QUASI-HYPERBOLIC DISCOUNTING

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STRUCTURE

- Motivation
  - Why one do you prefer?
  - Evidence
- Literature Review
  - Starting point
  - Problem
  - Solution
  - Current Research
- One of the solutions: IG

DEFINITIONS

- Hyperbolic discounting

$$\beta^H(t) = \frac{1}{1 - \alpha(t-1)}$$

- Quasi-hyperbolic discounting

$$\beta^Q(t) = \begin{cases} 1 & t = 0 \\ \beta\delta^t & t > 0 \end{cases}$$

MOTIVATION

WHICH ONE DO YOU PREFER?

- \$15 today or \$20 in one month?
- \$15 today or \$84 in six months?
- \$15 today or \$470 in one year?
- \$15 today or \$14,900 in two years?
- \$15 today or \$470,000,000 in five years?

UNDER GEOMETRIC DISCOUNTING

$u(\$15) > \beta u(\$20) \implies \beta \leq 0.75$

We can say (under assumptions on utility):

- $u(\$15) > \beta^6 u(\$84)$
- $u(\$15) > \beta^{12} u(\$470)$
- $u(\$15) > \beta^{24} u(\$14,900)$
- $u(\$15) > \beta^{60} u(\$470,000,000)$

SHORT-RUN IMPATIENCE AND LONG-RUN PATIENCE

SHORT-RUN IMPATIENCE AND LONG-RUN PATIENCE

Authors	Year	Journal	Data	Citation
Thaler	1981	EL	Questionnaire	3272
Benzion et al.	1989	MS	Experiment	1319
Laibson et al.	2024	RFS	Consumption	574
Frederick et al	2002	JEL	N/A	8712
Tymula et al	2024	Working paper	N/A	N/A

LITERATURE REVIEW

STARTING POINT OF LITERATURE REVIEW

- Strotz (1955) Myopia and Inconsistency in Dynamic Utility Maximization, RES, cited by 5346.
  - Calendar date can be importance
  - Revaluation of the policy leads to different policy
  - Pre-commitment

DAVID LAIBSON

- Laibson(1997): Golden Eggs and Hyperbolic Discounting, QJE, cited by 8326.
  - Consumption-Saving, discrete and deterministic, finite horizon
  - Use illiquid asset to pre-commit
  - Dynamic inconsistency $\implies$ use dynamic games to model (T-period game with T-plays(selves))
  - THM(Uniqueness): Restricting the choice set, finite horizon, unique SPNE (optimal policy)

LAIBSON MEETS RAMSEY

- Barro (1999): Laibson meets Ramsey in the neoclassical growth model, QJE, cited by 266
  - Application of Laibson (1997) to the neoclassical growth model (adding commitment and time-inconsistent preference)
  - Solve the model in continuous time with CRRA utility

KRUSELL AND SMITH 2003

- Krusell and Smith (2003): Consumption-saving decisions with quasi-geometric discounting, Econometrica, cited by 457
  - Discrete and Deterministic consumption-saving problem, Infinite horizon, all nice properties
  - THM (Indeterminacy): Continuum of equilibrium, each equilibrium with a continuum of optimal policies

SOLUTION METHOD: IG

- Harris and Laison (2013): Instantaneous gratification, QJE, cited by 194
  - Continuous and Stochastic consumption-saving problem, Infinite horizon
  - : IG: current self only exists at one instant
  - THM (Value function equivalence): Under their setting, v solves the IG Bellman equation $\iff$ v solves the Bellman equation of some dynamic consistent consumer.
  - Can extend to continuous and deterministic case

SOLUTION METHOD: SAVING OR DISSAVING?

- Cao and Werning (2018): Saving and Dissaving with Hyperbolic Discounting, Econometrica, cited by 51
  - Discrete and Deterministic consumption-saving problem, Infinite horizon
  - Still no uniqueness
  - THM: Under some conditions, all optimal policies are all saving or all dissaving.
  - THM: Determined by comparing the interest rate with a threshold related to the impatience parameter.

CURRENT RESEARCH: BEHAVIORAL NK AND HANK

- Gaibaix(2020): A behavioral New Keynesian model, AER, cited by 687
- Laibson, Maxted, Moll (2024): Present Bias Amplified the Household Balance-Sheet Channels of Macroeconomics Policy, QJE(accepted)

SOLUTION METHOD: IG IN DETAILS

CENTRAL IDEA

To have uniqueness, a new self is born every instant. The current self can deviate only instantaneously.

1. Present-Future model
   - transition from present to future with a constant hazard rate λ
2. IG model: $\lambda \to \infty$

PRESENT-FUTURE MODEL

Consider self n is born at time s_n, $\tau_n \sim \exp(\lambda)$

- Present from s_n to $s_n + \tau_n$ (controlled by self n)
- Future from $s_n + \tau_n$ to ∞ (controlled by future selves)

Self n has discount factor:

$$D_n(t) = \begin{cases} \delta^t & t \in [0, \tau_n) \\ \beta\delta^t & t \in [\tau_n, \infty) \end{cases}$$

PRESENT-FUTURE MODEL: FUTURE

Future selves use the same consumption policy $\tilde{c} : [0, \infty) \mapsto (0, \infty)$

Law of motion for wealth for future

$$dx_t = (\mu x_t + y - \tilde{c}(x_t))dt + \sigma x_t dz, \qquad z \sim \mathcal{W}$$

Continuation value of self *n* (beyond self *n*'s control)

$$v(x_{s_n+\tau_n}, \tilde{c}) = \mathbb{E}_{s_n+\tau_n}\left[\int_{s_n+\tau_n}^{\infty} e^{-\gamma(t-(s_n+\tau_n))} u(\tilde{c}(x_t)),\ dt\right]$$

where $\gamma = -\ln \delta > 0$ denote the long-run discount rate

PRESENT-FUTURE MODEL: PRESENT

For present self n, he has policy $c : [0, \infty) \mapsto (0, \infty)$

Law of motion of wealth for present:

$$dx_t = (\mu x_t + y - c(x_t))d_t + \sigma x_t dz, \qquad z \sim \mathcal{W}$$

Current value for self n (under self n's control)

$$w(x_{s_n}, c, \tilde{c}) = \mathbb{E}_{s_n}\left[\int_{s_n}^{s_n+\tau_n} e^{-\gamma(t-s_n)}u(c(x_t))\ dt + \beta e^{-\gamma\tau_n}v(x_{s_n+\tau_n}, \tilde{c})\right]$$

Objective for self n:

Let $BR(\tilde{c}) = \arg\max_c w(x_{s_n}, c, \tilde{c})$. We want the optimal policy c^* to be

$$c^* \in BR(c^*)$$

PRESENT-FUTURE MODEL: BELLMAN EQUATION

Under policy c, we have, HJB-Future

$$0 = \frac{1}{2}\sigma^2 x^2 v'' + (\mu x + y - c)v' - \gamma v + u(c)$$

HJB-Present:

$$0 = \frac{1}{2}\sigma^2 x^2 w'' + (\mu x + y - c)w' + \lambda(\beta v - w) - \gamma w + u(c)$$

Uniqueness:

- No dynamic inconsistency: $\lambda \to 0$, $\beta \to 1$
- Dynamic inconsistency: $\lambda \to \infty$, IG

IG-BELLMAN EQUATION

By $\beta v = w$, we obtain, HJB-IG

$$0 = \frac{1}{2}\sigma^2 x^2 v'' + (\mu x + y - c)v' - \gamma v + u(c)$$

Existence and Uniqueness by:

Theorem (Value Function Equivalence)
v is a value function of the IG consumer if and only if v is a value function of the û consumer.

where $\hat{u}$ consumer is dynamically consistent, hence we have existence and uniqueness.

WHAT IS $\hat{u}$ CONSUMER

For CRRA utility u with parameter ρ, $\hat{u}$ is a rescaling u,

$$\hat{u}(\hat{c}, x) = \begin{cases} \hat{u}_0(\hat{c}) = u(\hat{c}), & x = 0 \\ \hat{u}_+(\hat{c}) = \frac{\psi}{\beta} u\left(\frac{1}{\psi}\hat{c}\right) + \frac{\psi - 1}{\beta}, & x > 0 \end{cases}$$

where $\psi = \frac{\rho - (1-\beta)}{\rho}$.