth regime. Finally, suppose that another subset of instruments is highly correlated with e_{t+1}. The second-stage equation that Campbell and Mankiw estimate is then in effect,

$$\Delta \ln C_{t+1} = \alpha_0 + \alpha_1 E_t r_{t+1} + \lambda(\hat{g}_{t+1} + \hat{e}_{t+1}) + \varepsilon_{t+1},$$

where $\hat{g}_{t+1}$ reflects the correlation of their instruments with underlying "permanent" growth rate regime, and $\hat{e}_{t+1}$ reflects the correlation of their instruments with the predictable component of transitory income growth. The buffer-stock model implies that, across steady states, the coefficient on $\hat{g}_{t+1}$ should be one. It is less clear what the model would imply about the coefficient on $\hat{e}$, although the MPC out of transitory income should be an upper bound on the coefficient on $\hat{e}$. Although it is not clear what the coefficient on $\hat{g}$ would be during the transition period between the pre-1973 and post-1973 growth regimes, the relatively rapid convergence of the buffer-stock model under baseline parameter values suggests that this transition period would not last long, so the estimated coefficient on $\hat{g}_{t+1}$ should be close to one if separate coefficients on $\hat{g}$ and $\hat{e}$ were estimated. However, when the two terms are combined into a single term for predictable income growth, as Campbell and Mankiw do, any coefficient estimate between one and the coefficient on $\hat{e}$ could be consistent with a buffer-stock model, depending on, among other factors, the degree of correlation of the various instruments with the transitory and permanent components of growth.

Whether in practice a buffer-stock model predicts something like the 0.5 coefficient that Campbell and Mankiw [1989] typically found would depend in detail on the exact specification of the model; setting up and solving such a model is well beyond the scope of this paper (although it is an inviting project for future work). The argument here is only that a buffer-stock model at least has the *potential* to explain Campbell and Mankiw's results without assuming the existence of consumers who blindly set consumption equal to income in every period.

One important caveat in using the model to explain aggregate data is that, even if the typical household is a buffer-stock consumer, it is clear that at least some consumers do not behave according to a buffer-stock model (see the discussion of very wealthy households in subsection IV.C). To the extent that aggregate consumption reflects the behavior of these nonbuffer-stock consumers, the buffer-stock model may not be able to match aggregate data even if it is the right model for most consumers.

III.E. Other Implications

The results in Table I summarize other interesting characteristics of the steady-state solution of the buffer-stock model under a variety of parameter values.

First, the Marginal Propensity to Consume. The standard model under baseline parameter values implies an MPC out of transitory income of 2 percent. No commonly used set of parameter values in the standard model implies an MPC of greater than about 8 percent (the formula for the MPC in the standard models is given in subsection II.C). Table I shows that, over the entire range of parameter values considered in the table, the MPC is much greater than for the standard model. The average MPC for buffer-stock consumers is always at least 15 percent, and ranges up to 50 percent.

Another interesting question is what the model implies about the relationship between expected income growth and the personal saving rate. If the steady-state average net wealth ratio is w^* at income growth rate g, then (if the interest rate is zero) the personal saving rate necessary to make wealth grow at rate g (thus keeping the wealth/income ratio constant) is $s \approx g\ w^*$. If the target wealth ratio w^* were a constant, the model would obviously imply that $s \approx g\ w^*$ is higher when g is higher. However, w^* is a negative function of g; the standard term for the effect of g on consumption and saving decisions is the "human wealth effect." In practice, for all parameter values considered here, the human wealth effect is vastly smaller than in the standard model, so across steady states the elasticity of the saving rate with respect to the growth rate of income is positive.

In addition to its implications for the relationship between saving and growth, the drastic diminution of the human wealth effect compared with the standard model is also an interesting finding in itself. Several recent empirical tests can be interpreted as providing evidence that current consumption is affected much less by expected future income than the standard LC/PIH model implies. Campbell and Deaton [1989] calculate the implied effect on human wealth of an innovation in current income, and find that consumption greatly "underresponds" to innovations in human wealth. Carroll [1994] projects mean future income for a

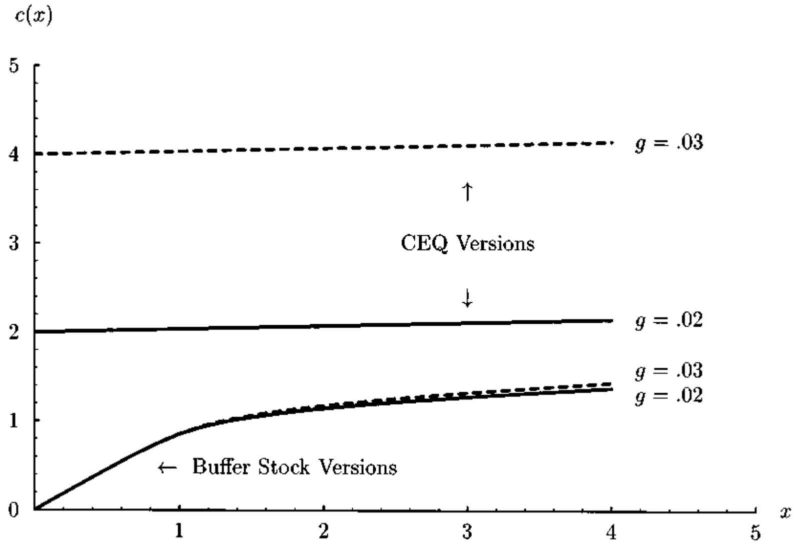

FIGURE III
Marginal Propensity to Consume out of Human Wealth

panel of households and finds no evidence that predictable future income growth affects current consumption at all. Viard [1993] shows that a standard LC/PIH model implies that the post-1973 slowdown in productivity growth in the United States should have sharply boosted saving rates; instead, saving rates have fallen.

Is the MPC out of human wealth in a buffer-stock model consistent with these results? Unfortunately, such a question is difficult to answer because, in contrast to the standard LC/PIH model, in precautionary saving models the MPC out of future income depends on the current level of physical wealth, the distribution of the future income, and the consumption rules expected to prevail over the entire remainder of the consumer's horizon. There is thus no general way to answer the question of how responsive consumption is to human wealth, because the answer will depend on the exact experiment. It is possible, however, to answer a specific question, such as how consumption would respond to an increase in the expected growth rate of income from 2 percent to 3 percent per year. Consider Figure III, which presents the converged consumption rules for infinite-horizon versions of the buffer-stock model and the certainty version of the

LC/PIH model in the case where the real interest rate is 4 percent, expected income growth is either 2 percent or 3 percent, and other parameters are at their baseline values.[23] The straight lines indicate the optimal consumption functions in the perfect certainty case; the curved functions represent the converged buffer-stock consumption rules. The dashed functions correspond to the $g = 0.02$ cases, and the solid functions to the $g = 0.03$ cases. When the growth rate of income increases from 2 to 3 percent, human wealth defined as the present discounted value of expected future labor income doubles,[24] boosting consumption substantially at all levels of gross wealth in the standard model. In the buffer-stock model, however, consumption rises far less: prudent buffer-stock consumers refuse to spend much out of expected higher future income, because that income just might not materialize.

The same point is made numerically by Table II. Under the chosen parameter values the MPC out of human wealth in the standard model is 3.8 percent. In the buffer-stock model the MPC rises with current assets but remains very small over the entire range. Although not precisely zero, it is easily small enough to be consistent with empirical estimates like those in Carroll [1994] that indicate that the MPC out of human wealth is close to zero.

Another way to measure the degree of responsiveness of current consumption to changes in expected future income is to calculate an implicit interest rate at which expected future income can be said to be "discounted" when the consumer decides how much of that future income to spend today. Appendix 3 describes how such a measure can be calculated; the results are presented in the column titled "Implied Discount Rate for Future Income" in Table II. The results are striking: at a gross wealth ratio of 0.2, the implied rate at which future income is discounted is 13,981 percent! This result arises because a consumer with gross wealth of only 0.2 is already consuming almost every penny, and when expected future income goes up, this consumer is unable to spend more than a tiny bit extra today.

Even at more moderate values of the gross wealth ratio, the implied discount rate is remarkably large. In particular, the tar-

23. The deviation of the interest rate from the baseline value of zero is motivated by the fact that the present discounted value of future income in the certainty case is infinite, making nonsense of the model.

24. This can be seen from the formula for human wealth in the infinite-horizon case in subsection II.C: $H = Y/(r - g)$. At $r = .04$, $g = .02$, $H = 50\ Y$; at $g = .03$, $H = 100\ Y$.

TABLE II

THE MARGINAL PROPENSITY TO CONSUME OUT OF HUMAN WEALTH

	Infinite horizon certainty model			Infinite horizon buffer-stock model			
Gross wealth ratio	Consumption $g = 2\%$	Consumption $g = 3\%$	MPC out of human wealth	Consumption $g = 2\%$	Consumption $g = 3\%$	MPC out of human wealth	Implied discount rate of future income
0.2	2.01	4.01	0.038	0.1858	0.1858	5.60E-07	139.8170
0.4	2.02	4.02	0.038	0.3689	0.3691	3.82E-06	40.1389
0.6	2.02	4.02	0.038	0.5457	0.5465	1.54E-05	9.5355
0.8	2.03	4.03	0.038	0.7114	0.7141	0.0001	2.4943
1.0	2.04	4.04	0.038	0.8419	0.8515	0.0002	0.6725
1.2	2.05	4.05	0.038	0.9253	0.9444	0.0004	0.3618
1.4	2.05	4.05	0.038	0.9778	1.0067	0.0006	0.2601
1.6	2.06	4.06	0.038	1.0159	1.0527	0.0007	0.2166
1.8	2.07	4.07	0.038	1.0451	1.0907	0.0009	0.1861
2.0	2.08	4.08	0.038	1.0698	1.1223	0.0010	0.1689
2.2	2.08	4.08	0.038	1.0886	1.1485	0.0012	0.1552
2.4	2.09	4.09	0.038	1.1075	1.1746	0.0013	0.1444
2.6	2.10	4.10	0.038	1.1247	1.1986	0.0014	0.1359
2.8	2.11	4.11	0.038	1.1402	1.2207	0.0015	0.1292
3.0	2.12	4.12	0.038	1.1558	1.2428	0.0017	0.1235

All parameters except the interest rate are equal to base values described in the text. The interest rate is assumed to be 4 percent. Calculation of the implied discount rate for future income is described in Appendix 3.

get gross wealth ratio in this model is around 1.6; the future income discount rate at a gross wealth ratio of 1.6 is about 22 percent.

IV. Buffer-Stock Saving and the Life Cycle: Resolving Three Empirical Puzzles

This section of the paper makes a systematic argument that a version of the LC/PIH model which implies that consumers engage in buffer-stock saving behavior over most of their working lifetimes fits the essential facts about the behavior of the typical household's consumption, income, and wealth better than either the standard LC/PIH model, a Keynesian alternative, or the hybrid of the Keynesian and standard LC/PIH models proposed by Campbell and Mankiw [1989].[25]

The buffer-stock version of the finite-horizon LC/PIH model will reflect the same baseline parameter values under which buffer-stock saving behavior emerges in the infinite-horizon context, but with a lifetime income process calibrated to actual U. S. household age/income profiles. The perfect-certainty finite-horizon model briefly described in subsection II.C is what I will call the standard LC/PIH model, although similar results would be obtained with a model with quadratic or Constant Absolute Risk Aversion utility. The Keynesian model will be taken to be a model of the form $C = \alpha_0 + \alpha_1 Y + u$, where consumption has a positive intercept α_0 and there is a constant marginal propensity to consume α_1 that is close to one (perhaps 0.9). The Keynesian model has not been taken seriously since the work of Friedman and Modigliani in the 1950s; it is examined here principally as an aid to understanding the nature of the evidence. The final model is the Campbell-Mankiw model which blends the standard LC/PIH and the Keynesian models by assuming that half of income goes to standard LC/PIH consumers and half goes to Keynesian consumers with $\alpha_0 = 0$ and $\alpha_1 = 1$.

Table III summarizes some of the principal points made below. I will argue that only a buffer-stock version of the LC/PIH model is consistent with the overall pattern of facts.

25. I do not consider a model with uncertain income and liquidity constraints, as in Deaton [1991], because I view that model and the model presented here as close substitutes. Most of the evidence that I find to be consistent with the buffer-stock model presented here would also be consistent with Deaton's model.

TABLE III
CONSISTENCY OF THE FOUR MODELS WITH THREE STYLIZED FACTS

	Stylized fact		
Model	Consumption/income parallel (Figure IV)	Consumption/income divergence (Table IV)	Wealth is small and positive (Table V)
Keynesian Model $C = \alpha_0 + \alpha_1 Y + u$ α_1 is near 1	Consistent Estimates using long-term low frequency aggregate data consistently find α_1 near one	Not consistent Estimates using short-term high-frequency household data find α_1 much less than one	Not consistent Model provides no reason why wealth should stay in a restricted range near zero
Standard life-cycle/ permanent income $C = k\,[W + H]$	Not consistent Lifetime profile of C should be unrelated to lifetime profile of Y	Not consistent Table IV implies an MPC out of transitory income of 0.2, which is too high	Not consistent Model provides no reason why wealth should stay in a restricted range near zero
Campbell-Mankiw $C = \lambda Y + (1 - \lambda)\, k[W + H]$, where $\lambda = 0.5$	Not consistent Figure IV suggests that $\lambda = 1$, not 0.5	Not consistent Table IV implies that $\lambda = 0.2$, not 0.5.	Not consistent Neither underlying model explains why wealth should stay positive and small
Buffer-stock LC/PIH model	Consistent See Figure IV	Consistent See Table I	Consistent See Figure VII

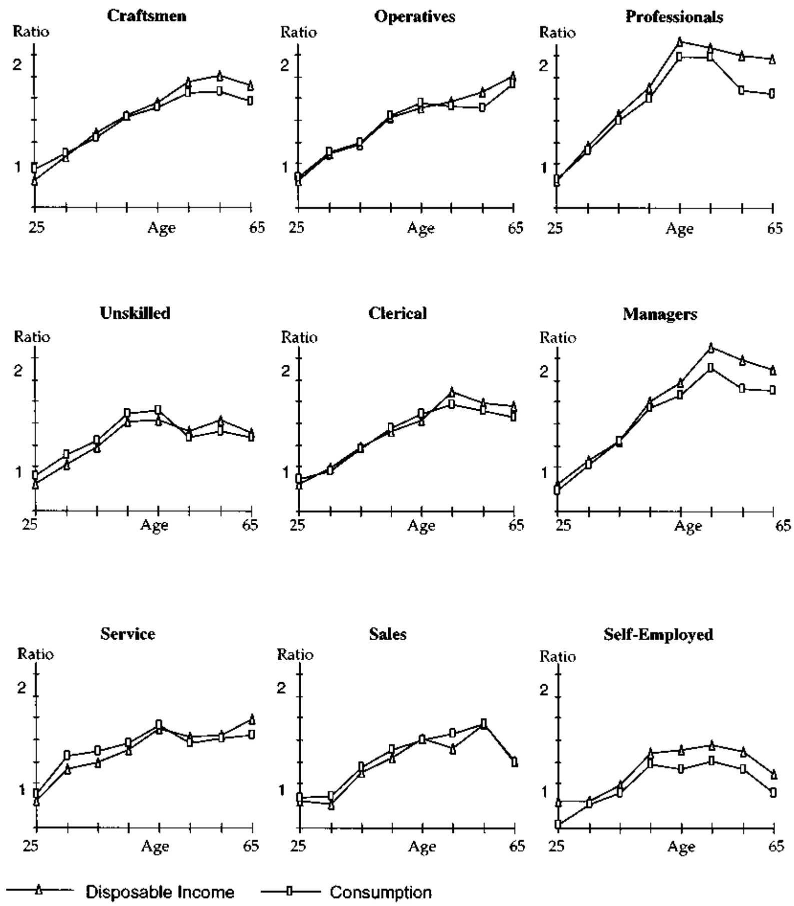

FIGURE IV
Lifetime Consumption and Income Profiles for Nine Occupation Groups

IV.A. The Consumption/Income Parallel in Low Frequency Data

Perhaps the most striking point of Carroll and Summers [1991] is made by Figure IV: across occupations, differences in age/income profiles are closely paralleled by differences in age consumption profiles.[26] The figure shows that consumption growth and income growth are very closely linked over periods of a few years or longer, a phenomenon we dubbed the "consump-

26. Carroll and Summers [1991] and Carroll [1994] also provide evidence that the profiles for different occupational groups remain relatively stable over time. Unlike the figure in Carroll and Summers [1991], income and consumption

tion/income parallel." This phenomenon is interesting because there is no explanation for it in the unconstrained standard LC/PIH framework. In that model the pattern of consumption growth is determined by tastes and is independent of the timing of income.

Of course, the Keynesian model $C = \alpha_0 + \alpha_1 Y + u$ can easily explain the consumption/income parallel if α_0 is small and α_1 is near one. Little of the evidence Carroll and Summers marshalled for the low-frequency consumption/income parallel would rule out such an explanation. However, evidence presented in the next section (along with a mountain of other evidence originally presented in the 1940s and 1950s by Modigliani, Friedman, and others) is much less favorable to the Keynesian model.

The Campbell-Mankiw model with $\lambda = 0.5$ has trouble explaining the consumption/income parallel because the parallel is simply too close. The ocular regression of consumption on income in Figure IV suggests a coefficient near 1, not near 0.5; the regressions of Carroll [1994] also suggest coefficients much nearer 1 than 0.5.

Can a plausibly parameterized buffer-stock version of the LC/PIH model explain the low frequency consumption/income parallel? To answer this question, I simulate a finite-horizon version of the model using income profiles calibrated to roughly match those in Figure IV. From Figure IV it appears that in some occupations, such as Unskilled Labor, labor income tends to stop growing relatively early in life, while for others, such as Managers, income tends to continue growing until late middle age. For other occupations, such as Operatives, income grows quickly until middle age, then slowly until retirement. For the simulations, I will consider three age/income profiles roughly calibrated using the data for Unskilled Laborers, Operatives, and Managers shown in Figure IV. Specifically, the Unskilled Labor profile income grows at 3 percent annually from ages 25 to 40, and is flat from age 40 to retirement at 65. For Operatives, labor income grows at 2.5 percent per year from age 25 to age 50, and then at 1 percent per year until retirement. Finally, for Managers, income grows at 3 percent a year from ages 25 to 55, and declines at 1

in this figure were adjusted for aggregate productivity growth by adding 1.5 percent to the growth of both the income and the consumption profiles in each year. Without this adjustment for aggregate productivity growth, Carroll and Summers found that for all occupations income reached a peak in middle age and then began declining, rapidly in some cases.

The productivity adjustment does not alter the principal conclusion Carroll and Summers [1991] drew from their figure (and from a variety of other evidence).

percent a year from 55 to 65. Postretirement income for all three categories is assumed to equal 70 percent of income in the last year of the working life, because empirical estimates suggest that the sum of pension, social security, and other noncapital income after retirement typically falls to roughly 70 percent of its preretirement level.

Solving the finite horizon version of the model by backwards recursion on (4) produces optimal consumption rules for each period of life. Estimates of average consumption and income were generated as for Figure II and Table I by randomly drawing income shocks according to the assumed income distributions, for 1000 consumers who start life with zero assets.[27] (The behavior of asset holding over the life cycle is discussed below.)

The results are shown in Figure V. A rough summary of the results of Figure V would be that consumers engage in buffer-stock saving behavior, and so consumption growth closely parallels income growth, until roughly age 45 or 50. Around that age consumers switch over to doing a bit of retirement saving, which allows the income profile to rise somewhat above the consumption profile in the years immediately before retirement.

It worth noting here that Friedman [1957] himself would not necessarily have interpreted Figure IV as evidence against his conception of the PIH. Indeed, he almost seems to anticipate it:

> For any considerable group of consumers the . . . transitory components tend to average out, so that if they alone accounted for the discrepancies between permanent and measured income, the mean measured income of the group would equal the mean permanent component, and the mean transitory component would be zero. . . .
>
> . . . It is tempting to interpret the permanent components as corresponding to average lifetime values and the transitory components as the difference between such lifetime values and the measured values in a specific time period. It would, however, be a serious mistake to accept such an interpretation . . . [pp. 23–23].

27. The slight upward fillip to consumption in the last two or three years of life occurs as consumers spend down their precautionary assets when they realize that the amount of uncertainty remaining is small. This feature is an artifact of the unattractive assumption of a certain date of death, and therefore is not one of the implications of the model on which I wish to concentrate. If the model were modified to incorporate length-of-life uncertainty by adding a probability of death in each year, it could likely replicate the results in Hubbard, Skinner, and Zeldes [1995], who find that consumption slopes downward throughout the entire retirement period.

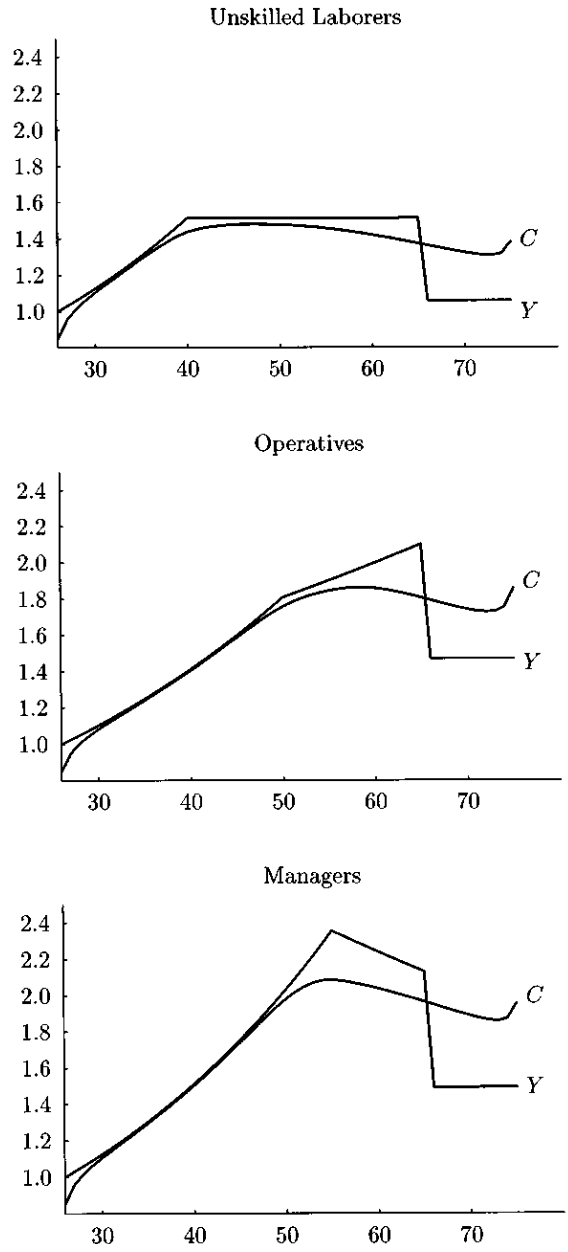

FIGURE V
Age Profiles of Consumption Predicted by the Model for Three Profiles of Lifetime Income

> The permanent income component is not to be regarded as expected lifetime earnings; it can itself be regarded as varying with age. It is to be interpreted as the mean income at any age regarded as permanent by the consumer unit in question, which in turn depends on its horizon and foresightedness [p. 93].[28]

Friedman [1957] explains that the principal reason interpreting permanent income as average lifetime income is a mistake is because such an interpretation prejudges the issue of the length of the horizon over which consumers calculate permanent income. Friedman later clarifies what he means by the horizon when he says that the typical household discounts future income at a rate of 33 1/3 percent. Although discounting future income at a 33 1/3 percent rate is very difficult to justify in the standard model, as Table II shows, in a buffer-stock model an apparent discount rate of 33 1/3 percent is not at all difficult to obtain.

IV.B. The Consumption/Income Divergence in High Frequency Data

It might seem that the low-frequency parallel between consumption and income suggests that a simple Keynesian model with α_0 near zero and α_1 near one is a better model than the comparatively complicated buffer-stock or standard LC/PIH models. The Keynesian model, however, was abandoned in the 1950s for a number of good reasons, perhaps the most important of which was the discrepancy between estimates of the model using long-term aggregate data, which found α_0 near zero and α_1 near one, and estimates using cross-section household surveys of consumption and income, which found α_0 to be substantially positive and α_1 to be much less than one.

Friedman's [1957] famous resolution of this puzzle was one of the persuasive pieces of evidence for the Permanent Income Hypothesis. He showed that if $C = P + u$ and $Y = P + v$, where P represents permanent income and Y represents observed income and u and v are stochastic, then a regression of C on Y would produce an estimated α_1 coefficient on *observed* income of far less than one even if the MPC out of *permanent* income is one. The logic is identical to the now-familiar errors-in-variables econometric problem in which the coefficient on a variable mea-

28. Another indication that Friedman regards mean income as a good indicator of permanent income is his statement that upon retirement permanent income typically falls by about 25 percent because pension income is less than working income.

sured with error is biased toward zero. To complete the case, Friedman argued that the reason α_1 was estimated to be much nearer one when long-term aggregate income data were used was because such long-term data are dominated by changes in permanent income.

Friedman [1957] noted that another implication of the model was that groups within the population for whom the variance of transitory income shocks is relatively greater should have a lower estimated marginal propensity to consume out of total income. As confirmation, he presented evidence that farmers and entrepreneurs had a lower MPC than other households.

A closely related implication of the model is that groups of consumers with a greater variance of transitory shocks should on average exhibit a greater divergence between consumption and income. The simple Keynesian model with α_0 close to zero and α_1 near one has no such implication. To see this, nest the two models by defining consumption as $C = P + \theta v + u$, where θ is the MPC out of transitory income v. If $\theta = 0$, this corresponds to the permanent income model, and if $\theta = 1$, it corresponds to the Keynesian model with $\alpha_0 = 0$ and $\alpha_1 = 1$. Now we have

$$
\begin{aligned}
\operatorname{var}\left(\frac{C}{Y}\right) &= \operatorname{var}\frac{P + \theta v + u}{P + v} \\
&= \operatorname{var}\left(\frac{1 + \theta v/P + u/P}{1 + v/P}\right) \\
&\approx \operatorname{var}\left(\left(1 + \theta\frac{v}{P} + \frac{u}{P}\right)\left(1 - \frac{v}{P}\right)\right) \\
&= \operatorname{var}\left(1 + \theta\frac{v}{P} + \frac{u}{P} - \frac{v}{P} - \theta\left(\frac{v}{P}\right)^2 - \left(\frac{v}{P}\right)\left(\frac{u}{P}\right)\right) \\
&\approx \operatorname{var}\left((\theta - 1)\left(\frac{v}{P}\right) + \left(\frac{u}{P}\right)\right),
\end{aligned}
$$

where the first approximation holds when v/P is not too far from zero, and the second approximation holds if $(v/P)^2$ and $(v/P)(u/P)$ are both close to zero, which will be true for plausible estimates of the magnitude of transitory shocks to income and consumption.[29]

29. The standard deviation of logarithmic transitory shocks to income, which can be identified here with e/p, is estimated in Carroll [1992] to be 10 percent annually. For a one standard-deviation shock, the error in the first approximation is given by comparing $1/1.1 \approx 0.91$ to 0.9. For the second approximation, if the standard deviation of consumption shocks is roughly the same size, both $(e/p)^2$ and $(e/p)(u/p)$ should be small enough to safely ignore.

TABLE IV
INCOME UNCERTAINTY AND THE CONSUMPTION/INCOME DIVERGENCE

Occupation of consumer	Variance of log transitory income	Variance of the C/Y ratio
Farmers and farm managers	0.129	0.092
Self-employed	0.096	0.038
Craftsmen and kindred	0.055	0.019
Operatives and laborers	0.054	0.022
Service workers	0.038	0.020
Managers and administrators	0.031	0.016
Professional, technical, and kindred	0.030	0.023
Clerical and sales	0.029	0.018

Source. Variances of transitory income innovations from Carroll and Samwick [1995a] computed using data from the PSID from 1981 through 1987. Variances of the consumption/income ratio were computed using data from the 1961–1962 *Consumer Expenditure Survey.* The consumption measure is total household expenditure. The income measure is disposable household income.

Assuming that the covariance between (v/P) and (u/P) is zero, this becomes

$$\text{(11)} \qquad \begin{aligned} \operatorname{var}(C/Y) &\approx \operatorname{var}((\theta - 1)(v/P)) + \operatorname{var}(u/P) \\ &= (\theta - 1)^2 \operatorname{var}(v/P) + \operatorname{var}(u/P). \end{aligned}$$

Thus, the Keynesian model ($\theta = 1$) implies that the variance of the consumption/income ratio is unrelated to the variance of transitory shocks to income, while the PIH model ($\theta = 0$) implies that the variance of the consumption/income ratio moves one-for-one with the variance of transitory shock to income.[30]

Table IV presents estimates of the variance of the consumption/income ratio by occupation group calculated from the 1960–1961 *Consumer Expenditure Survey,* along with estimates of the variance of transitory shocks to labor income taken from Carroll and Samwick [1995a].[31] It is clear that there is a strong positive

30. Although the simple Permanent Income model discussed here is not explicitly a finite-horizon model, similar results would apply for the standard finite-horizon model described in subsection II.C.

31. The data in both data sets are for consumers between the ages of 25 and 50′; that is, consumers in the age range where I argue that the infinite-horizon version of the buffer-stock model is a good approximation to the behavior of finite-horizon consumers.

Carroll and Samwick [1995a] used essentially the same technique to decompose shocks to income into transitory and permanent components that Carroll [1992] and Hall and Mishkin [1982] used. This technique is unable to distinguish transitory shocks to income from white noise measurement error in income. However, the point of the table is not related to the level of the transitory variance, but rather to the differences in estimated transitory income across groups. If the association between the two variances across occupation groups. A linear regression using the data in this table yields

$$\underset{}{\mathrm{var}(C/Y)} = \underset{(0.128)}{0.651}\ \mathrm{var}(v/P) - \underset{(0.0059)}{0.0072},$$

which implies a value for θ, the MPC out of transitory income, of about 0.2. This estimate is statistically significantly different from both zero and one at the 5 percent level of significance.

These results constitute stronger evidence against the Keynesian model than against the standard LC/PIH model because the usual errors-in-variables logic shows that any measurement error in the estimate of the variance of transitory income by occupation group would bias the regression coefficient on the transitory variance toward zero, the value implied by the Keynesian model. The coefficient is very significantly different from zero despite this bias toward zero, so the rejection of the Keynesian model is even stronger than implied by the simple t-statistic.

The same bias, however, weakens the case the regression makes against the standard LC/PIH model. It is conceivable that errors-in-variables bias is the only reason the MPC out of transitory income was estimated to be 0.2 rather than zero. Of course, this regression is not the only evidence that the MPC out of transitory income is too large to be consistent with the standard LC/PIH model. Early tests of the permanent income hypothesis found that the marginal propensity to consume out of unexpected, nonrecurring windfall payments was somewhere between 0.15 and 0.5 (see, e.g., Bodkin [1959], Kreinin [1961], and the discussion by Mayer [1972]).[32] Similar results have been found by many subsequent authors proceeding through Hall and Mishkin [1982] to very recent work by Souleles [1995].[33]

Table I showed that the average MPC out of transitory income is 15 percent or greater in the buffer-stock model for all

variance of measurement error is the same across occupation groups, the coefficient on the regression (and the implied estimate of the MPC out of transitory income) is unbiased.

32. Two natural experiments were examined in these papers: in the United States, the response of consumption to the "National Service Life Insurance Dividend of 1950," a special payment to World War II veterans, and in Israel, reparations payments to victims of German persecution during World War II. The authors argue that both were unanticipated and transitory shocks to income.

33. Although they interpret their results as suggesting that 20 percent of consumption is done by consumers who set consumption equal to income, an alternative interpretation is that all consumers have a marginal propensity to consume out of transitory income of 20 percent.

TABLE V
RATIO OF MEDIAN FINANCIAL ASSETS TO MEDIAN INCOME, BY AGE 1963, 1983, AND 1989 SURVEYS OF CONSUMER FINANCE

Age category	1963 Survey Median income	1963 Survey Financial asset ratio	1983 Survey Median income	1983 Survey Financial asset ratio	1989 Survey Median income	1989 Survey Financial asset ratio
25–34	23,285	0.02	25,366	0.05	24,000	0.06
35–44	24,999	0.06	34,285	0.09	35,000	0.11
45–54	27,980	0.11	32,849	0.12	35,000	0.17
55–64	14,919	0.26	27,674	0.31	26,000	0.32

Median income expressed in 1989 dollars, deflated using the PCE deflator. Financial assets are cash, checking and savings accounts, bonds, stock, mutual fund holdings, and trust accounts.

combinations of parameter values considered in the table. Similar MPCs arise over most of the working lifetime in the finite-horizon version of the buffer-stock model; results are not reported to conserve space. Of course, other parameter values could generate either larger or smaller MPCs. The essential point is that the buffer-stock model has no difficulty generating an MPC large enough to match even the larger empirical estimates, while, for plausible parameter values, the standard LC/PIH model is simply incapable of implying large values for the MPC out of transitory income.

It is again worth noting that Friedman would not have found these results inconsistent with his understanding of the Permanent Income Hypothesis. Friedman [1963] states that the Permanent Income Hypothesis implies a marginal propensity to consume out of purely transitory income of about 0.3.

IV.C. The Behavior of Wealth over the Lifetime

Diamond and Hausman [1984] called attention to the fact that the median household typically holds surprisingly small, but still positive, amounts of financial wealth over the entire working lifetime. Table V illustrates this point using data from the 1963, 1983, and 1989 *Surveys of Consumer Finances.* In all three surveys, at all ages before retirement, the ratio of median financial assets to median annual income is between 2 and 35 percent. In all three surveys the ratio rises modestly from ages 25 to 55, and then rises sharply in the last decade before the typical retirement age of 65.

The fact that the median ratio of wealth to income stays

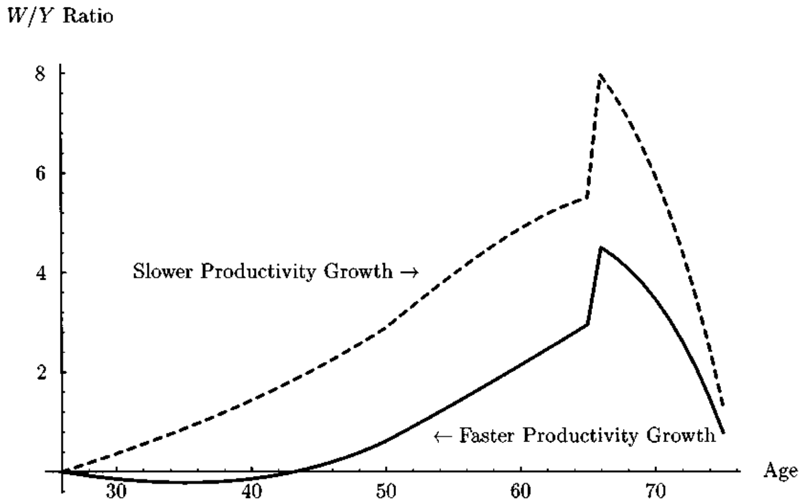

FIGURE VI
Effect of the Productivity Slowdown on the Predicted Age Profile of Wealth in the Standard LC/PIH Model

within a rather narrow range until just before retirement is not necessarily inconsistent with the standard LC/PIH model. Indeed, any particular pattern of wealth accumulation over the lifetime can be justified by some set of assumptions about tastes and the pattern of income over the lifetime. The stability of the Diamond-Hausman phenomenon between the early 1960s and the late 1980s, however, is troubling for the standard LC/PIH model because at some point between these two surveys there was a sharp slowdown in productivity growth and expected future productivity growth [Viard 1993].

To explore the implications of the standard and buffer-stock versions of the LC/PIH model for the age profile of the wealth ratio, I solved both models for the age-income profile labeled "Operatives" in Figure V, which was calibrated using the 1960s data. Under baseline parameter values, the standard LC/PIH model implies large negative wealth holding over most of the life cycle. However, there are obviously other parametric assumptions under which the model implies positive wealth. I experimented with the assumed interest rate and found that an interest rate of 8 percent per year produced the age/wealth profile that most closely resembled the pattern in Table V. The result is the age-wealth profile labeled "Faster Productivity Growth" in Figure VI.

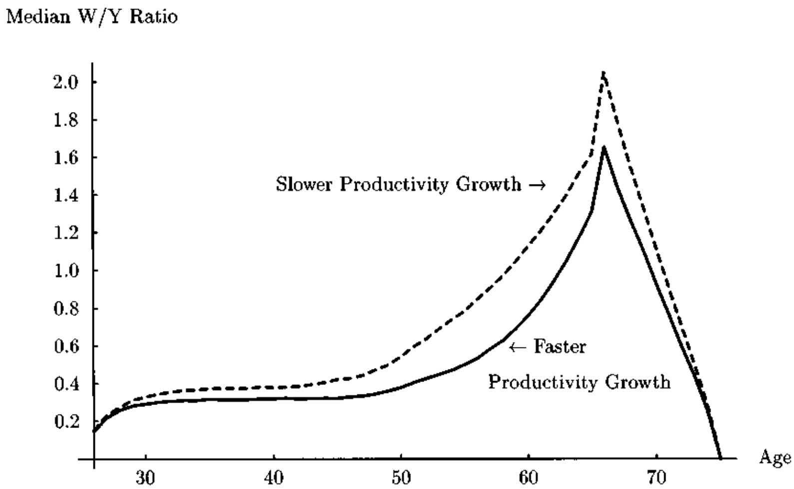

FIGURE VII
Effect of the Productivity Slowdown on the Predicted Age Profile of Wealth in the Buffer-Stock Version of the LC/PIH Model

I then solved the model again assuming that the post-1973 productivity growth slowdown resulted in a 1 percent slower growth rate of labor income over the working lifetime.[34] The resulting age/wealth profile is labeled "Slower Productivity Growth." The difference between the two curves is a rough measure of how the productivity slowdown should have affected the age/wealth profile for U. S. consumers if the standard LC/PIH model were correct. The figure shows that lower growth should have induced an enormous increase in household wealth at all ages greater than 30. On average, the "Slower" curve is higher than the "Faster" curve by an amount equal to roughly two years' worth of income. Comparison to Table V indicates that the model's predicted enormous increase in household wealth/income ratios is more than an order of magnitude greater than the actual increase in wealth/income ratios.

The results for the same experiment with the buffer-stock model are shown in Figure VII. The profile of the wealth ratio over the lifetime in this figure bears a strong resemblance to the profiles shown in Table V. Very early in life the wealth ratio is low. For the middle two decades it grows slightly, and then in the

34. See Carroll and Summers [1991] for some evidence that this is a good approximation of reality.

last decade before retirement the wealth ratio grows sharply. The ratio is uniformly a bit higher in the economy with slow productivity growth, but the difference between the two wealth profiles is not remotely so dramatic as in the standard model. The implication of a slightly higher wealth ratio in the slow-growth equilibrium is consistent with the pattern in Table V: for every age category the wealth ratios are a bit higher for the 1983 and 1989 surveys than for the 1963 survey.

Of course, the correspondence between the figure and the tables is not perfect. Under the chosen parameter values, the buffer-stock model implies a considerably larger buffer-stock than is evident in Table V. The model's predictions could be brought more in line with the results in Table V by assuming that consumers are more impatient, or less risk averse, that they face less uncertainty, or that they face a faster growth rate of income.

One objection to this line of argument might be that comparison to Table V is not appropriate, because the standard LC/PIH model's implications are about total net worth, not about financial assets. However, similar calculations comparing the ratio of net worth to income in the 1963, 1983, and 1989 survey produce similar conclusions: wealth-to-income ratios increased only slightly from the 1960s to the 1980s, rather than by the enormous amounts the standard model would imply.[35]

One feature of the model that appears to be strongly at variance with available evidence is its implication that wealth falls sharply after retirement, reaching zero in the year of death. This implication is a result of the assumptions that the date of death is known with certainty, that there is no bequest motive, and that forms of uncertainty other than labor income uncertainty (such as uncertain medical expenses) do not intervene to boost the saving rate as consumers age.[36] Because it does not address these issues, the buffer-stock model is probably less useful in understanding the behavior of the elderly than it is for describing the working population, particularly the working population well before retirement age.

35. It would not be appropriate to include, say, expected future pension benefits in the measure of wealth. As shown in Table II, in this model there is no single interest rate at which it is appropriate to discount uncertain expected future income.

36. The certain date of death accounts for the upward fillip in consumption in the last two or three years of life. As the certain end of life approaches, uncertainty about future income approaches zero, so remaining precautionary assets are spent.

Less formal evidence about the reasons for holding wealth may also be illuminating. As noted in the introduction, many more people cite emergencies than retirement as the most important reason for saving. Another informal source of evidence about how people think about uncertainty and savings is provided by personal financial planning guides. These guides commonly have passages that suggest that consumers should maintain a buffer stock of assets against uncertainty. The following is a typical passage: "It is generally held that your liquid assets should roughly equal four to six months' employment income. If you are in an unstable employment situation . . . the amount should probably be greater" [Touche Ross, 1989, p. 10].

As both this quotation and intuition suggest, one of the implications of a buffer-stock model is that consumers with higher income uncertainty should hold more wealth. Several recent papers have found empirical evidence that precautionary saving is statistically significant and economically important. Using wealth and income uncertainty data from the PSID, Carroll and Samwick [1995a, 1995b] find that wealth is substantially higher for consumers who face greater income uncertainty. Carroll [1994] provides some evidence that consumers with more variable incomes save more. Kazarosian [1990] finds, in a regression of wealth on demographic characteristics and income variability, that the degree of income variability is overwhelmingly significant.

One further category of evidence on household wealth accumulation patterns supports the buffer-stock interpretation of the LC/PIH model. This is observation by Avery and Kennickell [1989] that wealth holdings are extremely volatile, even over short periods. They first attempt to explain the changes in consumers' wealth between 1983 and 1986 with a statistical life cycle model, but the model performs poorly, explaining at most about 8 percent of the observed changes in wealth. The reason for the failure is that in the standard life cycle model wealth changes glacially, gradually accumulating or decumulating depending on life cycle stage, while in the SCF data wealth appears to fluctuate vigorously, just as would be expected in a model where the primary purpose of holding wealth is as a buffer against random shocks to income.

Even if the buffer-stock model provides a good description of the patterns of financial asset accumulation of the typical household, it is nevertheless clear that it is not a complete model of household wealth accumulation. For the typical household, housing equity constitutes a larger fraction of total net worth than do financial assets, and the buffer-stock model, with its single perfectly liquid, perfectly riskless asset, is a poor vehicle for understanding housing investments. One interpretation of the strong correspondence between the buffer-stock model's implications and observed patterns of financial asset holding and consumption behavior is that consumers separate the housing decision from other consumption-saving decisions. Once they have bought a house and committed themselves to a fixed monthly mortgage payment, they may subject the remaining "disposable" income stream to buffer-stock saving rules.

Finally, even as a model of financial asset holding, the buffer-stock model cannot be considered a complete description of the behavior of all households. This is illustrated by two related facts. First, if all consumers behaved according to a buffer-stock model under the baseline parameter values assumed here, the aggregate capital-income ratio would be far smaller than we observe it to be. Second, the distribution of financial asset holdings is far more concentrated than the model implies. For example, in 1983 the richest 1 percent of households in the United States held 64 percent of total financial assets held directly by the household sector. These people are clearly not buffer-stock savers, but it seems unlikely that they are life cycle savers either. To be complete, any description of the determinants of aggregate wealth must capture the behavior of these consumers.

V. Literature Survey

In the last few years a substantial literature has appeared examining the implications of models similar to the one in this paper. This section provides a brief discussion of some of the recent literature.

Two papers by Hubbard, Skinner, and Zeldes [1994, 1995] (hereinafter HSZ) examine the theoretical properties of a generalized version of the model in this paper. The principal generalizations are that they incorporate health risk and mortality risk, and they carefully model the U. S. social insurance system. They calibrate medical expense risk and mortality risk using empirical data, but find that neither health risk nor mortality risk has much effect on behavior, given the presence of labor income risk. Their empirical estimates of labor income risk are similar to those in this paper, and when my model is calibrated using the HSZ parametric assumptions, the two models generate similar predictions about lifetime age/wealth and age/consumption profiles.

The most important difference in parametric assumptions is that HSZ assume that consumers face substantially slower income growth. This is largely because their age/income profiles are estimated in a way that removes any aggregate productivity growth from their estimated household income process. Carroll and Summers [1991] provide a variety of evidence, however, that in the medium and long run, household income shares in aggregate productivity growth.

One advantage of the HSZ parameterization is that it causes the model to generate substantially larger estimates of aggregate wealth than it generates under my parameterization (henceforth, the buffer-stock parameterization). However, the implied *distribution* of wealth across households under HSZ parameter values differs greatly from the actual empirical distribution. In particular, the model under the HSZ parameterization substantially overpredicts the wealth of the median household over most of the lifetime, but greatly underpredicts the wealth of the richest households. While the buffer-stock parameterization also greatly underpredicts the wealth of the richest households, it appears to match median age-wealth profiles better than the HSZ parameterization. A related point is that the HSZ parameterization generates considerably less tracking of consumption to income over the lifetime than does the buffer-stock parameterization.

A very recent paper by Gourinchas and Parker [1995] uses synthetic cohort data from the U. S. *Consumer Expenditure Surveys* in a way that lets the available consumption and income data determine the period over which consumers engage in buffer-stock saving behavior. They find that consumers typically make the transition between buffer-stock saving and life cycle saving somewhere around age 45, at the low end of the range originally proposed in this paper. They also used a model essentially identical to the one in this paper to estimate time preference rate and risk aversion parameters by occupation and education group. They find substantial, and intuitive, differentials in time preference rates across groups.

Other evidence that supports the buffer-stock parameterization is provided in Carroll and Samwick [1995a]. We calculate the predictions of the model for the relationship between income uncertainty and wealth holding under the buffer-stock parameter values and the HSZ parameter values, and find that under the HSZ parameter values the model implies that wealth is roughly an order of magnitude more responsive to uncertainty in permanent income than under the buffer-stock parameter values. We then estimate the empirical relationship between saving and uncertainty, and find coefficients similar to those implied by the buffer-stock parameterization, and highly statistically different from those implied by the HSZ parameterization. Another recent paper by Carroll and Samwick [1995b] uses the buffer-stock model in combination with empirical methods to estimate that between one-third and one-half of the wealth of the typical working household is attributable to buffer-stock saving behavior.

Several other recent papers develop further implications of the infinite-horizon version of the model. Heaton and Lucas [1994] examine implications of the model for portfolio choice. Ludvigson [1996] develops a buffer-stock model with liquidity constraints that vary with the consumer's level of income, and uses the extended model to analyze the relationship between household balance sheet positions and aggregate consumption growth in the United States. Bird and Hagstrom [1996] use data from the *Survey of Income and Program Participation* (SIPP) to test the model's implication that more generous social insurance benefits reduce the target wealth stock. Gross [1995] adapts the model to study the investment decisions of liquidity-constrained firms.

VI. Conclusion

The standard version of the LC/PIH model remains the most commonly used framework for both micro and macro analysis of consumption behavior despite a large and growing body of evidence that it does a poor job explaining those data. This paper argues that a version of the LC/PIH model in which buffer-stock saving emerges is closer both to the behavior of the typical household and to Friedman's original conception of the Permanent Income Hypothesis model. The buffer-stock version of the model can explain why consumption tracks income closely when aggregated by groups or in whole economies, but is often sharply different from income at the level of individual households. Without imposing liquidity constraints, the model is consistent with recurring estimates of a much higher MPC out of transitory income than is implied by the standard LC/PIH model. And it provides an explanation for why median household wealth/income ratios are persistently small and have remained roughly stable despite a sharp slowdown in expected income growth. Insights from analysis of the buffer-stock version of the model may also help to explain many of the failures and anomalies of Euler equation models estimated using both household and aggregate data.

The buffer-stock model does not, of course, explain all behavior of all consumers. The *Surveys of Consumer Finances* show that a small number of wealthy consumers hold enormous financial assets. A buffer-stock model is clearly not a plausible description of the behavior of these people. Many other consumers explicitly engage in some form of life cycle saving behavior, particularly in the form of participation in pension plans. The model may also not be useful for understanding housing investments. Probably the appropriate place for the buffer-stock model is as an explanation of truly discretionary "high frequency" saving decisions of the median consumer. It seems plausible that many consumers ensure that retirement is taken care of by joining a pension plan, buy a house, and then subject the post-pension-plan, post-mortgage-payment income and consumption streams to buffer-stock saving rules.

The buffer-stock version of the LC/PIH model provides a new way of looking at both microeconomic and macroeconomic data on saving and consumption, and has many testable implications that differ from those of the standard LC/PIH model that has dominated empirical and theoretical work until recently. It promises to provide a fruitful framework for future work.

Appendix 1: Details of the Methods of Solution

The dynamic stochastic optimization problem solved in this paper is characterized by a fundamental equation (equation (4) in the text) of the form,

$$1 = R\beta E_t\left[\{c_{t+1}[R[x_t - c_t]/GN_{t+1}V_{t+1}]GN_{t+1}/c_t\}^{-\rho}\right],$$

or, returning to marginal utility notation and multiplying both sides by $u'(c_t)$,

$$u'(c_t) = R\beta E_t u'(c_{t+1}(R[x_t - c_t]/GN_{t+1}V_{t+1})GN_{t+1}).$$

As in Deaton [1991], the method of solution for the finite-horizon version of the model is to recursively solve backwards from the last period of life, in which the optimal plan is to consume all assets, $c_T(x_T) = x_T$. Given a value of x_{T-1}, and a method for computing the expectation (see the next paragraph) on the right-hand side of (A1), numerical algorithms can locate the c_{T-1} which satisfies (A1). In period $T - 1$, equation (A1) was solved numerically for the optimal value of consumption for a grid of m values for x, values x_i. The numerical approximation to the optimal consumption rule $c_{T-1}(x_{T-1})$ was then constructed by cubic interpolation (after experimenting with both linear and quadratic interpolation) between the values of the function at the m grid points. Given $c_{T-1}(x_{T-1})$, a grid of x values for period $T - 2$ was chosen, the numerical solution at each x_i was computed from equation (A1), and $c_{T-2}[x]$ was given by cubic interpolation, and so on.

As described in the text, I assumed that, with some probability p, income would be zero in period $t + 1$, $V_{t+1} = 0$. If income is not zero, then V_{t+1} and N_{t+1} are distributed lognormally with expected values $(1 - p)$ and 1, respectively. In solving the model, the lognormal distributions were truncated at three standard deviations from the mean, yielding minimum and maximum values $\underline{V}$, $\underline{N}$, $\overline{V}$, and $\overline{N}$. Full numerical integration is extremely slow, so the lognormal distributions were approximated by a ten-point discrete probability distribution. The distance $(\overline{V} - \underline{V})$ was divided into ten equal regions of size $(\overline{V} - \underline{V})/10$ with boundaries denoted B_j. Associated with each of these regions was the average value of V within the region, computed by calculating the numerical integral $\hat{V}_j = \int_{B_j}^{B_{j+1}} V\,dF(V)$. The probability of drawing a shock of value $\hat{V}_j$ is given by $F(B_{j+1}) - F(B_j)$. An analogous procedure was used to approximate the distribution of permanent shocks. This method is similar to the methods used by Deaton [1991], Hubbard, Skinner, and Zeldes [1995], and others working in this literature.

In solving the infinite horizon problem, a convergence criterion is needed to determine when successive $c(x)$ functions are sufficiently close that the consumption rules may be said to have converged. The criterion used was

$$\frac{1}{m}\sum |(c_t)(x_i) - c_{t+1}(x_i)| < 0.0005,$$

where i indexes the elements of the grid of m x's. I found that, for most purposes, a grid of twelve to fifteen values for the x's produced results indistinguishable from much finer grids, so most simulations reported in the paper were calculated using a twelve-point grid. Similarly, I found that the results do not change perceptibly even if the convergence criterion is tightened substantially relative to the 0.0005 criterion used for most reported results.

Appendix 2: The Numerical Method for Calculating the Ergodic Distribution of Net Worth

Standard methods for proving ergodicity do not appear to be applicable to the distribution of net worth in this problem, primarily because the state space is, in principle, unbounded from above. This problem can be solved by mapping the infinite range of x into a finite interval z using the mapping $z = x/(1 + x)$. It is then possible, using the numerical consumption rule and the assumptions on the distributions of shocks, to construct a discretized approximation to the transition matrix for z. The question of whether there is an ergodic distribution for z can then be answered numerically: if one of the eigenvalues of the transition matrix is 1, then an ergodic distribution for z exists, and will be given by the eigenvector associated with the eigenvalue of one.[37] With the ergodic distribution for z in hand, it is a simple matter to "unmap" it to obtain the corresponding steady-state distribution for x.

Appendix 3: The Implied Discount Rate for Future Income

This appendix describes how the implied discount rate for future income presented in Table II is calculated. In the infinite-horizon, perfect certainty version of the CRRA LC/PIH model, if income P is expected to grow according to $P_{t+1} = G\,P_t$ forever, and $G < R$, consumption in period t is given by

$$c_t = [1 - R^{-1}(R\beta)^{1/\rho}](W_t + H_t),$$

where (if $G < R$) human wealth $H_t = P_t/(1 - G/R)$ and where all other variables are as defined in the description of the basic

37. I am grateful to Angus Deaton for suggesting this method of finding the steady state numerically.

model. Consider changing the expected growth of income from $G_1 = 1 + g_1$ to $G_2 = 1 + g_2$. The change in consumption that results from this change in expected income growth is given by

$$\Delta c = [1 - R^{-1}(R\beta)^{1/\rho}]\left[\frac{P_t}{1 - G_2/R} - \frac{P_t}{1 - G_1/R}\right].$$

For a given value of Δc, for a particular set of taste parameter values, and for particular values of g_1 and g_2, one can search for the value of R, and therefore $r = R - 1$, which solves this equation. If such a value exists, it will correspond to the interest rate at which a consumer with certain future income would have to discount future income and consumption in order to justify the chosen Δc. Table II takes the values of Δc calculated from the buffer-stock model at various levels of the gross wealth ratio and presents the value of r that solves equation (A2).

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