MANAGEMENT SCIENCE
Vol. 35, No. 3, March 1989
Printed in U.S.A.

DISCOUNT RATES INFERRED FROM DECISIONS: AN EXPERIMENTAL STUDY*

URI BENZION, AMNON RAPOPORT AND JOSEPH YAGIL
Technion—Israel Institute of Technology, Haifa, Israel
Baruch College, City University of New York, New York, New York 10010
University of Haifa, Haifa, Israel
Department of Psychology, University of North Carolina, Chapel Hill, North Carolina 27514
University of Haifa, Haifa, Israel
New York University, New York, New York 10006

Two hundred and four students of economics and finance participated in an intertemporal choice experiment which manipulated three dimensions in a 4 × 4 × 4 factorial design: scenario (postponing a receipt, postponing a payment, expediting a receipt, expediting a payment), time delay (0.5, 1, 2, and 4 years), and size of cashflow ($40, $200, $1000, and $5000). Individual discount rates were inferred from the responses, and then used to test competitively four hypotheses regarding the behavior of discount rates. The classical hypothesis asserting that the discount rate is uniform across scenarios, time delays, and sums of cashflow was flatly rejected. A market segmentation approach was found lacking. The results support an implicit risk hypothesis according to which delayed consequences are associated with an implicit risk value, and an added compensation hypothesis which asserts that individuals require compensation for a change in their financial position.
(INTERTEMPORAL DECISIONS; TIME PREFERENCE; DISCOUNT RATES)

1. Introduction

Many decisions involve outcomes that are temporally remote, forcing the decision maker to make value comparisons between immediate and delayed consequences. Formal analyses of intertemporal decisions of this kind have typically relied on the concept of discounting. The discounting model was first proposed as a descriptive theory of intertemporal choice by Fisher (1930) and later derived axiomatically by Koopmans (1960), Lancaster (1963), Fishburn (1970), Fishburn and Rubinstein (1982), and others. A general assumption that underlies most of the models is that realization of a desirable outcome is preferred sooner to later, whereas realization of an undesirable outcome is preferred later to sooner. Another assumption generally made is that discount rates are constant in time. For example, the stationary axiom in the axiomatization of Fishburn and Rubinstein (1982) states that indifference between two time-dependent outcomes depends only on the difference between the times. If the two times are deferred or advanced by the same time period, the indifference between the outcomes is assumed to be preserved.

The discount rate has become one of the central concepts of finance. Some of its manifestations include familiar concepts such as opportunity cost, capital cost, borrowing rate, lending rate, and the rate of return on stocks or bonds. Due to its importance, the concept of discount rate has been examined intensively in the economic literature. However, little is presently known about how individuals actually discount future gains or losses and, in doing so, whether they conform to the implications derived from most theories of intertemporal choice.

Several exceptions to the statement made above should be noted. There have been several empirical attempts to estimate discount rates from individual saving behavior. However, as noted by Loewenstein (1987), such attempts are problematic. Because saving

* Accepted by Robert L. Winkler, former Departmental Editor; received November 30, 1987. This paper has been with the authors 3 months for 1 revision.

0025-1909/89/3503/0270$01.25

behavior depends on many factors other than discount rates, it is exceedingly difficult to isolate the sheer effect of time preference from the effects of other variables such as expected future income streams, projected needs, etc.

Because of the difficulties involved in estimating discount rates from saving behavior, behavioral economists and psychologists interested in intertemporal choice have turned from field studies to controlled laboratory experiments. Inspired by earlier work of Mischel and his coworkers on delay of gratification (Mischel 1961; Mischel and Metzner 1962; Mischel and Grusec 1967), Yates and Watts (1975) reported results supporting the position that when money outcomes are perceived as aversive, in a substantial amount of time individuals prefer deferred outcomes to more immediate ones. In an important contribution, Loewenstein (1987) asked subjects to assess the "most you would pay now" to obtain (avoid) both monetary and nonmonetary outcomes either immediately or following several time delays. He has argued convincingly for a model recognizing that anticipation of nonmonetary outcomes, like consumption itself, is a source of utility. In a series of four experiments, Stevenson (1986) reported consistent support for a ratio discounting function of time. Finally, Thaler (1981) tested and confirmed three hypotheses concerning the behavior of discount rates inferred from riskless choices. However, Thaler employed economically unsophisticated subjects with little or no knowledge of finance, allowed the subjects to check for consistency of their responses, and subjected his data to visual inspection rather than rigorous statistical analyses.

While attempting to overcome these methodological shortcomings, the present study examines how financially sophisticated students of economics and finance make intertemporal choices, and the extent to which the discount rates inferred from their choices depend on the time delay, size of cashflow, and sign of cashflow. §2 outlines the classical approach and several alternative hypotheses concerning the behavior of the discount rate, §3 describes the experimental design, §4 presents the experimental results, and §5 discusses their implications.

2. Theory and Hypotheses

A discount rate in economics is related to the marginal rate of substitution between current and future consumption; it is the ratio of the marginal utility of one unit of consumption in the future to that in the present. Thus, the discount rate may depend on factors which determine the marginal utility including time preferences. However, it is well known that with the existence of perfect capital markets where individuals can borrow and lend freely, the individual will make his or her intertemporal choice such that at the margin the discount rate is equal to the relevant interest rate in the market. Thus, the discount rate in finance is simply a measure of the relevant interest rate, and is independent of the individual utility function.

Our study is concerned with the discount rate in the economic sense. However, since our study involves monetary terms, it is plausible that an individual considers an option of borrowing or lending in response to a change in the amount of funds available to him or her in a given time period. Whereas in some parts of the paper we use the financial market interpretation to discuss the discount factors, this interpretation is not crucial to our analysis. Also, we cannot determine whether our subjects actually considered the existence of financial markets in determining their implicit rate of substitution.[1]

To infer discount rates from intertemporal decisions, we use the relationship between the present value of a cashflow X, denoted by P, and its future value, denoted by F. Formally,

$$F = P(1 + R)^t, \qquad (1)$$

[1] We discuss this issue further in §5 below.

where R is the discount rate and t is the time to be waited. In our experiment, either t and P are given and the individual has to specify F, or t and F are given and the individual has to specify P. The inferred discount rate is obtained from

$$R = (F/P)^{1/t} - 1. \tag{2}$$

Customarily, t is measured in years; therefore, R is the annual discount rate.

In correspondence with the experimental design to be described below, we make two fundamental distinctions concerning the discount rate. The first is between converting future cashflow for present cashflow and converting present cashflow for future cashflow. We shall denote the discount rates at which these conversions are made by R_P and R_F for the present-to-future and future-to-present conversions, respectively. The second distinction is between increasing and decreasing the immediate financial resources available to the individual. When her level of financial resources is reduced, the individual will reduce her consumption or increase her borrowing. On the other hand, when her level of financial resources is increased, she can increase her consumption and use extra money for saving. The discount rates in these two situations will be referred to here as rates of borrowing, R^B, and lending, R^L, respectively.[2]

Using the distinctions above, the discount rate can be considered a function of four variables: the magnitude of the cashflow, X, the length of time to be waited, t, the direction of the discount (present-to-future vs. future-to-present), and the direction of the change in the current cashflow.

Theoretical considerations or experimental evidence regarding intertemporal decisions give rise to four hypotheses concerning the *form* of the discount function. Three of these hypotheses summarize known approaches in corporate finance whereas the fourth is a new approach based on psychological and economic considerations.

2.1. *The Classical Approach*

This approach states that there is only one "market discount rate," which has the following two characteristics: (i) R is the same for all individuals and all situations; (ii) R is equal to the market riskless discount rate.

Formally, the classical approach asserts that $dR/dX = dR/dt = 0$, $R_F = R_P$, and $R^B = R^L$. These equations mean that the discount rates for borrowing and lending are identical, and that the relationship between P and F is (time) symmetric. Moreover, the magnitude of the gain or loss has no effect on the discount rate, and no individual differences are expected.

2.2. *The Market Segmentation Approach*

To evaluate which discount rate is used by an individual for a given sum of cashflow and a given point in time, a starting equilibrium position is assumed whereby the individual has some cashflow to receive or pay during the period. The change in the equilibrium position will be discounted by a rate which depends on the type of position change (i.e., delaying a receipt, delaying a payment, expediting a receipt, and expediting a payment). Denote the discount rates for these four position changes (scenarios) by $R(\mathrm{A})$, $R(\mathrm{B})$, $R(\mathrm{C})$, and $R(\mathrm{D})$, respectively. Then, according to the market segmentation approach,

$$R(\mathrm{A}) \neq R(\mathrm{B}) \neq R(\mathrm{C}) \neq R(\mathrm{D}). \tag{3}$$

However, for each type of position change, the discount rate is assumed to be independent of the time delay and magnitude of cashflow.

[2] It is usually observed in financial markets that $R^B > R^L$. This observation is consistent with a concave utility function having decreasing marginal utility of consumption even in the absence of financial markets.

Two economic arguments can be evoked in support of the market segmentation hypothesis. First, with the existence of financial markets we observe discrepancies between borrowing and lending rates and the absence of arbitrage between borrowers and lenders. Second, with a nonlinear (concave) utility function, we expect different responses in different scenarios, particularly between scenarios A and D (borrowing) and scenarios B and C (lending).[3]

2.3. *The Implicit Risk Approach*

According to the implicit risk approach (e.g., Mischel and Grusec 1967; Stevenson 1986), delayed consequences are associated with an implicit risk value. Delayed negative consequences are preferred and delayed positive consequences are avoided because both are less certain. The implicit risk approach assumes that the individual believes that there is a positive probability that delayed receipts will not be paid and delayed payments will not be collected. Even when the intertemporal decision problem is formulated in terms of certainty conditions, as in our experiment, the subjects "frame" it (Kahneman and Tversky 1979; Loewenstein 1986) as one involving some degree of risk.

Two extreme hypotheses concerning the effects of risk can be formulated within the implicit risk approach (e.g., Benzion and Yagil 1987; Robichek and Myers 1966). According to the *one-period-realization* (OPR) of risk hypothesis, risk depends on the time of the receipt or payment and not on the length of the time period. Consequently, there is a one-time discount rate factor for the implicit risk, denoted by d, in addition to the riskless discount rate, denoted by i. Formally, equation (1) assumes the following form:

$$F = P(1 + d)(1 + i)^t. \tag{4}$$

According to the *multiple-period-realization* (MPR) of risk hypothesis, risk increases proportionally in time, and equation (1) takes the form

$$F = P[(1 + d)(1 + i)]^t = P(1 + d)^t(1 + i)^t. \tag{5}$$

The difference between equations (4) and (5) is represented by the power of the risk factor $(1 + d)$: 1 and t for the OPR and MPR hypotheses, respectively.

Alternatively, the OPR and MPR hypotheses can be stated in terms of the (short-term) forward rates, r_t:

$$F = P(1 + R_1) \prod_{t=1}^{T} (1 + r_t), \tag{6}$$

where R_1 is the first year discount rate.

According to the OPR hypothesis,

$$R_1 = (1 + d)(1 + i) - 1 \qquad \text{and} \qquad r_t = i \qquad \text{for} \qquad t = 2, 3, \ldots, T.$$

The forward rates are derived in a straightforward manner from the relationship between long-term and short-term rates, implying that

$$r_t = [(1 + R_t)^t/(1 + R_{t-1})^{t-1}] - 1 \tag{7}$$

where R and r denote spot rate and forward rate, respectively.

The time patterns of the forward rate implied by the OPR and MPR hypotheses are different. The former hypothesis implies that risk will be reflected only in the first period discount rate. Subsequent discount rates will only reflect time value of money. In contrast,

[3] This hypothesis is consistent with the model of Loewenstein (1986), which incorporates the notion of reference point of Kahneman and Tversky's Prospect Theory (1979) and modifies the classical approach by substituting a Prospect Theory style value function for the utility function.

the MPR hypothesis implies that each discount rate contains both time value of money and risk premium.

2.4. *The Added Compensation Approach*

Changes in financial position require readjustment. The added compensation approach states that an individual will require a premium for readjustment or a *position change*. Specifically, the individual would require a compensation for postponing a receipt but would be willing to offer a premium for postponing a debt. Conversely, he or she would be willing to receive less when a receipt is expedited, but would also pay less when a debt is expedited. Moreover, the premium required for postponing a receipt will exceed the one offered for postponing a payment, and the premium offered for expediting a receipt will be smaller than the one required for expediting a payment. Therefore, the added compensation approach posits that the premium would be positive for postponing a receipt or a payment (scenarios A and B) and negative for expediting a receipt or a payment (scenarios C and D). The premium can be of constant magnitude or a variable magnitude that depends on the size of the cashflow.[4]

Incorporating the added compensation component into the relationship between future and present value given by equation (1) yields

$$F = P(1 + R)^t + B, \tag{8}$$

where B represents the added compensation component, and is dependent on the specific scenario.[5]

3. The Experimental Design

Subjects

Two hundred and eighty two undergraduate and graduate students of the University of Haifa and the Technion-Israel Institute of Technology participated in the experiment. The subjects were students of economics or finance. Having completed at least two-year-long courses of economics or finance, the subjects were well familiar with the basic economic theory of discount rate and with the economic conditions prevailing in Israel, in particular with the rate of inflation, the interest rate paid by banks, and short-term as well as long-term forecasts regarding the future of the Israeli economy. Having completed their military service, almost all the subjects had extensive experience with money management in an unstable economy with a three-digit inflation rate. Participation in the experiment was on a voluntary basis. The experiment was conducted during regular class hours; subjects not wishing to participate were free to leave the classroom.

Procedure

The subjects were presented individually with a detailed questionnaire about economic decision making. They were instructed in writing that the purpose of the experiment was to gather information about their preference for giving or receiving an amount of money in the present instead of giving or receiving another amount in the future.[6] They were further instructed that the questions presented to them did not have correct answers. Rather, the answers might vary from one individual to another depending on his or her present or future financial assets.

[4] The idea of different premiums for different position changes seems consistent with the theoretical investigation by Thaler and Shefrin (1981) of intertemporal choice as a problem in the economic theory of self-control.

[5] A possible extension is to view the added compensation as a function of time and amount of money.

[6] The sums of money involved in the experiment are in U.S. dollars rather than Israeli shekels in order to control for the effects of inflation, which was inordinately high when the experiment was being conducted. This is why most transactions in the economy were denominated in U.S. dollars.

Four scenarios were described in the instructions. Scenario A (postpone a receipt) concerns a case of a person who has just earned $\$y$ for his or her work in a financially solid public institute. Upon coming to receive the payment, the person is told that the institute is temporarily short of funds. Instead, he or she is assured payment of another amount of $\$x$ t time periods from now.

Scenario B (postpone a payment) concerns the case of a person who must pay immediately a debt of $\$y$ to the same public institute. Being temporarily short of funds, the person proposes instead to pay the institute $\$x$ t time periods from now. The person is willing to furnish the appropriate securities, and the institute may or may not accept his or her offer. Given the values of $\$y$ and t, the subject was asked to specify $\$x$, so that he or she would be indifferent between paying $\$y$ now or paying $\$x$ (with appropriate securities) t time periods from now.

Scenario C (expedite a receipt) concerns the case of a public institute which must pay its employee $\$y$ t time periods from now. The institute offers to expedite the payment of its debt and pay its employee now. Given the values of $\$y$ and t, the subject was asked to state $\$x$, so that he or she would be indifferent between receiving $\$y$ t time periods from now or receiving $\$x$ immediately.

Scenario D (expedite a payment) concerns the case of a person who must pay a debt of $\$y$ to a public institute t time periods from now. The institute already possesses the appropriate securities. Wishing to settle his or her debt immediately, the person offers to pay $\$x$ now. The institute may or may not accept his or her offer. The subject was asked to state an amount of money $\$x$ so that he or she would be indifferent between paying $\$y$ t time periods from now or paying $\$x$ immediately.

Scenarios A, B, C, and D were devised to estimate the four discount rates $R(\mathrm{A})$, $R(\mathrm{B})$, $R(\mathrm{C})$, and $R(\mathrm{D})$, respectively. A 4×4 factorial design was employed for each scenario separately with $t = \frac{1}{2}$, 1, 2, and 4 years, and $y =$ \$40, \$200, \$1,000, and \$5,000. This design allows examination of the effects of the time period and the amount of money involved on the discount rate. Sixty-four questions (4 scenarios times 16 questions per scenario) were thus constructed.

Each subject was given a questionnaire that included the 64 experimental questions and 16 filler questions. The questions were arranged in 16 blocks of 5 questions each. Each block included one question from scenarios A, B, C, and D, and a filler question. The order of the five questions within block was randomized. Similarly, the 16 blocks were presented in a random order. The subjects were instructed to answer each question separately and independently of the other questions.

4. Results

Of the 282 subjects who participated in the experiment, 78 did not complete the questionnaire or skipped one or more questions. We omitted these subjects and conducted all the analyses on the data of the remaining 204 subjects. The responses of each subject were organized such that scenarios A (postpone a receipt), B (postpone a payment), C (expedite a receipt), and D (expedite a payment) corresponded to questions 1–16, 17–32, 33–48, and 49–64, respectively. Means and standard deviations of the subjects' responses are presented in the Appendix.

4.1. *Effects of Independent Variables on Discount Rates*

Equation (1) was used to infer for each subject separately 64 discount rates, one for each question. The means and standard deviations of the individual discount rates—$R(\mathrm{A})$, $R(\mathrm{B})$, $R(\mathrm{C})$, and $R(\mathrm{D})$ for scenarios A, B, C, and D, respectively—are presented in Table 1. Each statistic is based on 204 observations.

The mean discount rates in Table 1 seem quite high for the smaller sums. Only for

TABLE 1
Means and Standard Deviations of Inferred Discount Rates

	Scenario A t				Scenario B t			
Sum	0.5	1	2	4	0.5	1	2	4
40	0.598 (0.613)	0.393 (0.341)	0.263 (0.196)	0.219 (0.189)	0.334 (0.378)	0.219 (0.218)	0.193 (0.157)	0.141 (0.102)
200	0.428 (0.408)	0.255 (0.251)	0.230 (0.202)	0.195 (0.142)	0.260 (0.263)	0.167 (0.195)	0.158 (0.149)	0.128 (0.104)
1000	0.407 (0.462)	0.275 (0.246)	0.200 (0.176)	0.185 (0.151)	0.217 (0.242)	0.155 (0.193)	0.152 (0.200)	0.121 (0.116)
5000	0.184 (0.192)	0.162 (0.203)	0.151 (0.172)	0.116 (0.115)	0.153 (0.176)	0.105 (0.150)	0.088 (0.126)	0.075 (0.095)
	Scenario C t				Scenario D t			
Sum	0.5	1	2	4	0.5	1	2	4
40	0.379 (0.503)	0.244 (0.259)	0.189 (0.223)	0.137 (0.126)	0.535 (0.660)	0.330 (0.455)	0.265 (0.261)	0.206 (0.182)
200	0.288 (0.430)	0.174 (0.212)	0.134 (0.120)	0.131 (0.118)	0.321 (0.433)	0.236 (0.264)	0.210 (0.218)	0.157 (0.127)
1000	0.217 (0.278)	0.157 (0.190)	0.122 (0.151)	0.123 (0.123)	0.310 (0.385)	0.219 (0.241)	0.166 (0.170)	0.163 (0.190)
5000	0.171 (0.246)	0.139 (0.175)	0.105 (0.168)	0.100 (0.107)	0.261 (0.419)	0.192 (0.226)	0.149 (0.197)	0.136 (0.133)

the larger sum of $5000 are the results reasonable. The mean discount rate for this sum computed over the four time delays and four scenarios is 0.143. This result compares favorably with the annual interest rates in Israel on bank loans and savings linked to the U.S. dollar, which were approximately 15% and 10%, respectively, at the time the experiment was conducted.

Table 1 shows that, with only minor exceptions, the mean discount rates decrease monotonically both in t and in the size of cashflow. Two-way analyses of variance (ANOVA) conducted separately on each scenario resulted in highly significant effects due to time, sum, and the time by sum interaction in each of the four cases ($p < 0.001$). The interaction effect is depicted graphically in Figure 1. Figure 1 shows that the discount rate changes inversely to the magnitude of cashflow involved when liquidity decreases (scenarios A and D) and when liquidity increases (scenarios B and C). However, for an individual with a concave utility function, the discount rate should decrease in the latter case (scenarios B and C) but increase in the former (scenarios A and D). This issue is discussed in the following section.

Two more findings are discerned from the mean discount rates in Table 1. First, it is seen that $R(\mathrm{A}) > R(\mathrm{B})$ and $R(\mathrm{C}) < R(\mathrm{D})$ in all the 32 comparisons between the corresponding means. The mean difference between the 16 $R(\mathrm{A})$ mean values in scenario A and the 16 $R(\mathrm{B})$ mean values in scenario B is 0.100, whereas the mean difference between the $R(\mathrm{D})$ and $R(\mathrm{C})$ mean values in scenarios D and C is 0.065. These results would be expected in the case of capital market imperfections where interest rates are not uniform.

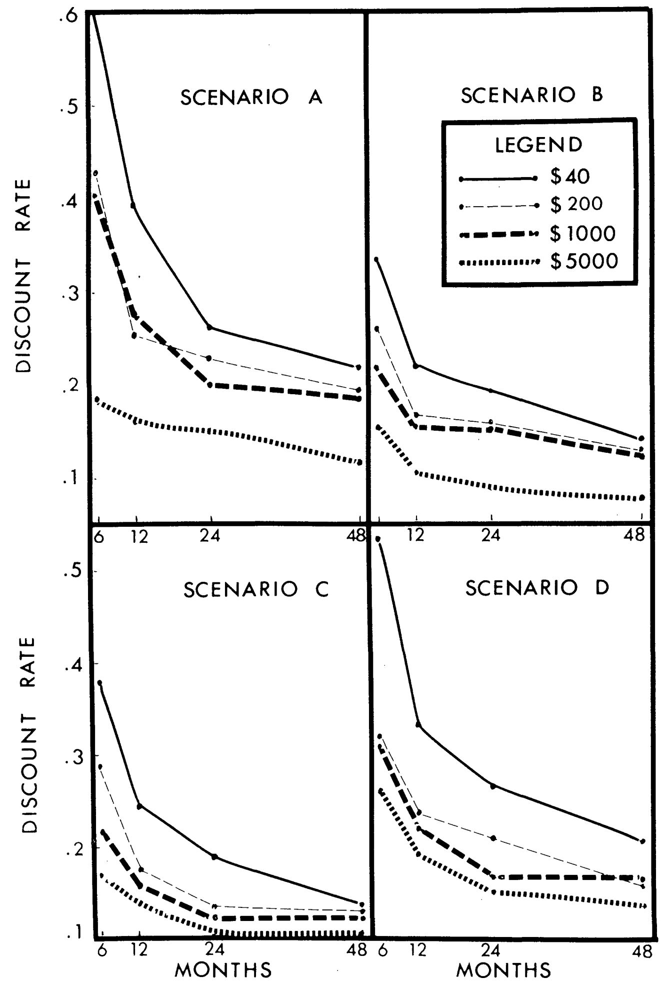

FIGURE 1. Mean Discount Rates for Different Time Delays by Sum of Cashflow and Scenario.

Table 2 presents the mean discount rates for time delay by scenario (Table 2.A), scenario by sum of cashflow (Table 2.B), and time delay by sum of cashflow (Table 2.C). In each table the results are averaged over one of the three factors of the experimental design. Very clear patterns are discerned. As shown by the bottom rows of Tables 2.A and 2.C (see also Figure 1), time has a depressing effect at a decreasing rate on the discount rate. As shown by the right hand columns of Tables 2.B and 2.C, the mean discount rate decreases in magnitude of cashflow. The scenario effect (right-hand column of Table 2.A and bottom row of Table 2.B) is consistent with a concave utility function or a financial market (borrowing and lending) interpretation.

To test for consistency within subjects, linear correlations between the discount rates across time periods were computed for each combination of scenario and size of cashflow separately. There are altogether six such correlations, each based on 204 pairs of observations. All the 96 correlations (6 correlations by 16 scenario by size of cashflow combinations) were positive and significantly different from zero ($p < 0.05$), with the size of the correlation typically decreasing as the difference between the two time periods grows larger. The 96 correlations ranged in value between 0.17 and 0.83 with a mean of 0.59. There is, therefore, strong evidence that individuals maintained their relative position in the frequency distributions of discount rates; for the same scenario and same size of

TABLE 2

Mean Discount Rates across Each of the Three Design Factors

TABLE 2.A. Mean Discount Rates across Sum by Time Period and Scenario

Scenario \ Time (years)	0.5	1	2	4	Mean Across Time
A	0.404	0.271	0.211	0.179	0.267
B	0.241	0.161	0.148	0.116	0.167
C	0.264	0.179	0.138	0.123	0.176
D	0.357	0.244	0.198	0.166	0.241
Means Across Scenario	0.317	0.214	0.174	0.146	0.213

TABLE 2.B. Mean Discount Rates across Time by Sum and Scenario

Sum ($) \ Scenario	A	B	C	D	Mean Across Scenario
40	0.368	0.222	0.237	0.334	0.290
200	0.277	0.178	0.182	0.231	0.217
1000	0.267	0.161	0.155	0.215	0.200
5000	0.153	0.105	0.129	0.185	0.143
Mean Across Sum	0.267	0.167	0.176	0.241	0.213

TABLE 2.C. Mean Discount Rates across Scenarios by Sum and Time Period

Sum ($) \ Time (years)	0.5	1	2	4	Mean Across Time
40	0.462	0.296	0.228	0.176	0.290
200	0.324	0.208	0.183	0.153	0.217
1000	0.288	0.201	0.160	0.148	0.200
5000	0.192	0.150	0.123	0.107	0.143
Mean Across Sum	0.317	0.214	0.174	0.146	0.213

cashflow, subjects who used a relatively high discount rate for one time period also used a relatively high, though typically different, discount rate for another.

The correlational analysis reported above was conducted for each scenario separately. To examine the correlations between discount rates for different scenarios (i.e., the correlations between all pairs of $R(\mathrm{A})$, $R(\mathrm{B})$, $R(\mathrm{C})$, and $R(\mathrm{D})$), we proceeded as follows. For each subject separately, we computed the mean discount rate over the 16 questions in each scenario. Then we computed the six pairwise linear correlations between the mean discount rates for the four different scenarios. The resulting correlations, each based on 204 pairs of observations, ranged between 0.67 and 0.83. All six correlations are positive and highly significant ($p < 0.001$). Together with the correlations reported above, they show a remarkable degree of intersubject consistency within and between scenarios.

Equation (7) was used to estimate for each subject separately the forward rates for questions 1 through 64, or, actually, only 48 forward rates because the first period 16 discount rates are spot rates. Table 3 presents the means and standard deviations (in parentheses) of the forward rates computed over subjects. As in Table 1, the results are

TABLE 3

*Means and Standard Deviations of Inferred Forward Rates**

	Scenario A *t*				Scenario B *t*			
Sum	0.5	1	2	4	0.5	1	2	4
40	0.598	0.300	0.169	0.199	0.334	0.182	0.179	0.100
	(0.613)	(0.553)	(0.234)	(0.379)	(0.378)	(0.662)	(0.202)	(0.139)
200	0.428	0.155	0.221	0.176	0.260	0.112	0.162	0.108
	(0.408)	(0.411)	(0.262)	(0.231)	(0.263)	(0.375)	(0.199)	(0.152)
1000	0.407	0.224	0.143	0.188	0.217	0.123	0.167	0.102
	(0.462)	(0.464)	(0.205)	(0.287)	(0.242)	(0.345)	(0.318)	(0.170)
5000	0.184	0.170	0.156	0.102	0.153	0.079	0.080	0.069
	(0.192)	(0.408)	(0.266)	(0.254)	(0.176)	(0.318)	(0.194)	(0.137)
	Scenario C *t*				Scenario D *t*			
Sum	0.5	1	2	4	0.5	1	2	4
40	0.379	0.180	0.161	0.105	0.535	0.242	0.256	0.186
	(0.503)	(0.410)	(0.332)	(0.194)	(0.660)	(0.648)	(0.420)	(0.367)
200	0.288	0.127	0.111	0.139	0.321	0.237	0.220	0.128
	(0.430)	(0.428)	(0.179)	(0.198)	(0.433)	(0.703)	(0.405)	(0.225)
1000	0.217	0.138	0.102	0.136	0.310	0.204	0.131	0.179
	(0.278)	(0.367)	(0.215)	(0.231)	(0.385)	(0.524)	(0.226)	(0.354)
5000	0.171	0.130	0.090	0.107	0.261	0.176	0.123	0.142
	(0.246)	(0.268)	(0.296)	(0.169)	(0.419)	(0.414)	(0.302)	(0.267)

* The first period rates are Spot rather than forward rates.

displayed as four 4 × 4 (time period by size of cashflow) tables, one for each scenario. Four two-way ANOVA's with repeated measures on both factors were computed, one for each scenario. In each case, the two main effects as well as the interaction effect were significant ($p < 0.001$). The interaction effect is seen most clearly in Figure 2.

The OPR hypothesis predicts that subsequent to a drop between the first and second time periods the forward rate will stay constant over time. In contrast, the MPR hypothesis predicts constant forward rate over all time periods. The results in Table 3 and Figure 2 clearly reject the MPR hypothesis. There is some support for the OPR hypothesis, especially for cashflows of $1000 and $5000. As shown in Figure 2, following a substantial drop between the first and second time periods, the behavior of the forward rates between the second, third, and fourth time periods is variable. The results suggest that the discount rate for the first time period (namely, the spot rates) reflects both the time value of money and other factors such as risk, whereas discount rates for subsequent time periods (namely, forward rates) primarily reflect the time factor.

4.2. *Model Testing*

The analyses reported above established the significant effects of the three variables that were experimentally manipulated, but did not address the models directly. Regression techniques are used to test each model separately.

The Classical and Market Segmentation Approaches. The major distinction between the classical and market segmentation approaches is that the former approach implies

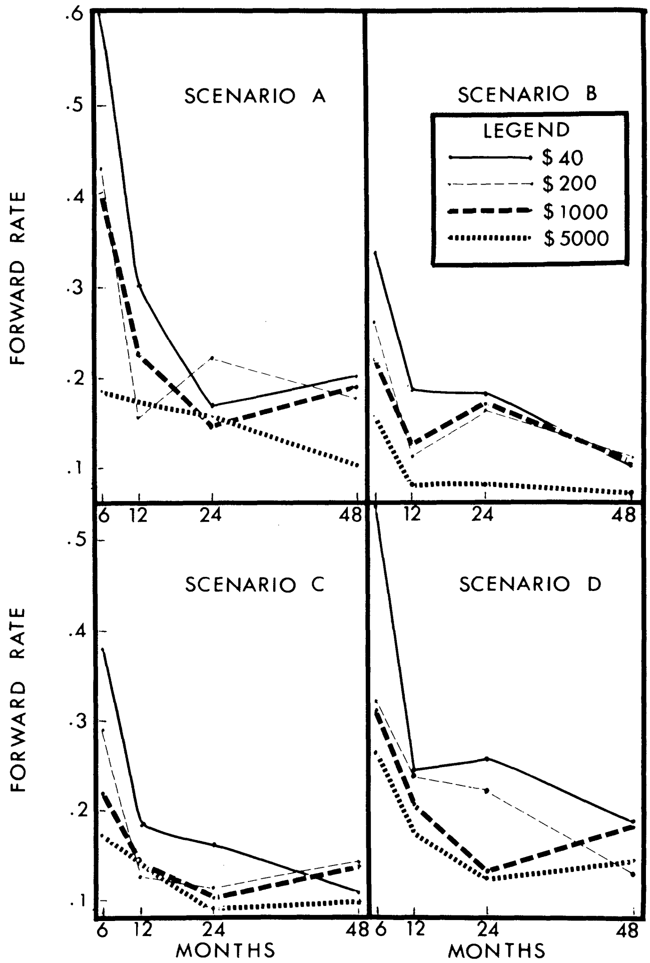

FIGURE 2. Mean Forward Rates for Different Time Delays by Sum of Cashflow and Scenario.

that discount rates are constant for all scenarios whereas the latter allows for variation across scenarios. We tested these two competing approaches by ANOVA; the results clearly reject the classical approach in favor of the market segmentation approach (see Figure 1).

In a more detailed analysis we also tested the dependency of the discount rate on the size of cashflow and delay time using the regression equation (9) below. It is to be noted that although the market segmentation model allows the discount rate to vary across the four scenarios, it predicts no effect due to size of cashflow or delay time. The appropriate regression model for testing this model is given by

$$K^m = \alpha + \beta_1 S_1 + \beta_2 S_2 + \beta_3 S_3 + \beta_4 T_1 + \beta_5 T_2 + \beta_6 T_3 + \beta_7 C_1 + \beta_8 C_2 + \beta_9 C_3 + e_j^m, \quad (9)$$

where α and β are the OLS coefficients, e is an error term, m denotes the subject, S, T, and C represent dummy variables for scenario, time period, and sum of cashflow, respectively, and K denotes a discount factor. Representing K in equation (9) by the discount rate (R) yields the following results: $\beta_1 = 0.025$, $\beta_2 = -0.074$, $\beta_3 = -0.065$, $\beta_4 = 0.171$, $\beta_5 = 0.068$, $\beta_6 = 0.028$, $\beta_7 = 0.147$, $\beta_8 = 0.074$, $\beta_9 = 0.056$, $\alpha = 0.105$, and a multiple correlation $R^2 = 0.11$ ($p < 0.001$).

The regression results indicate that in addition to varying across scenarios, the discount rate also varies across delay time and size of cashflow (see also Table 2). Additional regression analyses not reported here show that both delay time and size of cashflow have a strong negative impact on the discount rate (see Figure 1).

The regression analysis reported immediately above (equation (9)) was repeated with other estimates of K.[7] The results were essentially the same as reported above. Taken together, they imply the existence of additional variables affecting the discount rate to those postulated by the market segmentation approach.

The Implicit Risk Approach. A logarithmic transformation of equation (4) yields

$$\ln (F/P) = \ln (1 + d) + t \ln (1 + i). \tag{10}$$

The corresponding OLS regression model has the form

$$\ln (F/P)_j^m = \alpha_j + \beta_j t_j + e_j^m, \tag{11}$$

where α, β, e, and m are as defined above, and j denotes scenario. According to the OPR hypothesis, $\alpha_j = \ln (1 + d)$ and $\beta_j = \ln (i + 1)$, where i is the risk-free rate of interest.

The regression parameters α_j and β_j in equation (11) were estimated for each scenario separately. Additionally, the values of d—the risk premium—and i were also estimated. The results are summarized in Table 4.

The values of α_j in Table 4 show "scenario" values of 8.8%, 5.4%, 5.2%, and 7.7% for the risk premium d in scenarios A, B, C, and D, respectively. The corresponding "scenario" values of the risk-free interest rate, i, are 14.5%, 9.9%, 10.3%, and 13.3%. The risk-free interest rates are on the average 13.9% for scenarios A and D and 10.1% for scenarios B and C. These results are expected, given that scenarios A and D represent borrowing and B and C represent lending. Indeed, the mean estimates of i in Table 4 are of the order of magnitude of the interest rates in Israel on bank loans and savings which, as indicated above, were 15% and 10%, respectively, when the experiment was being conducted. These levels of risk-free rate and lending rate, in conjunction with a significantly positive α, lend support to the OPR in contrast to the MPR approach.

Another indication for these rates being reasonable estimates of the risk-free rate of interest is the substantial gap between the mean values of i in Table 4 and the mean discount rates in Table 2 (which were 26.7%, 16.7%, 17.6%, and 24.1% for scenarios A, B, C, and D, respectively). Table 4 also shows that the mean risk premium d varies in the same direction as i: both are higher for scenarios A and D than for B and C.

The Added Compensation Approach. After dividing both sides of Equation (8) by P, the regression model for testing the added compensation approach has the form

$$(F/P)_j^m = \alpha_j + \beta_j(1/P)^m + e_j^m, \tag{12}$$

where $\alpha_j = (1 + R)^t$, $\beta_j = B$, and e denotes error.

The parameters of equation (12) were estimated 16 times, once for each combination of time and scenario. All the parameters in all the 16 regressions were highly significant, with a multiple correlation of $R^2 = 0.33$. The means across time of the discount rates implied by the parameter α were found to be 22%, 14.5%, 12%, and 16% for scenarios A, B, C, and D, respectively. The means across time of the added compensation component (B) implied by the parameter β were, in turn, 13.0%, 5.4%, −2.7%, and −3.7% for scenarios A, B, C, and D, respectively. It may be recalled that the model predicts

[7] Equation (9) was used to estimate four other variants of the discount rate: the discount rate differential $GR = R - \bar{R}$ (where $\bar{R}$ is the subject's mean R across all 64 questions), the standardized discount rate $ZR = (R - \bar{R})/\sigma_R$ (where σ_R is the standard deviation of the R's), and the two logarithmic functions $\ln (1 + R)$ and $\ln (1 + GR)$.

TABLE 4
*Estimates of the Implicit Risk Premium (d) and Riskless Rate of Interest (i) by Scenario**

Scenario	α_j	β_j	R^2	$\hat{d}$	$\hat{i}$
A	0.084	0.136	0.27	0.088	0.145
B	0.053	0.094	0.23	0.054	0.099
C	0.052	0.098	0.21	0.052	0.103
D	0.073	0.125	0.24	0.077	0.133

* All parameter values of α_j, β_j and R^2 are significant at the 0.001 level.

positive premiums for scenarios A and B and negative premiums for scenarios C and D. It should also be noted that, as predicted, the positive premium is higher for A than for B, and the negative premium is lower for C than for D.

It is possible to generalize the added compensation approach by postulating two components—a fixed element, denoted again by B, which represents the idea of position change, and a variable element, denoted by b, which we assume to increase linearly in the amount of money involved. The relationship between future and present value is then given by

$$F = P(1 + R)^t + (B + bP). \tag{13}$$

Dividing both sides of equation (13) by P and presenting the result in a nonlinear regression form yields

$$(F/P)_j^m = \beta_{0,j} + (1 + \beta_{1,j})^t + \beta_{2,j}(1/P)^m + e_j^m, \tag{14}$$

where $\beta_{0,j} = b$, $\beta_{1,j} = R$, and $\beta_{2,j} = B$. Equation (14) thus allows the separation of the two factors R and b, which equation (12) combines in the intercept α_j. The parameters of equation (14) were estimated separately for each scenario. The values of β_0 were 0.037, 0.005, 0.037, and 0.050 for scenarios A, B, C, and D, respectively. The first two values of β_0 were not significantly different from zero, but the last two values of β_0 for scenarios C and D were. The values of β_1 and β_2 were highly significant in all four scenarios, and the values of R^2 ranged between 0.62 and 0.96 with a mean of 0.85. The estimates of both the constant element (B) of the added compensation factor and the discount rate (R) were consistent in both direction and magnitude with those obtained in the immediately preceding test of the linear regression model (equation (12)).

5. Discussion and Conclusions

The discount rates inferred from the riskless choices support the previous findings reported by Thaler (1981): The discount rates (i) decline as the time necessary to wait increases, (ii) decrease as the size of cashflow increases, and (iii) are smaller for losses than for gains. However, the discount rates in the present study are in general considerably smaller—and, as we argued above, more reasonable—than those reported by Thaler. We attribute the difference in the results to the difference in financial knowledge between the two populations of subjects and the difference between the two experimental designs.

Although our subjects constituted a relatively homogeneous group in terms of their formal training in economics and finance, Table 1 shows large individual differences in the implicit discount rates. These individual differences, however, were not due to carelessness in answering the questionnaire or some other form of response error. Rather, positive and significant correlations between discount rates were found both within and between the four scenarios. These correlations probably reflect individual differences in attitude toward risk as well as in information about the economy. They support our assumption that the subjects' responses in the present study reflected individual utilities rather than merely an interest rate (which is uniform).

Whereas the classical approach was flatly rejected, the significant scenario effect is consistent with psychological models based on the notion of a reference point (Kahneman and Tversky 1979; Loewenstein 1986). However, the significant effects due to time delay and sum of cashflow, which are not accounted for by the market segmentation approach, imply the existence of other variables that affect the discount rate.

One of these variables, the implicit risk associated with delayed consequences, is supported by the experimental findings. Additional supportive evidence was provided by Stevenson (1986), who interpreted the effect of time delay on the monetary consequences in her experiments as an implicit risk factor. In the present study the spot rate was found to contain a risk premium, whereas the subsequent discount rates (namely, the forward rates) were lower and mainly reflected the risk-free rate of interest. These results support the OPR hypothesis and reject the MPR hypothesis. However, the OPR hypothesis cannot account for the significant effects of the size of cashflow.

The added compensation approach, which postulates the existence of a premium for a position change, is consistent with the effects of the size of cashflow on the discount rates. It is also consistent with the decrease in discount rate as a function of delay time. Two additional findings support the added compensation approach. The first is the relatively high multiple correlations that were found when the added compensation approach was generalized to allow for both a fixed and a variable component. Another supporting finding is that, as predicted by that approach, positive premiums for position change were found for scenarios A and B and negative premiums for scenarios C and D.

Discount rates are usually assumed to depend on a variety of factors such as the person's capacity to imagine the future, his or her level of impatience, self control, etc. One of the reviewers has raised the objection that money outcomes are problematic for studying discount rates; only the rate of interest at which the individual borrows or lends money should matter in this case. We have argued in return that the large individual differences combined with the positive and highly significant correlations between subjects across scenario by size of cashflow combinations as well as the positive and highly significant correlations within subjects across scenarios are inconsistent with a financial market type of explanation. However, the present study does not provide a definite answer between a utility based and a financial market explanation. To distinguish between the two explanations, it is necessary to conduct intertemporal choice studies—preferably ones using within-subject design—that ask subjects to evaluate the present worth of receiving in the future both money outcomes and real consumption goods.

Our findings are intriguing because they are based on the responses of a large number of sophisticated students with extensive knowledge of economics and finance. However, a major shortcoming of the study, shared by most of the previous experimental investigations of intertemporal choice, is the restriction of the findings to responses given to hypothetical questions. It is, therefore, desirable to replicate the findings in choice experiments in which monetary payoff is contingent on decision. A successful replication of the present findings may have significant implications for axiomatic theories of intertemporal choice.[8]

[8] We wish to thank Mr. Nathan Mamias for his assistance in data analysis and two referees of this journal for excellent comments and helpful suggestions. We have also benefited from comments made by participants of seminars we have conducted at the University of Pittsburgh, University of Wisconsin at Milwaukee, University of Delaware, State University of New York at Buffalo, City University of New York, and Baruch College.

Appendix

Means and Standard Deviations of the Subjects' Response

	Scenario A *t*				Scenario B *t*			
Sum	0.5	1	2	4	0.5	1	2	4
40	49.8 (8.6)	55.7 (13.7)	65.4 (22.1)	107.5 (161.3)	45.8 (6.1)	48.8 (8.7)	58.0 (17.6)	71.6 (35.1)
200	236.9 (31.8)	251.0 (50.2)	310.8 (118.4)	451.6 (386.2)	223.5 (21.6)	233.5 (39.1)	272.9 (81.1)	344.1 (178.5)
1000	1174.4 (168.4)	1274.9 (246.4)	1471.8 (515.0)	2118.1 (1259.7)	1099.2 (96.5)	1155.2 (192.9)	1349.4 (695.8)	1720.2 (1512.5)
5000	5426.3 (410.8)	5810.3 (1019.3)	6684.5 (2307.8)	8328.4 (4628.7)	5356.1 (372.1)	5476.2 (784.4)	5993.8 (1811.4)	7031.4 (3886.9)

	Scenario C *t*				Scenario D *t*			
Sum	0.5	1	2	4	0.5	1	2	4
40	35.1 (4.2)	33.2 (5.3)	30.2 (9.4)	26.6 (11.4)	33.8 (5.1)	32.1 (6.6)	27.5 (8.7)	22.3 (10.1)
200	180.8 (21.3)	174.0 (21.4)	160.0 (29.8)	132.5 (42.7)	178.4 (21.3)	167.0 (25.2)	145.6 (36.7)	122.3 (41.1)
1000	919.8 (84.6)	882.8 (126.8)	823.9 (159.0)	680.3 (192.6)	893.0 (99.9)	839.8 (130.9)	771.3 (174.1)	617.7 (223.4)
5000	4678.9 (396.7)	4464.2 (499.3)	4271.8 (841.5)	3634.0 (929.2)	4552.4 (461.0)	4297.8 (564.6)	3994.2 (841.2)	3287.7 (1010.5)

References

BENZION, U. AND J. YAGIL, "On the Discounting Formula for a Stream of Independent Risky Cashflows," *The Engineering Economist*, 32 (1987), 337–345.

FISHBURN P. C., *Utility Theory and Decision Making*, Wiley, New York, 1970.

———, AND A. RUBINSTEIN, "Time Preference," *Internat. Economic Rev.*, 23 (1982), 677–694.

FISHER, I., *The Theory of Interest*, Macmillan, New York, 1930.

KAHNEMAN, D. AND A. TVERSKY, "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 47 (1979), 363–391.

KOOPMANS, T. C., "Stationary Ordinal Utility and Impatience," *Econometrica*, 28 (1960), 287–309.

LANCASTER, K. J., "An Axiomatic Theory of Consumer Time Preference," *Internat. Economic Rev.*, 4 (1963), 221–231.

LOEWENSTEIN, G., "Frames of Mind in Intertemporal Choice," Center for Decision Research, The University of Chicago, 1986.

———, "Anticipation and the Valuation of Delayed Consumption," *The Economic J.*, 97 (1987), 666–684.

MISCHEL W., "Preference for Delayed Reinforcement and Social Responsibility," *J. Abnormal and Social Psychology*, 62 (1961), 1–7.

——— AND J. GRUSEC, "Waiting for Rewards and Punishments: Effects of Time on Probability and Choice," *J. Personality and Social Psychology*, 5 (1967), 24–31.

——— AND R. METZNER, "Preference for Delayed Reward as a Function of Age, Intelligence, and Length of Delay Interval," *J. Abnormal and Social Psychology*, 64 (1962), 425–431.

ROBICHEK, A. A. AND S. C. MYERS, "Conceptual Problems in the Use of the Risk Adjusted Discount Rates," *J. Finance*, 21 (1966), 727–730.

STEVENSON, M. K., "A Discounting Model for Decisions with Delayed Positive or Negative Outcomes, *J. Experimental Psychology: General*, 115 (1986), 131–154.

THALER, R., "Some Empirical Evidence on Dynamic Inconsistency," *Economic Lett.*, 8 (1981), 201–207.

——— AND H. M. SHEFRIN, "An economic theory of self-control," *J. Political Economy*, 89 (1981), 392–406.

YATES, F. AND R. A. WATTS, "Preferences for Delayed Losses," *Organizational Behavior and Human Performance*, 13 (1975), 294–306.