Estimating Discount Functions with Consumption Choices over the Lifecycle*

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Abstract

We estimate β-δ time preferences and relative risk aversion (RRA) using a lifecycle model including stochastic income, liquid and illiquid assets, credit cards, dependents, Social Security, mortality, and bequests. Preference parameters are identified by cross-tabulating four lifecycle age intervals and four balance sheet moments: the share of households carrying (i.e., revolving) credit card debt, average carried credit card debt, average net wealth among households carrying credit card debt, and average net wealth among households not carrying credit card debt. The 16 moments are approximately matched by (MSM) parameter estimates $\beta = 0.53$, $\delta = 0.99$, and $RRA = 1.9$.
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Economic models predict that intertemporal preferences influence most important decisions, including human capital investment, work effort, nutritional choices, exercise, medical adherence, smoking, alcohol consumption, saving, and borrowing.

Most research on time preferences uses lab or field experiments in which the experimenter asks participants to make a series of choices between sooner, smaller rewards and later, larger rewards.[1] Experiments have the benefit of giving the researcher control over the options that are offered to the decision-maker (e.g., Augenblick, Niederle, and Sprenger 2015).

The current paper sacrifices that experimental control in return for the benefit of estimating time preferences using lifecycle consumption/saving/borrowing choices that, in aggregate, generate large-stakes consequences. We use a lifecycle model to structurally estimate time preferences by analyzing the evolution of four balance sheet moments: the proportion of households borrowing on their credit cards (i.e., holding some high-interest credit card debt that revolves from one month to the next), the average magnitude of credit card borrowing (i.e., the average magnitude of high-interest credit card debt that revolves from one month to the next, including zeroes in the average), total net wealth among households revolving credit card debt, and total net wealth among households not revolving credit card debt. We measure each of these four balance sheet moments in four age intervals: 21-30, 31-40, 41-50, and 51-60. Given a stochastic income process, these lifecycle balance sheet choices are the dual of the stochastic lifecycle consumption path. Our benchmark estimates study households with a completed high school degree and no college degree, as this is the largest educational category for the period we study.[2]

Our empirical moments, calculated from the Survey of Consumer Finances, suggest that

[1]For reviews, see, for example, Ainslie (1992), Frederick, Loewenstein, and O'Donoghue (2002), Ericson and Laibson (2019), and Cohen et al. (2020).

[2]Our Internet Appendix reports results for other educational groups.

households can appear to be either impatient or patient, depending on what part of the balance sheet the viewer is studying. On the one hand, we observe a high rate of credit card rollover borrowing (e.g., the majority of households aged 21 through 60 revolve a credit card balance). On the other hand, we also observe that households accumulate sizable amounts of (mostly illiquid) wealth over the lifecycle (e.g., even conditional on revolving a credit card balance, households aged 51-60 *simultaneously* hold average wealth of 4.6-times their average income). The observed rates and magnitudes of credit card borrowing and wealth formation may or may not pose a conceptual tension for classical economic models of household behavior. For example, some (classical) households will experience a sequence of shocks that lead to credit card borrowing that coincides with a large stock of (previously accumulated) illiquid wealth. To evaluate whether the population-level distribution of balance sheet data poses a quantitative tension, we use a calibrated structural model.

Lifecycle balance sheet choices depend complexly on financial constraints, preferences, and behavioral biases (Gomes, Haliassos, and Ramadorai 2021). To map field data to fundamental preference parameters, the structural lifecycle model that we use is designed to reflect key features of the institutional environment in which the field data was generated (DellaVigna 2018). We solve and simulate a model of lifecycle consumption and saving/borrowing choices, which builds on the "buffer stock" savings literature (e.g., Deaton 1991; Carroll 1992, 1997; Gourinchas and Parker 2002; Kaplan and Violante 2014; Kaplan, Moll, and Violante 2018; Choukhmane 2019). Our model – which adopts and extends the models of Angeletos et al. (2001) and Laibson, Repetto, and Tobacman (2003) – features liquid and illiquid assets, credit card debt, liquidity constraints, age-varying dependents (both children and older adults), Social Security, stochastic income, mortality, and bequests.

Our central departure from classical economic models is that we allow for a more flexible specification of time preferences. A large body of empirical research finds that a substantial fraction of economic agents have *present-focused preferences*, meaning that "agents are more likely in the present to choose an action that generates immediate experienced utility, than they would be if all the consequences of the actions in their choice set were delayed by the same amount of time. More informally, this amounts to people choosing more impatiently for the present than they do for the future" (Ericson and Laibson 2019, p. 3).

There are many ways to represent time preferences in general and present-focused preferences in particular. We use the two-parameter discount function introduced by Phelps and Pollak (1968) to study intergenerational discounting and then used by Laibson (1997) to study intrapersonal discounting: 1, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$, ... This discount function can be calibrated to generate exponential discounting ($\beta = 1$) or a particular form of present-focused preferences that is referred to as *present bias*: i.e., $\beta < 1$. When $\beta < 1$, the discount rate is greater in the short-run than the long-run.[3] Accordingly, by generalizing the discount function with present bias we allow for households' time preferences to reflect the potential tension between acting impatiently and acting patiently that is exhibited by our empirical moments.

Adopting the methodology of Gourinchas and Parker (2002), we use a two-stage Method of Simulated Moments (MSM) procedure to estimate three preference parameters in our model: β and δ (the time preference parameters), as well as a (constant) coefficient of relative risk aversion (RRA).[4] Our parameter estimates minimize a weighted loss function that compares the 16 empirical moments from U.S. household balance sheets to the same moments generated by simulating our lifecycle model.

When we impose an exponential discount function (i.e., constraining $\beta = 1$), the MSM procedure estimates an annual discount factor of $\hat{\delta} = 0.96$ and $\widehat{RRA} = 1.5$. When we

[3]Our main specifications assume that households are naive, meaning that they do not anticipate the self-control problems of future selves (Strotz 1955; Akerlof 1991; O'Donoghue and Rabin 1999a, 1999b). In most discrete-time consumption models, sophisticates and naifs behave similarly (Angeletos et al. 2001). In a model like ours, Maxted (2022) uses continuous-time methods to prove an observational equivalence between the consumption choices of sophisticates versus naifs. Section 6 extends our model to the case of sophistication, and the results are similar.

[4]See McFadden (1989), Pakes and Pollard (1989), and Duffie and Singleton (1993) for early references in the MSM literature.

estimate a present-biased discount function, the MSM procedure jointly estimates $\hat{\beta} = 0.53$, $\hat{\delta} = 0.99$, and $\widehat{RRA} = 1.9$. The standard error on $\hat{\beta}$ in the present-bias specification is 0.11. Using a 1% significance threshold, our estimate of $\hat{\beta}$ rejects the null hypothesis of exponential discounting ($\beta = 1$).

Together, our estimates of $\hat{\beta}$ and $\hat{\delta}$ allow our lifecycle model to match the tension between acting impatiently versus patiently that we observe in households' financial choices. Over the coming year, the *short-run* discount rate is approximated by $1 - \hat{\beta}\hat{\delta} = 48\%$. The high short-run estimated discount rate implies that households are likely to borrow on credit cards at high real interest rates; the real interest rate on credit cards is 11% in our benchmark calibration. This frequent and substantial borrowing at an 11% interest rate – consistent with our empirical credit card borrowing moments – makes households appear to be impatient.

For all subsequent years, the continuation discount rate is only $1 - \hat{\delta} = 1\%$ per year. This low *long-run* estimated discount rate implies that households are willing to accumulate substantial (illiquid) retirement wealth at a calibrated real interest rate of 5%.[5] Intuitively, when times are occasionally very good (e.g., a large positive transitory income shock), households put their extra liquidity into the illiquid asset (which pays a small premium over the liquid asset). Households use the illiquid asset to store wealth whether they are naive (our benchmark case) or sophisticated (an appendix case). Naive households put their extra funds in the illiquid asset because they anticipate that they will not need the liquidity in the medium run. On the other hand, sophisticated households are partially motivated to put their extra funds in the illiquid asset as a commitment device. Hence, whether households are naive or sophisticated they will hold substantial illiquid wealth and, nevertheless, maintain minimal liquidity buffers most of the time.[6]

[5] This is the calibrated historical real return on illiquid assets like housing. For example, Kaplan and Violante (2014) calibrate the long-run real return on housing to be 6.29%.

[6] In addition to fitting our empirical moments, these patterns are also consistent with empirical findings documented in other papers, notably Angeletos et al. (2001), Laibson, Repetto, and Tobacman (2003), Kaplan and Violante (2014), Kaplan, Moll, and Violante (2018), Laibson, Maxted, and Moll (2021), and Lee

Under present bias, the majority of simulated households borrow on credit cards in any given period of working life, consistent with our empirical findings. At an 11% real interest rate on credit card borrowing and a 5% real return on illiquid investment, *exponential discounting* cannot explain why most households roll over credit card debt and simultaneously accumulate high levels of illiquid wealth. However, *if* we assume different interest rates, most importantly, a lower real interest rate on credit card borrowing (e.g., 6% rather than our assumed value of 11%), the exponential model does match the balance sheet moments. Using a lower credit card borrowing rate we estimate $\hat{\beta}$ to be close to one. By varying our interest rate assumptions in our robustness analysis, we demonstrate the relationship between our model and exponential discounting models that assume different interest rates than we do (e.g., Kaplan and Violante 2014; Kaplan, Moll, and Violante 2018). Accordingly, our simulations show that frequent high-interest-rate borrowing is explained by present bias, whereas frequent low-interest-rate borrowing can be explained with exponential discounting.

A potential downside of structural modeling with complex household preferences, budget constraints, and state spaces, is that it is often difficult to understand the economic mechanisms driving parameter identification. We propose a novel identification analysis, which we refer to as *boundary analysis*, that explains how our lifecycle model identifies β, δ, and RRA. This graphical analysis draws boundaries around points in the (β, δ, RRA) parameter space that fit selected subsets of the household balance sheet moments that the model is attempting to match. In doing so, our boundary analysis provides a visualization of the parameter subspaces that are consistent with moment matching. For our benchmark calibration (with an 11% real interest rate on credit cards), the parameter subspace that fits the moments does not include any mass in the set of exponential discounting models (i.e., $\beta = 1$). The exponential model can fit the wealth moments *or* the credit card moments, but not both.[7]

and Maxted (2023).

[7]However, as noted above, the exponential model can fit all moments if we substantially lower the real interest rate on credit cards (or raise the return on illiquid assets). With a credit card interest rate that

Our parameter estimates are broadly consistent with other papers that have estimated present-biased time preferences with structural models, particularly when field data is collected from households with relatively lower income. Paserman (2008) uses variation in reservation wages and the length of unemployment spells to estimate β between 0.40 and 0.48 for workers with low and medium levels of income, and $\beta = 0.89$ for workers with high income. DellaVigna et al. (2017) also use a model of unemployment durations to estimate $\beta = 0.58$. Using data on welfare recipients, Fang and Silverman (2009) estimate $\beta = 0.36$. Skiba and Tobacman (2018) model payday loan borrowing to estimate $\beta = 0.50$. Jones and Mahajan (2015) run a field experiment with income tax refunds for households with low income and estimate $\beta = 0.34$. Ganong and Noel (2019) measure the trajectory of consumption among unemployed households, using consumption dynamics at the time of unemployment insurance exhaustion to identify time preferences. They estimate that one-quarter of households have $\beta = 0.5$, and the rest have $\beta = 0.9$. Lockwood's (2020) figure 2 summarizes the literature and reports a value of $\beta = 0.5$ for households with low income and 0.9 for households with high income.

Some papers with structural models and field data estimate higher levels of β, even for populations with low income. Allcott et al. (2022) estimate β between 0.74 and 0.83 for payday loan borrowers. For households with low income, Martinez, Meier, and Sprenger (2023) use the timing of tax filing to estimate $\beta = 0.86$ in their preferred specification.

There is a much larger literature that estimates β using laboratory/experimental evidence, which is reviewed in Ericson and Laibson (2019) and Cohen et al. (2020). Relative to the literature using structural models and field data, the literature using experimental evidence tends to find higher estimates of β.[8]

is close to the rate of return on the illiquid asset, the exponential discounting model can explain why most households simultaneously borrow on credit cards and accumulate large stocks of partially illiquid wealth.

[8] For example, Augenblick, Niederle, and Sprenger (2015) study college students and estimate $\beta = 0.97$ for money-earlier-or-later (MEL) choices and $\beta = 0.89$ for a time-stamped, effort-based task. Using similar time-stamped, effort-based experimental paradigms, Augenblick and Rabin (2019) and Fedyk (2022) also study

1 Wealth and Credit Card Data

We begin by empirically documenting household (net) wealth accumulation and credit card borrowing. Together these moments characterize the tension between acting patiently (i.e., saving for the long run) and acting impatiently (i.e., revolving high-interest credit card debt). We will use these moments in our structural model to estimate the discount function parameters that are needed to replicate the household balance sheet patterns that we observe in the data.

To capture households' financial behavior over the lifecycle, we estimate moments separately for households with heads aged 21-30, 31-40, 41-50, and 51-60.[9] All empirical moments are calculated using the Survey of Consumer Finances (SCF) from 1995 through 2013. Our analysis focuses on U.S. households whose head has a high school degree and no college degree (this group constitutes 57% of SCF households from 1995 through 2013).[10] Our moment estimates control for household demographics, business cycle effects, and cohort effects in order to make the characteristics of the SCF population analogous to our simulated data. Internet Appendix A contains a detailed description of the estimation procedures.

Before presenting our moment estimates, one important point to emphasize is that we only include households in our SCF sample if they *possess* a credit card. Relative to the raw SCF, this will increase our estimates of the share of households with revolving credit card debt. We make this restriction to match the SCF population with our modeled population, since our model assumes all households have a credit card.[11] There are clear benefits to possessing a credit card, and simply using a credit card does not require the cardholder to

college students and estimate $\beta = 0.83$ and $\beta = 0.82$, respectively. Goda et al. (2019) study a representative sample and use a MEL design to elicit time preferences; they find that the *average* β is close to one, and that variation in β across respondents strongly predicts the level of retirement savings.

[9] We view post-retirement financial decisions as sufficiently interesting that they call for specialized models, beyond the scope of what we study here, in order to realistically incorporate health shocks and intergenerational transfers (e.g., De Nardi, French, and Jones 2010; Ameriks et al. 2020).

[10] We discuss results for other education groups in Section 6.

[11] See also Lee and Maxted (2023) for a further discussion of this restriction.

revolve an interest-bearing balance. However, supply-side restrictions often inhibit financially fragile households from accessing credit cards (e.g., Bornstein and Indarte 2023). So, by restricting the SCF sample to households that have a credit card, we generate our empirical measurements with the types of households that we model (i.e., households that have enough financial strength to obtain a credit card in the first place).[12]

Our moment estimates are summarized in Table 1, and are described below. In Table 1 we adopt the notation $\bar{m}_{J_m}$ for the vector of moments and $se\left(\bar{m}_{J_m}\right)$ for their standard errors, consistent with the notation in Section 3 below.

Table 1: Second-stage moments

Name	$\bar{m}_{J_m}$	$se\left(\bar{m}_{J_m}\right)$
% Visa:		
21-30	0.640	0.021
31-40	0.629	0.026
41-50	0.588	0.029
51-60	0.503	0.037
mean Visa:		
21-30	0.111	0.012
31-40	0.096	0.014
41-50	0.110	0.017
51-60	0.104	0.020
wealth | debt:		
21-30	1.222	0.117
31-40	1.868	0.167
41-50	3.377	0.227
51-60	4.650	0.340
wealth | no debt:		
21-30	1.659	0.129
31-40	2.800	0.154
41-50	4.613	0.252
51-60	8.071	0.393

Estimates pertain to households with heads who have a high school diploma and no college degree. Standard errors are calculated using the procedure outlined in the 2013 SCF Codebook, and account for both imputation error and sampling error. See Internet Appendix A for estimation details. *Source:* Authors' estimation based on data from the Survey of Consumer Finances.

[12] Lee and Maxted (2023) also show that households without credit cards often substitute to higher-cost credit sources, such as payday loans, and these nonbank sources of credit are not well measured by the SCF (Zinman 2015).

The first statistic, $\%Visa$, is the share of households that borrow on credit cards.[13] For age groups [21-30, 31-40, 41-50, 51-60], we find that respectively [64.0%, 62.9%, 58.8%, 50.3%] of households carry high-interest credit card debt. Specifically, $\%Visa$ represents the fraction of households that report that they did not pay their credit card bills in full over the last billing cycle. Throughout, we restrict our measure of credit card debt to "high-interest" balances on which the household reports paying an interest rate above 5% in order to exclude promotional low-APR products, following Lee and Maxted (2023).

We construct the second statistic, $meanVisa$, by dividing the quantity of high-interest credit card debt (including zeros for households without such debt) by mean age-specific income. We then average this fraction over the decadal age bins. For age groups [21-30, 31-40, 41-50, 51-60] the average household has outstanding credit card debt equal, respectively, to [11.1%, 9.6%, 11.0%, 10.4%] of the mean income of its age cohort. This statistic is effectively a ratio of means. We use the ratio of means instead of the mean ratio since household-level income can take small values.[14,15]

The third statistic, $wealth|debt$, is also computed as a ratio of means. We divide average net worth among households that did not fully pay off their high-interest credit card debt over the last billing cycle by mean age-specific income.[16] For households with heads aged [21-30, 31-40, 41-50, 51-60], respectively, the resulting $wealth|debt$ measures equal [1.22,

[13]This is the fraction that borrows on any type of card, not just Visa cards.

[14]Compared to the Federal Reserve's G19 Consumer Credit Series and the Consumer Credit Panel of the New York Fed/Equifax, the SCF survey questions on credit card borrowing involve substantial underreporting of aggregate borrowing (Zinman 2009). We adjust the SCF magnitudes upwards to account for underreporting. See Internet Appendix A for details, and Table 4 for robustness to moment choices.

[15]Both of our credit card moments focus only on high-interest credit card debt, not on net liquid wealth. As a consequence, many of the households that are included in our $\%Visa$ and $meanVisa$ moments may still hold some stock of liquid assets simultaneously (i.e., the well-known "coholding puzzle"). Following similar arguments as in Lee and Maxted (2023), we focus on the debt side of household balance sheets because households' usage of revolving credit card debt reveals their marginal intertemporal price of consumption (even if they are also coholding). As we will discuss in Section 2, however, our model will not allow for the coholding of credit card debt and *liquid* assets, and hence we cannot rule out that many of the factors believed to underlie the coholding puzzle (outlined, e.g., in Gomes, Haliassos, and Ramadorai 2021) may also be partially at play in explaining the borrowing patterns we document.

[16]Most household wealth is held in illiquid assets (whether or not they have credit card debt). Over 80% of net wealth in our SCF sample is held in home equity, defined-contribution retirement accounts, and vehicles.

1.87, 3.38, 4.65]. Importantly, these $wealth|debt$ moments show that wealth accumulation is substantial, even for households who are simultaneously borrowing on their credit cards.

The final statistic, $wealth|no\ debt$, is similar to $wealth|debt$ but conditions on having no outstanding high-interest credit card balance. Such households hold more wealth than households with high-interest credit card debt. For households with heads aged [21-30, 31-40, 41-50, 51-60] we find $wealth|no\ debt$ equal to [1.66, 2.80, 4.61, 8.07], respectively.

2 Lifecycle Consumption-Saving Model

This section presents the lifecycle model. Our work extends the numerical simulation literature pioneered by Zeldes (1989), Deaton (1991), Carroll (1992, 1997), Hubbard, Skinner, and Zeldes (1995), Gourinchas and Parker (2002), and Cagetti (2003).[17] Our specific econometric analysis is most closely related to that of Gourinchas and Parker (2002), whose two-stage MSM procedure we employ (details in Section 3). Our structural model adopts and extends the models analyzed in Angeletos et al. (2001) and especially Laibson, Repetto, and Tobacman (2003). We use the same notation as the latter paper, when possible, to maximize comparability.

The first-stage parameters for this structural model can be found in Table 2, and are described here. Internet Appendix B provides additional details on our first-stage parameter estimation procedures. In the model, economic decision-making begins at age 20. Households have an age-dependent survival probability calibrated with data from Social Security Administration life tables. Household composition varies with age as children and adult dependents deterministically enter and leave the household.[18] Effective household size n_t is the

[17]We have commonalities with many recent papers including Becker and Shabani (2010), De Nardi, French, and Jones (2010), Bucciol (2012), Kaplan and Violante (2014), Kaplan, Violante, and Weidner (2014), Carroll et al. (2017), Pagel (2017), Auclert, Rognlie, and Straub (2018), Choukhmane (2019), Attanasio, Kovacs, and Moran (2020), De Nardi, Fella, and Paz-Pardo (2020), and Lee and Maxted (2023).

[18]Demographic profiles are estimated parametrically using the decennial census in 1980, 1990, and 2000, and the American Community Survey from 2001 through 2014.

number of household heads – which we assume to be two in our benchmark model – plus the number of dependent adults, plus 0.4 times the number of children under 18. Our robustness analysis studies alternative assumptions about returns to scale in household consumption.

Let Y_t represent income from wages and transfers in period t, net of taxes. Included in Y_t are income sources such as labor income, bequests, private defined-benefit pensions, and Social Security. We model $y_t = \ln(Y_t)$ as the sum of a cubic polynomial in age, family-size effects, an AR(1), and an i.i.d. shock. We approximate the AR(1) with a Markov process, and denote the Markov state ζ.[19] The income process is estimated with data from the Panel Study of Income Dynamics (PSID).

Let X_t represent the household's holdings of liquid assets at the start of period t, before receipt of income Y_t. If $X_t < 0$ then credit card debt was held between period $t-1$ and t. Households face an age-dependent credit limit of λ_t times average age-t income: $X_t \geq -\lambda_t \overline{Y}_t$. λ_t is a quadratic in age that we estimate from the SCF.[20] The model precludes consumers from simultaneously holding liquid assets and credit card debt, though such potentially suboptimal behavior has been documented among a subpopulation of consumers.[21]

Positive liquid asset holdings earn a risk-free real after-tax gross rate of return R. We calibrate R by calculating the inflation- and tax-adjusted average of AAA corporate bond yields from 1995-2013.[22] The model features a wedge between the return on borrowing and saving (i.e., a "soft constraint"). Specifically, households pay a real interest rate on credit card borrowing of R^{CC}. We refer to this simply as the credit card interest rate, but our computation of R^{CC} accounts for inflation and probabilistic bankruptcy, both of which

[19]Our benchmark specification does not model income using separate processes for the working lifetime and retirement. Hence, average income varies smoothly with age. Alternative assumptions are explored in the robustness section.

[20]This is a crude representation of the income-based credit limits that are common in the revolving credit market (Fulford and Schuh 2015).

[21]For more, see, for example, Gross and Souleles (2002), Telyukova and Wright (2008), Bertaut, Haliassos, and Reiter (2009), Telyukova (2013), and Gorbachev and Luengo-Prado (2019). See also the discussion in footnote 15.

[22]Specifically, we multiply bond yields by 0.75 to account for taxes, and then subtract the corresponding realized inflation rate.

lower consumers' effective interest payments.[23]

Let Z_t represent the (net) balance of illiquid assets at the start of period t, with $Z_t \geq 0$ for all t. Illiquid assets include durables, which generate consumption flows. For computational tractability, we assume that illiquid assets have no net capital gains (i.e., $R^Z = 1$) and generate an annual consumption flow of $\gamma Z_t = 0.05 \cdot Z_t$. Hence, the return from holding the illiquid asset is a 5% annual flow of consumption. We adopt the assumption that liquidating Z_t requires a proportional transaction cost, which declines as a logistic with age. Specifically, the proportional cost of liquidation at age t is $\kappa_t = \frac{1/2}{1+e^{(t-50)/10}}$. These choices about Z do not match the properties of any particular illiquid asset, though Z has some features of home equity and some features of defined-contribution pensions.

Two observations motivate our assumptions about the partial, age-dependent illiquidity of Z. First, despite increasing financial innovation, many household assets continue to be partially illiquid. Accessing equity in homes, cars, and retirement plans entails transaction costs and delays. Some of these frictions are by design: for example, early withdrawal penalties in defined-contribution plans and (essentially) complete illiquidity of defined-benefit plans.

Second, illiquid assets taken as a whole tend to become more liquid as a household ages. For example, IRS regulations establish that an employee with a 401(k) may only make a hardship withdrawal if (i) the employer allows such withdrawals, (ii) the household faces an immediate and heavy financial need, (iii) the employee couldn't reasonably obtain the funds from another source, and (iv) the withdrawal is limited to the amount necessary to satisfy that financial need.[24] To verify these preconditions, the household goes through a

[23]Specifically, from the quarterly credit card interest rates reported in the Federal Reserve Board's G-19 historical series, we subtract the realized inflation and bankruptcy rates. We calculate the latter by dividing the number of bankruptcies (American Bankruptcy Institute) by the number of households that have credit cards (approximated as 65% of households). This attributes all U.S. bankruptcies to households holding credit cards. Section 5.3 presents robustness checks using a wide range of credit card interest rates.

[24]Secure 2.0, signed into law in December 2022, marginally expands the liquidity of 401(k)s.

time-consuming application process. Even if this application is approved, the household still pays income taxes and a 10% penalty on the hardship distribution. By contrast, a mid-life worker who has *previous* employers, can roll over 401(k) balances from those previous employers into an IRA, which allows uncapped distributions without hardship restrictions; IRA distributions are still subject to income taxes and a 10% penalty (Beshears et al. 2022). At age $59\frac{1}{2}$ the early withdrawal penalty is eliminated.

Similar trajectories of rising liquidity characterize housing wealth. For example, a new homeowner will not be able to take out a home equity loan if the household has a high loan-to-value ratio. However, an older homeowner is likely to have a lower loan-to-value ratio due to the mechanical process of principal repayment and the probabilistic process of home value appreciation (Liu 2022). Accordingly, home equity tends to become less and less illiquid (on the margin) as households age. However, home equity loans generate transaction costs and inherent time delays, which may be enough to materially discourage their use by households seeking immediate gratification.

Ultimately, while our modeling of asset illiquidity qualitatively captures certain features of existing institutions, the illiquid Z account is a coarse modeling tool overall. Besides our reduced-form liquidity assumptions, another simplification is that we treat the illiquid account as net wealth, which prevents us from studying households' willingness to utilize secured debt (e.g., mortgages) to purchase illiquid assets. We also assume throughout that households have model-consistent expectations about the return on their illiquid assets (we provide a preliminary analysis of overoptimistic return expectations in Section 5.3). Though our robustness checks will evaluate the sensitivity of our results to the liquidity properties of Z, we view enriching the modeling of illiquid assets as an important task for future work.

Introducing notation for the household saving decisions in period t, we denote net investment into the liquid asset X by I_t^X. Likewise, we denote net investment into the illiquid asset Z by I_t^Z. Accordingly, the dynamic budget constraints are:

$$X_{t+1} = R^X \left(X_t + I_t^X\right), \text{ and} \tag{1}$$

$$Z_{t+1} = R^Z \left(Z_t + I_t^Z\right). \tag{2}$$

Since the interest rate on liquid wealth R^X depends on whether the consumer is holding positive liquid assets versus credit card debt,

$$R^X = \begin{cases} R^{CC} & \text{if} \quad X_t + I_t^X < 0 \\ R & \text{if} \quad X_t + I_t^X \geq 0 \end{cases}.$$

Once I_t^X and I_t^Z are chosen, flow consumption is also pinned down:

$$C_t = Y_t - I_t^X - I_t^Z + \kappa_t \min\left(I_t^Z, 0\right).$$

The state variables in this model are age (t), liquid wealth ($X_t + Y_t$), illiquid wealth (Z_t), and the value of the Markov process (ζ_t) that controls the AR(1) component of the income process. We denote the set of state variables by Λ_t.

The household has constant relative risk aversion denoted by ρ. For all periods of economic life, $t \in \{20, 21, ..., 90\}$,[25] self t has flow utility

$$u(C_t, Z_t, n_t) = n_t \cdot \frac{\left(\frac{C_t + \gamma Z_t}{n_t}\right)^{1-\rho} - 1}{1-\rho}.$$

The household accumulates utility through consumption until death, at which point the

[25] By solving our model with annual periods, we are implicitly assuming that the additional β discount factor kicks in after one year for present-biased households (Laibson and Maxted 2023). While one-year period lengths are likely longer than is psychologically realistic, we solve our model at an annual frequency for computational reasons. However, we conjecture that the choice of period length will affect our estimates of β to some extent, and in particular that credit card borrowing will be easier to generate for β values that are closer to 1 as the period length decreases (Maxted 2022).

household receives a (lump-sum) bequest payoff of $B(t, X, Z)$.[26]

The present-biased discount function $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ...\}$ corresponds to a short-run discount rate of $-\ln(\beta\delta)$ and a long-run discount rate of $-\ln(\delta)$. When $\beta = 1$ the household discounts exponentially.

Our benchmark in this paper follows Strotz (1955), Akerlof (1991), and O'Donoghue and Rabin (1999a, 1999b) in adopting the assumption of "naivete." Under naivete, self t erroneously believes that all future selves will make choices that are aligned with self t's preferences. We assume naivete for computational simplicity, and note that our economic setting is not a good domain for econometrically identifying sophistication versus naivete. Indeed, Maxted (2022) proves an observational equivalence between naifs and sophisticates in a two-asset model similar to the one studied here. For completeness, in Section 6.2 we extend our analysis to "sophistication," where the current self is aware of future selves' present bias. Results are qualitatively similar.

For the naivete benchmark case, let $V^E_{t,t+1}(\Lambda_{t+1})$ represent self t's erroneous expectation of self $t+1$'s value function. Self t's objective function is

$$u(C_t, Z_t, n_t) + \beta\delta E_t V^E_{t,t+1}(\Lambda_{t+1}). \tag{3}$$

Self t chooses $\left\{I^X_t, I^Z_t\right\}$ in state Λ_t to maximize this expression. Self t thinks that self $t+1$ will choose $\left\{I^{X,E}_{t+1}, I^{Z,E}_{t+1}\right\}$ to maximize an analogous expression, but with $\beta = 1$:

$$u(C_{t+1}, Z_{t+1}, n_{t+1}) + \delta E_{t+1} V^E_{t+1,t+2}\left(\Lambda_{t+2}\left(I^{X,E}_{t+1}, I^{Z,E}_{t+1}\right)\right). \tag{4}$$

[26]We assume that bequeathed wealth is consumed by heirs as an annuity. Let $A(t, X, Z) = \max\{0, (R-1)(X + (1-\kappa_t)Z)\}$ denote the flow annuity payment from bequeathed wealth. This assumes that liquidating illiquid wealth entails the same transaction cost κ_t faced by a living household of age t. Let $\bar{n}$ and $\bar{y}$ denote average effective household size and average labor income over the lifecycle, respectively. We set bequest payoff $B(t, X, Z) = \frac{\alpha}{1-\delta}\left[u(\bar{y} + A(t, X, Z), 0, \bar{n}) - u(\bar{y}, 0, \bar{n})\right]$, where α is the weight placed on the bequest motive. Intuitively, this bequest motive captures the value of a household of size $\bar{n}$ increasing consumption from $\bar{y}$ to $\bar{y} + A(t, X, Z)$. Our baseline calibration sets $\alpha = 0.5$. See Internet Appendix Table E4 for robustness.

For notational simplicity we denote $\Lambda_{t+2}\left(I_{t+1}^{X,E}, I_{t+1}^{Z,E}\right)$ as Λ_{t+2}^{E}. Continuation-value functions are determined recursively, where E superscripts reflect naive expectations:

$$V_{t-1,t}^{E}(\Lambda_t) = (1 - \mathbb{1}_t^{death})[u(C_t^E, Z_t, n_t) + \delta E_t V_{t,t+1}^{E}(\Lambda_{t+1}^{E})] + \mathbb{1}_t^{death} B(\Lambda_t), \tag{5}$$

where $\mathbb{1}_t^{death}$ indicates that the household dies between period $t-1$ and t. We solve for household choices using numerical backward induction.

We generate $J_s = 10,000$ independent streams of income realizations for J_s households, and we seed households with median age 20-24 wealth as calibrated from the SCF. Then we simulate lifecycle choices for these households, assuming they make decisions conditional on their state variables. From the simulated profiles of C, X, Z, and Y, we calculate the moments used in the MSM estimation procedure. Since the model cannot be solved analytically, its quantitative predictions are derived from the simulated lifecycle profiles.[27]

3 Two-Stage Method of Simulated Moments

We estimate the parameters of the model's discount function in the second stage of a Method of Simulated Moments procedure, closely following the methodology of Gourinchas and Parker (2002). MSM allows us to evaluate the predictions of our model and to formally test the nested null hypothesis of exponential discounting, $\beta = 1$.[28] This section describes our procedure. Internet Appendix C reviews the Gourinchas and Parker (2002) derivations.

Our MSM procedure has two stages. In the first stage, nuisance parameters, χ, are estimated using standard techniques (see Table 2). We estimate these N_χ parameters and their associated variances, Ω_χ. Internet Appendix B contains details.[29]

[27] See Maxted (2022) for a theoretical analysis of the effects of present bias on consumption-saving decisions.

[28] See McFadden (1989), Pakes and Pollard (1989), and Duffie and Singleton (1993) for early formulations of MSM.

[29] Included in χ are income level coefficients, income variability coefficients, effective household size coeffi-

Table 2: First-stage estimation results

Demographics			Liquid assets		
Number of children			*Credit limit* λ_t		
$k_t = \phi_0 exp(\phi_1 age - \phi_2 \frac{age^2}{100}) + \epsilon$			$\lambda_t = \phi_0 + \phi_1 age + \phi_2 \frac{age^2}{100} + \epsilon$		
ϕ_0	ϕ_1	ϕ_2	ϕ_0	ϕ_1	ϕ_2
0.003	0.358	0.508	0.167	-0.002	0.014
(4.21E-06)	(6.61E-05)	(8.69E-05)	(6.61E-02)	(3.18E-03)	(3.22E-03)
Number of dependent adults			*Real return on positive liquid assets* R		
$a_t = \phi_0 exp(\phi_1 age - \phi_2 \frac{age^2}{100}) + \epsilon$			1.0203		
			-		
ϕ_0	ϕ_1	ϕ_2			
4.59E-06	0.452	0.438	*Real credit card interest rate* R^{cc}		
(1.41E-08)	(1.24E-04)	(1.23E-04)	1.1059		
			-		

Illiquid assets	
Consumption flow as a fraction of assets γ	
0.05	
-	

Real income from wages and transfers

Income process

$$y_t = ln(Y_t) = \phi_0 + \phi_1 age + \phi_2 \frac{age^2}{100} + \phi_3 \frac{age^3}{10000} + \phi_4 Nheads + \phi_5 Nchildren + \phi_6 Ndep.adults + \xi_t$$

$$\xi_t = \eta_t + \nu_t = \psi \eta_{t-1} + \epsilon_t + \nu_t$$

ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ψ	σ_ϵ^2	σ_ν^2
7.563	0.135	-0.222	0.106	0.319	0.013	0.237	0.840	0.057	0.045
(0.256)	(0.017)	(0.037)	(0.024)	(0.023)	(0.008)	(0.012)	(0.047)	(0.005)	(0.004)

Estimates pertain to households with heads who have a high school diploma and no college degree. Standard errors are in parentheses. The constant of the deterministic component of income includes a birth-year cohort effect and a business cycle effect proxied by the unemployment rate. The income estimation includes household fixed effects. See Internet Appendix B for estimation details. All return parameters are assumed to be exactly known in the context of the first stage. This table only reports standard errors, but the full covariance matrix is used in the second stage of the MSM procedure. *Source:* Authors' estimation, following Laibson, Repetto, and Tobacman (2003), based on data from the PSID, SCF, IPUMS-USA, FRB, and American Bankruptcy Institute.

Given χ and Ω_χ, the second stage uses additional data and more of the model's structure to estimate N_θ additional parameters θ. The second stage, taking the first-stage parameters fixed at $\hat{\chi}$, chooses θ to minimize the distance between the empirical and the simulated moments. Specifically, we use the data from Section 1 on wealth accumulation and credit card borrowing over the lifecycle to estimate $\theta = (\beta, \delta, \rho)$ in the second stage. MSM, as we implement it, differs from a calibration exercise followed by a one-stage estimation in that it propagates uncertainty in the first-stage parameters into the standard errors of the second-stage parameter estimates. That is, the variance matrix of $\hat{\theta}$, denoted Ω_θ, depends on Ω_χ.[30] For parameters of the model that are not included in χ – like R, R^{CC}, γ, the parameters governing κ_t, and the bequest function – we perform additional robustness checks in Section 5.3.

Denote the empirical vector of N_m second-stage aggregate moments by $\bar{m}_{J_m}$. Let J_m be the numbers of empirical observations used to calculate the elements of $\bar{m}_{J_m}$. Denote the theoretical population analogue to $\bar{m}_{J_m}$ by $m(\theta, \chi)$ and let $m_{J_s}(\theta, \chi)$ be the simulation approximation to $m(\theta, \chi)$. Let $g(\theta, \chi) \equiv [m(\theta, \chi) - \bar{m}_{J_m}]$ and $g_{J_s}(\theta, \chi) \equiv [m_{J_s}(\theta, \chi) - \bar{m}_{J_m}]$. The moment conditions imply that in expectation

$$E[g(\theta_0, \chi_0)] = E[m(\theta_0, \chi_0) - \bar{m}_{J_m}] = 0,$$

where (θ_0, χ_0) is the true parameter vector. Define derivatives of the moment functions with respect to the parameters by $G_\theta \equiv \frac{\partial g(\theta_0, \chi_0)}{\partial \theta}$ and $G_\chi \equiv \frac{\partial g(\theta_0, \chi_0)}{\partial \chi}$. Let $\Omega_g \equiv E\left[g(\theta_0, \chi_0)\, g(\theta_0, \chi_0)'\right]$ be the variance of the second-stage moment estimates $\bar{m}_{J_m}$, which is estimated directly from sample data using bootstrapping.

cients, and credit limit coefficients.

[30]Our derivation of Ω_θ assumes that the first-stage moments and the second-stage moments have uncorrelated measurement error. Most of the data that we use to identify θ and χ come from separate data sets. The only exception is the credit limit.

Let W be a positive definite $N_m \times N_m$ weighting matrix. The loss function

$$q(\theta, \chi) \equiv g_{J_s}(\theta, \chi)' \cdot W \cdot g_{J_s}(\theta, \chi) \tag{6}$$

is the weighted sum of squared deviations of simulated (model-generated) moments from their corresponding empirical analogues. Our procedure is to fix χ at the value of its first-stage estimator, minimize the loss function $q(\theta, \hat{\chi})$ with respect to θ, and define the estimator as

$$\hat{\theta} = \arg\min_{\theta} \; q(\theta, \hat{\chi}). \tag{7}$$

As shown in Internet Appendix C (following Gourinchas and Parker 2002),

$$\Omega_\theta = Var\left(\hat{\theta}\right) = \left(G_\theta' W G_\theta\right)^{-1} G_\theta' W \left[\Omega_g + \Omega_g^s + G_\chi \Omega_\chi G_\chi'\right] W G_\theta \left(G_\theta' W G_\theta\right)^{-1}, \tag{8}$$

where $\Omega_g^s = \frac{J_m}{J_s}\Omega_g$ is the simulation correction. The first-stage correction is given by $G_\chi \Omega_\chi G_\chi'$. This correction increases with the uncertainty in our estimates of the first-stage parameters (Ω_χ) as well as the sensitivity of the second-stage moments to changes in the first-stage parameters (G_χ).

Equation (8) is used to calculate standard errors for our estimates of θ. All derivatives are replaced with numerical analogues, which we calculate using the model and simulation procedure. We estimate Ω_g and Ω_χ from sample data.

We use weighting matrix $W = diag\left(\hat{\Omega}_g\right)^{-1}$ for our baseline estimates. Many authors have found that optimally weighted GMM procedures lead to biased estimates in small samples (e.g., Altonji and Segal 1996; West, Wong, and Anatolyev 2009). An advantage of diagonal weighting matrices is that the contribution of each moment to $q(\theta, \hat{\chi})$ can be easily computed. We use this property in Section 5 to present a novel exploration of our model's identification, which we refer to as a *boundary analysis*. In robustness checks we find that our qualitative conclusions are not affected by the choice of weighting matrix.

4 Estimation Results

In this section we discuss the paper's main findings. We focus on estimates for the discount factors β and δ and the coefficient of relative risk aversion ρ, including the special case in which we impose $\beta = 1$. We also assess whether these models accurately predict key empirical regularities in the lifecycle literature summarized by the second-stage moments.

4.1 Benchmark case

We report our benchmark estimates in Table 3. In the unconstrained case (Column 1), the MSM procedure yields an estimate of $\hat{\beta} = 0.530$ with a standard error (s.e.(i) in the table) of 0.114. For this specification, $\hat{\beta}$ lies significantly below 1; the t-stat for the $\beta = 1$ hypothesis test is $t = 4.1$. The MSM procedure yields an estimate of $\hat{\delta} = 0.989$, with a standard error of 0.005, and $\hat{\rho} = 1.936$, with a standard error of 0.435. The estimated values of β and δ imply a short-run discount rate of $-\ln(0.530 \cdot 0.989) = 64.5\%$ and a long-run discount rate of $-\ln(0.989) = 1.1\%$.

At the estimated parameter values, the present-biased model generates the moment predictions reported in Column 1 of the lower panel of Table 3. Our estimated model has three free parameters, which are estimated with 16 debt and wealth moments. We can compare the model's simulated moments with the sample moments, which are reproduced in Column 3. Qualitatively, the present-biased model successfully matches the lifecycle patterns of both credit card borrowing and wealth accumulation. Quantitatively, the model underpredicts the fraction borrowing in every decadal age bin. The model is quantitatively more successful in matching average credit card borrowing. The model also performs reasonably well in matching the age-based pattern of wealth formation – both for households with and without

Table 3: Benchmark estimates

	(1) Present Biased	(2) Exponential	(3) Data
Parameter estimates			
$\hat{\beta}$	0.5305	1	-
s.e. (i)	(0.1143)	-	-
s.e. (ii)	(0.1128)	-	-
s.e. (iii)	(0.0536)	-	-
s.e. (iv)	(0.0503)	-	-
$\hat{\delta}$	0.9891	0.9601	-
s.e. (i)	(0.0051)	(0.0053)	-
s.e. (ii)	(0.0051)	(0.0053)	-
s.e. (iii)	(0.0022)	(0.0022)	-
s.e. (iv)	(0.0020)	(0.0020)	-
$\hat{\rho}$	1.9355	1.4663	-
s.e. (i)	(0.4350)	(0.2256)	-
s.e. (ii)	(0.4280)	(0.2228)	-
s.e. (iii)	(0.2188)	(0.1142)	-
s.e. (iv)	(0.2045)	(0.1085)	-
Second-stage moments			
$\%Visa$ *21-30*	0.605	0.309	0.640
$\%Visa$ *31-40*	0.585	0.287	0.629
$\%Visa$ *41-50*	0.523	0.299	0.588
$\%Visa$ *51-60*	0.475	0.257	0.503
mean $Visa$ *21-30*	0.103	0.044	0.111
mean $Visa$ *31-40*	0.117	0.051	0.096
mean $Visa$ *41-50*	0.124	0.059	0.110
mean $Visa$ *51-60*	0.116	0.034	0.104
wealth 21-30 | *debt*	0.913	0.880	1.222
wealth 31-40 | *debt*	1.412	0.999	1.868
wealth 41-50 | *debt*	2.640	1.941	3.377
wealth 51-60 | *debt*	4.723	3.811	4.650
wealth 21-30 | *no debt*	2.324	2.017	1.659
wealth 31-40 | *no debt*	3.248	2.850	2.800
wealth 41-50 | *no debt*	4.633	3.967	4.613
wealth 51-60 | *no debt*	7.475	5.358	8.071
Goodness-of-fit			
$q(\hat{\theta}, \hat{\chi})$	77.15	759.57	-
$\xi(\hat{\theta}, \hat{\chi})$	96.77	300.78	-
p value	0	0	-

This table reports estimates of the discount function under our benchmark assumptions. The top half of the table presents parameter estimates and standard errors. The bottom half of the table reports the moments used to identify the second-stage parameters. Four standard errors are shown: (i) includes both the first-stage correction and the simulation correction, (ii) includes just the first-stage correction, (iii) includes just the simulation correction, and (iv) includes neither correction.

credit card debt – though misses to some extent early in the lifecycle.

We also estimate δ and ρ, imposing the restriction that $\beta = 1$. This exponential discounting case yields the results in Column 2. We find $\hat{\delta} = 0.960$, implying a discount rate of 4.1%, with a standard error of 0.005. We estimate $\hat{\rho} = 1.466$, with a standard error of 0.226.

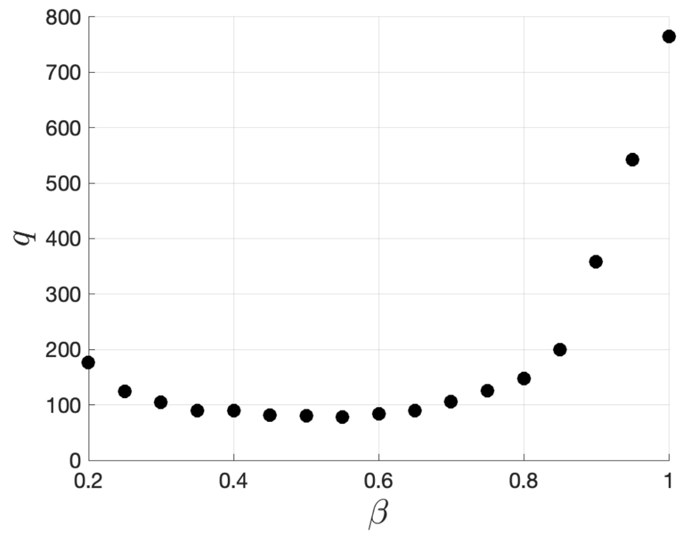
The exponential model matches the empirical facts about wealth accumulation over the lifecycle reasonably well, though it underpredicts wealth accumulation to some extent. Moreover, with such a low discount rate the model cannot account for observed credit card borrowing data. Instead, it predicts $\%Visa$ and $meanVisa$ to be relatively low at all ages. Intuitively, these credit card moments – which require a high discount rate – lose in the tug of war with the wealth moments – which require a low discount rate.[31] The best fit available under an exponential model predicts that most households have much less credit card debt than we observe in the data.

Figure 1 provides visual evidence of the exponential model's inability to match the full set of empirical moments. Figure 1 fixes the value of $\beta \in \{0.2, 0.25, 0.3, \ldots, 0.95, 1\}$ and reestimates the benchmark model for $\{\delta, \rho\}$ conditional on β. The vertical axis reports the q-value of best fit for each estimate. The model's ability to match the full set of credit card and wealth moments degrades as β approaches 1.

The standard errors reported as "s.e.(i)" in Table 3 and discussed above incorporate corrections for the first-stage estimation and for the simulation error. For comparison, we also report standard errors without these corrections: s.e.(ii) only includes the first-stage correction, s.e.(iii) only includes the simulation correction, and s.e.(iv) includes neither. The first-stage correction has a sizable effect on the standard errors. For example, if the first-stage parameters were known with certainty (comparing s.e.(i) and s.e.(iii)), then the standard error on β would roughly halve — falling from 0.114 to 0.054.

[31] As we discuss in Section 5.3, if the credit card interest rate is low enough, or the return to illiquid wealth (i.e., γ) is high enough, the exponential model can more successfully match the facts simultaneously.

Figure 1: q **on** β **(allowing** δ **and** ρ **to vary)**



This figure illustrates the sensitivity of the model fit to restrictions on the short-run discount factor β. The vertical axis lists the MSM objective function q. Each point comes from a separate estimate of δ and ρ, conditional on the indicated β.

Finally, our two-stage MSM procedure enables us to provide formal overidentification tests that combine all of the simulated moments. Despite the present-biased model's better fit of the empirical moments, both models are rejected by overidentification tests. This is not surprising, given that we have 3 free parameters and 16 precisely estimated moments. For the present-biased model, the (inverse) goodness-of-fit measure is $\xi\left(\hat{\theta},\, \hat{\chi}\right) = 97$. For the exponential discounting model, the (inverse) goodness-of-fit measure is $\xi\left(\hat{\theta},\, \hat{\chi}\right) = 301$. Under the null hypothesis that the model is correct, ξ is distributed chi-squared with degrees of freedom equal to the number of moments (16) minus the number of parameters (3 for the model with present bias and 2 for the exponential model). For reference, the 99% critical values of the chi-squared distribution with 13 and 14 degrees of freedom, respectively, are 27.7 and 29.1.

4.2 Benchmark case: Model properties

For our benchmark estimate with (naive) present bias, Figure 2 plots the average lifecycle profile of households in the model.[32] Average income peaks at age 47 before declining as households transition probabilistically into retirement. Even though income is declining, total consumption at the household level (which includes the consumption flow from the illiquid asset) remains approximately constant from age 45 until death. This is because the typical household accumulates illiquid wealth over the lifecycle. In particular, Internet Appendix Figure 5 shows that households deposit into the illiquid asset – which pays a small premium over the liquid asset – during spells of higher income.[33] While households may naively anticipate not needing that liquidity in the medium run, the liquid-assets curve in Figure 2 illustrates that households' subsequent (over)consumption instead often leads them into credit card debt. Thus, the lifecycle profile in Figure 2 captures the underlying tension between acting patiently and acting impatiently that is reflected in our empirical moments: households accumulate large stocks of illiquid wealth, but also frequently carry credit card debt.

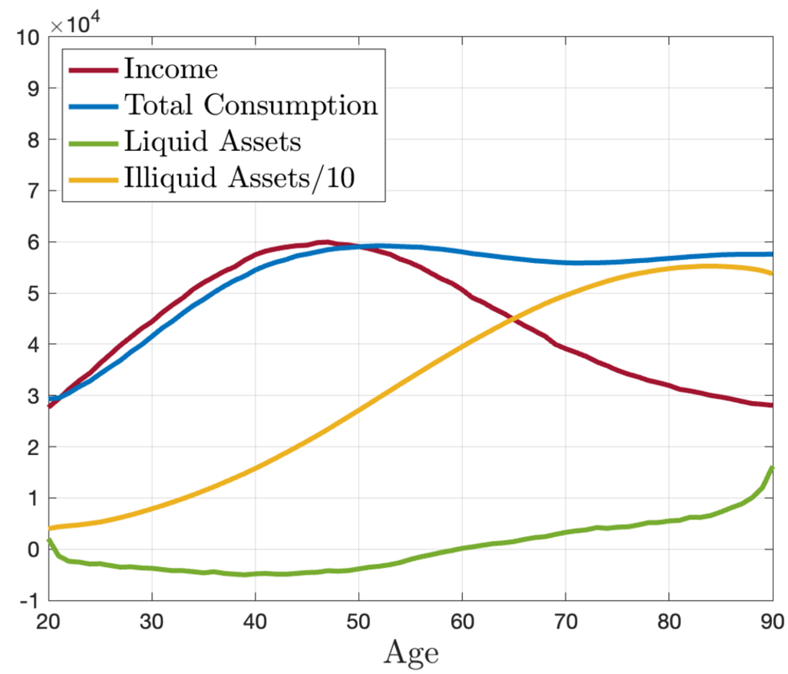
Our benchmark estimate also makes predictions about a household's marginal propensity to consume (MPC) over the lifecycle. In particular, we calculate the average one-year MPC out of a $1,000 windfall.[34] For age groups [21-30, 31-40, 41-50, 51-60] the average annual MPC is [0.31, 0.28, 0.19, 0.13], respectively. Using the method of Laibson, Maxted, and Moll (2022), we can also convert the model's MPCs into marginal propensities for expenditure (MPXs), which include total spending on both nondurables and consumer durables. For age groups [21-30, 31-40, 41-50, 51-60] the average annual MPX is [0.45, 0.41, 0.27, 0.18],

[32]For the $\beta = 1$ case, see Internet Appendix Figure 4.

[33]Note that our model predicts too strong a relationship between households' income state and their propensity to invest in the illiquid asset. For example, our model does not include various sorts of "forced savings" channels, such as monthly mortgage payments, that do not vary at high frequency (in part due to contractual features that may exist as an endogenous response to self-control problems).

[34]We use a nonuniform grid when solving the model numerically, and for some high-liquidity households the grid increment is greater than $1,000. In these cases, we impute the $1,000 MPC.

Figure 2: Average lifecycle profile for present-biased estimate



This figure plots the average lifecycle profile of income, total consumption, liquid assets, and illiquid assets (divided by 10 for scaling) for the benchmark estimate ($\hat{\beta} = 0.530$).

respectively.[35]

This pattern of declining MPCs and MPXs fits with the lifecycle predictions of the model. As households age they accumulate illiquid wealth and face a lower transaction cost to access this wealth, both of which decrease the share of hand-to-mouth households over the lifecycle.[36] The model's predicted MPX age dynamics are also consistent with the evidence in Fagereng, Holm, and Natvik (2021), who document a tendency for the MPX out of lottery winnings to decline with age.

[35] Laibson, Maxted, and Moll (2022) show that the one-period MPX can be approximated by:

$$MPX = \left(1 - s + \frac{s}{r + \mathcal{V}}\right) \times MPC,$$

where s is the durable share of consumption, $\mathcal{V}$ is the (annual) depreciation rate, and r is the interest rate. We set $r = 2.03\%$ (see Table 2), and follow the calibration of Laibson, Maxted, and Moll (2022) in setting $s = 0.125$ and $\mathcal{V} = 0.199$.

[36] Related to this prediction, Leth-Petersen (2010) and Sodini et al. (2023) show that young households are more responsive to credit supply increases.

5 Identification

5.1 Boundary analysis

A drawback to structural modeling is that the forces driving parameter identification are often opaque. In this section we present a novel strategy – which we call a *Boundary Analysis* – in order to address this identification challenge. The goal of our boundary analysis is to restrict the three-dimensional $\theta = (\beta, \delta, \rho)$ parameter space to areas where the simulated moments are close to their empirical counterparts. These boundaries help us to detect the underlying trade-offs that the MSM procedure makes as it chooses $\hat{\theta}$ optimally to fit the empirical moments.[37]

To conduct this analysis, we create a discrete three-dimensional grid of $\theta = (\beta, \delta, \rho)$ values and solve our model at each grid point θ_i. Our choice of a diagonal weighting matrix means that it is easy to determine the contribution of each individual moment (or set of moments) to $q(\theta_i, \hat{\chi})$.[38] We group our 16 moments into two categories – credit card borrowing moments and wealth moments – and determine each category's contribution to $q(\theta_i, \hat{\chi})$. Let $q_{cc}(\theta_i, \hat{\chi})$ denote the contribution of the credit card moments and let $q_{wealth}(\theta_i, \hat{\chi})$ denote the contribution of the wealth moments, such that $q(\theta_i, \hat{\chi}) = q_{cc}(\theta_i, \hat{\chi}) + q_{wealth}(\theta_i, \hat{\chi})$.

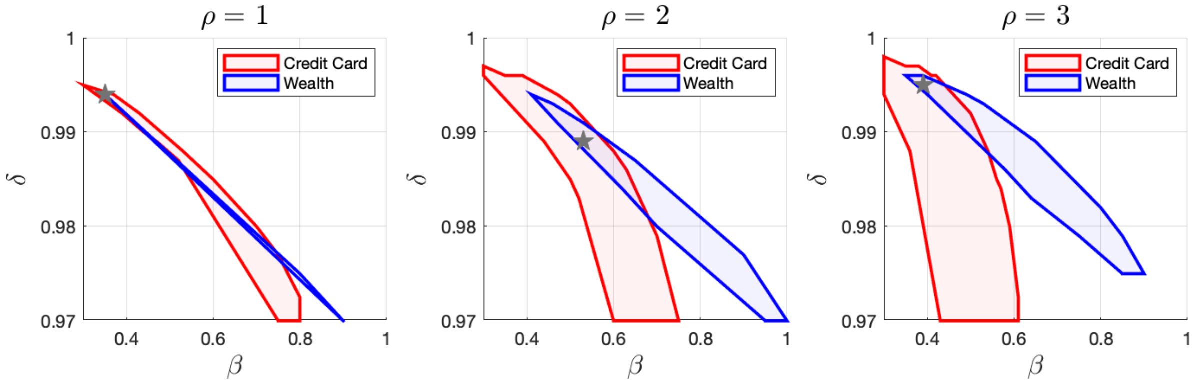
Figure 3 presents the boundary analysis. To visualize the three-dimensional grid of θ values, we show three two-dimensional plots in (β, δ)-space for $\rho \in \{1, 2, 3\}$. Each plot has two boundaries. The red boundary encases the set of grid points at which $q_{cc} < 77$, and the blue boundary encases the set of grid points at which $q_{wealth} < 77$. The threshold of 77 is chosen to align with the q of our baseline estimate in Table 3. The stars mark the point of

[37] The discussion of identification in this section will focus on how $\hat{\theta}$ is chosen to match the empirical moments of Section 1. Alternatively, a reader could ask how changes to the empirical moments affect the estimate of $\hat{\theta}$. To address this question, we report the Andrews, Gentzkow, and Shapiro (2017) sensitivity measure of $\hat{\theta}$ in Internet Appendix Table D1.

[38] Objective function $q(\theta, \hat{\chi}) = g_{J_s}(\theta, \hat{\chi})' \cdot W \cdot g_{J_s}(\theta, \hat{\chi})$, so W being diagonal implies that $q(\theta, \hat{\chi})$ is just a weighted sum of squared errors.

best fit conditional on ρ.

Figure 3: Boundary analysis



This figure shows the boundary analysis. The red locus marks the set of points in (β, δ)-space for which $q'_{cc} \cdot W_{cc} \cdot q_{cc} < 77$. The blue locus marks the set of points for which $q'_{wealth} \cdot W_{wealth} \cdot q_{wealth} < 77$. The star marks the point of best fit conditional on ρ.

We now use this boundary analysis to aid our discussion of the identification of $\hat{\theta}$. Starting with $\hat{\rho}$, Figure 3 illustrates two effects that occur as ρ increases. The first effect is that the two boundaries pull apart — the red credit card boundary shifts left and down toward lower values of β and δ while the blue wealth boundary shifts right and up toward higher values of β and δ. This first effect calls for the estimation of a low ρ in order to keep the two boundaries close together. On the other hand, the second effect of increasing ρ is that the area enclosed by each boundary increases. Put differently, the gradient of q_{cc} and q_{wealth} with respect to β and δ is decreasing in ρ. This second effect calls for the estimation of a high ρ. Balancing these two effects yields $\hat{\rho} \approx 1.9$ as is estimated in Table 3.

To understand the first effect of boundaries pulling apart, a higher value of ρ increases the household's precautionary savings motive. This discourages credit card borrowing in order to preserve that adjustment margin for a series of negative income shocks. As ρ increases the model can only match the empirical credit card borrowing moments with lower discount factors, hence the shift left and down of the red boundary. On the other hand, a higher value of ρ also discourages wealth accumulation beyond what is needed for self-insurance purposes. Wealth accumulation involves trading off current consumption for future consumption, and a higher ρ makes this trade-off less appealing. Thus, as ρ increases the blue boundary shifts right and up because higher discount factors are needed to continue fitting the empirical wealth moments.

For the second effect of decreasing gradients, recall that ρ is the inverse of the EIS, such that increasing ρ decreases the sensitivity of consumption growth to β and δ.[39] Thus, conditional on being at a grid point θ_i that is included in either the credit card or wealth boundary, larger changes to β and δ will be required for q_{cc} or q_{wealth} to change by enough to exit the boundary.

Now that we've identified ρ, we discuss the identification of $\hat{\beta}$ and $\hat{\delta}$ conditional on $\rho \approx 1.9$. Here we focus on the $\rho = 2$ subplot. The shape of both the red and blue areas implies that β and δ are partial substitutes: when δ is high then low values of β best match the empirical moments, and vice versa. Nonetheless, β and δ are separately identified because they are not perfect substitutes. The divergence of the red and blue boundaries as β approaches 1 shows that the effect of δ relative to β is larger for wealth accumulation than for credit card borrowing.[40] Matching the wealth moments relies on substantial *illiquid* asset accumulation, while matching the credit card moments relies on minimal *liquid* asset accumulation. Illiquid assets have a longer effective horizon than liquid assets, hence giving δ relatively more influence on them. As the $\rho = 2$ subplot shows, for β near 1 our model cannot fit both wealth and credit card moments, but by decreasing β (and increasing δ accordingly) our model is able to generate the simultaneous credit card borrowing and wealth accumulation required to match the patterns we observe in the data.[41]

[39] The EIS is not exactly equal to the inverse of ρ for two reasons. First, on the equilibrium path, households sometimes face binding liquidity constraints. Second, present bias can introduce a wedge between the EIS and the inverse of ρ (see, e.g., Laibson 1998).

[40] As β increases toward 1, δ needs to fall by only a little to continue fitting the wealth moments. Alternatively, δ needs to fall by a lot to continue fitting the credit card moments.

[41] See also Maxted (2022) and Lee and Maxted (2023) for further theoretical analysis of how β versus δ affect credit card borrowing and illiquid wealth accumulation.

5.2 Moment sets

As is well known from GMM theory, the choice of moments can be crucial to the outcome of the analysis. We chose to focus on the 16 moments discussed above – each of $\%Visa$, $meanVisa$, $wealth|debt$, and $wealth|no\ debt$, in four decadal age bins – because of their economic importance and their transparency. Since our model is overidentified, if it is correctly specified then omitting some of these moments should not change our estimated parameter values.

We pursue an initial examination of restrictions to the moment set in Table 4. Column 1 of Table 4 repeats the benchmark results. Columns 2-5 drop the $\%Visa$, $meanVisa$, $wealth|debt$, and $wealth|no\ debt$ groups of moments, respectively. While our estimates vary to some extent across these cases, we consistently estimate $\hat{\beta} < 1$.

Column 6 takes the more aggressive step of dropping both the $\%Visa$ and $meanVisa$ moment blocks simultaneously. In this case, we estimate $\hat{\beta} = 0.761$ and $\hat{\delta} = 0.980$. The wealth accumulation moments are matched well, but the model fails to match the credit card moments. In Column 7 we report estimates when all eight $wealth$ moments are dropped. This results in $\hat{\beta} = 0.483$ and $\hat{\delta} = 0.989$. Now, the credit card moments are matched well, but there is too little lifecycle wealth accumulation. Together with the boundary analysis above, Columns 6 and 7 help to illustrate that identification comes from the inclusion of both types of moments in order to generate a tension between short-run and long-run behavior.

5.3 Robustness

In Internet Appendix E we explore robustness to a variety of first-stage parameters and structural modeling assumptions. In summary, estimates of β reliably fall between roughly 0.3 and 0.8. Table E1 fixes the coefficient of relative risk aversion ρ to a variety of common values and reestimates β and δ conditional on the specified ρ. Table E2 examines robustness

Table 4: Sources of identification

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Benchmark	- $meanVisa$ $wealth\|debt$ $wealth\|no\ debt$	$\%Visa$ - $wealth\|debt$ $wealth\|no\ debt$	$\%Visa$ $meanVisa$ - $wealth\|no\ debt$	$\%Visa$ $meanVisa$ $wealth\|debt$ -	- - $wealth\|debt$ $wealth\|no\ debt$	$\%Visa$ $meanVisa$ - -	Data
Parameter estimates								
$\hat{\beta}$	0.5305	0.6593	0.5309	0.2723	0.4088	0.7612	0.4827	-
s.e. (i)	(0.1143)	(0.0760)	(0.0965)	(0.0602)	(0.0928)	(0.0871)	(0.1638)	-
$\hat{\delta}$	0.9891	0.9844	0.9886	0.9975	0.9925	0.9798	0.9893	-
s.e. (i)	(0.0051)	(0.0039)	(0.0042)	(0.0016)	(0.0031)	(0.0053)	(0.0036)	-
$\hat{\rho}$	1.9355	1.9548	1.7870	3.9063	1.1567	1.8133	2.4276	-
s.e. (i)	(0.4350)	(0.3582)	(0.6178)	(0.1327)	(0.1090)	(0.4516)	(1.5034)	-
Second-stage moments								
% Visa 21-30	0.605	0.504	0.612	0.642	0.611	0.439	0.629	0.640
% Visa 31-40	0.585	0.498	0.595	0.606	0.587	0.435	0.609	0.629
% Visa 41-50	0.523	0.479	0.540	0.524	0.527	0.442	0.564	0.588
% Visa 51-60	0.475	0.436	0.497	0.437	0.481	0.414	0.497	0.503
mean Visa 21-30	0.103	0.082	0.106	0.113	0.109	0.068	0.109	0.111
mean Visa 31-40	0.117	0.094	0.120	0.121	0.124	0.079	0.124	0.096
mean Visa 41-50	0.124	0.107	0.130	0.121	0.133	0.095	0.135	0.110
mean Visa 51-60	0.116	0.090	0.125	0.103	0.148	0.077	0.117	0.104
wealth 21-30 | debt	0.913	0.951	0.922	0.648	1.087	0.994	0.774	1.222
wealth 31-40 | debt	1.412	1.559	1.412	1.017	1.619	1.634	1.140	1.868
wealth 41-50 | debt	2.640	2.892	2.588	2.207	2.835	3.022	2.170	3.377
wealth 51-60 | debt	4.723	5.213	4.553	4.662	4.776	5.301	4.003	4.650
wealth 21-30 | no debt	2.324	2.236	2.348	2.069	2.515	2.210	2.188	1.659
wealth 31-40 | no debt	3.248	3.186	3.260	2.786	3.590	3.173	2.891	2.800
wealth 41-50 | no debt	4.633	4.689	4.628	4.288	4.937	4.649	4.128	4.613
wealth 51-60 | no debt	7.475	7.246	7.377	7.638	7.672	7.014	6.528	8.071
Goodness-of-fit								
$q(\hat{\theta}, \hat{\chi})$	77.15	53.22	72.85	25.61	29.04	43.18	8.17	-

This table reports estimates of the discount function under alternative moment sets. Columns 2 through 7 vary the set of empirical moments used for estimation by dropping the shaded moments, maintaining all other benchmark assumptions. In Columns 2, 3, 4, and 5, we remove each of the four moment sets individually. In Column 6, we estimate using two conditional wealth moments, $wealth|debt$ and $wealth|no\ debt$. In Column 7, we estimate using two credit card borrowing moments, $\%Visa$ and $meanVisa$.

to alternative assumptions on return parameters R, R^{CC}, and γ. Table E3 explores robustness to the model's income process, and Table E4 studies robustness to other structural modeling assumptions.[42] Estimates using alternative weighting matrices are presented in Table E5. Internet Appendix E also provides details on our choices for robustness checks.

The one robustness analysis that we discuss in the main text is the sensitivity of household savings behavior to joint assumptions on R^{CC} and γ. We base this analysis on the seminal model of Kaplan and Violante (2014), which also features a liquid asset and an illiquid asset, but is able to generate sizable borrowing with $\beta = 1$. Their calibration sets the real

[42]This includes the liquidity of the Z account, assumptions about household returns to scale, and the strength of the bequest motive.

return on liquid wealth to -1.48%, the real return on illiquid wealth to 2.29% plus a 4% consumption flow (for a total return of 6.29%), and the real cost of borrowing to 6%. In our model, this corresponds roughly to $R = 0.9852$, $\gamma = 6.29\%$, and $R^{CC} = 1.06$. In Table E6 we estimate our model under this alternative calibration, finding $\hat{\beta} = 1.04$ and $\hat{\delta} = 0.95$. These estimates are very similar to the calibration of Kaplan and Violante (2014), who set $\beta = 1$ and $\delta = 0.94$.[43] This result highlights that short-term borrowing can be rationalized if there is a small (or negative) wedge between the cost of borrowing and the return on illiquid wealth (e.g., $r^{CC} = 6\%$ and $\gamma = 6.29\%$). However, $\beta < 1$ allows the model to generate significant credit card borrowing even when there is a large wedge between the cost of borrowing and the return on illiquid wealth (e.g., $r^{CC} = 11\%$ and $\gamma = 5\%$). For additional intuition, Internet Appendix Figure 6 provides a boundary analysis with respect to the model's interest rates. This analysis further illustrates that the borrowing moments can be fit at higher values of β as the wedge between the cost of borrowing and the return on illiquid wealth declines.

We emphasize one interesting caveat to this discussion, which is that household beliefs may also be important. In particular, while the above analysis shows that empirically realistic levels of short-term borrowing can be generated without present bias if there is a small wedge between the cost of borrowing and the return on illiquid wealth, it may be sufficient for there to be a small wedge between households' *perceived* cost of borrowing and/or their *perceived* return on illiquid wealth. We explore a preliminary analysis of this by varying households' perceived return on illiquid wealth (while holding the realized return at 5%). As Table E7 shows, the $\beta = 1$ model's fit of the data improves as households' perceived return on the illiquid asset approaches the credit card borrowing rate.

[43] Kaplan and Violante (2014) set $\beta = 1$ and calibrate R^{CC} internally to match the credit card borrowing observed in the data. In this paper, we instead take R^{CC} from market data and allow β to adjust in order to match the credit card borrowing observed in the SCF.

6 Extensions

This paper's findings suggest several directions for future work. This section discusses three such directions and provides initial analyses. Internet Appendix F reports the results.

6.1 Heterogeneity

One important avenue to explore involves relaxing the assumption of homogeneous preferences.[44] Certainly there is substantial heterogeneity in the population. One might wonder whether a model with exponential consumers, but with heterogeneous δ's, could also resolve the empirical tensions discussed in this paper.

To explore this question, we fix $\beta = 1$ and $\rho \in \{0.5, 1, 2, 3\}$, and allow for preference heterogeneity over δ. In particular, let F_δ denote the CDF of the distribution over δ, which we assume follows a beta distribution.[45]

To estimate the parameters of F_δ, we follow a process analogous to the MSM procedure used throughout the paper. Let F denote an arbitrary CDF of the joint distribution of $\theta = (\beta, \delta, \rho)$. Let

$$m_h\left(F, \chi\right) = \int m_{J_s}\left(\theta, \chi\right) dF\left(\theta\right),$$

and

$$\hat{F} = \arg\min_F \left(m_h\left(F, \hat{\chi}\right) - \bar{m}_{J_m}\right)' W \left(m_h\left(F, \hat{\chi}\right) - \bar{m}_{J_m}\right).$$

The main challenge for implementation is computation of the integral defining m_h, since each calculation of $m_{J_s}\left(\theta, \chi\right)$ is numerically costly.[46] We approximate the integral by calculating

[44]For lifecycle models exploring preference heterogeneity, see, for example, Gomes and Michaelides (2005), Vestman (2019), and Calvet et al. (2021).

[45]We choose a beta distribution to ensure that $\delta \in [0, 1]$.

[46]Throughout this analysis, another limitation is the assumption that heterogeneity in (β, δ, ρ) is uncorrelated with unobserved heterogeneity in the first-stage parameters.

$m_{J_s}(\theta, \chi)$ over a dense grid of θ values and discretizing F over that grid.

Our estimates of the exponential heterogeneity cases are reported in Table F1. When $\rho = 2$ (the best-fitting case), the estimated beta distribution for δ has a mean of 0.87 with substantial dispersion. Allowing for heterogeneity in δ generates $q(\hat{F}, \hat{\chi}) = 364.27$. While this is a large improvement on the $\beta = 1$ single-point estimate (Column 2 of Table 3), the exponential model with heterogeneity still underperforms the single-point estimate that allows for $\beta < 1$. As Table F1 shows, the exponential heterogeneity model is simply unable to match the observed wealth levels for households carrying credit card debt. Though heterogeneity is obviously important, this suggests that some applications may achieve better performance at lower computational cost by incorporating present bias.

6.2 Sophistication

The benchmark model in this paper generalizes exponential discounting to allow for naive present bias. Under naivete the current self is unaware of future selves' present bias, and instead believes that future selves will discount exponentially. An alternative is to assume that the current self is (at least partially) aware of the present bias of future selves and hence perceives the self-control problem.

Harris and Laibson (2001) show that strategic interactions between the temporal selves of a sophisticated consumer induce pathological discontinuities in policy functions and kinks in value functions.[47] Solutions to the (finite horizon) lifecycle problem are still computable by backward induction, so we report estimates of $\hat{\beta}$ and $\hat{\delta}$ under the sophistication assumption in Table F2.

One consequence of the pathologies induced by sophistication is that $q(\theta, \hat{\chi})$ may not be continuous and therefore three caveats apply to the results in Table F2. First, the results are numerically more fragile than under naivete. Our estimates in Table F2 are

[47]See also Laibson and Maxted (2023) for a discussion.

presented conditional on ρ in order to offset this fragility by reducing the dimensionality of the minimization problem. Second, the numerical instability arising from pathologies means that any low-q points that are found could be partially a result of spurious fit. Third, our calculation of standard errors assumes differentiability of the objective function (see Internet Appendix C for details). Standard errors are grayed out in Table F2 to indicate that this assumption is not met under sophistication. With these caveats in mind, the results in Table F2 show that our identification of $\hat{\beta} < 1$ does not rely on the assumption of naivete.

6.3 Educational attainment

In this paper we focus on one educational category: high school graduates. By updating our first- and second-stage moments we are also able to estimate the discount functions of other educational populations.[48] Table F3 reports estimates of θ for the population without a high school degree, and the population with a college degree. Table F3 also includes the targeted empirical moments for these other educational populations, which are calculated using the same methodology as our benchmark moments. Our point estimates for $\hat{\beta}$ are less than 1 in both cases, though with larger standard errors than our baseline estimate (e.g., for the population with a college degree, we estimate $\hat{\beta} = 0.79$ with a standard error of 0.22).

7 Conclusion

This paper uses a structural lifecycle model to estimate household time preferences. In the data, U.S. households accumulate substantial illiquid wealth before retirement while simultaneously borrowing actively on credit cards. To explain these phenomena, our MSM procedure estimates $\hat{\beta} = 0.53$, $\hat{\delta} = 0.99$, and $\hat{\rho} = 1.94$. The low long-run discount rate

[48] Certain structural assumptions of our model were made with the high school graduate demographic in mind, so some of our assumptions do not map cleanly into other educational groups. For example, it would be preferable to start a model of college graduates at age 23 rather than age 20.

($-\ln\delta = 1.1\%$) accounts for observed levels of (illiquid) wealth accumulation. The high short-run discount rate ($-\ln\beta\delta = 64.5\%$) explains the observed levels of (high-interest) credit card borrowing. The MSM procedure rejects the restriction to exponential discounting ($\beta = 1$) in the benchmark parametrization and almost all robustness checks. The model with $\beta = 1$ can replicate either the wealth data or the credit card borrowing data, but not both.

The evidence reported here suggests that present bias improves the ability for consumption-saving models to match household balance sheet data over the lifecycle. Quantitatively, our parameter estimates are sensitive to some calibrational choices and our economic environment is highly stylized. One path for future research is to enrich the realism of our modeling framework. Additionally, counterfactual policy analysis in a model similar to the one studied here may be a fruitful avenue for research.

Code Availability: The replication code is available in the Harvard Dataverse at https://doi.org/10.7910/DVN/ZVAZVN.

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